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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

A Comparative Study of the Vasicek and the CIR Model of the Short Rate

Serkan Zeytun, Ankit Gupta

July 23, 2007

Abstract

In this work, we analyze two important and simple models of short rates, namely Vasicek and CIR models. The models are described and then the sensitivity of the models with respect to changes in the parameters are studied. Finally, we give the results for the estimation of the model parameters by using two different ways.

1 Introduction

The movements of interest rate play an important role in the decision of investment and risk management in the financial markets. One-factor models are a popular class of interest rate models which is used for these purposes, especially in the pricing of interest rate derivatives. One-factor models are represented by the following stochastic differential equation

$$dr(t) = \mu(t, r(t))dt + \sigma(t, r(t))dW(t) \tag{1}$$

where μ and σ are the drift and the diffusion term of the interest rate process, respectively, and W is a Brownian motion.

The Vasicek and CIR models are two important models of short rate in the class of one-factor models. Although there are many extensions of these models in the literature, they are still popular because of their tractability and their closed form solutions for various interest rate derivatives. In this work, we will focus on these two models.

2 Vasicek Model

Vasicek (1977) assumed that the instantaneous spot rate under the real world measure evolves as an Orstein-Uhlenbeck process with constant coefficients. For a suitable choice of the market price of risk, this is equivalent to assume that r follows an Ornstein-Uhlenbeck process with constant coefficients under the risk-neutral measure as well, that is under Q

$$dr(t) = \kappa[\theta - r(t)]dt + \sigma dW(t)), \quad r(0) = r_0, \tag{2}$$

where r_0,κ,θ and σ are positive constants. Under the physical measure Q_0, the model follows :

$$dr(t) = [\kappa\theta - (\kappa + \lambda\sigma)r(t)]dt + \sigma dW^0(t), \quad r(0) = r_0, \tag{3}$$

where λ is the market price of risk. Notice that for $\lambda = 0$ the two dynamics coincide, i.e. there is no difference between the risk neutral world and the subjective world. However, this process here assumes that the market price of risk has the form of

$$\lambda(t) = \lambda\ r(t) \tag{4}$$

which is a popular assumption in the literature. Under this choice we obtain a short rate process that is tractable in both measures. The market price of risk can be chosen as constant, i.e, $\lambda(\mathrm{t}) = \lambda$.

The most important feature which this model exhibits is the mean reversion,which means that if the interest rate is bigger than the long run mean (r$>\theta$), then the coefficient $\kappa(>0)$ makes the drift become negative so that the rate will be pulled down in the direction of θ. Similarly, if the interest rate is smaller than the long run mean (r$<\theta$), then the coefficient $\kappa(>0)$ makes the drift term become positive so that the rate will be pulled up in the direction of θ. Therefore, the coefficient κ is the speed of adjustment of the interest rate towards its long run level. There are also compelling economic arguments in favor of mean reversion. When the rates are high, the economy tends to slow down and borrowers require less funds. Furthermore, the rates pull back to its equilibrium value and the rates decline. On the contrary when the rates are low, there tends to be high demand for funds on the part of the borrowers and rates tend to increase. This feature is particularly attractive because without it, interest rates could drift permanently upward the way stock prices do and this is simply not observed in practice.

Other attractive features of the model is the availability of a closed form solutions for r_t which is obtained by integrating the risk-neutral equation mentioned above. The solution of the model is, for each $s \leq t$

$$r(t) = r(s)e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) + \sigma \int_s^t e^{-\kappa(t-u)} dW(u). \tag{5}$$

Here the interest rates are normally distributed and the expectation and variance are given by

$$E\{r(t)|F(s)\} = r(s)e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) \tag{6}$$

$$Var\{r(t)|F(s)\} = \frac{\sigma^2}{2\kappa}\left[1 - e^{-2\kappa(t-s)}\right] \tag{7}$$

One unfortunate consequence of a normally distributed interest rate is that it is possible for the interest rate to become negative. However, as t$\rightarrow\infty$, the limit of expected rate and variance, as long as $\kappa > 0$, will converge to θ and $\sigma^2/2\kappa$ respectively.

The Vasicek Model possess an affine term structure (ATS) i.e. the term structure has the form

$$p(t,T) = F(t, r(t); T) \tag{8}$$

where F has the form

$$F(t, r; T) = e^{(A(t,T)-B(t,T)r)}, \tag{9}$$

and where A and B are deterministic functions. It turns out that the existence of an affine term structure is extremely pleasing from an analytical and a computational point of view, so it is of considerable interest to see which model has an affine structure.

