TRAJECTORY PLANNING IN JOINT SPACE FOR FLEXIBLE ROBOTS WITH KINEMATICS REDUNDANCY

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Abstract: *The paper focuses on the problem of trajectory planning of flexible redundant robot manipulators (FRM) in joint space. Compared to irredundant flexible manipulators, FRMs present additional possibilities in trajectory planning due to their kinematics redundancy. A trajectory planning method to minimize vibration of FRMs is presented based on Genetic Algorithms (GAs). Kinematics redundancy is integrated into the presented method as a planning variable. Quadrinomial and quintic polynomials are used to describe the segments which connect the initial, intermediate, and final points in joint space. The trajectory planning of FRMs is formulated as a problem of optimization with constraints. A planar FRM with three flexible links is used in simulation. A case study shows that the method is applicable.*

Keywords: redundant robot, flexible-link, trajectory planning, vibration, genetic algorithms

1. INTRODUCTION

The use of a light-weight flexible robotic manipulator can increase load carrying capacity and operational speed. Other potential advantages of flexible robots include lower energy consumption, use of smaller actuators, safer operation due to reduced inertia, easier transport, and a more compliant structure for assembly. Work done on flexible robot manipulators in past decades involves mainly modeling, vibration control and trajectory tracking control, as reviewed and discussed by [1]. Since most of the robot tasks cannot be conducted without certain accuracy, the key problem of flexible robot manipulators is how to reduce the endpoint's vibrational error. This vibration remains even after the robot arm reaches the goal.

Several different approaches have already been reported and proven to be feasible on reducing vibrations of flexible arms with one or two links. Most of the work is based on (either open-loop or closed-loop) control strategies, for example [2,3]. However, the above research focuses on flexible robot manipulators with only one or two links.

It has been found recently that the redundancy, which is used in robot manipulators to achieve additional performance while tracking a given end-effector trajectory [4,5], has its special application in vibration reduction of flexible manipulators. Nguyen and Walker [6] made use of the self-motion to compensate for and damp out flexible deformation when the degree of redundancy is of the same number of the deformation modes to be controlled. Yue [7] presented an optimal method to choose the self-motion for reduction of the vibration of a FRM, in which joint flexibility is also taken into account. Kim and Park [8] employed the self-motion capability to solve the tracking control problem of a FRM, and developed an algorithm in which the self-motion is evaluated so as to nullify the dominant modal force of flexural motion induced by a rigid body motion. Nevertheless, trajectory planning of FRMs in joint space was not mentioned in the above papers.

Compared with one- or two-link flexible robot manipulators, it is reasonable to expect that kinematics redundancy can also provide additional possibilities for minimizing vibration in FRMs through trajectory planning in joint space. Moreover, the geometric problems with Cartesian paths related to workspace and singularities can be avoided if the point-to-point task is planned in joint space.

Genetic Algorithms (GAs), are population-based, stochastic, and global search methods [9]. As discussed in [4,5], global solution is quite difficult to reach using traditional methods. The global search ability of GAs provides a possibility for finding global solutions of redundancy. There has been little reported work on

applying GAs to trajectory planning of flexible robot manipulators.

In the following sections, a trajectory planning method for FRM based on GAs to minimize vibration while moving between the initial and final points is presented. First, a finite element model for describing dynamics of FRM is introduced (Section 2). Kinematics redundancy is integrated into the planning method as variables (Section 3). Quadrinomial and quintic polynomial are used to describe the paths which connect the initial, intermediate and final points in joint space (Section 4). The trajectory planning of FRM is formulated as a problem of optimization (Section 5). Suitable parameters of each polynomial between two points, suitable initial and final configurations are determined by using GAs (Section 6). Finally, a planar FRM with three flexible links is used in simulation and a case study is conducted and discussed (Section 7).

