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Vorwort

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Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Overview of Symbolic Methods in Industrial Analog Circuit Design

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Abstract

Industrial analog circuits are usually designed using numerical simulation tools. To obtain a deeper circuit understanding, symbolic analysis techniques can additionally be applied. Approximation methods which reduce the complexity of symbolic expressions are needed in order to handle industrial-sized problems.

This paper will give an overview to the field of symbolic analog circuit analysis. Starting with a motivation, the state-of-the-art simplification algorithms for linear as well as for nonlinear circuits are presented. The basic ideas behind the different techniques are described, whereas the technical details can be found in the cited references. Finally, the application of linear and nonlinear symbolic analysis will be shown on two example circuits.

Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index

1. Introduction to Symbolic Circuit Analysis

The motivation for applying symbolic techniques to the field of analog circuit design has been to gain insight into circuit behavior by interpreting analytic formulas instead of using traditional numerical design and simulation tools which lack in providing deeper design understanding. However, it becomes apparent quite quickly that exact symbolic analysis yields expressions which are too complex to be of any use.

For example, even a simple common-emitter amplifier consisting of only one BJT transistor (Figure 1) already results in a symbolic transfer function for the small-signal voltage gain with more than 130 terms as illustrated in Figure 2 (expression is given in Mathematica syntax). For this computation only a simplified PSpice small-signal transistor model was used – the full transistor model

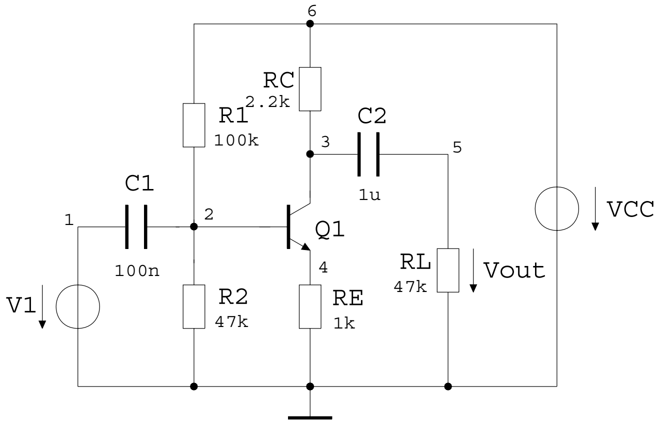

Figure 1: Common-emitter amplifier.

```
vac == (C1 C2 R1 R2 RC RL s^2 (RE - gm$Q1 Ro$Q1 Rpi$Q1 + Cbc$Q1 RE Ro$Q1 s + Cbc$Q1 RE Rpi$Q1 s +
      Cbe$Q1 RE Rpi$Q1 s + Cbc$Q1 Ro$Q1 Rpi$Q1 s + Cbc$Q1 gm$Q1 RE Ro$Q1 Rpi$Q1 s + Cbc$Q1 Cbe$Q1 RE Ro$Q1 Rpi$Q1 s^2 ) V1) /
  (R1 R2 RC + R1 R2 RE + R1 RC RE + R2 RC RE + R1 R2 Ro$Q1 + R1 RE Ro$Q1 + R2 RE Ro$Q1 + R1 RC Rpi$Q1 + R2 RC Rpi$Q1 +
    R1 RE Rpi$Q1 + R2 RE Rpi$Q1 + R1 Ro$Q1 Rpi$Q1 + R2 Ro$Q1 Rpi$Q1 + gm$Q1 R1 RE Ro$Q1 Rpi$Q1 + gm$Q1 R2 RE Ro$Q1 Rpi$Q1 +
    C1 R1 R2 RC RE s + C2 R1 R2 RC RE s + C2 R1 R2 RC RL s + C2 R1 R2 RE RL s + C2 R1 RC RE RL s + C2 R2 RC RE RL s + C2 R1 R2 RC Ro$Q1 s +
    Cbc$Q1 R1 R2 RC Ro$Q1 s + C1 R1 R2 RE Ro$Q1 s + Cbc$Q1 R1 R2 RE Ro$Q1 s + C2 R1 RC RE Ro$Q1 s + Cbc$Q1 R1 RC RE Ro$Q1 s +
    C2 R2 RC RE Ro$Q1 s + Cbc$Q1 R2 RC RE Ro$Q1 s + C2 R1 R2 RL Ro$Q1 s + C2 R1 RE RL Ro$Q1 s + C2 R2 RE RL Ro$Q1 s +
    C1 R1 R2 RC Rpi$Q1 s + Cbc$Q1 R1 R2 RC Rpi$Q1 s + Cbe$Q1 R1 R2 RC Rpi$Q1 s + C1 R1 R2 RE Rpi$Q1 s + Cbc$Q1 R1 R2 RE Rpi$Q1 s +
    Cbe$Q1 R1 R2 RE Rpi$Q1 s + C2 R1 RC RE Rpi$Q1 s + Cbc$Q1 R1 RC RE Rpi$Q1 s + Cbe$Q1 R1 RC RE Rpi$Q1 s + C2 R2 RC RE Rpi$Q1 s +
    Cbc$Q1 R2 RC RE Rpi$Q1 s + Cbe$Q1 R2 RC RE Rpi$Q1 s + C2 R1 RC RL Rpi$Q1 s + C2 R2 RC RL Rpi$Q1 s + C2 R1 RE RL Rpi$Q1 s +
    C2 R2 RE RL Rpi$Q1 s + C1 R1 R2 Ro$Q1 Rpi$Q1 s + Cbc$Q1 R1 R2 Ro$Q1 Rpi$Q1 s + Cbe$Q1 R1 R2 Ro$Q1 Rpi$Q1 s +
    C2 R1 RC Ro$Q1 Rpi$Q1 s + Cbc$Q1 R1 RC Ro$Q1 Rpi$Q1 s + C2 R2 RC Ro$Q1 Rpi$Q1 s + Cbc$Q1 R2 RC Ro$Q1 Rpi$Q1 s +
    Cbc$Q1 gm$Q1 R1 R2 RC Ro$Q1 Rpi$Q1 s + Cbe$Q1 R1 RE Ro$Q1 Rpi$Q1 s + Cbe$Q1 R2 RE Ro$Q1 Rpi$Q1 s + C1 gm$Q1 R1 R2 RE Ro$Q1 Rpi$Q1 s +
    Cbc$Q1 gm$Q1 R1 R2 RE Ro$Q1 Rpi$Q1 s + C2 gm$Q1 R1 RC RE Ro$Q1 Rpi$Q1 s + Cbc$Q1 gm$Q1 R1 RC RE Ro$Q1 Rpi$Q1 s +
    C2 gm$Q1 R2 RC RE Ro$Q1 Rpi$Q1 s + Cbc$Q1 gm$Q1 R2 RC RE Ro$Q1 Rpi$Q1 s + C2 R1 RL Ro$Q1 Rpi$Q1 s +
    C2 R2 RL Ro$Q1 Rpi$Q1 s + C2 gm$Q1 R1 RE RL Ro$Q1 Rpi$Q1 s + C2 gm$Q1 R2 RE RL Ro$Q1 Rpi$Q1 s + C1 C2 R1 R2 RC RE RL s^2 +
    C1 C2 R1 R2 RC RE Ro$Q1 s^2 + C1 Cbc$Q1 R1 R2 RC RE Ro$Q1 s^2 + C2 Cbc$Q1 R1 R2 RC RE Ro$Q1 s^2 + C2 Cbc$Q1 R1 R2 RC RL Ro$Q1 s^2 +
    C1 C2 R1 R2 RE RL Ro$Q1 s^2 + C2 Cbc$Q1 R1 R2 RE RL Ro$Q1 s^2 + C2 Cbc$Q1 R1 RC RE RL Ro$Q1 s^2 + C2 Cbc$Q1 R2 RC RE RL Ro$Q1 s^2 +
    C1 C2 R1 R2 RC RE Rpi$Q1 s^2 + C1 Cbc$Q1 R1 R2 RC RE Rpi$Q1 s^2 + C2 Cbc$Q1 R1 R2 RC RE Rpi$Q1 s^2 + C1 Cbe$Q1 R1 R2 RC RE Rpi$Q1 s^2 +
    C2 Cbe$Q1 R1 R2 RC RE Rpi$Q1 s^2 + C1 C2 R1 R2 RC RL Rpi$Q1 s^2 + C2 Cbc$Q1 R1 R2 RC RL Rpi$Q1 s^2 + C2 Cbe$Q1 R1 R2 RC RL Rpi$Q1 s^2 +
    C1 C2 R1 R2 RE RL Rpi$Q1 s^2 + C2 Cbc$Q1 R1 R2 RE RL Rpi$Q1 s^2 + C2 Cbe$Q1 R1 R2 RE RL Rpi$Q1 s^2 + C2 Cbc$Q1 R1 RC RE RL Rpi$Q1 s^2 +
    C2 Cbe$Q1 R1 RC RE RL Rpi$Q1 s^2 + C2 Cbc$Q1 R2 RC RE RL Rpi$Q1 s^2 + C2 Cbe$Q1 R2 RC RE RL Rpi$Q1 s^2 + C1 C2 R1 R2 RC Ro$Q1 Rpi$Q1 s^2 +
    C1 Cbc$Q1 R1 R2 RC Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 R1 R2 RC Ro$Q1 Rpi$Q1 s^2 + C2 Cbe$Q1 R1 R2 RC Ro$Q1 Rpi$Q1 s^2 +
    Cbc$Q1 Cbe$Q1 R1 R2 RC Ro$Q1 Rpi$Q1 s^2 + C1 Cbe$Q1 R1 R2 RE Ro$Q1 Rpi$Q1 s^2 + Cbc$Q1 Cbe$Q1 R1 R2 RE Ro$Q1 Rpi$Q1 s^2 +
    C2 Cbe$Q1 R1 RC RE Ro$Q1 Rpi$Q1 s^2 + Cbc$Q1 Cbe$Q1 R1 RC RE Ro$Q1 Rpi$Q1 s^2 + C2 Cbe$Q1 R2 RC RE Ro$Q1 Rpi$Q1 s^2 +
    Cbc$Q1 Cbe$Q1 R2 RC RE Ro$Q1 Rpi$Q1 s^2 + C1 C2 gm$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^2 + C1 Cbc$Q1 gm$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^2 +
    C2 Cbc$Q1 gm$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^2 + C1 C2 R1 R2 RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 R1 R2 RL Ro$Q1 Rpi$Q1 s^2 +
    C2 Cbe$Q1 R1 R2 RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 R1 RC RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 R2 RC RL Ro$Q1 Rpi$Q1 s^2 +
    C2 Cbc$Q1 gm$Q1 R1 R2 RC RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbe$Q1 R1 RE RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbe$Q1 R2 RE RL Ro$Q1 Rpi$Q1 s^2 +
    C1 C2 gm$Q1 R1 R2 RE RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 gm$Q1 R1 R2 RE RL Ro$Q1 Rpi$Q1 s^2 + C2 Cbc$Q1 gm$Q1 R1 RC RE RL Ro$Q1 Rpi$Q1 s^2 +
    C2 Cbc$Q1 gm$Q1 R2 RC RE RL Ro$Q1 Rpi$Q1 s^2 + C1 C2 Cbc$Q1 R1 R2 RC RE RL Ro$Q1 s^3 + C1 C2 Cbc$Q1 R1 R2 RC RE RL Rpi$Q1 s^3 +
    C1 C2 Cbe$Q1 R1 R2 RC RE RL Rpi$Q1 s^3 + C1 C2 Cbe$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^3 + C1 Cbc$Q1 Cbe$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^3 +
    C2 Cbc$Q1 Cbe$Q1 R1 R2 RC RE Ro$Q1 Rpi$Q1 s^3 + C1 C2 Cbc$Q1 R1 R2 RC RL Ro$Q1 Rpi$Q1 s^3 + C2 Cbc$Q1 Cbe$Q1 R1 R2 RC RL Ro$Q1 Rpi$Q1 s^3 +
    C1 C2 Cbe$Q1 R1 R2 RE RL Ro$Q1 Rpi$Q1 s^3 + C2 Cbc$Q1 Cbe$Q1 R1 R2 RE RL Ro$Q1 Rpi$Q1 s^3 + C2 Cbc$Q1 Cbe$Q1 R1 RC RE RL Ro$Q1 Rpi$Q1 s^3 +
    C2 Cbc$Q1 Cbe$Q1 R2 RC RE RL Ro$Q1 Rpi$Q1 s^3 + C1 C2 Cbc$Q1 gm$Q1 R1 R2 RC RE RL Ro$Q1 Rpi$Q1 s^3 +
    C1 C2 Cbc$Q1 Cbe$Q1 R1 R2 RC RE RL Ro$Q1 Rpi$Q1 s^4 )
```