This feature also made the model successful and famous because of the ease to calculate the term structure with given formulaes with no need of simulation of short rates. The model has the term structure given by :

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{10}$$

where

$$A(t,T) = exp\left\{\left(\theta - \frac{\sigma^2}{2\kappa^2}\right)[B(t,T) - T + t] - \frac{\sigma^2}{4\kappa}B(t,T)^2\right\} \tag{11}$$

$$B(t,T) = \frac{1}{\kappa}\left[1 - e^{-\kappa(T-t)}\right] \tag{12}$$

Jamshidian (1989) showed that the price at time t of a European option with strike K, maturity T an written on a pure discount bond maturing at time S can be easily calculated and is given by

$$ZBO(t,T,S,K) = w[P(t,S)\Phi(wh) - KP(t,T)\Phi(w(h-\sigma_p))], \tag{13}$$

where $w = 1$ for a call and $w = -1$ for a put, $\Phi(.)$ denotes the standard normal cumalative function, and

$$\sigma_p = \sqrt{\frac{1-e^{-2\kappa(T-t)}}{2\kappa}}B(T,S), \tag{14}$$

$$h = \frac{1}{\sigma_p}ln\frac{P(t,S)}{P(t,T)K} + \frac{\sigma_p}{2} \tag{15}$$

3 Cox-Ingersoll-Ross Model

In this model, under the risk-neutral measure Q, the instantaneous short rate follows the stochastic differential equation

$$dr(t) = \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t), \quad r(0) = r_0, \tag{16}$$

where κ, θ, σ, r_0 are positive constants, and $W(t)$ is a Brownian motion. When we impose the condition $2\kappa\theta > \sigma^2$ then the interest rate is always positive, otherwise we can only guarantee that it is non-negative (with a positive probability to terminate in zero).

The drift factor $\kappa(\theta - r(t))$ is same as in the Vasicek model. Therefore, the short rate is mean-reverting with long run mean θ and speed of mean-reversion equal to κ. The volatility term σ of the Vasicek model is multiplied with the term $\sqrt{r(t)}$, and this eliminates the main drawback of the Vasicek model, a positive probability of getting negatice interest rates. When the interest rate approaches zero then the volatility term $\sigma\sqrt{r(t)}$ approaches zero, cancelling the effect of the randomness, so the interest rate remains always positive. When the interest rate is high then the volatility is high and this is a desired property.

Under the objective measure Q_0, the model follows the process

$$dr(t) = [\kappa\theta - (\kappa + \lambda\sigma)r(t)]dt + \sigma\sqrt{r(t)}dW^0(t) \tag{17}$$

where λ is a constant and $W^0(t)$ is a Q_0-Brownian motion. The market price of risk has the form of

$$\lambda(t) = \lambda\sqrt{r(t)} \tag{18}$$

and, so it depends on the level of the interest rate. Again, this is a popular choice which we do not justify here.

If we work in the risk-neutral measure Q, then the conditional expectation and the conditional variance of the short rate are given by

$$E^Q\{r(t)|F(s)\} = r(s)e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) \tag{19}$$

$$Var\{r(t)|F(s)\} = r(s)\frac{\sigma^2}{\kappa}\left(e^{-\kappa(t-s)} - e^{-2\kappa(t-s)}\right) + \frac{\theta\sigma^2}{2\kappa}\left(1 - e^{-\kappa(t-s)}\right)^2 \tag{20}$$

In the CIR model, the short rate admits a noncentral chi-square distribution. The density function for the interest rate process is

$$p_{r(t)}(x) = p_{\chi^2(\nu,\lambda_t)/c_t}(x) = c_t p_{\chi^2(\nu,\lambda_t)}(c_t x) \tag{21}$$

where

$$c_t = \frac{4\kappa}{\sigma^2(1 - exp(-4\kappa))} \tag{22}$$

$$\nu = 4\kappa\theta/\sigma^2 \tag{23}$$

$$\lambda_t = c_t r_0 exp(-\kappa t) \tag{24}$$

and $\chi^2(\nu, \lambda)$ is the noncentral chi-squared distribution function with ν degrees of freedom and non-centrality parameter λ.

The CIR model provides explicit solutions for the bond prices. The model has affine term structure, so the price of a zero-coupon bond at time t with maturity T has the form

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{25}$$

where

$$A(t,T) = \left[\frac{2h exp\{(\kappa + h)(T - t)/2\}}{2h + (\kappa + h)(exp\{(T - t)h\} - 1)}\right]^{2\kappa\theta/\sigma^2}, \tag{26}$$

$$B(t,T) = \frac{2(exp\{(T-t)h\}-1)}{2h+(\kappa+h)(exp\{(T-t)h\}-1)}, \quad (27)$$

$$h = \sqrt{\kappa^2+2\sigma^2}. \quad (28)$$

Another important feature of the CIR model is that there is a closed form formula for the call option price written on a zero coupon bond. The price of a European call option at time t with maturity $T > t$, strike price K, written on a zero-coupon bond maturing at $S > T$, is

$$\begin{aligned} &ZBC(t,T,S,K) \\ &= P(t,S)\chi^2\left(2\overline{r}\left[\rho+\psi+B(T,S)\right];\frac{4\kappa\theta}{\sigma^2},\frac{2\rho^2 r(t)exp\{h(T-t)\}}{\rho+\psi+B(T,S)}\right) \\ &-KP(t,T)\chi^2\left(2\overline{r}\left[\rho+\psi\right];\frac{4\kappa\theta}{\sigma^2},\frac{2\rho^2 r(t)exp\{h(T-t)\}}{\rho+\psi}\right) \end{aligned} \quad (29)$$

where

$$\rho = \rho(T-t) := \frac{2h}{\sigma^2(exp\{h(T-t)\}-1)} \quad (30)$$

$$\psi = \frac{\kappa+h}{\sigma^2} \quad (31)$$

$$\overline{r} = \overline{r}(S-T) := \frac{ln(A(T,S)/K)}{B(T,S)} \quad (32)$$

4 Sensitivity of the models for the parameters

Vasicek and CIR are two of the historically most important short rate models used in the pricing of interest rate derivatives. Some of the parameters play a big role in the pricing of a financial derivatives or in the forecasting of a process, while some of them do not affect them so much. Therefore, depending on what we want to price or forecast, it is important to check the sensitivity of the models with respect to different parameters.

