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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

NUMERICAL EVIDENCE FOR THE NON–EXISTENCE OF SOLUTIONS OF THE EQUATIONS DESCRIBING ROTATIONAL FIBER SPINNING

THOMAS GÖTZ, AXEL KLAR, ANDREAS UNTERREITER, AND RAIMUND WEGENER

Abstract. The stationary, isothermal rotational spinning process of fibers is considered. The investigations are concerned with the case of large Reynolds ($\delta = 3/\mathrm{Re} \ll 1$) and small Rossby numbers ($\varepsilon \ll 1$). Modelling the fibers as a Newtonian fluid and applying slender body approximations, the process is described by a two–point boundary value problem of ODEs. The involved quantities are the coordinates of the fiber's centerline, the fluid velocity and viscous stress. The inviscid case $\delta = 0$ is discussed as a reference case. For the viscous case $\delta > 0$ numerical simulations are carried out. Transfering some properties of the inviscid limit to the viscous case, analytical bounds for the initial viscous stress of the fiber are obtained. A good agreement with the numerical results is found. These bounds give strong evidence, that for $\delta > 3\varepsilon^2$ no physical relevant solution can exist. A possible interpretation of the above coupling of δ and ε related to the die–swell phenomenon is given.

Keywords: Rotational Fiber Spinning, Viscous Fibers, Boundary Value Problem, Existence of Solutions

1. Introduction

Being an important technology, fiber spinning has recently gained considerable attention in the mathematical literature [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Describing a slender fluid jet, the model equations covering spinning processes are settled on balance laws for mass, momentum and energy. Due to slenderness cross–sectional averaging is performed to obtain spatially one–dimensional models [4, 11, 6, 7, 8].

In the present work we consider the isothermal, rotational spinning of fibers. As an example of industrial relevance, one might have the production of glass fibers in mind. In this situation energy balance a priorily holds and Coriolis and centrifugal forces have to be included, see [2, 3]. The model equations become in $\mathbb{R}^2$

$$\partial_t A + \partial_s(Au) = 0 \,, \tag{1.1a}$$

$$\partial_t v + u\partial_s v = \delta \frac{\partial_s(A\partial_s\gamma\partial_s u)}{A} + \frac{2}{\varepsilon}v^\perp + \frac{1}{\varepsilon^2}\gamma \,, \tag{1.1b}$$

$$\partial_t\gamma + u\partial_s\gamma = v \,, \tag{1.1c}$$

$$\|\partial_s\gamma\| = 1 \,. \tag{1.1d}$$

The independent variables are the arc–length $s \in [0, L]$ and the time $t \in [0, \infty)$. By $\gamma(s,t) \in \mathbb{R}^2$ and $v(s,t) \in \mathbb{R}^2$ we denote the position and the velocity of a point located on the fiber's centerline at time t. Additionally, $u(s,t) \in \mathbb{R}$ describes the tangential velocity of a fluid particle moving along the centerline. By $A = A(s,t)$ we denote the cross–sectional area of the fiber. The parameter $\delta = 3/\mathrm{Re}$ is related to the Reynolds number Re and ε is the Rossby number, i.e. the inverse of the scaled rotation frequency. Equation (1.1a) is the continuity equation. Equation (1.1b) the momentum equation including viscous forces and Coriolis and centrifugal forces.

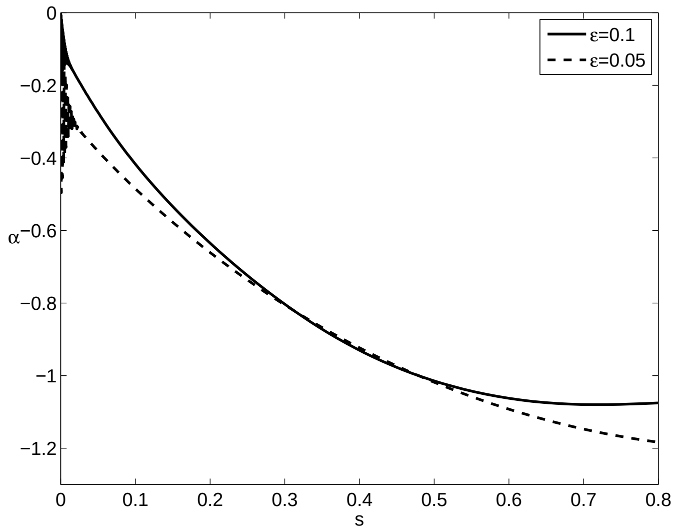

FIGURE 1.1. Numerical solution for the angle α for different values of ε. Note the instability close to $s = 0$.

