Entanglement of Atomic Ensembles by Trapping Correlated Photon States

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Abstract

We describe a general technique that allows for an ideal transfer of quantum correlations between light fields and metastable states of matter. The technique is based on trapping quantum states of photons in coherently driven atomic media, in which the group velocity is adiabatically reduced to zero. We discuss possible applications such as quantum state memories, generation of squeezed atomic states, preparation of entangled atomic ensembles and quantum information processing.

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One of the most intriguing aspects of quantum theory is the possibility to entangle quantum states of distinct objects. Recently these ideas led to many interesting new concepts such as quantum cryptography [1], teleportation [2] and quantum computation [3]. Photons are the fastest, simplest and very robust carriers of quantum information [4], but they are difficult to store.

This Letter describes a general technique that allows to transfer quantum correlations from traveling-wave light fields to collective atomic states and vice versa with nearly ideal efficiency. This is achieved by adiabatically reducing the group velocity of light fields to zero, thereby "trapping" the photons in a medium. Specifically, we here use the technique of intracavity Electromagnetically Induced Transparency (EIT) [5,6], in which the properties of a cavity filled with three-state Λ-type atoms can be manipulated by an external (classical) field.

Once the transfer is completed, the atomic ensemble "stores" all photons including their quantum correlations in metastable superpositions of many-atom states. Consequently a procedure of this kind can be used to generate non-classical states of the atoms, and, in particular, to entangle two or more distant atomic ensembles by mapping entangled photon wave-packets onto spatially separated atomic systems. In addition to fundamental aspects, this makes applications in low-noise spectroscopy [7] and quantum teleportation of collective atomic states [2] feasible. Furthermore, the atomic excitations can be coherently manipulated over a very long period of time, which opens up very interesting possibilities for quantum information processing [3]. Finally, the stored quantum states can be transfered back to light beams by simply reversing the adiabatic storage procedure.

The present contribution is motivated by recent experiments, in which EIT has been used to dramatically reduce the group velocity of light pulses in a coherently driven, optically dense ensemble of atoms [8]. Ideas involving adiabatic passage have already been considered for manipulation of single atoms in the context of cavity QED [9]. The method described here involves an optically dense many-atom system and does not require the usual strong-coupling regime of cavity QED. In the present system single photons couple to *collective* *excitations* associated with EIT, and the corresponding coupling strength can exceed that of an individual atom by orders of magnitude. In contrast to the approaches involving partial transfer of correlations by dissipative absorption of light [10], the present method is completely coherent and reversible.

To illustrate the essence of the technique, consider a single-mode cavity filled with a large number of coherently driven Λ-type atoms as shown in Fig. 1a. One transition is coupled by the quantum cavity field, whereas the other is driven by a classical coherent field of Rabi-frequency $\Omega(t)$. Under the condition of two-photon resonance the coherent driving field creates induced transparency [6] for the cavity field and the associated linear dispersion can substantially reduce its group velocity [8]. This leads to a dramatic enhancement of the effective storage time limited only by the lifetime of the dark state [5]. The basic Hamiltonian of the cavity + N-atom system can be written in terms of collective atomic operators $\hat{\Sigma}_{ab} = \sum_{i=1}^{N} \hat{\sigma}_{ab}^{i}$ and $\hat{\Sigma}_{ac} = \sum_{i=1}^{N} \hat{\sigma}_{ac}^{i}$ as

$$H = \hbar g \hat{a} \hat{\Sigma}_{ab} + \hbar \Omega(t) \hat{\Sigma}_{ac} + \text{h.c.}, \tag{1}$$

where $\hat{\sigma}_{\mu\nu}^{i} = |\mu\rangle_{ii}\langle\nu|$ is the flip operator of the ith atom between states $|\mu\rangle$ and $|\nu\rangle$. g is the coupling constant between the atoms and the field mode (vacuum Rabi-frequency) which for simplicity is assumed to be equal for all atoms. Here and below we work in a frame rotating with optical frequencies. This Hamiltonian has a family of dark states that are decoupled from both optical fields

