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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

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MULTIBODY DYNAMICS SIMULATION OF GEOMETRICALLY EXACT COSSERAT RODS

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Abstract. *In this paper, we present a viscoelastic rod model that is suitable for fast and sufficiently accurate dynamic simulations. It is based on Cosserat's geometrically exact theory of rods and is able to represent* extension, shearing *('stiff' dof),* bending *and* torsion *('soft' dof). For inner dissipation, a consistent damping potential from Antman is chosen. Our discrete model is based on a finite difference discretisation on a staggered grid. The right-hand side function f and the Jacobian $\partial f/\partial(q,v,t)$ of the dynamical system $\dot{q} = v$, $\dot{v} = f(q,v,t)$ – after index reduction from three to zero – is free of higher algebraic (e.g. root) or transcendent (e.g. trigonometric or exponential) functions and is therefore cheap to evaluate. For the time integration of the system, we use well established stiff solvers like* RADAU5 *or* DASPK. *As our model yields computation times within milliseconds, it is suitable for interactive manipulation in 'virtual reality' applications. In contrast to fast common VR rod models, our model reflects the structural mechanics solutions sufficiently correct, as comparison with* ABAQUS *finite element results shows.*

1 INTRODUCTION

The *Cosserat rod model* [1, 2, 3, 45, 46, 47] is an appropriate model for the geometrically exact simulation of deformable rods – i. e. slender one dimensional flexible structures – in space (statics) or space-time (quasistatics or dynamics). A Cosserat rod can be considered as the geometrically nonlinear generalisation of a Timoshenko-Reissner beam. In contrast to a Kirchhoff rod, which can be considered as a geometrically nonlinear generalisation of a Euler-Bernoulli beam, a Cosserat rod allows to model not only bending and torsion – these are 'soft dof' –, but as well extension and shearing – these are 'stiff dof'. For the Cosserat rod, like for the Kirchhoff rod, the overall deformation as response to moderate external loads, i. e. displacements, forces or moments, may become large, although locally the stresses and strains remain small. We remark that the Cosserat/Kirchhoff models are only 'skeleton' models. The reconstruction of the three dimensional displacement, stress and strain distributions through the rod cross sections can be conveniently carried out in a postprocessing by the use of 'warping functions' [20, 32].

This article is concerned with a discrete finite difference model of a Cosserat rod that is firmly based on structural mechanics and applicable to compute dynamical deformations very fast at sufficient accuracy. We continue the work [29, 30] from the ECCOMAS 2007, which considered the fast simulation of geometrically exact Kirchhoff rods in the quasistatic case.

The paper is structured as follows.

In section 2, we describe the basic equations for a Cosserat rod in the continuum, where we parametrise the rod directly with quaternions. Unit quaternions in the subgroup $\mathbb{S}^3 = \partial B_1^{\mathbb{H}}(0) = \{p \in \mathbb{H} : \|p\| = 1\} \subset \mathbb{H}$ are an appropriate way to describe (non-commutative spatial) rotations in $SO(3) = \{Q \in \mathbb{R}^{3\times 3} : QQ^T = Q^TQ = I, \det Q = 1\}$ for our purposes. This is analogous to unit complex numbers in the subgroup $\mathbb{S}^1 = \partial B_1^{\mathbb{C}}(0) = \{z \in \mathbb{C} : \|z\| = 1\} \subset \mathbb{C}$, which describe (commutative plane) rotations in $SO(2) = \{Q \in \mathbb{R}^{2\times 2} : QQ^T = Q^TQ = I, \det Q = 1\}$. Of course, other possibilities, such as Rodriguez parameters, rotation vectors, Euler or Cardan angles, exist [**?**, 18, 36, 41]. All of them have their pros and cons. So as a pro, gimbal locking can be avoided with quaternions. A con is that they must be kept at unity length.

In section 3, we present our discrete numerical model of the Cosserat rod, based on finite differences. In contrast to the finite element method, where the primary unknowns are situated at the nodes (= vertices), we suggest a staggered grid discretisation. This means that the translatory resp. the rotatory dof are ordered in an alternating fashion and that the trapezoidal resp. midpoint quadrature formulas form the basis for the approximation of the internal energy integrals. This construction allows to interpret the discrete Cosserat rod as a sequence of – almost rigid – cylinders with the primary rotatory dof situated at the cylinder centers. These cylinders are connected with appropriate springs and dampers. Here the 'springs' and 'dampers' are seriously derived from the continuous Cosserat strain and curvature relations. Whereas the extensional and shearing strains, which belong to such cylinders, are discretised via finite differences in the one and only one canonical standard fashion, several choices for the discrete bending and torsion curvature measures are possible. These curvatures belong to the vertices between the cylinders and are the results of more or less accurate and expensive rotation interpolations. We formulate the final discrete model as a constrained mechanical system, resulting in the well known Lagrangian DAE system of index three. Index reduction to zero plus introduction of Baumgarte penalty accelerations gives a universal ODE formulation, suitable for any ODE solver. As an alternative to the Baumgarte method, stabilisation by projection is as well convenient and cheap. For our approach, the inverse mass-constraint matrix is expli-

citly known and multiplication with the latter is exactly as expensive as multiplication with the mass-constraint matrix itself. The model can be implemented trigonometry and square root free, resulting in extremely cheap right hand side function and Jacobian evaluations. The extensible and inextensible Kirchhoff rod models can as well be conveniently fed into this framework.

In section 4, we present accurate numerical results, compared to ABAQUS finite element solutions. For bending and torsion scenarios e. g. in robotics or path planning simulations, we propose to impose strong damping on extension and shearing, which are of subordinate importance, and to solve the resulting stiff system via well established methods [22, 23, 35]. As our model allows computations within milliseconds, it is adequate for multibody dynamics simulations, especially for interactive manipulation applications.

Where can this work be situated within the state of the art rod models?

The handling of flexible objects in **multibody dynamics** simulations has been a long term field of research until today [5, 6, 7, 39, 42, 43, 44]. The standard approach, which is supported by today's commercial software packages such as SIMPACK, ADAMS or VIRTUALLAB, represents flexible structures by vibrational modes, e. g. of Craig and Bampton type [12], that are obtained from numerical modal analysis within the range of linear elasticity. Such methods are suitable and accurate to represent oscillatory response that results from *linear* response of the flexible structure. Unlike that, our approach in not of modal kind.

Our approach stands in contrast to the usual way in computational continuum mechanics, where the **finite element** approach is favored [6, 7, **?**, **?**, 14, 15, 20, 25, 26, 45, 46]. The reason for that is, that the main focus in FE is accuracy, not computational efficiency. The general problem in geometrically nonlinear FE is the proper interpolation of the finite rotations [38] such that objectivity of the strain measures is maintained, i. e. invariance under rigid body motions. This results in extremely technical and sophisticated models with expensive right hand side functions and Jacobians. Our approach is much simpler and objective is maintained by construction.

The main focus for rod models in the **physics based modeling / virtual reality / computer graphics** community is the numerical performance, rather than physical accuracy. Usually these models are too far away from continuum mechanics [48]. *'... It's good enough if it looks good ...'* and *'... fooling the eye ...'* is the main issue [37]. In our opinion, the best approach towards a physically serious rod model is the recent work [47]. However, here they did not publish total computational times.

2 GEOMETRICALLY EXACT COSSERAT RODS IN THE CONTINUUM

Our starting point for the continuous Cosserat rod model is the exposition in [45, 46]. For the constitutive material behaviour, we choose a simple linear viscoelastic one [1, 2, 3]. The elastic parameters can be straightforwardly deduced from material and geometric parameters [32]. Concerning the damping model, we note that it is macroscopic and phenomenological, it comprises not only pure material damping, but also miscellaneous damping mechanisms. We assume both the elastic and viscoelastic parts as diagonal. The generalisation to non-diagonal, symmetric and positively definite constitutive Hookean tensors or to nonlinear hyperelastic materials is straightforward and does not cause any harm in principle. We concentrate on the description of the internal potential, dissipation and kinetic energies in sections 2.1, 2.2 and 2.3 respectively, as these will be the basis for the discrete model later. In section 2.4 we give a detailed exposition of the dynamic equations of motion with the rotatory part in quaternion language.

