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S. Feldmann, P. Lang

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Fraunhofer-Institut für Techno- und
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Gottlieb-Daimler-Straße, Geb. 49

67663 Kaiserslautern

Telefon: +49 (0) 6 31/2 05-32 42
Telefax: +49 (0) 6 31/2 05-41 39
E-Mail: info@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

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Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Padé-like reduction of stable discrete linear systems preserving their stability

Sven Feldmann [a, 1], Patrick Lang [b, 2],

[a]Department of Mathematics, University of Kaiserslautern
[b]Fraunhofer Institute for Industrial Mathematics, ITWM, Kaiserslautern

submitted to *Linear Algebra and its Applications*

Abstract A new stability preserving model reduction algorithm for discrete linear SISO-systems based on their impulse response is proposed. Similar to the Padé approximation, an equation system for the Markov parameters involving the Hankel matrix is considered, that here however is chosen to be of very high dimension. Although this equation system therefore in general cannot be solved exactly, it is proved that the approximate solution, computed via the Moore-Penrose inverse, gives rise to a stability preserving reduction scheme, a property that cannot be guaranteed for the Padé approach. Furthermore, the proposed algorithm is compared to another stability preserving reduction approach, namely the balanced truncation method, showing comparable performance of the reduced systems. The balanced truncation method however starts from a state space description of the systems and in general is expected to be more computational demanding.

Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation

1 Introduction

Generally the problem of model reduction lies in the replacement of a given mathematical description Σ of a natural process by a description $\widehat{\Sigma}$, which is much smaller than Σ, but still, guarantees the main properties of the original process. For discrete linear systems

$$\Sigma : \begin{array}{rl} x_{k+1} & = \mathcal{A}x_k + \mathcal{B}u_k, \\ y_k & = \mathcal{C}x_k \end{array}, \ x_0 = o, \quad \mathcal{A} \in \mathbb{C}^{\nu\times\nu}, \mathcal{B}, \mathcal{C}^T \in \mathbb{C}^{\nu} \tag{1}$$

it makes sense to call an n-dimensional system $\widehat{\Sigma}$ *smaller* than Σ, if the inequality $n < \nu$ is satisfied. A typical property, which should be preserved by changing over to the reduced system, is stability. A discrete system Σ is said to be *stable*, if Σ has all its eigenvalues inside the open unit disc $\mathscr{D}$. Clearly, the reduced system $\widehat{\Sigma}$ should moreover fulfil certain approximation requirements. For discrete systems it is natural to demand the similarity of their impulse responses. More precisely, we are interested in the solution of the following problem: let a stable discrete linear single - input - single - output - system (SISO) (1) be given, we then are looking for a stable system $\widehat{\Sigma}$ of a lower state space dimension, such

[1]Corresponding author. E-mail address: sven@mathematik.uni-kl.de
[2]E-mail address: lang@itwm.fhg.de

that the quantity

$$\varepsilon := \frac{\|G - \widehat{G}\|_2}{\|G\|_2}, \qquad \|G\|_2 := \left(\frac{1}{2\pi}\int_0^{2\pi} |G(e^{\mathbf{i}\omega})|^2 d\omega\right)^{1/2},$$

is smaller as a prescribed error bound ε_0. Here, G and $\widehat{G}$ denote the transfer functions of Σ and $\widehat{\Sigma}$. Similar impulse responses are reflected by a small ε. A huge number of papers exists which deal with approximation problems of this nature. Two keywords are *Padé approximation* and *balanced truncation*. A good source to obtain an overview of the first topic is [4], and concerning balanced model reduction we refer the reader to [13], [14].

For the discrete time case the Padé approximation problem reads as follows: let a strictly proper rational function G of degree ν be given. Then the n-th Padé approximation problem of order N is to find a rational function $\widehat{G}$ of degree $n < \nu$, such that in a neighbourhood of ∞ the difference $G - \widehat{G}$ admits the representation

$$G(s) - \widehat{G}(s) = \sum_{k=N}^{\infty} d_k s^{-(k+1)}.$$

The context to the model reduction problem is based on the following two facts:

- the impulse response of (1) is given by the Laurent coefficient sequence $(h_k)_{k=0}^{\infty}$ of its transfer function $G(s) := \sum_{k=0}^{\infty} h_k s^{-(k+1)} := \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B}$, and
- the dimension of a minimal realization and the degree of a rational function $\widehat{G}$ coincide.

Consequently, $\widehat{G}$ admits a n-dimensional realization by a system with the same impulse response as Σ up to the N-th time step. In particular, the numbers h_k are called the *Markov parameters* of Σ. The n-th Padé approximation problem of order N is equivalent to the *partial realization problem*: let a finite complex number sequence $h := (h_k)_{k=0}^{N-1}$ be given. The problem is to find a matrix triple $\widehat{\Sigma} := (A, B, C)$, $A \in \mathbb{C}^{n\times n}$, $B, C^T \in \mathbb{C}^n$, such that $h_k = CA^kB$ holds true for $k = 0, \dots, N-1$. One deals with the *minimal partial realization* (MPR) *problem* if one is interested in solutions with minimal state space dimension n. For details we refer the reader to [3], [9], [10], [11], [5], [6]. In connection with model reduction problems the sequence h is defined by the first N Markov parameters of Σ, abbreviated by $h(\Sigma, N)$. For non stable systems and large N the calculation of $h(\Sigma, N)$ turns out to be impossible. Even for small N, if the associated Hankel matrix

$$H_N^n := \begin{bmatrix} h_0 & \dots & h_{n-1} \\ \vdots & & \vdots \\ h_{\ell-1} & \dots & h_{N-2} \end{bmatrix} \in \mathbb{C}^{\ell\times n}, \quad n + \ell = N, \tag{2}$$

exists, it then is ill conditioned. These numerical difficulties can be avoided by exploiting the connection between Padé approximants and the Lanczos process (Padé approximation via Lanczos process, PVL). For details attend [2], [7], [8], and the references therein.

The important drawback of Padé approximation methods is that they generally do not preserve stability. That means one can not ensure that all poles of $\widehat{G}$ are again in the open left half plane or in the open unit disc even if this holds for G. For the continuous time case an illustrating example can be found in [1], and for the discrete time case, we demonstrate this disadvantage with Example 1. One possibility to overcome this problem consists in the reduction of the order N of the Padé approximation problem to obtain a solution family of non-minimal partial realizations for $h(\Sigma, N)$ containing more than one element. The gained freedom then can be used to try to find a stable one. For example, one starts with an arbitrary stable denominator polynomial of degree n, and computes the corresponding numerator polynomial such that at least the first n members of $h(\Sigma, N)$ are matched. For details see [4], Section 12, and the references therein. Our reduction method is inspired by Padé approximation as well, however we overcome the stability problem by allowing small deviations between $\widehat{h}_k$ and h_k for all indices. For compensational reasons, we however try to approximate a large number N of Markov parameters h_k.

The n-th Padé approximation problem of order N for G is solved, if one is able to compute a vector $\xi \in \mathbb{C}^n$, such that

$$H_N^n \xi = h_N^n, \quad h_N^n := [h_n, \dots, h_{N-1}]^T. \tag{3}$$

Then the denominator polynomial of a fraction representation of $\widehat{G}$ is given by

$$Q(s) := s^n - [s^0, \dots, s^{n-1}]\xi. \tag{4}$$

However for $n \ll N$ it can not be expected, that the vector h_N^n belongs to the subspace span $H_N^n \subset \mathbb{C}^\ell$. In that case, it seems natural for us to set

$$\xi := (H_N^n)^\dagger h_N^n, \tag{5}$$

where $F^\dagger$ denotes the Moore-Penrose inverse of the matrix F. Our main result (Theorem 2) concerns the stability of the polynomial (4) for

$$\xi = \lim_{N\to\infty} (H_N^n)^\dagger h_N^n.$$

To measure the "similarity" of the impulse responses, the quantity $\|G - \widehat{G}\|_2$ is appropriate. For its estimation, the identification of the residual vector

$$\mathcal{E}^n := H_\infty^n \xi - h_\infty^n$$

as ℓ_2-sequence is useful, since there holds:

$$\|G - \hat{G}\|_2 \leq \|\mathcal{E}^n\|_2 (1-r)^{-n}.$$

Here, r denotes the maximum of the moduli of the zeros of Q.

