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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001



Fraunhofer Institut
Techno- und
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Heterogeneous Redundancy in Software Quality Prediction using a Hybrid Bayesian Approach

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Abstract

With the ever-increasing significance of software in our everyday lives, it is vital to afford reliable software quality estimates. Typically, quantitative software quality analyses rely on either statistical fault prediction methods (FPMs) or stochastic software reliability growth models (SRGMs). Adopting solely FPMs or SRGMs, though, may result in biased predictions that do not account for uncertainty in the distinct prediction methods; thus rendering the prediction less reliable. This paper identifies flaws of the individual prediction methods and suggests a hybrid prediction approach that combines FPMs and SRGMs. We adopt FPMs for initially estimating the expected number of failures for finite failure SRGMs. Initial parameter estimates yield more accurate reliability predictions until sufficient failures are observed that enable stable parameter estimates in SRGMs. Being at the equilibrium level of FPM and SRGM predictions we suggest combining the competing prediction methods with respect to the principle of heterogeneous redundancy. That is, we propose using the individual methods separately and combining their predictions. In this paper we suggest Bayesian model averaging (BMA) for combining the different methods. The hybrid approach allows early reliability estimates and encourages higher confidence in software quality predictions.

Keywords—Reliability Prediction, Fault Prediction, Non-homogeneous Poisson Process, Bayesian Model Averaging

MSC: 62F15, 62H12, 62C10

1 Introduction

Conventionally, quality prediction approaches use exclusively fault prediction methods (FPMs) or software reliability growth models (SRGMs). FPMs use software metrics and fault data from previous software releases or similar software projects to predict the number of faults of the software. SRGMs treat the software as a black-box and gather failure data throughout functional black-box testing; thereby, assuming software is exercised along its operational profile [1]. The observed failures are used to calibrate a stochastic model employed for predicting the reliability of the software.

Even though the competing models can complement each other, combining these two approaches is hardly implemented, if at all. Intuitively, FPMs render useful for initially estimating the number of finally expected failures of finite failure SRGMs, as long as not enough failures have been observed during testing. This combination enables early usage of SRGMs, even before testing has commenced. On the other hand, with more and more failures collected during testing the predictive performance of SRGMs is gradually improving. Yet SRGMs are always heavily dependent on the testing process and the failures collected during testing. This innate dependence causes biased predictions if the model assumptions are not properly met. FPMs are not as much dependent on testing. Considering only the software under test¹ the FPM approach is independent of testing. Only software measures are used to predict the number of faults. However, there is not the one software complexity measure that can be directly related to the number of faults expected. There is always a lot of uncertainty involved in building FPM models. Other sources for bias in the FPM models are the issue of transferability of a fault prediction model. Adopting a FPM model demands closeness of the current to the historical software projects, and even software development processes. SRGMs ignore any software measures or required closeness to former projects they are only dependent on the testing process.

The distinct software quality prediction approaches are quite different in use and in the data they draw on. Thus the individual prediction methods are prone to different kinds of errors. With the hybrid approach we suggest combining FPMs and SRGMs on different levels. We aim for:

- Enabling SRGM predictions from the very start of testing
- Supporting a priori model selection for SRGMs
- Providing higher confidence in reliability predictions
- Reducing bias inherent in the individual prediction methods
- Introducing more flexibility for adapting the prediction process to concrete projects.

¹Faults from previous or similar software projects may as well be found during testing.

To meet these aims we unveiled flaws of the distinct quality prediction approaches and identified coupling points. Drawing on these coupling points we suggest initial parameter estimates for SRGMs using FPMs. We propose a stopping rule for these initial parameter estimates when the prediction performance reaches a certain level. Confidence bounds are used to compare FPM with SRGM prediction performance. Having passed the stopping point for initial parameter estimates we propose model combination on the principle of heterogeneous redundancy. That is, using the individual prediction methods separately and combining their predictions. Since the approaches are entirely different, draw on different data, and are prone to different kinds of errors this combination is expected to yield more trustworthy reliability predictions. With the combination we are in average superior to solely employing the individual approaches, for we gain at least their prediction accuracy but are provided with a second independent prediction. This encourages higher confidence in the quality predictions. We use Bayesian model averaging (BMA) for combining the different FPM and SRGM models. Since neither FPM nor SRGM has one single optimal model for their predictions but in both distinct methods it is not possible to select the single best model a priori BMA does not only account for the bias innate in the distinct methods but also for the bias of the different models within the individual methods.

