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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

New Approaches to Hub Location Problems in Public Transport Planning

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Abstract

In this paper, a new mixed integer mathematical programme is proposed for the application of Hub Location Problems (HLP) in public transport planning. This model is among the few existing ones for this application. Some classes of valid inequalities are proposed yielding a very tight model. To solve instances of this problem where existing standard solvers fail, two approaches are proposed. The first one is an exact accelerated Benders decomposition algorithm and the latter a greedy neighborhood search. The computational results substantiate the superiority of our solution approaches to existing standard MIP solvers like CPLEX, both in terms of computational time and problem instance size that can be solved. The greedy neighborhood search heuristic is shown to be extremely efficient.

Key words: Integer programming, hub location, transportation, decomposition, heuristic

1 Introduction

In the last two decades, due to an enormous increase in the body of telecommunications, transportations and logistics, new and modern strategies are investigated and many studies are devoted to these areas. Hub Location Problems

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(HLP) have received a lot of attention but, the application in Public Transport not.
Hakimi (1964), showed that in order to find the optimal location of a single switching center that minimizes the total wire length in a communication network, one can limit oneself to finding the vertex median of the corresponding graph. Hakimi (1965), proved that optimal locations of switching center(s) in a graph of communication network are at p-medians of the corresponding weighted graph. In general he emphasized on the node optimality of one-median and p-median problems in a weighted graph.
Goldman (1969) proposed models for the problem of finding p centers and assigning flows to center(s) aiming to result in a minimum of transportation cost. His problem was in fact *hub median* problem while he used the word *center*, instead.
In a HLP network, the flow originated from an origin i and destined to node j is not shipped directly. Rather, it is sent through some intermediate nodes (called *hub nodes*) and maybe intermediate edges (called *hub edges*) connecting these hub nodes. When the hub nodes are selected, the non-hub nodes (called *spoke nodes*) will be allocated to them in order to send their flow via hub-level network. The allocation scheme can be single or multiple, based on the permission to allocate a spoke node only to a single node or at least one, respectively.
In classical HLP models, four main assumptions are always considered:

- Ass. a The hub-level network is a complete graph.
- Ass. b Using inter-hub connections has a lower price per unit than using spoke connections. That is, it benefits from a discount factor $\alpha, (0 < \alpha < 1)$.
- Ass. c Direct connections between the spoke nodes are not allowed.
- Ass. d Costs are proportional to the distance or in another word triangle inequality holds.

For the first time, O'Kelly (1986a,b) paved the way for the future study of hub location problems. On the discrete hub location problem, the first work is again due to O'Kelly (1987), where he proposed the first mathematical formulation (a quadratic model) for *Single Allocation p-Hub Median Problem* (SApHMP). This problem is also known as *Uncapacitated Single Allocation p-Hub Median Problem* (USApHMP).
There are some reviews devoted to HLPs on a discrete network. Among them, we refer readers to the two latest reviews (Campbell et al., 2002) and (Alumur and Kara, 2007) wherein one can also find more details about the other works and reviews.
In the formulation context as mentioned earlier, the first formulation is proposed by O'Kelly (1987) for SApHMP applied to airline passenger transport. But, the first linear integer programming for pHMP was proposed by Campbell (1994b) in 1994. Again, Campbell (1996) presented another integer programming (IP) formulations for SApHMP.

Skorin-Kapov et al. (1996), proposed a tight MIP formulation for the USApHMP. Ernst and Krishnamoorthy (1996) presented a new LP formulation for the SApHMP which required fewer variables and constraints than those available in literature. O'Kelly et al. (1996), tried to use other existing formulations and improve the linearization scheme for both single and multiple allocations. Ebery (2001) presented new MIP for USApHMP.

Other work by Sohn and Park (1998) deals with USApHMP. They studied the case when the unit flow cost is symmetric and proportional to the distance. They succeed to improve the formulation of (O'Kelly et al., 1996).
The first model for the multiple allocation problem was due to the work of Campbell (1992). After that, Campbell (1994b) realized that in the absence of capacity constraints, the total flow from each origin to each destination will be routed via the least cost hub pair. Therefore, it is not necessary for all the n^4 variables to be binary in MApHMP. He proposed formulations for UpHMP and UHLP. Again, Campbell (1996) presented another IP formulations for MApHMP. Skorin-Kapov et al. (1996) presented a new MIP for UMApHMP. A new model for UMApHMP was proposed by Ernst and Krishnamoorthy (1998a) based on the idea of their earlier work on the single allocation scheme. Sohn and Park (1998), proposed another model for the UMApHMP.
The answer to the question of the optimal number of hubs for a given set of interactions between a number of fixed nodes leads to incorporating new aspects in the problem. Trying to make the number of hubs an endogenous part of the problem, one can either make the operating cost of hub facilities explicit or consider an available budget. For the first time, O'Kelly (1992a) introduced incorporation of fixed costs as hub node setup cost. Campbell (1994b) also suggested using a threshold approach and incorporated fixed costs for spoke edges in pHMP. Sohn and Park (1998), proposed improved MIP formulations for UMApHMP and USApHMP where fixed cost for hub edges was considered.

To the best of our knowledge, Nickel et al. (2001) proposed the first model for HLPs. They assumed fixed costs, not only for hub facilities, but fixed costs for hub edges and spoke edges were also considered.
When the number of hubs is not fixed, in addition to having multiple and single allocations, there can be capacity policies. These cases can be studied in (Campbell, 1994b), (Aykin, 1994), (Ernst and Krishnamoorthy, 1999), (Ebery et al., 2000), (Labbé et al., 2005), (Yaman, 2005), (Yaman and Carello, 2005), (Marín, 2005a) and (Costa et al., 2007).
The rest of the paper is organized as follows: In the next section, we are going to present a new mathematical model for application of HLPs in public transport planning. We will compare our model with the comparable models

available in literature, in section 3. In section 4, we will propose different Benders decomposition schemes and accelerated versions. In addition to that, we will propose a greedy neighborhood search heuristic utilized with some improving strategies. The results of our heuristic are also used to evaluate the quality of a linear relaxation of the problem which is tightened by some classes of valid inequalities. In section 5, we will conclude our work and will propose some research directions.

2 A Hub Location Formulation for the Public Transport Planning Problem

To the best of our knowledge, Nickel et al. (2001) have been the first to propose MIP models for application of HLPs in urban traffic networks. They proposed two models which are known as Public Transport (PT) and Generalized Public transport. They relax some classical assumptions of HLPs and their models are customized for the public transport planning.

Here, we propose another MIP model which again emphasizes on the application in public transport. We refer to this model by Hub Location Model for Public Transport (HLPPT).