Figure 2: Small-signal voltage transfer function.

produces a transfer function with more than 1000 terms (by *term* we denote a single product of a sum given in sum-of-products form).

Even with exact computer algebra methods the generation of an interpretable formula from this transfer function remains unsuccessful. Obviously, for industrial circuits with more than just one transistor it is impossible to obtain useful results due to the extreme computational complexity of symbolic calculations. This contradicts the initial intention of symbolic analysis, namely to gain insight into unknown circuit behavior.

On the other hand it is a fact that industrial circuit designers rely on hand calculations where they are able to produce compact and interpretable expressions by using their expert knowledge of circuit behavior and design heuristics to simplify circuit equations before or during their solution. By that, one could derive the very simple formula

$$v_{ac} \approx -\frac{R_C}{R_E} \tag{1}$$

for the above common-emitter amplifier in contrast to the exact solution shown in Figure 2. This formula is actually valid with a little deviation only for the main operating region of the amplifier. Exactly these manual designer computations motivated the development of the algorithms which lead to a breakthrough in the field of symbolic circuit analysis. Nowadays it is possible to automatically reduce the transfer function from Figure 2 to formula (1).

1.1. Symbolic Simplification Algorithms

As shown above, symbolic analysis of large analog circuits seems to be senseless as long as the complexity problem has not been solved. Thus, in order to reduce the complexity of the symbolic expression, one needs to simplify it.

In general, the term *symbolic simplification* or *symbolic approximation* refers to a whole family of hybrid symbolic/numeric algorithms for expression simplification. These techniques require more numerical knowledge about the investigated circuit than manual simplifications but yield compact expressions with predictable error in a fully automated way. In manual circuit analysis the decisions on which expressions to keep and which ones to discard are based on vague and only qualitative assumptions, such as $R_1 \ll R_2$, that do not allow for assigning precise error figures to simplified expressions. For automating the designer's behavior within a computer program one needs exact figures to simplify an expression because qualitative relations between elements are not sufficient for determining the importance of a term especially if the expression to be simplified consists of non-trivial combinations of symbols.

The basic idea behind the simplification algorithms – in both the linear and nonlinear case – can be outlined as follows: starting with a symbolic equation system F describing the circuit's behavior, the user chooses one or more numerical reference solutions f_i as well as an error bound ε. The algorithms then apply symbolic simplifications to the system (e.g. the deletion of an entire expression in a sum) and solve this simplified system numerically. The hereby obtained solutions $\tilde{f}_i$ are compared to the reference solutions using an appropriate error norm: $\delta_i = \|f_i - \tilde{f}_i\|$. If the error bound is exceeded, i.e. $\min \delta_i > \varepsilon$, the simplification is undone. This is repeated until no more simplifications are possible without a violation of the error bound and the simplified symbolic system $\widetilde{F}$ is returned.

The simplification algorithms assure that the numerical behavior (with respect to the chosen references f_i) of the simplified system coincides with that of the original system within the user-given error bound. Depending on the analysis task, the reference solutions f_i can for example be a numerical transfer function, its poles and zeros, or a time-dependent solution.

Note: We want to point out that in our context *approximation* has nothing to do with common approximation techniques like Taylor series, Chebyshev polynomials, or numerical fitting. This is due to the fact that using symbolic approximation techniques the equation system itself is modified to reduce its complexity rather than being replaced by a different (unrelated) formulation. Moreover, the result is given in symbolic (parameterized) form.

The different simplification techniques are described in Section 2 for linear circuits and in Section 3 for nonlinear circuits.