Equation (1.1c) describes the fact that the total derivative of γ is given by v and finally, Equation (1.1d) is the condition for the arc length parametrization of the fiber which is defined by a normalized tangent.

In application relevant cases one is typically interested in stationary, i.e. time independent solutions for both δ and ε small. However, already for moderate values of δ, one faces severe numerical difficulties when solving the above system for small Rossby–numbers, e. g. $\varepsilon \sim 10^{-3}$, see Figure 1.1, where the angle α defined by $\partial_s\gamma = (\cos\alpha, \sin\alpha)$ is plotted, cf. [3].

The purpose of our present investigations is to identify in tractable terms those parameter regimes for δ and ε where physically relevant solutions exist.

This work is organized as follows: In Section 2, we rewrite the stationary version of the above system (1.1) in a more tractable form. Appropriate boundary conditions are specified. Section 3 is devoted to the inviscid case $\delta = 0$. The main result is contained in Section 4, where we characterize the solutions of the viscous problem $\delta > 0$ and show the non–existence of physically reasonable solution for some range of the parameters δ and ε. In Section 5 we present numerical simulations based on a solution of the boundary value problem in Eulerian coordinates. The obtained results confirm the analysis. Finally we draw some conclusions and suggest possible extensions of the model to overcome the problem of non–existing solutions.

2. THE STATIONARY CASE

In Eqn. (1.1a), the mass flux is given by Au. In the stationary case, the mass flux is constant and after appropriate scaling one gets $Au = 1$. Physical meaningful solutions are characterized by strictly positive u, hence $A = 1/u$. Furthermore, (1.1c) reads as $v = u\partial_s\gamma$ and inserting this into (1.1b) yields

$$\partial_s(u\,\partial_s\gamma) = \delta\partial_s\left(\frac{\partial_s u\,\partial_s\gamma}{u}\right) + \frac{2}{\varepsilon}\partial_s\gamma^\perp + \frac{1}{\varepsilon^2}\frac{1}{u}\gamma\,, \tag{2.1a}$$

$$\|\partial_s\gamma\| = 1\,. \tag{2.1b}$$

To eliminate the algebraic constraint (2.1b), we project $\partial_s\gamma$ onto the unit circle and parameterize $\partial_s\gamma := \tau = (\cos\alpha, \sin\alpha)$ by the angle $\alpha = \alpha(s)$. Furthermore, we use polar coordinates $\gamma = r(\cos\varphi, \sin\varphi)$ for the centerline and introduce $\beta = \alpha - \varphi$. Abbreviating derivatives with respect to s by a prime we transform (2.1a) via $q = u - \delta u'/u$ into a first order system which we supply with boundary conditions at $s = 0$ (nozzle) and $s = L$ (end point of the fiber):

$$\delta u' = u(u-q)\ , \qquad u(0) = 1\ , \tag{2.2a}$$

$$\varepsilon^2 u q' = r\cos\beta\ , \qquad q(L) = u(L)\ , \tag{2.2b}$$

$$r' = \cos\beta\ , \qquad r(0) = 1\ , \tag{2.2c}$$

$$\beta' = -\frac{2}{\varepsilon q} - \left(\frac{r^2}{\varepsilon^2 u q} + 1\right)\frac{\sin\beta}{r}\ , \qquad \beta(0) = 0\ . \tag{2.2d}$$

Remark 2.1. The above boundary conditions for u, r and β given at $s = 0$ expresses the fact that the fiber leaves the nozzle tangentially with a fixed speed. Due to the boundary condition for q at the end of the fiber, i.e. at $s = L$, the viscous stresses at the end of the fiber relax, i.e. $u'(L) = 0$. Note, that in the inviscid case $\delta = 0$, we have due to positive u the equation $u(s) = q(s)$ for all s.

Remark 2.2. The observable q is the internal energy of the fluid as the sum of the kinetic energy $Au^2 = u$ and the viscous stress energy $-\delta A u' = -\delta u'/u$. We observe from equation (2.2) that $q'(0) > 0$.

Remark 2.3. The polar angle φ of the centerline is determined via

$$\varphi' = \frac{\sin\beta}{r}\ , \qquad \varphi(0) = 0 \tag{2.3}$$

leading to $\varphi'(0) = 0$.

