Dark-State Polaritons in Electromagnetically Induced Transparency

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We identify form-stable coupled excitations of light and matter ("dark-state polaritons") associated with the propagation of quantum fields in Electromagnetically Induced Transparency. The properties of the dark-state polaritons such as the group velocity are determined by the mixing angle between light and matter components and can be controlled by an external coherent field as the pulse propagates. In particular, light pulses can be decelerated and "trapped" in which case their shape and quantum state are mapped onto metastable collective states of matter. Possible applications of this reversible coherent-control technique are discussed.

Dark resonances and electromagnetically induced transparency (EIT) [1,2] can be used to make a resonant, optically opaque medium transparent by means of quantum interference. Associated with the induced transparency is a dramatic modification of the refractive properties of the media. These can result, for instance, in very slow group velocities [3]. In the present contribution we study the propagation of quantum fields in EIT media. We demonstrate the existence of formstable quantum excitations associated with such propagation, which we term "dark-state polaritons". The polaritons are coherent superpositions of photonic and Raman-like matter branches. We show that their group velocity is directly related to the ratio of the two contributions. This ratio can be externally controlled by adiabatically changing a coherent control field as the pulse propagates. In particular, dark-state polaritons can be stopped and re-accelerated in such a way that their shape and quantum state are preserved. In this process the quantum state of light is ideally transfered to collective atomic excitations and vise versa.

The possibility to coherently control the propagation of quantum light pulses via dark-state polaritons opens up interesting applications involving the generation of nonclassical states of atomic ensembles (in squeezed or entangled states), reversible quantum memories for light waves [4–6], and high resolution spectroscopy [7]. Furthermore, the combination of the present technique with studies on few-photon nonlinear optics [8–12] can be used, in principle, for processing of quantum information stored in collective excitations of matter. Finally, the present technique may provide an interesting tool to study quantum scattering phenomena in systems involving coherent cold collisions. In this regard the present work opens a link between nonlinear optics for light waves and nonlinear atom optics. E.g. an interaction (or entanglement) between light waves can be induced by a collisional interaction of atoms (e.g. s-wave scattering); alternatively an interaction between atoms can be induced via optical nonlinearities.

We consider a medium consisting of Λ-type 3-level atoms with two meta-stable lower states as shown in Fig. 1. A quantum field described by the slowly-varying dimensionless operator

$$\hat{E}(z,t) = \sum_k a_k(t)\, \mathrm{e}^{ikz}\, \mathrm{e}^{-i\frac{\nu}{c}(z-ct)} \tag{1}$$

couples resonantly the transition between the ground state $|b\rangle$ and the excited state $|a\rangle$. $\nu = \omega_{ab}$ is the carrier frequency of the optical field. The upper level $|a\rangle$ is furthermore coupled to the stable state $|c\rangle$ via a coherent control field with the slowly-varying, real Rabi-frequency $\Omega(t)$. For the purposes of the present discussion the external field can be treated classically. We assume that initially (i.e before the quantum pulse arrives) all atoms are in their ground states $|b_j\rangle$. To describe the quantum properties of the medium, we use collective, slowly varying atomic operators, appropriately averaged over small but macroscopic volumes containing $N_z \gg 1$ particles at position z,

$$\hat{\sigma}_{\alpha\beta}(z,t) = \frac{1}{N_z}\sum_{j=1}^{N_z} |\alpha_j\rangle\langle\beta_j|\, \mathrm{e}^{-i\omega_{\alpha\beta}t}. \tag{2}$$

The interaction between light and atoms is governed by the Hamiltonian

$$\hat{V} = -N\int\frac{\mathrm{d}z}{L}\left(\hbar g\sum_k a_k \mathrm{e}^{ikz}\hat{\sigma}_{ab}(z) + \hbar\Omega\hat{\sigma}_{ac}(z)\right) + h.c. \tag{3}$$

Here $g = \wp\sqrt{\frac{\nu}{2\hbar\epsilon_0 V}}$ is the atom-field coupling constant with $\wp$ being the dipole moment of the $a-b$ transition and V the quantization volume. N is the number of atoms in this volume and L its length in z direction.