1.2. Ranking Methods

The order in which to simplify terms from the equation system is one of the crucial points: It is quite clear that those terms should be simplified first which have only a minor influence on the output behavior. Terms with a large influence should not be removed at all. To achieve a maximum number of simplifications and to avoid unnecessary modifications an optimized order, the so called *ranking*, should be used. For this, a ranking algorithm is needed which predicts the influence on the output a modification would cause. As the number of possible simplifications is very large it is inconvenient to exactly compute the influence and therefore estimation methods have to be used. The design of a good ranking algorithm is a trade off between accurate error prediction and computational efficiency.

1.3. Mathematical Background

The equation system describing the behavior of an analog circuit consists of equations originating from Kirchhoff's current and voltage laws as well as of the circuit element characteristics. The latter can be linear equations (like $U = R \cdot I$ for linear resistances), differential equations (like $I = C \cdot U'$ for linear capacitances), nonlinear equations (like $I = I_s(e^{U/nU_T} - 1)$ for diodes), or even a set of nonlinear equations and differential equations (e.g. the Gummel-Poon model for BJT transistors). It can be set up automatically using standard formulation methods such as the Modified Nodal Analysis or the Sparse Tableau Analysis. In general, the circuit equations are given by a differential-algebraic equation system (DAE system)

$$F = (f, g) = 0 \ , \tag{2}$$

where

$$f(x(t), x'(t), y(t), u(t)\,; p) = 0 \quad \text{for all } t \in I \tag{3}$$

$$g(x(t), y(t), u(t)\,; p) = 0 \quad \text{for all } t \in I \ . \tag{4}$$

Here, $u : \mathbb{R} \to \mathbb{R}^r$ denotes the inputs, $x = (v, i) : \mathbb{R} \to \mathbb{R}^k$ denotes the vector of dependent variables (usually voltages and currents), $y : \mathbb{R} \to \mathbb{R}^s$ denotes the outputs, and $I \subset \mathbb{R}$ denotes a time interval. Since we are working with symbolic equations, F is parameterized by symbolic element parameters $p = (p_1, \dots, p_N)$ (like a resistor value R_1, a voltage source value V_0, or a transistor parameter β_F).

If all circuit element characteristics are given by linear equations, the resulting DAE system can be Laplace transformed to the frequency domain resulting in a linear equation system formulated in the Laplace frequency variable s:

$$A(s\,;p)\hat{x} = b(\hat{u}, s\,;p) \ . \tag{5}$$

From this, the transfer function $H(s\,;p)$ can be computed.

Up to now, the equations are formulated symbolically using the vector p of symbolic element parameters. As motivated in Section 1.1, simplification of symbolic equations is not possible without additional numerical knowledge, because the DAE system (2) (or the linear system (5) respectively) has to be solved numerically as well. For this, one needs numerical values $\pi \in \mathbb{R}^N$ for the symbolic parameters p which have to be inserted into the equation system. The vector π is called *design point.*

The following standard numerical methods are applied to analyze the behavior of analog circuits: The *transient analysis* computes the time-dependent solution $(x(t), y(t))$, such that

$$F(x(t), x'(t), y(t), u(t)\,;\pi) = 0 \quad \text{for all } t \in T \tag{6}$$

for a given input signal $u(t)$. Omitting the dynamic behavior, the *DC analysis* computes the solution (x, y) of

$$F_{DC}(x, y, u\,;\pi) = F(x, 0, y, u\,;\pi) = 0 \quad \text{for an appropriate } x \in \mathbb{R}^k \tag{7}$$

for a given input value u. If several input values are given, we speak of a *DC-transfer analysis* or *DT analysis.* In the linear case, the *AC analysis* is used which computes magnitude and phase of the transfer function $H(s\,;p)$ for given frequency points ω_i along the imaginary axis:

$$h_i = H(j\omega_i\,;\pi) \quad \text{for all } i \ . \tag{8}$$

2. Linear Symbolic Analysis

The transfer function is the main object of interest in linear symbolic analysis. It allows for obtaining insights into the circuit's behavior and parameter dependencies. By post-processing the transfer function one can for example symbolically compute its poles and zeros to investigate the circuit stability. As Section 1 showed, this can only be achieved using symbolic simplification techniques.

The research on this topic started in the early 1990's (see for example Gielen and Sansen (1991) or Sommer (1994)). Basically, one distinguishes three types of linear simplification methods: Simplification Before Generation methods (SBG) simplify the matrix equations before computing the transfer function. Simplification During Generation methods (SDG) apply simplifications during the process of transfer function calculation. Simplification After Generation methods (SAG) simplify the transfer function directly. In the following, we will describe SAG and SBG methods only.

2.1. Simplification After Generation

This simplification technique (Gielen and Sansen, 1991) is based on the manipulation of the symbolic transfer function given as a rational expression

$$H(s\,;p) = \frac{\sum a_i(p)\, s^i}{\sum b_i(p)\, s^i} \ , \tag{9}$$

where the coefficients a_i and b_i are symbolic functions of the parameter vector p given in canonical sum-of-products form:

$$a_i(p) = \sum_j a_{ij}(p) \ , \quad b_i(p) = \sum_j b_{ij}(p) \ . \tag{10}$$

For a given error bound, those terms a_{ij} and b_{ij} are removed from the transfer function which cause a negligible deviation on a_i and b_i, respectively. By this, one can drastically reduce the symbolic complexity of the transfer function. On the other hand, it is not possible to achieve an order reduction of $H(s\,;p)$ with respect to s.

2.2. Simplification Before Generation

Even for circuits of small size it is not possible to calculate the full symbolic transfer function (9). For example, the $\mu A741$ operational amplifier yields a transfer function whose expanded denominator consists of more than 10^{34} terms (Hennig, 2000). Thus, the linear equation system itself has to be simplified before computing the symbolic transfer function. This can be done by rewriting each entry of the system matrix A (equation (5)) in sum-of-products form and sequentially removing terms from the matrix. The error is checked by computing the magnitude and phase of the (numerical) transfer function at certain frequency points. SBG methods reduce both the complexity of the transfer function as well as its polynomial order.

For SBG techniques a ranking method has been developed (Hennig, 2000) which makes use of the Sherman-Morrison formula. It allows for computing the influences for all terms in the matrix with a computational effort equivalent to that of a single numerical matrix inversion: the deviation $\Delta\hat{x}_k$ on the k-th component of the solution vector $\hat{x}$ when removing a term λ at position (i, j) from the matrix $A = (a_{ij})$ is given by

$$\Delta\hat{x}_k(\lambda) = \frac{\lambda\, a_{ki}^{-1}}{1 - \lambda\, a_{ji}^{-1}} \cdot \hat{x}_j \ . \tag{11}$$

2.3. Poles and Zeros

Symbolic calculation of poles and zeros plays an important role in the analysis of feedback amplifiers, but the extraction of symbolic expressions for poles and zeros is rarely possible without simplifications. In Hennig (2000), a matrix-based Simplification Before Generation method for direct approximation of a linear system with respect to a selected eigenvalue of a generalized eigenvalue problem

$$(A - \lambda B)\, u \;=\; 0 \tag{12}$$

$$v^H (A - \lambda B) \;=\; 0 \tag{13}$$

was presented. By means of eigenvalue sensitivity the symbolic parameters with negligible influence on the eigenvalue are discarded from the linear system resulting in a simplified generalized eigenvalue problem whose determinant yields a reduced-order approximation of the characteristic polynomial. To detect potentially false eigenvalue pairings during approximation, the modal assurance criterion (Friswell and Mottershead, 1995) is applied, which constitutes a measure for the correlation of two eigenvectors u_1 and u_2 and which is defined as

$$\mathrm{MAC}(u_1, u_2) = \frac{|u_1^H u_2|^2}{(u_1^H u_1)(u_2^H u_2)} \quad . \tag{14}$$

Since in this context one is interested in a single eigenvalue only, it is appropriate to use an iterative generalized eigenvalue problem solver like the Jacobi orthogonal correction method (Sleijpen *et al.*, 1996) instead of the QZ algorithm. As an additional benefit, the modal assurance criterion can be integrated within the Jacobi correction iteration. This results in a very efficient and reliable approximation method for the extraction of approximated symbolic poles and zeros. The application of the method will be shown in the section below.

2.4. Industrial Application

The application of the pole/zero extraction algorithm will be demonstrated on the CMOS folded-cascode operational amplifier shown in Figure 3. The frequency response of the operational amplifier's open-loop differential-mode voltage gain (see solid curve in Figure 4) shows a peak near 10 MHz, caused by a parasitic complex pole pair close to the imaginary axis. The analysis task is now to extract a symbolic expression for the parasitic pole pair which allows to determine those circuit parameters which have a dominant influence on the peak.

