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Contributions of the Geomathematics Group to the GAMM 76th Annual Meeting

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A: A Nonlinear Galerkin Scheme Involving Vectorial and Tensorial Spherical Wavelets for Solving the Incompressible Navier-Stokes Equation on the Sphere

B: Methods of Resolution for the Poisson Equation on the 3D Ball

C: Wavelets on the 3-dimensional Ball

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Abstract

The following three papers present recent developments in nonlinear Galerkin schemes for solving the spherical Navier-Stokes equation, in wavelet theory based on the 3-dimensional ball, and in multiscale solutions of the Poisson equation inside the ball, that have been presented at the 76th GAMM Annual Meeting in Luxemburg.

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A Nonlinear Galerkin Scheme Involving Vectorial and Tensorial Spherical Wavelets for Solving the Incompressible Navier-Stokes Equation on the Sphere

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The spherical Navier-Stokes equation plays a fundamental role in meteorology by modelling meso-scale (stratified) atmospherical flows. This article introduces a wavelet based nonlinear Galerkin method applied to the Navier-Stokes equation on the rotating sphere. In detail, this scheme is implemented by using divergence free vectorial spherical wavelets, and its convergence is proven. To improve numerical efficiency an extension of the spherical panel clustering algorithm to vectorial and tensorial kernels is constructed. This method enables the rapid computation of the wavelet coefficients of the nonlinear advection term. Thereby, we also indicate error estimates. Finally, extensive numerical simulations for the nonlinear interaction of three vortices are presented.

Preliminaries: The spherical, incompressible Navier-Stokes equation can be reduced in the weak sense to

$$\frac{\partial u}{\partial t} = -(u\cdot\nabla^*)u - 2\omega\wedge u + \nu\Delta^* u + f, \qquad \nabla^*\cdot u = 0, \qquad u(0) = u_0, \tag{1}$$

where u denotes the velocity field of the considered flow, ω is the rotational axes coinciding with the z-axis of an Earth-fixed reference frame, and ∇^* and Δ^*, respectively, denote the restriction of the gradient and the vectorial Laplace operator to the unit sphere Ω. Moreover, we consider an inhomogeneous flow, by letting f be a time depending external flow driving force. For more details on the considered function spaces, operators and further conditions we refer to [1]. Existence and uniqueness of a generalized weak solution of (1) is provided by [6].

Freeden et al. [5] introduce vectorial scaling functions and wavelets by applying the L^*-operator given by $L^*_\xi = \xi\wedge\nabla^*_\xi$ to scalar scaling functions and wavelets. Multiple illustrations and applications of spherical wavelets can be found in [3, 5]. The arising vectorial kernels are so-called type 3 vectorial scaling functions $\phi_J^{(3)}(\xi,\eta)$ and wavelets $\psi_J^{(3)}(\xi,\chi)$ located at a position $\eta,\chi\in\Omega$, see [1, 5]. In case of (band-)orthogonal, bandlimited kernels they read as

$$\phi_J^{(3)}(\xi,\eta_j) = \sum_{n=1}^{2^J}\Phi_J^\wedge(n)\frac{2n+1}{4\pi}P_n'(\xi\cdot\eta_j)\xi\wedge\eta_j, \qquad \psi_J^{(3)}(\xi,\chi_j) = \sum_{n=2^J+1}^{2^{J+1}}\Psi_J^\wedge(n)\frac{2n+1}{4\pi}P_n'(\xi\cdot\chi_j)\xi\wedge\chi_j,$$

where P_n denotes the Legendre polynomial of degree n, and $\eta_j, j=1,\ldots,N$ and $\chi_j, j=1,\ldots,M$ are elements of the grids $\Gamma_{v_J^{(3)}}\subset\Omega$ locating the scaling functions, and $\Gamma_{w_J^{(3)}}\subset\Omega$ the wavelets, respectively. By construction the scaling functions and wavelets are (band-)orthogonal and satisfy the (algebraic) constraint of incompressibility. Provided that $\Gamma_{v_J^{(3)}}$ is a fundamental system to $\mathrm{harm}^{(3)}_{1,\ldots,2^J}(\Omega)$, the span of all tangential and surface divergence free vectorial scaling functions of scale J forms a finite dimensional Hilbert space equipped with the canonical inner product on $l^2(\Omega)$.