The above mentioned stability problems do not arise if one reduces Σ via the *balanced truncation* method. The crucial step in this approach is the transformation of Σ into a balanced form, which requires the solution of two ν-dimensional Stein equations. This step however is computationally quite expensive for large ν and additionally often turns out to be ill conditioned. For details confer [12], and [14], Chapter 21.8. In Example 3 we will show, that our method leads to reduced systems, which are of the same approximation quality as systems obtained by balanced truncation.

The paper is organised as follows. First we repeat the solution procedure for the MPR-problem. Then, the difference of the Markov parameter sequences of Σ and its reduction $\widehat{\Sigma}$ is studied, where $\widehat{\Sigma}$ is obtained via the polynomial (4) with coefficient vector (5). In Section 4 the proof of Theorem 2 is prepared. Then we proceed with its formulation and discussion of certain consequences. In Section 6 a reduction example is given and in Section 7 the transfer function of $\widehat{\Sigma}$ with the transfer function of a system obtained by balanced truncation is compared.

2 The MPR-problem

A solution of the MPR-problem can be found by considering the Hankel matrix sequence $(H_{\ell,n})_{n=1}^{N}$, $\ell + n = N + 1$:

$$H_{\ell,n} = \begin{bmatrix} h_0 & \dots & h_{n-1} \\ \vdots & & \vdots \\ h_{\ell-1} & \dots & h_{N-1} \end{bmatrix} \in \mathbb{C}^{\ell \times n}.$$

Here, the polynomial spaces

$$\mathcal{H}_n := \{\psi(s,n)\xi : \xi \in \ker H_{\ell,n}, \ell + n = N + 1\} \subset \mathbb{C}[s],$$

are of special interest, where

$$\psi(s,n) := [s^0, \dots, s^{n-1}].$$

In [5] and [11] the following is proved.

PROPOSITION 1 *For any nonzero sequence $h := (h_k)_{k=0}^{N-1}$ there exist two integers d_1, d_2 with $d_1 \le d_2$, $d_1 + d_2 = N + 1$ and two monic polynomials $q_1 \in \mathcal{H}_{d_1+1}$, $q_2 \in \mathcal{H}_{d_2+1}$, such that for all $n = 1, \dots, N$:*

$$\mathcal{H}_n = \{q_1 p_1 + q_2 p_2 | p_i \in \mathbb{C}[s], \deg\, p_i < n - d_i, i = 1, 2\}.$$

Here we have to set $p_i \equiv 0$ if $n \le d_i$.

The polynomial system $\{q_1, q_2\}$ is called a *fundamental system* and the two integers d_1, d_2 are called the *characteristic* degrees of h. Theorem 1, taken from [11], relates the characteristic degrees of h to the dimension of its minimal partial realizations (MPRs).

THEOREM 1 *Let d_1, d_2 be the characteristic degrees, $\{q_1, q_2\}$ be a fundamental system, and ν be the state space dimension of a MPR of h. Then*

1. *If* $\deg q_1 = d_1$*, then* $\nu = d_1$*, else* $\nu = d_2$*.*
2. *If* $\nu = d_1$*, then* h *has a unique MPR up to similarity.*
3. *If* $\nu = d_2$*, then* h *has infinitely many MPRs with diagonalisable main operator* A*.*

Starting with

$$Q(s) := s^\nu - \textstyle\sum_{k=0}^{\nu-1} \xi_k s^k := \begin{cases} q_1(s), & \text{if } \deg q_1 = d_1, \\ q_2(s), & \text{if } \deg q_1 < d_1, \end{cases}$$

$$\xi := [\xi_0, \ldots, \xi_{\nu-1}],$$

a MPR $\widehat{\Sigma}$ of h is constructed as follows: to the polynomial Q a polynomial P is assigned according to $P(s) := s^{\nu-1}p_{\nu-1} + \ldots + p_0$, where

$$[p_{\nu-1}, \ldots, p_0]^T := T(h,\nu)[1, -\xi_{\nu-1}, \ldots, -\xi_1]^T, \quad T(h,\nu) := \begin{bmatrix} h_0 & & \\ \vdots & \ddots & \\ h_{\nu-1} & \ldots & h_0 \end{bmatrix}. \tag{6}$$

The polynomial P is said to be the *residual polynomial* of Q with respect to h, and the vector $\operatorname{res}(Q,h)$ is defined by $\operatorname{res}(Q,h) := [p_0, \ldots, p_{\nu-1}]$. Now, the matrix triple

$$\widehat{\Sigma} := (C_Q^T, e_\nu, \operatorname{res}(Q,h)), \; C_Q := \left[\begin{array}{c|c} 0 \ldots 0 & \xi \\ \hline I_{\nu-1} & \end{array} \right], \; e_\nu := [0, \ldots, 0, 1]^T \tag{7}$$

represents a MPR of h, and is known as the *controller form realization* of the rational function P/Q. The matrix C_Q is referred to as the *companion matrix* of Q. As already mentioned above, the Padé approximation may lead to non stable model reduction results, even if the original system is stable. This disadvantage is shown in Example 1.

EXAMPLE 1 For all $\alpha \in (0,1)$ the 3-dimensional system

$$\Sigma(\alpha): \; \mathcal{A} = \operatorname{diag}(\alpha, 0, -\alpha), \quad \mathcal{B} = [2, -1, -1]^T, \quad \mathcal{C} = [1, 1, 1]$$

is stable. With respect to the above described solution procedure, for certain $\alpha \in (0,1)$ we compute a MPR $\widehat{\Sigma}$ of $h(\Sigma(\alpha), 4)$, which turns out to be non stable and unique up to similarity. Hence, for the transfer function G_α of $\Sigma(\alpha)$ no stable Padé approximants of order 4, which can be realized by a system with state space dimension less than 3, exists. Obviously, $h_0 = 0$, $h_1 = 3\alpha$, $h_2 = \alpha^2$, $h_3 = 3\alpha^3$, and the solution of the equation system

$$H_4^2 x = h_4^2, \; H_4^2 = \begin{bmatrix} h_0 & h_1 \\ h_1 & h_2 \end{bmatrix}, \; h_4^2 = \begin{bmatrix} h_2 \\ h_3 \end{bmatrix},$$

is given by $\xi := [8\alpha^2/9, \alpha/3]^T$, where in particular for $\alpha \neq 0$ the Hankel matrix H_4^2 is regular. Consequently, the first characteristic degree d_1 of h is equal to 2, and the first characteristic polynomial q_1 is given by

$$q_1(s) := s^2 - \psi(s,2)\xi = s^2 - (\alpha/3)s - (8/9)\alpha^2$$

with the zeros $\frac{(\sqrt{33}\pm 1)}{6}\alpha$. Due to $1 < (\sqrt{33}+1)/6$, one is able to choose $\alpha \in (0,1)$, such that $1 < \frac{\sqrt{33}+1}{6}\alpha$ is fulfilled, namely $\alpha \in (6/(\sqrt{33}+1), 1)$. For these α the matrix C_{q_1} is non stable, which transfers to the non stability of $\widehat{\Sigma} := (C_{q_1}^T, e_2, \mathrm{res}(q_1, h))$. Theorem 1 states the uniqueness of $\widehat{\Sigma}$ up to similarity, since the dimension of $\widehat{\Sigma}$ coincides with d_1. $\diamond$

To overcome the stability problem in all that follows we loosen the demand for exact coincidence between h_k and $\widehat{h}_k$ for $k = 0, \ldots, N-1$. For compensational reasons, we however try to approximate a large number N of Markov parameters h_k.

3 Projection of h_N^n on span H_N^n

The condition $Q \in \mathcal{H}_{n+1}$, deg $Q = n$ is equivalent to the existence of a solution for

$$H_N^n x = h_N^n,$$

where H_N^n and h_N^n are defined as in (2) and (3). In the case where $h_N^n \notin \mathrm{span}\, H_N^n$, and one is satisfied with a vector ξ, such that

$$\mathcal{E}_N^n := H_N^n \xi - h_N^n, \ \|\mathcal{E}_N^n\|_2 \approx 0,$$

we will show that the system $\widehat{\Sigma} := (C_Q^T, e_n, \mathrm{res}(Q,h))$, $Q(s) := s^n - \psi(s,n)\xi$, becomes stable, if ξ is chosen to be

$$\xi := \lim_{N\to\infty} (H_N^n)^\dagger h_N^n.$$

Moreover, due to the properties of the Moore-Penrose inverse, the residual vector $\mathcal{E}_N^n$ satisfies the relation

$$\|\mathcal{E}_N^n\|_2 = \min_{x\in\mathbb{C}^n} \|H_N^n x - h_N^n\|_2,$$

where $\|.\|_2$ denotes the Euclidean norm in $\mathbb{C}^\ell$. Theorem 1 implies $\|\mathcal{E}_N^n\|_2 = 0$, if $\nu \leq n$, and $0 < \|\mathcal{E}_N^n\|_2$, if $n < \nu$, where ν denotes the dimension of the MPRs of $h(\Sigma, N)$. The following Proposition relates the chosen state space dimension $n < \nu$ to the differences of the Markov parameters from $\widehat{\Sigma}$ and Σ.

