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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

ON TWO-LEVEL PRECONDITIONERS FOR FLOW IN POROUS MEDIA

R.E. EWING, O.P. ILIEV, R.D. LAZAROV, AND I.V. RYBAK

ABSTRACT. Two-level domain decomposition preconditioner for 3D flows in anisotropic highly heterogeneous porous media is presented. Accurate finite volume discretization based on multipoint flux approximation (MPFA) for 3D pressure equation is employed to account for the jump discontinuities of full permeability tensors. DD/MG type preconditioner for above mentioned problem is developed. Coarse scale operator is obtained from a homogenization type procedure. The influence of the overlapping as well as the influence of the smoother and cell problem formulation is studied. Results from numerical experiments are presented and discussed.

Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner

1. Introduction

Multigrid, MG, and Domain Decomposition, DD, are the most advanced methods for solving large systems of linear algebraic equations arising in discretization of certain classes of PDEs. Here we are interested in elliptic PDEs, describing flow in porous media, heat conductivity, etc. Both methods, MG and DD, are extensively studied in the case when the coefficients of the elliptic equation are smooth, and/or when the media is isotropic. It is well known, that special attention has to be paid in the case when there are heterogeneities unresolved on the coarse grid, and when the full tensor discontinuous coefficients are considered. One approach to deal with highly oscillating coefficients, is to exploit homogenization techniques for building coarse grid equations. In the case of periodic isotropic media and clear separation of scales, such an approach is introduced and discussed in the pioneering work [8]. Here we study a combination of multigrid and homogenization approaches in conjunction with solving equations with tensor discontinuous coefficients, in the cases of periodic and non-periodic heterogeneities. Unlike using FEM and only primary grid, like in [8], we use finite volume, FV, discretization, and exploit a primary grid for the discretization and for the smoother, and a dual grid for the problem dependent prolongation. The usage of FV, namely of the Multi Point Flux Approximation based FV discretization, is especially important in the case of discontinuous tensor coefficients. MPFA does not only provide good discretization accuracy for the PDE under consideration, but it also does provide good approximation for the coarse scale equation, thus promoting a good convergence of the two-level algorithm. Furthermore, while the authors of [8] consider only cell problems with periodic boundary conditions in the upscaling procedure for the coarse grid equation, we discuss several possible formulations of cell problems, including such for non-periodic media. In fact, these formulations of the cell problems are known from the numerical upscaling, see [13], here we discuss them in conjunction with two-level iterative algorithm. It should be noted that DD community also has developed approaches to deal with rapidly oscillating heterogenieties. We refer, e.g., to [7, 6], and to references therein for a more detailed discussion of this issue.

The reminder of the paper is organized as follows. Next section concerns the statement of the problem. The Finite Volume discretization is described in section 3. The fourth section is devoted to the two-level iterative algorithm, in particular, to the choice of the intergrid operators, the choice of the coarse grid operator, etc. The fifth section presents results from numerical experiments. Finally, some conclusions are drawn.

2. Statement of the problem

In this paper, we consider steady state incompressible single phase flow in highly heterogeneous anisotropic porous media. Such flow is described by the equation for the unknown pressure p

$$-\nabla \cdot (K \nabla p) = f, \quad \text{in } \Omega, \tag{1}$$

subject to the following boundary conditions

$$p = g^D, \quad \text{on } \Gamma_D, \quad K \nabla p \cdot \mathbf{n} = g^N, \quad \text{on } \Gamma_N, \quad \partial\Omega = \Gamma_D \cup \Gamma_N. \tag{2}$$

This problem could be reformulated in a mixed form as a system of the equation $\nabla \cdot \mathbf{v} = f$ expressing mass conservation and the Darcy's law $\mathbf{v} = -K \nabla p$. Here the domain Ω is a parallelepiped with boundaries parallel to the coordinate planes, the set Γ_D is non-empty and had positive surface measure, the permeability tensor is full, symmetric, and uniformly positive definite in Ω:

$$K = \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{pmatrix}, \quad \begin{pmatrix} k_{xy} = k_{yx} \neq 0 \\ k_{xz} = k_{zx} \neq 0 \\ k_{yz} = k_{zy} \neq 0 \end{pmatrix}.$$

The entries of the permeability tensor K may have jump discontinuities along certain interfaces that are parallel to the coordinate planes and along these interfaces the following conditions are satisfied

$$[p] = 0, \quad [K \nabla p \cdot \mathbf{n}] = 0.$$

Here $[\xi] = \xi(x_{int} + 0) - \xi(x_{int} - 0)$ for the interface x_{int}.

3. Finite volume discretization

In this section, we derive a discretization for three-dimensional pressure equation (1) with discontinuous tensor coefficients. The discretization which we use for the 3-D case, is derived in the same way as the schemes in [4, 12], where 2-D problems were considered. The derivation is based on the finite (control) volume method and multipoint flux approximation of Aavatmark et al [1] and Edwards and Rogers [5] for multidimensional problems and general hexahedral meshes.

