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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

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MODEL REDUCTION OF NONLINEAR PROBLEMS IN STRUCTURAL MECHANICS

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Abstract. *This contribution presents a model reduction method for nonlinear problems in structural mechanics. Emanating from a Finite Element model of the structure, a subspace and a lookup table are generated which do not require a linearisation of the equations.*

The method is applied to a model created with commercial FEM software. In this case, the terms describing geometrical and material nonlinearities are not explicitly known.

1 INTRODUCTION

In the context of full vehicle simulation, numerical effort is an important issue. Due to the long simulation times when computing durability or comfort loading, the number of degrees of freedom needs to be kept small.

In this field, multibody simulation methods are extensively used and well established. By modelling most parts as rigid bodies, the behaviour of a vehicle can be described with very few degrees of freedom.
However, some applications require the inclusion of flexible components. In industrial applications, these are discretised by a Finite Element approach and treated by linear modal model reduction techniques like the Craig-Bampton method ([2]). In this procedure, the deformation of the flexible body is projected onto a subspace of relevant eigenmodes, which results in a reduced system of equations in the variable of modal participation factors.

These classical modal methods can only be used for linear systems. This assumption fails when large deformations or nonlinear materials are involved, which frequently occurs for rubber materials like bushings or tyres ([3]). The following sections present an approach to reduce Finite Element models with nonlinearities which consists of two parts: a projection subspace for reduction of the dimension and a lookup strategy for the nonlinear equations.

2 THE METHOD OF PROPER ORTHOGONAL DECOMPOSITION

One possibility to construct a subspace for the projection of nonlinear systems is the method of *Proper Orthogonal Decomposition (POD)*. Its flexibility in application is based on analysing a given data set to provide the reduced model.

2.1 Construction of a Subspace for Data Reduction

The method of POD can be regarded as an approach to approximate a given data set with a low dimensional subspace.

Let V be a Hilbert space with $dimV = N$ and let $\mathcal{Y} = \{y_1, \ldots, y_m\}$ be a data set $\subset V$ with $rank(\mathcal{Y}) = d$. Furthermore, let $V^l = span\{\varphi_1 \ldots \varphi_l\}$ be an l-dimensional subspace, $l \leq d, N$, with orthonormal basis $\{\varphi_i\}_{i=1,\ldots,l}$. Then the projection error of the data set onto the subspace is given by

$$PE(\mathcal{Y}, V^l) = \frac{1}{m} \sum_{k=1}^{m} \left\| y_k - \sum_{j=1}^{l} \langle y_k, \varphi_j \rangle_X \cdot \varphi_j \right\|_X^2 \tag{1}$$

where $\langle \cdot, \cdot \rangle_X$ denotes a scalar product in V.
In the following, the projection of y_k onto the subspace V^l is denoted by

$$P^l(y_k) = \sum_{j=1}^{l} \langle y_k, \varphi_j \rangle_X \cdot \varphi_j \tag{2}$$

The crucial idea behind the POD method is the construction of the subspace based on data.
In brief, we search for the basis $\{\varphi_i\}_{i=1,\ldots,l}$ of given dimension l which minimises the projection error (1). Then, the resulting subspace V^l can be seen as the best approximation to the set $\mathcal{Y}$ in a least squares sense.

Mathematically, this yields the following constrained optimisation problem:

$$\begin{aligned} \min J(\varphi_1,\dots,\varphi_l) & \qquad \text{over} \quad \varphi_i \in V && (3)\\ \text{with} \quad J(\varphi_1,\dots,\varphi_l) &= PE(\mathcal{Y}, span\{\varphi_1,\dots,\varphi_l\}) && (4)\\ \text{subject to} \quad \langle \varphi_i, \varphi_j \rangle &= \delta_{ij} && (5) \end{aligned}$$

Setting up the *Lagrange* functional and the respective *Karush-Kuhn-Tucker* equations for the system, we get ([5]):

$$\begin{aligned} L(\varphi_1,\dots,\varphi_l,\lambda_{11},\dots,\lambda_{ll}) &= J(\varphi_1,\dots,\varphi_l) + \sum_{i,j=1}^{l} \lambda_{ij} \left(\langle \varphi_i, \varphi_j \rangle - \delta_{ij} \right) \\ \frac{\partial L}{\partial \varphi_i} = 0 \quad &\Longleftrightarrow \quad \sum_{j=1}^{m} y_j \langle y_j, \varphi_i \rangle = \lambda_{ii} \varphi_i \\ & \qquad \text{and} \quad \lambda_{ij} = 0 \quad \text{for} \quad i \neq j \\ \frac{\partial L}{\partial \lambda_{ij}} = 0 \quad &\Longleftrightarrow \quad \langle \varphi_i, \varphi_j \rangle = \delta_{ij} \end{aligned}$$

