Fraunhofer Institut Techno- und Wirtschaftsmathematik

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Berichte des Fraunhofer ITWM, Nr. 112 (2007)

ISSN 1434-9973

Bericht 112 (2007)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

HYDRODYNAMIC LIMIT OF THE FOKKER–PLANCK EQUATION DESCRIBING FIBER LAY–DOWN PROCESSES

L. L. BONILLA, T. GÖTZ, A. KLAR, N. MARHEINEKE, AND R. WEGENER

Abstract. In this paper, a stochastic model [5] for the turbulent fiber lay-down in the industrial production of nonwoven materials is extended by including a moving conveyor belt. In the hydrodynamic limit corresponding to large noise values, the transient and stationary joint probability distributions are determined using the method of multiple scales and the Chapman-Enskog method. Moreover, exponential convergence towards the stationary solution is proven for the reduced problem. For special choices of the industrial parameters, the stochastic limit process is an Ornstein–Uhlenbeck. It is a good approximation of the fiber motion even for moderate noise values. Moreover, as shown by Monte–Carlo simulations, the limiting process can be used to assess the quality of nonwoven materials in the industrial application by determining distributions of functionals of the process.

Keywords. Stochastic Differential Equations, Fokker–Planck Equation, Asymptotic Expansion, Ornstein–Uhlenbeck Process

AMS Classification. 37H10, 34E13, 60H30, 65C05

1. Introduction

Nonwoven materials / fleece are webs of long flexible fibers that are used for composite materials (filters) as well as in the hygiene and textile industries. They are produced in melt-spinning operations: hundreds of individual endless fibers are obtained by the continuous extrusion of a molten polymer through narrow nozzles that are densely and equidistantly placed in a row at a spinning beam. The viscous / viscoelastic fibers are stretched and spun until they solidify due to cooling air streams. Before the elastic fibers lay down on a moving conveyor belt to form a

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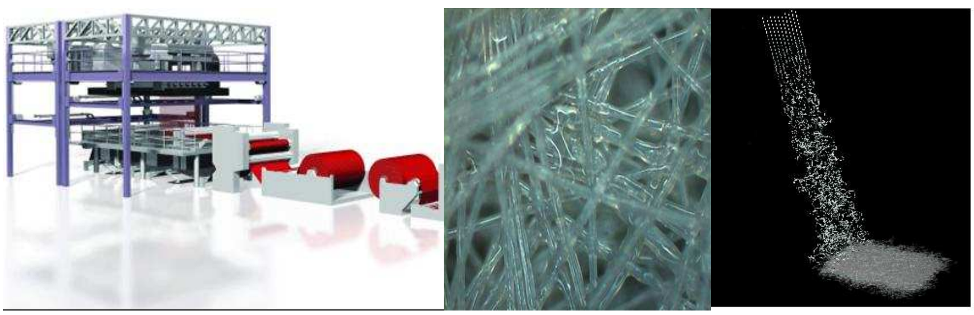

Figure 1.1. Production of nonwoven materials. Left to right: plant and fleece (Neumag, www.neumag.saurer.com), simulated process (computation by Fraunhofer ITWM (FIDYST), visualization by Fraunhofer IGD)

web, they become entangled and form loops due to the highly turbulent air flows. The homogeneity and load capacity of the fiber web are the most important textile properties for quality assessment of industrial nonwoven fabrics. The optimization and control of the fleece quality require modeling and simulation of fiber dynamics and lay-down. In addition, it is necessary to determine the distribution of fiber mass and directional arrangement in the web.

The software FIDYST, developed on basis of the mathematical model of [9] at the Fraunhofer ITWM, Kaiserslautern, enables numerical simulation of the spinning and deposition regime in the nonwoven production processes, cf. Figure 1.1. The interaction of the fiber with the turbulent air flows is described by a stochastic force in the momentum equation, which is derived, analyzed and experimentally validated in [11, 12]. The resulting force model depends on the flow velocity which is split into mean and random parts following Reynolds' idea for the averaged Navier-Stokes equations. The random force is modeled as white noise with a fluctuation-dependent amplitude that carries information of the kinetic turbulent energy, dissipation rate and correlation lengths. Due to the huge amount of physical details incorporated in FIDYST, the simulations of the fiber spinning and lay-down usually require an extremely large computational effort and high memory storage. Hence the optimization and control of the full process, and particularly of fleece quality, are difficult. Thus, a simplified stochastic model for the fiber lay-down process is presented in [5]. Under the assumption of a non-moving conveyor belt, this model describes the position of the fiber on the transport belt by a stochastic differential system containing parameters that characterize the process. For example, the effect of air turbulence has to be identified from the full model and adapted to be used in the reduced one. Parameter identification can be obtained from a FIDYST-simulation of a single, relatively short fiber whose computation time is short even using the more complex model. Then, the reduced model can be used to calculate fast and efficiently the performance of hundreds of long fibers for fleece production. In [5] the associated Fokker–Planck equation and stationary solution are investigated for the case of non-moving conveyor belt. In this case, the model without noise is conservative and its equations, Hamiltonian. For small turbulence noise, stochastic averaging can be used to derive a stochastic equation for the energy and related functionals of the stochastic process. Moreover, their distributions can be analyzed. An analytic investigation of the corresponding Fokker-Planck equation has been performed in [7], ergodicity of the process has been proven and explicit rates for the convergence to the stationary solution have been obtained.

In this paper, we extend the stochastic model of [5] to a more realistic fiber lay-down model with a moving transport belt, Section 2. In this case, the model equations are no longer Hamiltonian for zero noise. Both for moving and non-moving conveyor belts, we consider the case of large turbulence noise, $A \to \infty$, in which the probability density of the fiber becomes rapidly independent of the angle between the fiber and the direction of the conveyor's motion and the angle between the fiber and the position vector of its tip, respectively. In the case of a non-moving belt, Section 3 describes how to use the method of multiple scales in order to determine explicitly a reduced Smoluchowski equation for the fiber probability density, the stationary distribution and the transient joint probability distributions, all from the associated Fokker–Planck equation. For a moving belt, the same magnitudes are determined using the Chapman–Enskog method [4, 1] in Section 4. To leading order, the stationary distributions are of Gaussian type; in particular for special choices of the process parameters, Ornstein–Uhlenbeck processes turn out to be the limit solutions. In Section 5 exponential convergence towards the stationary solution of the reduced Fokker–Planck equation is proved by classical arguments. The numerical results in Section 6 show that direct Monte Carlo simulations of the fiber process agree quite well with the theoretical results even for moderate values of the noise strength A. In addition, certain functionals of the fiber (i.e. mass distributions) are essential for the quality assessment of nonwoven materials. We compare their distributions with the corresponding functionals for the limiting Ornstein–Uhlenbeck process.