The variables in this model are defined as follows: $x_{ijkl} = 1, i \neq j, k \neq l$ if the optimal path from i to j traverses the hub edge $k - l$ and 0, otherwise. Also, $a_{ijk} = 1, j \neq i, k \neq i, j$ if the optimal path from i to j traverses $i - k$, while i is not hub and 0, otherwise and $b_{ijk} = 1, j \neq i, k \neq i, j$ if the optimal path from i to j traverses $k - j$, while j is not hub and 0, otherwise. In addition, $e_{ij} = 1, i \neq j$ if the optimal path from i to j traverses $i - j$ while either i or j is hub and 0, otherwise. For the hub-level variables, $y_{kl} = 1, k < l$ if the hub edge $k - l$ is established and 0, otherwise and $h_k = 1$ if k is used as hub 0 otherwise.

The transportation cost for a given flow with origin i and destination j amounts to the sum of (i) the costs of sending the flow from i to the first hub node in the path to j, (ii) the costs incured by traversing one or more hub edges discounted by the discount factor α, $0 < \alpha < 1$, and (iii) the cost of transition on the last spoke edge. The proposed mathematical formulation turns out to be as follows:

(HLPPT)

$$
\begin{aligned}
Min \sum_i \sum_{j \neq i} \sum_k \sum_{l \neq k} \alpha W_{ij} C_{kl} x_{ijkl} + \sum_i \sum_{j \neq i} \sum_{k \neq i,j} W_{ij} C_{ik} a_{ijk} + \\
\sum_i \sum_{j \neq i} \sum_{k \neq i,j} W_{ij} C_{kj} b_{ijk} + \sum_i \sum_{j \neq i} W_{ij} C_{ij} e_{ij} + \\
\sum_k F_k h_k + \sum_k \sum_{l > k} I_{kl} y_{kl} \qquad (1)
\end{aligned}
$$

$$\begin{aligned}
s.t.\ & \sum_{l\neq i} x_{ijil} + \sum_{l\neq i,j} a_{ijl} + e_{ij} = 1, && \forall i, j \neq i, && (2)\\
& \sum_{l\neq j} x_{ijlj} + \sum_{l\neq i,j} b_{ijl} + e_{ij} = 1, && \forall i, j \neq i, && (3)\\
& \sum_{l\neq k,i} x_{ijkl} + b_{ijk} = \sum_{l\neq k,j} x_{ijlk} + a_{ijk}, && \forall i, j \neq i, k \neq i, j, && (4)\\
& y_{kl} \leq h_k, \quad y_{kl} \leq h_l, && \forall k, l > k, && (5)\\
& x_{ijkl} + x_{ijlk} \leq y_{kl}, && \forall i, j \neq i, k, l > k, && (6)\\
& \sum_{l\neq k} x_{kjkl} \leq h_k, && \forall j, k \neq j, && (7)\\
& \sum_{k\neq l} x_{ilkl} \leq h_l, && \forall i, l \neq i, && (8)\\
& e_{ij} \leq 2 - (h_i + h_j), && \forall i, j \neq i, && (9)\\
& a_{ijk} \leq 1 - h_i, && \forall i, j \neq i, k \neq i, j, && (10)\\
& b_{ijl} \leq 1 - h_j, && \forall i, j \neq i, l \neq i, j, && (11)\\
& a_{ijk} + \sum_{l\neq j,k} x_{ijlk} \leq h_k, && \forall i, j \neq i, k \neq i, j, && (12)\\
& b_{ijk} + \sum_{l\neq k,i} x_{ijkl} \leq h_k, && \forall i, j \neq i, k \neq i, j, && (13)\\
& e_{ij} + 2x_{ijij} + \sum_{l\neq j,i} x_{ijil} + \sum_{l\neq i,j} x_{ijlj} \leq h_i + h_j, && \forall i, j \neq i, && (14)\\
& x_{ijkl}, y_{kl}, h_k, a_{ijk}, b_{ijk}, e_{ij} \in \{0, 1\}. && && (15)
\end{aligned}$$

The objective (1) is the total cost of transportation plus hub nodes and edges setup costs. The constraints (2)-(4) are the flow conservation constraints. The constraints (5) ensures that both end-points of a hub edge are hub nodes. The constraints (6) ensure that a hub edge should exist before being used in any flow path. In (7) ((8)) it is ensured that only a flow with origin (destination) of hub type is allowed to select a hub edge to leave the origin (arrive to the destination). Constraints (9)-(11) check the end-points of spoke edges. Any flow from i to j, if enters to (depart from) a node other than i and j, that node should be a hub node. This is ensured by (12) ((13)). Selection of edges on the path between origin and destination (i and j) depends on the status of i and j: whether both, none or just one of them is a hub node. This has to be checked by (14). In an uncapacitated environment, as also mentioned in (Campbell, 1994b), only hub node and hub edge variables may need to be considered as binary variables. Therefore, the constraints (15) can be replaced by,

$$x_{ijkl}, a_{ijk}, b_{ijk}, e_{ij} \in (0, 1), h_k, y_{kl} \in \{0, 1\}. \qquad (16)$$

From now on, whenever we talk about the HLPPT we are referring to the model of (1)-(14) together with the constraint (16).

In our new model, we tried to emphasize on the real willingness of passengers who use public transport services. Some personal communications with them revealed new aspects. For instance, passengers who arrived to the hub level network do not like to change their vehicle (train) type inside this network. That is, if they enter to a hub node they are interested to use fast-lane as long as they did not reach to the last hub node where their destination is assigned to.
In addition to that Ass. a and Ass. d are relaxed in our model. In another word, by relaxing the Ass. a, there is no need to keep the completeness of hub-level network. But, the connectivity of hub-level will be guaranteed in the model in order to avoid changes in the types of edges inside it.
The model contains the following attributes:

- Attr. a Connected hub-level network.
- Attr. b The cost structure neither necessarily satisfies the triangle inequality nor any other special structure.
- Attr. c To ensure some levels of reliability, there will be possibility of multiple connections between a spoke node and the hub-level network.

While in HLPPT a connected hub-level network is assumed, in (Nickel et al., 2001), they did not introduce any alternative assumption for the relaxation of Ass. a. That means, the hub-level network is not necessarily connected. Moreover, in (Nickel et al., 2001), existence of spoke edges between hub nodes is allowed which can be a threat for connectivity of hub-level network.
Attr. c can guaranty the existence of an alternative path if due to an unpredicted failure in one transportation element in the spoke-level network a path is abandoned.
In HLPs with the assumptions of Ass. a - Ass. d, once the hub nodes are nominated, the hub-level configuration is known and the remaining problem in the multiple allocation will be to find the cheapest routes (although, in single assignment scheme it will be Quadratic Assignment Problem (QAP) which is again NP-hard problem but we are not dealing with the single assignment in our new model). The HLPPT does not follow this way.
In our model, HLPPT, the problem is first to locate the hub-nodes, second to choose the connecting hub edges so that results in a connected hub-level graph and then in the third step, routing the flows. In the second step, neither the number of hub edges is known nor the way in which they should be connected to make an optimal connected graph. In fact, the second can be considered as the problem of assigning an unknown and finite number of edges to pairs of hub nodes so that it results in a connected graph. Therefore, in the special case, it reduces to the QAP. When all these three steps should be solved simultaneously, it seems to be more difficult than the Single Allocation HLP

Table 1
HLPPT vs MAHLP and SAHLP

	HLPPT	MAHLP	SAHLP
Locating hubs	$\surd$	$\surd$	$\surd$
Selecting hub edges	$\surd^a$	$\times$	$\times$
Allocation	Polynomial	Polynomial	NP-hard[b]

[a] in special case reduces to QAP.

