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Simulation of quasistatic deformations  
using discrete rod models

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# Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



## FAST SIMULATION OF QUASISTATIC CABLE DEFORMATIONS USING DISCRETE ROD MODELS

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**Abstract.** *Recently we developed a discrete model of elastic rods with symmetric cross section suitable for a fast simulation of quasistatic deformations [33]. The model is based on Kirchhoff’s geometrically exact theory of rods. Unlike simple models of “mass & spring” type typically used in VR applications, our model provides a proper coupling of bending and torsion. The computational approach comprises a variational formulation combined with a finite difference discretization of the continuum model. Approximate solutions of the equilibrium equations for sequentially varying boundary conditions are obtained by means of energy minimization using a nonlinear CG method. As the computational performance of our model yields solution times within the range of milliseconds, our approach proves to be sufficient to simulate an interactive manipulation of such flexible rods in virtual reality applications in real time.*

## 1 Introduction

The work presented in this article concerns a method to derive discrete mathematical models of slender flexible bodies which are firmly based on structural mechanics and applicable to compute quasistatic deformations extremely fast at moderate accuracy. A very brief account on this approach has been given recently in [33] and is complemented here by further background information and methodical details.

The topics we discuss are located within the overlap of two different areas of simulation applications, namely (i) the visualisation and animation of deformable objects in computer graphics and (ii) the treatment of flexible structures in multibody simulation (MBS). The motion of rigid bodies takes a prominent role in the formulation of mathematical models within both areas and provides the common basis connecting these otherwise substantially different disciplines. Our method to model flexible structures favourably combines ideas from both areas in order to obtain the desired properties of the simulation models as mentioned above.

### Modelling of deformable bodies in multibody simulation

The handling of flexible objects in multibody simulation (MBS) models is both a long term research topic [1, 2] as well as an active area of current research [3, 4, 5, 6].

A standard approach supported by most commercial MBS software packages represents flexible bodies by means of *vibrational modes* (e.g. of *Craig–Bampton* type [7, 8]) computed by modal analysis within the framework of *linear* elasticity. This modal representation of flexible structures usually reduces the number of degrees of freedom drastically and thereby provides *reduced models* which can be readily included into MBS models without deteriorating computational performance in a severe way. However, such methods are suitable (as well as by definition restricted) to represent forced oscillations resulting from a *linear response* of the flexible structure to external excitations that effect merely *small* deformations, which in turn implies a certain compactness of the structure to yield methodical consistency of the whole procedure.

If the flexible bodies of interest possess special geometrical properties characterising them as *slender* (or *thin*) structures (i.e. rods, plates or shells), their overall deformation in response to moderate external loads may become *large*, although locally the stresses and strains remain *small*. Therefore models suitable to describe such *large deformations of slender structures* have to be capable to account for *geometric nonlinearities*. Compared to object geometries that require fully three dimensional volume modelling, the reduced dimensionality of rod or shell models is accompanied by a considerable reduction in the number of degrees of freedom, which makes the inclusion of appropriately discretised versions of the *full* models (in contrast to modally reduced ones as discussed above) into a MBS framework [4] computationally feasible.

A specific property of *Cosserat* type rod and shell models [28] which makes them especially attractive in view of their integration into MBS systems relies on the fact that locally such models carry kinematical degrees of freedom which are elements of  $\mathbb{R}^3 \times SO(3)$ , such that appropriately discretised versions of such one- or two-dimensional continuum models can be interpreted as chains or networks of rigid bodies interacting via special force elements. However, the standard numerical approach to treat such models of slender structures which seems to be almost exclusively followed in computational mechanics relies on *finite elements* — see [9, 10, 11, 12] and [13] for the numerical treatment of nonlinear rods by FE methods — and focusses on accuracy rather than computational efficiency, which makes them far too demanding if one is aiming at extremely short computation times at moderate accuracy requirements (as we do). Therefore it is worthwhile to consider alternative discretisation approaches.

## Modelling of flexible structures in computer graphics and VR

Computational efficiency is of primary importance in simulation applications of virtual reality, especially if the movement and deformation of objects are animated in *real time*. This capability is crucial for a seamless integration of simulator modules within VR (*virtual reality*) or FDMU (*functional digital mock up*) software packages used for *interactive simulation* (e.g. of assembly processes).