2. The model

Consider a slender, elastic, non-extensible and endless fiber in a lay-down regime. Let the fiber be produced with the spinning speed v_{spin}, excited into motion by a surrounding highly turbulent air flow and laid down on a conveyor belt moving with the velocity v_{belt}. Due to its slenderness, the fiber laid on the two-dimensional transport belt is described as a curve $\eta : \mathbb{R}_0^+ \to \mathbb{R}^2$. Choosing arc-length parameterization, the non-extensibility condition $\|d\eta/dt\| = 1$ holds by setting

$$d\eta = (\cos\alpha, \sin\alpha)\, dt$$

where α denotes the angle of the fiber relative to the direction of motion e_1 of the transport belt. The reference point of the spinning process determined by the position of the nozzle moves in the coordinate system of the transport belt in the direction $-e_1$. Thus,

$$\xi(t) = \eta(t) - (-\kappa t e_1)$$

describes the deviation of the fiber from the reference point as a function of the arc-length parameter t, where $\kappa = v_{belt}/v_{spin} \in [0, 1]$ is the ratio between the belt and spinning speeds. Generalizing [5], we model (ξ, α) by the following stochastic differential system

$$d\xi_1 = (\cos\alpha + \kappa)\, dt \tag{2.1a}$$

$$d\xi_2 = \sin\alpha\, dt \tag{2.1b}$$

$$d\alpha = c(\xi)\, (\xi_1 \sin\alpha - \xi_2 \cos\alpha)\; dt + A\, dW_t. \tag{2.1c}$$

Here, the change of the angle α is characterized by the deterministic buckling / coiling c of the fiber (that tends to turn it back to its reference point) and by the random fluctuations $A\, dW_t$ due to the interaction of the fiber with the external turbulent air flow. W denotes an one-dimensional Wiener process.

Remark 2.1. The general deterministic coiling behavior of flexible fibers has been studied for example in [10, 8]. The function c in our model prescribes its amplitude that depends on the lay-down process. c is a scalar-valued function for isotropic processes and a matrix-valued one for anisotropic processes, [5]. For reasons that will become clear later on, cf. Eq. (4.9), physically reasonable solutions can be expected only if $\exp(-B(\xi) - k\xi_1)$ is integrable for $k \in \mathbb{R}$, where $\partial_{\xi_i} B(\xi) = c(\xi)\xi_i$. A typical example satisfying this condition is $c(\xi) = 1$ since then $B(\xi) = (\xi_1^2 + \xi_2^2)/2$. □

Remark 2.2. The isotropic model considered here can be treated as dimensionless with $c(e_1) = 1$, for anisotropic lay-down processes with $1/2\,\mathrm{tr}(\mathrm{c}(\mathrm{e}_1)) = 1$. This corresponds to a scaled throwing (lay-down) range of order one. Consequently, the noise amplitude A characterizes the relation between stochastic and deterministic rates in the behavior of the system. □

To illustrate our previous considerations, realizations of the processes η and ξ are exemplified in Figure 2.1 (left) and 2.2, respectively, where the parameters (A, κ) are selected in the set $(A, \kappa) = \{(0.79, 0.1), (2.23, 0.1), (2.23, 0.8)\}$, and $c(\xi) = 1$ is fixed. Superposing many fibers, i.e. η-paths, generates a nonwoven material whose properties depend on the industrial control parameters A, κ and c, see Figure 2.1 (right)

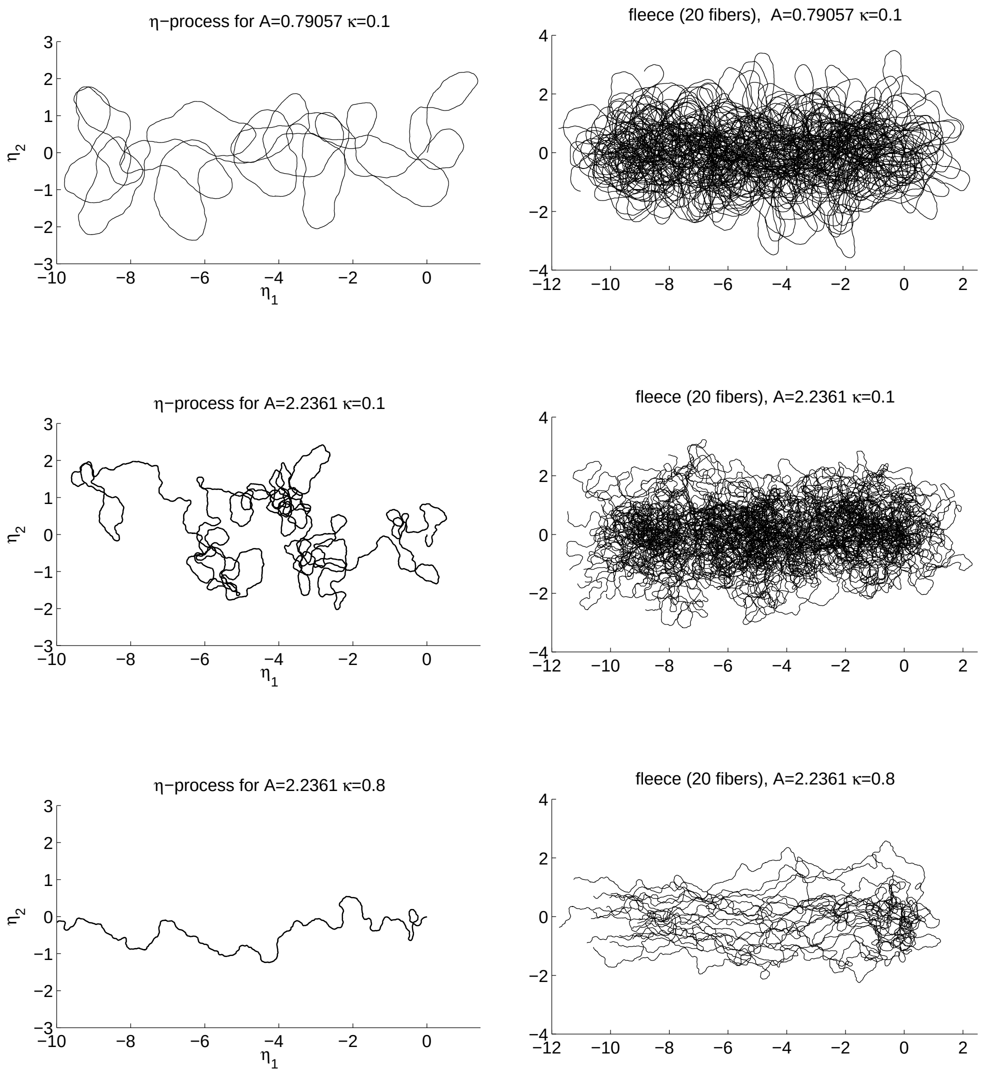

FIGURE 2.1. Left: η-path. Right: Associated fleece (20 fibers). Top to bottom: $(A, \kappa) = \{(0.79, 0.1), (2.23, 0.1), (2.23, 0.8)\}$

for 20 fibers. In this figure, the distance between two neighboring spinning nozzles is $d_{spin} = 2.5 \cdot 10^{-3}$, fleece length is $L_{fleece} = 10$ and fiber length is $T = L_{fleece}/\kappa$. For $\kappa \to 1$ the belt velocity coincides with the spinning speed such that the fibers lay down almost straight independent of turbulence noise. The smaller κ is, the more fiber material (length) can become entangled and form loops. The size of the loops is thereby determined by the amplitude of the turbulence noise A. For small A the deterministic coiling / buckling radius dominates the fiber behavior, whereas a finer entanglement on various scales arises for large A. For the industrial application, nonwoven materials with a homogeneous distribution of mass and fiber orientation are desirable, and they typically have these characteristics for small κ and larger A. To get a deeper insight into the probability density of the underlying ξ-process