Setting $\lambda_i = \lambda_{ii}$, we identify the vectors $\varphi_1,\dots,\varphi_l$ as the solution of the *eigenvalue problem*:

$$\sum_{j=1}^{m} y_j \langle y_j, \varphi_i \rangle = \lambda_i \varphi_i \quad \text{for} \quad i = 1,\dots,l. \tag{6}$$

This problem possesses $d = rank(\mathcal{Y})$ solutions $\lambda_1,\dots,\lambda_d$.

Combining equations (6) and (1), we find ([5]):

$$PE(\mathcal{Y}, \{\varphi_1,\dots,\varphi_l\}) = \frac{1}{m} \sum_{k=1}^{m} \left\| y_k - \sum_{j=1}^{l} \langle y_k, \varphi_j \rangle_X \cdot \varphi_j \right\|_X^2 = \sum_{i=l+1}^{d} \lambda_i \tag{7}$$

where $d < m$ denotes the dimension of the set $\mathcal{Y}$, i.e., $d = rank(\mathcal{Y})$.

Remark 1 *Equation* (7) *states that the projection error of the set* $\mathcal{Y}$ *onto the subspace* $V^l = span\{\varphi_1 \dots \varphi_l\}$ *can be expressed by the sum of eigenvalues* λ_i *corresponding to the eigenvectors* $\varphi_i, i = l+1,\dots,d$*, that are not included in the basis of* V^l.

Including the eigenvectors φ *of the* l *largest eigenvalues into the basis, we obtain the subspace* V^l *with the smallest projection error of all possible* l*-dimensional subspaces in* V *for the set* $\mathcal{Y}$ *in a least squares sense. This choice of basis vectors is called the* Karhunen-Loève *basis ([4]).*

2.2 The Snapshot POD Method for Model Reduction

If subspaces created by the POD method serve as the foundation for a Galerkin projection, the system can be used for the reduction of large models. In the following the procedure for the *snapshot POD method* ([4]) is described.

Let $y(t) \in V$ be defined as the solution of a dynamical system

$$\frac{\partial y(t)}{\partial t} = f\left(y(t), t\right), \quad t \in [0, T] \tag{8}$$

and $y_i = y(t_i)$ be snapshots of the solution at time instances $t_i, i = 1, \ldots, m$, $t_i \in [0, T]$. If the precision is sufficient for the corresponding application, these snapshots can also be taken from measurements or from computations with many degrees of freedom (e.g. large FEM models).

Defining the *snapshot matrix*

$$Y = [y_1, \ldots, y_m] \in V^m$$

the eigenvalue problem (6) to solve for the POD basis vectors $\varphi_1, \ldots, \varphi_l \in V$ can be written as

$$YY^*\varphi_i = \lambda_i \varphi_i \quad \text{for} \quad i = 1, \ldots, l \tag{9}$$

where Y^* denotes the transpose of Y.

Clearly, we get the same results from

$$Y^*Yv_i = \lambda_i v_i \quad \text{for} \quad i = 1, \ldots, l \quad \text{with} \quad \varphi_i = Y^* v_i. \tag{10}$$

Depending on the dimension of Y, we solve (9) if $N << m$ or (10) if $m << N$, where $N = dimV$.

The matrix $C = Y^*Y$ with $C_{ij} = \langle y_i, y_j \rangle$ is referred to as the *correlation matrix* of the snapshot set.

Using (10), each eigenvector v_k of the correlation matrix defines a basis vector φ_k of the POD subspace. Depending on the number of basis vectors used for the subspace $V^l = span\{\varphi_1, \ldots, \varphi_l\}$, the projection error for

$$P^l y := \sum_{j=1}^{l} \langle y, \varphi_j \rangle_X \cdot \varphi_j \tag{11}$$

is defined by:

$$\frac{1}{m} \sum_{k=1}^{m} \left\| y_k - \sum_{j=1}^{l} \langle y_k, \varphi_j \rangle_X \cdot \varphi_j \right\|_X^2 = \sum_{j=l+1}^{d} \lambda_j \tag{12}$$

where $d < n$ shall denote the dimension of the snapshot set Y and $l < d$ the number of POD basis vectors used for the projection.