The domain Ω is partitioned into blocks Ω_{ijk} so that the discontinuities of the permeability tensor K are aligned with cell boundaries. The centers of the cells Ω_{ijk} are denoted by (x_i, y_j, z_k) and the cell vertexes are the points $(x_i \pm \frac{1}{2}h_1, y_j \pm \frac{1}{2}h_2, z_k \pm \frac{1}{2}h_3)$. The mesh that will be used to approximate the pressure will include all cell centers (x_i, y_j, z_k). This mesh will be called *primary mesh* $\omega_h = \{(x_i, y_j, z_k) : \Omega_{ijk}\}$. Similarly we shall use also the mesh of all cell vertexes, called often *dual mesh*.

The continuity equation ($\nabla \cdot \mathbf{v} = f$) is integrated over control volume Ω_{ijk} and making use of the divergence theorem, we obtain

$$\int\limits_{\Omega_{ijk}} \nabla \cdot \mathbf{v} dx = \int\limits_{\Omega_{ijk}} f dx \quad \Rightarrow \quad \int\limits_{\partial\Omega_{ijk}} \mathbf{v} \cdot \mathbf{n} ds = \int\limits_{\Omega_{ijk}} f dx. \tag{3}$$

Using the Darcy's relation $\mathbf{v} = -K\nabla p$, the velocity $\mathbf{v}$ in (3) is replaced by certain approximation involving p, what results in a conservative method [9]. In this approximation we assume that the unknowns (or degrees of freedom) are the values of the pressure at the cell centers and then use these values to recover the velocity $\mathbf{v}$. According to the multi-point flux approximation (MPFA) this is done in the following manner. First we split each control volume

$$\Omega_{ijk} = (x_i - \frac{1}{2}h_1, x_i + \frac{1}{2}h_1) \times (y_j - \frac{1}{2}h_2, y_j + \frac{1}{2}h_2) \times (z_k - \frac{1}{2}h_3, z_k + \frac{1}{2}h_3)$$

into 8 subvolumes $\Omega_{ijk}^{1,1,1} = (x_i, x_i + \frac{1}{2}h_1) \times (y_j, y_j + \frac{1}{2}h_2) \times (z_k, z_k + \frac{1}{2}h_3)$, $\Omega_{ijk}^{-1,1,1} = (x_i - \frac{1}{2}h_1, x_i) \times (y_j, y_j + \frac{1}{2}h_2) \times (z_k, z_k + \frac{1}{2}h_3)$, etc. These are denoted by

$$\Omega_{ijk}^{\alpha\beta\gamma} = (x_i + \frac{\alpha}{2}h_1, x_i) \times (y_j + \frac{\beta}{2}h_2, y_j) \times (z_k + \frac{\gamma}{2}h_3, z_k),$$

where $\alpha, \beta, \gamma = \pm 1$. We take the pressure to be a linear function on each subvolume $\Omega_{ijk}^{\alpha\beta\gamma}$ so that

$$p_{ijk} = a_{ijk}^{\alpha\beta\gamma} x + b_{ijk}^{\alpha\beta\gamma} y + c_{ijk}^{\alpha\beta\gamma} z + d_{ijk}^{\alpha\beta\gamma}, \quad \alpha, \beta, \gamma = \pm 1. \tag{4}$$

The coefficients $a_{ijk}^{\alpha\beta\gamma}$, $b_{ijk}^{\alpha\beta\gamma}$, $c_{ijk}^{\alpha\beta\gamma}$ and $d_{ijk}^{\alpha\beta\gamma}$ in (4) are determined by the following conditions:
(1c) the pressure values at the volume centers;
(2c) the continuity of the pressure at the centers of the faces of the volume Ω_{ijk} and the pressure data on faces that are part of Γ_D;
(3c) the continuity of the normal component of the velocity $\mathbf{v}$ at the centers of the faces of the volume Ω_{ijk} and the boundary data for the normal velocity on faces on Γ_N.

These conditions are applied on a cell from the dual grid, i.e. a cell centered at a vertex point from the dual grid. These cells are of four categories corresponding to internal vertices, boundary vertices (but not on edges), vertices on the edges (but not in corner), and finally the 4 corner points of the domain Ω.

Consider an internal vertex, as shown on Figure 1, that is surrounded by eight subcells, and the centers of those subcells are connected together to form a graph with eight vertices and twelve edges (Fig. 1).