4.1 Sensitivity of short rate simulation for the model parameters

In this part we analyze the effect of the parameters, when we use either the Vasicek or the CIR model in simulation of the short rate.

Discretization of the models by the Euler method results in the discretized version of the Vasicek model as

$$r_V(t_{i+1}) = r_V(t_i) + \kappa[\theta - r_V(t_i)]\Delta t + \sigma\sqrt{\Delta t}Z_{i+1} \quad (33)$$

while the discretized version of the CIR model is

$$r_C(t_{i+1}) = r_C(t_i) + \kappa[\theta - r_C(t_i)]\Delta t + \sigma\sqrt{r_C(t_i)}\sqrt{\Delta t}Z_{i+1}. \quad (34)$$

If we change κ by an amount of δ^κ then our discretized models are

$$r_V^\kappa(t_{i+1}) = r_V(t_i) + (\kappa + \delta^\kappa)[\theta - r_V(t_i)]\Delta t + \sigma\sqrt{\Delta t}Z_{i+1} \tag{35}$$

and

$$r_C^\kappa(t_{i+1}) = r_C(t_i) + (\kappa + \delta^\kappa)[\theta - r_C(t_i)]\Delta t + \sigma\sqrt{r_C(t_i)}\sqrt{\Delta t}Z_{i+1}. \tag{36}$$

So, the change in the short rate for one unit of discretization in time is

$$\Delta_V^\kappa r_V(t_{i+1}) = r_V^\kappa(t_{i+1}) - r_V(t_{i+1}) = \delta^\kappa(\theta - r_V(t_i))\Delta t \tag{37}$$

for the Vasicek model, and

$$\Delta_C^\kappa r_C(t_{i+1}) = r_C^\kappa(t_{i+1}) - r_C(t_{i+1}) = \delta^\kappa(\theta - r_C(t_i))\Delta t \tag{38}$$

for the CIR model. This shows that, if we increase κ then the next value of the short rate increases by $\delta^\kappa(\theta - r(t_i)\Delta t)$ in both Vasicek and CIR models when the current short rate is below the long run mean, and decreases by the same amount when the current short rate is above the long run mean. So, the change in κ will not affect the short rate in long term, just effect the time which is necessary for the interest rate to come back to the long-run mean level. On the other hand, when the kappa is increased the variation of the interest rate decreases, so the volatility decreases. Therefore, kappa is important in the pricing of the financial instruments which are affected by the volatility, but are not dependent on the long term expected value of the simulated interest rate.

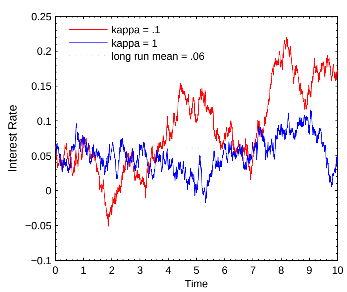

Figure 1: Sensitivity of Vasicek model for κ

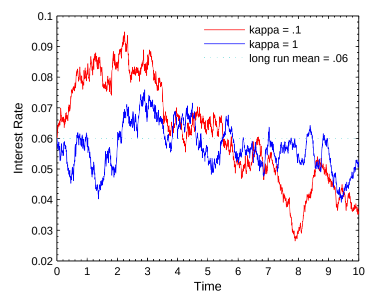

Figure 2: Sensitivity of CIR model for κ

The change in θ by δ^θ gives

$$r_V^\theta(t_{i+1}) = r_V(t_i) + \kappa[(\theta + \delta^\theta - r_V(t_i)]\Delta t + \sigma\sqrt{\Delta t}Z_{i+1} \tag{39}$$

for the Vasicek model, and

$$r_C^\theta(t_{i+1}) = r_C(t_i) + \kappa[(\theta + \delta^\theta - r_C(t_i)]\Delta t + \sigma\sqrt{r_C(t_i)}\sqrt{\Delta t}Z_{i+1} \tag{40}$$

for the CIR model. Then the changes in the short rate are

$$\Delta_V^\theta r_V(t_{i+1}) = \Delta_C^\theta r_C(t_{i+1}) = \delta^\theta\kappa\Delta t. \tag{41}$$