Applying the subspace projection to (8), we get the reduced surrogate model

$$\frac{\partial \alpha(t)}{\partial t} = \Phi f\left(\Phi^* \alpha(t), t\right), \quad t \in [0, T]. \tag{13}$$

where Φ denotes the POD projection matrix with columns $\varphi_k, k = 1 \ldots l$.

For linear second order equations, we get

$$\underbrace{\Phi^T M \Phi}_{=:\widetilde{M}} \ddot{p} + \underbrace{\Phi^T C \Phi}_{=:\widetilde{C}} \dot{p} + \underbrace{\Phi^T K \Phi}_{=:\widetilde{K}} p = \Phi^T \beta \tag{14}$$

Note that the matrices $\widetilde{M}, \widetilde{C}, \widetilde{K}$ are, in general, fully populated. Thus, the POD method transforms a large sparse system into a small dense system.

Remark 2 *The POD-reduced system is not an approximation of the original system itself, but of the system and its external excitation. Due to the subspace construction which is based on the snapshot data set, the reduction scheme depends on the previously computed setup. In the strict sense, only the computed solutions can be properly represented by the reduced system.*

In addition to the overall computation setup, the position of snapshots within the time-span is an important issue. Especially at time instances when the dynamics of the system are changing rapidly, the sampling rate for snapshots shall be increased.

States and phenomena not represented by the snapshot set can not be represented by the reduced system as well.

The procedure of snapshot computation depends on the model's later purpose of use and therefore requires thorough considerations.

3 LOOKUP TABLE APPROACH

Like all model reduction methods that are based on subspace projection, POD reduces the effort required to solve the linearised system of equations in each iteration step. For linear models, the reduced model is set up in the first step and can be used unchanged throughout the whole computation.

In nonlinear problems, the model equations depend on the current state u:

$$M\ddot{u} + C\dot{u} + R(u) = f_{ext} \tag{15}$$

$$P^T \cdot M \cdot (\ddot{P\alpha}) + P^T \cdot C \cdot (\dot{P\alpha}) + P^T \cdot R(P\alpha) = P^T \cdot f_{ext} \tag{16}$$

The effort to set up the nonlinear term $R(P\alpha)$ requires the transformation of the current state α to the full dimensional variable $u = P\alpha$. In general industrial problems - except in rare cases - the nonlinearity is not explicitly known or can not be computed in the reduced variable. Obviously, model reduction by POD is only helpful and sensible when most of the computational cost lies in the solution of the linearised problem and not in the composition of the equation system.

Due to the black-box character of commercial FEM codes, communication with these programs requires a cumbersome read- and write-procedure whenever the current equation system is needed. Some FEM programs only allow exchange of information by text files. This leads to computational costs which are far beyond any feasible time scale. Furthermore, the problem of licences can become a serious issue, as the solver of the FEM program is used in each time step to set up the equations.

For full exploitation of the capabilities of the POD method, the computation of the reduced problem needs to be decoupled from the commercial tool. In general, profound knowledge of the full system is needed in order to formulate the nonlinear equations with respect to the reduced variables only. However, black box tools do not allow such approaches as only few details of the underlying equations can be gained from commercial programs.

In the following sections, an approach is presented which uses the value of the nonlinear term and the derivative matrices of a full model to build up the decoupled reduced system.

3.1 Construction of the Lookup Table

We assume we have a nonlinear differential equation of second order

$$F(u, \dot{u}, \ddot{u}, t) = 0 \tag{17}$$

where F is continuous and differentiable with respect to u, $\dot{u}$ and $\ddot{u}$. Furthermore, it may depend nonlinearly on u and u', but linearly on $\dot{u}$ and $\ddot{u}$.

Applying a Finite Element discretisation in space we define the semi-discretised equations. We search for a vector $u(t) \in \mathbb{R}^N$ satisfying

$$\begin{aligned} M(u)\ddot{u} + C(u)\dot{u} + R(u) &= f_{ext}(t) \qquad (18) \\ \text{where } M(u) &:= \frac{\partial F}{\partial \ddot{u}} \\ C(u) &:= \frac{\partial F}{\partial \dot{u}} \end{aligned}$$

where the matrices $M, C \in \mathbb{R}^{N \times N}$ denote the mass and damping matrix, respectively. Both matrices may depend on the current deformation $u = u(t)$.