2. DYNAMIC MODEL

Finite element method is employed to build up the dynamic equations of a FRM with multiple flexible links. Without taking into account the joint flexibility, the dynamic equations of a flexible robot manipulator system can then be written as [10]:

$$[M]\{\ddot{\Phi}\}+[C]\{\dot{\Phi}\}+[K]\{\Phi\}=\{p\} \tag{1}$$

$$\{t\}=([D]+[J_Z])\{\ddot{q}\}+\{h\}+\{e\} \tag{2}$$

where **[M]** is the $n_u \times n_u$ global mass matrix, **[C]** is the $n_u \times n_u$ global damping matrix, **[K]** is the $n_u \times n_u$ global stiffness matrix, **{p}** is the $n_u \times 1$ inertia force matrix, $\{\Phi\}$, $\{\dot{\Phi}\}$ and $\{\ddot{\Phi}\}$ are the generalized coordinates vector, generalized velocity vector and generalized acceleration vector respectively, which describe the deformation behavior of the flexible links, n_u is the number of the generalized coordinates, **[D]** is the $n \times n$ inertia mass matrix, $\{q\}$ and $\{\ddot{q}\}$ are the vectors which describe the joint angle and angular acceleration respectively, **{h}** is the $n \times 1$ centrifugal, gravitational, coriolis of the rigid and flexible coupling terms, **{e}** is the $n \times 1$ link flexibility term, **[J$_z$]** is the $n \times 1$ rotor inertial mass matrix, **{t}** is the $n \times 1$ actuator torque matrix, and n is the number of joints in the robot system.

3. REDUNDANCY RESOLUTION of FRM

For a redundant robot, the number of degrees of freedom n of a manipulator is greater than the number of end-effector degrees of freedom m. That is, given a position/posture of the end-effector, there are infinite number of robot configurations available. In this study, redundancy of a robot is used to avoid the acute vibration when its end-effector moves from one point to another.

How can the redundancy of FRM be used to attain additional goals, such as avoiding acute vibration?
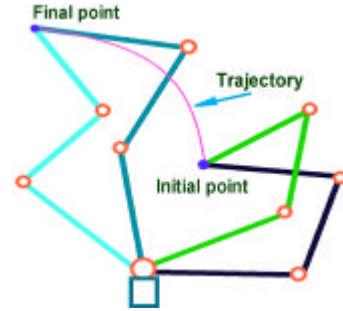

Figure1. Different configurations of a FRM correspond to initial and final point of trajectory

In the previous methods [6,7], the trajectory of a FRM was planned to reduce vibration when the prescribed trajectory was described in Cartesian space. However, the final results based on previous methods are local solutions [4,5] and are prone to various problems related to workspace and singularities, since the trajectory is given in Cartesian space [11]. As mentioned above, the presented method will conduct the planning problem of FRM in joint space. This means that the various problems in Cartesian space can be avoided easily.

Since only point-to-point trajectory is discussed in this paper, there are only two important points (i.e. the initial point and the final point of a possible path) that should be achieved. Corresponding to the two points, there are infinite possible configurations for each point due to kinematics redundancy, for example, shown in Figure 1. This means that the FRM can start with one suitable pose and end in a different pose, with the starting and final pose determined according to the task being performed.

Thus, the redundancy problem with respect to initial and final points involves determining the initial and final postures. These two postures can be described by *2(n-m)* parameters in joint space, where *(n-m)* is the redundant degree of freedom of the FRM. In our approach, as described in the following chapters, these *2(n-m)* variables can be determined using an optimization method. This means that the redundancy solution corresponding to initial and final points will be determined using GAs.

The problems of redundancy corresponding to intermediate via points will also be solved efficiently by planning in joint space instead of in Cartesian task space, as described in the following sections.