[b] QAP.

(MAHLP) as well as Multiple Allocation (SAHLP). Table 1 sheds some light on this fact.

Therefore, in terms of difficulty of problem, one can say that HLPPT is more difficult than MAHLP. Comparing with SAHLP, if not be more difficult is not easier. Because, allocation of $n-q$ spoke nodes to q hub nodes should not be more difficult than allocation of unknown and finite number of hub edges to pairs of hub nodes to make a connected graph. We expect it to be NP, too. It is well-known that the HLPs are NP-Hard problems which even small size instances cannot be solved to optimality in a reasonable amount of time. Our new HLPPT model, as we will show later, paves the way for preparing a good basis for exact decompositions as well as (meta-)heuristic algorithms. This may enable us to solve larger size instances to optimality or as good as possible of solutions, respectively.
To have a comparison between our new model and PT model of (Nickel et al., 2001), some modifications to the model should be taken into account. By adding new constraints and avoiding spoke connections between hub nodes, the following comparable model (CPT) is obtained.
(Comparable PT (CPT))

$$Min \sum_i \sum_{j \neq i} \sum_k \sum_{l \neq k} W_{ij} d_{kl} (\alpha X_{ijkl} + S_{ijkl}) + \sum_{k,l>k} I_{kl} Y_{k,l} + \sum_{k \in N} F_k H_k \quad (17)$$

$$s.t. \sum_{l \in N} (X_{ijkl} + S_{ijkl} - X_{ijlk} - S_{ijlk}) = \quad (18)$$

$$\begin{cases} +1, & \forall i,j,k \in V : k = i, i \neq j, \\ -1, & \forall i,j,k \in V : k = j, i \neq j, \\ 0, & \forall i,j,k \in V : k \neq i, k \neq j, \end{cases}$$

$$X_{ijkl} \leq Y_{kl} \qquad \forall i,j,\{k,l\} \in \mathcal{E}, \quad (19)$$

$$X_{ijlk} \leq Y_{kl} \qquad \forall i,j,\{k,l\} \in \mathcal{E}, \quad (20)$$

$$S_{ijik} \leq H_k \qquad \forall i,j,k,l : k \neq j, \quad (21)$$

$$S_{ijkj} \leq H_k \qquad \forall i,j,k,l : k \neq i, \quad (22)$$

$$S_{ijij} \leq H_i + H_j \qquad \forall i,j, \quad (23)$$

$$S_{ijkl} = 0 \qquad \forall i,j,k,l : k \neq i, l \neq j, \quad (24)$$

Table 2
Comparison between HLPPT and CPT

	number of constraints	Number of variables	
		binary	continuous
CPT	$6n^4 + 3n^3 + 2n^2$	$\frac{n(n-1)}{2} + n$	$2n^4$
HLPPT	$n^4 + 5n^3 + 7n^2$	$\frac{n(n-1)}{2} + n$	$n^4 + 2n^3 + n^2$

Table 3
Comparison between HLPPT and CPT on CAB instances

	CPT		HLPPT	
	r.n.g(%)	c.t.u (sec.)	r.n.g(%)	c.t.u (sec.)
CAB 5	27.08	0.50	*opt*	0.03
CAB 10	36.30	19.81	*opt*	0.42
CAB 15	64.35	461.63	*opt*	2.27
CAB 20	59.95	4596.49	*opt*	9.09
CAB 25	77.38	$\gg$	*opt*	28.23

$$Y_{kl} \leq H_k \qquad \forall \{k,l\} \in \mathcal{E}, \quad (25)$$
$$Y_{kl} \leq H_l \qquad \forall \{k,l\} \in \mathcal{E}, \quad (26)$$
$$S_{ijkl} + S_{ijlk} \leq 2 - H_k - H_l \qquad \forall i,j,k,l > k, \quad (27)$$
$$S_{ijkl}, X_{ijkl} \geq 0 \qquad \forall i,j,k,l, \quad (28)$$
$$Y_{kl}, H_k \in \{0,1\} \qquad \forall k,l. \quad (29)$$

As one can see in Table 2, in our new model HLPPT, the number of constraints is much less than CPT. Roughly speaking, it contains almost less than $\frac{1}{6}$ of constraints in CPT. With respect to the number of variables, though they both use the same number but the number of continuous variables in HLPPT is considerably less than in PT.

3 Computational Comparison

In this section, we are going to solve instances of AP and CAB dataset (from OR-Library) using both CPT and HLPPT models. We will compare *root node gaps(r.n.g)*, *cpu time usage(c.t.u)* and problem size that can be solved by each one in a given time limit.

As it is depicted in the Table 3, there is a considerable difference between the root node gaps of HLPPT and CPT on CAB instances. HLPPT solves all the instances of CAB dataset just in the root node to an integral optimal solu-

Table 4
Comparison between HLPPT and CPT on AP instances

	CPT		HLPPT	
	r.n.g(%)	c.t.u (sec.)	r.n.g(%)	c.t.u (sec.)
AP 5.2	*opt*	0.1	*opt*	0.03
AP 10.4	39.74	26.58	38.99	8.81
AP 15.6	39.87	1055.46	67.75	318.24
AP 20.8	39.89	12564.14	42.75	3683.07
AP 25.10	51.15	> 1 d[b]	44.55	56839.31
AP 30.12	N.A.[a]	N.A.	43.27[c]	N.A.

[a] N.A., Not able to solve the instance.

[b] day: 864000 seconds.

[c] The root node relaxation was solved.

tion. That is the LP relaxation bound coincides with the MIP optimal value. In Table 4, again the superiority of HLPPT to CPT with respect to the computational time is obvious. In general, the HLPPT shows to be superior to the CPT. For the problem size of 30, the CPT even could not load the model in the memory while it takes more than 1.4 GB of memory. But, HLPPT could load and did the primary computation and not only emphasized on the feasibility of problem but also the root relaxation was solved successfully and gap was reported. However, it failed when proceeded and needed extra memory.
With respect to the computational effort and CPU time usage, obviously HLPPT outperforms CPT when `CPLEX 9.1` is used to solve both instances of AP and CAB. Especially, in the case of CAB instances, HLPPT can be used to solve even more than 500 times faster for some instances in compare with CPT.
In our tables, the sign "$\gg$" states that the results have been much worse than the worst results in the rows. Analogously, "$>$" is used to show that the results are worse than the best results of the row in the table.