Assuming the matrices M, C, the nonlinearity $R(u)$ and its linearisation $K(u)$ can be extracted from the Finite Element software in each time step, the following procedure is set up:

(1) on each time level t_i, store the current state u_i, the nonlinear deformation expression $R(u_i)$ and its derivative $K(u_i) = \frac{\partial R}{\partial u}\Big|_{u_i}$

(2) (if coupled to a POD reduction scheme) project the data onto the given POD subspace

(3) if necessary, select few relevant states which define the look-up table

(4) in each iteration of the reduced model, construct approximation of the nonlinear term and its linearisations from the given data stored in the table

Note that steps $(1) - (3)$ are treated offline.

The computation steps of the reduced problem mainly consist of the solution of the projected equations and the treatment of the lookup table. Depending on the deformation variety and the nonlinearity of the system, the table can become rather large. Therefore, a simple search algorithm is needed.
Furthermore, during application of the reduced model, states u_i may occur which are not included in the table. The lookup algorithm should account for variations of external loads and therefore variations of the deformations within a reasonable range.

This implies that lookup strategies should allow for interpolation and extrapolation of the entries and still remain stable.

3.2 The Reduced Surrogate Model

Setting up a surrogate model, we have

$$\widehat{M}\ddot{u} + \widehat{C}\dot{u} + \widehat{R}(u) = \widehat{f}_{ext}(t) \tag{19}$$

where $\widehat{M}, \widehat{C}$ and $\widehat{f}$ can be identical to the corresponding quantities in the original model or suitable subspace approximations. In the computational treatment of model (19), the nonlinear part is approximated using the following lookup scheme.

Given the current state u_τ, the approximation of $R(u_\tau)$ and $K(u_\tau)$ is computed by data linearisation in the lookup table. Let the lookup data consist of states $u_i, i = 1, \dots, m$, their corresponding deformation terms $R(u_i)$ and linearisations $K(u_i) = \frac{\partial R}{\partial u}\big|_{u_i}$. In each iteration, we determine the approximated model by Taylor expansion in the neighbourhood of a given state u_i:

$$R(u_\tau) = R(u_i) + K(u_i)(u_\tau - u_i) + \mathbf{O}\left(\|u_\tau - u_i\|^2\right) \quad (20)$$
$$K(u_\tau) = K(u_i) + \mathbf{O}\left(\|u_\tau - u_i\|\right) \quad (21)$$

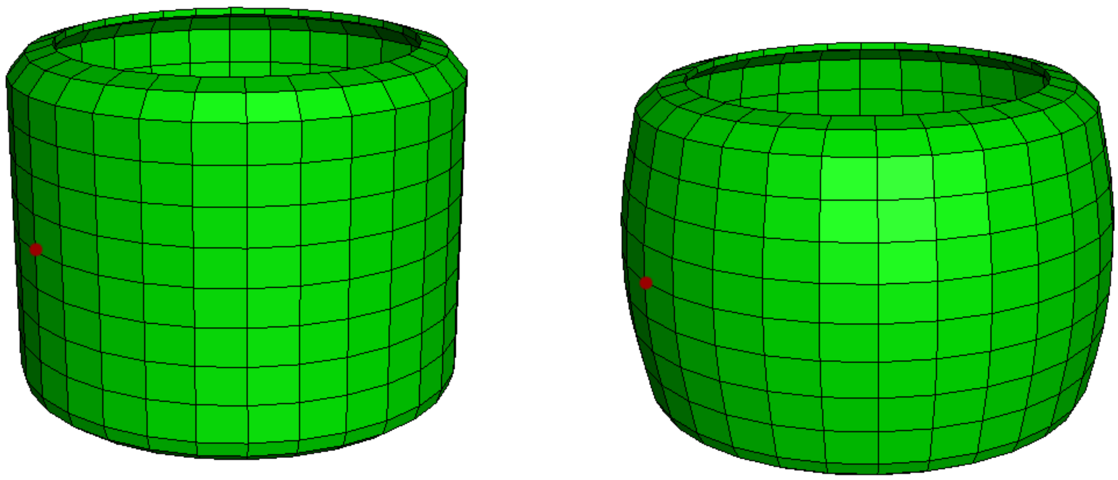
The state u_i is chosen as the nearest state to the current deformation u_τ measured in the L_2 norm.

In the following lookup method, Taylor expansion up to first order is used. When the nonlinear term and its linearisation are required, they are approximated by

$$R(u_\tau) \approx R(u_i) + K(u_i)(u_\tau - u_i) \quad (22)$$
$$K(u_\tau) \approx K(u_i)\,. \quad (23)$$

Additional considerations can be found in [3].

