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Vorwort

Das Tätigkeitsfeld des Fraunhofer Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Dieter Prätzel-Wolters

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Batch Presorting Problems. Models and Complexity Results

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Abstract

In this paper we consider short term storage systems. We analyze presorting strategies to improve the efficiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assignment problem, *i.e.*, it has an assignment problem kernel and some additional constraints.

We present different types of these presorting problems, introduce mathematical programming formulations and prove the *NP*-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.

Key words: Complexity theory, Integer programming, Assignment, Logistics

1 Introduction

The need for applying optimization tools arises in many aspects of industrial applications [1]. Especially mathematical programming is a very natural and powerful tool to solve problems in inhouse logistics (c.f. [2], [3], [4]) or company wide logistics (supply chain) [5]. One might argue that low structure systems can probably be handled well without optimization, but complex systems with many degrees of freedom and restrictions strongly require optimization in order to investigate a situation completely (to analyze problems which can appear, to perform solutions and to make safe conclusions...).

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Modern manufacturing systems require the use of high performance, short term storage. Conventional staker crane solutions providing the necessary output frequency are often not profitable. There are at least two main types of storages devices in industry, shelf rackes and carousel storage systems. For long term storage, usually one uses shelf rackes, where one can give precisely the two-dimensional coordinates of where to place and store objects, c.f. [6], [7]. Another type is a short term storage system which should allow sorting. Many papers are devoted to a wealth of interesting combinatorial problems, which arise in automated carousel storage systems, see, c.f. [8], [9].

In this paper we analyze the special carousel based storage system ROTASTORE [10] which not only allows storing but also sorting. Because sorting often has to be done in a short time period, this problem has an online aspect. In online situations objects are only known shortly in advance. A group of objects just arriving we call a batch. Once such a batch arrives we need to decide how to distribute the objects of this batch before the next batch of objects arrives. A decision made can not be changed later. More details on online optimization can be found in [11].

The paper is organzed as follows: at first we give a detailed description of the Rotastore. The Batch Presorting Problems which appear in this application are described in Section 3. The complexity status of one of the problems is investigated in Section 4, Section 5 presents numerical tests. Finally, the last section put results into perspective.

2 The Rotastore

In the following we describe the storage system Rotastore, produced by psb GmbH, Pirmasens, Germany [10]. This system overcomes the poor perfomance of stacker crane systems by implementing a modular multi-carousel principle. This principle allows parallel loading and unloading by means of elevators interfacing the vertically staked carousel layers. As shown in Fig. 1, a typical configuration consists of two elevators and a stack of carousels each with identical numbers of slots. Identically sized trays are transported to and from the lifts by conveyors. Each carousel and each elevator may move independently. As indicated by the arrows, two combs of punches push trays simultaneously on all layers in or out of the carousel stack. The whole assembly is controlled by a sophisticated high-level programmable computer system which keeps track of operations and inventory.

The system normally operates output-driven, *i.e.*, an input/output-cycle (I/O-cycle) starts with accepting a set of orders with one ordered tray on each layer. The carousels then place by rotating the ordered trays in front of the output

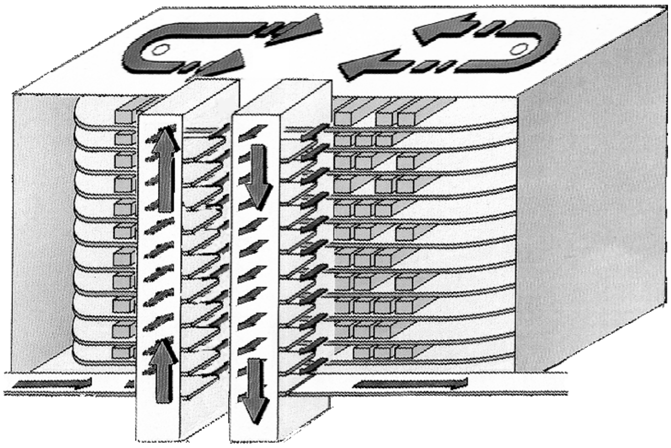

Fig. 1. Typical Rotastore configuration with elevators

elevator ("seeking"). Next, the punches push out all ordered trays onto the output elevator which immediately starts unloading them onto the leaving conveyer ("stepping"). The carousels move the now empty slots in front of the input elevator and the punch push in the waiting trays. While the elevators load and unload the next I/O-cycle starts with seeking the next set of orders. For the performance of the system the number of I/O-cycles per time unit is of course critical. The expected performance of a Rotastore with m slots on each of n layers was investigated in [12]. Practical experience shows that additional measures have to be taken into account to distribute the trays eventually with respect to the layers in order to realize the parallelizing speedup of the Rotastore. We will consider the case of a distribution center of a mayor German department store company. In this case, the trays are already partitioned into sets of orders when input into the Rotastore and the Rotastore may output a complete set of orders as soon as all trays of the set of orders are present. To improve the performance, the incoming trays are presorted such that each batch spreads over as many layers as possible. Conveyors transport the trays

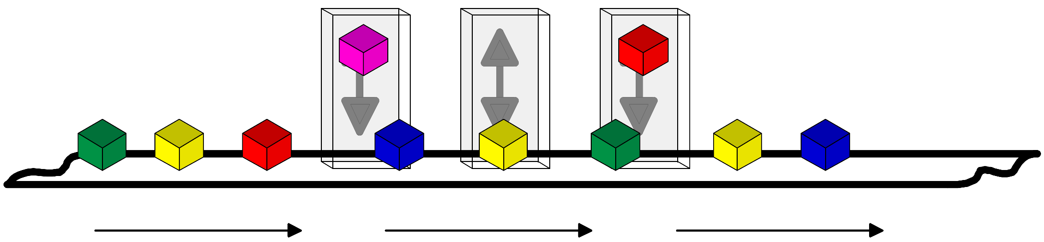

Fig. 2. Three stacker frames

throughout the system. Reordering is done online by stacker frames, small robotic devices which lift one tray high enough from the conveyor to let an arbitrary number of trays pass and set the lifted tray down again. As Fig. 2 illustrates, usually several stacker frames are placed in a row along the conveyer so a tray may overtake several preceding trays easily. The assembly is usually located directly in front of the Rotastore in the middle of sufficient length of accumulation conveyor (*e.g.*, roller conveyer).

3 Batch Presorting Problems

In the following we consider the problem of finding an sequence of trays to guarantee an optimal (even) distribution with respect to the layers, that can be achieved with the given hardware (stacker frame assambley). This problem will be called the Batch PreSorting Problem (BPSP), because the objects have to be sorted whithin one batch before they enter the storage system. For a more elegant presentation we prefer to speak of colors instead of orders and thus consider all trays of order k as having the same color k. We present three types of BPSP with differ in their objective. In BPSP_1 the objective is to minimize the total number of trays of the same color on one layer. In BPSP_2 the objective is to minimize the maximum number of trays of the same color on the same layer. Finally, BPSP_3 aims to minimize the sum of the maximum number of trays of the same color over all layers. Later we show that BPSP_1 is polynomial, but BPSP_2 is NP-complete. In Subsection 3.1 we present the optimization model of BPSP_1. In Subsection 3.2 we formulate BPSP_2 and BPSP_3 as decision problems and introduce optimization models.