Although the dominant paradigm to assess the quality of an animation or simulation within these application areas — as well as related ones like computer games or movies — used to be “...*It’s good enough if it looks good* ...” [14], such that a mere “*fake*” [15] of structure deformation is considered to be acceptable (at least for those applications were “...*fooling of the eye* ...” [14] is the main issue), the need for “*Physics-based modelling*” (PBM) approaches increases constantly, and the usage of models that are more [18] or less [16, 17, 19] based on ideas borrowed from classical structural and rigid body mechanics is not uncommon, especially if (apart from a “realistic” visual appearance) a minimum amount of physical information as well as an at least qualitatively correct behaviour are required. As a consequence, the usage of modelling ideas based on solid and fluid mechanics becomes increasingly popular within the computer graphics community, as such models have favourable properties concerning the simulation and rendering of “plausible” animations as well as the control of these by parameters which have a physical meaning.

Early work on the PBM approach to the animation of flexible objects using geometrically nonlinear structure models appeared about twenty years ago [20, 21]. In their SIGGRAPH lectures [22], which are a popular reference on PBM methods — an updated version [23] is hosted at *Pixar* (!) — Baraff and Witkin discuss the usage of rigid body kinematics as well as elementary dynamics simulation techniques to compute animations of moving objects, including the checking for and handling of collisions by simple contact algorithms. Although the models proposed there are based on procedures used in computational solid mechanics, the authors pointed out that “...*ultimate physical accuracy is not the ultimate goal* ...”. Consequently researchers active in the PBM community do not hesitate to take a considerable freedom to modify and/or simplify the models substantially according to their needs (as discussed e.g. in [16]).

Although the application of PBM to the animation and simulation of flexible objects was discussed already in [20] on a rather general level for all dimensionalities, the primary focus was on cloth simulation using Kirchhoff type membrane or shell models. Apart from the fact that cloth modelling might have been the most interesting topic for the research on PBM methods at that time, (we suppose) there also were technical reasons for the pronounced interest in 2D structure models: while full 3D models were computationally too demanding because of their large number of degrees of freedom, the kinematics of membrane models was readily accessible via elementary concepts of the differential geometry of surfaces in  $\mathbb{R}^3$ , as the director type d.o.f., which are inevitably present in geometrically nonlinear rod models and encode the coupling between bending and torsion, do not appear in unshearable membranes or shells.

This might be the cause for the fact that until today the kinematics of rod models still seems to be not well understood in the PBM community. A rare (maybe even singular) counter example is the paper of Pai [18], where (unlike the title of this paper suggest) an inextensible Kirchhoff rod model is used to simulate the spatial deformation of slender 1D objects. The method to solve the equilibrium equations explicitly utilizes the fact that the discretised rod locally carries the d.o.f. of rigid bodies. More frequently one encounters less sophisticated approaches that model flexible 1D objects as simple “mass & spring” systems (the articles [17] and [19]

are prototypical examples of this approach). While the obvious benefit of the “mass & spring” approach clearly is computational efficiency of such models due to their simple algebraic structure, the drawback of “mass & spring” systems is rooted in the method of their construction, which usually is done “bottom up”, introducing spring constants in an ad hoc way.

## Discrete rod models derived from structural mechanics

Looking at the various deformation energy terms of geometrically nonlinear continuum rod models one recognizes that finite difference discretisations of these models yield expressions whose algebraic form and complexity is comparable to those of “mass & spring” systems, with spring constants given by the material parameters of the continuum model and the grid constant used in the discretisation. In this way one obtains discrete rod models that are suitable for fast simulation as well as physically meaningful.

## 2 Cosserat and Kirchhoff rod models

In structural mechanics *slender objects* like cables, hoses etc. are described by one dimensional *beam* or *rod* models which utilise the fact that, due to the relative smallness of the linear dimension  $D$  of the cross section compared to the length  $L$  of a rod, the local stresses and strains remain small and the cross sections are almost unwarped, even if the overall deformation of the rod relative to its undeformed state is large. This justifies kinematical assumptions that restrict the cross sections of the deformed rod to remain *plane and rigid*.