To find the polynomial coefficients from (4), we use conditions (1c)–(3c). Let the coordinate origin be in the vertex node, so for the considered shifted control volume we have $-h_1/2 \le x \le h_1/2$, $-h_2/2 \le y \le h_2/2$, $-h_3/2 \le z \le h_3/2$. Condition (1c) gives us 8 equations by substitution of x, y, z and the pressure at the cell-centers (P, E, N, NE, P^p, E^p, N^p, NE^p) into equation (4). Taking into account the conditions (2c) of continuity the pressure at the centers of the faces of Ω_{ijk}, we get 12 equations, and finally considering the condition (3c) of continuity of the normal component of the velocity at the centers of the faces of the volume Ω_{ijk} we obtain last 12 equations needed to determine 32 polynomial coefficients from (4). Note, that it gives us the expressions for the velocity that is constant over each of the 8 subcells of the vertex-centered volume. These formulas are used to find $\mathbf{v} \cdot \mathbf{n}$ on $\partial\Omega_{ijk}$, as needed by the relation (3).

For vertex that is on the boundary the situation is simpler. In the case of Neumann boundary conditions the flux is given on the boundary, while in the case of Dirichlet boundary conditions we just do the same procedure as for the inner control volume, but in this case we have 4 interfaces only.

Combining this relationship for each neighboring vertex gives us a discrete pressure equation with a 27-point stencil.

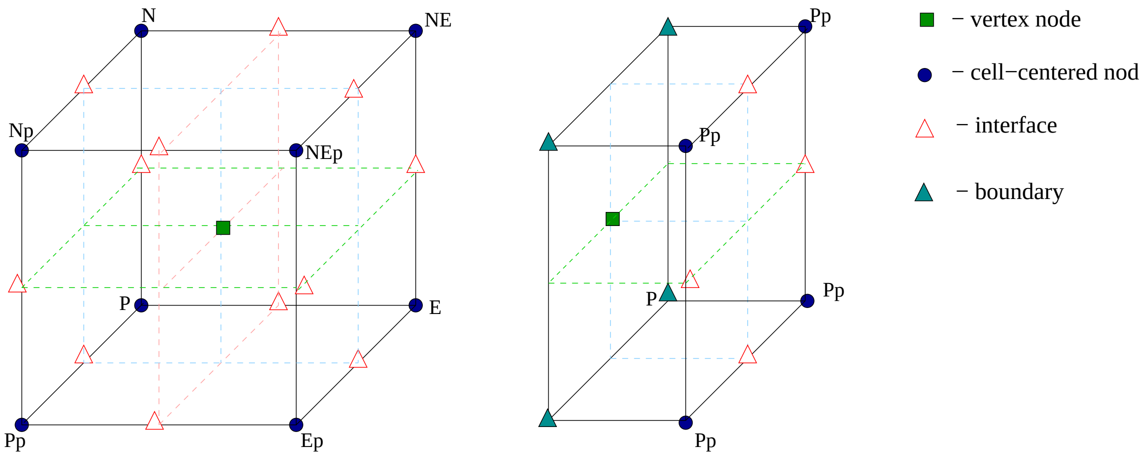

FIGURE 1. Shifted control volumes: inner and boundary cells

If the coefficient tensor K is constant, then this approximation recovers the linear pressure and therefore is at least of first order. However, on uniform meshes this scheme is of second order. The scheme could be written as the combination of the second order finite difference schemes from [9] with some $O(h^2)$ regularizator. For the simplicity, we write it in two-dimensional case

$$k_{xx}p_{x\bar{x}} + 0.5k_{xy}\left(p_{\bar{x}y} + p_{x\bar{y}} + p_{\bar{x}\bar{y}} + p_{xy}\right) + k_{yy}p_{y\bar{y}} + R = -f,$$

where

$$R = \frac{k_{xy}^2}{4}\left(\frac{h_x^2}{k_{xx}} + \frac{h_y^2}{k_{yy}}\right)p_{\bar{x}x\bar{y}y} = O\left(h^2\right).$$

Further, in the case of discontinuous permeability tensor K the approximation involves harmonic averaging of the coefficients, which in turn leads to a better scheme. Second order convergence of the developed algorithm is confirmed by our numerical experiments. For discontinuous diagonal tensor K the discretization reduces to 7-point stencil difference scheme with the harmonic averaging of the coefficients.

The discretization of the boundary-value problem (1), (2) on the domain $\bar{\Omega}$ gives us the following linear system

$$A_h p_h = f_h.$$

Let $A_h : U_h \to U_h$ be a matrix-valued operator, U_h is the space of fine grid functions with the inner product

$$(y_h, v_h)_{U_h} = \sum_{x\in\omega_h} y_h(x)v_h(x)h_1h_2h_3.$$

Operator A_h has the following properties:
(1p) symmetric;

(2p) positive definite.