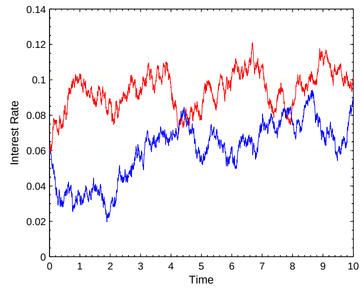

Figure 3: Sensitivity of Vasicek model for θ

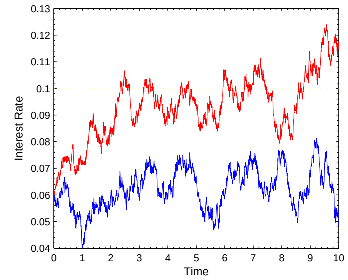

Figure 4: Sensitivity of CIR model for θ

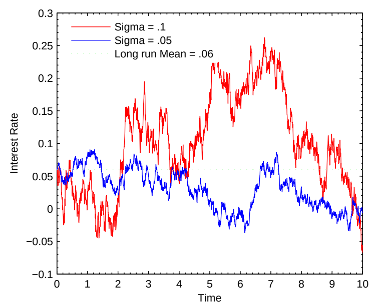

Figure 5: Sensitivity of Vasicek model for σ

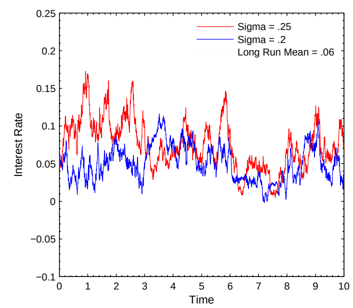

Figure 6: Sensitivity of CIR model for σ

Therefore, the increase in θ increases the next step value of short rate in the both models. So, it is a crucial parameter in the estimation of the model.

Following the same way for both models, a change in σ causes a change in interest rate

$$\Delta_V^\sigma r_V(t_{i+1}) = \delta^\sigma \sqrt{\Delta t} Z_{i+1} \tag{42}$$

for the Vasicek model, and

$$\Delta_C^\sigma r_C(t_{i+1}) = \delta^\sigma \sqrt{r_C(t_i)} \sqrt{\Delta t} Z_{i+1} \tag{43}$$

for the CIR model. So, the change in σ effects the short rate in both models. A fixed amount of change in σ effects the short rate simulation more in the Vasicek model than the CIR model, because in the CIR model the volatility contains the $\sqrt{r(t)}$ term and this term decreases the effect of a change in σ. We can realize that also from the Figure 5 and Figure 6. 0.05 increase in σ is quite visibile in the Vasicek model, but that is not the case for the CIR model. The change in short rate can be negative or positive depending on the sign of δ^σ and the random term. Also note that, the amount of change in the short rate is highly dependent on the random term. Because of the standard Brownian motion, in the long term, the effect of the change in σ does not effect the expected value of the interest rate, but it increases the variations. Therefore, if we are interested in the long term level of the interest rate, σ is not so crucial, but if we are interested in pricing of some financial instruments which are affected by the variations in the short rate dynamics in short term, then σ is a crucial parameter for us.

4.2 Sensitivity of bond prices for the model parameters

In this part the analysis is done to see how the variations in the various parameters affect the zero-coupon bond prices generated by the models. The method used is to fix two parameters and vary the third one to study the impact of varying parameter on the bond prices at a given time for different maturities. We use plots of bond prices for analysis as it is not easy to get formulas for the changes in bond prices caused by a change of a parameter.

Figure 7 and Figure 9 show the effect of κ to bond prices driven by Vasicek model for two different values of σ. When the σ is small then the effect of κ is less compared with the case of high σ. With a small σ we get nearly same bond prices for different κ values. An increase in κ decreases the bond prices and after a certain value of κ the effect of the κ in the bond prices is almost nothing. When we increase the σ, then the κ plays a bigger role in the pricing of the bonds (Figure 9). Figure 9 shows that with low κ value and high σ it is possible to get bond prices bigger than 1. This is because of the possibility of negative interest rates in Vasicek model. With smaller κ the probability of getting negative values of short rate is higher and a high volatility increases this probability further. A high κ value makes this probability negligible and we get more bond prices which are less than 1.

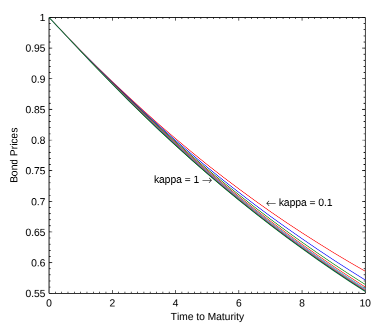

Figure 7: Vasicek bond prices with different κ's ($\sigma = 0.02$)

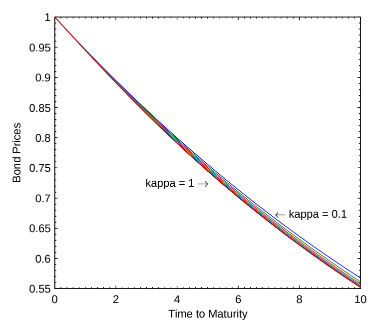

Figure 8: CIR bond prices with different κ's ($\sigma = 0.02$)

When we use CIR model, instead of the Vasicek model to price bonds, we get similar results except bond prices more than 1 (Figure 8 and Figure 10). This result because of the positive interest rates in CIR model. From the figures the variation of the bond prices with different κ values looks like smaller in the CIR model, but this is not the case. In the figures, we use same values of σ but typically values of σ are higher in the CIR model. When we increase the σ we can see the variation better.