In the following we give brief introduction to rod models of *Cosserat* and *Kirchhoff* type, the latter being a special case of the former. We do not present the most general versions of these models, which are discussed at length in the standard references [25] and [28]. The approach we finally use as a basis for the derivation of a generalized “*mass & spring*” type model by finite difference discretisation is an extensible variant of Kirchhoff’s original theory [24] as presented in [26] (see part II, §16–§19) for a hyperelastic rod with symmetric cross section subject to a constant gravitational body force.

### 2.1 Kinematics of Cosserat and Kirchhoff rods

A (*special*) *Cosserat rod* [28] is a *framed curve* [31] formally defined as a mapping  $s \mapsto (\varphi(s), \hat{\mathbf{F}}(s))$  of the interval  $\mathcal{I} = [0, L]$  into the configuration space  $\mathbb{R}^3 \times SO(3)$  of the rod, where  $L$  is the length of the undeformed rod. Its constituents are: (i) a space curve  $\varphi : \mathcal{I} \rightarrow \mathbb{R}^3$  that coincides with the *line of centroids* piercing the cross sections along the deformed rod at their geometrical center, and (ii) an “curve of frames”  $\hat{\mathbf{F}} : \mathcal{I} \rightarrow SO(3)$  with the origin of each frame  $\hat{\mathbf{F}}(s)$  attached to the point  $\mathbf{x}_s = \varphi(s)$ . The matrix representation of the frame  $\hat{\mathbf{F}}(s)$  w.r.t. a fixed global coordinate system  $\{\mathbf{e}^{(1)}, \mathbf{e}^{(2)}, \mathbf{e}^{(3)}\}$  of  $\mathbb{R}^3$  may be written as a triple of column vectors, i.e.  $\hat{\mathbf{F}}(s) = (\mathbf{d}^{(1)}(s), \mathbf{d}^{(2)}(s), \mathbf{d}^{(3)}(s))$ , obtained as  $\mathbf{d}^{(k)}(s) = \hat{\mathbf{F}}(s) \cdot \mathbf{e}^{(k)}$ . By definition  $\mathbf{d}^{(3)}$  coincides with the unit *cross section normal* vector located at  $\varphi(s)$ .

For simplicity we assume the undeformed rod to be *straight and prismatic* such that its initial geometry relative to  $\{\mathbf{e}^{(1)}, \mathbf{e}^{(2)}, \mathbf{e}^{(3)}\}$  is given by the direct product  $\mathcal{A} \times \mathcal{I}$  with a constant cross section area  $\mathcal{A}$  parallel to the plane spanned by  $\{\mathbf{e}^{(1)}, \mathbf{e}^{(2)}\}$ . Introducing coordinates  $(\xi_1, \xi_2)$  in the plane of the cross section  $\mathcal{A}$  relative to its geometrical center we may parametrise the material points  $\mathbf{X} \in \mathcal{A} \times \mathcal{I}$  of the undeformed rod geometry by

$$\mathbf{X}(\xi_1, \xi_2, s) = \sum_{k=1,2} \xi_k \mathbf{e}^{(k)} + s \mathbf{e}^{(3)},$$



and the deformation mapping  $\mathbf{X} \mapsto \mathbf{x} = \Phi(\mathbf{X})$  is given by the formula

$$\mathbf{x}(\xi_1, \xi_2, s) = \varphi(s) + \sum_{k=1,2} \xi_k \mathbf{d}^{(k)}(s) .$$

The kinematics of a framed curve as presented above determine the possible deformations of a *Cosserat* rod. These are: *stretching* (in the direction of the curve tangent), *bending* (around an axis in the plane of the cross section), *twisting* (of the cross section around its normal) and *shearing* (i.e. tilting of the cross section normal w.r.t. the curve tangent).