Table 1 lists the acronyms used throughout this paper. Section 2 provides a survey of existing quality prediction approaches and touches on their deficiencies. Section 3 is the main chapter and introduces the hybrid approach. It presents the initial parameter estimates, the stopping rule, and the model combination. Section 4 presents the conclusions and future work.

Table 1: Acronyms

SRG	software reliability growth
SRGM	software reliability growth model
FPM	fault prediction model
BMA	bayesian model averaging
(E)NHPP	(enhanced) non-homogeneous poisson process
ML	maximum likelihood
RSM	response surface model

2 Software quality prediction

2.1 Software Reliability Growth Models

Reliability is a key quality characteristic of software, which reflects the users view on software and favors trade-offs in release planning. Software reliability is the probability of failure-free operation for a specified period of time in a specified environment [2]. In order to predict software reliability we need to model the failure process of software. During testing and debugging we usually have reliability growth due to the (perfect) removal of faults causing the experienced failures.

Reliability prediction usually involves testing the software, and collecting the times of its failures², t_i . Using t_i we can calibrate the SRGMs and predict software's reliability. A comprehensive overview on SRGMs can be found in [3–6].

A large number of SRGMs have been developed over the years. Among the first were the binomial model of Jelinski and Moranda [7] and the Goel-Okumoto model [8] that describes the failure process as a NHPP (Non-Homogeneous Poisson Process). Since then, many other models have been suggested. For NHPP models Gokhale and Trivedi proposed a unifying framework, enhanced (E)NHPP, which also captures test-coverage [9]. However, the models rely on rather crucial assumptions. Software should be exercised along its operational profile, repairs should be effected instantly and without introducing new faults, software failures are independent, and so on. Beyond these issues, SRGMs can be employed only very late in the software development cycle, a priori model selection is not possible, and the models perform only well if sufficient failures have been observed. It is not possible, though, to pre-determine whether enough failure data has been collected. Having only insufficient failure data obstructs finding maximum likelihood estimates for the model parameters if it is at all possible. Even being provided with sufficient failure data maximum likelihood estimates are sometimes instable [10, 11]. With the hybrid approach we propose early parameter estimates using fault prediction models to enable early usage of SRGMs and to circumvent the problems of instable parameter estimates.

In practice many of the presented assumptions are often violated. So, software is commonly tested systematically with the purpose of finding faults. Also, there is often a correlation of successive failures [12] which comes along with testing for finding bugs. These violations lead to flawed reliability predictions. With the hybrid approach we intend to provide a back up for the predictions in order

²Collecting failure data in terms of grouped failure per time interval or collecting inter-failure times is also possible

to reduce the bias caused by violating the assumptions.

2.2 Fault Prediction Models

Faults inherent in software may cause failures while running the software. So estimates on the number of faults are quite interesting in measuring software's quality. For estimating this number of faults, fault prediction models search for attributes of the software or of the software development process to extract causes for faults. Based on these attributes a fault prediction model can be built. Diverse software metrics have been developed that in some way affect the number of faults and, thus, can be employed in fault prediction models³.

Using the collected metrics and faults from previous releases of similar software the fault prediction models can be built. Starting with simple univariate prediction models based solely on lines of code measures [13] up to complex prediction models like response surface models, multivariate adaptive regression splines (MARS), neuronal networks, and the like, there exists a broad variety of fault prediction models. A critical overview is provided in Fenton and Neil [14] further references are [15, 16].

Fault prediction models suffer from their uncertain or even lacking transferability. Having created a fault prediction model based on metrics and faults from historical projects it is not sure whether this model is applicable for a current project. Fenton and Neil [14] found that regression modeling alone is inadequate for software fault prediction. In the hybrid approach we want to combine fault prediction with reliability prediction that is solely based on data from the current project. Fault prediction is supposed to provide an independent back up for reliability prediction, and vice versa.