4. Two-grid method

In this section, we present two-level preconditioner (TGM) for solving problem (1), (2). To do this, we consider decomposition of the domain $\bar{\Omega}$ into the fine grid ω_h defined above and the coarse grid defined in a similar way

$$\omega_H = \{(x_I, y_J, z_K) : \Omega_{IJK} = m_x m_y m_z \Omega_{ijk}\},$$

where m_x, m_y, m_z is the number of fine grid blocks in a coarse one. Both grids are uniform and cell-centered, moreover, the interfaces of each coarse block match the interfaces of the fine blocks (Fig. 2).

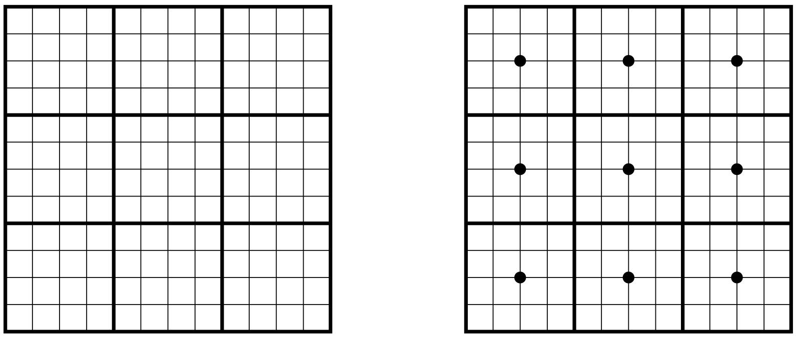

Figure 2. Fine and coarse grids

4.1. **Two-grid iteration.** The algorithm of TGM is similar to the multigrid approach [11], combining two processes - smoothing and coarse grid correction. If we knew the error on the coarse grid, then we could interpolate it back to the fine grid and use this as a correction. The error e_H on the coarse grid is unknown, but it could be calculated from the error equation $A_H e_H = r_H$, if the residual r_H on the coarse grid were known. The residual on the fine grid is known, so we can use it to approximate the residual on the coarse grid. For two-level algorithms see also [10, 8, 6].

To define TGM we need the smoother, the transfer operators and the coarse grid operator. Let $P : U_H \to U_h$ be the interpolation from the coarse grid to the fine grid, operator $R : U_h \to U_H$ be the restriction operator, $A_H : U_H \to U_H$ be a discrete form of the operator on the coarse grid, $S : U_h \to U_h$ and $\tilde{S} : U_h \to U_h$ be the smoothing iterations.

Suppose, that p_h^k is given, then one iteration of TGM is the following

$$p_h^{k+1} = TGM(p_h^k, A_h, f_h),$$

where p_h^{k+1} is defined by

Algorithm 4.1.

begin

pre-smoothing: $p_h^{k+1/3} = S(p_h^k, A_h, f_h)$,

coarse grid correction: $p_h^{k+2/3} = p_h^{k+1/3} + PA_H^{-1}R(f_h - A_h p_h^{k+1/3})$,

post-smoothing: $p_h^{k+1} = \tilde{S}(p_h^{k+2/3}, A_h, f_h)$.

end

That is, we smooth fine-grid solution, then calculate the residual on the fine grid, restrict it to the coarse grid, solve the coarse grid problem, interpolate the coarse grid solution to the fine grid, correct the fine grid solution and post-smooth it.

Remark. The choice of the intergrid (transfer) operators, as well as the choice of the coarse scale operator, are critical for the convergence of the method.

4.2. **Particular choices of two-grid ingredients.**

4.2.1. *Smoothing iterations (S, $\tilde{S}$).* In connection with the above mentioned TGM, we use Schwarz smoothers. Schwarz smoothers are simply one-level additive or multiplicative overlapping Schwarz Domain Decomposition iterative algorithms [10]. We use Dirichlet boundary condition on the interfaces.

Consider the decomposition of the domain Ω into overlapping subdomains Ω_i, $i = \overline{1,p}$. The domain Ω_i is Ω_{ijk} with some overlapping (see, for example, Fig. 3). The pure multiplicative Schwarz method involves the following substeps

$$\begin{aligned} u^{n+\frac{1}{p}} &\leftarrow u^n + B_1(f - Au^n), \\ u^{n+\frac{2}{p}} &\leftarrow u^{n+\frac{1}{p}} + B_2(f - Au^{n+\frac{1}{p}}), \\ &\dots \\ u^{n+1} &\leftarrow u^{n+\frac{p-1}{p}} + B_p(f - Au^{n+\frac{p-1}{p}}), \end{aligned}$$

where $B_i = R_i^T(R_iAR_i^T)^{-1}R_i$, and $A_i = R_iAR_i^T$ is merely the submatrix of A associated with the domain Ω_i. The residual at each fractional step, $f - Au^{\frac{i-1}{p}}$, need only to be be updated in Ω_i using values from Ω_i and its immediate neighbors. R_i is rectangular (restriction) matrix that returns the vector of coefficients defined in the interior of Ω_i. Note, that these matrices are never formed in practice. The matrix B_i restricts the residual to one subdomain, solves the problem on the subdomain to generate a correction, and then extends that correction back onto the entire domain. Again, B_i are never formed explicitely.