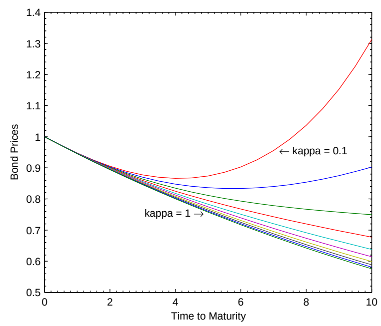

Figure 9: Vasicek bond prices with different κ's ($\sigma = 0.1$)

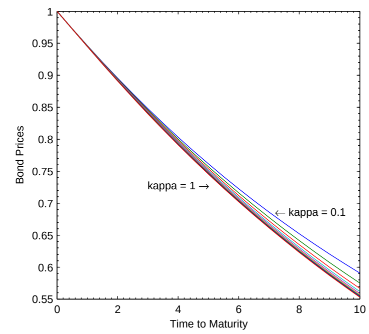

Figure 10: CIR bond prices with different κ's ($\sigma = 0.1$)

The effect of the θ for bond prices in Vasicek and CIR model is more straight (Figure 11 and Figure 12) compared with the other parameters. In both models, a increase in θ directly effect the bond prices and we get lower bond prices. The only difference is seen when we have low θ values. With a small θ value we can get bond prices greater than 1 in Vasicek model because of the possibility of negative interest rates, while it is not possible in CIR model.

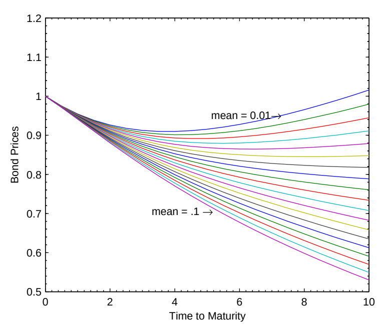

Figure 11: Vasicek bond prices with different θ's ($\sigma = 0.1$)

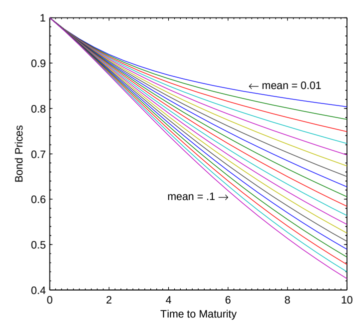

Figure 12: CIR bond prices with different θ's ($\sigma = 0.1$)

Figure 13 and Figure 14 show the different levels of bond prices for Vasicek and CIR model for different levels of σ. The variation of the bond prices w.r.t. same σ values are higher in the Vasicek model, because σ is typically higher in the CIR model (In the CIR model, volatility term contains $\sqrt{r(t)}$ in addition to volatility term of the Vasicek model. To compensate this difference, the σ term of the CIR should be higher). In both models, lower σ values give lower bond prices and higher σ values gives higher bond prices, while the sensitivity of bond prices for a change in σ is higher with high σ values. Also note that, in Vasicek model, a bond price higher than 1 (because of the possibility of negative interest rates)is more probable with high σ values.

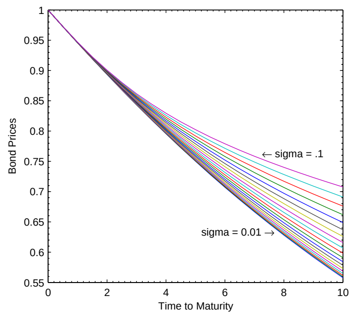

Figure 13: Vasicek bond prices with different σ's

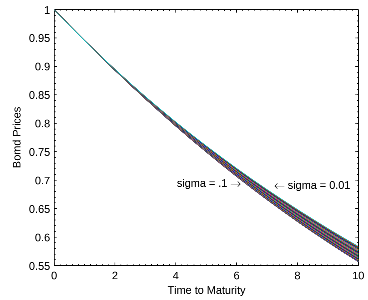

Figure 14: CIR bond prices with different σ's

5 Calibration and estimation of model parameters

5.1 Calibration of model parameters using bond prices

In this section, we calibrate Vasicek and CIR models to analyze the efficiency and stability of the models. To find the model parameters, we apply the classical Least-Square technique to the actual Canadian zero-coupon bond data. The data was taken from the Bank of Canada website. While zero-coupon rates are not themselves directly observable, the monthly data has been generated by them from the Svensson model as implemented by Bolder and Streliski (1999). This data correspond to the first day of each month, from January 1997 to December 2006. The market zero-coupon rates incorporated into our measurement system will include fourteen observations with 3-month, 6-month, 1-year, 1.5-year, 2-year, 3-year, 4-year, 5-year, 7-year, 10-year, 15-year, 20-year, 25-year and 30-year time to maturities. The optimization problem was solved using the algorithm Direct which finds the global minima of the objective function.

The code for the direct algorithm was taken from a free source available on the internet. The algorithm used by the Direct method is described here briefly. DIRECT is a sampling algorithm. That is, it requires no knowledge of the objective function gradient. Instead, the algorithm samples points in the domain, and uses the information it has obtained to decide where to search next. A global search algorithm like DIRECT can be very useful when the objective function is simulation or when the dimension of the objective function is small. The DIRECT algorithm will globally converge to the minimal value of the objective function. Unfortunately, this global convergence may come at the expense of a large and exhaustive search over the domain, and using a limited number of evaluations and iterations can cause to find a local minima. The name DIRECT comes from the shortening of the phrase "DIviding RECTangles", which describes the way the algorithm moves towards the optimum.

Because of the usage of a limited number of evaluations and iterations in the optimization algorithm we could have got local minima in some optimizations which made the standard error high in some cases. To see the trend of the parameters better we also included the approximations of the parameter values by applying a smoothing function to the graphs.