Using a SPICE Level 3 AC model for the MOS devices (Gray and Meyer, 1993) yields a system of 29 equations. The differential-mode voltage transfer function has 19 poles and 19 zeros and contains more than 5×10^{19} product terms. The symbolic approximation routines are applied to extract the parasitic pole pair at $s_p = (-2.1 \pm 8.3j) \times 10^7$ using a relative error bound $\varepsilon = 0.1$. The resulting simplified equation system can be algebraically reduced to a system of dimension 4 from which the wanted pole pair $s_p^{1,2}$ can be easily computed to the

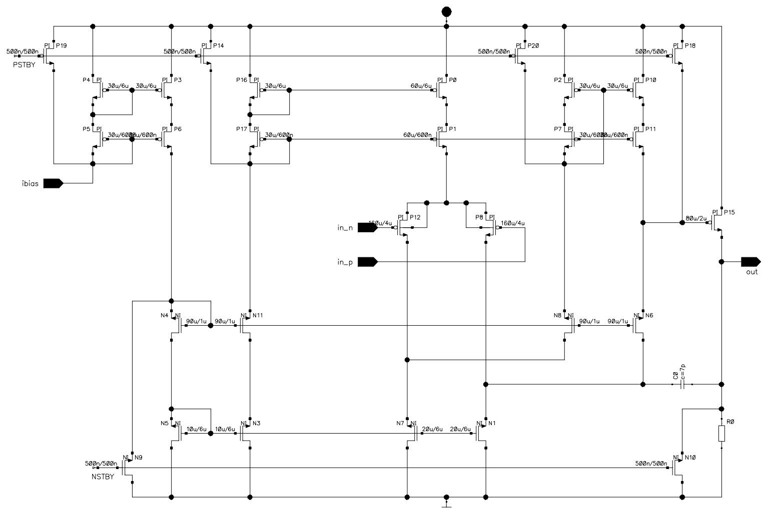

Figure 3: CMOS folded-cascode operational amplifier.

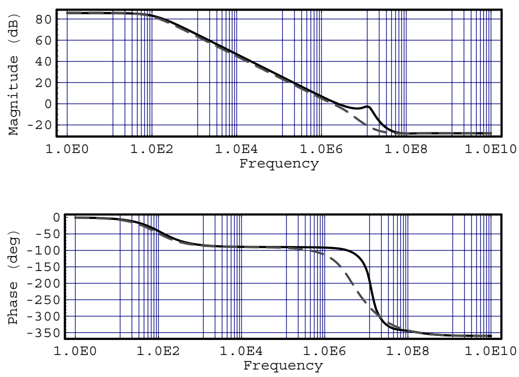

Figure 4: Original (solid) and compensated (dashed) open-loop voltage gain.

$$s_p{}^{1,2} == -\frac{(\mathrm{CC0}+\mathrm{CL})\ \mathrm{gm\$MN6}}{2\ \mathrm{CC0}\ \mathrm{CL}} \pm \frac{\sqrt{\mathrm{Cgs\$MP15}\ \mathrm{gm\$MN6}\ (\mathrm{Cgs\$MP15}\ (\mathrm{CC0}+\mathrm{CL})^2\ \mathrm{gm\$MN6} - 4\ \mathrm{CC0}^2\ \mathrm{CL}\ \mathrm{gm\$MP15})}}{2\ \mathrm{CC0}\ \mathrm{Cgs\$MP15}\ \mathrm{CL}}$$

Figure 5: Computed formula for the complex pole pair.

expression shown in Figure 5. The overall computation time (including netlist import and equation setup) to approximate the equation system and to extract this formula is about 8 seconds running the routines under Mathematica 4.0 on an AMD Athlon 1200 with 512 MB memory. By interpretation of the computed formula for the complex pole pair it turns out that an increased value for the gate-source capacitance `Cgs$MP15` of the transistor MP15 allows for decreasing the imaginary parts of the pole pair. As a consequence one could add an additional capacitor between the gate and the source terminals of the corresponding transistor and by that compensate the voltage gain of the operational amplifier (see dashed curve in Figure 4).

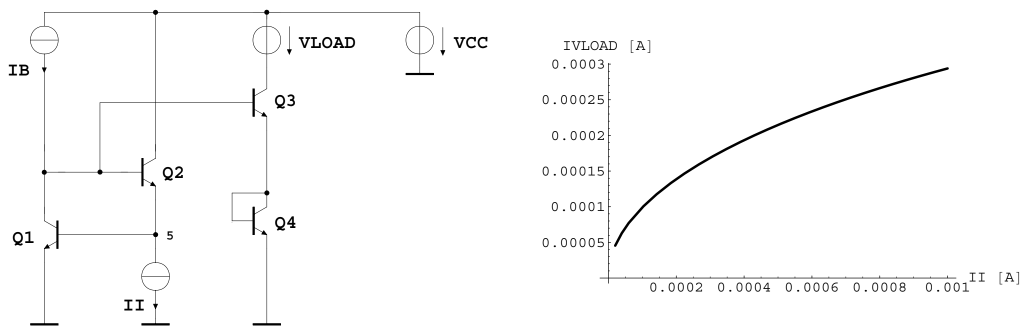

Figure 6: Schematic and DT characteristic of the square root function block.

3. Nonlinear Symbolic Analysis

In the previous section we presented simplification methods for linear analog circuits which have been successfully applied to industrial applications for several years. Now the question arises if these methods can be adapted to nonlinear circuits as well. As we will see in this section the answer is yes, but in general we can not expect to achieve explicit and interpretable formulas for the output variables as in the linear case. Research on this topic started a few years ago and is still in progress (Borchers (1998), Popp *et al.* (1998), Wichmann *et al.* (1999), Wichmann (2001)). Before explaining the nonlinear simplification methods we will demonstrate their application on an academic example circuit.

3.1. Academic Application

In Figure 6, a nonlinear analog circuit consisting of four bipolar transistors is shown. The input is given by the current `II`, the output is given by the current $I_{\texttt{VLOAD}}$ through the independent voltage source `VLOAD`. The circuit is known as *square root function block* since the output current is given by the square root of the input current. Performing a DT analysis by varying the input current from $20\,\mu$A to 1 mA results in an output current as shown in Figure 6. Text books on analog circuit design state the following approximate formula for the static input/output behavior (see for example Gray and Meyer (1993)):

$$I_0 = \sqrt{I_I}\,\sqrt{I_B}\,\sqrt{\frac{Is_3\,Is_4}{Is_1\,Is_2}}\ . \tag{15}$$

This formula was computed using hand approximations based on design knowledge. In the following we apply automated symbolic methods to analyze the square root function block.

Using the Gummel-Poon transistor model (Gray and Meyer, 1993) for the bipolar transistors, the behavior of the circuit can be described by a nonlinear equation system consisting of 27 equations and variables as well as 21 parameters. In total the equations comprise 113 terms (see Figure 7). Note that the equation