4. TRAJECTORY PLANNING STRATEGY

Here, trajectory refers to a time history of position, velocity, and acceleration for each degree of freedom. Suppose that the point-to-point trajectory consists of several segments with continuous acceleration at the intermediate via point (as shown in Figure 2). The position of each intermediate point is supposed to be unknown in the following part of the paper. Of course, the intermediate points can also be given as particular points that should be passed through. This is useful especially when there is an obstacle in the working area.
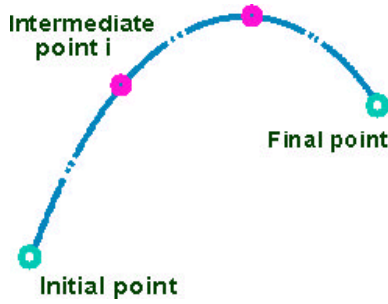

Figure 2.   Intermediate points on the point-to-point trajectory

If we wish to be able to specify the position, velocity, and acceleration at the beginning and end of a path segment, a quadrinomial and a quintic polynomial are required. Suppose that there are m_p intermediate via points between the initial and the final points.

Between the initial point to m_p intermediate via points, a quadrinomial is used to describe these segments as:

$$\boldsymbol{q}_{i,i+1}(t)=a_{i0}+a_{i1}t_i+a_{i2}t_i^2+a_{i3}t_i^3+a_{i4}t_i^4,\ (i=0,\cdots,m_p-1)\ \ (3)$$

where the constrains are given as

$$\boldsymbol{q}_i = a_{i0} \quad (4)$$
$$\boldsymbol{q}_{i+1} = a_{i0}+a_{i1}T_i+a_{i2}T_i^2+a_{i3}T_i^3+a_{i4}T_i^4 \quad (5)$$
$$\dot{\boldsymbol{q}}_i = a_{i1} \quad (6)$$
$$\dot{\boldsymbol{q}}_{i+1} = a_{i1}+2a_{i2}T_i+3a_{i3}T_i^2+4a_{i4}T_i^3 \quad (7)$$
$$\ddot{\boldsymbol{q}}_i = 2a_{i2} \quad (8)$$

where T_i is the executing time from point i to point $i+1$. The five unknowns can be solved as

$$a_{i0} = \boldsymbol{q}_i \quad (9)$$

$$a_{i1} = \dot{\boldsymbol{q}}_i \quad (10)$$
$$a_{i2} = \ddot{\boldsymbol{q}}_i/2 \quad (11)$$
$$a_{i3} = (4\boldsymbol{q}_{i+1}-\dot{\boldsymbol{q}}_{i+1}T_i-4\boldsymbol{q}_i-3\dot{\boldsymbol{q}}_iT_i-\ddot{\boldsymbol{q}}_iT_i^2)/T_i^3 \quad (12)$$
$$a_{i4} = (\dot{\boldsymbol{q}}_{i+1}T_i-3\boldsymbol{q}_{i+1}+3\boldsymbol{q}_i+2\dot{\boldsymbol{q}}_iT_i+\ddot{\boldsymbol{q}}_iT_i^2/2)/T_i^4 \quad (13)$$

The intermediate point $(i+1)$'s acceleration can be obtained as:

$$\ddot{\boldsymbol{q}}_{i+1} = 2a_{i2}+6a_{i3}T_i+12a_{i4}T_i^2 \quad (14)$$

The segment between the number m_p intermediate point and the final point can be described by a quintic polynomial as:

$$\boldsymbol{q}_{i,i+1}(t)=b_{i0}+b_{i1}t_i+b_{i2}t_i^2+b_{i3}t_i^3+b_{i4}t_i^4+b_{i5}t_i^5,\ (i=m_p)\ \ (15)$$

where the constraints are given as

$$\boldsymbol{q}_i = b_{i0} \quad (16)$$
$$\boldsymbol{q}_{i+1} = b_{i0}+b_{i1}T_i+b_{i2}T_i^2+b_{i3}T_i^3+b_{i4}T_i^4+b_{i5}T_i^5 \quad (17)$$
$$\dot{\boldsymbol{q}}_i = b_{i1} \quad (18)$$
$$\dot{\boldsymbol{q}}_{i+1} = b_{i1}+2b_{i2}T_i+3b_{i3}T_i^2+4b_{i4}T_i^3+5b_{i5}T_i^4 \quad (19)$$
$$\ddot{\boldsymbol{q}}_i = 2b_{i2} \quad (20)$$
$$\ddot{\boldsymbol{q}}_{i+1} = 2b_{i2}+6b_{i3}T_i+12b_{i4}T_i^2+20b_{i5}T_i^3 \quad (21)$$