4 Two Solution Approaches for the HLPPT

In this section, we propose two solution approaches namely, an exact Benders decomposition and a greedy neighborhood search.

4.1 Benders Decomposition

Benders algorithm was proposed by Benders (1962). This approach which is an iterative algorithm has been applied to many problems in combinatorial optimization. It exploits the decomposable structure of problems and tries to solve a decomposition of problem into a pair of master and sub-problem, iteratively. Master Problem (MP) prepares a lower bound and Sub-Problem (SP) an upper bound. The method makes use of exchange of information between these two smaller problems to reach the optimality. This is to be done by decreasing the upper bound and increasing the lower bound and eventually reaching to a optimality where these two values coincide. Details of these approach can be studied in (Benders, 1962).
Benders decomposition approaches for UMAHLP for the formulation of Hamacher et al. (2004) is considered by R.S. de Camargo et al.. They decomposed the problem following the Benders scheme and solve the sub-problem for each origin and destination by inspection.
Rodriguez-Martin and Salazar-Gonzalez also presented an MIP model and proposed methods is a Double Benders Decomposition.

4.1.1 Classical Benders algorithm

A classical Benders algorithm is a procedure that generates a single cut for the master problem in each iteration. This cut is generated from the solution to the dual of sub-problem (SPD). A choice for the MP and SP can lead to the following problems:

(MP1)

$$Min \sum_k F_k h_k + \sum_k \sum_{l>k} I_{kl} y_{kl} + \eta$$

$$s.t.\ y_{kl} \le h_k, \qquad \forall k, l > k, \quad (30)$$

$$y_{kl} \le h_l, \qquad \forall k, l > k, \quad (31)$$

$$\sum_{k,l>k} y_{kl}, \ge 1 \qquad y_{kl}, \forall k, l > k, \quad (32)$$

$$y_{kl}, h_k \in \{0,1\}, \eta \ge 0. \quad (33)$$

and for fixed values of h_k and y_{kl}, and also regarding the symmetry in the shortest paths, we will have:

(SP)

$$Min \sum_i \sum_{j>i} \sum_k \sum_{l\neq k} \alpha(W_{ij} + W_{ji}) C_{kl} x_{ijkl} + \sum_i \sum_{j>i} \sum_{k\neq i,j} (W_{ij} + W_{ji}) C_{ik} a_{ijk} +$$
$$\sum_i \sum_{j>i} \sum_{k\neq i,j} (W_{ij} + W_{ji}) C_{kj} b_{ijk} + \sum_i \sum_{j>i} (W_{ij} + W_{ji}) C_{ij} e_{ij} +$$

$$\sum_{k} F_k h_k + \sum_{k}\sum_{l>k} I_{kl} y_{kl} \tag{34}$$

$$s.t. \sum_{l\neq i} x_{ijil} + \sum_{l\neq i,j} a_{ijl} + e_{ij} = 1, \qquad \forall i, j > i, \tag{35}$$

$$\sum_{l\neq j} x_{ijlj} + \sum_{l\neq i,j} b_{ijl} + e_{ij} = 1, \qquad \forall i, j > i, \tag{36}$$

$$\sum_{l\neq k,i} x_{ijkl} + b_{ijk} = \sum_{l\neq k,j} x_{ijlk} + a_{ijk}, \qquad \forall i, j > i, k \neq i, j, \tag{37}$$

$$x_{ijkl} + x_{ijlk} \leq y_{kl}, \qquad \forall i, j > i, k, l > k, \tag{38}$$

$$\sum_{l\neq k} x_{kjkl} \leq h_k, \qquad \forall j, k < j, \tag{39}$$

$$\sum_{k\neq l} x_{ilkl} \leq h_l, \qquad \forall i, l > i, \tag{40}$$

$$e_{ij} \leq 2 - (h_i + h_j), \qquad \forall i, j > i, \tag{41}$$

$$a_{ijk} \leq 1 - h_i, \qquad \forall i, j > i, k \neq i, j, \tag{42}$$

$$b_{ijl} \leq 1 - h_j, \qquad \forall i, j > i, l \neq i, j, \tag{43}$$

$$a_{ijk} + \sum_{l\neq j,k} x_{ijlk} \leq h_k, \qquad \forall i, j > i, k \neq i, j, \tag{44}$$

$$b_{ijk} + \sum_{l\neq k,i} x_{ijkl} \leq h_k, \qquad \forall i, j > i, k \neq i, j, \tag{45}$$

$$e_{ij} + 2x_{ijij} + \sum_{l\neq j,i} x_{ijil} + \sum_{l\neq i,j} x_{ijlj} \leq h_i + h_j, \qquad \forall i, j > i, \tag{46}$$

$$x_{ijkl}, a_{ijk}, b_{ijk}, e_{ij} \in (0, 1). \tag{47}$$

and SPD would be:
(SPD)

$$\begin{aligned}
Max\ & - \sum_{i,j>i} (u_{ij} + v_{ij}) - \sum_{i,j>i}\sum_{k\neq i,j} (s_{ijk} + w_{ijk}) h_k - \sum_{j,k>j} p_{jk} h_k \\
& - \sum_{i,l>i} q_{il} h_l - \sum_{i,j>i} e_{ij}(2 - h_i - h_j) - \sum_{i,j>i} d_{ij}(h_i + h_j) \\
& - \sum_{i,j>i}\sum_{k\neq i,j} (a_{ijk}(1 - h_i) + b_{ijk}(1 - h_j)) - \sum_{i,j>i}\sum_{k,l>k} o_{ijkl} y_{kl} \\
s.t.\ & u_{ij} + v_{ij} + p_{ji} + q_{ij} + o_{ijij} + 2d_{ij} \geq -\alpha * (W_{ij} + W_{ji}) * C_{ij}, \qquad \forall i, j > i, \\
& v_{ij} + r_{ijk} + w_{ijk} + q_{ij} + o_{ijkj} + d_{ij} \geq -\alpha * (W_{ij} + W_{ji}) * C_{kj}, \quad \forall i, j > i, k \neq i, j, \\
& u_{ij} + p_{ji} + d_{ij} + s_{ijl} - r_{ijl} + o_{ijil} \geq -\alpha * (W_{ij} + W_{ji}) * C_{il}, \qquad \forall i, j > i, l \neq i, j, \\
& r_{ijk} - r_{ijl} + s_{ijl} + w_{ijk} + o_{ijkl} \geq -\alpha * (W_{ij} + W_{ji}) * C_{kl}, \qquad \forall i, j > i, k, l \neq i, j, \\
& u_{ij} - r_{ijk} + s_{ijk} + a_{ijk} \geq -(W_{ij} + W_{ji}) * C_{ik}, \qquad \forall i, j > i, k \neq i, j, \\
& v_{ij} + r_{ijk} + w_{ijk} + b_{ijk} \geq -(W_{ij} + W_{ji}) * C_{kj}, \qquad \forall i, j > i, k \neq i, j, \\
& u_{ij} + v_{ij} + d_{ij} + e_{ij} \geq -(W_{ij} + W_{ji}) * C_{ij}, \qquad \forall i, j > i, \\
& d_{ij}, e_{ij}, p_{ij}, q_{ij}, a_{ijk}, b_{ijk}, s_{ijk}, w_{ijk}, o_{ijkl} \in \mathbb{R}^+, \\
& u_{ij}, v_{ij}, r_{ijk} \text{ free in sign.}
\end{aligned}$$