3.1 Mathematical Formulation of BPSP_1

An instance of the BPSP_1 has the following input data:

- A sequence of N^O colored trays. These trays are indexed by i or j. $\mathcal{S}_k$ is the set of trays of color k and $i \in \mathcal{S}_k$ means that the tray at position i in the sequence (also called "ith tray" or "tray i" for short) has color k.
- N^K is the number of colors, indexed by k;
- N^L is the number of layers, indexed by l;
- N^S is the number of stacker frames.

A feasible solution to BPSP_1, *i.e.,* the sequence of trays after the stacker frame assembly, is given by a permutation δ of the index set $\{1, \ldots, N^O\}$, *i.e.,* $\delta(i) = j$, if tray i is placed onto position j. In [12] it was shown that exactly the permutations δ with

$$\delta(i) \geq i - N^S \tag{3.1}$$

can be achieved by the given hardware. Note further that exactly the trays at the permuted positions $\delta(i) = j$ with $j \equiv l \bmod N^L$ will be placed onto layer l. In addition we introduce the following notation:

$$\delta_{ij} = \begin{cases} 1, & \text{if } \delta(i) = j \\ 0, & \text{otherwise} \end{cases}$$

$$C_{ij} = \begin{cases} 1, & \text{if layer } l \equiv j \bmod N^L \text{ has a tray of the same order with tray } i \\ 0, & \text{otherwise} \end{cases}$$

The optimal permutation can be constructed from the solution of the following linear program:

$$\min \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} C_{ij}\delta_{ij} \tag{3.2}$$

$$\sum_{j=1}^{N^O} \delta_{ij} = 1, \ \forall i = \{1, ..., N^O\} \tag{3.3}$$

$$\sum_{i=1}^{N^O} \delta_{ij} = 1, \ \forall j = \{1, ..., N^O\} \tag{3.4}$$

$$\delta_{ij} = 0, \ \forall \{(i,j) =| \ j < i - N^S\} \tag{3.5}$$

$$\delta_{ij} \in \{0,1\}, \forall \{i,j\} \tag{3.6}$$

The objective function (3.2) minimizes the total number of trays of the same color on one layer. Unfeasible permutations are excluded (in dependence on N^S) a priori by (3.5).

It is well known that this kind of integer program is total unimodular (c.f. [13]) and may be solved efficiently by some versions of the simplex algorithm. Many special matching algorithms solve the problem in polynomial time (c.f. [14]). The simplicity of the formulation (3.2)-(3.6) lends itself to extensions. In applications where the Rotastore needs to output all trays of a set of orders consecutively, performance depends on the number of I/O cycles needed to completely output a set. For a given set of orders, the number of I/O-cycles needed for output is the maximum number of trays of this set of orders found on a single layer. This problem is presented next.

In this section we present formulations of the BPSP_2 and BPSP_3 as decision problems. We keep most of the notations used in the previous section. But we change C_{ij} to C_{kl}, which is the number of trays of color k already present on layer l. Additionally we define:

- an integer bound B,
- constants:

$$S_{ik} = \begin{cases} 1, & \text{if } i \in \mathcal{S}_k \\ 0, & \text{otherwise} \end{cases}$$

$$M_{jl} = \begin{cases} 1, & \text{if } j \equiv l \mod N^L \\ 0, & \text{otherwise.} \end{cases} .$$

This enables us to define $D_{ikjl} := S_{ik} M_{jl}$.

Now we can formulate the following decision problems:

D-BPSP$_2$: Is there a permutation $\delta : \{1, ..., N^O\} \to \{1, ..., N^O\}$, satisfying $\delta(i) = j \Longrightarrow j \geq i - N^S$ such that the maximal cost

$$\max_{k=1,\dots,N^K} \max_{l=1,\dots,N^L} \left(C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl} \delta_{ij} \right) \tag{3.7}$$

does not exceed B?

D-BPSP$_3$: Is there a permutation $\delta : \{1, ..., N^O\} \to \{1, ..., N^O\}$, satisfying $\delta(i) = j \Longrightarrow j \geq i - N^S$ such that the total cost

$$\sum_{k=1}^{N^K} \max_{l=1,\dots,N^L} \left(C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl} \delta_{ij} \right) \tag{3.8}$$

does not exceed B?

Remarks: The term $D_{ikjl}\delta_{ij}$ becomes 1 exactly if an additional tray of color k is placed onto layer l by realizing permutation δ. As C_{kl} denotes the number of trays of color k already present on that layer, the cost $C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ij}$

gives the number of trays after the permuted trays have all entered the Rotastore. In other words, D-BPSP$_2$ is the problem of finding a permutation of trays such that the maximum number of trays of the same color on any layer is less than or equal to B for all colors. To output all trays of a certain color, the Rotastore needs at least as many I/O-cycles as there are trays of that color on any layer, so the total cost term of D-BPSP$_2$ can be interpreted as a worst-case estimation of the performance. The total cost of D-BPSP$_3$ analogously represents the average performance over all colors.

3.2.1 An optimization version of BPSP$_2$

Since the objective is to minimize the maximal cost (3.7) we now formulate the decision problem as an optimization problem:

$$\min \max_{k=1,\ldots,N^K} \max_{l=1,\ldots,N^L} \left(C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ij} \right)$$

We replace the *minimax* objective function by an equivalent linear formulation. For this reason new variables are introduced: u_k – the maximal number of trays of color k on any layer, *i.e.*,

$$u_k = \max_{l=1,\ldots,N^L} \left(C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ij} \right) \tag{3.9}$$

and $y = \max_{\forall k=\{1,\ldots,N^K\}} \{u_k\}$.

That leads us to the transformed objective function

$$\min y, \tag{3.10}$$

and to some additional constraints

$$u_k \leq y, \ \forall k = \{1, ..., N^K\}. \tag{3.11}$$

$$C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ij} \leq u_k, \forall \{k, l\} \tag{3.12}$$

$$\sum_{j=1}^{N^O} \delta_{ij} = 1, \ \forall i = \{1, ..., N^O\} \tag{3.13}$$

$$\sum_{i=1}^{N^O} \delta_{ij} = 1, \ \forall j = \{1, ..., N^O\} \tag{3.14}$$

$$\delta_{ij} = 0, \ \forall\{(i,j) \mid j < i - N^S\} \tag{3.15}$$

$$\delta_{ij} \in \{0,1\}, \forall\{i,j\} \tag{3.16}$$

Let us make a few remarks related to those constraints:

Note that (3.10) and (3.11) establish the identity $y = \max_{\forall k=\{1,\ldots,N^K\}} \{u_k\}$. The inequalities (3.12) express that the number of trays of color k which are already on the layer l plus a number of trays of color k assigned onto this layer cannot be more than the maximal number u_k of trays of color k on any layer. The assignment constraints (3.13)-(3.14) account for the permutation of trays, *i.e.*, each tray can take only one position in a new order of trays and each new position can be filled only by one tray. Certain permutations (in dependence on N^S) can be excluded a priori by (3.15).

In some situations when the difference between the number of trays of different orders is very big it may not be advisable to minimize just the maximum number of trays of this set of orders found on a single layer. Instead it is more efficient to minimize the total amount of output cycles. We treat this approach in the optimization problem BPSP_3 introduced below.