The nonlinear Galerkin Scheme: The nonlinear Galerkin approximation

$$u_J:\mathbb{R}_0^+\to v_J^{(3)} = \mathrm{span}\{\phi_J^{(3)}(\cdot,\eta_1),\ldots,\phi_J^{(3)}(\cdot,\eta_N)\}, \qquad t\mapsto u_J = \sum_{j=1}^N u_j(t)\phi_J^{(3)}(\cdot,\eta_j),$$

is obtained by solving

$$\begin{aligned}\frac{\partial}{\partial t}\langle u_J,\phi\rangle_{l^2(\Omega)} - \nu\langle\Delta^* u_J,\phi\rangle_{l^2(\Omega)} + 2\langle\omega\wedge u_J,\phi\rangle_{l^2(\Omega)} + \langle u_J\cdot\nabla^* u_J,\phi\rangle_{l^2(\Omega)}&\\ +\langle z_J\cdot\nabla^* u_J,\phi\rangle_{l^2(\Omega)} + \langle u_J\cdot\nabla^* z_J,\phi\rangle_{l^2(\Omega)} &= \langle f,\phi\rangle_{l^2(\Omega)},\\ -\nu\langle\Delta^* z_J,\psi\rangle_{l^2(\Omega)} + 2\langle\omega\wedge z_J,\psi\rangle_{l^2(\Omega)} + \langle u_J\cdot\nabla^* u_J,\psi\rangle_{l^2(\Omega)} &= \langle f,\psi\rangle_{l^2(\Omega)},\end{aligned} \tag{2}$$

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for all $\phi \in v_J^{(3)}$, all $\psi \in w_J^{(3)}$, and $u_J(0) = u_0|_{v_J^{(3)}}$, where

$$z_J : \mathbb{R}_0^+ \to w_J^{(3)} = \mathrm{span}\{\psi_J^{(3)}(\cdot, \chi_1), \dots, \psi_J^{(3)}(\cdot, \chi_M)\}, \qquad t \mapsto z_J = \sum_{j=1}^{M} z_j(t)\psi_J^{(3)}(\cdot, \chi_j).$$

The function z_J can be interpreted as a high-frequent perturbation of the flow. Above noted nonlinear Galerkin scheme yields an ordinary differential equation in u_J, which possesses a unique solution with domain of convergence $[0, +\infty)$. Moreover, a proof of convergence for the limit $J \to +\infty$, such that u_J converges to u in a strong topology is given by [1] in connection with [7]. The arising coupling terms in (2) involving the vectorial radial basis functions can be stated explicitly in terms of Wigner-$3j$ coefficients (see [1]). Although one could exploit Wigner-$3j$ selection rules, a full “spectral code” scales with $O(N^7)$, if $N = 2^J$ denotes the maximal spherical harmonic degree to be resolved. The reason is obviously the “quadratic” advection term. To improve algorithmic efficiency we write for the advection $(u \cdot \nabla^*)u = (\nabla^* \otimes u)^T u$, and generalize the concept of spherical panel clustering algorithms [4] to vectorial and tensorial radial basis functions. The details can be found in [1]. The idea is similar to the concept of pseudo spectral methods, see, for instance, [1, 2, 9]. Hence, one evaluates rapidly the velocity field u as well as the tensorial part $(\nabla^* \otimes u)^T$ on a Gauss-Legendre grid in space domain. The subsequent numerical integration of the coupling integral and the solution of the normal equations can be also performed with a modified scalar panel clustering algorithm. For some a priori given relative accuracy ε the algorithmic effort grows with $O(|\log \varepsilon|^4 N^2)$, where $N = 2^J$. This is asymptotically the same effort as required by locally supported kernels, and outperforms the exact, classical pseudo spectral transform involving (vectorial) spherical harmonics scaling with $O(N^3)$, see [2, 9].

It is interesting to note, that this technique is also able to separate vector fields of mixed type since it is based on a polynomial exact integration. This is useful when considering other partial differential equations from meteorology, where the assumption on surface incompressibility is no longer appropriate. Moreover, our extensions of the spherical panel clustering method covers all analytic scalar kernels, which includes also the exponential convergence of the far-field approximation. Numerical realizations including a comparison to a pseudo spectral code can be found in [1].