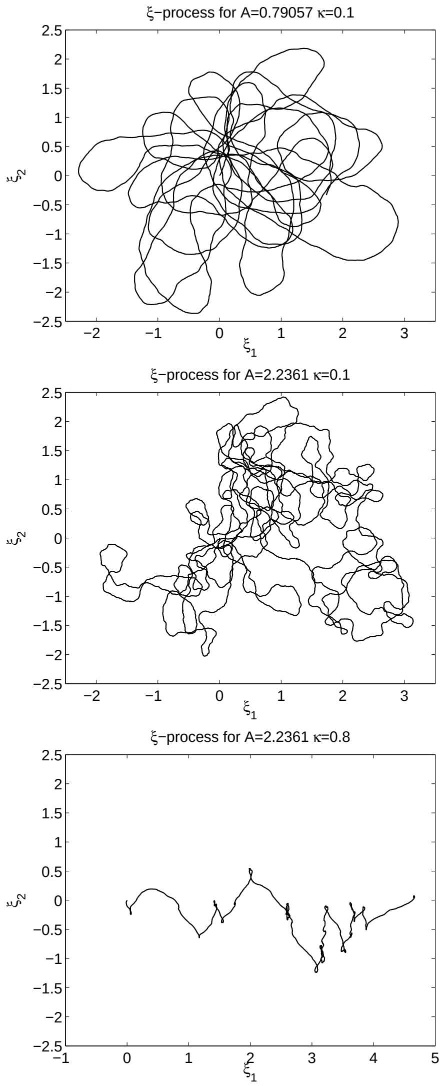

FIGURE 2.2. ξ-path, corresponding to Fig. 2.1. Top to bottom: $(A, \kappa) = \{(0.79, 0.1), (2.23, 0.1), (2.23, 0.8)\}$

(2.1), $p = p(\xi_1, \xi_2, \alpha, t)$, we consider its associated Fokker–Planck equation

$$\partial_t p + (\cos\alpha + \kappa)\,\partial_{\xi_1} p + \sin\alpha \partial_{\xi_2} p - \partial_\alpha \left[c(\xi)(-\xi_1 \sin\alpha + \xi_2 \cos\alpha)p\right] = \frac{A^2}{2}\partial_\alpha^2 p \,. \tag{2.2}$$

Remark 2.3. In the case of a non-moving conveyor belt ($\kappa = 0$), the processes η and ξ coincide. Then, it is advantageous to introduce polar coordinates $\xi_1 = r\cos\varphi$, $\xi_2 = r\sin\varphi$ and $\beta = \alpha - \varphi$ and to define $b(r) = \|\xi\|\, c(\|\xi\|)$ as done in [5]. The resulting system reduces then to two dimensions and the associated Fokker–Planck equation for (r, β) reads

$$\partial_t p + \cos\beta \partial_r p + \left(b(r) - \frac{1}{r}\right) \partial_\beta \left(p \sin\beta\right) = \frac{A^2}{2}\partial_\beta^2 p \tag{2.3}$$

□

In the following we determine the evolution and the stationary solution of the Fokker–Planck equations (2.2), (2.3) in the limit as $A \to \infty$. Note, since we embed our model in the context of dynamical systems and stochastic processes, we refer occasionally to the notation and interpretation of time for the fiber arc-length t.

3. The non–moving conveyor belt

We start our investigation with the case of a non–moving belt. This case is quite instructive and allows to introduce the main ideas to tackle also the case of a moving belt. Let $\varepsilon = 1/A^2 \ll 1$. As already mentioned above, we introduce polar coordinates and obtain the following Fokker–Planck equation:

$$\partial_t p + \cos\beta \partial_r p + \left(b(r) - \frac{1}{r}\right) \partial_\beta (p \sin\beta) = \frac{1}{2\varepsilon} \partial_\beta^2 p \tag{3.1a}$$

for the density distribution $p(r, \beta, t)$ subject to the normalization condition

$$\int_{\mathbb{R}_+ \times [-\pi,\pi]} p(r, \beta, t)\, dr\, d\beta = 1 \tag{3.1b}$$

and the initial condition

$$p(r, \beta, 0) = p_0(r, \beta). \tag{3.1c}$$

Note that the stochastic term only appears in the angular coordinate. Hence, for dominating stochastic forcing, i.e. $\varepsilon \ll 1$, we expect a fast averaging over the β–coordinate. Dominant balance between diffusion and the time derivative of p implies a fast time scale $\tau = t/\varepsilon$. The relaxation to the stationary distribution will take much longer.

To capture the fast averaging over β and the slower convergence to the stationary solution, we use the method of multiple scales. Let us introduce two time scales: the fast scale $\tau = t/\varepsilon$ and a slow scale $T = \varepsilon t$. For the distribution function $p = p(r, \beta, t; \varepsilon)$ (which is 2π-periodic in β), we propose the following ansatz:

$$p = p^{(0)}(r, \beta, \tau, T) + \varepsilon p^{(1)}(r, \beta, \tau, T) + \varepsilon^2 p^{(2)}(r, \beta, \tau, T) + \dots \tag{3.2}$$

Inserting (3.2) into (3.1) and equating equal powers of ϵ in the resulting equations, we obtain a hierarchy of problems for the $p^{(m)}$. As we shall see, secular terms appear only in the equation for $p^{(2)}$, and their elimination requires the introduction of the slow scale $T = \epsilon t$. To leading order, we have to solve

$$Lp^{(0)} = 0 \tag{3.3a}$$

$$\int_{\mathbb{R}_+ \times [-\pi,\pi]} p^{(0)}\, dr\, d\beta = 1 \tag{3.3b}$$

$$p^{(0)}(r, \beta, 0, 0) = p_0(r, \beta) \tag{3.3c}$$

where $L = \partial_\tau - \partial_\beta^2/2$ denotes the diffusion operator in the angular direction. Solving the parabolic equation (3.3a) yields

$$p^{(0)}(r, \beta, \tau, T) = \frac{1}{2\pi} \mathcal{P}(r, T) + \sum_{j \in \mathbb{Z} \setminus \{0\}} e^{ij\beta - j^2 \tau/2} C_j(r) \tag{3.4a}$$

where

$$C_j(r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ij\beta} p_0(r, \beta)\, d\beta \tag{3.4b}$$

and

$$\mathcal{P}(r, 0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} p_0(r, \beta)\, d\beta \tag{3.4c}$$

are the Fourier–coefficients of the initial condition.

In the case of a rotational symmetric initial distribution $p_0 = p_0(r)$, all the coefficients C_j vanish identically. If the initial distribution is not symmetric, the angular components $C_j e^{ij\beta - j^2\tau/2}$ are exponentially decaying with τ, i.e. the angular dependence of p is averaged out on the fast time scale τ. The relaxation to the stationary solution is determined by the behavior of $\mathcal{P}(r,T)$ on the long time scale T. Therefore, we will neglect the exponentially small terms $C_j e^{ij\beta - j^2\tau/2}$ in the following.