This master problem, (MP1), cannot guaranty a feasible hub-level configuration at each iteration. Moreover, the number of infeasible configuration is higher than feasible ones which are the connected hub-level networks. Therefore, another master problem should be considered to be replaced by. These infeasible hub-level configurations are disconnected graphs and each one leads to generating a cut. Since the cardinality of the set of infeasible hub-level configurations is much higher than the set of feasible ones, the algorithm may visit too many of them during the solution process (even tens of them between two consecutive feasible configurations). This not only compels solving an MIP1 master problem for each of them but also as the iterations proceed it makes a very hard-to-solve MP1 with the added cuts. We suggest to replace the MP1 with the following one to ensure that the added cuts are only from the extreme points rather than extreme rays.
Let $G(V, E)$ be a connected graph, where $V = \{1, 2, 3, \ldots, n\}$ is the set of nodes or vertices and E the set of edges. Let $G_d = (V, A)$ be a directed graph derived from G, where $A = \{(i, j), (j, i) | \{i, j\} \in E\}$, that is, each edge u is associated with two arcs (i, j) and $(j, i) \in A$. Two new graphs $G^0 = (V_0, E_0)$ and $G^0_d = (V_0, A_0)$ where $V_0 = V \cup \{0\}$, $E_0 = E \cup \{\{0, j\} | j \in V\}$, $A_0 = A \cup \{\{0, j\} | j \in V\}$, are defined.
Let $h = (h_i)_{i \in V} \in \{0, 1\}^{|V|}$, $y = (y_u)_{u \in E_0} \in \{0, 1\}^{|E_0|}$ two $0 - 1$ vectors, and $z^k_{ij} \geq 0, (i, j) \in A_0$, $k \in V'$, where V' is a subset of V, and z^k_{ij} is a real flow in the arc $(i, j) \in A_0$, having 0 as source and k as destination. $E(i)$ is considered as the set of edges $u \in E$ such that an endpoint is i, $\Gamma^+(i) = \{j | (i, j) \in A_0\}$ and $\Gamma^-(i) = \{j | (j, i) \in A_0\}$, $m = |E|$ and $n = |V|$ (Maculan et al., 2003).
From now on, we will refer to the following model as MP.

(MP)

$$
\begin{aligned}
& Min \sum_k F_k h_k + \sum_k \sum_{l>k} I_{kl} y_{kl} + \eta \\
& s.t. \sum_{j \in \Gamma^+(0)} z^k_{0j} - h_k = 0, && \forall k \in V, (48) \\
& \sum_{j \in \Gamma^+(i)} z^k_{ij} - \sum_{j \in \Gamma^-(i)} z^k_{ji} = 0, && \forall i \in V - \{k\}, k \in V, (49) \\
& \sum_{j \in \Gamma^+(k)} z^k_{kj} - \sum_{j \in \Gamma^-(k)} z^k_{jk} + h_k = 0, && \forall k \in V, (50) \\
& z^k_{ij} \leq y_{ij}, && \forall \{i, j\} \in E_0, k \in V, (51) \\
& z^k_{ji} \leq y_{ij}, && \forall \{i, j\} \in E_0, k \in V, (52) \\
& y_{ij} \leq x_i, && \forall \{i, j\} \in E, (53) \\
& y_{ij} \leq x_j, && \forall \{i, j\} \in E, (54) \\
& \sum_{j \in V} y_{0j} = 1, && \forall i, j = 1, \ldots, n, (55) \\
& z^k_{ij} \geq 0, && \forall (i, j) \in A_0, k \in V, (56) \\
& y_{ij} \in \{0, 1\}, \{i, j\} \in E_0, h_k \in \{0, 1\}, k \in V, \eta \geq 0. && (57)
\end{aligned}
$$

A cut for MP is in the form of the following constraint:

$$-\sum_{i,j\neq i}\Bigg((u_{ij}+v_{ij})+\sum_{k\neq i,j}(s_{ijk}+w_{ijk})h_k+p_{ji}h_i+q_{ij}h_j+\sum_{k,l>k}o_{ijkl}y_{kl}$$
$$+\sum_{k\neq i,j}(a_{ijk}(1-h_i)+b_{ijk}(1-h_j))+d_{ij}(h_i+h_j)+e_{ij}(2-h_i-h_j))\Bigg)\leq\eta,$$

where the LHS is composed of $\frac{n(n-1)}{2}$ different independent terms. If we look at it more carefully, each of $\frac{n(n-1)}{2}$ terms is generated from the corresponding $\text{SPD}_{i,j>i}$. In each of those problems, for a given $i, j > i$ we can generate the following part of the cut:

$$-\Bigg((u_{ij}+v_{ij})+\sum_{k\neq i,j}(s_{ijk}+w_{ijk})h_k+p_{ji}h_i+q_{ij}h_j+\sum_{k,l>k}o_{ijkl}y_{kl}$$
$$+\sum_{k\neq i,j}(a_{ijk}(1-h_i)+b_{ijk}(1-h_j))+d_{ij}(h_i+h_j)+e_{ij}(2-h_i-h_j))\Bigg)\leq\eta_{ij},$$

where $\eta_{ij}\geq 0, \eta=\sum_{ij>i}\eta_{ij}$.
Briefly speaking, instead of solving one large LP problem as SPD and generate the corresponding cut, one can generate such a cut by solving $\frac{n(n-1)}{2}$ easier sub-SPDs. Such a cut is in the form of:

$$-\Bigg((u+v)+\sum_{k}(s_k+w_k)h_k+ph_i+qh_j+\sum_{k,l>k}o_{kl}y_{kl}+d(h_i+h_j)$$
$$+\sum_{k}(a_{ijk}(1-h_i)+b_{ijk}(1-h_j))+e(2-h_i-h_j))\Bigg)\leq\eta_{ij}. \tag{58}$$

If this single cut is produced instantaneously (traditional Benders algorithm), we call the approach SC1 and if it is assembled from sub-cuts of form (58) we call it SC2.
SP is well-known due to its degenracy and therefore SPD due to its multiple optimality. As a results, these cuts may be the cuts dominated by the other ones corresponding to the other solutions. Therefore, we apply the concept of pareto-optimal cut proposed in (Magnanti and Wong, 1981) for both SC1 and SC2 and call them SC1^d and SC2^d, respectively. The computational results are reported in Table 5. In Table 5, SC1 obviously outperforms SC2. That means the single cuts in their aggregated from are not as strong as the original cuts. This can also be deduced from the superscript which stands

Table 5
Comparison between SCs.