Numerical Results: We consider three vortices located on the northern hemisphere. Two of them in the North rotate clockwise while the third one rotates anti-clockwise. A similar test procedure is proposed by [8] on the torus. We consider a viscosity ν of 10^{-3} and resolve all scales of motion up to scale 4. The linear system in (2) is highly ill conditioned which does not allow us to increase the resolution at the moment. It is content of our future work to improve this point. As time-stepping scheme we apply a Runge-Kutta-4 method with step-size $h = 10^{-4}$. The velocity of the flow is illustrated in Fig. 1-4 and the results are in accordance to a pseudo spectral code involving type 3 vector spherical harmonics, see [1].

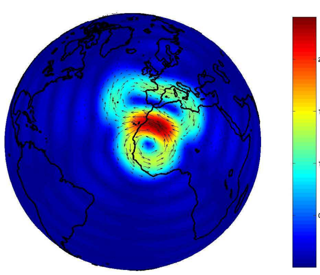

Fig. 1 Velocity at t=0.

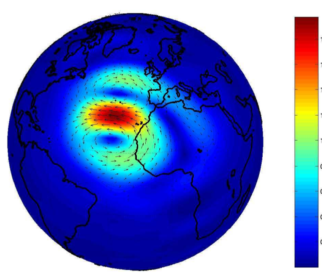

Fig. 2 Velocity at t=0.9 .

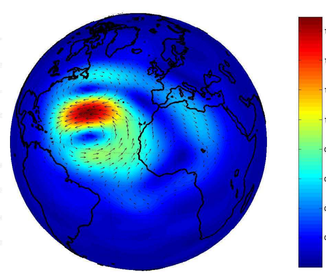

Fig. 3 Velocity at t=1.1.

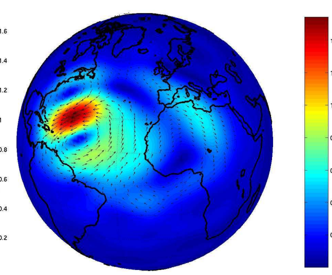

Fig. 4 Velocity at t=1.3 .

References

[1] M. J. FENGLER: *Vector Spherical Harmonic and Vector Wavelet Based Non-Linear Galerkin Schemes for Solving the Incompressible Navier-Stokes Equation on the Sphere.* PhD-thesis (submitted), University of Kaiserslautern, Department of Mathematics, Geomathematics Group, 2005.

[2] M. J. FENGLER, W. FREEDEN: *A Nonlinear Galerkin Scheme Involving Vector and Tensor Spherical Harmonics for Solving the Incompressible Navier-Stokes Equation on the Sphere.* SIAM Journal of Scientific Computing (accepted), 2004.

[3] W. FREEDEN: *Multiscale Modelling of Spaceborne Geodata.* B. G. Teubner, Stuttgart, Leipzig, 1999.

[4] W. FREEDEN, O. GLOCKNER, M. SCHREINER: J. of Geodesy, Vol. 72, pp. 586-599, 1998.

[5] W. FREEDEN, T. GERVENS, M. SCHREINER: *Constructive Approximation on the Sphere (With Applications to Geomathematics).* Oxford Science Publications, Clarendon, Oxford, 1998.

[6] A. A. IL'IN, A. N. FILATOV: Soviet Math. Dokl., Vol. 38, No. 1, pp. 9-13, 1989.

[7] M. MARION, R. TEMAM: SIAM J. Numer. Anal., Vol. 26, No. 5, pp. 1139-1157, 1989.

[8] K. SCHNEIDER, N. K.-R. KEVLAHAN, M. FARGE: J. Theor. Comput. Fluid Dynamics, Vol. 9, pp. 191-206, 1997.

[9] S. A. ORSZAG: J. Atmos. Sci., Vol. 27, pp. 890-895, 1970.

Methods of Resolution for the Poisson Equation on the 3D Ball

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Within the article at hand, we investigate the Poisson equation solved by an integral operator, originating from an ansatz by Greens functions. This connection between mass distributions and the gravitational force is essential to investigate, especially inside the Earth, where structures and phenomena are not sufficiently known and plumbable. Since the operator stated above does not solve the equation for all square-integrable functions, the solution space will be decomposed by a multiscale analysis in terms of scaling functions. Classical Euclidean wavelet theory appears not to be the appropriate choice. Ansatz functions are chosen to be reflecting the rotational invariance of the ball. In these terms, the operator itself is finally decomposed and replaced by versions more manageable, revealing structural information about itself.