We start with the **kinematics** for the Cosserat rod. The Cosserat rod is completely determined by its *centerline* of mass centroids

$$x : [0, L] \times [0, T] \to \mathbb{R}^3, \qquad (s, t) \mapsto x(s, t)$$

and its *unit quaternion field*

$$p : [0, L] \times [0, T] \to \mathbb{S}^3 = \partial B_1^{\mathbb{H}}(0) \hookrightarrow \mathbb{H}, \qquad (s, t) \mapsto p(s, t).$$

The quaternion field uniquely determines its *orthonormal frame field*

$$R \circ p : [0, L] \times [0, T] \xrightarrow{p} \mathbb{S}^3 = \partial B_1^{\mathbb{H}}(0) \xrightarrow{R} SO(3), \qquad (s, t) \mapsto R\big(p(s, t)\big).$$

The situation is depicted in Figure 1. Any point of the deformed rod in space s and time t is addressed by the deformation map

$$[0, L] \times [0, T] \times \mathcal{A} \ni (s, t, (\xi_1, \xi_2)) \mapsto x(s, t) + \xi_1 d^1(p(s, t)) + \xi_2 d^2(p(s, t)).$$

The parameter $s \in [0, L]$ is the arc length of the undeformed rod centerline, $L > 0$ is the total undeformed arc length and $\mathcal{A} \subset \mathbb{R}^2$ is a bounded, connected coordinate domain for the coordinates (ξ_1, ξ_2) in the cross section, which is assumed rigid and plane throughout the deformation. In classical differential geometry, the object $(x(\cdot, t), (R \circ p)(\cdot, t))$ constitutes a so-called 'framed curve'. For a quaternion $p = p_0 + \hat{p} = \Re(p) + \Im(p) = (p_0;\, p_1, p_2, p_3)^T \in \mathbb{H}$ the frame $R(p)$ is given by the Euler map

$$R : \mathbb{H} \to \mathbb{R}SO(3), \quad p \mapsto \big(d^1(p) \big| d^2(p) \big| d^3(p)\big) = \big(2p_0^2 - \|p\|^2\big)\mathcal{I} + 2\hat{p} \otimes \hat{p} + 2p_0\mathcal{E}(\hat{p})$$

with the alternating skew tensor $\mathcal{E}$, which identifies skew tensors in $so(3)$ with their corresponding axial vectors in $\mathbb{R}^3$ via

$$\mathcal{E} : \mathbb{R}^3 = \Im(\mathbb{H}) \to so(3), \qquad \mathcal{E}(u)v = u \times v \quad \text{for } u, v \in \mathbb{R}^3.$$

We write $u \simeq \mathcal{E}(u)$ for $u \in \mathbb{R}^3$. It is convenient to identify $\Im(\mathbb{H}) = \mathbb{R}^3$. The directors $d^1(p)$ and $d^2(p)$ span the rigid cross section of the rod. The third director $d^3(p)$ is always normal to the cross section and is kept close to the tangent $\partial_s x$ of the centerline by stiff shearing springs. It holds that $R(\lambda p) = \lambda^2 R(p)$ for each $p \in \mathbb{H}$ and $\lambda \in \mathbb{R}$. This property makes R sensitive with respect to stretching of p. Especially, R maps $\mathbb{S}^3$ into $SO(3)$. It holds $R(-p) = R(p)$, which implies that p and its antipode $-p$ describe the same rotation. It is well known that for each rotation Q in $SO(3)$ there exist exactly two unit quaternions – necessarily antipodes – that produce Q. Via R, the unit sphere $\mathbb{S}^3$ covers $SO(3)$ exactly two times [17, 19, 27]. Stretched rotation can be expressed via quaternions as $R(p)v = pv\bar{p}$ (forward) and $R(p)^T v = \bar{p}vp$ (backward) for $p \in \mathbb{H}$ and $v \in \Im(\mathbb{H}) = \mathbb{R}^3$, especially $d^\nu(p) = pe^\nu\bar{p} = R(p)e^\nu$ for each of the space fixed Euclidean base vectors e^1, e^2 and e^3 (classically denoted by 'i', 'j' and 'k') of $\Im(\mathbb{H}) = \mathbb{R}^3$. Recall that the quaternion product is defined by

$$pq = p_0q_0 - \langle \hat{p}, \hat{q} \rangle + p_0\hat{q} + q_0\hat{p} + \hat{p} \times \hat{q} \qquad \text{for } p,\, q \in \mathbb{H}.$$

We use the symbols $p_0 = \Re(p)$ resp. $\hat{p} = \Im(p)$ to denote the real resp. the imaginary (= vector) part and $\bar{p} = p_0 - \hat{p}$ to denote the conjugate of a quaternion $p \in \mathbb{H}$. Note that $\bar{p} = \|p\|^2 p^{-1}$, where p^{-1} is the multiplicative inverse of p. Thus unit quaternions yield pure rotations without stretching. For details on the Hamilton quaternion division algebra, see [17].

Roughly speaking, the Cosserat rod possesses six degrees of freedom, three translatory and three rotatory ones. These equivalently correspond to the three strain and three curvature components.

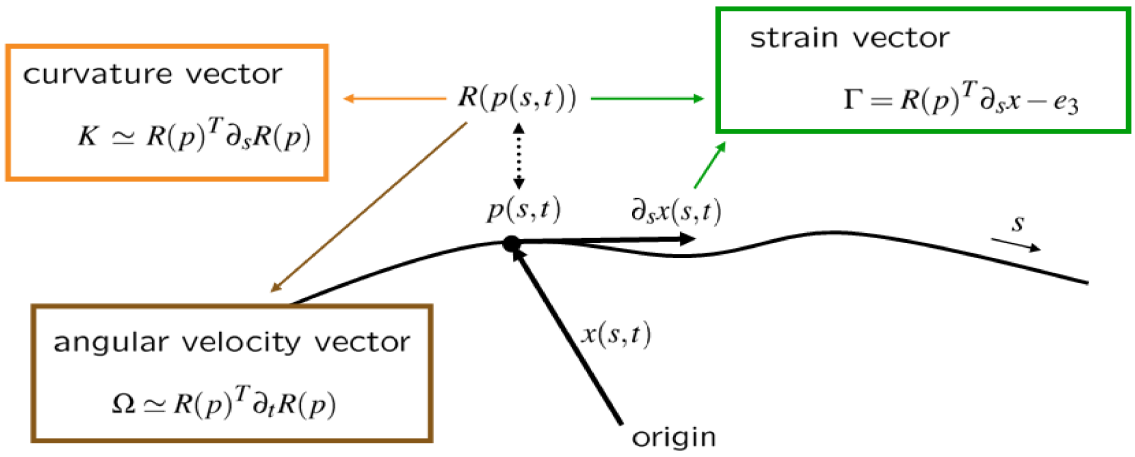

Figure 1: *The continuous Cosserat rod*

Remark 2.1 (Kirchhoff rod models) An *'extensible Kirchhoff rod'* additionally satisfies the shearing constraints $\langle d^1, \partial_s x\rangle = \langle d^2, \partial_s x\rangle = 0$, i. e. the cross sections stay always orthogonal to the centerline tangent. It therefore has only four dof. Extension is still allowed. An *'inextensible Kirchhoff rod'* additionally satisfies $\|\partial_s x\| = 1$, this means that the rod remains parametrised by arc length all the time during deformation. Then the centerline unit tangent $\tau = \partial_s x / \|\partial_s x\|$ is equal to $\partial_s x$, and the number of dof is equal to three. □

2.1 Continuous potential energies and strain measures

The total potential energy $\mathcal{V} = \mathcal{V}_{SE} + \mathcal{V}_{BT}$ is additively decomposed into extensional and shearing energy $\mathcal{V}_{SE}$ and bending and torsion energy $\mathcal{V}_{BT}$. In $\mathcal{V}$, we include only the internal elastic energies, other conservative forces such as the gravitational force can simply be added as additional external forces. on the right hand side of the balance equations. The **potential extensional** and **shearing energy** is given by

$$\mathcal{V}_{SE} = \frac{1}{2}\int_0^L \Gamma^T C^\Gamma \Gamma \mathrm{d}s, \qquad C^\Gamma = \mathrm{diag}\left(GA_1, GA_2, EA\right), \tag{1}$$

where the *material strains* are defined by

$$\Gamma = R(p)^T \partial_s x - e^3. \tag{2}$$

Γ^1 resp. Γ^2 are the strains corresponding to shearing in d^1- resp. d^2-direction, Γ^3 is the strain corresponding to extension in d^3-direction. In components, we have

$$\Gamma^1 = \langle d^1(p), \partial_s x\rangle, \qquad \Gamma^2 = \langle d^2(p), \partial_s x\rangle, \qquad \Gamma^3 = \langle d^3(p), \partial_s x\rangle - 1.$$

$E > 0$ denotes Young's modulus and $G > 0$ the shear modulus of the material, $A = \iint_{\mathcal{A}} \mathrm{d}(\xi_1, \xi_2)$ is the area of the rigid cross section, $A_1 = \kappa_1 A$ and $A_2 = \kappa_2 A$ are some effective cross section areas with some Timoshenko shear correction factors $0 < \kappa_1, \kappa_2 \leq 1$, cf. [11]. The **potential bending** and **torsion energy** is

$$\mathcal{V}_{BT} = \frac{1}{2}\int_0^L K^T C^K K \,\mathrm{d}s, \qquad C^K = \mathrm{diag}\left(EI_1, EI_2, GJ\right), \tag{3}$$

where the *material curvature vector* (or the *'Darboux' vector*) is given by

$$K \simeq \mathcal{E}(K) = R(p)^T \partial_s R(p). \tag{4}$$

K^1 resp. K^2 are the curvatures corresponding to bending around the d^1- resp. d^2-axis, K^3 is the curvature corresponding to torsion around the d^3-axis. $I_1 = \iint_{\mathcal{A}} \xi_2^2 \,\mathrm{d}(\xi_1, \xi_2)$ and $I_2 = \iint_{\mathcal{A}} \xi_1^2 \,\mathrm{d}(\xi_1, \xi_2)$ are the geometric moments of inertia of the rigid cross section. $J = I_3 = \iint_{\mathcal{A}} (\xi_1^2 + \xi_2^2) \,\mathrm{d}(\xi_1, \xi_2) = I_1 + I_2$ denotes its polar moment. If the cross section is symmetric, then we have $I_1 = I_2$ and $J = I_3 = 2I_1 = 2I_2$. The *conservative elastic forces* F^Γ and *moments* M^K are derived from the potential energy as $F^\Gamma = C^\Gamma \Gamma = \frac{1}{2}\partial_\Gamma(\Gamma^T C^\Gamma \Gamma)$ and $M^K = C^K K = \frac{1}{2}\partial_K(K^T C^K K)$. Clearly, any other more general form of hyperelastic constitutive material behaviour could be used instead [45]. If the rod possesses non-vanishing precurvature $K^0 : [0, L] \to \mathbb{R}^3$ in the undeformed configuration, K can simply be replaced by $K - K^0$ in (3) throughout the model [45].