and these constraints specify a linear set of six equations with six unknowns whose solution is

$$b_{i0} = \boldsymbol{q}_i \quad (22)$$
$$b_{i1} = \dot{\boldsymbol{q}}_i \quad (23)$$
$$b_{i2} = \ddot{\boldsymbol{q}}_i/2 \quad (24)$$
$$b_{i3} = (20\boldsymbol{q}_{i+1}-20\boldsymbol{q}_i-(8\dot{\boldsymbol{q}}_{i+1}+12\dot{\boldsymbol{q}}_i)T_i-(3\ddot{\boldsymbol{q}}_i-\ddot{\boldsymbol{q}}_{i+1})T_i^2)/2T_i^3 \quad (25)$$
$$b_{i4} = (-30\boldsymbol{q}_{i+1}+30\boldsymbol{q}_i+(14\dot{\boldsymbol{q}}_{i+1}+16\dot{\boldsymbol{q}}_i)T_i+(3\ddot{\boldsymbol{q}}_i-2\ddot{\boldsymbol{q}}_{i+1})T_i^2)/2T_i^4 \quad (26)$$
$$b_{i5} = (12\boldsymbol{q}_{i+1}-12\boldsymbol{q}_i-(6\dot{\boldsymbol{q}}_{i+1}+6\dot{\boldsymbol{q}}_i)T_i-(\ddot{\boldsymbol{q}}_i-\ddot{\boldsymbol{q}}_{i+1})T_i^2)/2T_i^5 \quad (27)$$

As formulated above, the total parameters that should be determined are the joint angles of each intermediate via point ($n{\times}m_p$ parameters), the joint angular velocities of each intermediate point ($n{\times}m_p$ parameters), the executing time of each segment (m_p+1 parameters) and the initial and final posture of FRM ($2(n\text{-}m)$ parameters). Therefore, there are a total of (($2n+1$) $m_p+2(n\text{-}m)+1$) to be determined. It should be pointed out that joint angular acceleration at each intermediate point can be obtained through equation (14). If all the intermediate points are connected by quintic polynomials, there will be (($3n+1$) $m_p+2(n\text{-}m)+1$) parameters to be determined. This would be more time-consuming, so we choose both quadrinomial and quintic polynomial to generate the segments.

5. OPTIMIZING TRAJECTORY USING GAs

For a point-to-point trajectory planning problem for a FRM, the vibration deformation amplitude is one of the most important factors to be considered in the optimization. It is reasonable to assume that the vibrational deformation amplitude is the objective in the path planning process. Since the limitations of joint angles, joint angular velocities, joint angular accelerations and joint torques are considered in the optimization process, the objective and constraints are finally written as:

$$f_m = a_{vib} \rightarrow \min \tag{28}$$

$$s.t. \quad q_{i,\min} \leq q_i \leq q_{i,\max} \quad (i = 1, \cdots, n) \tag{29}$$

$$w_{i,\min} \leq w_i \leq w_{i,\max} \quad (i = 1, \cdots, n) \tag{30}$$

$$e_{i,\min} \leq e_i \leq e_{i,\max} \quad (i = 1, \cdots, n) \tag{31}$$

$$t_{i,\min} \leq t_i \leq t_{i,\max} \quad (i = 1, \cdots, n) \tag{32}$$

where a_{vib} is the largest amplitude of vibrational deformation during the point-to-point path, $q_{i,min}$ and $q_{i,max}$ are the lower and upper permitted angle of ith joint, $w_{i,min}$ and $w_{i,max}$ are the lower and upper permitted angular velocities of ith joint, $e_{i,min}$ and $e_{i,max}$ are the lower and upper permitted angular accelerations of ith joint, $t_{i,min}$ and $t_{i,max}$ are the lower and upper computed torque of ith joint, respectively.

Since the basic formulation of point-to-point motion is given, if the parameters of the motion have been determined, then the optimal trajectory can be easily determined. We use genetic algorithms to search for those parameters. GA programs have been described in detail [9].