Remark 2.4 (Equations in Lagrangian coordinates)**.** For describing the regimes for δ and ε where physically relevant solutions of (2.2) exist, it is convenient to introduce Lagrangian coordinates. The passage from Eulerian to Lagrangian coordinates is settled on the transformation

$$\frac{ds(t)}{dt} = u(s(t)) > 0, \qquad s(0) = 0$$

mapping the Euler position s of a material point to the Lagrangian parameter t. Starting from the nozzle it takes a fluid particle t time units to approach position $\gamma(s)$. It takes a particle T time units to travel from the nozzle to the endpoint $s = L$.

After some elementary manipulations we obtain the Lagrangian system

$$\delta \dot{u}_L = u_L^2(u_L - q_L)\ , \qquad u_L(0) = 1\ , \tag{2.4a}$$

$$\varepsilon^2 \dot{q}_L = r_L\cos\beta_L\ , \qquad q_L(T) = u_L(T)\ , \tag{2.4b}$$

$$\dot{r}_L = u_L\cos\beta_L\ , \qquad r_L(0) = 1\ , \tag{2.4c}$$

$$\dot{\beta}_L = -\frac{2u_L}{\varepsilon q_L} - \left(\frac{r_L^2}{\varepsilon^2 q_L} + u_L\right)\frac{\sin\beta_L}{r_L}\ , \qquad \beta_L(0) = 0\ . \tag{2.4d}$$

Let $u_L(t) = u(s(t))$ be the velocity in the Lagrangian framework and mutatis mutandis for the other variables. By a dot, e.g.

$$\dot{u}_L = \frac{du_L(t)}{dt} = \frac{du(s(t))}{ds}\frac{ds(t)}{dt} = u'u\ ,$$

we denote the derivative with respect to the Lagrangian parameter.

3. Formal limits $\delta \to 0$ and $\varepsilon \to 0$

As outlined in the Introduction, we are in particular interested in cases where δ and ε are small. In this Section we shall exploit this fact in a formal manner.

In particular let us check whether or not the limits $\delta \to 0$ and $\varepsilon \to 0$ formally commute.

As small ε corresponds to a high rotation frequency one expects $u = O(1/\varepsilon)$ at least in the fiber's interior. Due to the definition of q one also expects $q = O(1/\varepsilon)$. Therefore it is convenient to introduce the re–scaled variables

$$v = \varepsilon u\ , \qquad w = \varepsilon q\ .$$

In terms of v, w, r and β the model equations (2.2) become

$$\delta\varepsilon v' = v(v - w)\ , \qquad v(0) = \varepsilon\ , \tag{3.1a}$$

$$vw' = r\cos\beta\ , \qquad v(L) = w(L)\ , \tag{3.1b}$$

$$r' = \cos\beta\ , \qquad r(0) = 1\ , \tag{3.1c}$$

$$\beta' = -\frac{2}{w} - \left(\frac{r^2}{vw} + 1\right)\frac{\sin\beta}{r}\ , \qquad \beta(0) = 0\ . \tag{3.1d}$$

In the — formal — inviscid limit $\delta = 0$ the system (3.1) together with the assumed positivity of v yields

$$v = w\ .$$

This is compatible with the boundary condition at $s = L$. Inserting $v = w$ into the equations we obtain after some elementary manipulations

$$r' = \cos\beta\ , \qquad r(0) = 1\ , \tag{3.2a}$$

$$\beta' = -\frac{2}{\sqrt{r^2 + \varepsilon^2 - 1}} - \frac{2r^2 + \varepsilon^2 - 1}{r^2 + \varepsilon^2 - 1}\frac{\sin\beta}{r}\ , \qquad \beta(0) = 0 \tag{3.2b}$$

with $v = \sqrt{r^2 - 1 + \varepsilon^2}$. We consider next the formal limit $\varepsilon = 0$ for system (3.2)

$$r_0' = \cos\beta_0\ , \qquad r_0(0) = 1\ , \tag{3.3a}$$

$$\beta_0' = -\frac{2}{\sqrt{r_0^2 - 1}} - \frac{2r_0^2 - 1}{r_0^2 - 1}\frac{\sin\beta_0}{r_0}\ , \qquad \beta_0(0) = 0\ . \tag{3.3b}$$

in terms of r_0 and β_0. This system can be solved analytically

$$r_0(s) = \sqrt{2s + 1}\ , \tag{3.4a}$$

$$\beta_0(s) = -\sqrt{2s} - \arctan\frac{\sqrt{2s}\cos\sqrt{2s} - \sin\sqrt{2s}}{\cos\sqrt{2s} + \sqrt{2s}\sin\sqrt{2s}}\ . \tag{3.4b}$$

A phase portrait of system (3.2) for ε small exhibits a similar behaviour as for system (3.3) in the formal limit $\varepsilon = 0$ which is shown in Figure 3.1.