4 MODEL DESCRIPTION



(a) Undeformed configuration

(b) Deformed configuration after static step (inner pressure)

Figure 1: Abaqus model geometry of an airspring, output node marked

As a moderately large industrially motivated example an airspring model was chosen (Figure 1). The Finite Element mesh is composed of 450 linear 8-node continuum elements yielding 960 nodes and therefore 2880 degrees of freedom. The entire spring cushion consisted of a nonlinear *Neo-Hooke* rubber material with the parameters $C_{10} = 2.9 \cdot 10^5$ and $D_1 = 3.5 \cdot 10^{-7}$

(see, for example, [1]) and a density of $\rho = 1.1 \cdot 10^3 kg/m^3$. Furthermore, mass proportional Rayleigh damping ($\alpha = 100$) was included in the material.

The computation consists of two steps. In the first static step, an inner pressure of $4kPa$ is applied to the inner surface. Notice that this type of load induces difficulties in the geometrically nonlinear case, as pressure forces have to be perpendicular to the inner surface. This leads to deformation dependent external forces $f_{ext}(u,t)$ which require additional iteration routines within the computations of the reduced model. For simplification, inner pressure was only applied to six element rows in the middle of the spring cushion (see Figure 2), as these are assumed to not undergo excessive rotation.

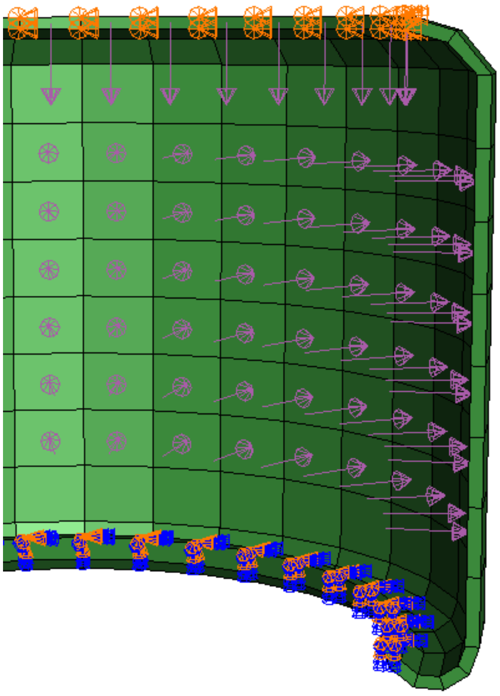


Figure 2: Position and direction of inner pressure and boundary conditions

The second step consists of the dynamic computations. In this step a vertical load varying in magnitude and sign is applied to the upper element ring (Figures 2, 3). The dashed line denotes the run used for the setup of the reduced models, whereas the continuous line denotes the computation used for comparison of the model performance. Like in the static step, the nodes of the lower boundary are fixed in all three directions. The nodes of the upper boundary are allowed to move in vertical direction only (see Figure 2).

The computation is accomplished by an implicit transient analysis in Abaqus as described in [1]:

$$M\ddot{u}_i + C\dot{u}_i + I\left(u_i\right) - P_i + L_j = 0 \tag{24}$$

with constant mass and damping matrices M and $C = \alpha \cdot M$. The vector I denotes the inner forces in the element, P_i the external excitation at time t_i and L_j the Lagrangian forces induced by boundary conditions at node j.

In general nonlinear computations, the inner forces term $I\left(u_i\right)$ as well as its linearisation, the stiffness matrix $K\left(u_i\right) = \frac{\partial I}{\partial u}|_{u_i}$, depend on the current deformation u. The damping term may

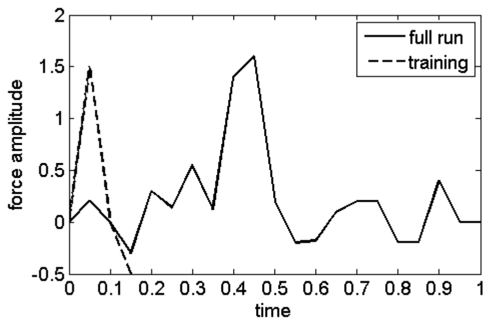


Figure 3: Amplitude of the external load over time: training input and input for the reference computation

also include several nonlinear relations, depending on the underlying material law. For the airspring model, the simplest form of Rayleigh damping was used.

Unless otherwise stated, the following results refer to the solution at the output reference node number 718 (Figure 1).

