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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Using the Sharp Operator for Edge Detection and Nonlinear Diffusion

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Abstract

In this paper we investigate the use of the sharp function known from functional analysis in image processing. The sharp function gives a measure of the variations of a function and can be used as an edge detector. We extend the classical notion of the sharp function for measuring anisotropic behaviour and give a fast anisotropic edge detection variant inspired by the sharp function. We show that these edge detection results are useful to steer isotropic and anisotropic nonlinear diffusion filters for image enhancement.

1 Introduction

The sharp function is a well-known functional analytic concept to measure the oscillatory behaviour of functions. It goes back to the *maximal function* which was introduced by Hardy and Littlewood [13] in 1930 to solve a problem in the theory of functions of complex variable. Based on this idea John and Nirenberg [16] introduced the concept of *bounded mean oscillation (BMO) functions.* In 1972, Fefferman and Stein [9] introduced the *sharp function* (denoted by $f^{\#}$) and found that a function $f \in BMO$ is equivalent with $f^{\#} \in L^{\infty}$. The theory of H^p spaces (Hardy Spaces) received impetus from the work of Fefferman and Stein. Their work resulted in the identification of a dual of H^1 with BMO. The idea of applying the sharp operator to measure the oscillation and classification of images was first proposed by Ahmad and Siddiqi [2] where it was used to find a suitable compression technique.

Nonlinear diffusion filtering is a well-established tool for image denoising and simplification. Starting with the pioneering work by Perona and Malik [25] in 1990, it has attracted the attention of many researchers working in mathematics and image processing (see [6, 28, 30, 24, 23, 8], for example). This filter class makes it possible to smooth images while the edges as main source of information are preserved. This leads to an adaptive simplification that can be useful for image understanding and interpretation. Among the most effective extensions of the basic method are the anisotropic filters [28] that offer the possibility to remove noise very well and enhance flow-like structures.

In this paper, we study the Hardy-Littlewood maximal function and the sharp operator to measure the oscillatory behaviour of images following ideas in [2]. Further, we propose an alternative way to steer nonlinear diffusion filters via the sharp operator without using derivatives to measure edges. We show that the results of these diffusion filters are comparable to classical versions while the underlying sharp operator has a rich theoretical background. Motivated by the available diffusion processes in image processing, we propose an extension of the sharp operator for measuring anisotropic structures. To use this to steer anisotropic diffusion processes, we show how a fast variant of it can be implemented and used in practice.

The paper is organized as follows. In Section 2, we shortly describe the aspects of the theory for the maximal function, bounded mean oscillation functions, and the sharp function, that are necessary for this paper. Section 3 gives a review of classical nonlinear diffusion filters for image processing. The main idea of this paper, namely the use of the sharp operator in nonlinear diffusion filters, is presented in Section 4. These results are generalised to the anisotropic setting in Section 5. To evaluate the methods in practice, Section 6 describes some computational experiments. A summary and an outlook conclude the paper in Section 7.

2 The Sharp Operator

In this section we give a short introduction to the sharp operator and its background. There is a rich theory behind it, and we are going to point out the main results connected to it.

The Hardy-Littlewood maximal function was developed to solve a problem in the theory of functions of complex variable. The analogue for integrals, which is required for the function theoretic applications, is also derived by Hardy and Littlewood [13].

Definition 2.1 *Let $\mathbb{R}^n$ be the n-dimensional Euclidean space and $f(x)$ be a real valued measurable function on $\mathbb{R}^n$. For such a function f on $\mathbb{R}^n$ its Hardy-Littlewood maximal function is defined by the formula*

$$Mf(x) = \sup_Q \left\{ \frac{1}{\lambda(Q)} \int_Q |f(y)| dy \; : \; Q \subset \mathbb{R}^n, x \in Q \right\} \tag{2.1}$$

where the supremum ranges over all finite cubes Q in $\mathbb{R}^n$ and $\lambda(Q)$ is the Lebesgue measure of Q.

The function $Mf(x)$ has the following properties:

(i) $0 \leq Mf(x) \leq \infty$

(ii) $M(f+g)(x) \leq Mf(x) + Mg(x)$

(iii) $M(\alpha f)(x) = |\alpha| Mf(x)$

where f, g are measurable functions on $\mathbb{R}^n$ and α is some scalar quantity.

It is easy to find a function whose maximal function is un-bounded.

Example 2.2 *For $f(x) = |x|^t$ with $t > 0$, we get $Mf(x) = \infty$ for each $x \in \mathbb{R}$.*

Now we state a Hardy-Littlewood maximal theorem.