$$
\begin{aligned}
&\{\mathtt{I\$CS\$Q2} + \mathtt{I\$VDC} + \mathtt{I\$VLOAD} == -\mathtt{IB},\ \mathtt{I\$BS\$Q2} + \mathtt{I\$BS\$Q3} + \mathtt{I\$CS\$Q1} == \mathtt{IB},\ \mathtt{I\$BS\$Q4} + \mathtt{I\$CS\$Q4} + \mathtt{I\$ES\$Q3} == 0,\\
&\ \mathtt{I\$BS\$Q1} + \mathtt{I\$ES\$Q2} == -\mathtt{II},\ \mathtt{I\$CS\$Q3} - \mathtt{I\$VLOAD} == 0,\ -\mathtt{I\$BS\$Q1} - \mathtt{I\$CS\$Q1} - \mathtt{I\$ES\$Q1} == 0,\\
&\ \mathtt{I\$BS\$Q1} == \mathtt{ib\$Q1},\ -\mathtt{I\$BS\$Q1} - \mathtt{I\$ES\$Q1} == \mathtt{ic\$Q1},\ \mathtt{ic\$Q1} == \mathtt{AREA\$Q1}\,(-1 + e^{38.6635\,\mathtt{V\$5}})\,\mathtt{IS\$Q1} +\\
&\quad \mathtt{GMIN}\,\mathtt{V\$5} - \left(1 + \frac{1}{\mathtt{BR\$Q1}}\right)(\mathtt{AREA\$Q1}\,(-1 + e^{38.6635\,(-\mathtt{V\$3}+\mathtt{V\$5})})\,\mathtt{IS\$Q1} + \mathtt{GMIN}\,(-\mathtt{V\$3} + \mathtt{V\$5})),\\
&\ \mathtt{ib\$Q1} == \frac{\mathtt{AREA\$Q1}\,(-1 + e^{38.6635\,\mathtt{V\$5}})\,\mathtt{IS\$Q1} + \mathtt{GMIN}\,\mathtt{V\$5}}{\mathtt{BF\$Q1}} + \frac{\mathtt{AREA\$Q1}\,(-1 + e^{38.6635\,(-\mathtt{V\$3}+\mathtt{V\$5})})\,\mathtt{IS\$Q1} + \mathtt{GMIN}\,(-\mathtt{V\$3} + \mathtt{V\$5})}{\mathtt{BR\$Q1}},\\
&\ -\mathtt{I\$BS\$Q2} - \mathtt{I\$CS\$Q2} - \mathtt{I\$ES\$Q2} == 0,\ \mathtt{I\$BS\$Q2} == \mathtt{ib\$Q2},\\
&\ -\mathtt{I\$BS\$Q2} - \mathtt{I\$ES\$Q2} == \mathtt{ic\$Q2},\ \mathtt{ic\$Q2} == \mathtt{AREA\$Q2}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$5})})\,\mathtt{IS\$Q2} -\\
&\quad \left(1 + \frac{1}{\mathtt{BR\$Q2}}\right)(\mathtt{AREA\$Q2}\,(-1 + e^{38.6635\,(-\mathtt{V\$1}+\mathtt{V\$3})})\,\mathtt{IS\$Q2} + \mathtt{GMIN}\,(-\mathtt{V\$1} + \mathtt{V\$3})) + \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$5}),\\
&\ \mathtt{ib\$Q2} == \frac{\mathtt{AREA\$Q2}\,(-1 + e^{38.6635\,(-\mathtt{V\$1}+\mathtt{V\$3})})\,\mathtt{IS\$Q2} + \mathtt{GMIN}\,(-\mathtt{V\$1} + \mathtt{V\$3})}{\mathtt{BR\$Q2}} +\\
&\quad \frac{\mathtt{AREA\$Q2}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$5})})\,\mathtt{IS\$Q2} + \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$5})}{\mathtt{BF\$Q2}},\ -\mathtt{I\$BS\$Q3} - \mathtt{I\$CS\$Q3} - \mathtt{I\$ES\$Q3} == 0,\\
&\ \mathtt{I\$BS\$Q3} == \mathtt{ib\$Q3},\ -\mathtt{I\$BS\$Q3} - \mathtt{I\$ES\$Q3} == \mathtt{ic\$Q3},\ \mathtt{ic\$Q3} == \mathtt{AREA\$Q3}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$4})})\,\mathtt{IS\$Q3} +\\
&\quad \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$4}) - \left(1 + \frac{1}{\mathtt{BR\$Q3}}\right)(\mathtt{AREA\$Q3}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$OUT})})\,\mathtt{IS\$Q3} + \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$OUT})),\\
&\ \mathtt{ib\$Q3} == \frac{\mathtt{AREA\$Q3}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$4})})\,\mathtt{IS\$Q3} + \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$4})}{\mathtt{BF\$Q3}} +\\
&\quad \frac{\mathtt{AREA\$Q3}\,(-1 + e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$OUT})})\,\mathtt{IS\$Q3} + \mathtt{GMIN}\,(\mathtt{V\$3} - \mathtt{V\$OUT})}{\mathtt{BR\$Q3}},\\
&\ -\mathtt{I\$BS\$Q4} - \mathtt{I\$CS\$Q4} - \mathtt{I\$ES\$Q4} == 0,\ \mathtt{I\$BS\$Q4} == \mathtt{ib\$Q4},\ -\mathtt{I\$BS\$Q4} - \mathtt{I\$ES\$Q4} == \mathtt{ic\$Q4},\\
&\ \mathtt{ic\$Q4} == 0.\ \mathtt{AREA\$Q4}\left(1 + \frac{1}{\mathtt{BR\$Q4}}\right)\mathtt{IS\$Q4} + \mathtt{AREA\$Q4}\,(-1 + e^{38.6635\,\mathtt{V\$4}})\,\mathtt{IS\$Q4} + \mathtt{GMIN}\,\mathtt{V\$4},\\
&\ \mathtt{ib\$Q4} == \frac{0.\ \mathtt{AREA\$Q4}\,\mathtt{IS\$Q4}}{\mathtt{BR\$Q4}} + \frac{\mathtt{AREA\$Q4}\,(-1 + e^{38.6635\,\mathtt{V\$4}})\,\mathtt{IS\$Q4} + \mathtt{GMIN}\,\mathtt{V\$4}}{\mathtt{BF\$Q4}},\ \mathtt{V\$1} - \mathtt{V\$OUT} == \mathtt{VLOAD},\ \mathtt{V\$1} == \mathtt{VDC}\}
\end{aligned}
$$

Figure 7: Original equation system of the square root function block.

$$
\begin{aligned}
&\{-\mathtt{II} + \mathtt{AREA\$Q2}\,e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$5})}\,\mathtt{IS\$Q2} == 0,\\
&\ \mathtt{IB} - \mathtt{AREA\$Q1}\,e^{38.6635\,\mathtt{V\$5}}\,\mathtt{IS\$Q1} == 0,\ -\mathtt{AREA\$Q3}\,e^{38.6635\,(\mathtt{V\$3}-\mathtt{V\$4})}\,\mathtt{IS\$Q3} + \mathtt{I\$VLOAD} == 0,\\
&\ \mathtt{IB} - \mathtt{AREA\$Q1}\,e^{38.6635\,\mathtt{V\$5}}\,\mathtt{IS\$Q1} - \mathtt{AREA\$Q4}\,e^{38.6635\,\mathtt{V\$4}}\,\mathtt{IS\$Q4} + \mathtt{I\$VLOAD} == 0\}
\end{aligned}
$$

Figure 8: Simplified equation system of the square root function block.

$$
\mathtt{I\$VLOAD[II]} == \sqrt{\frac{\mathtt{AREA\$Q3}\ \mathtt{AREA\$Q4}}{\mathtt{AREA\$Q1}\ \mathtt{AREA\$Q2}}}\ \sqrt{\frac{\mathtt{IS\$Q3}\ \mathtt{IS\$Q4}}{\mathtt{IS\$Q1}\ \mathtt{IS\$Q2}}}\ \sqrt{\mathtt{IB}}\ \sqrt{\mathtt{II}}
$$

Figure 9: Explicit solution of output current.

system does not involve dynamic expressions since we analyze the static behavior of the circuit.

Obviously, the equation system is too complex to allow a deeper understanding of the circuit. It is not possible to explicitly solve for the output variable. For this reason we apply nonlinear simplification techniques (which will be described below) with a maximum error $\varepsilon = 25\,\mu\mathrm{A}$ within the given range for `II`. The algorithm returns an equation system of only 4 equations and variables, 10 parameters, and a total number of 10 terms (see Figure 8).

This equation system is simple enough to be explicitly solved for the output current using standard symbolic solving techniques. The resulting expression is shown in Figure 9. The overall computation time (including netlist import and equation setup) to approximate the equation system and to extract this formula is about 6 seconds running the routines under Mathematica 4.0 on an AMD Athlon 1200 with 512 MB memory.

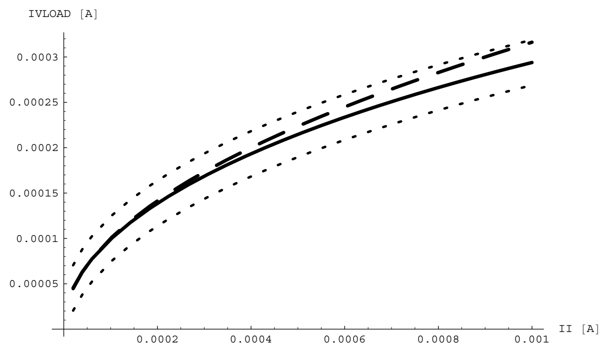

Figure 10: DT solution of original (solid) and simplified (dashed) equations.