Additive Schwarz method

$$u^{n+1} \leftarrow u^n + \sum_i B_i(f - Au^n).$$

4.2.2. *Restriction transfer operator (R).* Restriction operator could be derived from the condition

$$\int_{\Omega_{ijk}} r_h dx = \int_{\Omega_{ijk}} R r_h dx,$$

where midpoint rule is used to approximate the integrals. So, we use volume averaging as a restriction operator

$$r_H = \frac{1}{m} \sum_{i=1}^{m_x} \sum_{j=1}^{m_y} \sum_{k=1}^{m_z} r_h^{ijk},$$

where r_H is the residual on the coarse grid, r_h^{ijk} is the residual on the fine grid, and $m = m_x m_y m_z$ is the number of fine grid blocks in the coarse grid block Ω_{ijk}.

4.2.3. *Coarse grid operator (A_H).* We discretize and solve on a coarse grid the equation for the correction. The choice of the coarse scale operator is critical for the convergence of the method. So, it is important that the coarse grid problem represents the fine grid problem well enough. To define the coarse grid discretization we will use homogenization techniques [3, 13, 2, 14]. The homogenization of equations with variable coefficients often yields coarse scale equations of a different form than the original fine scale equations. Homogenization procedures presented here allow the coarse scale pressure equation to be of the same form as equation (1), but with the permeability K replaced by the coarse scale or effective permeability tensor $\tilde{K}$:

$$-\nabla \cdot (\tilde{K} \nabla p_H) = f_H. \tag{5}$$

Different definitions of $\tilde{K}$ have been proposed [3, 13]. Solutions of the local flow problems in each coarse grid block are postprocessed in order to upscale the permeability tensor. The main differences among various formulations are the boundary conditions imposed on the local flow equation and the averaging processes for computing $\tilde{K}$. Let us first shortly discuss the approach from [3], and after that to summarize the discussions in [13].

Consider a cubic grid block V. Following [3], to define $\tilde{K}$ in V we write the coarse scale Darcy's law

$$\langle \mathbf{v} \rangle_V = -\tilde{K} \langle \nabla p \rangle_V \,, \tag{6}$$

where p and $\mathbf{v}$ are fine scale solutions of the problem $v = -K \cdot \nabla p$, $\nabla \cdot v = 0$ in the grid block V with appropriate boundary conditions. Note that the source term is set to zero because effective permeability (if it exists) should be independent on the source term f and on the boundary conditions posed on the boundary $\partial\Omega$ of the domain of interest. Here $< . >_V$ is the volume average over V:

$$\langle . \rangle_V = \frac{1}{V} \int_V (.) dx.$$

In three-dimensional case, three fine scale flow solutions are necessary in order to determine $\tilde{K}$ from (6), provided that the volume averages of the pressure gradients are linearly independent. So, we need to solve three local problems in each coarse block

$$\langle \mathbf{v}_i \rangle_V = -\tilde{K} \langle \nabla p_i \rangle_V \,, \quad i = \overline{1,3}. \tag{7}$$

The subscript of $\mathbf{v}$ and ∇p designates the flow problem (1 corresponds to flow in x direction, 2 to flow in y, and 3 to flow in z). From these three flow problems the components of the full tensor $\tilde{K}$ can be computed.

Upscaled permeability tensor $\tilde{K}$ computed via equations (7) will not in general be symmetric. Various procedures can be applied to enforce symmetry. The simplest approach is to set each of the cross terms equal to $(\tilde{k}_{xy}+\tilde{k}_{yx})/2$, $(\tilde{k}_{xz}+\tilde{k}_{zx})/2$, $(\tilde{k}_{yz}+\tilde{k}_{zy})/2$.

Unfortunately, the mentioned above approach doesn't guarantee the positive definiteness of the upscaled permeability tensor $\tilde{K}$. There exists another method of computing the upscaled permeability tensor [13], which leads to symmetric and positive definite $\tilde{K}$:

$$\mathbf{e}_i \cdot \tilde{K}\mathbf{e}_j = \langle \nabla p_i \cdot K \nabla p_j \rangle_V ,$$

where $\mathbf{e}_i$ is the unit vector in the ith direction. But the disadvantage of this approach is that it could not be applied to certain types of local boundary conditions which will be described later.