The evolution of the Heisenberg operator corresponding to the optical field can be described in slowly varying amplitude approximation by the propagation equation

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = igN\,\hat{\sigma}_{ba}(z,t). \tag{4}$$

The atomic evolution is governed by a set of Heisenberg-Langevin equations

$$\frac{\partial}{\partial t}\hat{\sigma}_{\mu\nu} = -\gamma_{\mu\nu}\sigma_{\mu\nu} + \frac{i}{\hbar}[\hat{V}, \hat{\sigma}_{\mu\nu}] + F_{\mu\nu}, \tag{5}$$

where $\gamma_{\mu\nu}$ are the transversal decay rates and $\hat{F}_{\mu\nu}$ are δ-correlated Langevin noise operators.

We now assume that the Rabi-frequency of the quantum field is initially much smaller than Ω and that the number of photons in the input pulse is much less than the number of atoms. We will show that the Rabi-frequency of the quantum field will then be much smaller than Ω at all times. In such a case the atomic equations can be treated perturbatively in $\hat{E}$. In zeroth order only $\hat{\sigma}_{bb} = \mathbf{1}$ is different from zero and in first order one finds

$$\hat{\sigma}_{ba} = -\frac{i}{\Omega(t)}\frac{\partial}{\partial t}\hat{\sigma}_{bc}, \tag{6}$$

$$\hat{\sigma}_{bc} = -\frac{g\hat{E}}{\Omega} - \frac{i}{\Omega}\left[\left(\frac{\partial}{\partial t} + \gamma_{ba}\right)\left(-\frac{i}{\Omega}\frac{\partial}{\partial t}\hat{\sigma}_{bc}\right) + \hat{F}_{ba}\right]. \tag{7}$$

In the above equations we disregarded a (small) decay of the Raman coherence (γ_{bc}).

The propagation equations simplify considerably if we assume a sufficiently slow change of Ω, i.e. adiabatic conditions [8,10]. Introducing a normalized time $\tilde{t} = t/T$ where T is a characteristic time scale and expanding the r.h.s. of (7) in powers of $1/T$ we find in lowest non-vanishing order

$$\hat{\sigma}_{bc}(z,t) = -g\frac{\hat{E}(z,t)}{\Omega(t)}. \tag{8}$$

Note that $\langle \hat{F}_x(t)\hat{F}_y(t')\rangle \sim \delta(t-t') = \delta(\tilde{t}-\tilde{t}')/T$. Thus in the perturbative and adiabatic limit the propagation of the quantum light pulse is governed by the equation

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = -\frac{g^2N}{\Omega(t)}\frac{\partial}{\partial t}\frac{\hat{E}(z,t)}{\Omega(t)}. \tag{9}$$

If Ω is constant, the term on the r.h.s. simply leads to a modification of the group velocity of the quantum field according to $v_g = c/(1+\frac{g^2N}{\Omega^2})$. In the general case the field equation of motion will acquire an additional term proportional to $(\dot{\Omega}/\Omega)\,\hat{E}$ which describes reversible changes in quantum amplitudes due to stimulated Raman scattering.

One can obtain a very simple solution of eq.(9) by introducing a new quantum field $\hat{\Psi}(z,t)$ via the canonical transformation

$$\hat{\Psi}(z,t) = \cos\theta(t)\,\hat{E}(z,t) - \sin\theta(t)\sqrt{N}\,\hat{\sigma}_{bc}(z,t), \tag{10}$$

$$\cos\theta(t) = \frac{\Omega(t)}{\sqrt{\Omega^2(t)+g^2N}}, \quad \sin\theta(t) = \frac{g\sqrt{N}}{\sqrt{\Omega^2(t)+g^2N}}.$$