PROPOSITION 2 *Let h be the Markov parameter sequence of a ν-dimensional system Σ. Consider for $n < \nu$ the monic polynomial Q and the system $\widehat{\Sigma}$, defined by $Q(s) := s^n - \psi(s,n)(H_N^n)^\dagger h_N^n$ and $\widehat{\Sigma} := (C_Q^T, e_n, \mathrm{res}(Q,h))$. Then the differences $d_k = \widehat{h}_k - h_k$,*

$k = 0, \ldots, N-1$, can be represented via the residual vector $\mathcal{E}_N^n$ of the projection of h_N^n on $\operatorname{span}(H_N^n)$. *More precisely they are given by the output of the system*

$$\begin{array}{ll} x_{k+1} & = C_Q^T x_k + e_n \varepsilon_k \\ d_k & = e_n^T x_k \end{array}, x_0 = o, \tag{8}$$

with respect to the input $\mathcal{E} = [\varepsilon_0, \ldots, \varepsilon_{n-1}, (\mathcal{E}_N^n)^T]^T \in \mathbb{C}^N$, defined by

$$\varepsilon_0 = \ldots = \varepsilon_{n-1} = 0, \quad \mathcal{E}_N^n = H_N^n (H_N^n)^\dagger h_N^n - h_N^n.$$

Proof. Because the third component of $\widehat{\Sigma}$ is defined by $\operatorname{res}(Q, h)$, for $k < n$ we have $h_k = \widehat{h}_k$, which means $d_0 = \ldots = d_{n-1} = 0$. To prove for $n \leq k$ the validity of (8), we consider the equation system

$$\begin{bmatrix} \widehat{h}_n \\ \vdots \\ \widehat{h}_{N-1} \end{bmatrix} = \begin{bmatrix} \widehat{h}_0 & \ldots & \widehat{h}_{n-1} \\ \vdots & & \vdots \\ \widehat{h}_{\ell-1} & \ldots & \widehat{h}_{N-2} \end{bmatrix} \begin{bmatrix} \xi_0 \\ \vdots \\ \xi_{n-1} \end{bmatrix}, \; n + \ell = N,$$

which follows immediately by the definition of $\widehat{h}_k$. Replacement of $\widehat{h}_k$ by $h_k + d_k$ yields

$$\widehat{h}_{n+k} = \sum_{i=0}^{n-1} \widehat{h}_{k+i} \xi_i = \sum_{i=0}^{n-1} (h_{k+i} + d_{k+i}) \xi_i = \sum_{i=0}^{n-1} h_{k+i} \xi_i + \sum_{i=0}^{n-1} d_{k+i} \xi_i = (h_{n+k} + \varepsilon_k) + \sum_{i=0}^{n-1} d_{k+i} \xi_i,$$

which means that the differences d_{n+k} fulfil the recurrence relation

$$d_{n+k} = \sum_{i=0}^{n-1} d_{k+i} \xi_i + \varepsilon_k, \; d_0 = \ldots = d_{n-1} = 0,$$

which is equivalent to (8). □

To prove the stability of $\widehat{\Sigma}$, for $n < \nu$ we associate the vector ξ with the solution of a certain Stein equation.

4 The associated Stein equation

Let $\Sigma = (\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a ν-dimensional stable system, $h := h(\Sigma, \infty)$ be its Markov parameter sequence, and let the n-dimensional system $\widehat{\Sigma}$ be defined by

$$\widehat{\Sigma} := (C_{Q_n}^T, e_n, \operatorname{res}(Q_n, h)), \quad Q_n(s) := s^n - \psi(s, n)\xi, \quad \xi := \lim_{N \to \infty} (H_N^n)^\dagger h_N^n. \tag{9}$$

The polynomial Q_n exists, because Σ is supposed to be stable. In the following for $n = \nu - 1$ we use the term *single-step-reduction*, and for $n < \nu - 1$ the term *multi-step-reduction.*

It turns out, that for a convenient vector $W \in \mathbb{C}^\nu$, the coefficient vector ξ of the polynomial Q_n can be represented as linear combination of the columns of a matrix M, that solves

$$X - C_u X C_u^* = WW^*, \quad C_u \in \mathbb{C}^{\nu\times\nu},\ u(s) = \det(sI_\nu - \mathcal{A})\ , \tag{10}$$

where C_u denotes the companion matrix of the polynomial $u(s)$ as defined in (7).

Proposition 3 *For the ν-dimensional, stable, minimal system $\Sigma = (\mathcal{A}, \mathcal{B}, \mathcal{C})$ and $n < \nu$, let the polynomial Q_n and the matrix M be defined by*

$$Q_n(s) = s^n - \psi(s,n)(H_\infty^n)^\dagger h_\infty^n \in \mathbb{C}_n[s], \quad M = ((H_\infty^\nu)^* H_\infty^\nu)^{-1} \in \mathbb{C}^{\nu\times\nu}.$$

If M is partitioned according to $M = [M_{ij}]_{ij=1}^2$, $M_{11} \in \mathbb{C}^{n\times n}$, then Q_n and M are related by

$$Q_n(s) = \psi(s,\nu) M \begin{bmatrix} o_n \\ \mu \end{bmatrix},\ \mu = M_{22}^{-1} e_1. \tag{11}$$

Here, e_1 denotes the first unit vector of appropriate length.

Proof. Using the abbreviations $M_n := ((H_\infty^n)^* H_\infty^n)^{-1} \in \mathbb{C}^{n\times n}$, and $c_n := (e_n^T M_n e_n)^{-1}$ one obtains $M = M_\nu$ and the recurrence relation

$$M_{n+1} = \left[\begin{array}{c|c} M_n^{-1} & (H_\infty^n)^* h_\infty^n \\ \hline (h_\infty^n)^* H_\infty^n & (h_\infty^n)^* h_\infty^n \end{array}\right]^{-1}.$$

Consequently,

$$\begin{aligned} M_{n+1} e_{n+1} c_{n+1} = & \begin{bmatrix} -M_n (H_\infty^n)^* h_\infty^n \\ 1 \end{bmatrix} = \begin{bmatrix} -(H_\infty^n)^\dagger h_\infty^n \\ 1 \end{bmatrix}, \\ Q_n(s) = & \ \psi(s, n+1) M_{n+1} e_{n+1} c_{n+1}. \end{aligned}$$

Let the vector τ, the matrices N_{ij}, and the constant c be defined by

$$\tau := M \begin{bmatrix} o_n \\ \mu \end{bmatrix}, \quad \left[\begin{array}{c|c} M_{n+1}^{-1} & N_{12} \\ \hline N_{21} & N_{22} \end{array}\right] := M^{-1}, \quad c := e_1^T \mu.$$

Then $\left[M_{n+1}^{-1}, N_{12}\right] \tau = e_{n+1} c$, and because τ is of the form

$$\tau = \begin{bmatrix} \vartheta \\ o \end{bmatrix} \in \mathbb{C}^\nu,\ \vartheta := \begin{bmatrix} \mu \\ 1 \end{bmatrix} \in \mathbb{C}^{n+1},$$

one obtains $M_{n+1}^{-1} \vartheta = e_{n+1} c$. Hence, $M_{n+1} e_{n+1} = \vartheta c^{-1}$, and

$$\begin{aligned} Q_n(s) &= \psi(s, n+1) M_{n+1} e_{n+1} c_{n+1} \\ &= \psi(s, n+1) \vartheta c^{-1} c_{n+1} \\ &= \psi(s, \nu) \tau c^{-1} c_{n+1}. \end{aligned}$$

By definition Q_n is monic, and the last component of ϑ is equal to 1. Hence $c_{n+1} = c$. □

Assuming M solves an equation of the form (10), then Proposition 4 ensures the stability of polynomials of the form (11).