A number of local flow boundary conditions are used in practice, see the deep discussion in [13]. Periodic conditions can be formulated as

$$p_i = x_i + \varepsilon, \quad \text{periodic on } V, \tag{8}$$

linear pressure drop conditions

$$p_i = x_i, \quad \text{on } \partial V, \tag{9}$$

pressure drop no-flow conditions

$$p_i = x_i, \quad \text{on } \partial\Gamma_i, \qquad \mathbf{n} \cdot v_i = 0, \;\; \text{on } \partial\Gamma_j,\; i \neq j, \tag{10}$$

where Γ_i are the faces of ∂V normal to $\mathbf{e}_i$.

Oscillatory boundary conditions can also be posed

$$p_i = x_i, \quad \text{on } \partial\Gamma_i, \qquad p_i = P_{1d}(x_i), \;\; \text{on } \partial\Gamma_j,\; i \neq j. \tag{11}$$

The operator P_{1d} will be defined in next section.

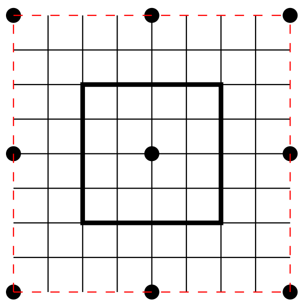

FIGURE 3. Extended coarse block

Remark. Local flow problems could also be solved in some extended local subdomain (see Fig. 3), these are the approaches known as oversampling.

4.2.4. *Prolongation transfer operator (P).* Since we deal with discontinuous and strongly varying by several orders of magnitude coefficients in the domain of interest, the problem dependent prolongation operator has to be used [11].

Suppose that the pressure values $p_C = p_{IJK}$ are known in the coarse grid nodes (x_I, y_J, z_K), and these values p_C have to be interpolated to the fine grid. Consider 8 neighboring coarse grid nodes forming a cube (Fig. 4).

To build the interpolation operator P, the following three-stage algorithm is used. First, we solve 12 one-dimensional problems

$$-\nabla \cdot (K \nabla p_h^{edge}) = 0, \quad \text{in the edges}, \tag{12}$$

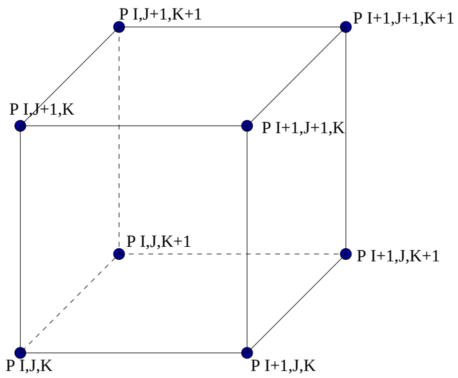

Figure 4. Local subdomain

$$p_h^{edge} = p_H^{corner}, \quad \text{in the corners}\,, \tag{13}$$

where the points $(x_I,\, y_J,\, z_K)$, $(x_I + I_x H_x,\, y_J + I_y H_y,\, z_K + I_z H_z)$ are the corners, the permeability is scalar $K = k_{xx} I_x + k_{yy} I_y + k_{zz} I_z$, and

$$I_x I_y I_z = 0, \quad I_x + I_y + I_z = 1, \quad I_x, I_y, I_z \in \{0, 1\}.$$

We discretize problem (12), (13) by means of the finite volume method (harmonic average scheme with 3-point stencil), and solve the discrete problem directly using Thomas algorithm. The solutions of one-dimensional problems (12), (13) give us fine grid pressure values along 12 edges of the considered local subdomain

$$p_h^{edge} = P_{1d}(p_H^{corner}).$$

After that we solve 6 two-dimensional problems of the type

$$-\nabla \cdot (K \nabla p_h^{plane}) = 0, \quad \text{in the planes,} \tag{14}$$

$$p_h^{plane} = p_h^{edge}, \quad \text{on the edges,} \tag{15}$$

where the permeability is given by

$$K = \begin{pmatrix} k_{xx} & 0 \\ 0 & k_{yy} \end{pmatrix} I_{xy} + \begin{pmatrix} k_{xx} & 0 \\ 0 & k_{zz} \end{pmatrix} I_{xz} + \begin{pmatrix} k_{yy} & 0 \\ 0 & k_{zz} \end{pmatrix} I_{yz},$$

$$I_{xy} I_{xz} I_{yz} = 0, \quad I_{xy} + I_{xz} + I_{yz} = 1, \quad I_{xy}, I_{xz}, I_{yz} \in \{0, 1\}.$$