³Typically size measures like lines of code are strongly correlated to faults.

3 Hybrid Reliability Prediction

As listed, sole use of FPMs or SRGMs is tainted with much uncertainties and tends to yield predictions that are sometimes not satisfactory. The individual techniques rely on many assumptions that impede their adoption in specific projects or render their results inaccurate without any further information about their prediction accuracy. If data of one of the individual prediction techniques is flawed all models within this prediction method suffer from these data. The hybrid approach that incorporates two distinct prediction techniques based on different data and assumptions remedies these deficiencies.

Intuitively, the notion of combining different prediction techniques seems straightforward. Yet, only little research has been conducted on this topic. So far, combining FPMs and SRGMs targets at alleviating the prediction incapability of SRGMs at the very start of testing when only little failures have been observed. With ENHPP Gokhale and Trivedi [9] stated the possibility of initially estimating the number of expected failures of SRGMs using FPMs. Xie et al. [17] proposed a method for early estimating the fault detection rate based on former projects. They make rather strict assumptions on the similarity of the involved projects, though. Musa [4] proposed the fault exposure ratio for providing early parameter estimates for SRGMs. This was further investigated by Malaiya et al., e.g. [18]. A different approach for failure estimates using FPMs was made by Nagappan et al. [19,20]. They proposed early estimation of post-release failures based on historical failures and product and process metrics.

Besides these approaches to support SRGMs using FPMs there are approaches that state superior predictions for combining different models of a single prediction method. Lyu et al. [21] proposed a linear combination of different SRGMs to obtain improved results. They found that the combination, even in its simplest format, appeared to provide more accurate predictions than the individual SRGMs alone [22]. The combination of solely SRG models, though, is prone to the same kind of errors that afflict individual SRGMs, e. g. since individual SRGMs depend on testing so does their combination. Apparently, homogeneous redundancy improves reliability predictions, for it moderates the flaws of the individual SRG models. However, it does not address the deficiencies within the prediction technique itself. Generally in forecasting, model combination has attracted much attention. A summary can be found in [23].

The main challenge for the hybrid approach is the identification of appropriate coupling techniques of FPMs with SRGMs. Fig. 1 depicts the combination and coupling respectively of FPMs and SRGMs.

We identified four main coupling ideas:

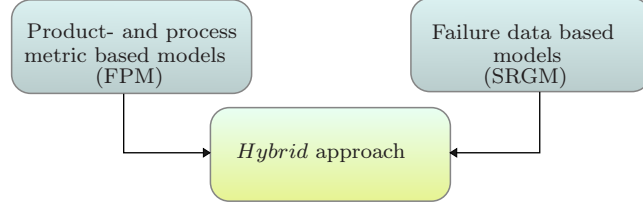


Figure 1 Overview of the Hybrid approach

- Specific SRG model parameter estimation via FPM model
- SRG model class estimation via FPM model
- SRG model fault intensity function class estimation via FPM model
- On principle of heterogeneous redundancy based coupling technique.

In this paper we concentrate on the combination of the first and fourth coupling, which addresses the parameter estimation problem, the problem of estimating conditional confidence bounds, and the problem of combining distinct modeling approaches.

3.1 Initial Parameter Estimates

With the hybrid approach we resume research on providing early parameter estimates for SRGMs. FPMs can be used to estimate the number of expected failures of SRGMs given infinite testing time. That is, we identify a suitable fault prediction model, f_{FPM} , based on software measures, X , and fault data from previous or similar software

$$\hat{a} = f_{FPM}(X, \theta) + \epsilon. \quad (1)$$

Here \hat{a} is the estimated expected number of failures given infinite testing time. We use as the function approximator the so called response surface model with second order interactions,

$$f_{FPM}(X, \theta) = \theta_0 + \sum_{i=1}^N \theta_i X_i + \sum_{k < j} \theta_{kj} X_k X_j, \quad (2)$$

with parameter vector $\theta \in R^{N+m+1}$ and the number of summands in the second term of equation 2 equals $m = \binom{N}{2}$. $X \in R^N$ is the vector of software measures available, and $\epsilon \sim N(0, I\sigma^2)$ is the independently identically distributed term of random errors with zero expectation and homoscedastic variance covariance matrix $I\sigma^2$.