It is well known that the strain measures (2) and (4) are frame indifferent.

2.2 Continuous dissipation energies and strain rates

Dissipation is not explained in [45]. For our dissipation potential, we follow [1, 2, 3] and choose friction forces resp. moments that are proportional to the strain rates resp. curvature rates. The total dissipation energy $\mathcal{D} = \mathcal{D}_{SE} + \mathcal{D}_{BT}$ consists of dissipative extensional and shearing energy $\mathcal{D}_{SE}$ plus dissipative bending and torsion energy $\mathcal{D}_{BT}$. The **dissipative extensional** and **shearing energy** is

$$\mathcal{D}_{SE} = \int_0^L \dot{\Gamma}^T C^{\dot{\Gamma}} \dot{\Gamma} \mathrm{d}s, \qquad C^{\dot{\Gamma}} = \mathrm{diag}\left(c_1^{\dot{\Gamma}}, c_2^{\dot{\Gamma}}, c_3^{\dot{\Gamma}}\right) \tag{5}$$

with the *material strain rates* $\dot{\Gamma} = \partial_t R(p)^T \partial_s x + R(p)^T \partial_{st}^2 x$. The **dissipative bending** and **torsion energy** is given by

$$\mathcal{D}_{BT} = \int_0^L \dot{K}^T C^{\dot{K}} \dot{K} \,\mathrm{d}s, \qquad C^{\dot{K}} = \mathrm{diag}\left(c_1^{\dot{K}}, c_2^{\dot{K}}, c_3^{\dot{K}}\right) \tag{6}$$

with the *material curvature rates* $\dot{K} \simeq \partial_t \mathcal{E}(K) = \partial_t R(p)^T \partial_s R(p) + R(p)^T \partial_{st}^2 R(p)$. The nonnegative constants $c_\nu^{\dot{\Gamma}}$ and $c_\nu^{\dot{K}}$ for $\nu = 1, 2, 3$ denote some viscoelastic material parameters. The *dissipative damping forces* $F^{\dot{\Gamma}}$ and *moments* $M^{\dot{K}}$ are derived from the dissipation potential as $F^{\dot{\Gamma}} = 2C^{\dot{\Gamma}}\dot{\Gamma} = \partial_{\dot{\Gamma}}(\dot{\Gamma}^T C^{\dot{\Gamma}} \dot{\Gamma})$ and $M^{\dot{K}} = 2C^{\dot{K}}\dot{K} = \partial_{\dot{K}}(\dot{K}^T C^{\dot{K}} \dot{K})$. Of course, any other more general form of consistent viscoelastic constitutive material behaviour could replace these assumptions.

2.3 Continuous kinetic energies

The total kinetic energy $\mathcal{T} = \mathcal{T}_T + \mathcal{T}_R$ consists of two parts, the **translatory** $\mathcal{T}_T$ and the **rotatory** $\mathcal{T}_R$ **kinetic energy**, given by

$$T_T = \frac{\varrho A}{2} \int_0^L \|\dot{x}\|^2 \mathrm{d}s, \qquad T_R = \frac{\varrho}{2} \int_0^L \Omega^T I \Omega \,\mathrm{d}s, \qquad I = \mathrm{diag}\left(I_1, I_2, J\right). \tag{7}$$

Here the *material angular velocity vector* (or the *'vorticity' vector*) is

$$\Omega \simeq \mathcal{E}(\Omega) = R(p)^T \partial_t R(p). \tag{8}$$

Here $\varrho > 0$ is the material density, I_1, I_2 and $J = I_3$ are as above, and we identify $\Omega \simeq \mathcal{E}(\Omega)$. We rewrite the rotatory kinetic energy with the identity $\Omega^T I \Omega = \dot{p}^T \mu(p) \dot{p}$ and the p dependent 4×4 quaternion mass matrix [24]

$$\mu(p) = 4\mathcal{Q}(p) I \mathcal{Q}(p)^T, \qquad \mathcal{Q}(p) = \left(\begin{array}{c|c} p_0 & -\hat{p}^T \\ \hline \hat{p} & p_0 \mathcal{I} + \mathcal{E}(\hat{p}) \end{array} \right). \tag{9}$$

The corresponding computations are carried out in detail in [41]. The mass matrix satisfies the symmetry property $\mu(-p) = \mu(p)$, which is a consequence of the fact that both p and $-p$ describe the same rotation $R(p) = R(-p)$. The kernel of $\mu(p)$ is given by $\ker \mu(p) = \mathbb{R}p$, consequently we have $\mathrm{rk}\, \mu(p)$ equal to three. $\mu(p)$ is positively semi-definite with the one singular dimension in direction p.

The reader should note that the situation for K and Ω is completely symmetric from a two dimensional field view. For our discrete model, evolution (4) is the basis for spatial discretisation. Evolution (8) is solved 'continuously' in time, of course.

2.4 Continuous equations of motion

It can be shown from a two dimensional variational principle, that the Cosserat rod, for given exterior material force densities $\hat{F} = \hat{F}(t)$ (per length) and given exterior material moment densities $\hat{M} = \hat{M}(t)$ (per length), satisfies the following nonlinear system of partial differential equations

$$\left\{ \begin{array}{rcl} \varrho A \ddot{x} & = & \partial_s \big(pF\bar{p} \big) + p\hat{F}\bar{p} \\ \varrho \Big[\mu \ddot{p} - \frac{1}{2} \partial_p \big(\dot{p}^T \mu \dot{p} \big) + \partial_p \big(\mu \dot{p} \big) \dot{p} \Big] & = & 2x' pF + \partial_s \big(2pM \big) + 2p' M + 2p\hat{M} - \lambda p \\ 0 & = & \|p\|^2 - 1 \end{array} \right. , \tag{10}$$

where $' = \partial_s$ and $\dot{} = \partial_t$. The viscoelastic forces and moments are given by $F = C^\Gamma \Gamma + 2C^{\dot{\Gamma}} \dot{\Gamma}$ and $M = C^K K + 2C^{\dot{K}} \dot{K}$. Together with appropriate initial and boundary conditions, this is the system that has to be solved for the unknowns x, p and λ.

Originally in [45, 46], averaging the normal Piola-Kirchhoff tractions and corresponding torques over the cross section of the deformed rod, it was shown that the Cosserat rod must satisfy the following form of the balance equations of motion, namely

$$\left\{ \begin{array}{rcl} \varrho A\, \ddot{x} & = & \partial_s f + \hat{f} \\ \varrho \big(i\dot{\omega} + \omega \times i\omega \big) & = & \partial_s m + \partial_s x \times f + \hat{m} \end{array} \right. . \tag{11}$$

This original formulation of the equations of motion, which is probably more familiar to the reader, can as well be found in [1, 2, 3]. Here the spatial quantities $\gamma = R\Gamma$, $k = RK$, $\omega = R\Omega$, $i = RIR^T$, $f = RF$ and $m = RM$ are obtained from the corresponding material ones by a push forward via R. The components of any of these spatial quantities in the moving coordinate system (d^1, d^2, d^3) are identical to the components of the corresponding material quantities, measured in the fixed global coordinate system (e^1, e^2, e^3).

It can be shown that the systems (10) and (11) are equivalent.

3 DISCRETE COSSERAT RODS VIA FINITE DIFFERENCES

Here we present our discrete version of the Cosserat model. Sections 3.1, ..., 3.4 are the discrete counterparts of sections 2.1, ..., 2.4 respectively. In contrast to finite element methods,

we propose the following staggered grid discretisation. We subdivide the arc length interval $[0, L]$ into N segments $[s_{n-1}, s_n]$ with the vertices

$$0 = s_0 < s_1 < \ldots < s_{N-1} < s_N = L.$$

Together with the midpoints $s_{n-1/2} = (s_{n-1} + s_n)/2$, we have the staggered grid

$$0 = s_0 < s_{1/2} < s_1 < \ldots < s_{N-1} < s_{N-1/2} < s_N = L.$$

Now we let the discrete **translatory degrees of freedom** $x_n : [0, T] \to \mathbb{R}^3$, i. e. the cross section centroids, live on the vertices,

$$x_0(\cdot) \approx x(s_0, \cdot), \qquad \ldots, \qquad x_N(\cdot) \approx x(s_N, \cdot),$$

and the discrete **rotatory degrees of freedom** $p_{n-1/2} : [0, T] \to \mathbb{H}$, i. e. the quaternions specifying the frame orientations, on the segment midpoints,

$$p_{1/2}(\cdot) \approx p(s_{1/2}, \cdot), \qquad \ldots, \qquad p_{N-1/2}(\cdot) \approx p(s_{N-1/2}, \cdot).$$

In order that the quaternions remain in $\mathbb{S}^3$, we have to introduce the discrete **Lagrange multipliers** $\lambda_{n-1/2} : [0, T] \to \mathbb{R}$ of dimension 'energy', situated on the midpoints,

$$\lambda_{1/2}(\cdot) \approx \lambda(s_{1/2}, \cdot), \qquad \ldots, \qquad \lambda_{N-1/2}(\cdot) \approx \lambda(s_{N-1/2}, \cdot)$$

and N constraints $2g_{n-1/2}(q) = \|p_{n-1/2}\|^2 - 1$, which are situated here as well. The corresponding frames $R(p_{n-1/2})$ and the directors $d^\nu(p_{n-1/2})$ live as well on the midpoints.