Solutions of local problems (12), (13) are used as Dirichlet boundary conditions (15). We discretize and solve on a fine grid problem (14), (15). The discretization is based on the finite volume approach (harmonic average scheme with 5-point stencil). By solving two-dimensional problems (14), (15), we obtain fine grid pressure on 6 planes of the local subdomain under consideration

$$p_h^{plane} = P_{2d}(p_h^{edge}) = P_{2d} P_{1d}(p_H^{corner}).$$

To interpolate the rest of the values we solve three-dimensional problem

$$-\nabla \cdot (K \nabla p_h) = 0, \quad \text{in the cube,} \tag{16}$$

$$p_h = p_h^{plane}, \quad \text{on the planes,} \tag{17}$$

where the permeability tensor is defined in the following way

$$K = \begin{pmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{pmatrix}.$$

The solutions of problems (14), (15) are used as Dirichlet boundary conditions. Problem (16), (17) is discretized by the finite volume method. The solution of this three-dimensional problem gives us fine grid pressure values in the considered local subdomain

$$p_h = P_{3d}(p_h^{plane}) = P_{3d}P_{2d}P_{1d}(p_H^{corner}).$$

Obviously, by applying this procedure to all local subdomains, it is easy to obtain the matrix associated with the prolongation operator P. So, we get $p_h = P_{3d}P_{2d}P_{1d}p_H$, and $P = P_{3d}P_{2d}P_{1d}$.

Note. To interpolate the pressure values in the near boundary subdomains, we solve local problems with the boundary conditions defined on the boundary of the domain of interest $\partial\Omega$.

5. Numerical experiments

In this section, we apply TGM to several example cases. In some of the examples the permeability tensor is isotropic on a fine grid while it is full on a coarse grid, in the other cases both, fine and coarse scales are anisotropic. We will consider three-dimensional periodic medium with cubic inclusions (see Fig. 6 for its two-dimensional analog), periodic medium with L-shaped inclusions (Fig. 5) and non-periodic medium with random cubic inclusions (Fig. 7).

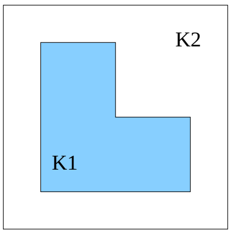

Figure 5. *L*shape

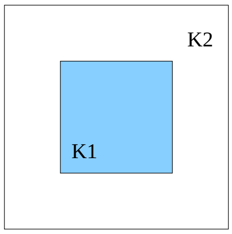

Figure 6. Cubic

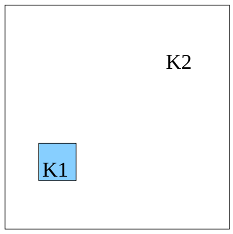

Figure 7. Random

In the paper, both soft and stiff inclusions are studied. The permeability tensor of the inclusion is K_1, while K_2 is the permeability tensor of the surroundings. For stiff inclusion the permeability tensor is defined in the following way

$$K_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 10000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{pmatrix}, \tag{18}$$

while for soft inclusions we have

$$K_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix}. \tag{19}$$

The influence of the overlapping as well as the influence of the smoother and cell problem formulation is investigated.

For all experimental results, cell-centered uniform grids are used. The number of coarse grid blocks is equal in every direction $N_x = N_y = N_z = N$, the number of fine grid blocks in each coarse one is also the same for every direction $n_x = n_y = n_z = n$.

The convergence of TGM is attained when the residual in the L_2 norm was reduced by q orders of magnitude

$$\frac{\|r^n\|}{\|r^0\|} \leq 10^{-q}, \text{ or } \frac{\|r^n\|}{\|r^1\|} \leq 10^{-q}.$$

The initial guess for TGM iterations is the null vector, the right-hand side is constant vector.

5.1. **One-level DD and TGM.** To compare one-level DD and TGM, we consider the example with $4 \times 4 \times 4$ periodic stiff cubic inclusions. One-level DD is simply additive Schwarz method S, so we execute only first step of Alg. 4.1, while for TGM we perform all the three steps with 2 pre- and 2-post-smoothings. We define the overlap by the number of grid points included in it. So, if we refine the grid, the overlapping becomes smaller. That is why we need more DD and TGM iterations for the finer grids. In Tab. 1 we give the number of one-level DD and TGM iterations needed to reduce the initial residual by the factor of 10^{-4}.

N	n	# DD iter.	# TGM iter.
4	8	49 (16 sec)	4 (5 sec)
4	16	83 (162 sec)	4 (41 sec)
4	32	136 (2187 sec)	8 (312 sec)
8	4	78 (20 sec)	3 (11 sec)
8	8	133 (256 sec)	4 (64 sec)
8	16	216 (3371 sec)	5 (441 sec)

TABLE 1. Number of DD and TGM iterations needed to reduce the initial residual by the factor of 10^{-4}

The results in the Table demonstrate the better performance of the TGM, compared to the one level DD iterative method.

5.2. **TGM components.** We study the influence of the smoothing iterations and the coarse grid correction on the convergence of the two-grid method.

5.2.1. *Smoothing procedure.* For TGM we use Schwarz smoothers as well as ILU smoother. Schwarz smoothers are simply one-level additive or multiplicative overlapping Schwarz preconditioners with Dirichlet boundary conditions on the interfaces.