Proposition 4 *Let u be a stable monic polynomial, and the solution M of*

$$X - C_u X C_u^* = WW^* \tag{12}$$

be positive definite. Let M be partitioned according to $M = [M_{ij}]_{ij=1}^2$, $M_{11} \in \mathbb{C}^{n\times n}$, and the polynomial Q_n be defined by

$$Q_n(s) := \psi(s,\nu)\tau, \quad \tau := M\varrho, \quad \varrho := \begin{bmatrix} o_n \\ \mu \end{bmatrix}, \quad \mu := M_{22}^{-1} e_1.$$

Then Q_n is stable.

Proof. Assume that $Q_n(\delta) = 0$, $u(\delta) \neq 0$, and $\eta \in \mathbb{C}^\nu$ is defined by $\eta := (\delta I - C_u^*)^{-1}\varrho$. We show that η and τ are orthogonal. With the abbreviations

$$S := C_{s^\nu}^T, \quad \widetilde{u}(s) := u(s^{-1})s^\nu, \quad e(s) := s^0 + \ldots + s^{\nu-2},$$

the equation

$$(\delta I - C_u)^{-1} = \widetilde{u}(S)\psi(\delta^{-1},\nu)^T\delta^\nu u(\delta)^{-1}\psi(\delta,\nu) - Se(\delta S)$$

holds true, hence,

$$\begin{aligned} \eta^* = \varrho^*(\delta I - C_u)^{-1} = & \underbrace{\varrho^*\widetilde{u}(S)\psi(\delta^{-1},\nu)^T\delta^\nu u(\delta)^{-1}}_{=:\varphi\in\mathbb{C}}\psi(\delta,\nu) - \underbrace{[o_n^*,\mu^*]Se(\delta S)}_{=:[o_{n+1}^*,\omega^*]} \\ = & \ \varphi\psi(\delta,\nu) - [o_{n+1}^*,\omega^*]. \end{aligned}$$

Because τ is of the form $\tau = \begin{bmatrix} \vartheta \\ o \end{bmatrix}$, $\vartheta \in \mathbb{C}^{n+1}$, one obtains

$$\eta^*\tau = (\varphi\psi(\delta,\nu) - [o_{n+1}^*,\omega^*])\tau = \varphi\psi(\delta,\nu)\tau = \varphi Q_n(\delta) = 0.$$

The equations $WW^* = M - C_u M C_u^*$, $\eta^* C_u = \delta\eta^* - \varrho^*$ and the definition $\kappa := \eta^* M\eta > 0$ now imply

$$\begin{aligned} 0 \ \leq & \ \kappa - \eta^* C_u M C_u^*\eta = \kappa - (\delta\eta^* - \varrho^*)M(\overline{\delta}\eta - \varrho) \\ = & \ \kappa(1-|\delta|^2) + 2\Re(\delta\underbrace{\eta^*\tau}_{=0}) - \varrho^* M\varrho = \kappa(1-|\delta|^2) - \varrho^* M\varrho < \kappa(1-|\delta|^2). \end{aligned}$$

Consequently, $\delta \in \mathscr{D}$. □

It remains to construct a vector W, such that the matrix M as defined in Proposition 3 solves the equation (12).

For pairwise distinct complex numbers $\{\alpha_1,\ldots,\alpha_\nu\} \subset \mathscr{D}$ we define

$$u(s) := \prod_{i=1}^\nu (s-\alpha_i), \ \hat{u}(s) := \prod_{i=1}^\nu (1-\overline{\alpha_i}s), \ u'(s) := \tfrac{du(s)}{ds},$$

$$r(s) := \tfrac{\hat{u}(s)}{u'(s)}, \ \alpha := [\alpha_1,\ldots,\alpha_\nu]^T, \ r(\alpha) := [r(\alpha_1),\ldots,r(\alpha_\nu)]^T$$

$$D(r(\alpha)) := \mathrm{diag}(r(\alpha)), \ V := [\alpha_i^{j-1}]_{i,j=1}^\nu, \ e := [1,\ldots,1].$$

Proposition 5 *For the minimal system* $\Sigma = (\mathcal{A},\mathcal{B},\mathcal{C})$, $\sigma(\mathcal{A}) = \{\alpha_1,\dots,\alpha_\nu\} \subset \mathscr{D}$, $\mathcal{A} \in \mathbb{C}^{\nu\times\nu}$, *let the matrix* M *be defined by* $M = ((H_\infty^\nu)^* H_\infty^\nu)^{-1} \in \mathbb{C}^{\nu\times\nu}$. *Then* M *satisfies*

$$M - C_u M C_u^* = WW^*, \quad W := V^{-1} D(g)^{-1} D(r(\alpha))e. \tag{13}$$

Here, $\sum_{i=1}^{\nu} \frac{g_i}{s-\alpha_i}$, *represents the transfer function of* Σ, *and*

$$D(g) := \mathrm{diag}(g),\ g := [g_1,\dots,g_\nu]^T.$$

Proof. In order to prove (13), depending on $\sigma(\mathcal{A}) \subset \mathscr{D}$ the Cauchy matrix

$$\Omega := \left[(1-\overline{\alpha_i}\alpha_j)^{-1}\right]_{i,j=1}^{\nu} \in \mathbb{C}^{\nu\times\nu}$$

is introduced. Since all eigenvalues of $\mathcal{A}$ are pairwise distinct, Ω is regular. The non negativity of $\Omega^{-1} - D(\alpha)\Omega^{-1}D(\alpha)^*$ is responsible for the definition (13) of W. To make that evident, let $\overline{\Omega} := [(1-\alpha_i\overline{\alpha_j})^{-1}]_{i,j=1}^{\nu}$. Then $\Omega^{-1} = D(r(\alpha))\overline{\Omega}D(r(\alpha))^*$, and together with $\overline{\Omega} - D(\alpha)\overline{\Omega}D(\alpha)^* = ee^T$, one concludes

$$\begin{aligned} \Omega^{-1} - D(\alpha)\Omega^{-1}D(\alpha)^* &= D(r(\alpha))(\overline{\Omega} - D(\alpha)\overline{\Omega}D(\alpha)^*)D(r(\alpha))^* \\ &= D(r(\alpha))ee^T D(r(\alpha))^*. \end{aligned} \tag{14}$$

Since $\mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B} = \sum_{j=1}^{\nu} \frac{g_j}{s-\alpha_j}$, $g_j \neq 0$, the Markov parameters of Σ take the form $h_i = \sum_{j=1}^{\nu} g_j \alpha_j^i$. Hence, the matrix $(H_\infty^\nu)^* H_\infty^\nu$ $(= M^{-1})$ admits the factorisation

$$(H_\infty^\nu)^* H_\infty^\nu = V^* D(g)^* \Omega D(g) V, \tag{15}$$

which makes it easy to check (13), namely by virtue of $VC_u = D(\alpha)V$, and (14) one gets

$$\begin{aligned} M - C_u M C_u^* &= (V^*D(g)^*\Omega D(g)V)^{-1} - C_u(V^*D(g)^*\Omega D(g)V)^{-1}C_u^* \\ &= V^{-1}D(g)^{-1}\Omega^{-1}D(g)^{-*}V^{-*} - C_uV^{-1}D(g)^{-1}\Omega^{-1}D(g)^{-*}V^{-*}C_u^* \\ &= V^{-1}D(g)^{-1}\Omega^{-1}D(g)^{-*}V^{-*} - V^{-1}D(\alpha)D(g)^{-1}\Omega^{-1}D(g)^{-*}D(\alpha)^*V^{-*} \\ &= V^{-1}D(g)^{-1}(\Omega^{-1} - D(\alpha)\Omega^{-1}D(\alpha)^*)D(g)^{-*}V^{-*} \\ &= \underbrace{V^{-1}D(g)^{-1}D(r(\alpha))e}_{=W}\,\underbrace{e^TD(r(\alpha))^*D(g)^{-*}V^{-*}}_{=W^*} = WW^*. \end{aligned}$$

□

As usual let ℓ_2 be the normed vector space of all square-summable complex number sequences, and $L_2(\mathbb{T})$ be the normed vector space of all Lebesgue square-integrable functions on $\mathbb{T} := \partial\mathscr{D}$, more precisely

$$\begin{aligned} a := (a_k)_{k=0}^{\infty} \in \ell_2 \quad &\Leftrightarrow \quad \|a\|_2 := \left(\textstyle\sum_{k=0}^{\infty} |a_k|^2\right)^{1/2} < \infty, \\ G \in L_2(\mathbb{T}) \quad &\Leftrightarrow \quad \|G\|_2 := \left(\frac{1}{2\pi}\int_0^{2\pi} |G(e^{\mathrm{i}\omega})|^2 d\omega\right)^{1/2} < \infty. \end{aligned}$$

For stable systems it is not difficult to show, that the ℓ_2-distance between h_∞^n and span H_∞^n is finite. Therefore the residual vector $\mathcal{E}^n := H_\infty^n \xi - h_\infty^n$ can be used to define an $L_2(\mathbb{T})$-function, which is then appropriate to estimate the norm of $G - \widehat{G}$, where G and $\widehat{G}$ denote the transfer functions of Σ and $\widehat{\Sigma}$.