3.2.2 An optimization version of $BPSP_3$

The objective function wich corresponds to (3.8) is:

$$\min \sum_{k=1}^{N^K} \max_{l=1,\ldots,N^L} \left(C_{kl} + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ij} \right)$$

Using the new variables u_k (3.9), we get:

$$\min \sum_{k=1}^{N^K} u_k. \tag{3.17}$$

subject to (3.12)-(3.16).

As one can see, BPSP_2 and BPSP_3 have BPSP_1 as a kernel. Unfortunately the polyhedron of BPSP_2 is not integral. The following section is devoted to the complexity of BPSP_2.

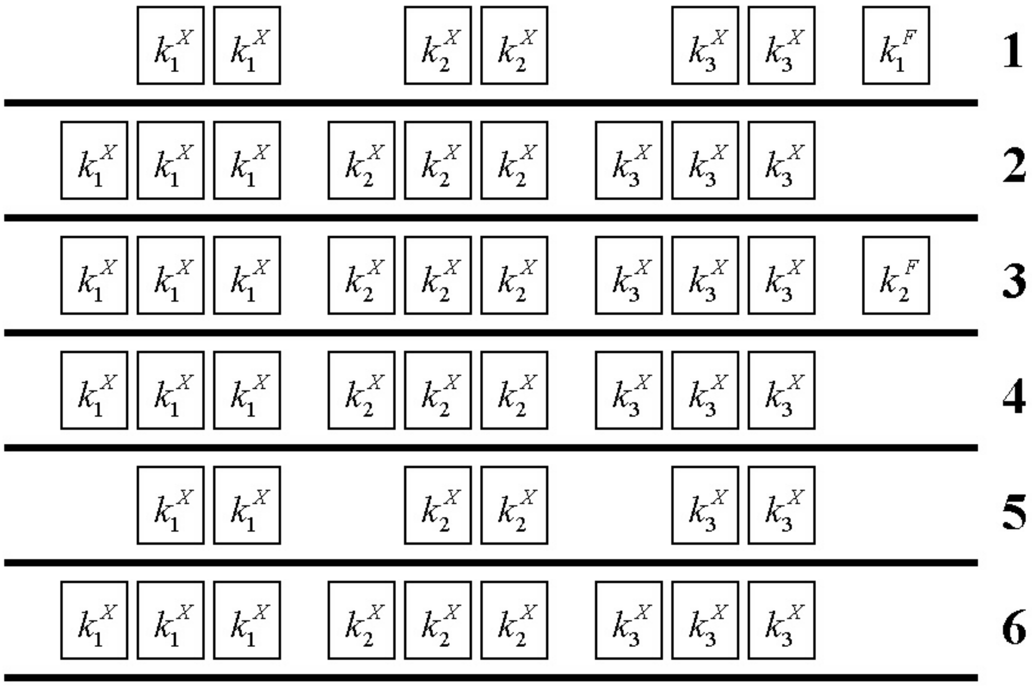

Fig. 3. The content of the Rotastore before distribution trays from *seq*.

4 Complexity Results

Regarding the complexity of $BPSP_2$ we can prove the following theorem.

Theorem 1 *Problem $BPSP_2$ is NP-complete.*

Proof. We proceed by showing that $BPSP_2$ can be reduced to the well-known *NP*-complete *3-SAT (3-Satisfiability)* problem. Concerning the complexity issues of the 3-SAT problem we refer the reader to [15]. Below we give the definition of the Satisfiability problem ([16]).

Definition 1 *Let $X = \{x_1, x_2, ..., x_n\}$ be a set of n Boolean variables. A literal y_i is either a variable x_i or its negation $\bar{x}_i$. A clause F_j is a disjunction of literals. Let formula $F = F_1 \wedge F_2 \wedge ... \wedge F_m$ be a conjunction of m clauses. The formula F is satisfiable if and only if there is a truth assignment $t : X \to \{0, 1\}$, which simultaneously satisfies all clauses F_j in F. The Satisfiability problem is the problem to decide whether for a given instance (X, F) there is a truth assignment for X that satisfies F. The 3-SAT problem is a restriction of the Satisfiability problem where each clause contains exactly 3 literals. The 3-SAT problem is still NP-complete.*

For an arbitrary instance (X, F) of *3-SAT* we define an instance of the sequencing problem $BPSP_2$, such that there exists a feasible permutation of the trays $\delta : \{1, ..., N^O\} \to \{1, ..., N^O\}$, satisfying $\delta(i) = j \Longrightarrow j \geq i - N^S$ with

$$\max_{k=1,...,N^K} \max_{l=1,...,N^L} \left(C(k,l) + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ikjl} \right) \leq B$$

if and only if there is a truth assignment $t : X \to \{0, 1\}$ satisfying F.

We choose $B = 3$ and $N^S = 1$ (this means that a tray can move forward at most one position). The number of layers is

$$N^L := 2(m + 1), \tag{4.1}$$

the number of trays is

$$N^O := 2nN^L, \tag{4.2}$$

and the number of sets $\mathcal{S}_k$ is

$$N^K := 2n + m + 2mn. \tag{4.3}$$

Specifically, we have

- $2n$ colors k_i^X, k_i^L, one for each variable x_i,
- m colors k_j^F, one for each clause F_j,
- mn additional colors k_{ij}^{XF} and
- mn additional colors $k_{ij}^{\bar{X}F}$.

The sequence seq consists of $2n$ subsequent parts seq_i of N^L trays each, *i.e.*, $seq := (seq_1,\ seq_2,\ ...,\ seq_n,\ seq_{n+1},\ seq_{n+2},\ ...,\ seq_{2n})$. Each seq_i has $2(m + 1)$ trays, *i.e.*, $seq_i := (q_{i,1}, q_{i,2}, ..., q_{i,2m+2})$. Now we define the sets $\mathcal{S}_k$, *i.e.*, the color for each tray of the sequence seq_i :

the first and last trays of seq_{2i-1} and seq_{2i} always have the colors of the variable x_i, *i.e.*,

$$q_{2i-1,1}, q_{2i,1} \in \mathcal{S}_{k_i^X} \tag{4.4}$$

$$q_{2i-1,N^L}, q_{2i,N^L} \in \mathcal{S}_{k_i^L}. \tag{4.5}$$

The colors of the $2m$ trays in between are defined depending on the occurrence of variable x_i in the clauses F_j as follows:

$$x_i \in F_j \Longrightarrow \begin{array}{r} q_{2i,2j} \in \mathcal{S}_{k_j^F} \\ q_{2i,2j+1} \in \mathcal{S}_{k_{ij}^{\bar{X}F}} \\ q_{2i-1,2j}, q_{2i-1,2j+1} \in \mathcal{S}_{k_{ij}^{XF}} \end{array} \tag{4.6}$$

$$\bar{x}_i \in F_j \Longrightarrow \begin{array}{r} q_{2i-1,2j} \in \mathcal{S}_{k_j^F} \\ q_{2i-1,2j+1} \in \mathcal{S}_{k_{ij}^{XF}} \\ q_{2i,2j}, q_{2i,2j+1} \in \mathcal{S}_{k_{ij}^{\bar{X}F}} \end{array} \tag{4.7}$$

$$x_i,\ \bar{x}_i \notin F_j \Longrightarrow \begin{array}{r} q_{2i-1,2j}, q_{2i-1,2j+1} \in \mathcal{S}_{k_{ij}^{XF}} \\ q_{2i,2j}, q_{2i,2j+1} \in \mathcal{S}_{k_{ij}^{\bar{X}F}} \end{array} \tag{4.8}$$