$$|D, n\rangle = \sum_{k=0}^{n} \sqrt{\frac{n!}{k!(n-k)!}} \frac{(-g)^k N^{k/2} \Omega^{n-k}}{(g^2 N + \Omega^2)^{n/2}} |n-k\rangle |c^k\rangle \tag{2}$$

composed of cavity field states containing $|n-k\rangle$ photons and symmetric Dicke-like atomic states $|c^k\rangle$ containing k atoms in level $|c\rangle$, and all others in the ground state $|b\rangle$:

$$|c^0\rangle \equiv |b_1 ... b_N\rangle, \quad |c^1\rangle \equiv \sum_{i=1}^{N} \frac{-1}{\sqrt{N}} |b_1 ... c_i ... b_N\rangle, \tag{3}$$

$$|c^2\rangle \equiv \sum_{i \neq j = 1}^{N} \frac{1}{\sqrt{2N(N-1)}} |b_1 ... c_i ... c_j .... b_N\rangle, \text{ etc.} \tag{4}$$

We here assumed that the number of atoms is much larger than the number of photons in the light field.

The essence of the adiabatic transfer is the asymptotic behavior of the dark states (2) in the two limiting cases:

$$|D,n\rangle \to |n\rangle|c^0\rangle, \text{ when } \Omega \gg g\sqrt{N}, \tag{5}$$

$$|D,n\rangle \to |0\rangle|c^n\rangle, \text{ when } \Omega \ll g\sqrt{N}. \tag{6}$$

For a sufficiently strong coherent driving field the atoms do not interact with light, and the dark state coincides with the "bare" cavity mode where all atoms remain in the ground state. In this limit photons can "leak" in and out of the cavity as if it would be empty. In the opposite limit, the dark state is a purely atomic (metastable) state with no photons in the cavity. In the latter case the lifetime of excitations will not be sensitive to cavity decay; it will be limited solely by the decay of the metastable atomic states. It is most important that by varying the strength of the driving field $\Omega(t)$, and consequently by changing the linear dispersion in the medium, the state of the combined atom+cavity system can be changed from cavity-like (in which excitation is mostly of photon nature) to the atom-like (in which excitations are shared among the atoms). Since all dark states are orthogonal to each other, the ideal storing procedure (as discussed below) will transform any superposition of photon states into corresponding superpositions of atomic states:

$$\sum_i \alpha_i\, |i\rangle|c^0\rangle \to \sum_i \alpha_i\, |0\rangle|c^i\rangle. \tag{7}$$

Before proceeding with a detailed description of the specific adiabatic technique we note that the above results can be easily generalized to the case of two atomic ensembles, which can be entangled by trapping two entangled photon fields. In this case the atomic ensembles are placed either within the same or two different optical cavities (Fig.1b). The dark states are then the direct product of those corresponding to the subsystems: $|D,n,m\rangle = |D_r,n_r\rangle|D_l,m_l\rangle$. Hence the following operation can be accomplished:

$$\sum_{nm} \alpha_{nm}\, |nm\rangle|c_r^0\rangle|c_l^0\rangle \to \sum_{nm} \alpha_{nm} |0\rangle|c_r^n\rangle|c_l^m\rangle. \tag{8}$$

It is clear that trapping perfectly entangled photon states will result in perfectly entangled atomic ensembles.

Yet another related situation that we wish to mention involves trapping the states of two distinct fields within *one* atomic species. Here the two fields interact with more complex atoms, such as those shown in Fig.1c. By using essentially the same arguments as above one finds that a perfect state-transfer of the two fields to the atoms yields:

$$\sum_{nm} \alpha_{nm} \, |nm\rangle |c_r^0 c_l^0\rangle \rightarrow \sum_{nm} \alpha_{nm} \, |0\rangle |c_r^n c_l^m\rangle. \tag{9}$$

The potential significance of the last scheme is that the correlations and the entanglement of the two fields can be manipulated, since they are now stored within the same atomic ensemble. This is of prime importance in quantum information processing, in particular, for quantum logic devices and quantum computation.