1 Introduction

Several PDEs are solved by integral operators using Green's functions. This is especially true for the Poisson equation, though only for piecewise Hölder-continuous densities possible. Expanding this operator on a Hilbert space, equipped with an orthonormal system or a multiresolution analysis, we can decompose the operator, revealing detail information and its structural connection between the different scale spaces. For an application, a radially symmetric Earth density model is used.

2 Constructive Approximation

The region under consideration is the open ball $\mathcal{B}$ in $\mathbb{R}^3$ with radius $R > 0$, the function space is given by the Hilbert space $L^2(\mathcal{B})$. For considerations of orthonormal systems, scaling functions and wavelets in $L^2(\mathcal{B})$, see [6]. Thus, let $\{\Phi_J\}_{J\in\mathbb{N}_0} \subset L^2(\mathcal{B} \times \mathcal{B})$ be the family of scaling functions,

$$P_J : L^2(\mathcal{B}) \to L^2(\mathcal{B})\,,\; F \mapsto \Phi_J * \Phi_J * F$$

be the corresponding low-pass fi lter with

$$\lim_{J\to\infty} \|P_J F - F\|_{L^2(\mathcal{B})} = 0$$

for all $F \in L^2(\mathcal{B})$. For detailed considerations of wavelets on spheres or analoguous general Hilbert space approaches, see [2, 3, 4, 5].

3 Operator Decomposition

Let $T : L^2(\mathcal{B}) \to L^2(\mathcal{B})$ be a linear, bounded operator. Since defi nition and image space can be expanded in approximations by scaling functions, we obtain

Theorem 3.1 *The approximation $T_J := P_J T P_J$ converges at least pointwise to T.*

Considering $\Phi_J * F$ as scaling coeffi cients of F, these are transformed by the operator, and afterwards reconstructed by convolutions with the scaling functions.

Theorem 3.2 *In terms of a multiresolution analysis, the operator T has the shape*

$$T_J : L^2(\mathcal{B}) \to P_J(L^2(\mathcal{B}))\,,\quad F \mapsto T_J F = ((\Phi_J * F) * (\Phi_J * T\Phi_J)) * \Phi_J$$

Investigating an operator, it is mainly important to investigate the kernels used for transforming the scaling coeffi cients. Thus,

$$\Phi_J * T\Phi_J : \mathcal{B} \times \mathcal{B} \to \mathbb{R}$$

will be considered furtheron.

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4 Application

For solving the Poisson equation, a density model for the right-hand side is necessary. As a first approximation, a radially symmetric 13-layer Earth Model (PREM, cf. [1]) has been used. Since the operator retains this structure, the kernel functions can be chosen radially symmetric again and depend, therefore, just on two radial paremeters. Figure 1 presents the density model and a multiscale approximation by a band-limited kernel function. It is obvious that the huge discontinuity at the core-mantle boundary is the greatest task for a smoothing approximation. Also level 6 would need more information to be accurate.

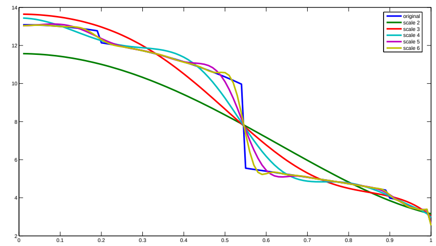
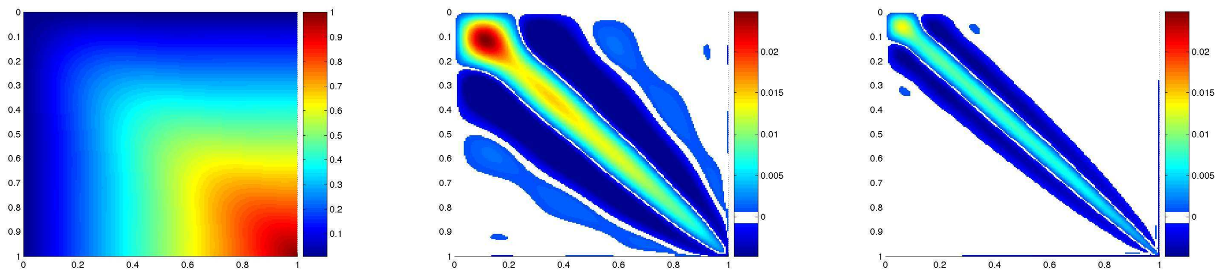


Fig. 1 Earth density with multiscale approximations.