The values of the cost function $C(k,l) := C_{kl}$ (which reflects the content of the Rotastore) are defined for any $\{k,l\}$ as:

$$C_{k_i^X,l} = \begin{cases} 2, & \text{if } l \in \{1, N^L - 1\} \\ 3, & \text{else} \end{cases}, \ \forall i \in \{1,...,n\} \tag{4.9}$$

$$C_{k_j^F,l} = \begin{cases} 1, & \text{if } l = 2j-1 \\ 0, & \text{else} \end{cases}, \ \forall j \in \{1,...,m\} \tag{4.10}$$

$$C_{k_{ij}^{XF},l}, C_{k_{ij}^{\bar{X}F},l}, C_{k_i^L,l} = 0, \forall \{i,j,l\}. \tag{4.11}$$

Fig. 3 illustrates these coefficients for example with $n = 3$ and $m = 2$. In the remaining part of the proof we show that a feasible permutation of the trays from seq with

$$\max_{k=1,...,N^K} \max_{l=1,...,N^L} \left(C(k,l) + \sum_{i=1}^{N^O} \sum_{j=1}^{N^O} D_{ikjl}\delta_{ikjl} \right) \leq 3$$

exists if and only if F is satisfiable, *i.e.*, there is a truth assignment $t : X \to \{0,1\}$ that satisfies each clause in F.

First, we assume that F is satisfiable. We show that there exists a feasible permutation of trays with no more than 3 trays of the same color on any layer. Recall that:

- for each color k_i^X we have already exactly three trays on the layers $2, ..., N^L - 2, N^L$, two trays on the layers $1, N^L - 1$, one tray in seq_{2i-1} and one tray in seq_{2i};
- for each color k_j^F we have one tray on layer $2j - 1$ and exactly three trays in seq;
- for each color k_{ij}^{XF}, $k_{ij}^{\bar{X}F}$we have at most two trays in seq;
- for each color k_i^L only two trays exist in seq.

Now consider the first tray of subsequence seq_{2i-1}. This tray has the color k_i^X. It has to be sent to layer $N^L - 1$ or has to remain on the first layer, because all other layers are already occupied by three trays of this color. Layers 1 and $N^L - 1$ already contain two trays of color k_i^X and can accommodate only one additional tray each. Therefore, the second tray of color k_i^X – the first tray of seq_{2i} – has to be sent to the layer not used by the first tray. This will be done by moving the trays the minimal distance (*i.e.*, to the first possible layer), such that $\delta(seq_i) = seq_i$. That means we can discuss each subsequence independently. We will use this alternation to map the truth assignment onto the permutation δ:

$$x_i \mapsto^t 1 \Longrightarrow \begin{cases} q_{2i-1,1} \mapsto^\delta q_{2i-1,N^L-1} \\ q_{2i-1,j} \mapsto^\delta q_{2i-1,j-1} \quad , \quad 1 < j < N^L \\ q_{2i-1,N^L} \mapsto^\delta q_{2i-1,N^L} \\ q_{2i,j} \mapsto^\delta q_{2i,j} \quad , \quad 1 \leq j \leq N^L \end{cases} \tag{4.12}$$

$$x_i \mapsto^t 0 \Longrightarrow \begin{cases} q_{2i,1} \mapsto^\delta q_{2i,N^L-1} \\ q_{2i,j} \mapsto^\delta q_{2i,j-1} \quad , \quad 1 < j < N^L \\ q_{2i,N^L} \mapsto^\delta q_{2i,N^L} \\ q_{2i-1,j} \mapsto^\delta q_{2i-1,j} \quad , \quad 1 \leq j \leq N^L \end{cases} \tag{4.13}$$

In this way, if t assigns 1 to x_i, then from (4.12) the first tray of seq_{2i-1} moves to layer $N^L - 1$, and all other trays in seq_{2i-1} move one layer up (except for the last one which stays on the last layer). All trays of seq_{2i} keep their position. Otherwise, it follows from (4.13) that if t assigns 0 to x_i then all trays from seq_{2i-1} keep their positions and the first tray of seq_{2i} moves to the layer $N^L - 1$. All other trays in seq_{2i} move one layer up (except for the last one which stays on the last layer).

Assume there would be a layer l with 4 or more trays of the same color. Let us first discuss the color. It cannot be one of $k_{ij}^{XF}, k_{ij}^{\bar{X}F}, k_i^L$ since there are at most two of those and C_{kl} is zero for them. Without loss of generality we assume that there are 4 trays of color k_j^F on layer l. Since $|\mathcal{S}_{k_l^F}| = 3$, $C_{k_j^F,l}$ must be 1 and because of (4.10) it follows that $l = 2j - 1$. Now consider one of the 3 trays. From (4.6) and (4.7) we know that $\mathcal{S}_{k_l^F} \subset \{q_{2i,2j}, q_{2i-1,2j}\}$. We distinguish two cases:

Case 1:

$$q_{2i,2j} \in \mathcal{S}_{k_l^F} \Longrightarrow^{(4.6)} x_i \in F_j \wedge q_{2i,2j} \mapsto^{\delta} q_{2i,2j-1} \tag{4.14}$$

$$\text{since } l = 2j - 1 \Longrightarrow^{(4.13)} x_i \mapsto^{t} 0 \tag{4.15}$$

Case 2:

$$q_{2i-1,2j} \in \mathcal{S}_{k_l^F} \Longrightarrow^{(4.6)} \bar{x}_i \in F_j \wedge q_{2i-1,2j} \mapsto^{\delta} q_{2i-1,2j-1} \tag{4.16}$$

$$\text{since } l = 2j - 1 \Longrightarrow^{(4.12)} \bar{x}_i \mapsto^{t} 0 \tag{4.17}$$

Therefore, in both cases the truth value of the literal of x_i in F_j is false, so F_j contains one false literal. The same reasoning holds for the other two trays. Thus, F_j contains three false literals and is therefore false itself. This is a contradiction to the precondition that t satisfies (X, F).

For the missing direction of the proof, we assume that (X, F) is unsatisfiable. Our goal is to prove that for any permutation δ there exists at least one color for which 4 trays are located on the same layer. Barring trivial cases, that will be color k_j^F.

Consider the permutations, where $\delta(q_{2i,1}) \equiv l \bmod N^L$ with $2 \leq l < N^L - 1$ or $l = N^L$, *i.e.*, when the first tray of a subsequence $2i$ is not moved to the first or last-but-one layer. Because of (4.9) $C_{k_i^X,l} = 3$, layer l holds four trays of the color k_i^X. Therefore we just need to consider the remaining cases, *i.e.*, the first tray moved to the first or to the last-but-one layer.