To determine the stationary solution $\mathcal{P}$, we proceed with the next terms of the expansion (3.2). The $\mathcal{O}(\varepsilon)$–problem reads as

$$Lp^{(1)} = -\frac{\cos\beta}{2\pi}\left[\partial_r\mathcal{P} + \left(b(r) - \frac{1}{r}\right)\mathcal{P}\right]$$

$$\int_{\mathbb{R}_+\times[-\pi,\pi]} p^{(1)}\,dr\,d\beta = 0$$

$$p^{(1)}(r,\beta,0,0) = 0.$$

Again, solving the above parabolic problem, yields

$$p^{(1)} = \frac{\mathcal{A}(r,T)}{2\pi} - \frac{\cos\beta}{\pi}\left[\partial_r\mathcal{P}(r,T) + \left(b - \frac{1}{r}\right)\mathcal{P}(r,T)\right],$$

where $\mathcal{A}(r,T)$ is a solution of the homogeneous problem, $L\mathcal{A} = 0$, such that $\int_0^\infty \mathcal{A}(r,T)\,dr = 0$ (normalization condition). At this order, we have two functions, $\mathcal{P}$ and $\mathcal{A}$, not yet determined. Hence, we proceed to the second order

$$\begin{aligned}
Lp^{(2)} = {} & \frac{\cos^2\beta}{\pi}\partial_r\left[\partial_r\mathcal{P} + \left(b - \frac{1}{r}\right)\mathcal{P}\right] \\
& + \left(b - \frac{1}{r}\right)\left[\partial_r\mathcal{P} + \left(b - \frac{1}{r}\right)\mathcal{P}\right]\partial_\beta\frac{\sin\beta\cos\beta}{\pi} - \frac{1}{2\pi}\partial_T\mathcal{P} \\
& - \frac{\cos\beta}{2\pi}\left[\partial_r\mathcal{A} + \left(b(r) - \frac{1}{r}\right)\mathcal{A}\right] \\
= {} & \frac{1+\cos 2\beta}{2\pi}\partial_r\left[\partial_r\mathcal{P} + \left(b - \frac{1}{r}\right)\mathcal{P}\right] - \frac{1}{2\pi}\partial_T\mathcal{P} \\
& + \left(b - \frac{1}{r}\right)\left[\partial_r\mathcal{P} + \left(b - \frac{1}{r}\right)\mathcal{P}\right]\frac{\cos 2\beta}{\pi} \\
& - \frac{\cos\beta}{2\pi}\left[\partial_r\mathcal{A} + \left(b(r) - \frac{1}{r}\right)\mathcal{A}\right]
\end{aligned}$$

To ensure the boundedness of $p^{(2)}$, the average of the right hand side of the preceding equation over β should vanish. Otherwise a secular term proportional to T would be part of the solution $p^{(2)}$. This solvability condition yields

$$\partial_T\mathcal{P} = \partial_r\left[\partial_r\mathcal{P} + \left(b(r) - \frac{1}{r}\right)\mathcal{P}\right] \tag{3.5a}$$

where $\mathcal{P}$ also satisfies the normalization condition

$$\int_{\mathbb{R}_+} \mathcal{P}(r,T)\,dr = 1 \tag{3.5b}$$

the initial condition (3.4c) and

$$\mathcal{P}(0,T) = \mathcal{P}(\infty,T) = 0\,. \tag{3.5c}$$

Equation (3.5) is the reduced Fokker–Planck (Smoluchowski) equation, which determines the leading order approximation to the solution of the system (3.1), in the limit as $\varepsilon \to 0$, i.e. for dominating stochastic forcing.

The stationary solution $\mathcal{P}_s(r)$ satisfying (3.5) is given by

$$\mathcal{P}_s(r) = k\, re^{-B(r)} \tag{3.6}$$

where $B'(r) = b(r)$ and k is the normalization constant. Note that $\mathcal{P}_s(r)$ is independent of the noise strength A, and is also the stationary solution of the full Fokker–Planck equation (3.1). The limiting stochastic differential equation (SDE) associated to (3.5) reads

$$dr = -\left(b(r) - \frac{1}{r}\right) dT + \sqrt{2}\, dW_T.$$

Remark 3.1. In the generic case $b(r) = r$, we obtain $B(r) = r^2/2$ and the stationary solution

$$\mathcal{P}_s(r) = re^{-r^2/2}\ .$$

that is a rotational symmetric Gaussian distribution centered at the origin with variance 1. The solution of its associated SDE

$$dr = -\left(r - \frac{1}{r}\right) dT + \sqrt{2}\, dW_T$$

is a radially symmetric Ornstein–Uhlenbeck process. This can be concluded from the Fokker–Planck equation of the reduced process (3.5). Defining the function $\tilde{\mathcal{P}}(\xi) = \mathcal{P}(r)/r$ for $\xi = (\xi_1, \xi_2)$ and $r = \sqrt{\xi_1^2 + \xi_2^2}$ we obtain

$$\partial_T \tilde{\mathcal{P}} = \nabla_\xi \cdot (\nabla_\xi + c(\xi)\xi)\, \tilde{\mathcal{P}} \tag{3.7}$$

with the associated SDE

$$d\xi = -c(\xi)\xi\, dT + \sqrt{2}\, dW_T.$$

For our special case $c(\xi) = 1$, the solution is the Ornstein–Uhlenbeck process. The probability density for this case can be calculated explicitely, as we will do in the next section. □

Remark 3.2. A direct solution of equation (3.5) for $b(r) = r$ can be performed in terms of a series expansion in Laguerre polynomials. For a normalized initial distribution we obtain

$$\mathcal{P} = re^{-\frac{r^2}{2}} + a_1 e^{-2T} r(1 - \frac{r^2}{2})e^{-r^2/2} + \sum_{\nu=2}^{\infty} a_\nu e^{-2\nu T} re^{-r^2/2} L_\nu(\frac{r^2}{2})$$

where the expansion coefficients are determined by the initial distribution:

$$a_\nu = \frac{\int_{\mathbb{R}_+ \times [-\pi,\pi]} p_0(r,\beta)\, L_\nu(\frac{r^2}{2})\, dr d\beta}{2\pi \int_0^\infty e^{-x} [L_\nu(x)]^2\, dx}$$

□

Remark 3.3. We consider the full Fokker–Planck equation (3.1). Even with a rotationally symmetric initial condition and the rotationally symmetric stationary solution (3.6), terms depending on the angle β appear at intermediate times. This can be seen by computing the next term in the expansion (3.2)

$$p^{(1)} = p^{(1)}(r, \beta, T) = -\frac{\cos\beta}{\pi} \left[\partial_r \mathcal{P} + \left(b(r) - \frac{1}{r}\right) \mathcal{P}\right],$$

which depends on β even though the initial condition and the stationary solution do not. This could have been already anticipated from the full Fokker–Planck equation, which does not admit time–dependent rotationally symmetric solutions. □

4. The case of a moving conveyor belt

In the case of a moving belt, the Fokker–Planck equation (2.2) reads as

$$\partial_t p + ((s + \kappa e_1)\cdot \nabla_\xi)\, p - \partial_\alpha \left[c(\xi)\,(n\cdot\xi)\, p\right] = \frac{1}{2\varepsilon}\partial_\alpha^2 p\ . \tag{4.1}$$

where $s = (\cos\alpha, \sin\alpha)$ and $n = \partial_\alpha s = (-\sin\alpha, \cos\alpha)$ as well as $\varepsilon = 1/A^2$ are introduced to simplify the notations. The density distribution p satisfies the normalization condition

$$\int_{\mathbb{R}^2\times[-\pi,\pi]} p(\xi,\alpha,t)\, d\xi\, d\alpha = 1\ .$$

Additionally we have the initial condition

$$p(\xi,\alpha,0) = p_0(\xi,\alpha)\ .$$

In the case of strong stochastic influence, i.e. $\varepsilon \ll 1$, we would like to follow the main ideas of the previous case for $\kappa = 0$, i.e. the non–moving belt. However, the term proportional to κ generates secular terms in the equation for $p^{(1)}$. This indicates that the slow scale needed to rid of the secular terms should be t. To leading order, the method of multiple scales would then give a hyperbolic reduced equation that does not describe the even slower relaxation towards a stationary solution on the scale $T = \epsilon t$. We need a perturbation method that yields a reduced equation with terms of different order in ϵ: the Chapman-Enskog method. As explained in [4] and [1], the Chapman-Enskog ansatz for the probability density is

$$p(\xi,\alpha,t;\epsilon) = \frac{1}{2\pi}\,\mathcal{P}(\xi,t;\epsilon) + \epsilon\, p^{(1)}(\xi,\alpha;\mathcal{P}) + \epsilon^2 p^{(2)}(\xi,\alpha;\mathcal{P}) + o(\epsilon^2). \tag{4.2}$$