Following [31] we denote a frame  $\hat{\mathbf{F}}(s)$  as *adapted* to the curve  $\varphi(s)$  if  $\mathbf{d}^{(3)}(s)$  coincides with the *unit tangent* vector  $\mathbf{t}(s) = \partial_s \varphi(s) / \|\partial_s \varphi(s)\|$  along the curve. An adapted frame satisfies the *Euler–Bernoulli* hypothesis which states that the cross sections remain always orthogonal to the centerline curve also in a deformed state. Curves with adapted frames describe the possible deformations of (extensible) *Kirchhoff* rods. Compared to Cosserat rods the kinematics of Kirchhoff rods are restricted further, as they do *not* allow for shear deformations. The *inextensibility* condition  $\|\partial_s \varphi\| = 1$  constitutes an additional kinematical restriction and yields Kirchhoff’s rod model in the form most frequently presented in the literature (see e.g. §16–§20 in Landau’s book [26]).

Measuring the slenderness of a rod of cross section diameter  $D$  and length  $L$  in terms of the small parameter  $\varepsilon = D/L$  and assuming (hyper)elastic material behaviour it can be shown that the potential energy terms corresponding to bending and torsion are of the order  $\mathcal{O}(\varepsilon^4)$ , while the energy terms corresponding to stretching and shearing scale as  $\mathcal{O}(\varepsilon^2)$ . In this way the latter effectively act as *penalty terms* that enforce the kinematical restrictions  $\|\partial_s \varphi\| = 1$  and  $\mathbf{d}^{(3)}(s) = \mathbf{t}(s)$ . This explains how Kirchhoff rods appear as a natural limit case of Cosserat rods subject to moderate deformations provided  $\varepsilon$  is sufficiently small.

## 2.2 Hyperelastic Kirchhoff rods with cross section symmetry

As we are interested in a rod model that is suitable for the simulation of moderate cable deformations, both the Cosserat as well as the Kirchhoff approach would fit for our purpose. A characteristic feature of the Cosserat model consists in the description of the bending and torsion of the rod in terms of the frame variables, while the bending of the centerline curve  $\varphi(s)$  is produced only indirectly by the shearing forces that approximately align the curve tangent to the cross section normal. In contrast to that, Kirchhoff’s model [30] encodes bending strain directly by the curvature of  $\varphi(s)$  and therefore provides a direct pathway to discrete models formulated in terms of (discrete) d.o.f. of the centerline.

Averaging the normal Piola–Kirchhoff tractions and corresponding torques over the cross section surface of the deformed rod located at  $\varphi(s)$  yields *stress resultants*  $\mathbf{f}(s)$  and *stress couples*  $\mathbf{m}(s)$ , i.e. resultant force and moment vectors per unit reference length [29]. If the rod is in a *static equilibrium* state, these vectors satisfy the differential balance equations of forces and moments

$$\partial_s \mathbf{f} + \mathbf{G} = \mathbf{0} , \quad \partial_s \mathbf{m} + \partial_s \varphi \times \mathbf{f} = \mathbf{0} , \quad (1)$$

where  $\mathbf{G}$  represents a (not necessarily constant) body force acting along the rod, and we assumed that no external moment is applied in between the rod boundaries. Although the form of the equilibrium equations (1) is identical for the Cosserat and Kirchhoff model, the nature of the stress resultant  $\mathbf{f}$  is different in both cases. In the Cosserat model all components of  $\mathbf{f}$  and  $\mathbf{m}$  are related to corresponding strain measures via a constitutive function. In an inextensible Kirchhoff rod only the components of the stress couple  $\mathbf{m}$  are determined constitutively,

while the stress resultant  $\mathbf{f}$  is a Lagrange multiplier which has to be determined from the equilibrium equations (1).

In the case of an extensible Kirchhoff rod which in its undeformed state has the form of a straight cylinder with symmetric cross section, the assumption of a hyperelastic material behaviour yields the expression [26]

$$\mathbf{m}(s) = EI \mathbf{t}(s) \times \partial_s \mathbf{t}(s) + GJ \Omega_t \mathbf{t}(s) \quad (2)$$

for the stress couple, where  $E$  is Young's modulus,  $G$  the shear modulus,  $I$  measures the geometrical moment of inertia of the cross section (e.g.  $I = \frac{\pi}{4} R^4$  for a circular cross section of radius  $R$ ) and  $J = 2I$ . The quantities  $EI$  and  $GJ$  determine the stiffness of the rod w.r.t. bending and torsion. According to (2) the stress couple is given as the sum of the *bending moment*  $\mathbf{m}_b = EI \mathbf{t} \times \partial_s \mathbf{t}$  and the *torsional moment*  $\mathbf{m}_t = GJ \Omega_t \mathbf{t}$ .