With the same reasoning we can assume that $\delta(q_{2i-1,1}) \equiv \pm 1 \bmod N^L$. So we only deal with cases where

$$\delta(q_{2i,1}) \equiv 1 \bmod N^L \text{ and } \delta(q_{2i-1,1}) \equiv -1 \bmod N^L \tag{4.18}$$

or

$$\delta(q_{2i,1}) \equiv -1 \bmod N^L \text{ and } \delta(q_{2i-1,1}) \equiv 1 \bmod N^L \tag{4.19}$$

Let us define a truth assignment t_δ by

$$x_i \mapsto^{t_\delta} \begin{cases} 1 \text{ if } \delta(q_{2i,1}) \equiv 1 \bmod N^L \\ 0 \text{ if } \delta(q_{2i,1}) \equiv -1 \bmod N^L \end{cases} \tag{4.20}$$

Furthermore, (3.1) tells us that $\delta(j) \geq j - 1$ as $N^S = 1$ by construction. Since (X, F) is unsatisfiable by assumption, there must be some clause F_j which is not satisfied by t_δ.

Case 1: F_j contains a non-negated literal x_i.

Then $x_i \mapsto^{t_\delta} 0$, so from (4.20) we know that $\delta(q_{2i,1}) \equiv -1 \bmod N^L$ and from (4.18) that $\delta(q_{2i-1,1}) \equiv 1 \bmod N^L$. Because of $\delta(q_{2i,1}) \geq q_{2i,1} - 1 \equiv 0 \bmod N^L$, so $\delta(q_{2i,1}) \geq q_{2i,1} + N^L - 2$ since $\delta(k) > k - 2$, i.e., the stacker frame lifted the tray $q_{2i,1}$ at least for $N^L - 2$ positions. Therefore, $\delta(q_{2i,k}) = q_{2i,k} - 1$ for $2 \leq k \leq N^L - 1$, especially for $k = 2j$. From (4.6) we know that $q_{2i,2j} \in \mathcal{S}_{k_j^F}$, hence there is one additional tray of color k_j^F in layer $2j - 1$.

Case 2: F_j contains a negated literal $\bar{x}_i$.

Then $x_i \mapsto^{t_\delta} 1$, so from (4.20) we know that $\delta(q_{2i,1}) \equiv 1 \bmod N^L$ and from (4.19) that $\delta(q_{2i-1,1}) \equiv -1 \bmod N^L$. As in the first case we conclude that $\delta(q_{2i-1,1}) \geq q_{2i-1,1} + N^L - 2$ and therefore, it follows that $\delta(q_{2i-1,2j}) = q_{2i-1,2j} - 1 \equiv 2j - 1 \bmod N^L$. Since (4.7) $q_{2i-1,2j} \in \mathcal{S}_{k_j^F}$ there is an additional tray of color k_j^F on layer $2j - 1$.

Hence, for each of the three literals of F_j there is one tray of color k_j^F on layer $2j - 1$ and $C_{2j-1,k_j^F} = 1$ according to (4.10). Therefore, layer $2j - 1$ contains four trays of color k_j^F total and the proof is complete. ■

We illustrate Theorem 1 with the following example.

Example 1 *Suppose we have the following instance of 3-SAT problem:* $F = (x_1 \vee x_2 \vee x_3)(\bar{x}_1 \vee x_2 \vee \bar{x}_3)$, *i.e.,* $n = 3$ *and* $m = 2$. *Now we define all data needed for constructing the sequence seq. By (4.1)-(4.3) we obtain* $N^L = 6$, $N^O = 36$, *and the number* N^K *of sets* $\mathcal{S}_k$ *is* $N^K = 20$, *i.e.,*

- k_i^X, k_i^L, $i = 1, 2, 3$;
- k_j^F, $j = 1, 2$;
- k_{ij}^{XF}, $k_{ij}^{\bar{X}F}$, $i = 1, 2, 3$, $j = 1, 2$;

Fig. 3 shows the content of the Rotastore with respect to formulas (4.9) and (4.10) before the trays from seq are distributed in. Fig. 4 displays the sequence $seq = (seq_1, seq_2, ..., seq_6)$. *In addition, this picture illustrates the different* *assignments, e.g., for x_1, described by formulas (4.12) and (4.13) and the subsequent distribution on the layers of the Rotastore. Similar transformation will apply to the sequences seq_3, seq_4, seq_5, seq_6 (the first two correspond to x_2, the others to x_3).*

5 Numerical Tests

To investigate the behavior of the models BPSP_2 and BPSP_3 several numerical experiments have been carried out. At first, we were interested in knowing which model produces a solution with minimal number of output cycles for the Rotastore. In both models, u_k are the variables connected to output cycles.