A staggered grid discretisation was already proposed in [16, 28] for the case of an inextensible Kirchhoff rod. We extend this idea to the shear and extensional flexible Cosserat rod with quaternions. The staggered grid approach allows it to interpret the rod as a sequence of almost rigid cylinders, connected with appropriate 'bushings' for bending and torsion. We consequently use the notation $\cdot_n$ for quantities at the vertices s_n; here n ranges from 0 to N. We use the notation $\cdot_{n-1/2}$ for quantities that are situated on the midpoints $s_{n-1/2}$; here n ranges from 1 to N, if not otherwise explicitly stated. The situation is depicted in Figure 2.

In the sequel, we discretise the continuous internal Cosserat energy integrals $\mathcal{V}$, $\mathcal{T}$ and $\mathcal{D}$ by the use of either midpoint or trapezoidal quadrature, depending on where which magnitude is 'at home'. The weight factors for the midpoint rule are the segment lengths $\Delta s_{n-1/2} = s_n - s_{n-1}$. Likewise, the weights for the trapezoidal rule are the lengths of the bucked segments $2\delta s_0 = \Delta s_{1/2}$, $2\delta s_n = \Delta s_{n-1/2} + \Delta s_{n+1/2}$ and $2\delta s_N = \Delta s_{N-1/2}$. Then, with $q = (x, p)$, the discrete potential energy $\mathcal{V}$, the discrete kinetic energy $\mathcal{T}$, the discrete dissipation energy $\mathcal{D}$, the Lagrangian function $\mathcal{L} = \mathcal{T} - \mathcal{V} - g^T\lambda$ and exterior forces $\psi(t)$, the variational principle

$$\delta \int_0^T \mathcal{L}\,\mathrm{d}t - \int_0^T \partial_{\dot q}\mathcal{D}\,\delta q\,\mathrm{d}t + \int_0^T \psi\,\delta q\,\mathrm{d}t = 0$$

yields the Euler-Lagrange equations as the well-known *index three* differential algebraic system of equations

$$\left\{ \begin{array}{rcl} \mathcal{M}(q)\ddot q & = & \mathcal{F}(q, \dot q, t) - \mathcal{G}(q)^T\lambda \\ 0 & = & g(q) \end{array} \right.$$

with the right hand side forces given by $\mathcal{F}(q, \dot q, t) = \psi(t) - \partial_q\mathcal{V} - \partial_{\dot q}\mathcal{D} + \partial_q\mathcal{T} - \partial_q(\mathcal{M}(q)\dot q)\dot q$, cf. [4, 23].

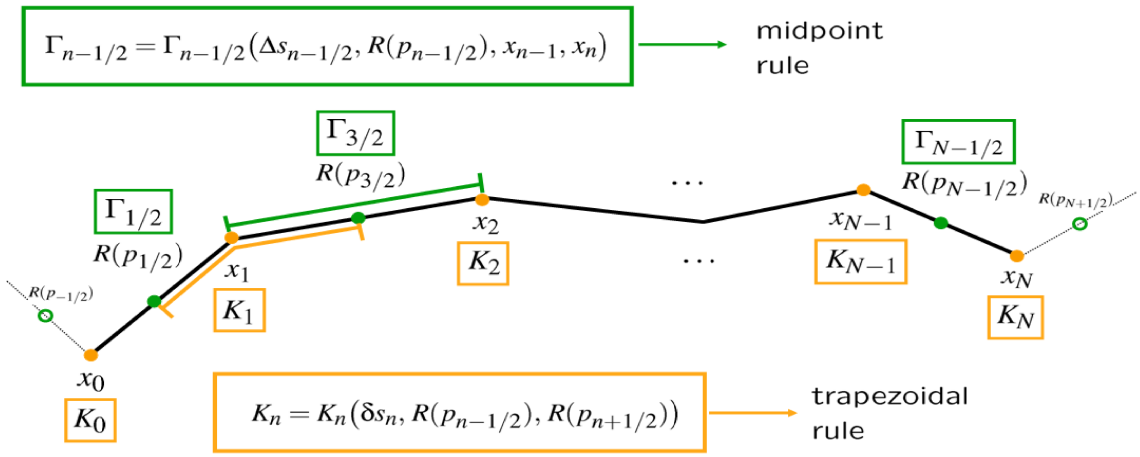

Figure 2: *Our discrete Cosserat rod*

How to solve the dynamical equations? One method is to use eight local charts $p_k = \pm(1 - \sum_{j\neq k} p_j^2)^{1/2}$ for $k = 0, \ldots, 3$ that cover the unit sphere $\mathbb{S}^3 = \partial B_1^{\mathbb{H}}(0) \subset \mathbb{H}$. Numerical approaches using local charts exist [23], however, changing charts is a tedious task and it is much easier to formulate the equations of motion as a system of differential algebraic equations, where we keep the quaternion unity condition $2g = \|p\|^2 - 1 = 0$ as a hard algebraic constraint. It is well known that the numerical solution of the index three system involves difficulties such as poor convergence of Newton's method [4, 18, 21, 23, 44], so we reduce the index to one

$$\begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathcal{M}(q) & \mathcal{G}(q)^T \\ \mathcal{G}(q) & 0 \end{pmatrix}^{-1} \begin{pmatrix} \mathcal{F}(q,\dot{q},t) \\ -g_q''(\dot{q},\dot{q}) \end{pmatrix} \tag{12}$$

and solve the index zero subsystem $\ddot{q} = \ddot{q}(q, \dot{q}, t)$. Stabilisation of the quaternion unity constraints in the index one/zero case is especially easy and cheap.

Remark 3.1 (Boundary conditions) In order to apply clamped boundary rotations properly, i. e. $p_0 = p_0(t)$ and $p_N = p_N(t)$, we introduce virtual *ghost* quaternions $p_{-1/2}$ and $p_{N+1/2}$, which is a standard technique [33]. They are situated beyond the boundary and defined as the spherical linear extrapolation of $p_{1/2}$ via $p_0(t)$ or $p_{N-1/2}$ via $p_N(t)$ respectively. The handling of free boundary conditions with vanishing boundary curvature and moment is clear. □

3.1 Discrete potential energies and strain measures

We start with the discrete version of the **potential extensional** and **shearing energy**. The strains $\Gamma_{n-1/2}$ are situated on the midpoints. Therefore, the integral (1) is approximated with the midpoint rule,

$$\mathcal{V}_{SE} = \frac{1}{2}\sum_{n=1}^{N} \Delta s_{n-1/2} \Gamma_{n-1/2}^T C^\Gamma \Gamma_{n-1/2}, \qquad \Gamma_{n-1/2} = R(p_{n-1/2})^T \frac{\Delta x_{n-1/2}}{\Delta s_{n-1/2}} - e^3, \tag{13}$$

where $\Gamma_{n-1/2}$ denote the *discrete material strains*. They locally depend on x_{n-1}, x_n and $p_{n-1/2}$, see Figure 2. Of course, finite difference schemes of higher order are possible [31].

We continue with the discrete version of the **potential bending** and **torsion energy**. The curvatures K_n are situated on the vertices. Consequently, we approximate (3) with the trapezoidal

rule

$$\mathcal{V}_{BT} = \frac{1}{2}\sum_{n=0}^{N} \delta s_n K_n^T C^K K_n, \qquad K_n = K_n\big(\delta s_n, R(p_{n-1/2}), R(p_{n+1/2})\big), \tag{14}$$

where the *discrete material curvatures* K_n depend on $R(p_{n-1/2})$ and $R(p_{n+1/2})$, see Figure 2. Frame indifference requires that K_n is in fact a function of $R(p_{n-1/2})^T R(p_{n+1/2})$. And again it is clear that higher order finite difference approximations are possible [31].