$\hat{\Psi}$ obeys the following equation of motion

$$\left[\frac{\partial}{\partial t} + c\cos^2\theta(t)\frac{\partial}{\partial z}\right]\hat{\Psi}(z,t) = 0, \tag{11}$$

which describes a shape-preserving propagation with velocity $v = v_g(t) = c\cos^2\theta(t)$:

$$\hat{\Psi}(z,t) = \hat{\Psi}\left[z - c\int_0^t \mathrm{d}\tau\cos^2\theta(\tau), t=0\right]. \tag{12}$$

Several interesting properties of the new field should be noted. First of all, by introducing a plain-wave decomposition $\hat{\Psi}(z,t) = \sum_k \hat{\Psi}_k(t)\,\mathrm{e}^{ikz}$ one finds that the mode operators $\hat{\Psi}_k$ and $\hat{\Psi}_k^\dagger$ obey the commutation relations

$$[\hat{\Psi}_k, \hat{\Psi}_{k'}^+] = \delta_{k,k'}\left[\cos^2\theta + \sin^2\theta\frac{1}{N}\sum_j(\hat{\sigma}_{bb}^j - \hat{\sigma}_{cc}^j)\right]. \tag{13}$$

In the linear limit considered here, where the number density of photons is much smaller than the density of atoms, $\hat{\sigma}_{bb}^j \approx 1, \hat{\sigma}_{cc}^j \approx 0$. Thus the new field possesses bosonic commutation relations and we can associate with it bosonic quasi-particles (polaritons). Furthermore one immediately verifies that all number states created by $\hat{\Psi}_k^\dagger$ are dark-states [2,5]:

$$|D_n^k\rangle = \frac{1}{\sqrt{n!}}\left(\hat{\Psi}_k^\dagger\right)^n|0\rangle|b_1...b_N\rangle, \tag{14}$$

where $|0\rangle$ denotes the field vacuum. In particular, the states $|D_n^k\rangle$ do not contain the excited atomic state and are thus immune to spontaneous emission. Furthermore, they are eigenstates of the interaction Hamiltonian with eigenvalue zero, $\hat{V}\,|D_n^k\rangle = 0$. For these reasons we call the quasi-particles "dark-state polaritons".

To summarize, we have found a shape-preserving, polariton-like superposition $\hat{\Psi}$ of an electromagnetic field and collective Raman coherences. This excitation is not of soliton type since no special pulse-shape or pulse area is required. It is related to the classical adiabaton solutions of pulse-pair propagation in Λ-type media [13–15] in the limit of one strong and one weak field. We emphasize however that the field can here be in any quantum state. In particular it does not need to have a coherent component with a well defined phase.

One of the most interesting aspects of dark-state polaritons is the possibility to coherently control their properties by changing $\Omega(t)$. For example, by adiabatically rotating $\theta(t)$ from 0 to $\pi/2$ one can decelerate and stop an input light pulse. It is remarkable that in this process pulse shape and quantum state of the initial light pulse are mapped onto collective, metastable states of matter in which they are stored. Likewise the dark-state polariton can be re-accelerated to the vacuum speed of light; in this process the stored quantum states is transferred back to the field. This is illustrated in Fig. 2, where we have shown the coherent amplitude of a dark-state polariton which results from an initial light pulse as well as the corresponding field and matter components. One recognizes that the pulse shape is preserved and that the stopping corresponds to a transfer from field to atomic Raman excitations. Explicitly, the mapping of the quantum states corresponds to the following unitary transformation:

$$\left(\sum_{k,l,m...}\xi_{k,l,m...}a_k^\dagger a_l^\dagger a_m^\dagger...|0\rangle\right)|b_1...b_N\rangle \leftrightarrow \tag{15}$$

$$\left(\sum_{k,l,m...}\xi_{k,l,m...}\sqrt{N}\sigma_{cb}^k\sqrt{N}\sigma_{cb}^l\sqrt{N}\sigma_{cb}^m...|b_1...b_N\rangle\right)|0\rangle,$$

as can be verified using expression (14) for the polariton state vectors.