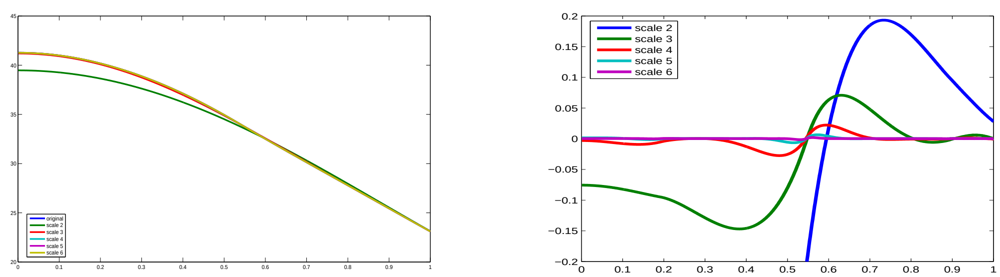
Considering the transformation kernel, we observe in Figure 2a a smoothing effect. The differences between two scales (3 and 4 in Figure 2b and 4 and 5 in Figure 2c) demonstrate the amount of information the kernel is gaining. Since this is localized more and more around the diagonal, we can state that the operator smoothes out high-frequent details and improves the lower degrees. Thus, higher scales of the operator need not be computed.



a. Level 3. b. Difference between level 3 and 4. c. Difference between level 4 and 5.

Fig. 2 Structure of the transformation kernels $r_x r_y (\Phi_J * T\Phi_J)(r_x, r_y)$, weighted for better insight. The axis denote in horizontal and vertical direction the radial parameters r_x and r_y.

For comparing this to the result we investigate the gravitational potential . Here, we observe the same effect, from scale 4 the error (though obviously maximal at the core-mantle boundary) is negligible compared to the potential values.



a. Approximation levels 2 to 6. b. Error within approximation levels.

Fig. 3 Gravitational potential, multiresolution analysis.

References

[1] A. Dziewonski, D.L. Anderson: Phys. Earth Planet. Inter., 25, 297-356 (1981).
[2] W. Freeden, U. Windheuser: Advances in Computational Mathematics, 5, 51-94 (1999).
[3] W. Freeden, T. Gervens, M. Schreiner: Constructive Approximation on the Sphere – With Application to Geomathematics (Clarendon Press, Oxford, 1998).
[4] W. Freeden, O. Glockner, R. Litzenberger: Numer. Funct. Anal. Optimiz., 20, 853-879 (1999).
[5] V. Michel: A Multiscale Approximation for Operator Equations in Separable Hilbert Spaces – Case Study: Reconstruction and Description of the Earth's Interior, Habilitation Thesis, University of Kaiserslautern (Shaker Verlag, Aachen, 2002).
[6] V. Michel: contribution to the 2005 GAMM Proceedings.

Wavelets on the 3–dimensional Ball

V. Michel*

In this article wavelets on a ball in $\mathbb{R}^3$ are introduced. Corresponding properties like an approximate identity and decomposition/reconstruction (scale step property) are proved. The advantage of this approach compared to a classical Fourier analysis in orthogonal polynomials is a better localization of the used ansatz functions.

1 Orthogonal Polynomials

The following systems of orthogonal polynomials on a three-dimensional ball are known (see, for example, [1, 6, 10]).

Theorem 1.1 *Two complete orthonormal systems in the Hilbert space* $\mathrm{L}^2(\mathcal{B})$*, where* $\mathcal{B} := \{x \in \mathbb{R}^3 \,|\, |x| < \beta\}$, $\beta > 0$*, are given by*

$$G^{\mathrm{I}}_{n,j,m}(x) \;:=\; \sqrt{\frac{4m+2n+3}{\beta^3}}\, P_m^{(0,n+1/2)}\left(2\,\frac{|x|^2}{\beta^2}-1\right)\left(\frac{|x|}{\beta}\right)^n Y_{n,j}\left(\frac{x}{|x|}\right); \quad n,m \in \mathbb{N}_0;\ j = 1,...,2n+1$$

$$G^{\mathrm{II}}_{n,j,m}(x) \;:=\; \sqrt{\frac{2m+3}{\beta^3}}\, P_m^{(0,2)}\left(2\,\frac{|x|}{\beta}-1\right) Y_{n,j}\left(\frac{x}{|x|}\right); \quad n,m \in \mathbb{N}_0;\ j = 1,...,2n+1.$$