Instance	**SC1s** (sec.)		**SC2s** (sec.)	
	SC1	$SC1^d$	SC2	$SC2^d$
AP5.2	$0.31^{(8)}$	$0.11^{(4)}$	$2.72^{(16)}$	$0.33^{(4)}$
AP10.4	$42.42^{(30)}$	$3.52^{(8)}$	$6529.39^{(367)}$	$6.02^{(8)}$
AP15.6	$14155.78^{(97)}$	$48.48^{(8)}$	$\gg$	$47.50^{(9)}$
AP20.8	$>$ 1d	$854.08^{(14)}$	$\gg$	$820.83^{(12)}$
AP25.10	$\gg$	55902.82^{24}	$\gg$	$45801.05^{(22)}$

for the number of iterations of algorithms. However, in the pareto-optimality approach, although the Benders algorithm was still performing very poor, yet we observed that for a given machine specification, $SC2^d$ can solve larger instances when compared to $SC1^d$ in a given time limit. When the problem size grows, $SC2^d$ is superior to the $SC1^d$. Therefore, $SC2^d$ is clearly superior to all the other three schemes. With respect to the number of iterations, it is also clear that in the pareto-optimality approach, cuts are stronger. This idea may help us in the next section to propose a new cutting strategy. Here, we set the time limit to 48 hours.

4.1.2 Accelerated Benders algorithm

Instead of adding just one cut in each iteration, it can also be possible to add more than one cut. That is, we add the sub-cuts to the MP without assembling them. So, we will have MC1 when $n-1$ cuts by aggregating for each i all the cuts of ij-th sub-SPDs and another one, MC2, which adds all the $\frac{n(n-1)}{2}$ cuts. Moreover, both of them can be subjected to strengthening of cuts and make the $MC1^d$ and $MC2^d$. The computational results depicted in Table 6 revealed the superiority of $MC2^d$ to all other three variants.

As one can see the pareto-optimal cut approaches work much better with respect to the computational time needed to solve the problem to optimality. However, $MC2^d$ is absolutely superior to the $MC1^d$ with respect to the efficiency of resolution of the problem. The number of iterations after which the optimality is met are considerably reduced and computational time are strictly less. Yet, in a given time limit of 10 hours $MC2^d$ can solve larger instances.

Table 6
Comparison between MCs.

Instance	MC1s(sec.)		MC2s(sec.)	
	MC1	MC1^d	MC2	MC2^d
AP5.2	$0.34^{(5)}$	$0.25^{(3)}$	$0.22^{(4)}$	$0.25^{(4)}$
AP10.4	$9.61^{(11)}$	$4.30^{(6)}$	$5.47^{(7)}$	$3.77^{(5)}$
AP15.6	$97.30^{(10)}$	$16.11^{(4)}$	$21.34^{(5)}$	$12.00^{(4)}$
AP20.8	$933.88^{(11)}$	$240.17^{(8)}$	$223.05^{(8)}$	$153.61^{(6)}$
AP25.10	$26332.39^{(15)}$	$3351.60^{(9)}$	$2672.11^{(7)}$	$1855.69^{(6)}$
AP30.12	$\gg$	$8209.14^{(9)}$	$9083.13^{(10)}$	$3350.86^{(6)}$
AP35.14	$\gg$	>10 hrs	$\gg$	$120206.82^{(7)}$
AP40.16	$\gg$	$\gg$	$\gg$	> 6 d

4.1.3 Extra Accelerated Benders algorithm

By adding some classes of valid (improving) inequalities, one can obtain a very tight formulation of the problem which can be used to solve the LP relaxation and accelerating the resolution of MIP problem.
In this approach, while the LP relaxation is not solved to optimality, the cuts corresponding to the fractional values of relaxed modified MP (i.e. AMP) are used to generate the cuts and be added to the relaxed AMP. When the LP is solved, AMP will be un-relaxed and from a very small integrality gap the MC2^d proceeds. We refer to this approach by AMC2^d. The AMP will be as it follows:

(AMP)

$$Min \sum_k F_k h_k + \sum_k \sum_{l>k} I_{kl} y_{kl},$$

$$s.t. \ (48), (49), (50), (51), (52), (53), (54), (55), (56),$$

$$\sum_k y_{0k} \leq 1, \tag{59}$$

$$\sum_k y_{k0} = 1, \tag{60}$$

$$\sum_{k,l>k} y_{kl} \geq 1, \tag{61}$$

$$\sum_k h_k \geq 2, \tag{62}$$

$$\sum_{l>k} y_{kl} \geq \sum_k h_k - 1, \tag{63}$$

$$h_k \leq \sum_{l\neq k} (y_{kl} + y_{lk}), \quad \forall k, \tag{64}$$

Table 7
Final Comparison.

Instance	CPLEX 9.1	SC2^d	MC2^d	AMC2^d(sec.)	
	(sec.)	(sec.)	(sec.)	OptLP	nonOptLP
AP5.2	0.03	$0.33^{(4)}$	$0.25^{(4)}$	$0.53^{(6)}$	$0.52^{(6)}$
AP10.4	0.09	$6.02^{(8)}$	$3.77^{(5)}$	$3.30^{(6)}$	$4.23^{(6)}$
AP15.6	2.3	$47.50^{(9)}$	$12.00^{(4)}$	$12.19^{(7)}$	$13.09^{(6)}$
AP20.8	62.02	$820.83^{(12)}$	$153.61^{(6)}$	$56.47^{(8)}$	$40.34^{(7)}$
AP25.10	900.50	$45801.05^{(22)}$	$1855.69^{(6)}$	$172.89^{(8)}$	$134.19^{(7)}$
AP30.12	5530.00	$\gg$	$3350.86^{(6)}$	$675.09^{(13)}$	$534.56^{(8)}$
AP35.14	N.A.	$\gg$	$120206.82^{(7)}$	$4238.14^{(12)}$	$2771.99^{(8)}$
AP40.16	N.A.	$\gg$	> 6 d	$25676.55^{(14)}$	$14181.73^{(8)}$
AP45.18	N.A.	$\gg$	$\gg$	$87130.57^{(15)}$	$99483.25^{(10)}$
AP50.20	N.A.	$\gg$	$\gg$	$566360.33^{(14)}$	$528663.42^{(10)\,a}$

[a] $\simeq$ 146.85 hrs (approximately 6 days).

$$y_{kl} \leq \sum_{m \neq k}(y_{mk} + y_{km}) + \sum_{m \neq l}(y_{ml} + y_{lm}), \qquad \forall k, l > k, \quad (65)$$

$$y_{ij} \in (0,1), \{i,j\} \in E_0, h_k \in (0,1), k \in V, \quad (66)$$

A natural question might arise here: whether it is necessary to solve the relaxed problem to the optimality. Here, we proposed two approaches. The first one solves that LP to optimality before un-relaxing the AMP and the second one to a gap of less than %0.05 between lower and upper bounds of our Benders algorithm. The first one will be referred by OptLP and the latter by nonOptLP.