The coherent transfer of quantum states between light and matter opens interesting prospectives for the generation of non-classical atomic ensembles in squeezed and entangled states, high-precision spectroscopy with resolution beyond the standard quantum limit [7] as well as reversible quantum memories. Furthermore, by trapping correlated photons in separate media entangled states of separated atomic ensembles can be created. With respect to these applications the present paper is complementary to our earlier studies in which we showed that quantum states of light can be mapped onto Dicke-like collective states of an EIT medium in an optical resonator [4,5]. The quantum states of matter generated in the case of the present paper are more complicated; however trapping the light in a traveling-wave geometry does not require special shaping of the classical driving pulses (quantum impedance matching), which is necessary in a cavity configuration.

We also note related studies on quantum memories for light involving mapping the quantum state of the field onto atoms by dissipative absorption [6,16]. In contrast to these approaches the adiabatic passage technique [17] used here allows for a complete and reversible excitation transfer of arbitrary quantum wavepackets.

Finally, our approach is also different from the mechanism suggested recently in [18], in which "freezing" of the light pulse in a laboratory frame was proposed using moving atoms.

The above analysis involves a perturbation expansion, an adiabatic approximation and disregards the decay of Raman coherence. In what follows the validity of these approximations is discussed. First of all, we note that making use of (8) one finds: $g^2\hat{E}^+\hat{E}/|\Omega|^2 = \hat{\sigma}_{cb}\hat{\sigma}_{bc}$. I.e., the ratio of the average intensities of quantum and control field is proportional to that of the matter field $\langle\hat{\sigma}_{cc}\rangle$. If the initial number of photons in the quantum field is much less than the number of atoms, $\langle\hat{\sigma}_{cc}\rangle$ is always much smaller than unity. Therefore the mean intensity of the quantum field remains small compared to that of the control field even when the latter is turned to zero.

In order to check the validity of the adiabatic approximation we consider the first correction to $\hat{\sigma}_{bc}$:

$$\hat{\sigma}_{bc} \approx -\frac{g\hat{E}}{\Omega} + \frac{1}{\Omega}\left(\frac{\partial}{\partial t} + \gamma_{ba}\right)\frac{1}{\Omega}\frac{\partial}{\partial t}\frac{g\hat{E}}{\Omega} + \cdots \tag{16}$$

The non-adiabatic correction in (16) leads to a spectral narrowing (pulse spreading) of the quantum field due to the finite bandwidth of the transparency window [10], which results in a "pulse"-matching of quantum and classical control field [19,15]. Using the adiabatic solution (12), one can verify that these corrections are small for propagation distances:

$$z \ll z_{max} = \frac{g^2N}{\gamma_{ab}} \times \frac{L_p^2}{c}, \tag{17}$$

where L_p is the length of the input pulse. Hence, in order to trap a pulse with negligible losses, it is required that

$$\frac{g^2NL_p}{c\gamma_{ab}} \gg 1. \tag{18}$$

This condition contains the number of atoms which is a signature of collective interactions. It should be contrasted to the strong-coupling condition corresponding to a quantum state transfer in cavity QED [20]. We note, in particular, that in the optically dense medium the adiabatic condition (18) is much easier to implement.

The effect of the Raman coherence decay can be easily estimated using the explicit expression for the generated matter states (15). It is clear that the collective states containing n_e atomic excitations will dephase at a rate $\gamma_{bc}\,n_e$. Hence, the time of the storage should be limited to $t_s \ll (\gamma_{bc}\,n_e)^{-1}$ to avoid decoherence [5].