The first term in this equation solves the leading order problem $\partial_\alpha^2 p = 0$. We have anticipated that after a transient in the fast scale $\tau = \epsilon t$, the slowly-varying density $\mathcal{P}$ becomes independent on α, as shown by the method of multiple scales. Of course, this ignores an initial layer that can be inferred from (3.4a): An additional term corresponding to $\sum_{j\in\mathbb{Z}\setminus\{0\}} e^{ij\beta - j^2 t/(2\epsilon)} C_j(r)$ in (3.4a) should be added to (4.2) to account for the effect of initial conditions, so that the probability density becomes

$$\begin{aligned} p(\xi,\alpha,t;\epsilon) = \frac{1}{2\pi}\,\mathcal{P}(\xi,t;\epsilon) + \sum_{j\in\mathbb{Z}\setminus\{0\}} \frac{e^{ij\alpha - j^2 t/(2\epsilon)}}{2\pi} \int_{-\pi}^{\pi} e^{-ija} p_0(\xi,a)\, da \\ +\epsilon\, p^{(1)}(\xi,\alpha;\mathcal{P}) + \epsilon^2 p^{(2)}(\xi,\alpha;\mathcal{P}) + o(\epsilon^2). \end{aligned} \tag{4.3}$$

The higher order terms $p^{(m)}$ depend on time only through their dependence on $\mathcal{P}$. Moreover, up to terms of order ϵ^2, we have

$$\partial_t \mathcal{P} = F^{(0)} + \varepsilon F^{(1)}\ . \tag{4.4}$$

$F^{(m)}$ are functionals of $\mathcal{P}$ to be determined so that the $p^{(m)}$ are bounded and 2π-periodic in α. Inserting (4.2) and (4.4) in (4.1), we find a hierarchy of problems. To ensure that $\mathcal{P}$ contains all the contributions from the homogeneous equations in the hierarchy, we have to impose the additional constraints

$$\int_{-\pi}^{\pi} p^{(m)}\, d\alpha = 0, \quad m = 1, 2, \ldots . \tag{4.5}$$

The following problem corresponds to the terms of order $\mathcal{O}(\varepsilon)$:

$$-\frac{1}{2}\,\partial_\alpha^2 p^{(1)} = -\,(s\cdot\nabla_\xi)\,\mathcal{P} - \kappa\partial_{\xi_1}\mathcal{P} - \partial_\alpha(c(\xi)\,(s\cdot\xi)\,\mathcal{P}) - F^{(0)}\ ,$$

together with (4.5). This problem has a normalized solution which is 2π-periodic in α provided the average over one period of the right hand side of the linear equation vanishes. This solvability condition yields $F^{(0)}$:

$$0 = \kappa \partial_{\xi_1} \mathcal{P} + F^{(0)} \tag{4.6}$$

This condition means that the transport of $\mathcal{P}$ with the belt velocity κ in the ξ_1–direction occurs on the original time scale t. Furthermore, we get

$$p^{(1)} = -2 \left[s \cdot (\nabla_\xi + c(\xi)\xi)\, \mathcal{P} \right],$$

which satisfies (4.5) for $m = 1$. Note that we have not added a term $\mathcal{A}(\xi, t)/(2\pi)$ to the right hand side of this equation because of the condition (4.5) ensuring that all solutions of the homogeneous equation $\partial_\alpha^2 \mathcal{A} = 0$ are included in $\mathcal{P}(\xi, t; \epsilon)$.

To determine the reduced Fokker–Planck equation in analogy to (3.5), we have to consider again the problem provided by terms of order $\mathcal{O}(\varepsilon^2)$

$$\begin{aligned} -\frac{1}{2}\, \partial_\alpha^2 p^{(2)} = &- (s \cdot \nabla_\xi)\, p^{(1)} - \kappa \partial_{\xi_1} p^{(1)} - \partial_\alpha \left[c(\xi)\, (n \cdot \xi)\, p^{(1)} \right] \\ &- F^{(1)} + 2 \left[s \cdot (\nabla_\xi + c(\xi)\xi)\, F^{(0)} \right], \end{aligned}$$

together with (4.5). The solvability condition that the average of the right hand side over one period in α should vanish yields $F^{(1)}$:

$$0 = \nabla_\xi \cdot (\nabla_\xi + c(\xi)\xi)\, \mathcal{P} - F^{(1)} \tag{4.7}$$

Inserting the conditions (4.6) and (4.7) in equation (4.4) yields the reduced equation

$$\partial_t \mathcal{P} = \nabla_\xi \cdot (\varepsilon \nabla_\xi + \varepsilon c(\xi)\xi - \kappa e_1)\, \mathcal{P}\,. \tag{4.8}$$

This is the analogon to (3.7), the difference lies in the transport term $\kappa \partial_{\xi_1} \mathcal{P}$. The stationary solution $\mathcal{P}_s(\xi)$ is characterized by

$$\nabla \cdot (\varepsilon \nabla + \varepsilon c(\xi)\xi - \kappa e_1)\, \mathcal{P}_s = 0$$

together with the normalization condition

$$\int_{\mathbb{R}^2} \mathcal{P}_s \, d\xi = 1\,.$$

The solution of this linear PDE is given by

$$\mathcal{P}_s(\xi) = k e^{-B(\xi) - \kappa \xi_1/\varepsilon} \tag{4.9}$$

where $\nabla B(\xi) = c(\xi)\xi$ and k is the normalization constant. The associated SDE is

$$d\xi = -\epsilon c(\xi)\xi dt + \kappa e_1 dt + \sqrt{2\epsilon}\, dW_t$$

Remark 4.1. In the case of a moving conveyor belt, the stationary distribution (4.9) depends on the noise, as $A = 1/\sqrt{\varepsilon}$. This contrasts with the case of the non-moving belt, $\kappa = 0$, in which the stationary distribution is the same for deterministic ($A = 0$) or stochastic ($A > 0$) dynamics. Obviously, we obtain a stationary distribution independent of ε in the limit as $\varepsilon \to 0$ only if κ is proportional to $\varepsilon = 1/A^2$. This means, we deal with the case of large A and small κ, and the turbulence noise happens to be of order $1/\sqrt{\kappa}$. □

Remark 4.2. As in the case of the non-moving belt, we consider the special case $c(\xi) = 1$, i.e. $b(r) = r$. Then, $B(\xi) = \xi_1^2/2 + \xi_2^2/2$ and we obtain the Ornstein–Uhlenbeck type process prescribed by

$$d\xi = -\epsilon \xi \, dt + \kappa e_1 \, dt + \sqrt{2\epsilon}\, dW_t \tag{4.10}$$

or respectively

$$\partial_t \mathcal{P} = \nabla \cdot (\varepsilon \nabla + \varepsilon \xi - \kappa e_1)\, \mathcal{P}\,.$$