In the procedure of GAs, initialization generates the first host population P_0 randomly. The population P_N of the Nth generation is formed by survivors of the last generation and new individuals generated through mutation and crossover. Single-point crossover is used to form the new generation. The point-to-point trajectory is finally selected when the termination condition is satisfied. The termination condition of the procedure can be maximum generations or a certain value according to different demand.

6. CASE STUDY

A planar FRM with three flexible links is used for numerical simulation in case studies, as shown in Figure 1. The robot has one redundant freedom only in terms of positioning. Its parameters are given as: length of each link 250m*m*, height of cross section 3mm and width 4mm,

elastic modules 7.1×10^{10}Pa, shear modules 2.6×10^{10}Pa, lumped mass at each distal end of link 40g, lumped mass at endpoint 20g, moment of inertia of base joint 1.5×10^{-5}Kgm^2, second joint 1.0×10^{-5}Kgm^2, third joint 0.5×10^{-5}Kgm^2. The material for each link is aluminum. The case is simulated in the horizontal plane. There is no obstacle in the working area.

The constraints used in the following cases are: joint angle $q_i \in [-2\pi, 2\pi]$rad, angular velocities $w_i \in [-8,8]$ rad/s, angular accelerations $e_i \in [-40,40]$rad/s^2, computed torques $t_1 \in [-2.5,2.5]$Nm, $t_2 \in [-1.5,1.5]$Nm and $t_3 \in [-1.0, 1.0]$Nm, respectively.
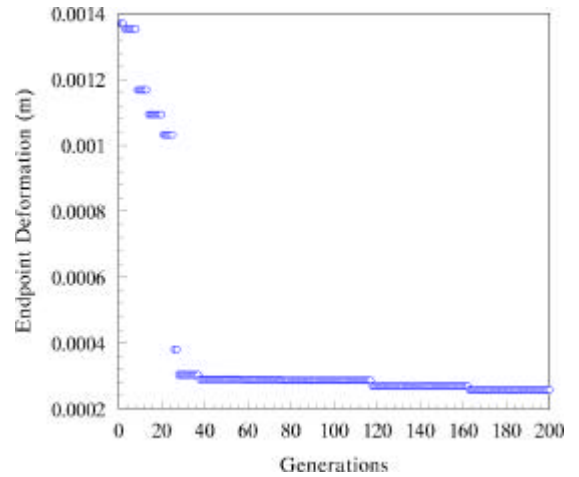

Figure 3. Endpoint deformation amplitude of FRM versus number of generations

In this case study, only the vibrational amplitude is assumed to be the objective of optimization. One intermediate via point is assumed. It is also assumed that the executing time of each segment is one second. The termination condition used here is 200 generations. All the selected trajectories start from the same initial configuration. There are seven parameters to be determined in this case; they are: configuration at final point, the three joint angles and the three joint angular velocities at the intermediate point. The optimization process took about 30 minutes with a Pentium II 500MHz computer. The results are shown in the following figures.

It is found that the vibrational amplitude is reduced obviously (shown in Figure 3). Vibration at different generations can be found in details in Figure 4. The vibrational amplitude decreased rapidly between these generations. This reflects the ability of GAs to find solutions with efficiency.
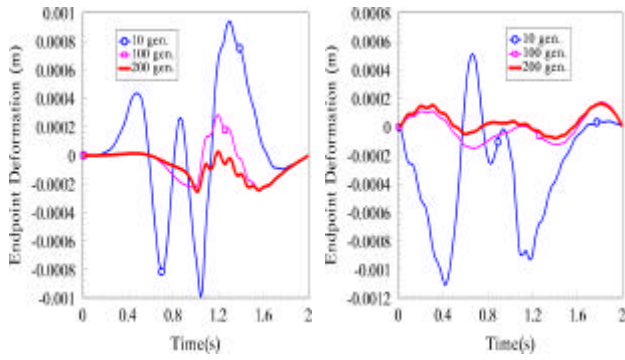
Figure 4. Endpoint deformation curves at 10th,100th and 200th generations in X direction (left) and Y direction (right)