We estimate the SRGM with a modified ENHPP model in the following manner. Let $N(t)$ be the accumulated number of detected failures until time t , which is

a random variable with Poisson distribution, i. e.

$$P(N(t) = k) = \frac{[m(t)]^k}{k!} e^{-m(t)}, k = 0, 1, \dots, \infty. \quad (3)$$

The NHPP expected number of failures is expressed as

$$m(t) = \hat{a}Kc(t).$$

Here $K \in [0, 1]$ is the efficiency of failure detection in case of exercising the respective fault site, and $c(t)$ is the test coverage function. Because the test coverage function $c(t)$ is hardly to be determined by the testing crew we have decided to estimate the coverage growth by the logistic function

$$c(t) = \frac{1}{1 + u \exp(-vt)}.$$

Then we estimate the parameters u and v by maximum likelihood. Estimating the expected number of failures using FPM enables reliability predictions with only little failure data, or even before testing has commenced. Furthermore, we circumvent the problem of instable parameter estimates and the problem of instable reliability growth trends [10, 11]. For the coverage function, $c(t)$, it is as well possible to use other coverage functions like the exponential coverage function which is used in the Goel-Okumoto [8] model. This and other coverage functions are listed in [9].

3.2 Stopping Rule for Initial Parameter Estimates

From different publications mentioned above it is clear that at the beginning of testing the SRG models have poor predictive performance. Therefore at initial state of testing the hybrid model, with the initial parameter estimates, seems to be the only possibility to achieve appropriate estimates of the failure content of the software. At this stage combining FPMs and SRGMs on the heterogeneous redundancy principle is not reasonable. However, with increasing testing duration the SRG model performance improves due to the fact that more and more data is available for the maximum likelihood (ML) estimation of the parameters in the model. Conversely, the FPMs do not depend on the current failure finding process, and therefore their performance is constant over the entire testing period. Having increasing model performance of the SRGMs and constant performance of the FPMs it becomes evident that we need to define a stopping rule for the initial parameter estimates. This stopping rule is also the starting point for combining FPMs and SRGMs on the principle of heterogeneous redundancy. When the predictions of SRGMs are of equal performance as the predictions of FPMs this combination assures improved prediction accuracy. Figure 2 displays the notion of the testing time dependent coupling and combination implemented in the hybrid approach.

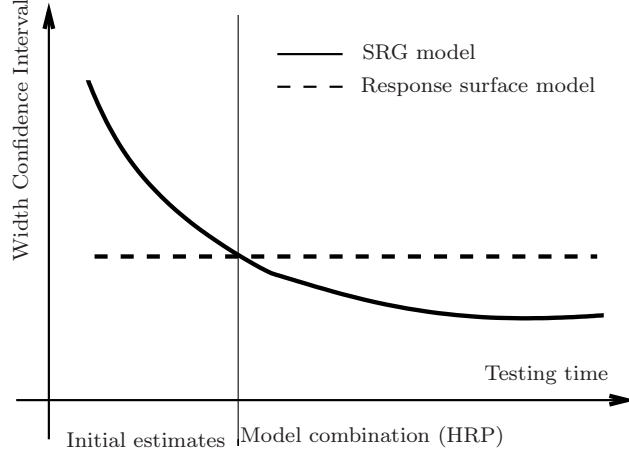


Figure 2 Stopping rule for initial parameter estimates

It is convenient to take the width of the estimated confidence interval of the interesting model parameter as model performance measure, e. g. the confidence interval of the expected number of failures in the code by infinite time of testing. The confidence interval for \hat{a} from the response surface model (see formula 2) for a particular vector of software complexity measure, X , can be estimated by

$$B_{RSM}(X) = \left[\hat{a} \pm t_{n-2; \alpha/2} s \sqrt{\frac{1}{N} + \frac{(X - \bar{X})^2}{(n-1)s_x}} \right] \quad (4)$$

where $t_{n-2; \alpha/2}$ is the $\alpha/2$ quantile of the student's t-distribution

$$s^2 = \frac{1}{N-2} \sum_{i=1}^N (a_i - \hat{a}_i)^2, s_x^2 = \sum_{i=1}^N (X_i - \bar{X}_i)^2.$$