We consider the limit–equation (3.3) in the phase–space $D = [1, \infty) \times [-\pi/2, 0]$. Note, that on D, we have that r_0 is increasing and β_0 is decreasing. For the parameter $s \to \infty$, the solution tends to $(r_0, \beta_0) = (\infty, -\pi/2)$. Interestingly, there exists a separatix β_s in D given by $\beta_s' = 0$, i.e. $\beta_s(r) = -\arcsin\frac{2r\sqrt{r^2-1}}{2r^2-1}$. Furthermore, the solution (3.4) has singular derivatives at $s = 0$. To resolve this singularity, we carry out an asymptotic expansion for $\beta \ll 1$ and get

$$\beta_0' \simeq -\frac{\beta_0}{2s} - \frac{\sqrt{2}}{\sqrt{s}}$$

and hence

$$\beta_0 \simeq -\sqrt{2s}\ .$$

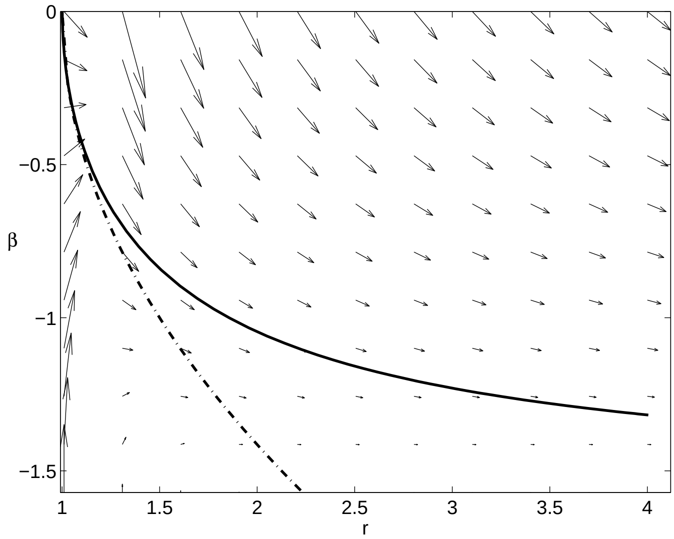

FIGURE 3.1. Phaseportrait of the $r_0\beta_0$–system for $\varepsilon = 0$. The solution trajectory is given by the solid line ('—') and the dash–dotted line ('– .') corresponds to the asymptotic expansion of the solution for $\beta_0 \ll 1$.

Now let us consider the opposite order of limits. If we set $\varepsilon = 0$ in (3.1) we will obtain a limiting system which is independent of δ. Based on the positivity of v in the interior of the fiber domain, we conclude again $v = w$ and hence end up formally again with system (3.3).

Remark 3.1. [Concavity of the velocity in Eulerian coordinates for $\delta = 0$] Considering system (3.2) the expression for u'' reads as

$$u'' = \frac{\left(\varepsilon^2 u^2 + 2\varepsilon r u \sin\beta - r^2\cos(2\beta)\right)}{\varepsilon^4 u^3}$$

and especially

$$u''(0) = \frac{\varepsilon^2 - 1}{\varepsilon^4} < 0$$

for $\varepsilon < 1$, i.e. in the case of high rotation frequencies.

Remark 3.2. [Convexity of u_L in Lagrangian coordinates for $\delta = 0$] We also investigate the second derivative of the Lagrangian velocity u_L, i.e. $\ddot{u}_L$. Using (2.4) with $\delta = 0$, we get

$$\varepsilon^2 \ddot{u}_L = u_L \left(1 + \frac{r\sin\beta}{\varepsilon u_L}\right)^2 > 0$$

provided $u_L > 0$. However, at the nozzle we always have

$$\ddot{u}_L(0) = \frac{1}{\varepsilon^2} > 0\,.$$

Hence, in the Lagrangian framework, the velocity is a convex function at least close to the nozzle. Note, that $\ddot{u} > 0$ implies that the acceleration of a fluid particle increases with its flight time. This agrees with the fact, that the acting forces, i.e. the centrifugal and Coriolis forces, also increase the further outward the fluid particle travels.