The joint angle, angular velocity, angular acceleration and computed joint torque of each joint at 10th, 100th and 200th generations are compared in Figures 5 through 7. It is noticeable that first and second joint angular accelerations at 100th or 200th generation are quite smooth; however, the third joint angular acceleration at the same generation has a sharp peak. This means that the third link swings to avoid acute inertia. Since the computed torque can be influenced by vibration, the results at different generations are also quite different, even at the end of trajectories (as shown in Figure 7).
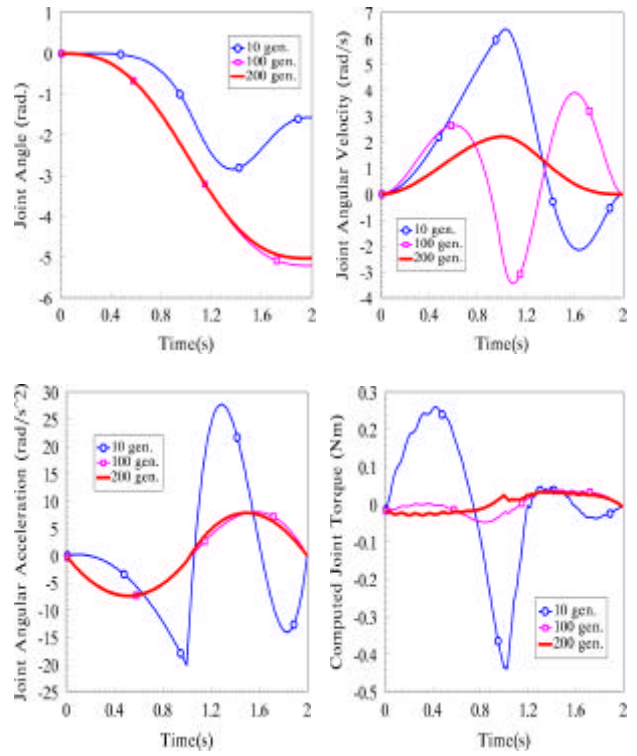

Figure 6. The angle, angular velocity, angular acceleration and Computed joint torque of the second joint at 10th ,100th and 200th generation
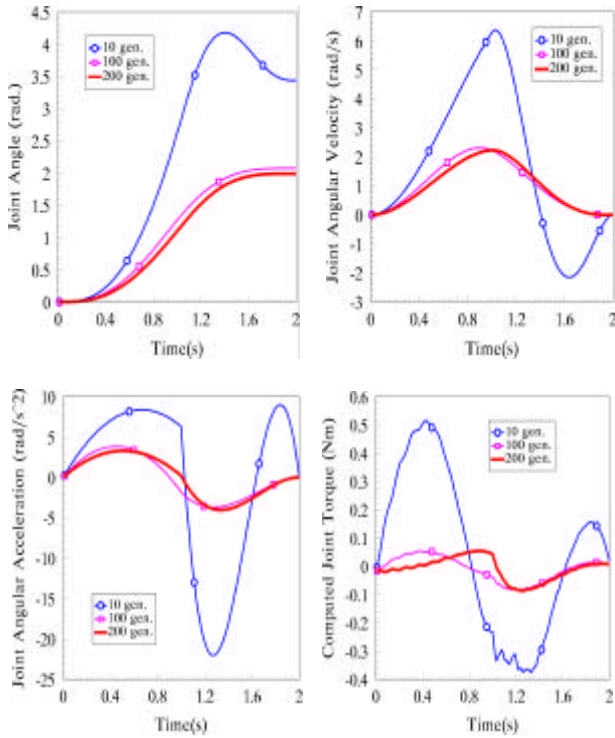