In both cases, $P_m^{(a,b)}$ *represents the Jacobi polynomials (see* [9] *for further details) and* $Y_{n,j}$ *stands for a spherical harmonics orthonormal basis (see, for instance,* [2]*) in* $\mathrm{L}^2(\mathrm{S}^2)$*, where* S^2 *is the unit sphere in* $\mathbb{R}^3$*.*

If simply $G_{n,j,m}$ is written in this paper, then system I as well as system II could be chosen.
Since those function systems are polynomial in the radius $|x|$ and in the unit vector part $\frac{x}{|x|}$ they inherit the well–known disadvantages of no localization in space. Approximation models always have to be calculated globally and cannot be adapted to local changes of the data effectively. We, therefore, introduce here an alternative method of wavelets on the 3-dimensional ball, which is related to the wavelets on the sphere S^2 in [2, 4] and the general Hilbert space approaches in [3, 7].

2 Scaling Functions and Wavelets

Definition 2.1 A function $K \in \mathrm{L}^2(\mathcal{B}\times\mathcal{B})$ is called product kernel if it has the form

$$K(x,y) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=1}^{2n+1} K^\wedge(n,m) G_{n,j,m}(x) G_{n,j,m}(y); \quad x,y \in \mathcal{B};$$

where the double–series converges in the $\mathrm{L}^2(\mathcal{B}\times\mathcal{B})$–sense, i.e. $\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{\infty}\left(K^\wedge(n,m)\right)^2 < +\infty$. The sequence $\{K^\wedge(n,m)\}_{n,m\in\mathbb{N}_0}$ is called symbol of K. Moreover, if $F \in \mathrm{L}^2(\mathcal{B})$ then we define

$$K * F := \int_{\mathcal{B}} K(\cdot,y)F(y)\,\mathrm{d}y.$$

Note that the addition theorem for spherical harmonics

$$\sum_{j=1}^{2n+1} Y_{n,j}(\xi)Y_{n,j}(\eta) = \frac{2n+1}{4\pi}\, P_n^{(0,0)}(\xi\cdot\eta); \quad \xi,\eta \in \mathrm{S}^2,$$

essentially simplifies the evaluation of the kernel.
Now the following main result can be formulated.

Theorem 2.2 *Let the scaling function* $\{\Phi_J\}_{J\in\mathbb{N}_0}$*, the primal wavelet* $\{\Psi_J\}_{J\in\mathbb{N}_0}$*, and the dual wavelet* $\{\tilde{\Psi}_J\}_{J\in\mathbb{N}_0}$ *be product kernels in* $\mathrm{L}^2(\mathcal{B}\times\mathcal{B})$ *whose symbols satisfy*

1. $0 \le \Phi_J^\wedge(n,m) \le \Phi_{J+1}^\wedge(n,m)$ *for all* $n,m,J \in \mathbb{N}_0$*,*

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2. $\lim_{J\to\infty} \Phi_J^\wedge(n,m) = 1$ *for all* $n, m \in \mathbb{N}_0$,
3. $\tilde{\Psi}_J^\wedge(n,m)\Psi_J^\wedge(n,m) = \Phi_{J+1}^\wedge(n,m) - \Phi_J^\wedge(n,m)$ *for all* $n, m, J \in \mathbb{N}_0$.

Then $\{\Phi_J\}_{J\in\mathbb{N}_0}$ *represents an approximate identity, i.e.*

$$\lim_{J\to\infty} \|F - \Phi_J * F\|_{\mathrm{L}^2(\mathcal{B})} = 0 \quad \textit{for all } F \in \mathrm{L}^2(\mathcal{B}).$$

Moreover, the wavelets provide a decomposition and reconstruction of the signal, i.e.

$$\Phi_{J_2} * F = \Phi_{J_1} * F + \sum_{j=J_1}^{J_2-1} \tilde{\Psi}_j * \Psi_j * F \tag{1}$$

for all $J_1, J_2 \in \mathbb{N}_0$ *with* $J_1 < J_2$.