5 RESULTS OF THE REDUCED MODEL

In a first run with the full model, the lookup table is generated. The airspring is objected to a large tensile force followed by a large compressive force to cover the relevant range of deformation. The step is finished at time $t = 0.15$ after a rather generous number of 150 equally sized increments ($\Delta t = 10^{-3}s$).

For the lookup table, it is not necessary to include all snapshots, as the deformation states are likely to repeat themselves. If possible, the deformation range should be checked and double states be dismissed. For the airspring, this task is relatively simple as the deformation of the nodes in the upper ring can only move in one direction. Yet, the displacement of the nodes in the inner part of the airspring can be different depending on the preceding load history due to dynamic effects.

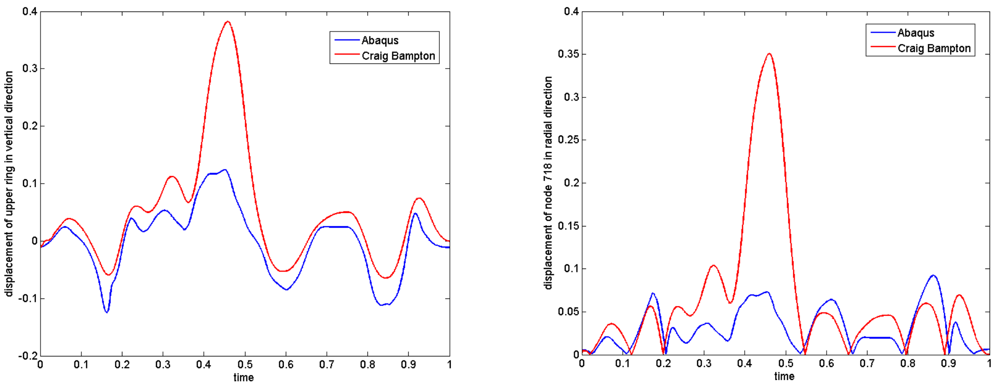
In this example, the lookup table consists of the states $55 - 151$ of the dynamic computation step, as this range covers the transition from the largest tension to the largest compression deformation of the upper ring (see Figure 3). The computation of the POD basis is done with all deformation snapshots $u(t_i)$ using the L_2 norm. The reduced mass and damping matrices $\widetilde{M}$, $\widetilde{C} = \alpha\widetilde{M}$ and the lookup table $\{p_i, \widetilde{R}_i, \widetilde{K}_i\}_{i=1,\ldots,97}$ are constructed by projection of the respective quantities onto the POD subspace.

This setup defines the input for the reduced computations.

The static step is not reproduced by the POD model. From the full model, the final state of the inflation step is projected onto the POD subspace to provide the initial conditions $p_0, \dot{p}_0$ for the dynamic computations with the reduced one.

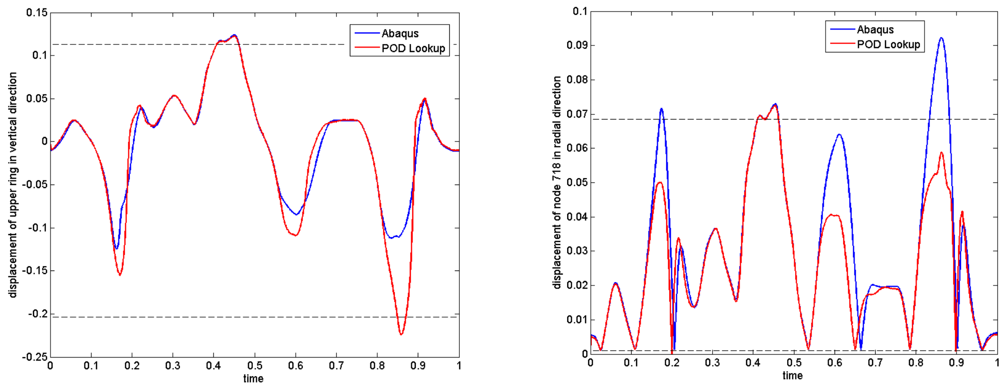
In the following result presentation, three different setups are compared: the full solution computed with Abaqus, the POD reduced lookup table approach and a Craig-Bampton method. For the latter one, the model was linearised at the beginning of the dynamic step, i.e., after inflation of the airspring. The Abaqus simulation serves as the reference solution for the reduced models. For the reduced bases, POD and Craig-Bampton, 15 basis vectors were chosen.

To study the extrapolation abilities of the reduced approaches, the external excitation is adjusted in order to provoke larger deformations than the lookup table includes.