Using the symbolic approximation algorithm it was possible to automatically derive exactly the text book formula (15) – without any circuit design knowledge and with assured error control. Note that in (15) the area parameters of the transistors are included in the saturation current parameters $Is_1, \ldots, Is_4$. To verify that the error bound was met, Figure 10 shows the numerical solution of both the original equations and the approximated formula.

3.2. Simplification Techniques

As already mentioned, in general we can not expect to obtain explicit formulas for nonlinear circuits. In contrast to linear symbolic analysis, nonlinear simplification techniques are therefore mainly used for automated behavioral model generation. However, the square root function block showed that for small circuits it can be possible to obtain an interpretable result with assured error control (compared to hand calculations).

Behavioral model generation is a technique for speeding up numerical simulation of large circuits. The idea is to replace each frequently used subblocks in the circuit by a single simplified model description and by that reducing the complexity of the whole circuit and decreasing the computation time. Nonlinear simplifications can be used to automatically generate behavioral models from netlist descriptions of the subblocks (Näthke *et al.*, 2002).

To simplify the notation given in Section 1.3, we will not distinguish input, state, and output variables and suppress the symbolic parameters p, thus equations (3) and (4) simplify to

$$f(x, x') = 0 \tag{16}$$
$$g(x) = 0 \ . \tag{17}$$

Moreover, in our context it is often sufficient to assume the following structure for F:

$$F(x, x') = A(x)x' + h(x) \ , \tag{18}$$

where A is a matrix-valued function and $A(x)$ is singular. DAE systems of this form are often called *quasilinear.*

The general outline of the nonlinear simplification algorithms is described in Section 1.1. The following simplification techniques have been developed to reduce the complexity of nonlinear DAE systems:

Elimination. This simplification technique attempts to solve equations explicitly for one variable in order to substitute this variable in the remaining system, thus reducing the number of equations. Additionally, those equations can be removed from the system which are not needed to solve for the given output variables. Elimination is a mathematically exact operation, therefore no error calculation is necessary.

Deletion of terms. Written in expanded sum-of-products form, each equation of the DAE system consists of a large number of terms. But often only some terms contribute significantly to the overall sum whilst the others can be omitted without a major influence on the result. Those "small" terms can therefore be neglected within a certain error bound. We call this procedure *deletion of terms.*

Substitution of terms by constant values. Deleting a term can also be interpreted as substituting this term by the constant expression 0. Of course, a term could also be substituted by any other nonzero value. For example, a variable which denotes the node voltage not lying on the signal path can be replaced by the constant supply-voltage value. This technique is called *substitution of terms by constant values.*

Simplification of piecewise functions. Nearly all existing transistor models are defined using piecewise functions in order to model the different operating regions of a transistor. But usually a certain transistor within a circuit is driven in one operating region only (depending on the range of the input signals). Thus, branches of piecewise functions which model unused operating regions can be omitted without error (again depending on the range of the input signals).

And others. Of course, there is a wide range of additional simplification methods, like linearization or power expansion of terms. But in our implementations and applications it turned out that the above mentioned techniques are usually the most effective ones.

3.3. Nonlinear Ranking Methods

In principle, nonlinear ranking methods have to be designed for each simplification method and each analysis method separately. In the following we will briefly describe two ranking methods for the deletion of terms when using DC and transient simulation. Additional ranking methods can be found for example in Popp *et al.* (1998) and Wichmann *et al.* (1999).

DC ranking. Given a DAE system F and a simplified system $\widetilde{F}$ the task of the DC ranking is to estimate the distance $\varepsilon = \|x - \tilde{x}\|$, where x and $\tilde{x}$ are the solutions of F_{DC} and $\widetilde{F}_{\mathrm{DC}}$, without calculating $\widetilde{x}$. Usually, the DC solution is computed using a Newton iteration starting in an initial guess. The idea of the DC ranking is now to estimate $\tilde{x}$ and thus ε (since x is known) by computing only the first step of a Newton iteration on $\widetilde{F}_{\mathrm{DC}}$ starting in x:

$$x_1 = x - J_{\widetilde{F}}(x)^{-1}\widetilde{F}(x) \ . \tag{19}$$

Using the Sherman-Morrison formula for the inverse of the Jacobian, this first step can be computed quite efficiently. Details of the DC ranking method can be found in Wichmann *et al.* (1999). In our examples it turned out that the DC ranking is on the one hand fast to compute and on the other hand the norm $\|x_1 - x\|$ yields satisfactory estimations for ε.

Transient ranking by one-step solver. Roughly speaking, the transient solution of a DAE system F is computed as follows: At a time point t_i, the differentials are replaced by a finite difference expression resulting in a static nonlinear system F_{S} which is solved using Newton iteration. Then, a new time step t_{i+1} is computed usually by some sort of time-step control. This is repeated until the upper time limit is reached. We modify this general procedure in two ways: First, we turn off the time-step control but compute the solutions exactly at those time points which were used to calculate the original solution $x(t_i)$ of F. Secondly, we do not perform a complete Newton iteration at each time instance but stop after the first iteration, starting in the original system's solution at this time point (similar to the DC ranking described above):

$$x_1(t_i) = x(t_i) - J_{\widetilde{F}_{\mathrm{S}}}(x(t_i))^{-1}\widetilde{F}_{\mathrm{S}}(x(t_i)) \ . \tag{20}$$

This results in estimations $x_1(t_i)$ for the true solution at each time instance which are finally interpolated to yield the overall estimation of the transient solution. In our applications this ranking method yields satisfactory error estimates with moderate computational effort.

3.4. Index Calculation

The index plays an important role in the theory of DAE systems. Roughly speaking, it describes the "distance" (given by an integer value) of a DAE system from being an ODE system. For an introduction to DAE systems and the index concept we refer to Brenan *et al.* (1989). It is well known that numerical solving of systems with an index higher than 1 is an ill-posed problem, thus higher-index systems should be avoided. However, during symbolic simplification of the nonlinear DAE system the index may increase. As a consequence we want to observe possible index changes during simplification.

There are a number of different index concepts in the literature, for example the differential index (Gear, 1988), the geometrical index (Reich, 1990), the perturbation index (Hairer *et al.*, 1989), the tractability index (Griepentrog and März, 1986), the strangeness index (Kunkel and Mehrmann, 1998), or the involutive index (Hausdorf and Seiler, 2001). In the following, we will concentrate on the tractability index.

The tractability index is based on the linearization of a DAE system and requires only weak smoothness assumptions. The index-1 case is defined for quasilinear systems in the following way:

Definition: Let $F(x,x') = A(x)x' + h(x)$ where A is a matrix-valued function and $A(x)$ is singular for each x and of constant rank. Furthermore, we assume that $N = \ker A(x)$ does not depend on x. Let $B(x,x') = D_x F$ and let Q denote a projector onto N. Then the DAE system F is called *index-1 tractable* (or *transferable*) if the matrix $A(x) + B(x,x')Q$ is regular.

In Schwarz and Tischendorf (2000) structural properties have been stated an electrical network has to fulfill in order to assure transferability. By means of C-V-loops and L-I-cutsets it is possible to predict the index based on the network topology without analyzing the equation system. But this result is not suitable for our algorithm: After some simplification steps the resulting system of equations may not be re-interpreted as an electrical network. For example, Kirchhoff's current law may be violated for some nodes. Therefore, in our case the index has to be computed based on the equation system.

According to the definition this requires two steps: calculation of the projector matrix Q and the singularity test on the matrix $A_1 = A(x) + B(x,x')Q$. The Gram-Schmidt orthonormalization or the singular value decomposition can for example be used to calculate kernel projectors. For this, let $A \in \mathbb{R}^{n\times n}$ be singular. Using Gram-Schmidt orthonormalization we compute a decomposition of A such that

$$(A^T\ I) = (V_1 \,|\, V_2)\, R\,, \tag{21}$$

where the non-zero columns of $V = (V_1 \,|\, V_2) \in \mathbb{R}^{n\times 2n}$ are orthonormal and $R \in \mathbb{R}^{2n\times 2n}$ is upper triangular. Then

$$Q = V_2\, V_2^T \tag{22}$$

is a projector onto $\ker A$. Alternatively, let the singular value decomposition of A be given by

$$A = U^T \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}\,, \tag{23}$$

where $U \in \mathbb{R}^{n\times n}$, $\Sigma = \mathrm{diag}(\sigma_1,\ldots,\sigma_l) \in \mathbb{R}^{l\times l}$, $V_1 \in \mathbb{R}^{l\times n}$, $V_2 \in \mathbb{R}^{(n-l)\times n}$ and $\sigma_i > 0$ for all i. Then

$$Q = V_2^T\, V_2 \tag{24}$$

is a projector onto $\ker A$. Both statements can be easily seen using the properties of the Gram-Schmidt orthonormalization and the singular value decomposition.