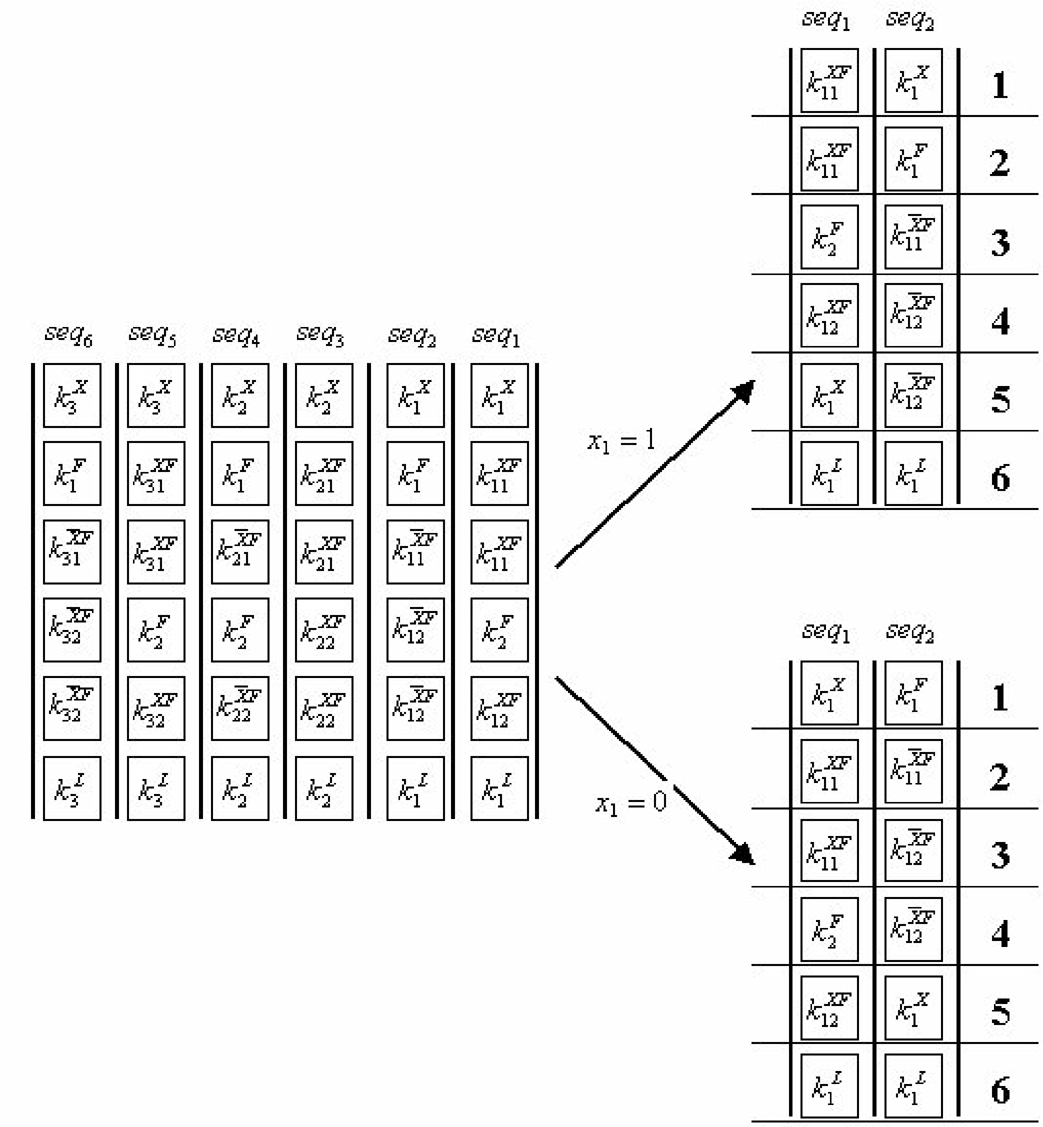

Fig. 4. Changes in the sequence after assigning a value to x_1

The total number of output cycles is calculated as the sum over all these variables: $\sum_{k=1}^{N^K} u_k$. The second point we are interested in is the average CPU time needed.

Computational experiments with randomly generated examples of the sets $\mathcal{S}_k$ were carried out using ILOG's OPL-studio [17] with the following input data:

- $N = 900$ is total number of trays;
- $N^O = 60$ is a number of known trays;
- $N^L = 15$;
- $N^K = 125$;
- $N^S = [1, N^L - 1]$.

It needs $\lceil N/N^O \rceil$ times to solve a problem in order to distribute all trays among the layers of the Rotastore (each time only N^O trays are available, next N^O trays become known only after distribution of the first N^O trays). Then N^K sets $\mathcal{S}_k$ were generated. For example, if $N^K = 4$, $N^O = 7$, then the generated sets could be:

$$\mathcal{S}_1 = \{1, 4\}, \quad \mathcal{S}_2 = \{\emptyset\}, \quad \mathcal{S}_3 = \{2, 5, 6, 7\}, \quad \mathcal{S}_4 = \{3\}.$$

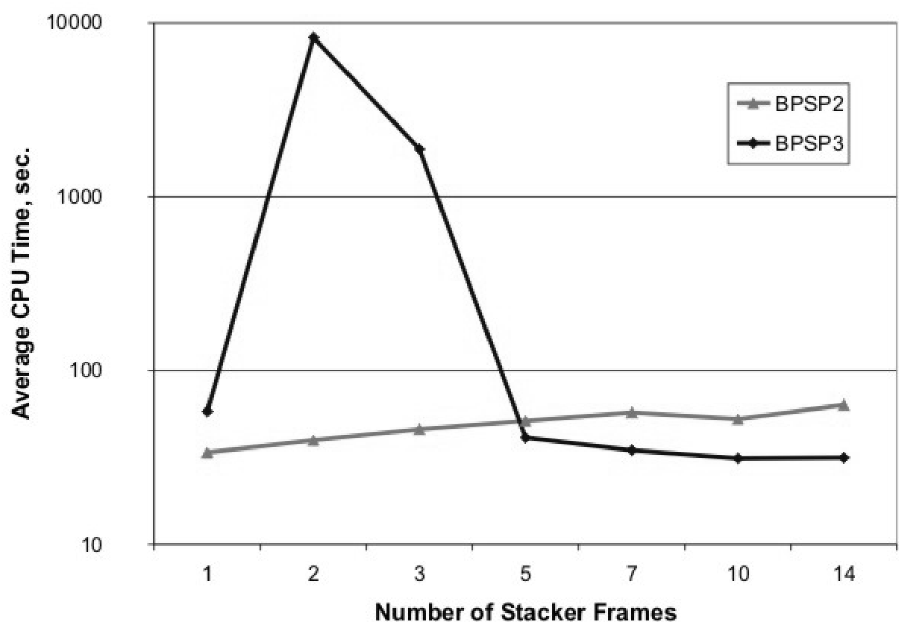

Fig. 5. The running time versus number of stacker frames.

For each value N^S, ten runs were made (each one starting with an empty Rotastore, *i.e.*, $C_{kl} = 0 \ \forall\{k, l\}$) for different inputs $\mathcal{S}_k$. Figures 5 and 6 show the average values of CPU time (in seconds) and the number of output cycles, respectively.

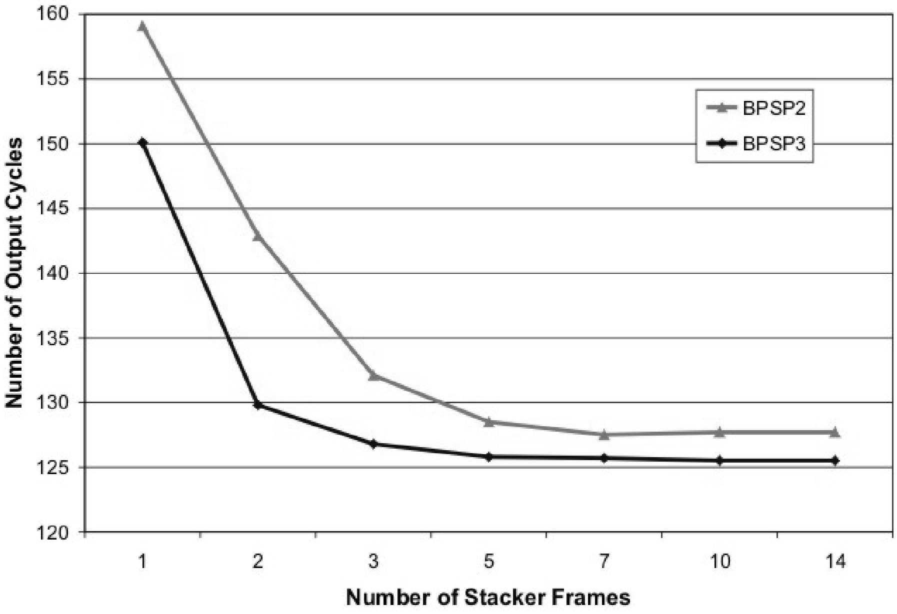

Fig. 6. The number of output cycles versus the number of stacker frames.