We now describe and analyze an adiabatic procedure by which an input traveling-wave quantum field can be captured, stored and released. To this end we consider a quasi-1D system, include the continuum of the free-space plane-wave modes (with creation operators b_k^+) and model the coupling of these modes with the cavity by an effective Hamiltonian $V = \hbar \sum_k \kappa \hat{a}^\dagger \hat{b}_k + \text{h.c.}$. κ is a coupling constant. The initial state of the free field is taken to be $|\Psi_{\text{in}}\rangle = \sum_k \xi_k^1 |1_k\rangle + \sum_{k,m} \xi_{k,m}^2 |1_k 1_m\rangle + \ldots$. It is convenient to work with correlation amplitudes, i.e. Fourier transforms of $\xi_{k...l}^j$:

$$\Phi_j(t_1 ... t_j) = \langle 0| \hat{E}(t_1) ... \hat{E}(t_j) |\Psi\rangle, \tag{10}$$

where $\hat{E}(t) = L/(2\pi c) \int d\omega_k \exp(i\omega_k t) \hat{b}_k$, and L is quantization length. E.g. Φ_1 describes the envelope of a single photon wave packet, Φ_2 is the coincidence amplitude etc. We now consider a broad class of pulsed fields that are characterized by a single envelope $h(t)$ such that

$$\Phi_j(t_1, t_2, ... t_j) = \alpha_j \sqrt{j!} h(t_1) h(t_2) ... h(t_j). \tag{11}$$

A quantum state of such pulses can be described by a density matrix $\rho_{nm} = \alpha_n^* \alpha_m$. The corresponding mode function (envelope) is a superposition of plane waves proportional to

$h(z/c) = \int \mathrm{d}\omega_k\, \xi_k\, \mathrm{e}^{i\omega_k z/c}$. When the pulses interact with the combined system of cavity mode and atoms the states $\alpha_j|c^j\rangle$ are excited. We proceed by deriving the equations of motion for the probability amplitudes in the basis of dark and orthogonal bright states. The bright states as well as the excited states (containing components of states $|a^i..\rangle$) are then adiabatically eliminated. The remaining amplitudes of dark states and free-field components form a Dicke-like ladder. The ladder states are coupled to each other with the time-dependent coupling strength $\kappa\cos\theta(t)$ determined by the mixing angle $\cos\theta(t) = \Omega(t)/\sqrt{\Omega(t)^2 + g^2 N}$.

In the case when only single-photon pulses are involved the evolution equations are [11]:

$$\dot{D}_1(t) = i\kappa\cos\theta(t)\sum_k \xi_k(t), \tag{12}$$

$$\dot{\xi}_k(t) = -i\Delta_k\, \xi_k(t) + i\kappa\cos\theta(t)\, D(t). \tag{13}$$

We proceed by formally integrating Eq.(13), substituting the result into Eq.(12) and invoking the standard Markov approximation. Assuming that no photons arrive to the cavity before t_0 we find for the dark state amplitude $D_1(t) = -i\alpha_1 d(t)$ with

$$\begin{aligned} d(t) = \sqrt{\gamma\frac{c}{L}} \int_{t_0}^{t} \mathrm{d}\tau\, \cos\theta(\tau)\, h(\tau) \\ \times \exp\left\{ -\frac{\gamma}{2}\int_{\tau}^{t} \mathrm{d}\tau'\, \cos^2\theta(\tau') \right\}, \end{aligned} \tag{14}$$

and for the input-output relation

$$h_{\mathrm{out}}(t) = h(t) - \sqrt{\gamma L/c}\, d(t), \tag{15}$$

where $h_{\mathrm{out}}(t)$ is a pulse-shape of the outgoing wave packet. Here we have introduced the empty-cavity decay rate $\gamma = \kappa^2 L/c$. In order to trap photons we require $h_{\mathrm{out}}(t) = \dot{h}_{\mathrm{out}}(t) = 0$. Differentiating Eqs.(14,15) yields