The question arises, how to interpolate the two quaternions $p_{n-1/2}$ resp. $p_{n+1/2}$, living on the segment midpoints $s_{n-1/2}$ resp. $s_{n+1/2}$, to a quaternion p_n, living on the vertex at s_n, and to choose an appropriate difference expression in order to evaluate $\mathcal{E}(K) = R(p)^T \partial_s R(p)$ at the vertex s_n. So for example, the simple ansatz in [47] chooses linear midpoint interpolation $p_n = (p_{n-1/2} + p_{n+1/2})/2$, the secant $\delta p_n = p_{n+1/2} - p_{n-1/2}$ and $\delta s_n = s_{n+1/2} - s_{n-1/2}$, irregardless of the fact that p_n violates the unity constraint. This results in an expression for K_n that is proportional to $\Im(\bar{p}_{n-1/2} p_{n+1/2})$, which is non-physically soft and suffers from very poor stability properties for large bending or torsion angles. Phenomena such as 'quaternion flipping' are the consequence.

The details of more sophisticated approaches go beyond the scope of the present paper. For thorough discussions, how to interpolate the rotations, we refer to [38].

3.2 Discrete dissipation energies and strain rates

The discretisation of the dissipation potential has to be consistent with the discretisation of the potential energies. We start with the **dissipative extensional** and **shearing energy**. The strain rates $\dot{\Gamma}_{n-1/2}$ are situated at the midpoints. Thus, the continuous integral (5) is approximated with the midpoint rule,

$$\mathcal{D}_{SE} = \sum_{n=1}^{N} \Delta s_{n-1/2} \dot{\Gamma}_{n-1/2}^T C^{\dot{\Gamma}}\, \dot{\Gamma}_{n-1/2}, \qquad \dot{\Gamma}_{n-1/2} = \frac{\partial}{\partial t}\Gamma_{n-1/2},$$

where the *discrete material strain rates* are given by the time derivative of (13). They depend both on the positions x_{n-1}, x_n, $p_{n-1/2}$ and the velocities $\dot{x}_{n-1}$, $\dot{x}_n$, $\dot{p}_{n-1/2}$. Concerning the discrete **dissipative bending** and **torsion energy**, the curvature rates $\dot{K}_n$ are situated – like the curvatures themselves – on the vertices. Thus, (6) is approximated with a trapezoidal sum,

$$\mathcal{D}_{BT} = \sum_{n=0}^{N} \delta s_n \dot{K}_n^T C^{\dot{K}} \dot{K}_n, \qquad \dot{K}_n = \frac{\partial}{\partial t} K_n,$$

where the *discrete material curvature rates* are the time derivative of (14), depending on $p_{n-1/2}$, $p_{n+1/2}$ and $\dot{p}_{n-1/2}$, $\dot{p}_{n+1/2}$.

3.3 Discrete kinetic energies

As the centroids x_n are situated at the vertices s_n, we discretise the **translatory kinetic energy** integral in (7) by the trapezoidal rule and, as the quaternions are situated on the midpoints $s_{n-1/2}$, the **rotatory kinetic energy** integral (7) is approximated with the midpoint rule

$$\mathcal{T}_T = \frac{\varrho A}{2}\sum_{n=0}^{N} \delta s_n \|\dot{x}_n\|^2, \qquad \mathcal{T}_R = \frac{\varrho}{2}\sum_{n=1}^{N} \Delta s_{n-1/2} \dot{p}_{n-1/2}^T \mu(p_{n-1/2}) \dot{p}_{n-1/2}. \tag{15}$$

Thus, the translatory mass is situated at the vertices and the rotatory quaternion mass belongs to the segments. The *discrete material angular velocities* $\mathcal{E}(\Omega_{n-1/2}) = R(p_{n-1/2})^T \partial_t R(p_{n-1/2})$ as well belong to the midpoints. The mass matrix $\mathcal{M}(q)$ of the system is block diagonal with alternating 3×3 (translatory, diagonal and constant) and 4×4 (rotatory, position-dependent) blocks.

Each summand in (15) can be interpreted as the rotatory energy of a rigid body with moments of inertia equal to $\bar{I}^1 = \varrho \Delta s I_1$, $\bar{I}^2 = \varrho \Delta s I_2$ and $\bar{I}^3 = \varrho \Delta s I_3$, see [36, 41]. These are in fact the physical moments of inertia of a disc with vanishing thickness. Now we consider the rod as decomposed into N – almost rigid – cylinders with moments of inertia, which we denote by I^1, I^2 and I^3. Comparison shows that $I^\nu - \bar{I}^\nu = \mathcal{O}(\Delta s^3)$ for $\nu = 1, 2$, and $I^3 - \bar{I}^3 = 0$. For fine discretisations, these defects may be neglected. Otherwise, a more detailed analysis might be necessary.

For each centroid x_n with mass $\varrho A \delta s_n$, the translatory mass-matrix block is given by a 3×3 diagonal, constant, state independent block $\varrho A \delta s_n \mathcal{I}$. For the rotatory part, we fix a segment $\Delta s = \Delta s_{n-1/2}$ and its quaternion $p = p_{n-1/2}$. The constraints of position, velocity and acceleration are written $2g = \|p\|^2 - 1 = 0$, $\dot{g} = G(p)\dot{p} = \langle p, \dot{p} \rangle = 0$ and $\ddot{g} = G(p)\ddot{p} + g_p''(\dot{p}, \dot{p}) = \langle p, \ddot{p} \rangle + \|\dot{p}\|^2 = 0$ respectively, where $G(p) = \partial_p g(p) = p^T$. Thus, the rotatory quaternion mass-constraint-matrix 5×5 block is given by

$$\left(\begin{array}{c|c} \varrho \Delta s \mu(p) & G(p)^T \\ \hline G(p) & 0 \end{array}\right) = \left(\begin{array}{c|c} \varrho \Delta s \mu(p) & p \\ \hline p^T & 0 \end{array}\right) \tag{16}$$

with the singular quaternion mass $\mu(p)$ from (9). The inverse of (16) exists iff $p \neq 0$ and can be explicitly algebraically computed. It has exactly the same structure as (16) and can therefore be calculated at exactly the same numerical cost.

The question arises, why to use $p_{n-1/2}$ and $\dot{p}_{n-1/2}$ as the primary unknowns and not $p_{n-1/2}$ and $\Omega_{n-1/2}$, as it is frequently recommended [41, 47]. The reason is that many standard ODE or DAE solvers such as RADAU5, SEULEX or RODAS [21, 23, 34] support sparse / banded linear algebra that is specially adapted to second order systems of the form $\dot{q} = v$, $\dot{v} = \dot{v}(q, v, t)$.

Remark 3.2 (Condition) It can be shown that the condition number of the mass-constraint matrix (16) is equal to the ratio $\max M / \min M$, where $M = \{\varrho \Delta s I_1, \varrho \Delta s I_2, \varrho \Delta s I_3, 1/4\}$. So, for typical material parameters and discretisations, the system is rather ill-conditioned.

> *'In the case of ill-conditioned linear systems, scaling of the constraints ... should be tried.'* [44].

By scaling the constraint equation by a factor of $c > 0$, the condition number of any quaternion mass-constraint block (16) can be influenced. If g is replaced by cg, G has to be replaced by cG and g_p'' by cg_p''. For the special case of symmetric cross sections for example, where $I_1 = I_2 = I$, $J = I_1 + I_2 = 2I$, any choice $c \in 4\varrho \Delta s[I, J]$ leads to a condition number of two. Note that the Lagrange multiplier λ scales with $1/c$, so that the constraint force $G^T \lambda$, which is normal to the unit sphere in accordance to d'Alembert's principle and keeps the quaternions on its $\mathbb{S}^3$ orbit, remains unchanged. □

For the index zero/one formulation (12), the position q drifts quadratically from the constraint manifold $\{q : g(q) = 0\}$. Subsequent projection $p \mapsto p/\|p\|$ of the quaternion position and $\dot{p} \mapsto \dot{p} - \langle p, \dot{p} \rangle p$ of the quaternion velocity is especially cheap, it may be applied even after each successful integration step. However, easy and efficient implementations of this method are

restricted to one step integration methods, excluding BDF methods with order of at least two. Another stabilisation technique – already on the model level – is the Baumgarte method [23]. Imposing the linear combinations $\ddot{g} + 2r\dot{g} + \omega^2 g = 0$ as constraints with $\omega, r > 0$, the index one Baumgarte formulation is obtained. The equation for the Lagrange multiplier λ remains unchanged. The additional penalty accelerations are needed to pull p back to $\mathbb{S}^3$, if the velocity constraint $\langle p, \dot{p} \rangle \neq 0$ or the position constraint $\|p\| \neq 1$ is violated. The choice $r = \omega$ leads to critical damping of the constraint defect g. One might argue that the Baumgarte method is energy dissipating. This is true, but the projection method is as well, see [23, 24]. Another frequent objection is that the Baumgarte method introduces additional stiffness into the system. This is true, but the Cosserat shearing and extensional 'springs' are already stiff, so that this does not mean a drawback from the practical viewpoint.