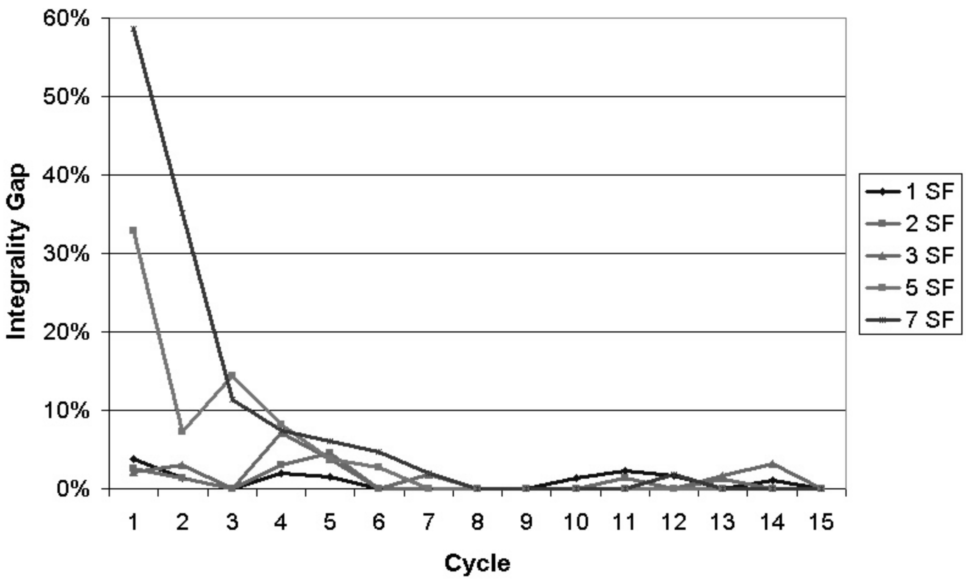

Fig. 7. The integrality gap versus the filling of the Rotastore for different number of staker frames (SF).

It is obvious that $BPSP_3$ gives us a better value for the number of output cycles (which was expected, because the objective (3.17) of $BPSP_3$ is to minimize the number of output cycles). But at the same time, for some examples, the average running time is very large. Analogous results were obtained for various values of N^L, N^K, N^S and N. The model shows the worst running time when $N^S = [1, 3]$. What could be the reason for this? To answer that we investigated how the integrality gap Δ (in percents)

$$\Delta = \frac{z_1^{IP} - z_1^{LP}}{z_1^{IP}} \cdot 100\%$$

depended on the degree of filling of the Rotastore (see Fig. 7) with various N^S for BPSP$_3$. On average, one observes that Δ decreases when the Rotastore fills (C_{kl} becomes larger). This is explained by the fact that the feasible region becomes smaller, and therefore, there are less possible combinations of the binary variables δ_{ij}. As a consequence of this, the number of active nodes in the Branch-and-Bound algorithm becomes smaller, which in turn decreases the running time. So, we conclude that BPSP$_3$ is better in use when the Rotastore is not empty. However, in practical situations this happens very seldom. In some rare cases, when the Rotastore is empty, we can fill it using BPSP$_2$, and continue with BPSP$_3$.

In addition, we observe that it is not useful to have more than 5 stacker frames because this does not lead to a smaller number of output cycles. This fact was also confirmed for different input sequences.

6 Summary & Conclusions

In this paper we have considered the BPSP which appears in many applications of storage systems. We introduced three different types of the BPSP and presented their optimization models. BPSP$_1$ is polynomial, because it is appears as an assignment problem. Although BPSP$_2$ and BPSP$_3$ have an assignment problem kernels, BPSP$_2$ is shown to be NP-hard. The complexity status of BPSP$_3$ is unknown. In some cases when we have an additional knowledge on the input sequence, the problem seems to become easier to solve. The direction for the future work is to reformulate the problem in case we have full knowledge on data and to investigate its complexity status for some particular cases.

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 pages, 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 pages, 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 pages, 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 pages, 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 pages, 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way. The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the EleyRideal and LangmuirHinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 pages, 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multiphase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the nonconvolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stressfield from known properties of the components. This is done by the extension of the asymptotic homogenization technique known for pure elastic nonhomogeneous bodies to the nonhomogeneous thermoviscoelasticity of the integral noncon-

volution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type. SanchezPalencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integralmodeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the spaceoperator kernels.
(20 pages, 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time. In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely. If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems. Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 pages, 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 pages, 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds

for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures
Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e.g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacita*ted Multiple Allocation Hub Location Problem*
We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation
Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:
The Finite-Volume-Particle Method for Conservation Laws
In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along pre- scribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions. After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management
The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords: facility location, software development, geographical information systems, supply chain management.
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art
This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation
A Grid free method for solving the Poisson equation is presented. This is an iterative method. The method is based on the weighted least squares approximation in which the Poisson equation is enforced to be satisfied in every iterations. The boundary conditions can also be enforced in the iteration process. This is a local approximation procedure. The Dirichlet, Neumann and mixed boundary value problems on a unit square are presented and the analytical solutions are compared with the exact solutions. Both solutions matched perfectly.
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier

Simulation of the fiber spinning process

To simulate the influence of process parameters to the melt spinning process a fiber model is used and coupled with CFD calculations of the quench air flow. In the fiber model energy, momentum and mass balance are solved for the polymer mass flow. To calculate the quench air the Lattice Boltzmann method is used. Simulations and experiments for different process parameters and hole configurations are compared and show a good agreement.
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis

On interaction of a liquid film with an obstacle

In this paper mathematical models for liquid films generated by impinging jets are discussed. Attention is stressed to the interaction of the liquid film with some obstacle. S. G. Taylor [Proc. R. Soc. London Ser. A 253, 313 (1959)] found that the liquid film generated by impinging jets is very sensitive to properties of the wire which was used as an obstacle. The aim of this presentation is to propose a modification of the Taylor's model, which allows to simulate the film shape in cases, when the angle between jets is different from 180°. Numerical results obtained by discussed models give two different shapes of the liquid film similar as in Taylors experiments. These two shapes depend on the regime: either droplets are produced close to the obstacle or not. The difference between two regimes becomes larger if the angle between jets decreases. Existence of such two regimes can be very essential for some applications of impinging jets, if the generated liquid film can have a contact with obstacles.
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner

Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids

The filling process of viscoplastic metal alloys and plastics in expanding cavities is modelled using the lattice Boltzmann method in two and three dimensions. These models combine the regularized Bingham model for viscoplastic with a free-interface algorithm. The latter is based on a modified immiscible lattice Boltzmann model in which one species is the fluid and the other one is considered as vacuum. The boundary conditions at the curved liquid-vacuum interface are met without any geometrical front reconstruction from a first-order Chapman-Enskog expansion. The numerical results obtained with these models are found in good agreement with available theoretical and numerical analysis.
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert

»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«

Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001

Was macht einen guten Hochschullehrer aus? Auf diese Frage gibt es sicher viele verschiedene, fachbezogene Antworten, aber auch ein paar allgemeine Gesichtspunkte: es bedarf der »Leidenschaft« für die Forschung (Max Weber), aus der dann auch die Begeisterung für die Lehre erwächst. Forschung und Lehre gehören zusammen, um die Wissenschaft als lebendiges Tun vermitteln zu können. Der Vortrag gibt Beispiele dafür, wie in angewandter Mathematik Forschungsaufgaben aus praktischen Alltagsproblemstellungen erwachsen, die in die Lehre auf verschiedenen Stufen (Gymnasium bis Graduiertenkolleg) einfließen; er leitet damit auch zu einem aktuellen Forschungsgebiet, der Mehrskalenanalyse mit ihren vielfältigen Anwendungen in Bildverarbeitung, Materialentwicklung und Strömungsmechanik über, was aber nur kurz gestreift wird. Mathematik erscheint hier als eine moderne Schlüsseltechnologie, die aber auch enge Beziehungen zu den Geistes- und Sozialwissenschaften hat.
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari

Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations

A Lagrangian particle scheme is applied to the projection method for the incompressible Navier-Stokes equations. The approximation of spatial derivatives is obtained by the weighted least squares method. The pressure Poisson equation is solved by a local iterative procedure with the help of the least squares method. Numerical tests are performed for two dimensional cases. The Couette flow, Poiseuelle flow, decaying shear flow and the driven cavity flow are presented. The numerical solutions are obtained for stationary as well as instationary cases and are compared with the analytical solutions for channel flows. Finally, the driven cavity in a unit square is considered and the stationary solution obtained from this scheme is compared with that from the finite element method.
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel

Optimal Portfolios with Fixed Consumption or Income Streams

We consider some portfolio optimisation problems where either the investor has a desire for an a priori specified consumption stream or/and follows a deterministic pay in scheme while also trying to maximize expected utility from final wealth. We derive explicit closed form solutions for continuous and discrete monetary streams. The mathematical method used is classical stochastic control theory.
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel

Optimal portfolios with a loan dependent credit spread

If an investor borrows money he generally has to pay higher interest rates than he would have received, if he had put his funds on a savings account. The classical model of continuous time portfolio optimisation ignores this effect. Since there is obviously a connection between the default probability and the total percentage of wealth, which the investor is in debt, we study portfolio optimisation with a control dependent interest rate. Assuming a logarithmic and a power utility function, respectively, we prove explicit formulae of the optimal control.
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz

The Euler number of discretized sets - on the choice of adjacency in homogeneous lattices

Two approaches for determining the Euler-Poincaré characteristic of a set observed on lattice points are considered in the context of image analysis { the integral geometric and the polyhedral approach. Information about the set is assumed to be available on lattice points only. In order to retain properties of the Euler number and to provide a good approximation of the true Euler number of the original set in the Euclidean space, the appropriate choice of adjacency in the lattice for the set and its background is crucial. Adjacencies are defined using tessellations of the whole space into polyhedrons. In R 3 , two new 14 adjacencies are introduced additionally to the well known 6 and 26 adjacencies. For the Euler number of a set and its complement, a consistency relation holds. Each of the pairs of adjacencies (14:1; 14:1), (14:2; 14:2), (6; 26), and (26; 6) is shown to be a pair of complementary adjacencies with respect to this relation. That is, the approximations of the Euler numbers are consistent if the set and its background (complement) are equipped with this pair of adjacencies. Furthermore, sufficient conditions for the correctness of the approximations of the Euler number are given. The analysis of selected microstructures and a simulation study illustrate how the estimated Euler number depends on the chosen adjacency. It also shows that there is not a uniquely best pair of adjacencies with respect to the estimation of the Euler number of a set in Euclidean space.
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

A generalized lattice Boltzmann model to simulate free-surface is constructed in both two and three dimensions. The proposed model satisfies the interfacial boundary conditions accurately. A distinctive feature of the model is that the collision processes is carried out only on the points occupied partially or fully by the fluid. To maintain a sharp interfacial front, the method includes an anti-diffusion algorithm. The unknown distribution functions at the interfacial region are constructed according to the first order Chapman-Enskog analysis. The interfacial boundary conditions are satis-

fied exactly by the coefficients in the Chapman-Enskog expansion. The distribution functions are naturally expressed in the local interfacial coordinates. The macroscopic quantities at the interface are extracted from the least-square solutions of a locally linearized system obtained from the known distribution functions. The proposed method does not require any geometric front construction and is robust for any interfacial topology. Simulation results of realistic filling process are presented: rectangular cavity in two dimensions and Hammer box, Campbell box, Sheffield box, and Motorblock in three dimensions. To enhance the stability at high Reynolds numbers, various upwind-type schemes are developed. Free-slip and no-slip boundary conditions are also discussed.
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

In the present paper a kinetic model for vehicular traffic leading to multivalued fundamental diagrams is developed and investigated in detail. For this model phase transitions can appear depending on the local density and velocity of the flow. A derivation of associated macroscopic traffic equations from the kinetic equation is given. Moreover, numerical experiments show the appearance of stop and go waves for highway traffic with a bottleneck.
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

To a network N(*q*) with determinant D(*s*;*q*) depending on a parameter vector *q* Î R^r via identification of some of its vertices, a network N^ (*q*) is assigned. The paper deals with procedures to find N^ (*q*), such that its determinant D^ (*s*;*q*) admits a factorization in the determinants of appropriate subnetworks, and with the estimation of the deviation of the zeros of D^ from the zeros of D. To solve the estimation problem state space methods are applied.
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

A spectral theory for stationary random closed sets is developed and provided with a sound mathematical basis. Definition and proof of existence of the Bartlett spectrum of a stationary random closed set as well as the proof of a Wiener-Khintchine theorem for the power spectrum are used to two ends: First, well known second order characteristics like the covariance can be estimated faster than usual via frequency space. Second, the Bartlett spectrum and the power spectrum can be used as second order characteristics in frequency space. Examples show, that in some cases information about the random closed set is easier to obtain from these characteristics in frequency space than from their real world counterparts.
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

We present a unified approach of several boundary conditions for lattice Boltzmann models. Its general framework is a generalization of previously introduced schemes such as the bounce-back rule, linear or quadratic interpolations, etc. The objectives are two fold: first to give theoretical tools to study the existing boundary conditions and their corresponding accuracy; secondly to design formally third- order accurate boundary conditions for general flows. Using these boundary conditions, Couette and Poiseuille flows are exact solution of the lattice Boltzmann models for a Reynolds number *Re* = 0 (Stokes limit).
Numerical comparisons are given for Stokes flows in periodic arrays of spheres and cylinders, linear periodic array of cylinders between moving plates and for Navier-Stokes flows in periodic arrays of cylinders for *Re* < 200. These results show a significant improvement of the overall accuracy when using the linear interpolations instead of the bounce-back reflection (up to an order of magnitude on the hydrodynamics fields). Further improvement is achieved with the new multi-reflection boundary conditions, reaching a level of accuracy close to the quasi-analytical reference solutions, even for rather modest grid resolutions and few points in the narrowest channels. More important, the pressure and velocity fields in the vicinity of the obstacles are much smoother with multi-reflection than with the other boundary conditions.
Finally the good stability of these schemes is highlighted by some simulations of moving obstacles: a cylinder between flat walls and a sphere in a cylinder.
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Im Rahmen dieser Arbeit soll eine elementar gehaltene Einführung in die Aufgabenstellungen und Prinzipien der modernen Finanzmathematik gegeben werden. Insbesondere werden die Grundlagen der Modellierung von Aktienkursen, der Bewertung von Optionen und der Portfolio-Optimierung vorgestellt. Natürlich können die verwendeten Methoden und die entwickelte Theorie nicht in voller Allgemeinheit für den Schulunterricht verwendet werden, doch sollen einzelne Prinzipien so heraus gearbeitet werden, dass sie auch an einfachen Beispielen verstanden werden können.
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

In this paper we consider short term storage systems. We analyze presorting strategies to improve the effiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assigment problem, i. e., it has an assigment problem kernel and some additional constraints. We present different types of these presorting problems, introduce mathematical programming formulations and prove the NP-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

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