Figure 5. The angle, angular velocity, angular acceleration and computed joint torque of the first joint at 10th ,100th and 200th generation
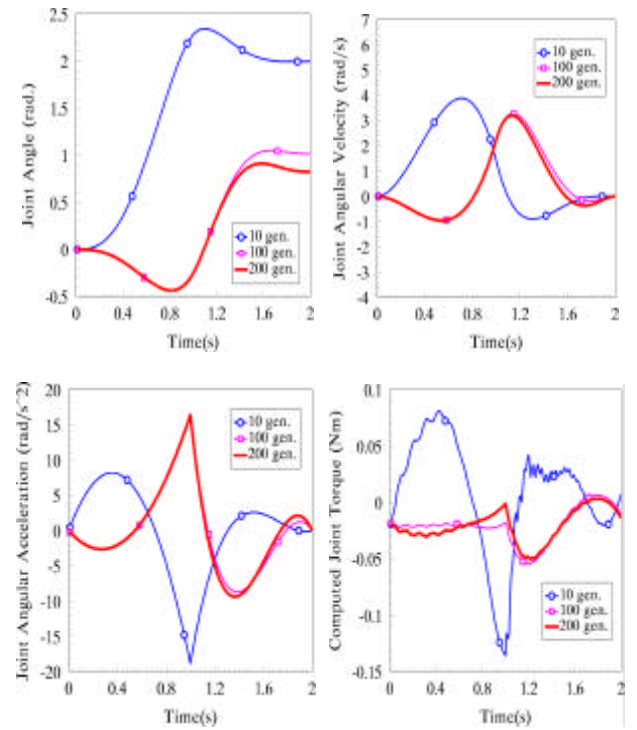

Figure 7. The angle, angular velocity, angular acceleration and Computed joint torque of the third joint at 10th ,100th and 200th generation

The configurations at 10th, 100th and 200th generations are shown in Figure 8. It is found that the 100th and 200th generation's moving areas are confined to one side and have shorter trajectories compared with 10th generation. However, the straight line from the initial point to the final point is the shortest one, but is far from the best one according to the GA optimization results.
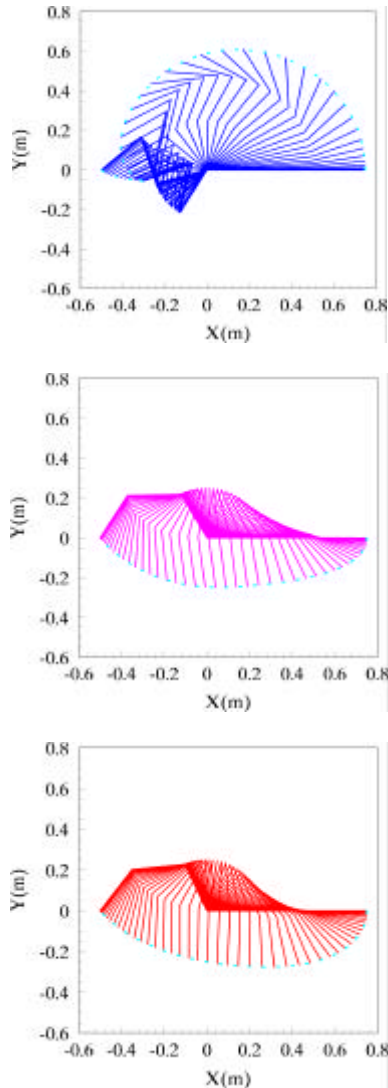

Figure 8. The configurations of FRM at 10th generation (above), 100th generation (middle) and 200th generation (below) respectively, the trajectory starts from right to left

7. CONCLUSIONS

In the above sections, the problem of trajectory planning for FRM was studied in detail. A trajectory planning method for FRMs based on GAs to minimize vibration was presented. Kinematics redundancy was considered as a planning variable in the presented method. Quadrinomial and quintic polynomials were used to describe the segments which connect the initial, intermediate, and final points in joint space. Suitable parameters of each polynomial between two points and suitable initial and final configurations were determined using GAs. Case study of a planar FRM with three flexible links showed that the method is effective. The geometric problems with Cartesian paths related to workspace and singularities were avoided by using the presented method. It should be noted that joint elasticity was not taken into account in this method.

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