Calibration results for the κ in the Vasicek and CIR models are given in Figure 15 and Figure 16, respectively. Results for both models are unstable. However, this is not particularly a big problem, because, as we concluded in the sensitivity of the bond prices for κ in the previous section, that kappa is a crucial parameter only when the σ is high and κ is small. Only in that situation the bond prices are highly sensitive to κ. If we compare the calibration results for κ and σ then we can observe that when the κ is small then the corresponding σ is also small, and this eliminates the effect of the κ.

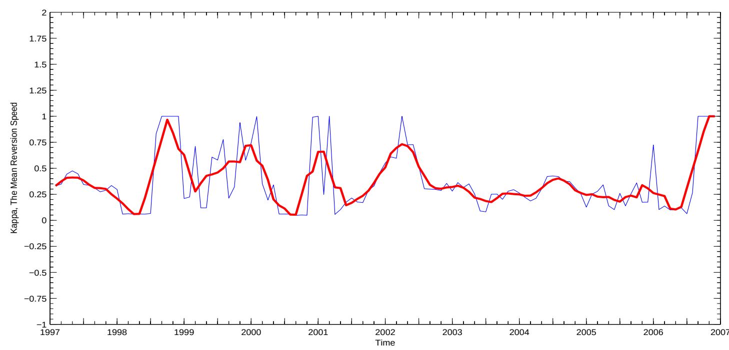

Figure 15: Calibration results for κ in Vasicek model

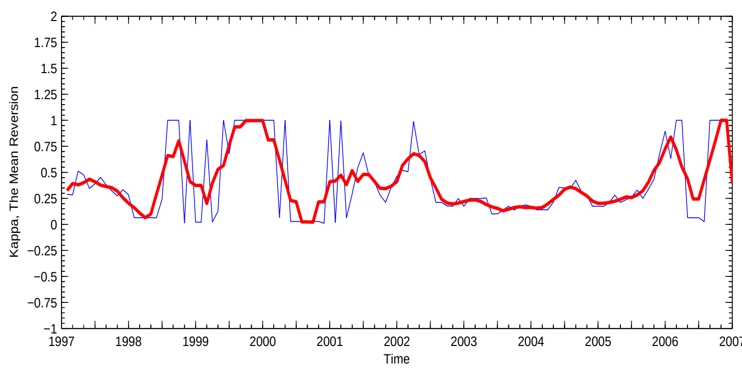

Figure 16: Calibration results for κ in CIR model

Figure 17 and Figure 18 show the calibration results for θ. From the figures, it is clear that the results in both models are inline with each other. There is no big differences among both models for the estimation of θ.

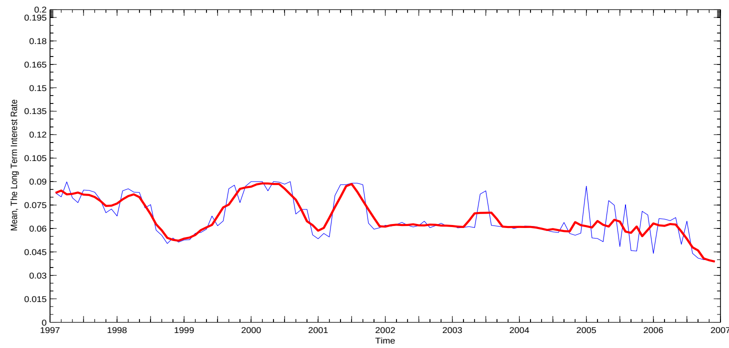

Figure 17: Calibration results for θ in Vasicek model

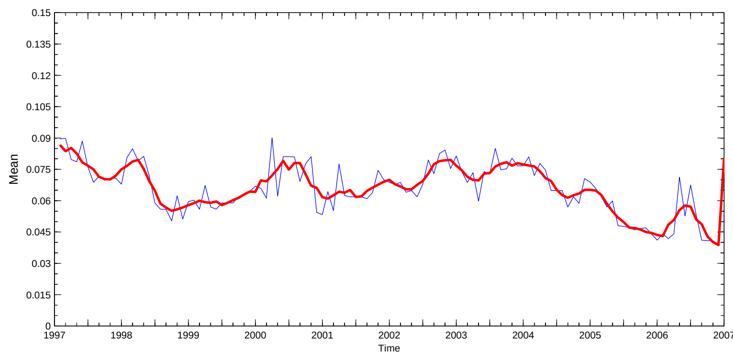

Figure 18: Calibration results for θ in CIR model

Calibration results of the models for σ are given in Figure 19 and 20. Calibration of the CIR model gives higher value of σ as what was expected. The Vasicek model gives more stable σ estimates compared with the CIR model, so this model can be prefered to infer the volatility structure of the Canadian bond prices.

Summing up all conclusions for the calibration of the models using bond prices we see that for both Vasicek and CIR, estimates κ and θ were similar, while Vasicek has more stable σ estimates for the Canadian bond prices than the CIR model.