In our applications it turned out that the singular value decomposition yields the best numerical results for computing the tractability index. The Gram-Schmidt orthonormalization can also be used to calculate the index condition symbolically, but the resulting expressions are too complex even for small circuits. To check the singularity of A_1, again the singular value decomposition yields the most reliable results.

Currently, we are working on an implementation of the strangeness index as described in (Kunkel and Mehrmann, 1998). The strangeness index is well suited for being computed numerically and is closely coupled with the numerical integration method. We are going to analyze how the index computation can be combined with the one-step solver ranking. Additionally, research is in progress for computing the differential and involutive index using Groebner base techniques.

4. Conclusions

We have presented an overview on symbolic techniques for the analysis and design of analog circuits. It was motivated that due to the complexity problem simplification methods are indispensable for handling industrial-sized problems. The basic ideas behind these simplification methods – a combination of symbolic and numeric algorithms – have been shown for the linear and the nonlinear case.

The described simplification techniques are integrated in the software package *Analog Insydes* (Halfmann *et al.* (2001), `www.analog-insydes.de`). Analog Insydes is an add-on package to the computer-algebra system Mathematica (Wolfram, 1999) for the analysis, modeling, sizing, and optimization of linear and nonlinear circuits of industrial size. Based on a hierarchical netlist-description language, symbolic circuit equations can be set up automatically. Besides the simplification algorithms, standard electrical engineering analysis and graphics methods are available within Analog Insydes. For being accepted by circuit designers the tool had to be integrated into industrial design frameworks and therefore several interfaces have been established to exchange data with commercial circuit simulators like PSpice™, Eldo™, or Saber™.

During several years of application the symbolic simplification algorithms have proven to be applicable to industrial-sized problems and by that making symbolic analysis a powerful technique in industrial analog circuit design.

Acknowledgments

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 pages, 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 pages, 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 pages, 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 pages, 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 pages, 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way. The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the EleyRideal and LangmuirHinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 pages, 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multiphase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the nonconvolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stressfield from known properties of the components. This is done by the extension of the asymptotic homogenization technique known for pure elastic nonhomogeneous bodies to the nonhomogeneous thermoviscoelasticity of the integral noncon-

volution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type. SanchezPalencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integralmodeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the spaceoperator kernels.
(20 pages, 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding “good” cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time. In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely. If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems. Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton’s intersection formulae and Hadwiger’s recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 pages, 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 pages, 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell’s Corpuscle Problem

Wicksell’s corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell’s problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds

for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e.g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacita*ted Multiple Allocation Hub Location Problem*

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:
The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along pre- scribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions. After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords: facility location, software development, geographical information systems, supply chain management.
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation

A Grid free method for solving the Poisson equation is presented. This is an iterative method. The method is based on the weighted least squares approximation in which the Poisson equation is enforced to be satisfied in every iterations. The boundary conditions can also be enforced in the iteration process. This is a local approximation procedure. The Dirichlet, Neumann and mixed boundary value problems on a unit square are presented and the analytical solutions are compared with the exact solutions. Both solutions matched perfectly.
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier

Simulation of the fiber spinning process

To simulate the influence of process parameters to the melt spinning process a fiber model is used and coupled with CFD calculations of the quench air flow. In the fiber model energy, momentum and mass balance are solved for the polymer mass flow. To calculate the quench air the Lattice Boltzmann method is used. Simulations and experiments for different process parameters and hole configurations are compared and show a good agreement.
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis

On interaction of a liquid film with an obstacle

In this paper mathematical models for liquid films generated by impinging jets are discussed. Attention is stressed to the interaction of the liquid film with some obstacle. S. G. Taylor [Proc. R. Soc. London Ser. A 253, 313 (1959)] found that the liquid film generated by impinging jets is very sensitive to properties of the wire which was used as an obstacle. The aim of this presentation is to propose a modification of the Taylor's model, which allows to simulate the film shape in cases, when the angle between jets is different from 180°. Numerical results obtained by discussed models give two different shapes of the liquid film similar as in Taylors experiments. These two shapes depend on the regime: either droplets are produced close to the obstacle or not. The difference between two regimes becomes larger if the angle between jets decreases. Existence of such two regimes can be very essential for some applications of impinging jets, if the generated liquid film can have a contact with obstacles.
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner

Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids

The filling process of viscoplastic metal alloys and plastics in expanding cavities is modelled using the lattice Boltzmann method in two and three dimensions. These models combine the regularized Bingham model for viscoplastic with a free-interface algorithm. The latter is based on a modified immiscible lattice Boltzmann model in which one species is the fluid and the other one is considered as vacuum. The boundary conditions at the curved liquid-vacuum interface are met without any geometrical front reconstruction from a first-order Chapman-Enskog expansion. The numerical results obtained with these models are found in good agreement with available theoretical and numerical analysis.
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert

»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«

Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001

Was macht einen guten Hochschullehrer aus? Auf diese Frage gibt es sicher viele verschiedene, fachbezogene Antworten, aber auch ein paar allgemeine Gesichtspunkte: es bedarf der »Leidenschaft« für die Forschung (Max Weber), aus der dann auch die Begeisterung für die Lehre erwächst. Forschung und Lehre gehören zusammen, um die Wissenschaft als lebendiges Tun vermitteln zu können. Der Vortrag gibt Beispiele dafür, wie in angewandter Mathematik Forschungsaufgaben aus praktischen Alltagsproblemstellungen erwachsen, die in die Lehre auf verschiedenen Stufen (Gymnasium bis Graduiertenkolleg) einfließen; er leitet damit auch zu einem aktuellen Forschungsgebiet, der Mehrskalenanalyse mit ihren vielfältigen Anwendungen in Bildverarbeitung, Materialentwicklung und Strömungsmechanik über, was aber nur kurz gestreift wird. Mathematik erscheint hier als eine moderne Schlüsseltechnologie, die aber auch enge Beziehungen zu den Geistes- und Sozialwissenschaften hat.
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari

Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations

A Lagrangian particle scheme is applied to the projection method for the incompressible Navier-Stokes equations. The approximation of spatial derivatives is obtained by the weighted least squares method. The pressure Poisson equation is solved by a local iterative procedure with the help of the least squares method. Numerical tests are performed for two dimensional cases. The Couette flow, Poiseuelle flow, decaying shear flow and the driven cavity flow are presented. The numerical solutions are obtained for stationary as well as instationary cases and are compared with the analytical solutions for channel flows. Finally, the driven cavity in a unit square is considered and the stationary solution obtained from this scheme is compared with that from the finite element method.
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel

Optimal Portfolios with Fixed Consumption or Income Streams

We consider some portfolio optimisation problems where either the investor has a desire for an a priori specified consumption stream or/and follows a deterministic pay in scheme while also trying to maximize expected utility from final wealth. We derive explicit closed form solutions for continuous and discrete monetary streams. The mathematical method used is classical stochastic control theory.
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel

Optimal portfolios with a loan dependent credit spread

If an investor borrows money he generally has to pay higher interest rates than he would have received, if he had put his funds on a savings account. The classical model of continuous time portfolio optimisation ignores this effect. Since there is obviously a connection between the default probability and the total percentage of wealth, which the investor is in debt, we study portfolio optimisation with a control dependent interest rate. Assuming a logarithmic and a power utility function, respectively, we prove explicit formulae of the optimal control.
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz

The Euler number of discretized sets - on the choice of adjacency in homogeneous lattices