The strain measure related to the bending moment is given by the vector

$$\mathbf{t} \times \partial_s \mathbf{t} = \frac{\partial_s \boldsymbol{\varphi} \times \partial_s^2 \boldsymbol{\varphi}}{\|\partial_s \boldsymbol{\varphi}\|^2} = \|\partial_s \boldsymbol{\varphi}\| \kappa \mathbf{b} \quad (3)$$

which is proportional to the *Frenet curvature*  $\kappa(s)$  of the centerline and (if  $\kappa > 0$ ) points in the direction of the binormal vector  $\mathbf{b}(s)$ . The strain measure related to the torsional moment is determined by the *twist*

$$\Omega_t(s) = \mathbf{t}(s) \cdot [\mathbf{d}(s) \times \partial_s \mathbf{d}(s)] \quad (4)$$

where  $\mathbf{d}(s)$  is any unit normal vector field to the centerline given as a fixed linear combination  $\mathbf{d} = \cos(\alpha_0) \mathbf{d}^{(1)} + \sin(\alpha_0) \mathbf{d}^{(2)}$  of the frame vectors  $\mathbf{d}^{(1)}(s)$  and  $\mathbf{d}^{(2)}(s)$  for some constant angle  $\alpha_0$ . Note that the special constitutive relation (2) implies that in equilibrium the twist  $\Omega_t$  is constant.

As a Kirchhoff rod is (by definition) unshearable, only the *tangential* component of the stress resultant  $\mathbf{f}(s)$  is constitutively determined by the *tension*

$$\mathbf{t}(s) \cdot \mathbf{f}(s) =: T(s) = EA (\|\partial_s \boldsymbol{\varphi}\| - 1) \quad (5)$$

related to the elongational strain  $(\|\partial_s \boldsymbol{\varphi}\| - 1)$ . The resistance of the rod w.r.t. stretching is determined by  $EA$  where  $A = |\mathcal{A}|$  is the size of the cross section area (in our case:  $A = \pi R^2$ ). The *shearing force* acting parallel to the cross section is given by  $\mathbf{f}_{sh}(s) = \mathbf{f}(s) - T(s) \mathbf{t}(s)$ . It is not related to any strain measure but has to be determined from the equilibrium equations a Lagrange parameter corresponding to the internal constraint  $\mathbf{d}(s) \cdot \mathbf{t}(s) = 0$ .

To determine the deformation of the rod in static (or likewise quasistatic) equilibrium one has to solve the combined system of the eqns. (1) to (5) for a suitable set of *boundary conditions*, e.g. like those discussed in [30]. (This issue will not be discussed here.) Equivalently one may obtain the centerline  $\boldsymbol{\varphi}(s)$  and the unit normal vector field  $\mathbf{d}(s)$  that represents the adapted frame of the rod by *minimization of the potential energy*

$$W_{pot}[\boldsymbol{\varphi}, \mathbf{d}] = \int_0^L w_{el}(s) ds - \int_0^L \mathbf{G}(s) \cdot \boldsymbol{\varphi}(s) ds \quad (6)$$

According to eqns. (2) to (5) the *elastic energy density* is a quadratic form in the various strain measures given by

$$w_{el}(s) = \frac{EI}{2} (\mathbf{t} \times \partial_s \mathbf{t})^2 + \frac{GJ}{2} \Omega_t^2(s) + \frac{EA}{2} (\|\partial_s \boldsymbol{\varphi}\| - 1)^2 \quad (7)$$