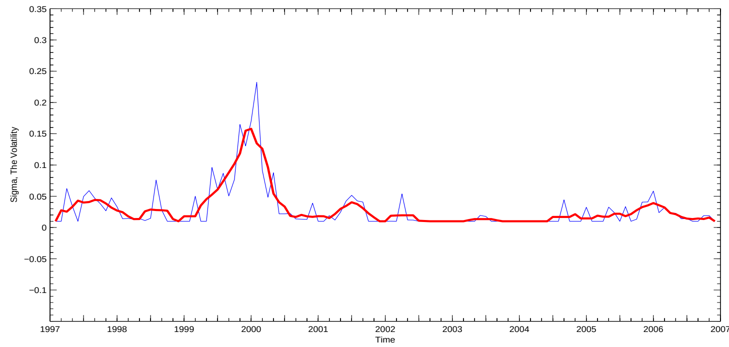

Figure 19: Calibration results for σ in Vasicek model

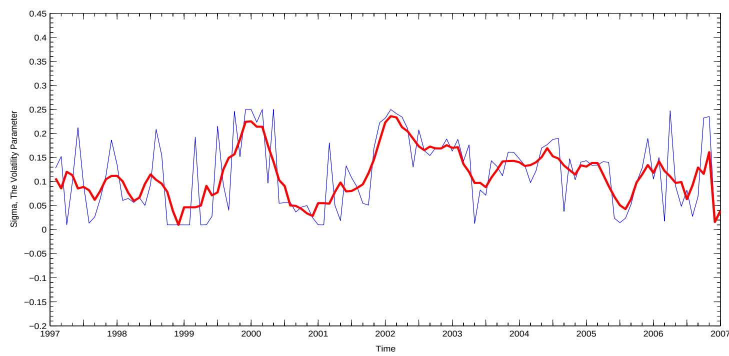

Figure 20: Calibration results for σ in CIR model

5.2 Estimation of the parameters using historical short rates and calibration of market price of risk

In this section, we assume short rate under the subjective measure Q_0 in the Vasicek model follows

$$dr(t) = \kappa^*(\theta^* - r(t))dt + \sigma^* dW(t) \tag{44}$$

and in the CIR model follows

$$dr(t) = \kappa^*(\theta^* - r(t))dt + \sigma^* \sqrt{r(t)} dW(t). \tag{45}$$

where λ is a constant which features the market price of risk. Using historical data for the interest rates we can estimate the parameters of the models under the objective measure. Changing the measure by Girsanov's Theorem we can go back to the risk-neutral measure, and under this measure our parameters are

$$\kappa = \kappa^* - \lambda\sigma^*, \quad \theta = \frac{\kappa^*\theta^*}{\kappa^* - \lambda\sigma^*}, \quad \sigma = \sigma^*$$

.
Then, using these parameters which depend on λ only, we can calibrate the model for λ by using the current bond prices. This λ can then be used to calculate model parameters under the risk-neutral measure and the market price of risk for the models.

To estimate the real world parameters we used the historical overnight rates data for Canada starting from the beginning of 1994 to the end of 2004. We used 8-year data to estimate the parameters. For estimation of the parameters from the historical time series data, we used two-stage estimation method of Philips P. C. B. and Yu J. (2005). This method estimates parameters of the diffusion term and drift term separately. In the first step, the parameters of the diffusion term are found by a quadratic variation type estimator. In the second step, an approximate likelihood function is maximized to find a maximum likelihood estimator of the parameters of drift term.

After estimating the real world parameter, last day's zero coupon bond prices were used to calibrate the models for λ. This λ is then used to change the measure and find the parameters of risk-neutral world. After calculating the parameters for the first 8-year data, we shifted the time frame one month further and used the last 8-year data and applied the same procedure to this data. This process was repeated for 36 observations,i.e. for each month till the end of 2004. The results are given in Figure 21, 22, 23 and 24 (blue curves are for the Vasicek model while red ones are for the CIR model).

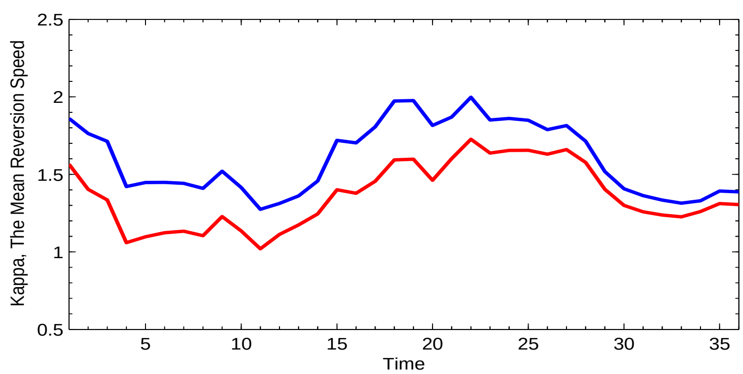

Figure 21: Estimation results for κ in Vasicek and CIR models

The Vasicek and CIR models gives almost the same results for all parameters. The long rum mean is the same in both models while the speed of mean reversion is a little bit higher in the Vasicek model. σ follows the same trend in both models and the difference in the level of σ comes from the inclusion of $\sqrt{r(t)}$ term in the volatility term of the CIR model. Figure 24 gives the market price of risk for the models, i.e. $\lambda(t) = \lambda r(t)$ for the Vasicek model, and $\lambda(t) = \lambda\sqrt{r(t)}$ for the CIR model. We have similar results also for the market price of risk of the models.