Two approaches for determining the Euler-Poincaré characteristic of a set observed on lattice points are considered in the context of image analysis { the integral geometric and the polyhedral approach. Information about the set is assumed to be available on lattice points only. In order to retain properties of the Euler number and to provide a good approximation of the true Euler number of the original set in the Euclidean space, the appropriate choice of adjacency in the lattice for the set and its background is crucial. Adjacencies are defined using tessellations of the whole space into polyhedrons. In R 3 , two new 14 adjacencies are introduced additionally to the well known 6 and 26 adjacencies. For the Euler number of a set and its complement, a consistency relation holds. Each of the pairs of adjacencies (14:1; 14:1), (14:2; 14:2), (6; 26), and (26; 6) is shown to be a pair of complementary adjacencies with respect to this relation. That is, the approximations of the Euler numbers are consistent if the set and its background (complement) are equipped with this pair of adjacencies. Furthermore, sufficient conditions for the correctness of the approximations of the Euler number are given. The analysis of selected microstructures and a simulation study illustrate how the estimated Euler number depends on the chosen adjacency. It also shows that there is not a uniquely best pair of adjacencies with respect to the estimation of the Euler number of a set in Euclidean space.
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

A generalized lattice Boltzmann model to simulate free-surface is constructed in both two and three dimensions. The proposed model satisfies the interfacial boundary conditions accurately. A distinctive feature of the model is that the collision processes is carried out only on the points occupied partially or fully by the fluid. To maintain a sharp interfacial front, the method includes an anti-diffusion algorithm. The unknown distribution functions at the interfacial region are constructed according to the first order Chapman-Enskog analysis. The interfacial boundary conditions are satis-

fied exactly by the coefficients in the Chapman-Enskog expansion. The distribution functions are naturally expressed in the local interfacial coordinates. The macroscopic quantities at the interface are extracted from the least-square solutions of a locally linearized system obtained from the known distribution functions. The proposed method does not require any geometric front construction and is robust for any interfacial topology. Simulation results of realistic filling process are presented: rectangular cavity in two dimensions and Hammer box, Campbell box, Sheffield box, and Motorblock in three dimensions. To enhance the stability at high Reynolds numbers, various upwind-type schemes are developed. Free-slip and no-slip boundary conditions are also discussed.
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

In the present paper a kinetic model for vehicular traffic leading to multivalued fundamental diagrams is developed and investigated in detail. For this model phase transitions can appear depending on the local density and velocity of the flow. A derivation of associated macroscopic traffic equations from the kinetic equation is given. Moreover, numerical experiments show the appearance of stop and go waves for highway traffic with a bottleneck.
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

To a network N(q) with determinant D(s;q) depending on a parameter vector q Î R^r via identification of some of its vertices, a network N^ (q) is assigned. The paper deals with procedures to find N^ (q), such that its determinant D^ (s;q) admits a factorization in the determinants of appropriate subnetworks, and with the estimation of the deviation of the zeros of D^ from the zeros of D. To solve the estimation problem state space methods are applied.
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

A spectral theory for stationary random closed sets is developed and provided with a sound mathematical basis. Definition and proof of existence of the Bartlett spectrum of a stationary random closed set as well as the proof of a Wiener-Khintchine theorem for the power spectrum are used to two ends: First, well known second order characteristics like the covariance can be estimated faster than usual via frequency space. Second, the Bartlett spectrum and the power spectrum can be used as second order characteristics in frequency space. Examples show, that in some cases information about the random closed set is easier to obtain from these characteristics in frequency space than from their real world counterparts.
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

We present a unified approach of several boundary conditions for lattice Boltzmann models. Its general framework is a generalization of previously introduced schemes such as the bounce-back rule, linear or quadratic interpolations, etc. The objectives are two fold: first to give theoretical tools to study the existing boundary conditions and their corresponding accuracy; secondly to design formally third- order accurate boundary conditions for general flows. Using these boundary conditions, Couette and Poiseuille flows are exact solution of the lattice Boltzmann models for a Reynolds number $Re = 0$ (Stokes limit).
Numerical comparisons are given for Stokes flows in periodic arrays of spheres and cylinders, linear periodic array of cylinders between moving plates and for Navier-Stokes flows in periodic arrays of cylinders for $Re < 200$. These results show a significant improvement of the overall accuracy when using the linear interpolations instead of the bounce-back reflection (up to an order of magnitude on the hydrodynamics fields). Further improvement is achieved with the new multi-reflection boundary conditions, reaching a level of accuracy close to the quasi-analytical reference solutions, even for rather modest grid resolutions and few points in the narrowest channels. More important, the pressure and velocity fields in the vicinity of the obstacles are much smoother with multi-reflection than with the other boundary conditions.
Finally the good stability of these schemes is highlighted by some simulations of moving obstacles: a cylinder between flat walls and a sphere in a cylinder.
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Im Rahmen dieser Arbeit soll eine elementar gehaltene Einführung in die Aufgabenstellungen und Prinzipien der modernen Finanzmathematik gegeben werden. Insbesondere werden die Grundlagen der Modellierung von Aktienkursen, der Bewertung von Optionen und der Portfolio-Optimierung vorgestellt. Natürlich können die verwendeten Methoden und die entwickelte Theorie nicht in voller Allgemeinheit für den Schulunterricht verwendet werden, doch sollen einzelne Prinzipien so heraus gearbeitet werden, dass sie auch an einfachen Beispielen verstanden werden können.
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

In this paper we consider short term storage systems. We analyze presorting strategies to improve the effiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assigment problem, i.e., it has an assigment problem kernel and some additional constraints. We present different types of these presorting problems, introduce mathematical programming formulations and prove the NP-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn

On the frame-invariant description of the phase space of the Folgar-Tucker equation

The Folgar-Tucker equation is used in flow simulations of fiber suspensions to predict fiber orientation depending on the local flow. In this paper, a complete, frame-invariant description of the phase space of this differential equation is presented for the first time.
Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel

A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects

In this article, we consider the problem of planning inspections and other tasks within a software development (SD) project with respect to the objectives quality (no. of defects), project duration, and costs. Based on a discrete-event simulation model of SD processes comprising the phases coding, inspection, test, and rework, we present a simplified formulation of the problem as a multiobjective optimization problem. For solving the problem (i.e. finding an approximation of the efficient set) we develop a multiobjective evolutionary algorithm. Details of the algorithm are discussed as well as results of its application to sample problems.
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
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43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus

Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem -

Radiation therapy planning is always a tight rope walk between dangerous insufficient dose in the target volume and life threatening overdosing of organs at risk. Finding ideal balances between these inherently contradictory goals challenges dosimetrists and physicians in their daily practice. Today's planning systems are typically based on a single evaluation function that measures the quality of a radiation treatment plan. Unfortunately, such a one dimensional approach can-

not satisfactorily map the different backgrounds of physicians and the patient dependent necessities. So, too often a time consuming iteration process between evaluation of dose distribution and redefinition of the evaluation function is needed.
In this paper we propose a generic multi-criteria approach based on Pareto's solution concept. For each entity of interest - target volume or organ at risk a structure dependent evaluation function is defined measuring deviations from ideal doses that are calculated from statistical functions. A reasonable bunch of clinically meaningful Pareto optimal solutions are stored in a data base, which can be interactively searched by physicians. The system guarantees dynamical planning as well as the discussion of tradeoffs between different entities.
Mathematically, we model the upcoming inverse problem as a multi-criteria linear programming problem. Because of the large scale nature of the problem it is not possible to solve the problem in a 3D-setting without adaptive reduction by appropriate approximation schemes.
Our approach is twofold: First, the discretization of the continuous problem is based on an adaptive hierarchical clustering process which is used for a local refinement of constraints during the optimization procedure. Second, the set of Pareto optimal solutions is approximated by an adaptive grid of representatives that are found by a hybrid process of calculating extreme compromises and interpolation methods.
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
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44. T. Halfmann, T. Wichmann

Overview of Symbolic Methods in Industrial Analog Circuit Design

Industrial analog circuits are usually designed using numerical simulation tools. To obtain a deeper circuit understanding, symbolic analysis techniques can additionally be applied. Approximation methods which reduce the complexity of symbolic expressions are needed in order to handle industrial-sized problems.
This paper will give an overview to the field of symbolic analog circuit analysis. Starting with a motivation, the state-of-the-art simplification algorithms for linear as well as for nonlinear circuits are presented. The basic ideas behind the different techniques are described, whereas the technical details can be found in the cited references. Finally, the application of linear and nonlinear symbolic analysis will be shown on two example circuits.
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
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