5 The Main Theorem

The Propositions 2, 3, 4, 5 lead us to our main Theorem.

THEOREM 2 *Let $\Sigma = (\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a minimal stable system with diagonalisable main operator. Then the system* (9) *is again stable. In particular,*

$$\|G - \widehat{G}\|_2 \leq \frac{\|\mathcal{E}^n\|_2}{\min_{s\in\mathbb{T}} |Q_n(s)|} \leq \frac{\|\mathcal{E}^n\|_2}{(1-r)^n},$$

where $\mathcal{E}^n := H^n_\infty \xi - h^n_\infty \in \ell_2$ and r is defined by the maximum of the moduli of the zeros of Q_n.

Proof. By virtue of Propositions 3, 4, 5 we know that Q_n is stable.

To prove the inequalities, we first note that due to (15) the columns of H^n_∞ as well as h^n_∞ can be interpreted as elements of ℓ_2. Consequently the residual vector $\mathcal{E}^n$, is interpretable as ℓ_2-element, too. Let $d(s) = \widehat{G}(s) - G(s) = \sum_{k=0}^\infty d_k s^{-(k+1)}$, and $\mathcal{E}(s) := \sum_{k=n}^\infty \varepsilon_k s^{1-k}$, where $\mathcal{E}^n =: (\varepsilon_k)_{k=n}^\infty$. Due to $\mathcal{E}^n \in \ell_2$, the relation $\mathcal{E}(s) \in L_2(\mathbb{T})$ can be stated. With respect to Proposition 2 one obtains

$$d(s) = e_n^T(sI - C_{Q_n}^T)^{-1} e_n \mathcal{E}(s) = s^{n-1} Q_n(s)^{-1} \mathcal{E}(s).$$

Hence,

$$\|G - \widehat{G}\|_2 = \|Q_n^{-1}\mathcal{E}\|_2 \leq \max_{s\in\mathbb{T}} |Q_n(s)^{-1}| \, \|\mathcal{E}^n\|_2 = \frac{\|\mathcal{E}^n\|_2}{\min_{s\in\mathbb{T}} |Q_n(s)|}.$$

□

REMARK 1 It is natural to ask for the minimality of $\widehat{\Sigma}$. The following example shows, that a single step reduction does not necessarily lead to a minimal system. As example we consider the degenerated case where the transfer function of Σ is of the form

$$G(s) = \sum_{k=0}^{\nu-1} (s - \alpha_k)^{-1}, \ \alpha_k = \rho e^{\mathrm{i}\frac{2\pi}{\nu}k}, \ \rho < 1.$$

Then the transfer function $\widehat{G}$ of $\widehat{\Sigma}$ is equal to ν/s, which means $\widehat{G}$ can be realized by the one dimensional system $\widetilde{\Sigma} := (0, 1, \nu)$. Whereby the made assumption leads to

$$h_k = \begin{cases} 0, & \text{if} \quad k \notin \nu\mathbb{N}_0, \\ \nu\rho^k, & \text{if} \quad k \in \nu\mathbb{N}_0. \end{cases}$$

Consequently, for every non vanishing component of $h^{\nu-1}_\infty$ the corresponding row of $H^{\nu-1}_\infty$ vanishes and vice versa, every non-vanishing row of $H^{\nu-1}_\infty$ corresponds with a zero component of $h^{\nu-1}_\infty$. Therefore, $h^{\nu-1}_\infty \in (\operatorname{span} H^{\nu-1}_\infty)^\perp$ which is equivalent to $\xi = (H^{\nu-1}_\infty)^\dagger h^{\nu-1}_\infty = o$. Thus, $Q(s) = s^{\nu-1}$, and $\operatorname{res}(Q, h) = \nu e^T_{\nu-1}$. Finally

$$\widehat{G}(s) = \operatorname{res}(Q, h)(sI - C_Q^T)^{-1} e_\nu = \nu s^{\nu-2}/s^{\nu-1} = \nu/s.$$

REMARK 2 It is natural to ask for the similarity of the systems Σ_2 and $\widetilde{\Sigma}_2$, where the first one is obtained by two consecutive single-step-reductions, whereas the second one is obtained by a two-step-reduction. The following example shows, that, in general, non-similar systems are obtained. Let Σ be given by:

$$\mathcal{A} = \rho \operatorname{diag}(\mathbf{i}, -\mathbf{i}, 1),\ \rho = 0.99,\ \mathcal{B} = \mathcal{C}^T,\ \mathcal{C} = [1, 1, 1].$$

Then $h_k = \rho^k (1 + \mathbf{i}^k + (-\mathbf{i})^k)$ and the consecutive single-step-reductions of Σ lead to the systems

$$\Sigma_1 := (A_1, B_1, C_1),\quad A_1 = \begin{bmatrix} 0 & 1 \\ -0.4852 & 0.4899 \end{bmatrix}, B_1 = [0,1]^T, C_1 = [-0.4798, 3],$$

$$\Sigma_2 := (\alpha, 1, h_0),\qquad \alpha := 0.2501 = ((\widetilde{H}^1_\infty)^T \widetilde{H}^1_\infty)^{-1} (\widetilde{H}^1_\infty)^T \widetilde{h}^1_\infty,\ \widetilde{h}_k := C_1 A_1^k B_1.$$

On the other side, the two-step-reduction leads to

$$\widetilde{\Sigma}_2 = (\widetilde{\alpha}, 1, h_0),\ \widetilde{\alpha} := 0.3236 = ((H^1_\infty)^T H^1_\infty)^{-1} (H^1_\infty)^T h^1_\infty.$$

Since $\alpha \neq \widetilde{\alpha}$, Σ_2 and $\widetilde{\Sigma}_2$ are non similar.

6 Comparison of consecutive single-step-reductions to multi-step-reductions

According to Theorem 2 for every stable, diagonizable system Σ a $(\nu - 1)$-dimensional stable system $\widehat{\Sigma}$ exists with Markov parameters, which are"similar" to the original ones. In particular, $\widehat{\Sigma}$ generically again fulfils the reduction assumptions, such that $\widehat{\Sigma}$ itself can be reduced. Thus one obtains a sequence of stable systems with descending state space dimensions and similar impulse responses. Based on the Nyquist plots, we now compare the approximation quality related to consecutive single-step- versus multi-step-reductions for one concrete example. For stable discrete systems the complex number set

$$G(\mathbb{T}) = \{G(e^{\mathbf{i}\omega}) : \omega \in [0, 2\pi]\} \subset \mathbb{C}$$

relates the output $(y_k)_{k=1}^\infty$ to an input of the form $(u_k)_{k=0}^\infty$, $u_k = \cos(\omega k)$, according to

$$y_k \approx |G(e^{\mathbf{i}\omega})| \cos(\omega k + \varphi(\omega)),\ \varphi(\omega) = \arg G(e^{\mathbf{i}\omega}).$$

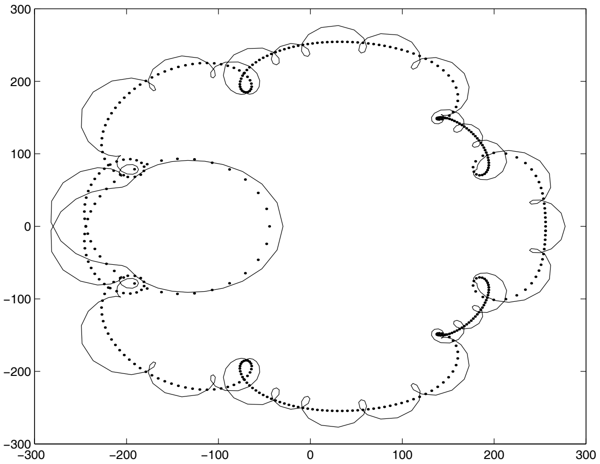
Therefore, the quantity $||G - \widehat{G}||_2$ yields a measure for the preservation of the IO-behaviour by the passage from Σ to $\widehat{\Sigma}$.