3.4 Discrete equations of motion

Carrying out the details of the preceding sections, adding exterior material force densities $\hat{F}_\cdot(t)$ and exterior material moment densities $\hat{M}_\cdot(t)$ and plugging all the pieces together, we obtain the discrete translatory and rotatory balance equations in *index one* formulation. Letting $n = 0, \ldots, N$ and $\nu = 1/2, \ldots, N - 1/2$, the topological structure of the system is seen to be

$$\left\{ \begin{array}{rcl} \ddot{x}_n & = & \ddot{x}_n(x_{n-1}, p_{n-\frac{1}{2}}, x_n, p_{n+\frac{1}{2}}, x_{n+1};\ \dot{x}_{n-1}, \dot{p}_{n-\frac{1}{2}}, \dot{x}_n, \dot{p}_{n+\frac{1}{2}}, \dot{x}_{n+1};\ t) \\ \ddot{p}_\nu & = & \ddot{p}_\nu(p_{\nu-1}, x_{\nu-\frac{1}{2}}, p_\nu, x_{\nu+\frac{1}{2}}, p_{\nu+1};\ \dot{p}_{\nu-1}, \dot{x}_{\nu-\frac{1}{2}}, \dot{p}_\nu, \dot{x}_{\nu+\frac{1}{2}}, \dot{p}_{\nu+1};\ t) \\ \lambda_\nu & = & \lambda_\nu(p_{\nu-1}, x_{\nu-\frac{1}{2}}, p_\nu, x_{\nu+\frac{1}{2}}, p_{\nu+1};\ \dot{p}_{\nu-1}, \dot{x}_{\nu-\frac{1}{2}}, \dot{p}_\nu, \dot{x}_{\nu+\frac{1}{2}}, \dot{p}_{\nu+1};\ t) \end{array} \right. .$$

This results in upper and lower bandwidths of the Jacobians $\partial\ddot{q}/\partial q$ and $\partial\ddot{q}/\partial\dot{q}$ equal to ten. The corresponding index zero subsystem $\dot{u} = f(u, t)$, $u = (q, v)$, $q = (x, p)$, $\dot{q} = v$, which is obtained by discarding the equation for the multipliers, can be solved by any ODE integrator.

It can be shown that our approach yields consistent discretisations of (10).

The reader should observe that, intrinsically in the model, there are many common terms and (skew-)symmetries. This is one reason, why the model can be implemented with very few arithmetic operations. So, the model needs about $174N$ additions, $111N$ subtractions, $289N$ multiplications and $3N$ divisions, where N is the number of segments. All in all, the evaluation cost for f and $\partial f/\partial u$ is thus extremely small.

Remark 3.3 (Kirchhoff rod models) Usually in robotics applications for example, one is interested solely in the proper modeling of bending and torsion. The stiff extensional and shearing springs only deteriorate the computational performance.

'If stiff components are not really necessary, avoid them by model reduction.' [44]

The (in)extensible Kirchhoff rods can be incorporated easily by the introduction of additional constraints $g^\nu_{n-1/2} = \Gamma^\nu_{n-1/2}$, $\nu = 1, 2(, 3)$. For the boundary conditions, the handling of t-dependent constraints is explained in [4]. The advantage is, that for coarse discretisations, a significant increase in step size can be achieved. But the approach has some disadvantages. The first disadvantage is that an easy ODE formulation $\dot{u} = f(t, u)$ is problematic, since the inverse of the mass-constraint matrix is full, so that an $\mathcal{O}(N)$ multibody formalism [18, 40] has to be applied. The second is that stabilisation of the constraints is not trivial and numerically more expensive as it is for our discrete Cosserat model. The third concerns the inextensible Kirchhoff rod model, which allows the largest step sizes: The mass-constraint matrix becomes singular,

for example when the rod is straight. (This is often the initial configuration!) Therefore, additional inconvenient treatments are necessary. To sum up, in the treatment of the stiff extensional and shearing springs, we rather follow another advise,

'Dissipation in the elastic model alleviates the numerical difficulties ... .' [44].

Our tests yield that the full Cosserat rod model with consistent strong damping on tension and shearing performs better than the (in)extensible Kirchhoff one, if the constraints are introduced as above. However, a pure bending and torsion model in quaternion minimal coordinates will be the topic of further research. □

4 NUMERICAL EXAMPLES

In this section, we compare the solutions of our model to finite element solutions, computed with ABAQUS both 1D beam and 3D continuum elements. We further demonstrate the performance of our model for two simple scenarios with strong damping on the extension and shearing dof.

Test 1 vs. ABAQUS (dynamic) In a first test, we compare a $L = 1\text{m}$ long dynamically swinging pendulum rubber rod, subdivided into $N = 10$ segments, circular cross section with radius $r = 5.0\text{E}^{-3}\text{m}$ and end centroid $x_0(t) \equiv x_0$ fixed under gravity load with $g = 9.81\text{m}\,s^{-2}$. The scenario is depicted in Figure 3. The rubber material parameters are chosen as $E = 5.0\text{E}^{+6}\text{N}\,\text{m}^{-2}$, $\nu = 0.5$ and $\varrho = 1.10\text{E}^{+3}\text{kg}\,\text{m}^{-3}$. All the examples in this section have been performed with shear correction factors equal to $\kappa_1 = \kappa_2 = 1$.

Figure 4 shows excellent agreement compared to the corresponding ABAQUS 1D solution, computed with B31 Timoshenko shear flexible beam elements. It is seen that not only the positions/displacements, but as well the forces and moments are reflected accurately. The same applies to the frame positions, the (angular) velocities and the (angular) accelerations, which are not plotted here. Small model differences occur, since in ABAQUS shearing and extension are decoupled, which is not true for the Cosserat model.

Test 2 vs. ABAQUS (quasistatic) In a second test, we compare the quality of our discretisation scheme with the full 3D ABAQUS solution. To that end, a 3D continuum FE model of the rubber beam of Test 1 has been set up in ABAQUS. It is discretised with 160 (in longitudinal direction) $\times$ 12 (in the cross section) continuum elements of type C3D8. (In total, these are 1920 elements.) We consider the scenario that is plotted in Figure 5. The scenario includes nontrivial coupling of bending and torsion. The fully clamped boundary conditions $(x_0(t), p_0(t))$ and $(x_N(t), p_N(t))$ are chosen in the way that the rod traverses the shape of the Greek letter γ (front view) or the Greek letter Ω (side view). Figure 6 shows excellent agreement of our model results. We emphasise that for our discrete Cosserat rod model, we took only $N = 10$ segments.

Performance Now we introduce some damping into the model, slight damping for bending and torsion, $c_1^{\dot{K}} = c_2^{\dot{K}} = 2.0\text{E}^{-4}\text{kg}\,\text{m}^3$, $c_3^{\dot{K}} = 8.0\text{E}^{-6}\text{kg}\,\text{m}^3$, and strong damping for shearing and extension $c_1^{\dot{\Gamma}} = c_2^{\dot{\Gamma}} = 1.0\text{E}^{-1}\text{kg}\,\text{m}$, $c_3^{\dot{\Gamma}} = 2.0\text{E}^{+2}\text{kg}\,\text{m}$. As we apply the multistep solver DASPK, we prefer the Baumgarte stabilisation technique to the projection technique. For the Baumgarte parameters, we choose $\omega = r = 1.0\text{E}^2\text{s}^{-1}$, thus the constraint defect is critically damped without introducing significant additional stiffness into the model.