Its stationary density distribution is Gaussian, centered at $\mu = (\kappa/\varepsilon, 0)$ with variance $\sigma^2 = 1$

$$\mathcal{P}_s(\xi) = \frac{1}{2\pi} e^{-(\xi_1 - \kappa/\varepsilon)^2/2 - \xi_2^2/2} \,. \tag{4.11}$$

□

To investigate the relaxation to the stationary solution in more detail, we focus on the case $c(\xi) = 1$. To compute the density of the process explicitely, we assume, that the initial distribution is a Dirac delta at some point $\mu_0 \in \mathbb{R}^2$. We make the following ansatz for the transient distribution

$$\mathcal{P}(\xi, t) = \frac{f(t)}{2\pi} e^{-(\xi - \mu(t)/\varepsilon)^2/(2\sigma(t))} \,,$$

i.e. a Gaussian with moving center $\mu(t)$, variance $\sigma^2(t)$ and normalization constant $f(t)$. Plugging this ansatz into the reduced Fokker–Planck equation (4.8) and equating for all ξ_1, ξ_2 yields after some calculations

$$\begin{aligned} \frac{d\mu}{dt} &= \varepsilon\,(\kappa e_1 - \mu) \\ \frac{d\sigma}{dt} &= 2\varepsilon\,(1 - \sigma) \\ \frac{df}{dt}\sigma + f\frac{d\sigma}{dt} &= 0 \end{aligned}$$

Together with the initial conditions $\mu(0) = \mu_0$, $\sigma(0) = 0$ and $f(0) = 1$, we obtain $f = 1/\sigma$ and the following motions of the mean and the standard deviation

$$\begin{aligned} \mu(t) &= \kappa e_1 (1 - e^{-\varepsilon t}) + \mu_0 e^{-\varepsilon t} \\ \sigma(t) &= 1 - e^{-2\varepsilon t}. \end{aligned}$$

Compare this result with the explicit solution formulas for linear stochastic differential equations in [3].

Remark 4.3. Note that the relaxation to the stationary solution, i.e. $\mu = \kappa e_1$ and $\sigma = 1$, happens on the slow time scale $T = \varepsilon t$. Furthermore the decay rate for the standard deviation is twice the decay rate of the mean value. □

5. Convergence of the reduced Fokker–Planck equation

In the previous section we have derived the reduced Fokker–Planck equation (4.8)

$$\partial_t \mathcal{P} = \nabla \cdot (\varepsilon \nabla \mathcal{P} + (\varepsilon c \xi - \kappa e_1)\,\mathcal{P})$$

in the case of dominating stochastic forcing $A^2 = 1/\varepsilon \gg 1$. The "relative velocity" κ of the lay-down process as well as the function $c = c(\xi)$ governing the deterministic fiber bending are still arbitrary. The stationary distribution $\mathcal{P}_s$ of (4.9) is of Gaussian type

$$\mathcal{P}_s(\xi) = k e^{-B(\xi) - \kappa \xi_1/\varepsilon}$$

with $\nabla B(\xi) = c(\xi)\xi$.

The convergence against this stationary solution can be proven by classical arguments, see e.g. [2] for a recent discussion. Let us introduce the Kullback-Leibler relative entropy

$$S = \int \mathcal{P} \ln \frac{\mathcal{P}}{\mathcal{P}_s} \,. \tag{5.1}$$

Clearly, $S \geq 0$. The rate of dissipation of the entropy is given by

$$\partial_t S = \int \partial_t \mathcal{P} \ln \frac{\mathcal{P}}{\mathcal{P}_s} = \int \ln \frac{\mathcal{P}}{\mathcal{P}_s} \nabla \cdot [\varepsilon \nabla \mathcal{P} + (\varepsilon c \xi - \kappa e_1)\mathcal{P}]$$

and after integration by parts

$$\partial_t S = -\int \left[\nabla \ln \frac{\mathcal{P}}{\mathcal{P}_s} \right] \cdot [\varepsilon \nabla \mathcal{P} + (\varepsilon c \xi - \kappa e_1)\mathcal{P}]$$

Using the fact, that $\varepsilon \nabla \mathcal{P}_s = -(\varepsilon c \xi - \kappa e_1)\mathcal{P}_s$, we get

$$\partial_t S = -\varepsilon \int \mathcal{P} \left(\nabla \ln \frac{\mathcal{P}}{\mathcal{P}_s} \right)^2 \leq 0$$

Hence, the entropy is monotonically decaying in time and $S = 0$ if and only if $\mathcal{P} = \mathcal{P}_s$.

Applying the logarithmic Sobolev inequality [6], we obtain

$$\partial_t S \geq -2\varepsilon S \tag{5.2}$$

and hence a decay rate of $e^{-2\varepsilon t}$ for the entropy S. Using the Csiszar-Kullback inequality yields a decay rate of $e^{-\varepsilon t}$ for the $\mathcal{L}_1$–distance of $\mathcal{P}$ and $\mathcal{P}_s$.

6. Approximation quality of Ornstein–Uhlenbeck process

In this section we investigate the process (2.1) with $c(\xi) = 1$ numerically and compare it with the limiting process for $A \to \infty$, i.e. (4.10):

$$d\xi = -\epsilon \xi \, dt + \kappa e_1 \, dt + \sqrt{2\epsilon} \, dW_t.$$

Its stationary probability density,

$$\mathcal{P}_s(\xi) = \frac{1}{2\pi} e^{-(\xi_1 - \kappa/\varepsilon)^2/2 - \xi_2^2/2} ,$$

is independent of ε for $\kappa A^2 = k$, $k \in \mathbb{R}$. To test how well $\mathcal{P}_s$ approximates the numerically obtained stationary probability distribution of the process (2.1), we compare both distributions for different values of A. Figure 6.1 shows the stationary marginal probability distributions for the components ξ_1 and ξ_2 when $k = 0.5$. The distributions are computed from 15000 Monte–Carlo simulations of the ξ-process (2.1). Whereas the distribution functions for $A < 1$ are quite different from the marginals of $\mathcal{P}_s$, they are qualitatively similar for $A = 1$ and show good agreement for $A > 2$. The $\mathcal{L}^\infty$– and $\mathcal{L}^2$–errors are less than 2% for $A > 2$ as illustrated in Figure 6.2. For $A > 2$ and $N = 15000$ Monte–Carlo simulations, the deviations of the stationary marginal probability distributions from the limiting marginals are within the range of the approximation error, of order $1/\sqrt{N} \sim 10^{-2}$. Consequently, the limit distribution is a good approximation of the true distributions – already for moderate values of A. However, we should note that the resulting "limit process" of our fiber model for $A \to \infty$, the Ornstein–Uhlenbeck process, is only continuous, not differentiable. Hence, its associated η-process $\eta(t) = \xi(t) - \kappa t e_1$, is not parameterized by arc-length and the lack of differentiability obviously affects the non-extensibility condition. In Figure 6.3 realizations of the Ornstein-Uhlenbeck (ξ-process of (4.10)) and its associated η-process are depicted and compared to our differentiable fiber process of Section 2, assuming an initial value $\xi(0) = (0, 0)$, final time $T = 100$ and parameter values $\kappa = 0.1$, $A = 2.23$. Note that the same amount of fiber mass is laid down.