(a) Vertical displacement (b) Radial displacement

Figure 4: Vertical displacement of upper ring and radial displacement of node 718, computed with Abaqus and Craig-Bampton method, using 15 basis vectors



(a) Vertical displacement (b) Radial displacement

Figure 5: Vertical displacement at upper ring and radial displacement at node 718, computed with Abaqus and POD Lookup, using 15 basis vectors
The horizontal dashed lines border the range covered by the lookup table

Figure 4 shows the vertical displacement of the upper ring and the radial displacement of the reference node 718 for the Craig Bampton method. The resulting deformation is entirely different from the full solution. Obviously, a linear approach is not justified in this model setup.

Compared to that, the POD Lookup method captures the nonlinear behaviour in a large portion of the time span. However, both the vertical and radial displacements show that the method fails to reproduce the deformations correctly, when the lookup table range is exceeded (Figure 5). This effect is seen in the periods around $t = 0.18s$, $t = 0.6s$ and $t = 0.85s$.

In other computations, where the deformations remained within the lookup table range, the results of the reduced models were nearly identical to the full solution for all three computed POD and lookup setups.

The following section takes a closer look at the deformation of the full system at time $t = 0.18s$, where the lookup methods do not reproduce the displacements correctly. Around this time period, the airspring undergoes a large compression followed by a rapid change of force direction. Due to inertia effects, the central belt rows of the airspring bulge downwards and develop a deformation that is not included in the training setup (see Figure 6). Therefore, neither the POD basis nor the lookup table contain information on this state.

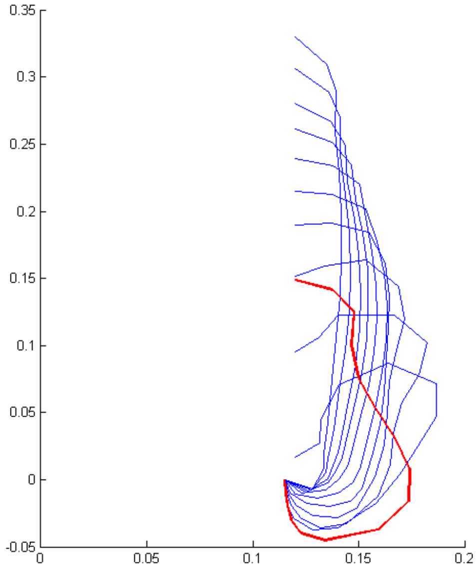
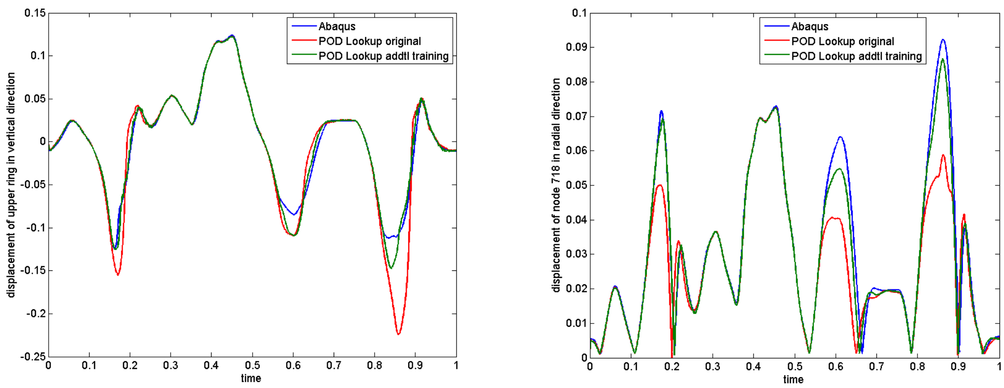


Figure 6: Training deformation states $u_i, i = 61, 71, \ldots, 151$ (blue lines) and deformation state of the full solution at time $t = 0.18s$ (red line) in a vertical line along the outer surface of the airspring

To deal with this shortcoming, an additional training input is set up. The new load case covers a time span of $0.2s$ in 200 time steps and contains a short period of tensile loading after compression. With this, the structure deforms similarly as the full solution at time $t = 0.18s$ of the dynamic step. A new reduced model is defined using 201 snapshots and a lookup table of 147 entries $\{p_i, \widetilde{R}_i, \widetilde{K}_i\}_{i=1,\ldots,147}$ for the states $55 - 201$.