As depicted in Table 7, in general, Benders approaches are capable of solving larger instances where `CPLEX 9.1` fails. Yet, multiple cut approaches (MC2^d and AMC2^{d}s) and specially the accelerated multiple cut schemes, AMC2^{d}s, seem to be capable of solving much larger instances. In terms of computational time, obviously MC2^d and AMC2^{d}s are superior. For some instances like AP30.12, AMC2^d in the second variant, solves the instance more than 10 times faster than `CPLEX 9.1`.
Among the multiple cut approaches, with respect to the problem instance size which is solved, AMC2^{d}s outperform MC2^d. While it takes more than 6 days to solve AP40.16 with MC2^d, the larger instance of AP50.20 can be solved by AMC2^d (the second variant), in such an amount of time (approximately 146 hrs). However, the absolute superiority of AMC2^d to other methods both in terms of computational time and the instance size that can solve is obvious in

Table 7.

4.2 Greedy Neighborhood search

A simple greedy neighborhood search will be proposed in this section which as we will show later is extremely efficient to solve instances of HLPPT in a reasonable amount of time and to high quality solutions.
As mentioned earlier, our problem can be re-stated to be the problem of finding a connected hub-level network and consequently a minimum flow cost problem. Obviously, the second part is a function of the first part. That means, how the flow should be transferred is induced by the hub-level network configuration. Therefore, without loss of generality we concentrate on the search for the best (or as good as possible) hub-level configuration.

Definition 1 (Edge Vector) *An edge vector* $\boldsymbol{a}$*, is an* $\frac{n(n-1)}{2}$ *vector of 0-1 values, where* $\boldsymbol{a}_i = 1$ *if the edge corresponding to i-th element of vector receives a hub edge and 0, otherwise.*

Now, we translate our problem into the necessary components of a greedy algorithm.

- Set of all edges as the *set of candidates*,
- $\Delta = f^{new} - f^{cur}$ as the *selection function*,
- a functionality for checking the connectivity, to act as a *feasibility function*,
- and the objective function of HLPPT (hub-level network setup cost plus the flow cost) as the *objective function*.

In fact this greedy algorithm is a Hill Climbing algorithm on a neighborhood induced by the Hamming metric on the set of *edge vectors*. In this algorithm internal loop iteratively checks for a new neighbor with distance of 1.
Although, the size of this neighborhood is $\frac{n\times(n-1)}{2}$, however, not all of them can result in a connected hub-level network. In this algorithm we merge the *feasibility function* and the *objective function* and let *Eval* to return ∞ if the resulted trial point is infeasible and the objective value, elsewhere. That is, in order to improve the performance of algorithm, our concern would not be to move from a feasible solution to other feasible ones and examining them to find the best one. Rather, this is to be done by objective function whether it is infeasible or not and return corresponding value.

Note 1 *By flattening out a 2D edge array and taking into account the indirectedness of the hub-level graph, the hub edge* $k-l$ $(l > k)$ *corresponds to the* $(k \times n - k \times (k-1)/2 + l - k)$*-th entry of a the linear edge vector. From now on we will always refer to this 1D vector as the edge array.*

Algorithm 1: A simple greedy algorithm for HLPPT

```
Input: init_sol
Output: x*
x̄:=Create_initial_solution();
min := Eval(x̄);
last_min := ∞;
repeated_min := 0;
while (repeated_min = 0) do
    f̄ := Eval(x̄);
    if f̄ ≤ min then
        min := f̄;
        x* = x̄;
    end
    foreach i = 1  to  nrLocations * (nrLocations − 1)/2 do
        Δf := 0;
        x' := x̄;
        x'_i := 1 − x'_i;
        if  is_not_feasible(x') then
            Δf := ∞;
        else
            Δf := Eval(x') − min;
        end
        if Δf < 0 then
            x* := x' ;
            min := Eval(x');
        end

    end
    if min = last_min then
        repeated_min := repeated_min + 1;
    end
    last_min := min;
    x̄ = x*;
end
stop.
```

The greedy algorithm is depicted in Algorithm 1.

4.2.1 Initial Solution

As we can see in Algorithm 1, an initial solution is needed for our algorithm to proceed. Our experiments have revealed that starting with a random initial solution may not be the best idea. Actually, this is the nature of COPs which have many local optimums that may work as whirlpools to stop the search in one of them. Therefore, it would be worthwhile to have a more prudent strategy in order to create an initial solution.

From the experiences with the instances of HLPPT, we observed that:

- the number of hubs in the optimal solution is an unknown function of the discount factor. That is, the number of hubs has a direct relationship with the discount considered for using of hub edges; The higher discount, the higher tendency to having more hub edges and subsequently hub nodes,
- it is more likely for the most center oriented and busiest (in terms of total flow arriving to and departing from) locations to receive hub (in our experience with our data, there was at least one hub node in a set composed of $n \times 0.2$ most central nodes in union with $n \times 0.2$ busiest nodes).

For example, for $\alpha = 0.5$, we try to select $max(n \times 0.2, 2)$ of the most central nodes and (higher income-outgoing) $max(n \times 0.2, 2)$ of busiest locations as initial hubs. Preferably, the hub level network should be a complete graph of these selected locations which we call them hubs from now on. This initial solution will be passed to the main process of the algorithm.

4.2.2 Complexities

In this subsection we try to shed some light on the complexity of algorithm and the size of neighborhood used.

Since the hub-level network is an undirected graph, we will have $\frac{n\times(n-1)}{2}$ possible hub edges. In the other hand, we assume two configurations to be neighbors if they have distance of 1 with respect to the Hamming metric. As a result, the cardinality of the set of neighbors of a given configuration is in general $\frac{n\times(n-1)}{2}$. Therefore, the size of the neighborhood in the worst case is $\frac{n\times(n-1)}{2}$, that is of $\mathcal{O}(n^2)$. At each iteration of external loop, the internal loop checks the best and feasible move from among maximum $\frac{n\times(n-1)}{2}$ moves. Therefore, in each iteration at most one move will take place. How long the external loop will iterate, is not known in anticipation. But, from the experiences, it is much less than neighborhood size. Therefore we cannot say it to be of $\mathcal{O}(N^2)$, as we cannot say it to be of $\mathcal{O}(N)$, either. On the other hand, for each *feasible* neighbor (a feasible neighbor is a neighbor with the connected hub-level graph) and for each pair of origin-destination $i-j_{(j>i)}$ a shortest path Dijkstra's algorithm is applied. The complexity of each Dijkstra's algorithm is $\mathcal{O}(|E|+|V|log|V|)$, where $|E| \leq q+p(n-p) \leq \frac{n\times(n-1)}{2}$ and $|V| = n$, where q is the number of hub edges and p is the number of hub nodes in the feasible neighbor. This procedure should be considered for $n-1$ nodes and as a result the complexity is of $\mathcal{O}(|V|(|E|+|V|log|V|)) \leq \mathcal{O}((n-1)(\frac{n\times(n-1)}{2}) + n\log n) = \mathcal{O}(n^3)$.