On the other side we need the confidence interval of the parameter \tilde{a} of the NHPP model, which is estimated by the maximization of the corresponding ML equation. A standard result says that the ML estimator has a limiting normal distribution

$$\tilde{a} \xrightarrow{n \rightarrow \infty} \mathcal{N}(\tilde{a}, I(\tilde{a})^{-1})$$

where

$$I(\tilde{a}) = - \sum_{i=1}^n \frac{\partial^2 \ln(L_i)}{\partial \tilde{a}^2}$$

is the local Fisher information. The logarithmic ML equation with respect to the model described in formula 3, is expressed as follows

$$\ln(L_i) = \ln \left[\frac{[m(t_i) - m(t_{i-1})]^{f_i}}{f_i!} e^{[m(t_i) - m(t_{i-1})]} \right],$$

for any $i = 1, \dots, n$, where f_1, \dots, f_n are the numbers⁴ of detected failures respective to the time intervals $(t_0 = 0, t_1), (t_1, t_2), \dots, (t_{n-1}, t_n)$. With this result we can construct the asymptotic variance of the ML estimator of the mentioned parameter, which is the inverse of the local Fisher information

$$\text{var}(\tilde{a}) = \frac{1}{I(\tilde{a})}.$$

With this knowledge we can compute the confidence interval of the SRG model, which is described as follows

$$B_{SRG}(t) = \left[\tilde{a} \pm Z_{\alpha/2} \sqrt{\text{var}(\tilde{a})} \right], \quad (5)$$

where $Z_{\alpha/2}$ is the $100(1 + \alpha)/2$ percentile of the standard normal distribution.

Using these confidence intervals we can estimate the breaking time (BT) (vertical line on Fig. 2), where we change the hybrid model from initial parameter estimates to the heterogeneous redundancy principle based model. Setting the y-axis of Fig. 2 as the width of the confidence intervals of the two models. Break time is the time t for which the following holds

$$|B_{SRG}(t)| = |B_{RSM}(X_c)| \quad (6)$$

that is the time t under the condition that $|B_{SRG}(t)| = |B_{RSM}(X_c)|$, where X_c is the vector of software complexity measure for the current software under testing. The confidence interval widths can be computed by the formulas 4 and 5.

3.3 Model combination on principle of heterogeneous redundancy

Being at the equilibrium level of FPMs and SRGMs, that is having passed the breaking time (BT) (see equation 6) both prediction methods yield reasonable quality predictions. With the hybrid approach we suggest combining these predictions to account for the uncertainties innate in the individual methods. The notion of using redundant approaches to gain higher trust in systems is pervasive in fault tolerant system. Thus using different FPM or SRG models is straightforward and alleviates problems as model selection and the like. For SRGMs this approach has been successfully employed by Lyu et al. [21]. Yet, since all the models within either FPM or the SRG method suffer from the same kind of errors, e.g. all draw on the same data, homogeneous redundancy will not suffice. Thus we suggest combining FPM and SRGM predictions.

Model combination is often considered an obstacle in finding the true model [24]. We argue, that in case of software quality prediction there is no such

⁴We collected grouped failure data.

one true model that generates the fault data or describes the failure process of software. Conversely, using multiple approaches and combining them on principal of heterogeneous redundancy yields more trustworthy predictions, for the individual predictions are not suppressed but are used as information for building a more accurate model.