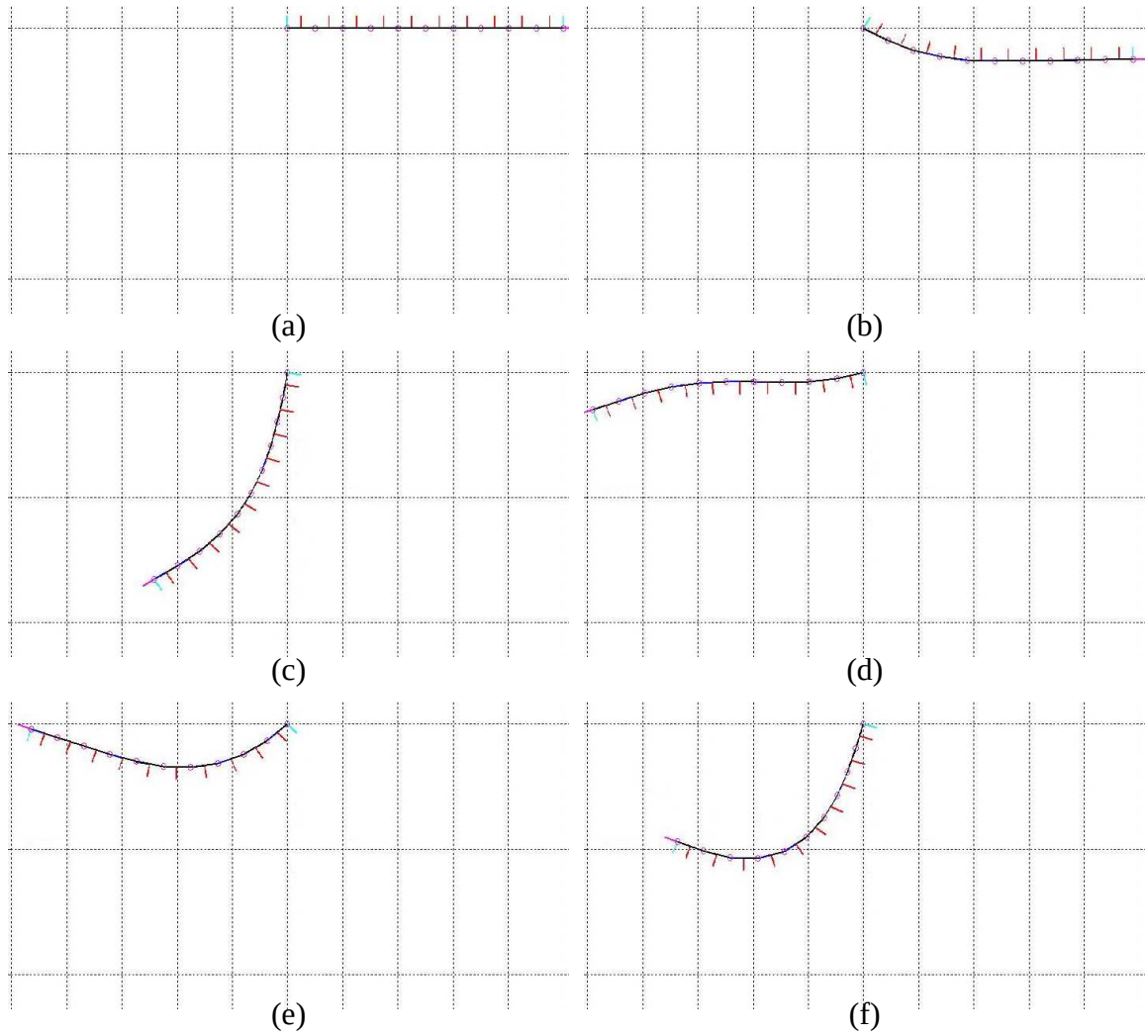

Figure 3: *Snapshots of the dynamic Test 1.*

For a *rubber example*, we choose the same swinging pendulum with fixed end centroid $x_0(t) \equiv x_0$ as in the comparative Test 1 vs. ABAQUS above. For a *steel string example*, we choose a string of length $L = 1.0\text{m}$ and radius $1.0\text{E}^{-3}\text{m}$ without precurvature, clamp in fully at $s = 0$, i. e. $x_0(t) \equiv x_0$ and $p_0(t) \equiv p_0$, and subject it to gravity load. The material parameters for steel are set to $E = 2.1\text{E}^{+6}\text{N}\,\text{m}^{-2}$, $\nu = 0.2$, $\varrho = 7.85\text{E}^{+3}\text{kg}\,\text{m}^{-3}$. In both examples, the number of unknowns in q is equal to 70. Figure 7 shows the computational times for the solvers RADAU5 (an implicit Runge-Kutta method), SEULEX (an extrapolation method), RODAS (a Rosenbrock method), DOPRI5 (an explicit Runge-Kutta method) from [21, 22, 23, 34] and DASPK (= DASSL, a multistep BDF method) from [35] at several tolerances. For all the computations we choose TOL = ABSTOL = RELTOL, discarding the error control for the Lagrange multipliers. Clearly, the problem is stiff even for rubber material because of the presence of the stiff extensional and shearing springs. Thus, DOPRI5 fails, the corresponding step sizes indicate that it runs at the stability limit. In contrast to that, the four stiff solvers reveal satisfactory step size behaviour. For RADAU5, we discovered no significant difference between the classical and the Gustafsson step size strategy. For the solvers RADAU5, SEULEX and RODAS, we chose sparse linear algebra, adapted to second order ODEs, with upper resp. lower bandwidths MU-

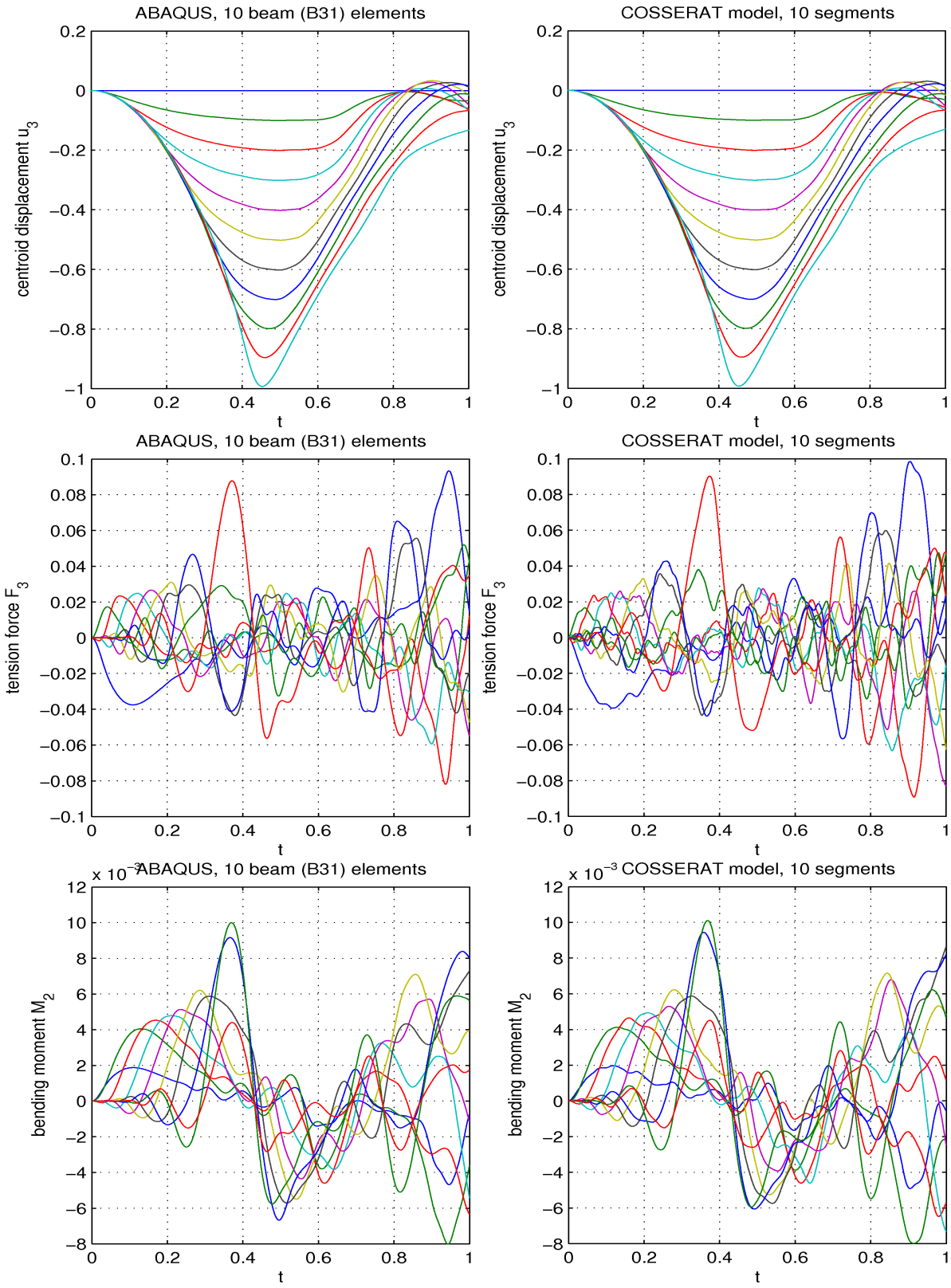

Figure 4: *Dynamical comparison with* ABAQUS *1D finite element solution (Test 1)*