EXAMPLE 2 Let $\nu = 40$, $\rho = 0.93$, $\mathcal{A} = \operatorname{diag}(\rho e^{\frac{2\pi \mathbf{i}}{\nu} k})_{k=0}^{\nu-1} \in \mathbb{C}^{\nu \times \nu}$, $\mathcal{B} = [1, \ldots, 1]^T \in \mathbb{R}^\nu$, and

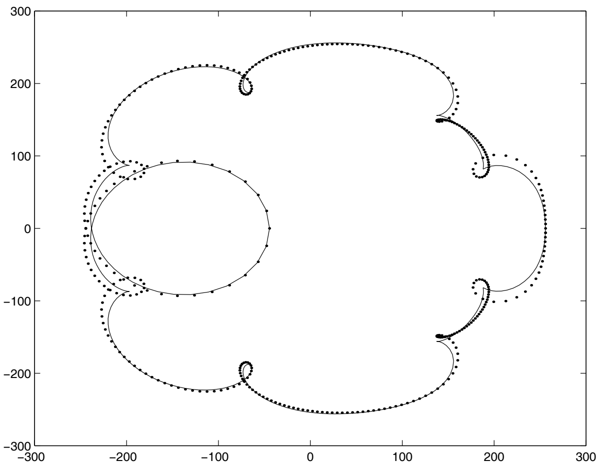
$$\mathcal{C} = [c_1, c_2, \ldots, c_{20}, c_{21}, c_{20}, \ldots, c_2],$$

$$[c_1, \dots, c_{21}] := [8, 8, 2, 6, 5, 5, 3, 8, 8, 8, 6, 7, 2, 6, 8, 10, 4, 4, 10, 3, -9].$$

The following figure (Nyquist plot) compares $G(\mathbb{T})$ (solid line) with $\widehat{G}(\mathbb{T})$ (dotted line), where $\widehat{G}$ represents the transfer function of the 10-dimensional system $\widehat{\Sigma}$, obtained after 30 consecutive single-step-reductions. In every reduction step 500 Markov parameters have been used.



The next figure compares $\widehat{G}(\mathbb{T})$ (dotted line) to $\widetilde{G}(\mathbb{T})$ (solid line), where $\widetilde{G}$ represents the transfer function of the system $\widetilde{\Sigma}$, obtained by reduction of 30 state space dimensions in one step.



Obviously both reduction procedures lead to a satisfying reproduction of the original Nyquist plot. By virtue of the second figure it seems that some of the small loops are better preserved by the consecutive single-step-reduction. Such phenomena will be investigated by the authors in a future article. $\diamond$

7 Comparison of multi-step-reductions to reductions obtained by balanced truncation

A stable discrete system $\Sigma = (\mathcal{A}, \mathcal{B}, \mathcal{C})$ is said to be *balanced*, if a non negative diagonal matrix $X =: \operatorname{diag}(\sigma_1, \ldots, \sigma_\nu)$, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_\nu \geq 0$ exists, which simultaneously solves the Stein equations

$$X - \mathcal{A}X\mathcal{A}^* = \mathcal{B}\mathcal{B}^*, \quad X - \mathcal{A}^*X\mathcal{A} = \mathcal{C}^*\mathcal{C}.$$

The decreasingly ordered numbers σ_i are called the *singular values* of Σ. For every stable system a transformation matrix T exists, such that

$$\Sigma_b := (A_b, B_b, C_b) := (T\mathcal{A}T^{-1}, T\mathcal{B}, \mathcal{C}T^{-1})$$

is balanced. To find T, assume that Σ is minimal, and that $\mathcal{P}$ and $\mathcal{Q}$ satisfy

$$\mathcal{P} - \mathcal{A}\mathcal{P}\mathcal{A}^* = \mathcal{B}\mathcal{B}^*, \quad \mathcal{Q} - \mathcal{A}^*\mathcal{Q}\mathcal{A} = \mathcal{C}^*\mathcal{C}.$$

Then $\mathcal{P} = \sum_{k=0}^{\infty} \mathcal{A}^k\mathcal{B}\mathcal{B}^*(\mathcal{A}^*)^k$, and $\mathcal{Q} = \sum_{k=0}^{\infty} (\mathcal{A}^*)^k\mathcal{C}^*\mathcal{C}\mathcal{A}^k$, which means that $\mathcal{P}$ and $\mathcal{Q}$ are positive definite, and the transformation matrix T is given by $T := \widehat{X}^{1/4}U^*R^{-*}$. Here, R is obtained by the Cholesky decomposition of $\mathcal{P} =: R^*R$. The diagonal matrix $\widehat{X}$ and the unitary matrix U are obtained by the diagonalization of the product $R\mathcal{Q}R^* =: U\widehat{X}U^*$. In particular, the singular values of Σ are the square roots of the eigenvalues of $R\mathcal{Q}R^*$: $X = \widehat{X}^{1/2}$. Now, truncation of Σ_b yields a stable system $\Sigma^{\text{trunc}} := (A_{11}, B_1, C_1)$, where

$$A_b = [A_{ij}]_{i,j=1}^2,\ B_b = [B_1^T, B_2^T]^T,\ C_b = [C_1, C_2],\ A_{11} \in \mathbb{C}^{n\times n},\ C_1^T, B_1 \in \mathbb{C}^n.$$

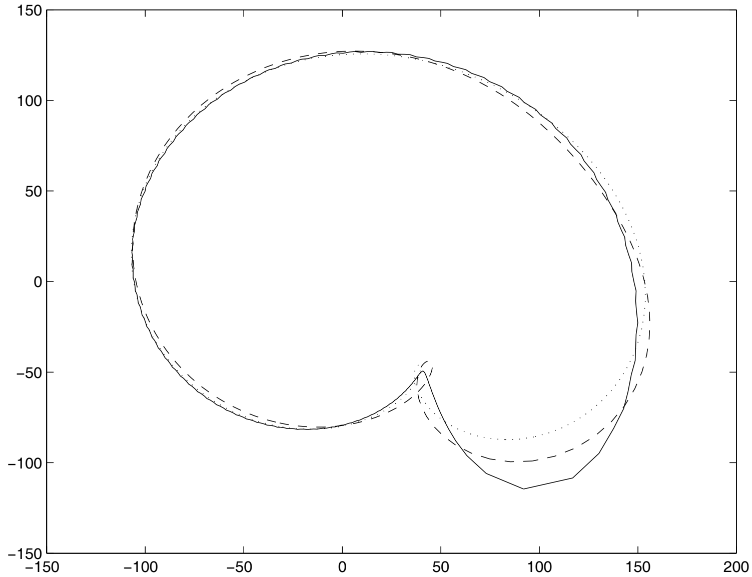
Estimates for $\max_{\omega\in[0,2\pi]} |G(e^{\mathbf{i}\omega}) - G^{\text{trunc}}(e^{\mathbf{i}\omega})|$ can be found in [12], and in [14], Chapter 21.8.

To transform Σ into balanced form one has to solve two ν-dimensional matrix equations and to realize two decompositions. On the other hand for our multi-step-reduction procedure one has to determine the Markov parameter sequence $(\mathcal{C}\mathcal{A}^k\mathcal{B})_{k=0}^{N-1}$, and to invert the n-dimensional Hankel matrix product $(H_N^n)^*H_N^n$ for sufficient large N. Hence, for $n \ll \nu$ our approach avoids the numerical solution of ν-dimensional matrix problems. The following example tries to illustrates, that the obtained reduction results are very similar despite their very different computational efforts.

Example 3 Let the ν-dimensional system Σ be given by

$$\mathcal{A} = \rho\operatorname{diag}(e^{2\pi\mathbf{i}k/\nu})_{k=1}^{\nu},\ \mathcal{B} = [1, \ldots, 1]^T,\ \mathcal{C} = \operatorname{row}(k/\nu)_{k=1}^{\nu}.$$

For $\nu = 200$, $\rho = 0.97$, $N = 400$, and reduced model state space dimension $n = 2$, the following Nyquist plot compares $G(\mathbb{T})$ (solid), $\widehat{G}(\mathbb{T})$ (dotted), and $G^{\text{trunc}}(\mathbb{T})$ (dashed):



With respect to the low degree of $\widehat{G}$ and G^{trunc} both obviously approximate G in a quite satisfying manner.