$$-\frac{\mathrm{d}}{\mathrm{d}t}\ln\,\cos\theta(t) + \frac{\mathrm{d}}{\mathrm{d}t}\ln h(t) = \frac{\gamma}{2}\cos^2\theta(t), \tag{16}$$

and with the asymptotic condition $\cos\theta \to 0$ the output field remains zero. The above condition corresponds to a quantum or dynamical impedance matching [11]. The term on the r.h.s. is the effective cavity decay rate reduced due to intracavity EIT [5]. The first term on the l.h.s. of Eq.(16) describes internal "losses" due to coherent Raman adiabatic passage and the second term is due to the time-dependence of the input field. As in the case of classical impedance matching [12], Eq. (16) reflects the condition for complete destructive interference resulting in a vanishing outgoing wave. Solving Eq.(16) yields

$$\cos^2\theta(t) = \frac{h^2(t)}{\gamma\int_{-\infty}^{t} \mathrm{d}\tau h^2(\tau)}, \tag{17}$$

which corresponds to $d(t \to +\infty) \to 1$ (see Fig.2a). Hence, by suitable variation of the classical driving field any single-photon pulse can be trapped ideally, if its pulse length is longer than the bare-cavity decay time.

Generalizations of the above considerations to multi-photon states can proceed along the same lines, but involve more tedious algebra. In particular, for the two-photon states one finds $D_2(t) = -\alpha_2 d(t)^2$, and in general

$$D_k(t) = (-i)^k \alpha_k d(t)^k \tag{18}$$

can be proved. Under conditions of quantum impedance matching $D_k(t \to \infty) \to (-i)^k a_k$ for arbitrary k. Hence pulsed fields in a generalized single mode with arbitrary quantum state can be mapped onto the atomic ensemble.

Releasing the stored quantum state into a pulse of desired shape can be accomplished in a straightforward way. A simple reversal of the time dependence of the control field at a later time t_d leads to a perfect mirror-image of the initial pulse. This can be verified directly from Eqs. (14,15), see also Fig.2a. However, $\cos\theta$ can also be rotated back to its original value in another way, which allows to "tailor" the pulse-shape of the outgoing wave-packet while retaining the quantum state.

We next examine the factors limiting the performance of the photon trapping scheme. Adiabatic following occurs if the population in the excited and bright states is small at all times. The corresponding conditions can be derived by substituting the adiabatic solutions into the exact equations and requiring that the coupling of dark states to all other states is small [13]. For a pulse duration T, a linewidth of the excited state γ_a and a cavity width γ, the adiabaticity conditions are

$$\Omega(t)^2 + g^2 N \gg \max\left[\gamma\gamma_a,\ \frac{\gamma_a}{T},\ \sqrt{\frac{\gamma}{T}}\gamma_a\right]. \tag{19}$$

Since the characteristic input-pulse length and thus the characteristic times T have to be larger or equal to the bare cavity decay time γ^{-1}, the first condition is the most stringent one. Therefore adiabatic following is possible provided that $g^2 N \gg \gamma\gamma_a$.

In the discussion above we have disregarded the finite lifetime of the metastable state, γ_0^{-1}. If γ_0 is small, its influence during the loading and unloading periods can be neglected but needs to be taken into account during the storage interval. Collective states such as $|c^n\rangle$ will dephase at a rate $\gamma_n = n\gamma_0$, which sets the upper limit on the longest storage time. To illustrate the effect of this damping we have plotted in Fig. 2b the fidelity of the quantum state storage, defined as $f = \mathrm{Tr}\left\{\rho_{\mathrm{in}}\rho_{\mathrm{out}}\right\}$, as function of the storage time t_s (the time between capture and release) for input pulses in a number state and a squeezed vacuum state. It is apparent that the maximum storage time is on the order of the single-atom decay time divided by the characteristic number of input photons. We note that in alkali-vapor cells with buffer gas and/or wall coatings dark state lifetimes on the order of seconds are observed e.g. by Budker et al. [8].