4. The viscous case

After the investigation of the inviscid case $\delta = 0$, we turn our attention to the boundary value problem (2.2) governing the viscous case

$$\delta u' = u(u-q)\ , \qquad u(0) = 1\ , \tag{4.1a}$$

$$\varepsilon^2 u q' = r\cos\beta\ , \qquad q(L) = u(L)\ , \tag{4.1b}$$

$$r' = \cos\beta\ , \qquad r(0) = 1\ , \tag{4.1c}$$

$$q\beta' = -\frac{2}{\varepsilon} - \left(\frac{r^2}{\varepsilon^2 u} + q\right)\frac{\sin\beta}{r}\ , \qquad \beta(0) = 0\ . \tag{4.1d}$$

Remark 4.1. From the above system, we directly obtain

$$q'(0) = \frac{1}{\varepsilon^2} \quad \text{and} \quad \beta'(0) = -\frac{2}{\varepsilon q_0} \tag{4.2}$$

with $q_0 = q(0)$.

Assumption 1. *We assume*

$$u'(0) \geq 0\ , \tag{4.3}$$

$$\beta'(0) < 0\ . \tag{4.4}$$

for the following physical reasons: the assumption (4.3) *assures, that the fiber is not accelerated into the nozzle. Due to* (4.3)*, we get*

$$0 \leq u'(0) = \frac{u}{\delta}(u-q)\Big|_0 = \frac{1-q_0}{\delta}$$

and hence

$$q_0 \leq 1\ .$$

From Remark 2.3 we observe that assumption (4.4) *is equivalent to* $\alpha'(0) = \beta'(0) - \varphi'(0) = \beta'(0) < 0$*. Hence the fiber initially bends* in *the sense of rotation of the drum. The opposite condition* $\alpha'(0) > 0$ *would imply, that the fiber initially bends* against *the direction of rotation of the drum. This is obviously counterintuitive and* physically not reasonable*. From* (4.4) *and* (4.2) *we obtain*

$$q_0 > 0\ .$$

Summarizing we have

$$q_0 \in (0,1]\ . \tag{Q}$$

Definition 1 (Physically relevant solutions)**.** *We call a solution* (u, q, r, β) *of the boundary value problem* (4.1) physically relevant*, if it satisfies the assumption* (Q)*.*

Remark 4.2. The solution of the inviscid case, i.e. the solution of (3.2) together with the according solutions for u and q is physically relevant.

To gain some insight into the behaviour of the solutions for small values of δ and ε, we carry out simulations for $\delta = 0.1$ and 0.01 and decrease ε. The numerical results are obtained with *Maple* using a midpoint rule with Richardson extrapolation on an adaptive mesh. As an initial guess for the solution we use the inviscid case $\delta = 0$. Table 4.1 documents the obtained results.

For $\delta = 0.1$ and $\delta = 0.01$ but smaller values of ε, our numerical method fails to solve the boundary value problem (4.1). Interestingly the numerical results indicate that $q_0(\varepsilon, \delta) \to 0$ and hence we may expect for fixed δ a limiting value $\varepsilon = \overline{\varepsilon}(\delta)$ such that $q_0(\overline{\varepsilon}(\delta), \delta) = 0$. At this point our assumption (Q) would be violated.

$\delta = 0.1$				$\delta = 0.01$			
ε	$q_0(\varepsilon,\delta)$	$u''(0)$	$\ddot{u}(0)$	ε	$q_0(\varepsilon,\delta)$	$u''(0)$	$\ddot{u}(0)$
1	0.9085	-0.0141	0.823	1	0.9901	-0.0073	0.973
0.5	0.7109	-2.7298	5.629	0.5	0.9623	-8.7741	5.440
0.3	0.3796	-10.584	27.90	0.25	0.8675	-99.145	76.479
0.25	0.2021	-16.534	47.14	0.1	0.42	-836.95	$2.53{\cdot}10^3$
0.23	0.1104	-20.925	58.22	0.0725	0.0761	-1251.0	$7.28{\cdot}10^3$
0.21	0.0102	-29.815	68.15	0.0705	0.0395	-1288.0	$7.94{\cdot}10^3$

Table 4.1. Results of the numerical simulations using Maple.