P r o o f. From the Parseval identity we get the series

$$\|F - \Phi_J * F\|^2_{\mathrm{L}^2(\mathcal{B})} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=1}^{2n+1} \left(1 - \Phi_J^\wedge(n,m)\right)^2 \left(F, G_{n,j,m}\right)^2_{\mathrm{L}^2(\mathcal{B})}$$

which is uniformly convergent with respect to J since $0 \leq (1 - \Phi_J^\wedge(n,m))^2 \leq 1$ and $F \in \mathrm{L}^2(\mathcal{B})$ such that

$$\lim_{J\to\infty} \|F - \Phi_J * F\|^2_{\mathrm{L}^2(\mathcal{B})} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=1}^{2n+1} \lim_{J\to\infty} \left(1 - \Phi_J^\wedge(n,m)\right)^2 \left(F, G_{n,j,m}\right)^2_{\mathrm{L}^2(\mathcal{B})} = 0.$$

Moreover, condition 3. directly implies (1). □

Finally, it is easy to check that conditions 1. and 2. imply that the scale spaces $\mathcal{V}_J := \{\Phi_J * F \mid F \in \mathrm{L}^2(\mathcal{B})\}$ constitute a multiresolution analysis in $\mathrm{L}^2(\mathcal{B})$.
The advantage of this wavelet approach compared to classical Fourier analysis in an orthonormal (polynomial) basis is clearly the strong localization of the wavelet functions (see Figure 1).

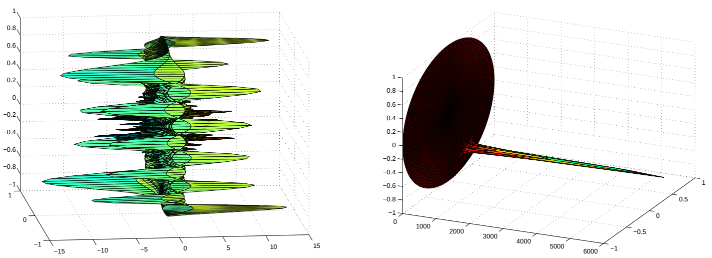

Fig. 1 $G^{\mathrm{I}}_{20,11,60}(y)$ (left) and $\Phi_6(x,y)$ (right), where $x = (0, 0.4, -0.7)^{\mathrm{T}}$ is fixed, plotted in the $y_1 = 0$ plane; the chosen scaling function has the symbol $\Phi_J^\wedge(n,m) = \exp(-2^{-J}(n(2^{-J}n+1) + m(2^{-J}m+1)))$ and uses the basis G^{II} where the series were truncated at $n = m = 1000$

Note that by taking only a part of the basis system $\{G_{n,j,m}\}_{n,j,m}$ for the construction of the kernels we can analyse certain subsets of $\mathrm{L}^2(\mathcal{B})$. If we take, for example, only $\{G^{\mathrm{I}}_{n,j,0}\}_{n\in\mathbb{N}_0; j=1,\ldots,2n+1}$ then we obtain a multiresolution analysis of the solution space of the Laplace equation $\Delta F = 0$ on the ball (cf. [8]). On the other hand, $\{G^{\mathrm{I}}_{0,1,m}\}_{m\in\mathbb{N}_0}$ and $\{G^{\mathrm{II}}_{0,1,m}\}_{m\in\mathbb{N}_0}$ provide us with tools for the analysis of purely radially dependent structures such as the Earth model PREM.
For numerical results the author refers to [5].

References

[1] L. Ballani, J. Engels, E.W. Grafarend: Manuscripta Geodaetica, **18**, 99-114 (1993).
[2] W. Freeden, T. Gervens, M. Schreiner: Constructive Approximation on the Sphere — With Applications to Geomathematics (Clarendon Press, Oxford, 1998).
[3] W. Freeden, O. Glockner, R. Litzenberger: Numerical Functional Analysis and Optimization, **20**, 853-879 (1999).
[4] W. Freeden, U. Windheuser: Advances in Computational Mathematics, **5**, 51-94 (1996).
[5] D. Michel: contribution to the 2005 GAMM Proceedings.
[6] V. Michel: A Multiscale Method for the Gravimetry Problem — Theoretical and Numerical Aspects of Harmonic and Anharmonic Modelling, PhD Thesis, University of Kaiserslautern (Shaker Verlag, Aachen, 1999).
[7] V. Michel: A Multiscale Approximation for Operator Equations in Separable Hilbert Spaces — Case Study: Reconstruction and Description of the Earth's Interior, Habilitation Thesis, University of Kaiserslautern (Shaker Verlag, Aachen, 2002).
[8] V. Michel: Inverse Problems, **21**, 997-1025 (2005).
[9] G. Szegö: Orthogonal Polynomials (AMS Colloquium Publications, Volume XXIII, Providence, Rhode Island, 1939).
[10] C.C. Tscherning: Math. Geology, **28**, 161-168 (1996).

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