Table 8
Overall Comparison.

Instance	AMC2^d (sec.)	Greedy Algorithm (sec.)	Gap (%)
AP5.2	0.52	0.00	0_{opt}
AP10.4	4.23	0.01	0_{opt}
AP15.6	13.09	0.08	0_{opt}
AP20.8	40.34	0.44	0_{opt}
AP25.10	134.19	1.33	0_{opt}
AP30.12	534.56	4.64	0_{opt}
AP35.14	2771.99	7.91	0.01
AP40.16	14181.73	20.72	0.01
AP45.18	99483.25	31.89	0.01
AP50.20	528663.42	135.99	2.67

4.2.3 Computational Results

As is reported in the Table 8, insofar as the optimal solution for HLPPT instances (i.e. sizes of 5...50) are known, our heuristic except for one case either reached to the optimal solution or for a few cases to a gap of less than %0.01.
The Benders algorithm successfully found the optimal solution of HLPPT instances in a smaller amount of time compared to `CPLEX 9.1` and also could solve larger size instances within a specific time limit. The following table shows the computational time comparison between the best of Benders algorithms and that of our greedy heuristic. Table 8 shows that except for one case, either the solutions reported by heuristic were optimal or the gap was less or equal to %0.01 . Furthermore, our heuristic could find them in a fraction of cpu time that Benders algorithm needs. However, they are not guaranteed to be optimal as far as the optimal is not found by our Benders algorithm.

4.3 More Exploration for Better Solutions

Due to the metaheuristic nature of the method and myopic characteristic of greedy algorithms, this possibility always exists that the search process gets stuck in a local optimum as it is the case for example for the problems of AP35.14 until AP50.20 for which the optimums are known. This is always worthwhile to try a prudent diversification of the search rules and directions with the hope of reaching to a new better solution.

Definition 2 (Neighborhood I) *Due to the metaheuristic nature of the method and myopic characteristic of greedy algorithms, this possibility always exists that the search process gets stuck in a local optimum as it is the case for example for the problems of AP35.14 until AP50.20 for which the optimums are known. This is always worthwhile to try a prudent diversification of the search rules and directions with the hope of reaching to a new better solution.*

The process continues like this: there exist a spoke node as the **best** of p-th level best spoke nodes corresponding to any of the hubs that can be replaced by that hub (resulted configuration may have worst objective function but is a locally best choice which imitates the diversification process; if the new trial point is worst, maybe it is standing on a non-previously explored peak which can drop to a deeper narrow if become subjected to a neighborhood search with respect to the original neighborhood). By moving to this neighbor regarding the idea of neighborhood I and clashing this new structure to the original neighborhood structure by delivering this new trial point to the greedy search, we may have a new better hub-level structure. That is, greedy search may remove some components in favor of other ones (In this case we just used $p = 1$).

Alternatively, there can be another neighborhood structure to be used in the case that the Neighborhood I gets stuck in local optimum or even it was not able to find a better solution than what has been found by the basic greedy algorithm.

Definition 3 (Neighborhood II) *For a given hub-level structure and for a given hub node i, the new structure resulted by replacing the given hub with the p-th($p = 1 \dots 3$) closest non-hub node to i and switching the assignment of all the incoming and outgoing edges of the given hub to this p-th closest non-hub is called the p-th level neighbor of i with respect to the Neighborhood II for the existing hub-level structure.*

In the case of the AP instances, the first neighborhood was sufficient to find the optimal solution of those instances with known optimal solution. However, there have been some other instances of AP for which the second neighborhood could improve the solution compared to the first one.

4.3.1 Quality of Solutions

Here, we examine the quality of our heuristic by comparing the best-known solution of heuristic with that of LP relaxation of HLPPT (HLPPT^{LP}). In Table 9, the gap between the solution of our heuristic and the LP relaxation is measured by $\frac{UB-LB}{LB} \times 100$ and is depicted in the column titled by Gap^{LP} (%). The gap between the optimal solution of AMC2^d and that of our heuristic is

Table 9
Quality of $greedy^+$ solutions

Instance	$greedy^+$ (sec.)	HLPPT^{LP}	Gap^{LP} (%)	Gap^{opt} (%)
AP5.2	0.03	0.41	0	0_{opt}
AP10.4	0.14	2.44	0	0_{opt}
AP15.6	0.27	11.50	0	0_{opt}
AP20.8	0.95	32.98	0.1	0_{opt}
AP25.10	2.53	96.16	1.8	0_{opt}
AP30.12	7.13	451.06	1.4	0_{opt}
AP35.14	18.39	1024.25	3.9	0_{opt}
AP40.16	34.66	2475.16	3.9	0_{opt}
AP45.18	70.22	4098.70	4.8	0_{opt}
AP50.20	176.13	6188.42	5.9	-
AP55.22	264.05	12070.44	6.9	-
AP60.24	565.91	28774.47	7.7	-
AP65.26	663.86	54109.12	12.0	-

reported in the last column (Gap^{opt}(%)).

One may conclude that our heuristic is extremely satisfactory and also LP relaxation is good approximations of optimal solution of HLPPT.

4.3.2 Computational Results

Table 10 reports the computational results for some instances of the AP dataset. As mentioned earlier, insofar as the optimal solution is known the metaheuristic algorithm was able to reach it.

In a time limit of $\simeq$4.5 hrs, $greedy^+$ can meet the termination criteria for all the problem instances up to size 100 as depicted in Table 10.

5 Summary and Conclusion

We proposed a new HLP model customized for public transport application in which some of the classical assumptions of the HLPs have been relaxed in favor of achieving a more realistic model. Several variants of Benders decomposition are proposed. We have also shown how the model can be tightened by means of

Table 10
greedy$^+$ Algorithm run-time report

Instance	T. Cpu(s)	Instance	T. Cpu(s)
AP5.2	0.03	AP55.22	264.05
AP10.4	0.14	AP60.24	565.91
AP15.6	0.27	AP65.26	663.86
AP20.8	0.95	AP70.28	1407.46
AP25.10	2.53	AP75.30	1785.86
AP30.12	7.13	AP80.32	3142.24
AP35.14	18.39	AP85.34	4274.56
AP40.16	34.66	AP90.36	6934.25
AP45.18	70.22	AP95.38	8601.04
AP50.20	176.13	AP100.40	15370.72

valid inequalities improving the performance of our exact Benders algorithm. Moreover, the presented heuristic is successful in finding mostly the optimal solution of problem as far as it is known.

Taking into account other issues such as capacity policies and multiple criteria aspects, are possibilities for future research on HLPs in public transport planning. Other aspects like, reliability, congestion and re-routing scenarios are among the areas which also deserve more attention in connection with public transport planning.

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