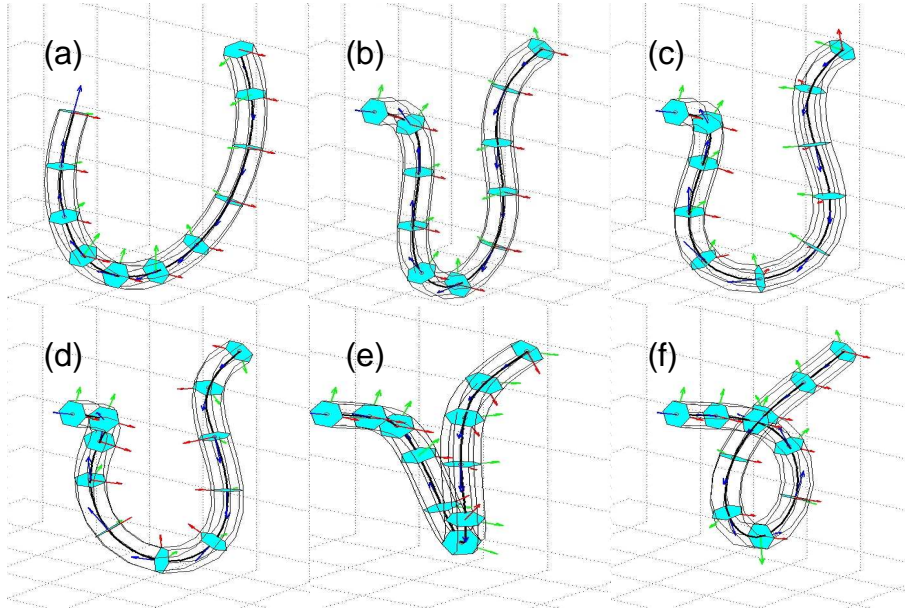


Figure 1: Sequential deformation of a discrete, hyperelastic Kirchhoff rod of symmetric cross section

and determines the *stored energy function*  $W_{el}[\boldsymbol{\varphi}, \mathbf{d}] = \int_0^L w_{el}(s) ds$  containing the internal part of  $W_{pot}$ . A specific choice of boundary conditions may be accounted for by modified expressions for  $w_{el}(0)$  and  $w_{el}(L)$  which are obtained from (7) by fixing combinations of the kinematical variables  $\boldsymbol{\varphi}(s)$  and  $\mathbf{d}(s)$  and their derivatives at prescribed values (as required by the b.c.) and substituting these into  $w_{el}(s)$ .

### 3 A discrete Kirchhoff rod model

The final step of our approach towards a model of flexible rods suitable for the fast computation of quasistatic rod deformations is the discretisation of the potential energy by applying standard (e.g. central) finite difference stencils to the elastic energy density (7) and corresponding quadratur rules (e.g. trapezoidal) to the energy integrals (6). Boundary conditions are treated in the way described at the end of the previous section. This procedure results in a discrete model of an extensible Kirchhoff rod which has a similar structure like the simple “mass & spring” type models presented in [17] and [19].

However, as a benefit of the systematic derivation procedure on the basis of a proper continuum model, our discrete rod model is able to capture the rather subtle coupling of bending and torsion deformation correctly, as the sequential deformation of the discrete Kirchhoff rod shown in **Fig. 1** illustrates: (a) Starting from a circle segment, the tangents of the boundary frames are bent inward to produce (b) an (upside down)  $\Omega$ -shaped deformation of the rod at zero twist. To demonstrate the effect of mutual coupling of bending and torsion in the discrete model, the boundary frame at  $s = L$  is twisted counterclockwise by an angle of  $2\pi$  while the other boundary frame at  $s = 0$  is held fixed. The pictures (c)–(f) show snapshots of the deformation state taken at multiples of  $\frac{\pi}{2}$ . We compute approximate solutions of the equilibrium equations for sequentially varying boundary conditions by a minimization of the discrete potential energy using a standard nonlinear CG method [32] without preconditioning. The overall deformation from (a) to (f) was split up into a sequence of 25 consecutive changes of the boundary conditions defined by the terminal frames of the rod. For a discretization of the cable into 10 segments, the simulation took 150 ms on 1 CPU of an AMD 2.2 GHz double processor PC, which amounts

to an average computation time of 6 *ms* per step, which indicates the computational efficiency of our approach.

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