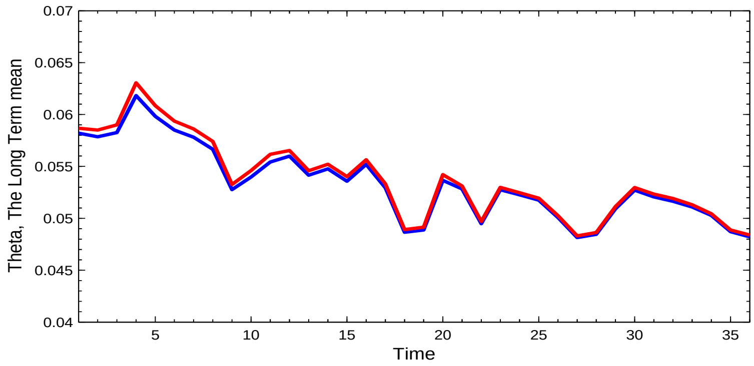

Figure 22: Estimation results for θ in Vasicek and CIR models

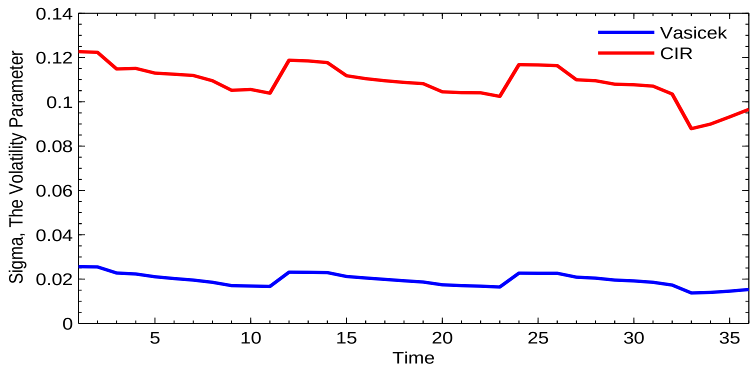

Figure 23: Estimation results for σ in Vasicek and CIR models

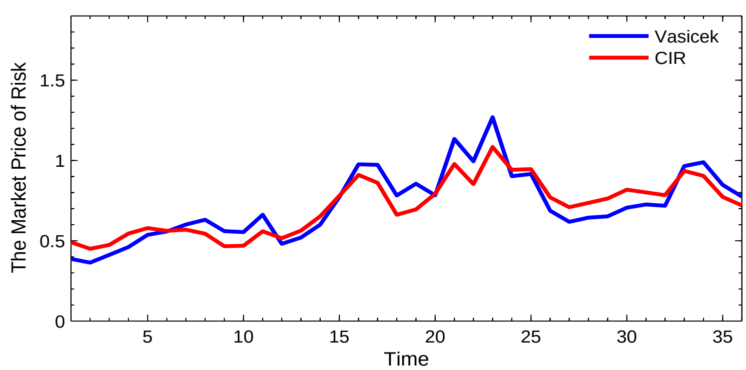

Figure 24: Market price of risk for Vasicek and CIR models

Therefore, we can not decide which model is better for the historical short rate data of Canada as both methods gives similar results for all parameters.

6 Conclusions

This work has been focused on the study of two endogenous one factor models, namely the CIR and Vasicek model. In the first part, we studied the basic features of the two models and did a comparative study to see how the various parameters affect the dynamics and pricing of bonds evaluated using the two models. The analysis showed that the two models were quite similar in the way they react to changes in the parameters. However, due to the multiplication of the square root term in the CIR model, a particular change in σ did not affect the prices of the bonds as it did in the Vasicek model. Moreover the value of sigma in the CIR model was generally high, which can be misleading at times. On the other hand, a high sigma in the Vasicek model could result in negative interest rates which is not observable in reality.

In the second part of the work, we applied the models to a given data set, in an attempt to study and analyze the nature of fits of the models. First, we calibrated the models using the bond prices, thus getting the parameters in the risk neutral world. The results of the two models seemed quite similar, but again the calibration gave a higher sigma in the CIR model, which was rather unstable. However, the results could also have been questionable due to the optimization procedure we used. We applied two different optimization algorithms to check the correctness of the results and the two methods yielded similar results. The parameters of the models were also estimated using the historical overnight interest rates. The two stage method was used in the estimation procedure. The parameter values driven by the estimation had exactly the same variations over time for the two models, again showing that for the given data set, the two models were quite similar. However, the Vasicek model seemed to perform better due to the more stable volatility parameter.

In the given data set it is quite clear that the models exhibit quite nice fit. The question of choosing a model is answerable in terms of what we want to price. In general if the interest rates are far away from zero, the Vasicek model can be given an edge over the CIR model, due to the tractability of the model and also the availability of closed form solutions even for more complex financial interest rate derivatives. However, if the interest rates are closer to zero then working with the Vasicek model can become cumbersome due to the possibility of negative interest rates which can yield some illogical results and prices. However, in data which are not as smooth and well-behaved as the given data set, time dependent models would be much better in explaining the features of the term structure. Typically, the Vasicek and CIR models do not effectively handle a complex term structure. In this case, a smaller number of parameters prevents a satisfactory calibration to the market data and the zero coupon curve is likely to be badly reproduced, and also the shapes like the inverted yield curves cannot be explained by these models. In that case time dependent models which fit the current term structure perfectly would be better.

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