Combining two distinct prediction approaches, however, demands that these approaches predict the same variable. For simplicity we assume that a fault predicted with FPMs causes finally a failure. This assumption holds when the FPMs are calibrated with faults that were observed as failures, i.e. failures found during testing and after release are used to calibrate FPM models. Thus with FPM (see equation 1) we can compute the number of expected failures, \hat{a} . On the site of SRGMs we have the mean value function, $\mu(t)$, the number of expected failures up to time t . Thus, with $t \rightarrow \infty$ we obtain the eventually expected number of failures. Let F be the expected number of failures then we have $F = \hat{a}$ on site of the FPM models and $F = \lim_{t \rightarrow \infty} \mu(t)$ on site of SRGMs.

For the combination of these prediction results we chose Bayesian Model Averaging (BMA) [25]. BMA provides a coherent mechanism to account for different model uncertainties. It averages over the different predictions, weighted by the posterior model probability. Let $M_{FPM_1}, \dots, M_{FPM_I}$ be the fault prediction models and $M_{SRGM_{I+1}}, \dots, M_{SRGM_k}$ be the reliability prediction models. Then we have M_1, \dots, M_k different models that predict the number of failures, F . The posterior distribution given data, D , is

$$p(F|D) = \sum_{i=1}^k p(F|M_i, D)p(M_i|D), \quad (7)$$

where $p(F|M_i, D)$ is the posterior distribution of F under model M_i and $p(M_i|D)$ is the posterior probability of model M_i given D . Since we have different kinds of data for fault and reliability prediction models, equation 7 forms to

$$\begin{aligned} p(F|RGD, FPD) &= \sum_{j=1}^I p(F|M_{FPM_j}, FPD)p(M_j|FPD) \\ &+ \sum_{j=I+1}^k p(F|M_{SRGM_j}, RGD)p(M_j|RGD), \end{aligned} \quad (8)$$

where FPD and RGD stand for the metrics, i.e. the data for the fault prediction models, and the failure data, i.e. the data for the SRGMs, respectively. Thereby it is indifferent whether we compute the posterior probability $p(M|D)$ for FPMs or for SRGMs. The posterior probabilities of the k considered models have to add up to 1

$$\sum_{i=1}^k p(M_i|D) = 1.$$

The BMA point prediction of the number of experienced failures is

$$\hat{F}_{BMA} = \sum_{i=1}^k \hat{F}_i p(M_i|D),$$

where \hat{F}_i is the estimated number of faults of model i and the posterior probability for model M_i is given by

$$p(M_i|D) = \frac{p(D|M_i)p(M_i)}{\sum_{j=1}^k p(D|M_j)p(M_j)},$$

with

$$p(D|M_i) = \int p(D|\theta_i, M_i)p(\theta_i|M_i)d\theta_i. \quad (9)$$

Equation 9 is the integrated likelihood of model M_i where θ_i is the vector of parameters of model M_i , $p(\theta_i|M_i)$ is the prior density of θ_i under model M_i , $p(D|\theta_i, M_i)$ is the likelihood, and $p(M_i)$ is the prior probability that M_i is the true model [25].

The integrated likelihood from equation 9 for the FP model can be estimated in the following way. For simplicity we reject from the equations the conditional information on models. Let $g(\theta_i) = \log(p(D|\theta_i)p(\theta_i))$ and let $\tilde{\theta}_i = \arg \max_{\theta \in \Theta_i} g(\theta)$. Then after Taylor series expansion truncated at the second term we obtain:

$$g(\theta_i) \approx g(\tilde{\theta}_i) + 1/2(\theta_i - \tilde{\theta}_i)^T g''(\tilde{\theta}_i)(\theta_i - \tilde{\theta}_i).$$

It follows

$$\begin{aligned} p(D|M_i) &= \int e^{g(\theta_i)} d\theta_i \\ &= e^{g(\tilde{\theta}_i)} \int e^{1/2(\theta_i - \tilde{\theta}_i)^T g''(\tilde{\theta}_i)(\theta_i - \tilde{\theta}_i)} d\theta_i. \end{aligned}$$