It is known that the multiplicative method converges approximately two times faster than the additive one [10]. We studied the influence of the smoother on the convergence of the overall TGM. We consider stiff periodic cubic inclusions (Fig. 6), and execute TGM with 2 pre- and 2 post-smoothings by different smoothers. In Tab. 2, we give the number of TGM iterations needed to reduce the initial residual by the factor of 10^{-5} for different smoothers. It is easy to see that the multiplicative Schwarz method is better smoother for TGM. We investigated also the influence of the overlapping on the convergence of overall TGM. We fix the grid and use several choices for overlap. From Tab. 2 it is easy to see, the more points in overlapping are then the faster TGM converges. Here $H = \frac{1}{4}$, $h = \frac{1}{64}$, $c = 4^3$, c - number of inclusions, β - size of overlapping.

Smoother	$\beta = \dfrac{1}{128}$	$\beta = \dfrac{3}{128}$	$\beta = \dfrac{7}{128}$
AS	**23** (213 sec)	**7** (74 sec)	**5** (75 sec)
MS	**11** (102 sec)	**5** (53 sec)	**3** (46 sec)
ILU	**8** (44 sec)	**8** (44 sec)	**8** (44 sec)

TABLE 2. # TGM iterations

We also studied the influence of the number of pre- and post-smoothings on the convergence of TGM. Consider again stiff periodic. Here $H = \frac{1}{8}$, $h = \frac{1}{64}$, number of inclusions $c = 8^3$, size of overlapping$\beta = \frac{1}{128}$, accuracy 10^{-5}. Tab. 3 shows, that in the case of highly varying permeability tensor it is better to perform both pre- and post-smoothings.

Smoother	0-pre, 2-post	2-pre, 2-post	0-pre, 4-post
AS	35	**14**	19
MS	17	**7**	10
ILU	26	**9**	12

TABLE 3. # TGM iterations

5.2.2. *Coarse grid operator.* To define the coarse grid operator we need to know the effective permeability tensor in the coarse blocks. Note that effective permeability can be full tensor, even if fine scale permeability is isotropic. Different boundary conditions could be posed for local flow problems, as well as oversampling could be used or not. Here, we studied the influence of the cell problem formulation on the convergence of TGM.

There exist two possibilities: to solve local flow problems exactly in the coarse grid block or in extended subdomain (Fig. 3). Note, that in both cases the volume average of the pressure gradient ∇p and the velocity $\mathbf{v}$ are calculated exactly in the considered coarse block.

Suppose that we have periodic porous media, for example, L-shape inclusions (Fig. 5). When we calculate the effective permeability tensor by solving local flow problems with no oversampling in the coarse block, we will have the same effective permeability tensor for each coarse cell. However, if we use oversampling and solve local flow problems in extended subdomain, the effective permeability tensor in inner coarse grid cells will differ from permeability tensor in near the boundary coarse blocks. Nevertheless, these two approaches require almost the same number of TGM iterations for the particular problem which we are solving here.

5.3. **Different geometries.** We investigated the convergence of TGM for different geometries (Fig. 6–7). From Tab. 5.3 we see that the convergence of TGM depends on the geometry of inclusions.

Local flow	N	n	# TGM iter.
No oversampling	10	12	18
Oversampling	10	12	17

Table 4. Number of TGM iterations needed to reduce the initial residual by the factor of 10^{-4}.

Geometry	N	n	# TGM iter.
Periodic L-shape	10	12	14
Periodic cubic	10	12	11
Random cubic	10	12	9

Table 5. Number of TGM iterations needed to reduce the initial residual by the factor of 10^{-6}

5.4. **Soft and stiff inclusions.** We study the convergence of TGM for both soft (19) and stiff (18) inclusions. While a very good convergence is observed in the case of stiff inclusions, there is no convergence for some of the examples in the case when the inclusions are soft, see Tab. 5.4.

Geometry	N	n	# TGM it. (stiff)	# TGM it. (soft)
Periodic L-shape	10	12	14	∞
Periodic cubic	10	12	11	∞
Random cubic ($r = 1$)	10	12	9	9

Table 6. Number of TGM iterations needed to reduce the residual by the factor of 10^{-6}.

5.5. **Concluding remarks.** Two level domain decomposition algorithm for 3D flows in anisotropic heterogeneous porous media is presented. Finite volume discretization based on MPFA is used on the fine, as well as on the coarse scales. Additive and multiplicative Schwarz DD iterative methods, as well as ILU factorization, are implemented as smoothers for the two-level algorithm. The coarse scale operator is obtained from numerical upscaling, different formulations of the cell problems in the coarse blocks are discussed. The influence of the overlapping, of the choice of the smoother, of the number of subdomains, on the convergence of TGM is studied. Applicability of the proposed method for non-periodic media is considered.

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