The numerical results documented in Table 4.1 indicate that the viscous solutions have common properties with the inviscid limit case $\delta = 0$:

$$u''(0) < 0, \tag{P1}$$

compare Remark 3.1 and

$$\ddot{u}_L(0) = u''(0)\,u(0)^2 + u'(0)^2\,u(0) > 0\ . \tag{P2}$$

compare Remark 3.2. Furthermore, using (4.3), (P1) assures

$$q'(0) = u'(0) - \delta\frac{u''(0)}{u(0)} + \delta\frac{u'(0)^2}{u(0)^2} > 0$$

as we already noted in Remark 2.2.

Let us briefly discuss (P1) and (P2).

Via differentiation we deduce from (4.1) the equation

$$\delta^2 u'' = \delta(2u-q)u' - \delta u q' = u(u-q)(2u-q) - \frac{\delta}{\varepsilon^2} r\cos\beta\ . \tag{4.5}$$

As a consequence we obtain at the nozzle, i.e. for $s = 0$

$$\delta^2 u''(0) = (1-q_0)(2-q_0) - \frac{\delta}{\varepsilon^2}\ , \tag{4.6}$$

such that via $q_0 \in (0,1]$,

$$\delta^2 u''(0) < 2 - \frac{\delta}{\varepsilon^2}\ ,$$

and therefore $u''(0) < 0$ for all δ and ε with $2\varepsilon^2 < \delta$. As this statement holds for fixed δ for all "small" parameters ε, inequality (P1) is herewith rigorously justified for small ε. In particular, for fixed δ (P1) does not result in a restiction on the values of ε as ε tends to 0.

We proceed similarly in the Lagrangian framework to obtain

$$\delta^2 \ddot{u}_L(0) = 2(1-q_0)^2 + (1-q_0) - \frac{\delta}{\varepsilon^2}\ , \tag{4.7}$$

such that the observed inequality $\ddot{u}_L(0) > 0$, see (P2), restricts due to $q_0 \in (0,1]$, the possible range of ε via

$$\frac{\delta}{\varepsilon^2} < 3\ .$$

In this case we obtain from (P2) a restriction on the possible values of ε as ε tends to 0. The problem ceases to be well posed, for small values of ε if we require condition (P2) for a solution. Interestingly, for the limiting cases in Table 4.1 we have

$$\delta/\varepsilon^2 = \frac{0.1}{0.21^2} \approx 2.268\ , \qquad \frac{0.01}{0.0705^2} \approx 2.012\ ,$$

which are both close to the critical value 3.

A somewhat more rigorous investigation is formulated as follows.

Theorem 4.3. *Assume* (u, q, r, β) *to be a physically relevant solution. Furthermore, assume that it satisfies additionally the inequalities* (P1) *and* (P2)*. Then, the following estimate for the initial value* q_0 *holds:*

$$\max\left(0, \underline{q_0}\right) \le q_0 \le \min\left(1, \overline{q_0}\right)$$

where

$$\underline{q_0} = \frac{3 - \sqrt{1 + 4\delta\varepsilon^{-2}}}{2} \, ,$$
$$\overline{q_0} = \frac{5 - \sqrt{1 + 8\delta\varepsilon^{-2}}}{4} \, .$$

Proof. Due to (4.6) we have

$$\delta^2 u''(0) = (1 - q_0)(2 - q_0) - \frac{\delta}{\varepsilon^2} = \mu \, .$$

To meet the inequality (P1), we require $\mu < 0$ and obtain

$$\underline{q_0} = \frac{3 - \sqrt{1 + 4\delta\varepsilon^{-2}}}{2} < q_0 < \frac{3 + \sqrt{1 + 4\delta\varepsilon^{-2}}}{2} \ge 1 \, .$$

Analogously, due to (4.7)

$$\delta^2 \ddot{u}_L(0) = 2(1 - q_0)^2 + (1 - q_0) - \frac{\delta}{\varepsilon^2} = \lambda \, .$$

Now, $\lambda \ge 0$ leads to

$$q_0 \ge \frac{5 + \sqrt{1 + 8\delta\varepsilon^{-2}}}{4} > 1 \quad \text{or} \quad q_0 \le \overline{q_0} = \frac{5 - \sqrt{1 + 8\delta\varepsilon^{-2}}}{4} \, .$$

Summarizing the individual estimates shows the assertion. □

Corollary 4.4 (Non–existence of physically relevant solutions)**.** *If*

$$3\varepsilon^2 < \delta$$

no physically relevant solution satisfying (P2) *exists.*

Proof. If $\overline{q_0} = 0$, physically relevant solutions cannot exist. □

Remark 4.5. The result of the above corollary can be partly understood, if the parameters are reinterpreted in their physical context. We have, that $\delta = 3/\mathrm{Re}$ is proportional to the kinematic viscosity ν. Furthermore ε, being the Rossby–number, is proportional to the inverse of the rotation frequency ω of the spinning drum. With this interpretation in mind, Corollary 4.4 states, that with some scaling constant $c > 0$,

- for $\nu < \overline{\nu} := \frac{c}{\omega^2}$ physically relevant solutions may exist and
- for $\nu > \overline{\nu}$ physically relevant solutions cannot exist.