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 pages, 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 pages, 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 pages, 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 pages, 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 pages, 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way. The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the EleyRideal and LangmuirHinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 pages, 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multiphase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the nonconvolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stressfield from known properties of the components. This is done by the extension of the asymptotic homogenization technique known for pure elastic nonhomogeneous bodies to the nonhomogeneous thermoviscoelasticity of the integral noncon-

volution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type. SanchezPalencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integralmodeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the spaceoperator kernels.
(20 pages, 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time. In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely. If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems. Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 pages, 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 pages, 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i.e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds

for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker
A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e.g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn
Polyhedral Properties of the Uncapacit*ated Multiple Allocation Hub Location Problem*

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel
Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:
The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along pre- scribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions. After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel
Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords: facility location, software development, geographical information systems, supply chain management.
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra
Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari
Grid free method for solving the Poisson equation

A Grid free method for solving the Poisson equation is presented. This is an iterative method. The method is based on the weighted least squares approximation in which the Poisson equation is enforced to be satisfied in every iterations. The boundary conditions can also be enforced in the iteration process. This is a local approximation procedure. The Dirichlet, Neumann and mixed boundary value problems on a unit square are presented and the analytical solutions are compared with the exact solutions. Both solutions matched perfectly.
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier

Simulation of the fiber spinning process

To simulate the influence of process parameters to the melt spinning process a fiber model is used and coupled with CFD calculations of the quench air flow. In the fiber model energy, momentum and mass balance are solved for the polymer mass flow. To calculate the quench air the Lattice Boltzmann method is used. Simulations and experiments for different process parameters and hole configurations are compared and show a good agreement.
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis

On interaction of a liquid film with an obstacle

In this paper mathematical models for liquid films generated by impinging jets are discussed. Attention is stressed to the interaction of the liquid film with some obstacle. S. G. Taylor [Proc. R. Soc. London Ser. A 253, 313 (1959)] found that the liquid film generated by impinging jets is very sensitive to properties of the wire which was used as an obstacle. The aim of this presentation is to propose a modification of the Taylor's model, which allows to simulate the film shape in cases, when the angle between jets is different from 180°. Numerical results obtained by discussed models give two different shapes of the liquid film similar as in Taylors experiments. These two shapes depend on the regime: either droplets are produced close to the obstacle or not. The difference between two regimes becomes larger if the angle between jets decreases. Existence of such two regimes can be very essential for some applications of impinging jets, if the generated liquid film can have a contact with obstacles.
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner

Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids

The filling process of viscoplastic metal alloys and plastics in expanding cavities is modelled using the lattice Boltzmann method in two and three dimensions. These models combine the regularized Bingham model for viscoplastic with a free-interface algorithm. The latter is based on a modified immiscible lattice Boltzmann model in which one species is the fluid and the other one is considered as vacuum. The boundary conditions at the curved liquid-vacuum interface are met without any geometrical front reconstruction from a first-order Chapman-Enskog expansion. The numerical results obtained with these models are found in good agreement with available theoretical and numerical analysis.
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert

»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«

Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001

Was macht einen guten Hochschullehrer aus? Auf diese Frage gibt es sicher viele verschiedene, fachbezogene Antworten, aber auch ein paar allgemeine Gesichtspunkte: es bedarf der »Leidenschaft« für die Forschung (Max Weber), aus der dann auch die Begeisterung für die Lehre erwächst. Forschung und Lehre gehören zusammen, um die Wissenschaft als lebendiges Tun vermitteln zu können. Der Vortrag gibt Beispiele dafür, wie in angewandter Mathematik Forschungsaufgaben aus praktischen Alltagsproblemstellungen erwachsen, die in die Lehre auf verschiedenen Stufen (Gymnasium bis Graduiertenkolleg) einfließen; er leitet damit auch zu einem aktuellen Forschungsgebiet, der Mehrskalenanalyse mit ihren vielfältigen Anwendungen in Bildverarbeitung, Materialentwicklung und Strömungsmechanik über, was aber nur kurz gestreift wird. Mathematik erscheint hier als eine moderne Schlüsseltechnologie, die aber auch enge Beziehungen zu den Geistes- und Sozialwissenschaften hat.
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari

Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations

A Lagrangian particle scheme is applied to the projection method for the incompressible Navier-Stokes equations. The approximation of spatial derivatives is obtained by the weighted least squares method. The pressure Poisson equation is solved by a local iterative procedure with the help of the least squares method. Numerical tests are performed for two dimensional cases. The Couette flow, Poiseuelle flow, decaying shear flow and the driven cavity flow are presented. The numerical solutions are obtained for stationary as well as instationary cases and are compared with the analytical solutions for channel flows. Finally, the driven cavity in a unit square is considered and the stationary solution obtained from this scheme is compared with that from the finite element method.
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel

Optimal Portfolios with Fixed Consumption or Income Streams

We consider some portfolio optimisation problems where either the investor has a desire for an a priori specified consumption stream or/and follows a deterministic pay in scheme while also trying to maximize expected utility from final wealth. We derive explicit closed form solutions for continuous and discrete monetary streams. The mathematical method used is classical stochastic control theory.
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel

Optimal portfolios with a loan dependent credit spread

If an investor borrows money he generally has to pay higher interest rates than he would have received, if he had put his funds on a savings account. The classical model of continuous time portfolio optimisation ignores this effect. Since there is obviously a connection between the default probability and the total percentage of wealth, which the investor is in debt, we study portfolio optimisation with a control dependent interest rate. Assuming a logarithmic and a power utility function, respectively, we prove explicit formulae of the optimal control.
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz

The Euler number of discretized sets - on the choice of adjacency in homogeneous lattices

Two approaches for determining the Euler-Poincaré characteristic of a set observed on lattice points are considered in the context of image analysis { the integral geometric and the polyhedral approach. Information about the set is assumed to be available on lattice points only. In order to retain properties of the Euler number and to provide a good approximation of the true Euler number of the original set in the Euclidean space, the appropriate choice of adjacency in the lattice for the set and its background is crucial. Adjacencies are defined using tessellations of the whole space into polyhedrons. In R 3 , two new 14 adjacencies are introduced additionally to the well known 6 and 26 adjacencies. For the Euler number of a set and its complement, a consistency relation holds. Each of the pairs of adjacencies (14:1; 14:1), (14:2; 14:2), (6; 26), and (26; 6) is shown to be a pair of complementary adjacencies with respect to this relation. That is, the approximations of the Euler numbers are consistent if the set and its background (complement) are equipped with this pair of adjacencies. Furthermore, sufficient conditions for the correctness of the approximations of the Euler number are given. The analysis of selected microstructures and a simulation study illustrate how the estimated Euler number depends on the chosen adjacency. It also shows that there is not a uniquely best pair of adjacencies with respect to the estimation of the Euler number of a set in Euclidean space.
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

A generalized lattice Boltzmann model to simulate free-surface is constructed in both two and three dimensions. The proposed model satisfies the interfacial boundary conditions accurately. A distinctive feature of the model is that the collision processes is carried out only on the points occupied partially or fully by the fluid. To maintain a sharp interfacial front, the method includes an anti-diffusion algorithm. The unknown distribution functions at the interfacial region are constructed according to the first order Chapman-Enskog analysis. The interfacial boundary conditions are satis-

fied exactly by the coefficients in the Chapman-Enskog expansion. The distribution functions are naturally expressed in the local interfacial coordinates. The macroscopic quantities at the interface are extracted from the least-square solutions of a locally linearized system obtained from the known distribution functions. The proposed method does not require any geometric front construction and is robust for any interfacial topology. Simulation results of realistic filling process are presented: rectangular cavity in two dimensions and Hammer box, Campbell box, Sheffield box, and Motorblock in three dimensions. To enhance the stability at high Reynolds numbers, various upwind-type schemes are developed. Free-slip and no-slip boundary conditions are also discussed.
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

In the present paper a kinetic model for vehicular traffic leading to multivalued fundamental diagrams is developed and investigated in detail. For this model phase transitions can appear depending on the local density and velocity of the flow. A derivation of associated macroscopic traffic equations from the kinetic equation is given. Moreover, numerical experiments show the appearance of stop and go waves for highway traffic with a bottleneck.
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

To a network N(q) with determinant D(s;q) depending on a parameter vector q Î R^r via identification of some of its vertices, a network N^ (q) is assigned. The paper deals with procedures to find N^ (q), such that its determinant D^ (s;q) admits a factorization in the determinants of appropriate subnetworks, and with the estimation of the deviation of the zeros of D^ from the zeros of D. To solve the estimation problem state space methods are applied.
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