We conclude by summarizing the main results and outlining the possible avenues of future studies opened by the present work. We demonstrated that is possible to map ideally the quantum states of light fields onto metastable states of atomic ensemble. In particular, this allows for the generation of non-classical (e.g. squeezed) states of atoms (see Eq.7). These states are precisely of the form required to achieve spectroscopic sensitivity beyond the usual quantum limit [7]. We have shown that by trapping two correlated fields it is possible to generate entangled states of two distinct atomic ensembles (see Eq.8). Prospects for quantum teleportation of collective atomic states are hence feasible. Finally, by trapping two fields within the same multi-state species (see Eq. 9) it seems possible to create good conditions for coherent manipulation of field correlations, entanglement, etc. This opens up interesting prospectives for quantum information processing and, in particular, for quantum logic operations. It is essential that all of the transfer operations can be achieved without invoking the strong coupling regime of single-atom cavity QED. Specifically, we have shown that under conditions of quantum impedance matching free fields can be ideally transfered back and forth provided the excitation rate of the collective mode ($\sim g^2N/\gamma_a$) exceeds the cavity decay rate (see Eq.19). Potentially, this can be used to considerably improve the fidelity of quantum processing.

At the same time we note that several interesting questions remain open and need to be explored. For example, we have not considered here any specific schemes to perform quantum logic gates with trapped photons. Possible ways include cavity QED techniques [9], direct nonlinear interactions of photons via e.g. resonantly enhanced Kerr nonlinearities [14] or, alternatively, atom-atom interactions. Another way of manipulating the quantum statistics involves the regime in which, contrary to the situation considered here, the characteristic number of photons exceeds the number of atoms. We further note that although the present analysis involves electronic degrees of freedom, it can be used to excite states of the center-of-mass motion of cold atomic samples in BECs [15]. Here again atom-atom collisions need to be taken into account. This adds new interesting dimensions to the present studies and will discussed elsewhere.

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FIGURES

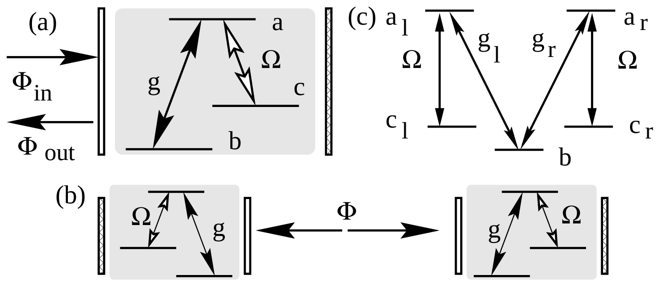

FIG. 1. (a) Optical cavity with single out-coupling mirror filled with large number of Λ-type atoms. External coherent field of Rabi-frequency $\Omega(t)$ is used to dynamically control properties of resonator system. Input field state is transferred back and forth to atomic system via coherent Raman adiabatic passage from states $|b\rangle$ to state $|c\rangle$. (b) Generation of entangled atomic ensembles in distant cavities using correlated photons. (c) Multi-state atoms for trapping correlated photons of right and left circular polarizations.

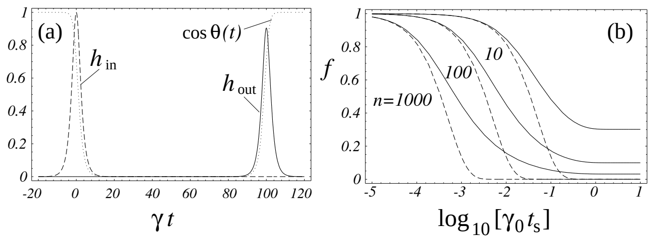

FIG. 2. (a) Storage of a hyperbolic secant pulse. Shown are normalized input (dashed line) and output pulse (full line) as a function of time as well as time dependence of $\cos\theta(t)$, optimized for the input field. Time unit is decay time of bare-cavity γ^{-1}. Dark-state decay rate is $\gamma_0 = 10^{-3}\gamma$. (b) Fidelity of storage for Fock state (dashed line) and squeezed vacuum state (solid line) inputs as function of storage time t_s. n denotes mean number of photons.