(a) Vertical displacement

(b) Radial displacement

Figure 7: Radial displacement at upper ring and vertical displacement at node 718, computed with Abaqus, POD Lookup with original input and POD Lookup with additional input, using 15 basis vectors

Figure 7 compares the vertical and radial displacements of the full solution with the reduced one for both the original and the additional training input. The original lookup table consists of 97 entries and the one with the additional input of 147 states. In both cases, the POD Lookup method was used with a subspace dimension of 15. Both figures show a significant improvement not only at position $t = 0.18s$, but also around the critical regions $t = 0.6$ and $t = 0.85$. The listing of errors in Table 1 confirms this observation.

Method	Number of DOF	Number of lookup entries	relative error, L_2 norm	relative error, L_∞ norm
POD Lookup	15	97	22.1%	25.7%
POD Lookup	15	147	4.7%	13.3%

Table 1: Comparison of errors: The reduced models are based on 15 POD basis vectors with 151 or 201 snapshots and 97 or 147 Lookup table entries, respectively

Method	Number of DOF	CPU Time
Abaqus	2880	$810.4s$
POD Lookup	15	$1.7s$
Craig-Bampton	15	$0.6s$

Table 2: Comparison of CPU time

Table 2 shows a comparison of CPU time for the different approaches. All tested reduction methods run at least two orders of magnitude faster in Matlab than the original model in Abaqus.

Despite the computational savings obtained by the reduced methods, the CPU time for the model setup needs to be taken into account (see Table 3). The effort basically consists of the communication with Abaqus and the computation of the POD basis. Note that these computations can be done offline and need to be undertaken only once for the model setup and not for each computation.

Due to the slow communication with Abaqus via text files, the read steps require most effort - especially the reading and assembling procedure for the full stiffness matrices. This cost can be reduced if an efficient positioning of the lookup table states within the time interval is performed. Then only the relevant matrices need to be output from Abaqus and imported in Matlab. Furthermore, if the POD basis is determined in advance from a preceding run, the element matrices can be read in and projected onto the POD subspace within the same step. Assembly of the matrices is done by adding the projected element matrices.

Procedure	CPU Time
Read and assemble stiffness matrices and mass matrix	$1015.4s$
Read deformations and inner forces	$70.9s$
Compute POD basis	$4.7s$

Table 3: CPU time for the Abaqus communication and offline computations

6 CONCLUSION

This work shows a method to reduce the computational effort of nonlinear flexible bodies.

The structure is described by a Finite Element Method combined with nonlinear model reduction. Unlike most model reduction methods - as the frequently used Craig-Bampton approach - the method of Proper Orthogonal Decomposition (POD) offers a projection basis suitable for nonlinear models.

The reduction of nonlinear problems introduces additional difficulties. The projection-based method of POD reduces the effort needed for solution of the model equations, but not for the function evaluations required for the equation setup. Especially in the case of collaboration with black-box software, the effort for equation setup exceeds the savings obtained from reduction of the degrees of freedom.

To decouple the reduced surrogate system from the full model, a lookup table approach is presented. It makes use of the preceding computation step with the full model necessary to set up the POD basis. The nonlinear term of inner forces and the stiffness matrix are output and stored in a lookup table for the reduced system.

The method is applied to an airspring computed in Abaqus. It is shown that effort reductions of two orders of magnitude are possible within a reasonable error tolerance. The classical Craig-Bampton method is unable to reproduce any nonlinear effects and yields completely wrong results.

Furthermore, the example illustrates the influence of training excitation on the quality of the reduced model. The reduced solution strongly deviates from the full one when dynamical effects occur that were not included in the training. An adapted training input accounting for the missing states yields considerable improvements.

Overall, the computations show that the Lookup method combined with a POD subspace projection constitutes a good method to massively reduce the computational effort of large nonlinear structures.

REFERENCES

[1] Abaqus Theory Manual, Version 6.6

[2] R. R. Craig, Jr.: Coupling of Substructures for Dynamic Analyses: an Overview, *AIAA-2000-1573*, 2000

[3] S. Herkt: Model Reduction of Nonlinear Problems in Structural Mechanics: Towards a Finite Element Tyre Model for Multibody Simulation, PhD Thesis, TU Kaiserslautern, 2009

[4] W.H.A. Schilders, H. van der Vorst: Model Order Reduction: Theory, Research Aspects and Applications, Springer, 2008

[5] S. Volkwein: Interpretation of proper orthogonal decomposition as singular value decomposition and HJB-based feedback design, *Proceedings of the Sixteenth International Symposium on Mathematical Theory of Networks and Systems (MTNS)*, Leuven, Belgium, July 5-9, 2004

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