In the discussion above we have considered the case where the control field only depends on time. This is valid, for instance, when the control field propagates in a direction perpendicular to that of the quantum field. In experiments involving hot atomic vapors co-propagation is required, however, in order to cancel Doppler broadening of the two-photon transition. In this case propagation effects of the control field need to be considered. If the quantum field is weak, the control field propagates as in free space and thus $\Omega(z,t) = \Omega(t - z/c)$. In this case one finds:

$$\left(\frac{\partial}{\partial t} + c\cos^2\theta(z,t)\frac{\partial}{\partial z}\right)\frac{\hat{E}(z,t)}{\Omega(z,t)} = 0. \tag{19}$$

Since the group velocity is now also z-dependent, trapping of the pulse does not preserve the shape exactly. Nevertheless it is evident that trapping and a reversible transfer of the quantum state from light to atoms are still possible. In experiments, however, a more practical approach can be taken in which a light pulse enters the medium already with $v_g^0 \ll c$. In such a case retardation of the control field can be ignored and one has $\Omega(t - z/c) \approx \Omega(t)$. Since the index of refraction is close to unity there will be no reflection losses at the entrance plane. However the polariton pulse becomes spatially compressed according to $L_p/L_p^0 = v_g^0/c$, and its amplitude grows according to the boundary condition $\hat{\Psi}(0,t) = \sqrt{c/v_g^0}\hat{E}(0,t)$. In this way, the total energy of the polariton field inside the medium is equval to the energy of the light field outside. After entering the medium the polaritons can be manipulated as discussed above.

In conclusion we have shown that it is possible to control the propagation of quantum pulses in optically thick Λ-type media. This coherent control mechanism is based on dark-state polaritons associated with EIT. In particular, a quantum light pulse can be "trapped", in which case its shape and quantum state are preserved in stationary atomic excitations. The matter-like polariton can then be re-accelerated and converted back into a photon pulse. These properties of dark-state polaritons can be used for squeezing and entanglement transfer from light to atoms. Furthermore, we anticipate interesting applications involving nonlinear interactions between such polaritons.

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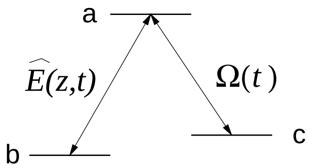

FIG. 1. 3-level Λ-type medium resonantly coupled to a classical field with Rabi-frequency $\Omega(t)$ and quantum field $\hat{E}(z,t)$.

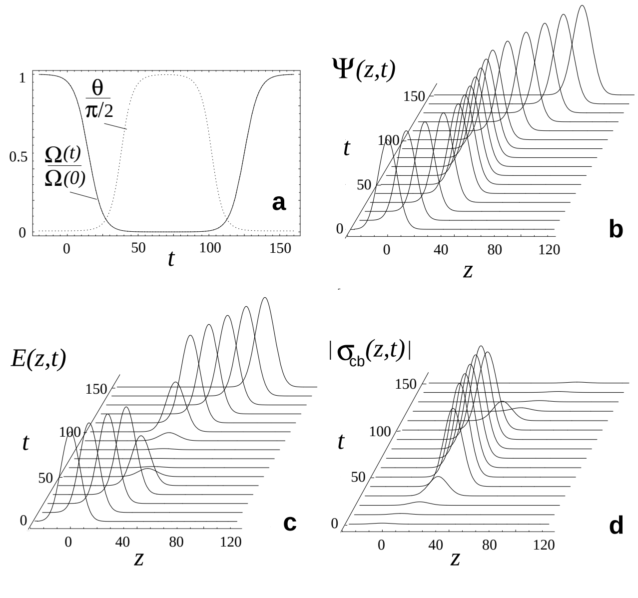

FIG. 2. Propagation of a dark-state polariton with envelope $\exp\{-(z/10)^2\}$. The mixing angle is rotated from 0 to $\pi/2$ and back according to $\cot\theta(t) = 100(1-0.5\tanh[0.1(t-15)] + 0.5\tanh[0.1(t-125)])$ as shown in (a). The coherent amplitude of the polariton $\Psi = \langle\hat{\Psi}\rangle$ is plotted in (b) and the electric field $E = \langle\hat{E}\rangle$ and matter components $|\sigma_{cb}| = |\langle\hat{\sigma}_{cb}\rangle|$ in (c) and (d) respectively. Axes are in arbitrary units with $c = 1$.