For the industrial application, it is important to know and control the mass distribution or other distributions of functionals of ξ. These distributions shed light into

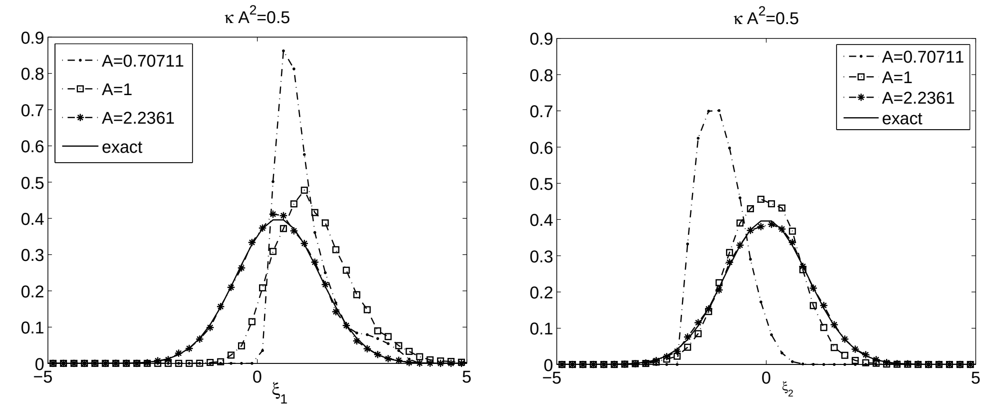

FIGURE 6.1. Stationary marginal distributions of ξ-components for $c = 1$, $\kappa A^2 = 0.5$ and several values of A

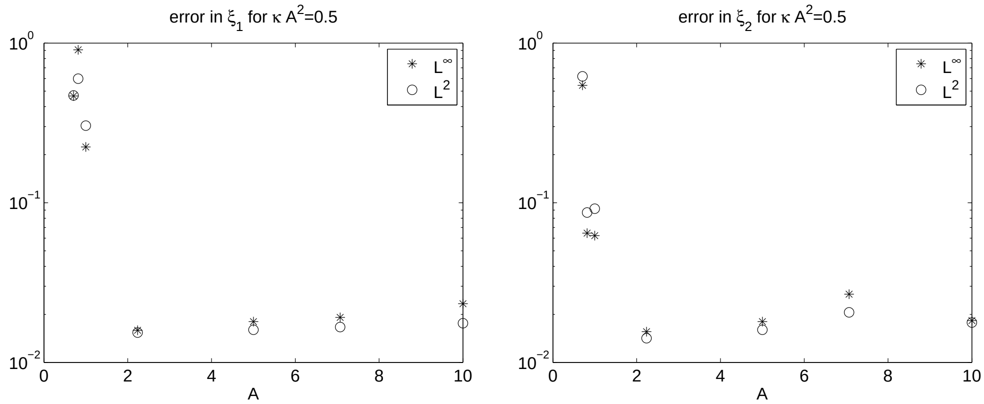

FIGURE 6.2. $\mathcal{L}^\infty$–error and $\mathcal{L}^2$–error between the stationary marginal distributions and the limiting ($A \to \infty$) stationary marginal distribution for different A.

the structure of the fleece material and therefore may serve to assess its quality. The fiber mass that lies in a prescribed spatial domain D can also be interpreted as the time the process stays in that domain. It is described by the distribution of the random variable

$$M = \int_{t_0}^{T} \chi_D(\eta(t))\, dt \tag{6.1}$$

for fixed T, $T > t_0$ with χ_D denoting the characteristic function of D. In the following we compare the distribution of (6.1) for the original fiber process given by (2.1) and the limit process (4.10). We evaluate the distribution of M numerically for the two processes and compare them using Monte–Carlo simulations for fixed $\kappa = 0.1$, $A = 2.23$. Figure 6.4 shows the probability distribution function (pdf) for the relative time that the respective ξ-processes ((2.1) and (4.10)) spend in a square domain D. The square is centered at a point in the set $K = \{(0,0), (0,1), (1,0)\}$, its length may vary in the set $L = \{1, 0.5, 0.25\}$, initially at time $t_0 = 0$, $\xi(0) = (0,0)$ and the final time is $T = 100$. The respective means differ only by 1% which is within the order of the approximation error of the Monte–Carlo simulations. In contrast to this, the relative error of the standard deviations depends on the chosen size of the test domain: the smaller the domain, the higher the error – up to 14% for $L = 0.25$, but only 2% for $L = 1$.

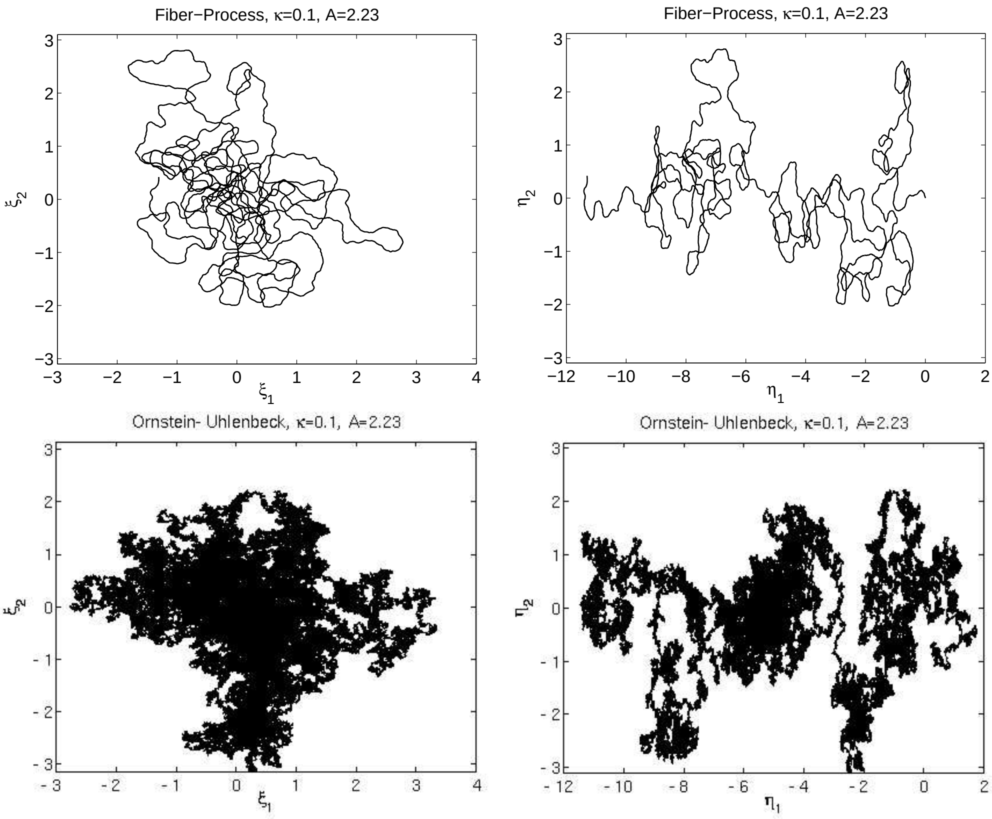

FIGURE 6.3. Differentiable fiber process (top) versus continuous Ornstein–Uhlenbeck limit process (bottom)

Figure 6.5 compares the distributions of the mass of a single fiber laid down in a nonwoven web. This means we consider the ditribution of (6.1) for the η-processes. We observe the same trend as for the ξ-processes for the relative time spent in a square D: very good agreement for larger test domains and poor agreement for smaller domains. The symmetry axis for the η-processes is $\eta_2 = 0$. Hence, we consider domains with a certain distance d_{sym} from the center point to the symmetry axis: the larger d_{sym}, the lower the probability that mass lies in D. This tendency is amplified by the size of D: the smaller the test domain, the lower the probability. In contrast to this trend, the probability that mass is accumulated in small domains D is much higher for the Ornstein–Uhlenbeck process than for our fiber process. The reason is that a realization of the continuous Ornstein–Uhlenbeck can move more easily, whereas the differentiable fiber process stays longer in certain regions and therefore other regions are not covered.