Hence, for a given rotation frequency ω, fibers with a viscosity $\nu > \overline{\nu}$ cannot be spun.

Remark 4.6. Based on our definition of physically relevant solutions, we have $q_0 > 0$ and $q'(0) > 0$. For all numerical solutions we obtained $q > 0$ for all values of s. Suppose we would have a solution with $u > 0$ but $q < 0$. In this situtation, we had the bound

$$\delta u' = u^2 - uq > u^2 \, , \quad u(0) = 1$$

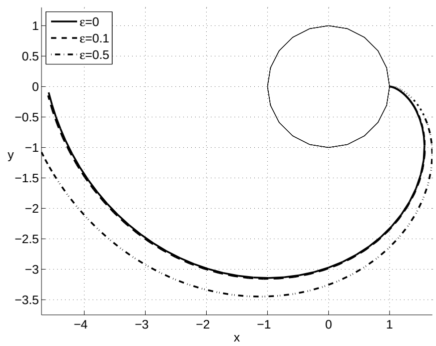

FIGURE 5.1. Trajectories of an inviscid fiber leaving the rotating drum for different values of ε.

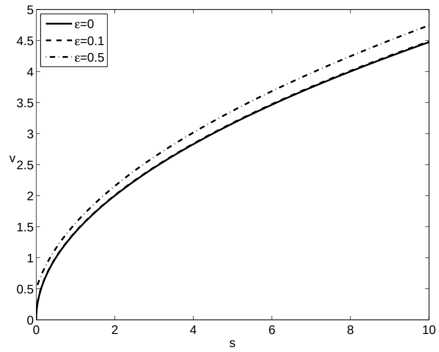

FIGURE 5.2. Velocity $v = \varepsilon u$ of an inviscid fiber for different values of ε.

and therefore

$$u(s) > \tilde{u}(s) = \frac{1}{1 - s/\delta}$$

where $\tilde{u}$ has a singularity at $s = \delta$. That means the solution does not exist on the interval $[0, L]$.

Remark 4.7. The crucial point for the non–existence of physically relevant solutions in the sense of Definition 1 is Assumption (4.4). An analogous model to (4.1) for a *non–rotating* process with a spatially increasing forces is

$$\delta u' = u(u - q) \ , \qquad u(0) = 1 \ ,$$
$$\varepsilon^2 u q' = 1 + s \ , \qquad q(L) = u(L) \ ,$$

where there force $1 + s$ models the effect of the spatially increasing centrifugal force $r \cos \beta$ in the rotational case. A analysis of this system yields the same bounds for q_0, except that there is *no reason* to assume $q_0 > 0$. However, for parameters ε and δ in the range of Corollary 4.4 one still finds solutions which are close to the inviscid solution, but where $q_0 < 0$. E.g. for $\delta = 0.01$ and $\varepsilon = 0.05$ and $L = 10$ one obtains $q(0) = -0.554$ from numerical computations.

5. Numerical results

The Figures 5.1 and 5.2 show the trajectories and velocities of the inviscid fiber for different values of ε. The graphs for $\varepsilon = 0$ and $\varepsilon = 0.1$ are almost indistinguishable and the result for $\varepsilon = 0.5$ is still close to the limit $\varepsilon = 0$.

Figure 5.3 shows the parameter values, for which the viscous simulations reported in Table 4.1 are carried out. The dotted area bounded by the curve $\delta = 3\varepsilon^2$ is the region, where according to corollary 4.4 no physical relevant solution exists. Obviously the estimate fits very well to the observed numerical behaviour. Figure 5.4 illustrates the behavior of q_0 for the two choices of δ and varying ε. The symbols indicate the numerical results, whereas the lines show the bounds for q_0 due to Theorem 4.3. As q_0 approaches 0 the values of ε^2 approach the limiting value $\delta/3$.