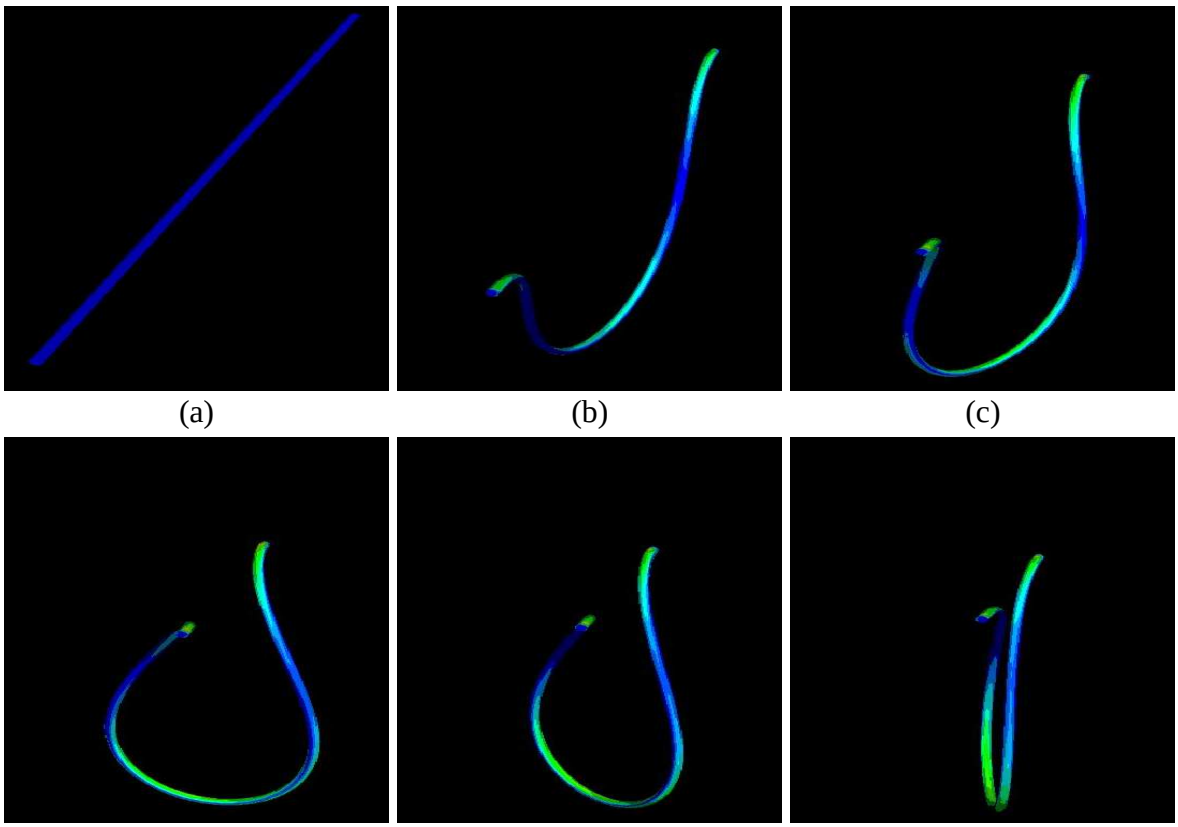

Figure 5: *Snapshots of the quasistatic Test 2. (Colored is the Mises stress.)*

JAC resp. MLJAC of the Jacobians $\partial f/\partial q$ and $\partial f/\partial v$ equal to ten. In both examples, we used a very robust discretisation of the curvature terms, motivated by the work [8]. RADAU5 spent 38% of the total computational time in order to evaluate f and $\partial f/\partial u$, SEULEX 43%, RODAS 61% and DOPRI5 90%; the remaining percentage is needed for (non)linear algebra. Roughly, an evaluation of f needs about 1.06E^{-5}s, which is comparable to [47], but with a much more robust curvature model, an evaluation of $\partial f/\partial u$ needs about 1.02E^{-4}s, this is about ten times larger. Clearly, for coarse discretisations and rough error control during time integration, RODAS performs best. Here, for the rubber pendulum example, the factor to the real physical time is 47, for the steel string example 17. For more stringent tolerances, RADAU5 performs best, since it is a high order method.

The computations have been performed on a 2.19 GHz Dual Core AMD (Opteron) machine.

5 CONCLUSIONS

We presented an alternative discretisation approach for Cosserat rods that allows *both* very fast computations within milliseconds *and* accurate results compared to detailed finite element solutions. Our ansatz was based on quaternion parametrisation of rotations and a finite difference scheme on a staggered grid. The semi-discrete equations of motion were obtained as the Euler-Lagrange equations from a discrete variational principle. They yielded a consistent semi-discretisation to the continuous dynamical Cosserat partial differential equations of motion. For their numerical solution, we applied standard ODE / DAE solvers.

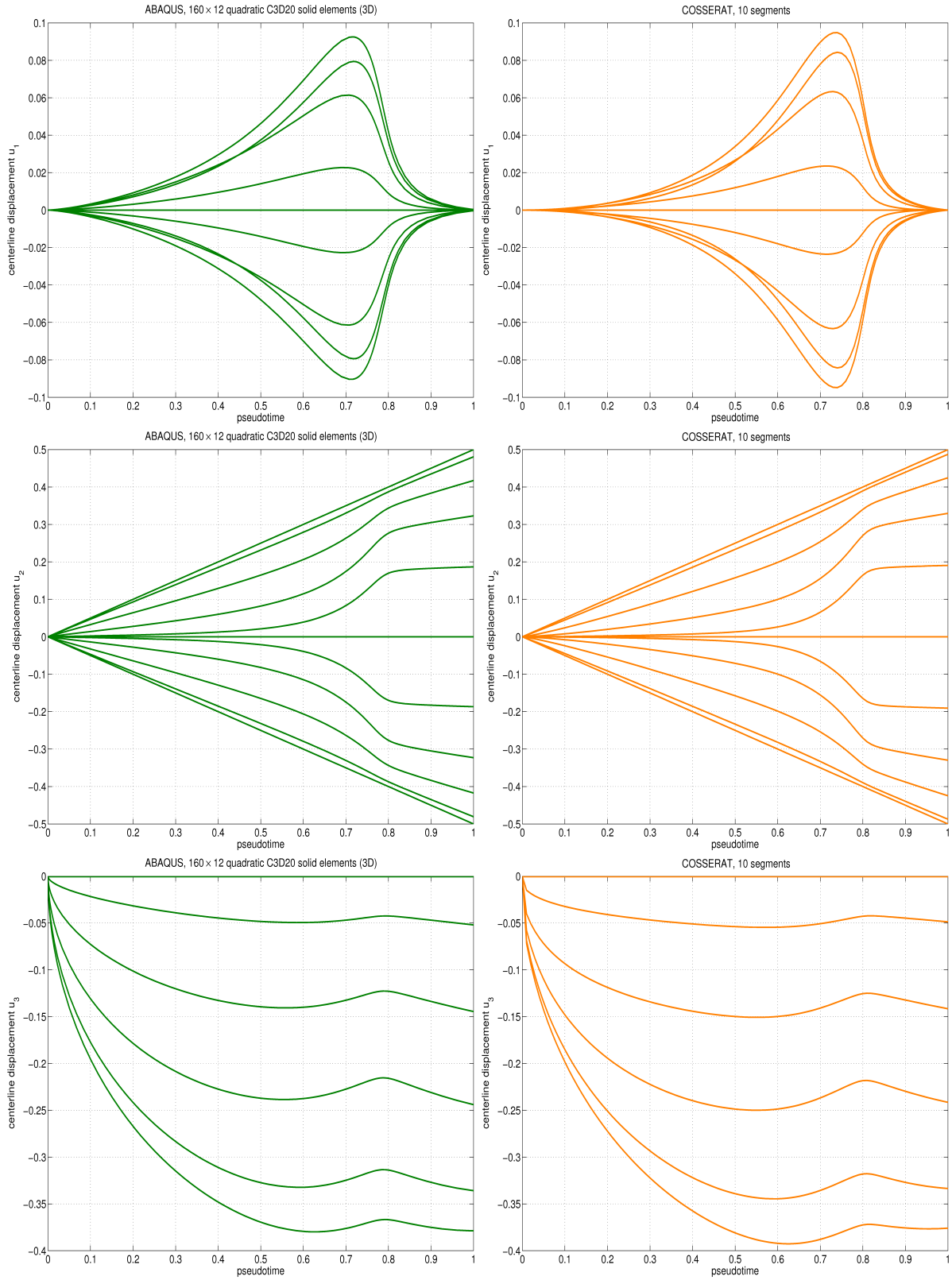

Figure 6: *Quasistatic comparison with* ABAQUS *3D finite element solution (Test 2)*

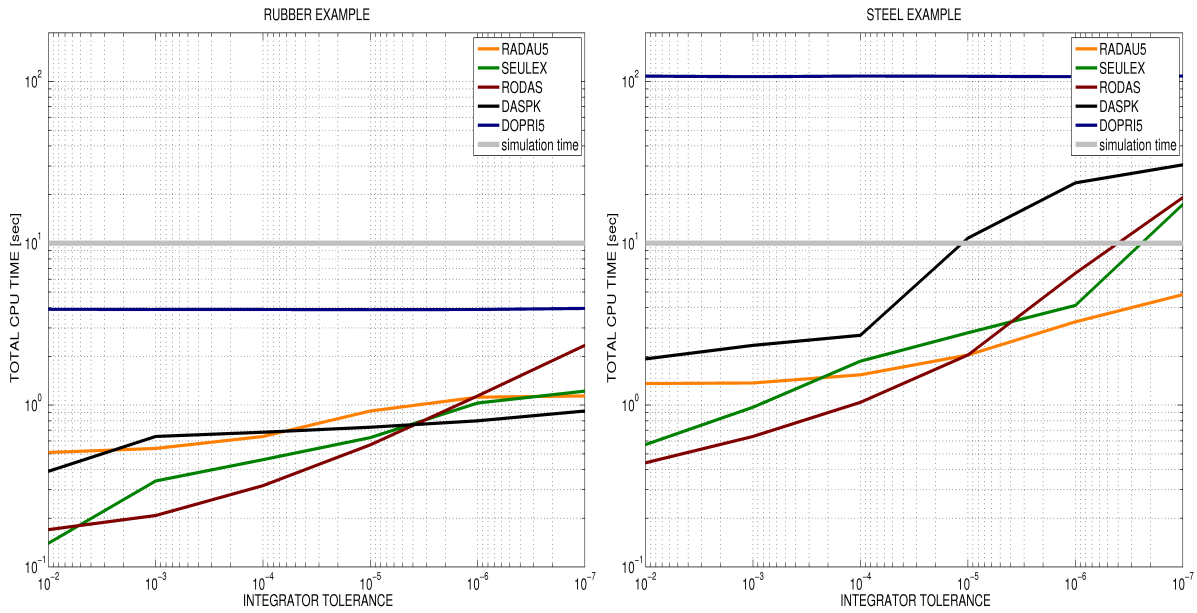

Figure 7: *Computational times for different solvers.*

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