Summarizing, the Ornstein–Uhlenbeck limit process approximates our fiber process well – not only as regards the joint probability distribution but also the mass distributions for test domains of size 1 which corresponds to the size of the throwing (lay-down) range of the fiber, but not for smaller domains.

7. Conclusion

In this work we have presented an extended stochastic model for the fiber lay-down regime in a nonwoven production process that contains a moving conveyor

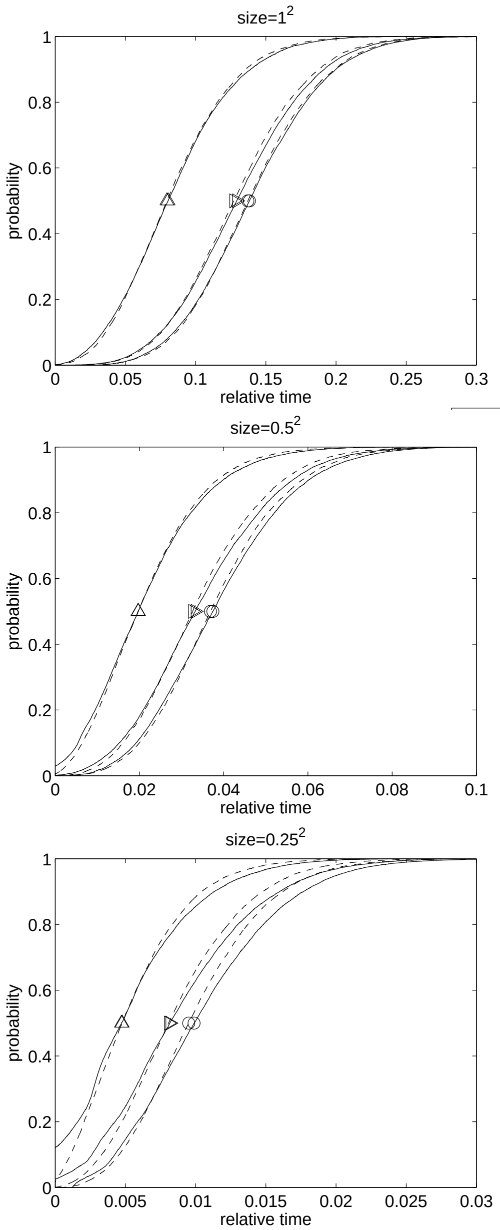

FIGURE 6.4. Pdf for the relative time that the fiber ξ-process (–) and the Ornstein–Uhlenbeck process (- -) spend in a square of size $L^2 = \{1^2, 0.5^2, 0.25^2\}$ (top to bottom), centered at $K = \{(0,0),(0,1),(1,0)\}$ (marked by $\circ$, $\triangleright$, $\triangle$)

belt. From the associated Fokker–Planck equation and using the method of multiple scales or the Chapman-Enskog technique, we have explicitely determined the limit processes and the stationary and transient joint probability distributions in the hydrodynamic limit, as $A \to \infty$. Quite generally and to leading order of these perturbation methods, we have found that the limiting stationary distribution (as $A \to \infty$) approaches a Gaussian-type function. For the special choice $c = 1$ of the fiber coiling function, the limiting process is a Ornstein–Uhlenbeck process, and the mean of its stationary Gaussian distribution depends on the relation of "relative process velocity" and turbulence noise, κA^2. Already for moderate values of A,

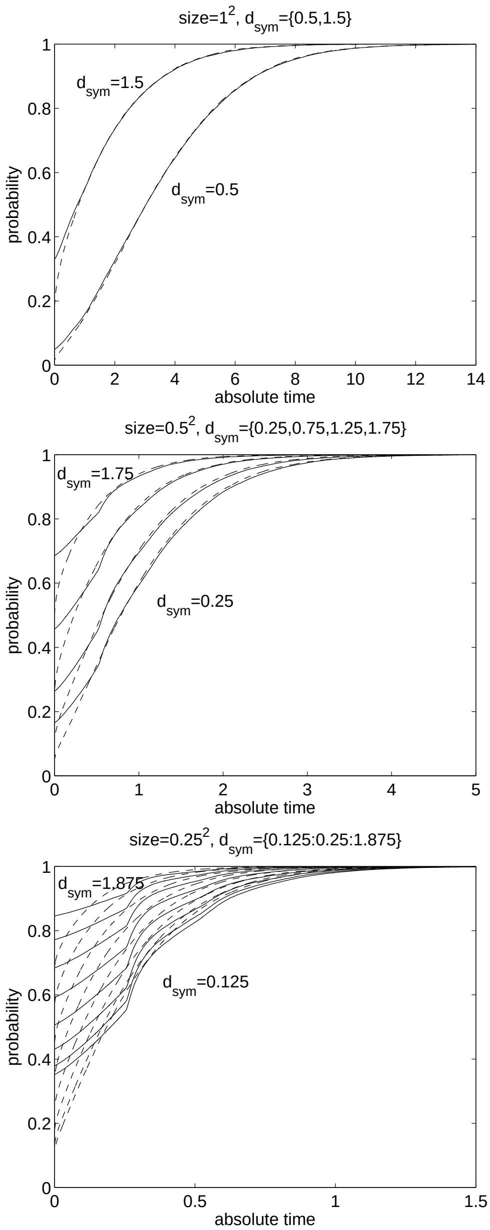

FIGURE 6.5. Pdf for the mass M of the fiber η-process (–) and the associated Ornstein–Uhlenbeck η-process (- -) laid in a square D, $|D| = L^2 = \{1^2, 0.5^2, 0.25^2\}$ (top to bottom) with different distances d_{sym} to the symmetry axis

i.e. $A > 2$, this limiting distribution turns out to be a very good approximation according to our numerical simulations. Moreover, important distributions of functionals of the process, such as the mass distribution, are well approximated by the Ornstein–Uhlenbeck process for test squares D of the size of the typical throwing (lay-down) range of the fibers.

For the control and optimization of the production and quality of nonwoven materials, the parameters characterizing our model, c, A, κ and samples sizes D, should be identified from FIDYST-simulations of the complete physical production process as well as from experimental data. If the ranges of these parameters are such that the limiting process studied in this work describes well the physical production, the fiber mass distribution in a fleece material could be determined from the superposition of many Ornstein–Uhlenbeck η-processes.

Acknowledgments

We thank Vincenzo Capasso for having brought us together to collaborate in this topic. The authors acknowledge support from the Rheinland–Pfalz Excellence Cluster "Dependable Adaptive Systems and Mathematical Modelling".

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