A spectral theory for stationary random closed sets is developed and provided with a sound mathematical basis. Definition and proof of existence of the Bartlett spectrum of a stationary random closed set as well as the proof of a Wiener-Khintchine theorem for the power spectrum are used to two ends: First, well known second order characteristics like the covariance can be estimated faster than usual via frequency space. Second, the Bartlett spectrum and the power spectrum can be used as second order characteristics in frequency space. Examples show, that in some cases information about the random closed set is easier to obtain from these characteristics in frequency space than from their real world counterparts.
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

We present a unified approach of several boundary conditions for lattice Boltzmann models. Its general framework is a generalization of previously introduced schemes such as the bounce-back rule, linear or quadratic interpolations, etc. The objectives are two fold: first to give theoretical tools to study the existing boundary conditions and their corresponding accuracy; secondly to design formally third- order accurate boundary conditions for general flows. Using these boundary conditions, Couette and Poiseuille flows are exact solution of the lattice Boltzmann models for a Reynolds number $Re = 0$ (Stokes limit).
Numerical comparisons are given for Stokes flows in periodic arrays of spheres and cylinders, linear periodic array of cylinders between moving plates and for Navier-Stokes flows in periodic arrays of cylinders for $Re < 200$. These results show a significant improvement of the overall accuracy when using the linear interpolations instead of the bounce-back reflection (up to an order of magnitude on the hydrodynamics fields). Further improvement is achieved with the new multi-reflection boundary conditions, reaching a level of accuracy close to the quasi-analytical reference solutions, even for rather modest grid resolutions and few points in the narrowest channels. More important, the pressure and velocity fields in the vicinity of the obstacles are much smoother with multi-reflection than with the other boundary conditions.
Finally the good stability of these schemes is highlighted by some simulations of moving obstacles: a cylinder between flat walls and a sphere in a cylinder.
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Im Rahmen dieser Arbeit soll eine elementar gehaltene Einführung in die Aufgabenstellungen und Prinzipien der modernen Finanzmathematik gegeben werden. Insbesondere werden die Grundlagen der Modellierung von Aktienkursen, der Bewertung von Optionen und der Portfolio-Optimierung vorgestellt. Natürlich können die verwendeten Methoden und die entwickelte Theorie nicht in voller Allgemeinheit für den Schulunterricht verwendet werden, doch sollen einzelne Prinzipien so heraus gearbeitet werden, dass sie auch an einfachen Beispielen verstanden werden können.
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

In this paper we consider short term storage systems. We analyze presorting strategies to improve the effiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assigment problem, i.e., it has an assigment problem kernel and some additional constraints. We present different types of these presorting problems, introduce mathematical programming formulations and prove the NP-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn

On the frame-invariant description of the phase space of the Folgar-Tucker equation

The Folgar-Tucker equation is used in flow simulations of fiber suspensions to predict fiber orientation depending on the local flow. In this paper, a complete, frame-invariant description of the phase space of this differential equation is presented for the first time.
Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel

A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects

In this article, we consider the problem of planning inspections and other tasks within a software development (SD) project with respect to the objectives quality (no. of defects), project duration, and costs. Based on a discrete-event simulation model of SD processes comprising the phases coding, inspection, test, and rework, we present a simplified formulation of the problem as a multiobjective optimization problem. For solving the problem (i.e. finding an approximation of the efficient set) we develop a multiobjective evolutionary algorithm. Details of the algorithm are discussed as well as results of its application to sample problems.
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
(29 pages, 2003)

43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus

Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem -

Radiation therapy planning is always a tight rope walk between dangerous insufficient dose in the target volume and life threatening overdosing of organs at risk. Finding ideal balances between these inherently contradictory goals challenges dosimetrists and physicians in their daily practice. Today's planning systems are typically based on a single evaluation function that measures the quality of a radiation treatment plan. Unfortunately, such a one dimensional approach can-

not satisfactorily map the different backgrounds of physicians and the patient dependent necessities. So, too often a time consuming iteration process between evaluation of dose distribution and redefinition of the evaluation function is needed.
In this paper we propose a generic multi-criteria approach based on Pareto's solution concept. For each entity of interest - target volume or organ at risk a structure dependent evaluation function is defined measuring deviations from ideal doses that are calculated from statistical functions. A reasonable bunch of clinically meaningful Pareto optimal solutions are stored in a data base, which can be interactively searched by physicians. The system guarantees dynamical planning as well as the discussion of tradeoffs between different entities.
Mathematically, we model the upcoming inverse problem as a multi-criteria linear programming problem. Because of the large scale nature of the problem it is not possible to solve the problem in a 3D-setting without adaptive reduction by appropriate approximation schemes.
Our approach is twofold: First, the discretization of the continuous problem is based on an adaptive hierarchical clustering process which is used for a local refinement of constraints during the optimization procedure. Second, the set of Pareto optimal solutions is approximated by an adaptive grid of representatives that are found by a hybrid process of calculating extreme compromises and interpolation methods.
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
(31 pages, 2003)

44. T. Halfmann, T. Wichmann

Overview of Symbolic Methods in Industrial Analog Circuit Design

Industrial analog circuits are usually designed using numerical simulation tools. To obtain a deeper circuit understanding, symbolic analysis techniques can additionally be applied. Approximation methods which reduce the complexity of symbolic expressions are needed in order to handle industrial-sized problems.
This paper will give an overview to the field of symbolic analog circuit analysis. Starting with a motivation, the state-of-the-art simplification algorithms for linear as well as for nonlinear circuits are presented. The basic ideas behind the different techniques are described, whereas the technical details can be found in the cited references. Finally, the application of linear and nonlinear symbolic analysis will be shown on two example circuits.
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
(17 pages, 2003)

45. S. E. Mikhailov, J. Orlik

Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites

Asymptotic homogenisation technique and two-scale convergence is used for analysis of macro-strength and fatigue durability of composites with a periodic structure under cyclic loading. The linear damage accumulation rule is employed in the phenomenological micro-durability conditions (for each component of the composite) under varying cyclic loading. Both local and non-local strength and durability conditions are analysed. The strong convergence of the strength and fatigue damage measure as the structure period tends to zero is proved and their limiting values are estimated.
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions
(14 pages, 2003)

46. P. Domínguez-Marín, P. Hansen, N. Mladenović, S. Nickel

Heuristic Procedures for Solving the Discrete Ordered Median Problem

We present two heuristic methods for solving the Discrete Ordered Median Problem (DOMP), for which no such approaches have been developed so far. The DOMP generalizes classical discrete facility location problems, such as the p-median, p-center and Uncapacitated Facility Location problems. The first procedure proposed in this paper is based on a genetic algorithm developed by Moreno Vega [MV96] for p-median and p-center problems. Additionally, a second heuristic approach based on the Variable Neighborhood Search metaheuristic (VNS) proposed by Hansen & Mladenovic [HM97] for the p-median problem is described. An extensive numerical study is presented to show the efficiency of both heuristics and compare them.
Keywords: genetic algorithms, variable neighborhood search, discrete facility location
(31 pages, 2003)

47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto

Exact Procedures for Solving the Discrete Ordered Median Problem

The Discrete Ordered Median Problem (DOMP) generalizes classical discrete location problems, such as the N-median, N-center and Uncapacitated Facility Location problems. It was introduced by Nickel [16], who formulated it as both a nonlinear and a linear integer program. We propose an alternative integer linear programming formulation for the DOMP, discuss relationships between both integer linear programming formulations, and show how properties of optimal solutions can be used to strengthen these formulations. Moreover, we present a specific branch and bound procedure to solve the DOMP more efficiently. We test the integer linear programming formulations and this branch and bound method computationally on randomly generated test problems.
Keywords: discrete location, Integer programming
(41 pages, 2003)

48. S. Feldmann, P. Lang

Padé-like reduction of stable discrete linear systems preserving their stability

A new stability preserving model reduction algorithm for discrete linear SISO-systems based on their impulse response is proposed. Similar to the Padé approximation, an equation system for the Markov parameters involving the Hankel matrix is considered, that here however is chosen to be of very high dimension. Although this equation system therefore in general cannot be solved exactly, it is proved that the approximate solution, computed via the Moore-Penrose inverse, gives rise to a stability preserving reduction scheme, a property that cannot be guaranteed for the Padé approach. Furthermore, the proposed algorithm is compared to another stability preserving reduction approach, namely the balanced truncation method, showing comparable performance of the reduced systems. The balanced truncation method however starts from a state space description of the systems and in general is expected to be more computational demanding.
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation
(16 pages, 2003)

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