By recognizing the integrand as proportional to the multivariate normal density and using the Laplace method for integrals we obtain:

$$p(D|M_i) = e^{g(\tilde{\theta}_i)} (2\pi)^{d_i/2} |A|^{-1/2}, \quad (10)$$

where d_i is the number of parameters in the model M_i , and $A = -g''(\tilde{\theta}_i)$. It can be shown that for large N : $\tilde{\theta}_i \approx \hat{\theta}_i$, where $\hat{\theta}_i$ is the MLE, and $A \approx NI$. Thereby I is the expected Fisher information matrix. It is a $d \times d$ matrix whose (l, j) elements are given by:

$$I_{lj} = -E \left[\sum_{l=1}^N \frac{\partial^2 \log(p(a_l|\theta))}{\partial \theta_l \partial \theta_j} \Big|_{\theta=\hat{\theta}_i} \right]. \quad (11)$$

If we take the logarithm of equation 10 we obtain

$$\begin{aligned}\log(p(D|M_i)) &= \log p(D|\hat{\theta}_i) + \log p(\hat{\theta}_i) + d/2 \log(2\pi) \\ &- d/2 \log(N) - 1/2 \log |I| + \mathcal{O}(N^{-1/2}).\end{aligned}$$

Now if we retain only the terms which do not vanish for $N \rightarrow \infty$ we get the following final formula

$$\log(p(D|M_i)) = \log p(D|\hat{\theta}_i) - d/2 \log(N) + \mathcal{O}(1). \quad (12)$$

Combining FPMs and SRGMs we have two types of model classes in equation 9. For FPM we have chosen response surface models, and for SRGMs NHPP based models with different types of Poisson process intensity functions. The log likelihood function in the Fisher information matrix presented in equation 11 for the response surface models and under normality assumption, i. e. $a_1, \dots, a_N \sim \mathcal{N}(F_{FPM}(M, \theta), 1)$ are

$$\log(p(a|\theta)) = -N \log \sqrt{2\pi} - 0.5 \sum_{i=1}^N (a_i - f_{FPM}(M, \theta))^2.$$

The log likelihood function for the models based on the Poisson process can be expressed as follows, we observe a non-homogeneous Poisson process $f = f_1, \dots, f_n$, then the log likelihood is given by:

$$\begin{aligned}\log(p(f|\theta)) &= \sum_{i=1}^n f_i \log[m(t_i) - m(t_{i-1})] \\ &- [m(t_i) - m(t_{i-1})] - \log(f_i!),\end{aligned}$$

where f_i is the number of detected failures in the interval $t_i - t_{i-1}$, $m(t_i)$ is the expectation function of the process, and the $t_i, i = 1, \dots, n$ are time interval boundaries in which the number of failures have been counted.

4 Conclusion

In software industry is a strong demand for early and accurate software quality predictions that can be trusted. Combining two entirely distinct prediction approaches, the resulting hybrid approach meets this demand. Using redundancy is a wide-spread method in fault tolerant systems to improve dependability of such systems. This notion is also straightforward in software quality prediction. Being equipped with two independent quality predictions decisions whether releasing or further testing software can be made with more confidence. The hybrid approach uses initial parameter estimates for providing early reliability predictions and provides a tool to decide how long this initial parameter estimates are useful.

Before FPM and SRGM predictions have reached this point combining them in accord with the redundancy principle would degrade prediction performance. Yet, as we have defined a criterion to decide how long initial parameter estimates are beneficial and when fault prediction and reliability prediction models are of equal performance we know when to adopt FPM and SRGM predictions with respect to the principle of heterogeneous redundancy. With model combination using Bayesian model averaging (BMA) we not only fortify the predictions with a second independent one, but provide a combined model that accounts for the uncertainty in the distinct modeling approaches. Since we adopt FPM and SRGM prediction independently we achieve at least their prediction accuracy but tend to be superior to the individual predictions and are in any case equipped with a second independent prediction. Furthermore, using the stopping point rule we can, even without model combination, put more trust in one method than in the other.

Reviving the last comment, next steps are using BMA together with the stopping rule algorithm to account for the increasing modeling performance of SRGMs over testing time. Also we want to employ Monte Carlo Markov Chains for calculating the posterior probability of a model to overcome the constant error factor in equation 12.

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