The Figures 5.5 and 5.6 show a comparison of the inviscid case $\delta = 0$ and the viscous case with $\delta = 0.1$, both for $\varepsilon = 0.25$. In Fig. 5.5 we plot the trajectories and Fig. 5.6 shows the velocities.

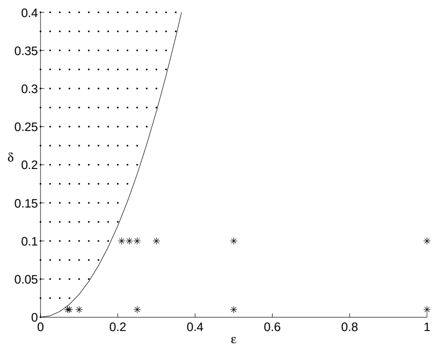

FIGURE 5.3. The ε and δ parameters for the simulations indicated by stars. In the dotted area no physical relevant solution can exist.

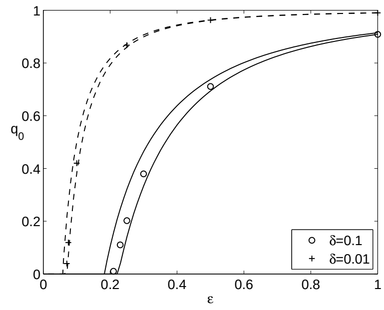

FIGURE 5.4. Plot of q_0 vs. ε for $\delta = 0.1$ and 0.01.

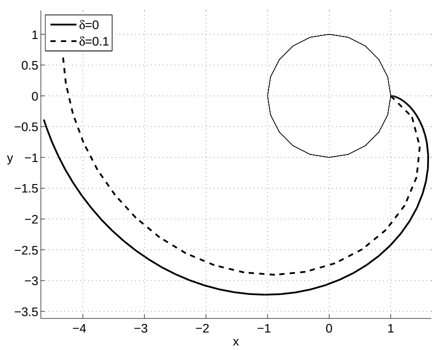

FIGURE 5.5. Trajectories in the inviscid $\delta = 0$ and the viscous case $\delta = 0.1$ for $\varepsilon = 0.25$.

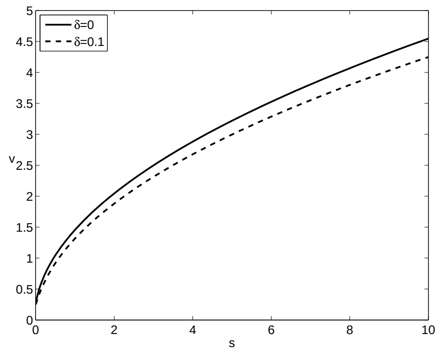

FIGURE 5.6. Velocity $v = \varepsilon u$ in the inviscid $\delta = 0$ and the viscous case $\delta = 0.1$ for $\varepsilon = 0.25$.

6. CONCLUSIONS

There is strong numerical evidence that the regime where physically relevant solutions exist for small values of δ and ε is characterized by the inequality

$$\delta < 3\varepsilon^2 \ . \tag{6.1}$$

Hence one cannot consider δ and ε as independent parameters but one has to couple δ and ε such that (6.1) holds. A possible choice is to require a functional dependence like

$$\delta = \delta(\varepsilon) = \kappa\varepsilon^2 \tag{6.2}$$

for some $\kappa \in (0, 3)$.

Naturally, one has to interpret a coupling of δ and ε like (6.2) in unscaled physical terms. Let us recall

$$\delta = \frac{3}{\mathrm{Re}} = \frac{3\nu}{UR} \ , \qquad \varepsilon = \frac{U}{\omega R} \ ,$$

where ν is the kinematic viscosity of the fluid, U is – in our setting – the exit velocity at the nozzle, R is the radius of the rotating drum and ω is the rotational frequency. Thus, in terms of unscaled quantities we deduce from (6.2) the identity

$$U = \sqrt[3]{3\nu\omega^2 R/\kappa}\,.$$

Consequently, U is determined by the kinematic viscosity, by the rotational frequency and the drum radius. Recalling the well–known effect of die–swell for liquid jets, see [8, 12], the above relation might give a hint on the fluid velocity and hence also on the radius of the fluid jet in the swelling region.

In future work, extensions of the model equations (1.1) will have to be taken into account. Whether including effects like gravity, surface tension and/or aerodynamic forces alters or eliminates the coupling of δ and ε will be a subject of futher research.

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