Theorem 2.3 *For each function $f \in L^1(\mathbb{R}^n)$ we have*

$$\lambda(\{x : Mf(x) > t\}) \leq 6^n t^{-1} \|f\|_1, \;\; t > 0 \, .$$

Proof. See [33], page 142.

An interesting application of the maximal theorem is a version of the Lebesgue differentiation theorem.

Theorem 2.4 *(Lebesgue Differentiation Theorem). Let $f \in L^1(\mathbb{R}^n)$. For almost all $x \in \mathbb{R}^n$ and for every decreasing sequence of cubes $(Q_j)_{j=1}^{\infty}$, such that $\cap_{j=1}^{\infty} Q_j = \{x\}$, we have*

$$\lim_{j\to\infty} \frac{1}{\lambda(Q_j)} \int_{Q_j} f(y)dy = f(x). \tag{2.2}$$

Proof. See [26], page 81.

The space BMO, i.e. bounded mean oscillation of functions is introduced by John and Nirenberg [16].

Definition 2.5 *A measurable function f on $\mathbb{R}^n$ has bounded p-mean oscillation, $1 \leq p < \infty$, if*

$$\|f\|_{BMO_p} = \sup_Q \left(\frac{1}{\lambda(Q)} \int_Q |f(x) - f_Q|^p dx \right)^{\frac{1}{p}} < \infty, \tag{2.3}$$

where the supremum ranges over all finite cubes Q in $\mathbb{R}^n$ and

$$f_Q = \frac{1}{\lambda(Q)} \int_Q f(x)dx$$

is the mean value of the function f on the cube Q.

The set of all functions of bounded p-mean oscillation is denoted by $BMO_p(\mathbb{R}^n)$. $\|f\|_{BMO_p}$ is "almost" a norm since it has the following properties.

(i) $\|f+g\|_{BMO_p} \leq \|f\|_{BMO_p} + \|g\|_{BMO_p}$

(ii) $\|\alpha f\|_{BMO_p} = |\alpha| \cdot \|f\|_{BMO_p}$

(iii) $\|f\|_{BMO_p} = 0$ if and only if $f=$ constant almost every where,

where f, g are measurable functions on $\mathbb{R}^n$ and α is some scalar quantity.

If we define

$$\|f\|_{BMO_p'} = \left| \int_{\mathbb{R}^n} f(x)dx \right| + \sup_Q \left(\frac{1}{\lambda(Q)} \int_Q |f(x) - f_Q|^p dx \right)^{\frac{1}{p}},$$

we get a norm of f and BMO'_p becomes a Banach Space. On the other way we can say, $\|f\|_{BMO_p}$ becomes a norm if we identify functions which differ by a constant. With this identification $BMO_p(\mathbb{R}^n)$ becomes a normed space, and ultimately a Banach space.

Fefferman and Stein [9] introduced "sharp function" $f^{\#}$ that mediates between BMO_p and L^p spaces. It is defined as follows.

Definition 2.6 *Let f be a locally integrable function on $\mathbb{R}^n$. The sharp function $f^{\#}(x)$ is represented by the formula,*

$$f^{\#}(x) = \sup_{Q:x\in Q} \left(\frac{1}{\lambda(Q)} \int_Q |f(y) - f_Q|^p dy \right)^{\frac{1}{p}}. \tag{2.4}$$

Of course, $f \in BMO_p$ is identical with $f^{\#} \in L^{\infty}$. It is also observed that there are unbounded functions in $BMO_p(\mathbb{R})$.

Example 2.7 *The function $f(x) = \ln|x|$ on $\mathbb{R}$ is in $BMO_1(\mathbb{R})$.*

After calculation it comes out to be $\|\ln|x|\|_{BMO_1} \leq 2$. So, the unbounded function $\ln|x|$ is in $BMO_1(\mathbb{R})$.

It is important to note that it does not matter in which L^p norm we measure the oscillation. This is clear from the following corollary.

Corollary 2.8 *For each p, $1 \leq p < \infty$, there exists a constant C_p such that for each $f \in BMO_p(\mathbb{R}^n)$ we have*

$$\|f\|_{BMO_1} \leq \|f\|_{BMO_p} \leq C_p \|f\|_{BMO_1}$$

Proof. See [33], page 156.

In view of the above corollary the spaces $BMO_p(\mathbb{R}^n)$ are equivalent for all p, $1 \leq p < \infty$.

We have given a short introduction to the sharp operator and the maximal function and their properties. In the next section we will describe the tool of diffusion filtering in image processing as both the techniques are combined later on.

3 Classical Nonlinear Diffusion Filters

Diffusion is interesting as image processing tool since it is a physical process that equilibrates concentration without creating or destroying mass. The idea behind the use of the diffusion equation in image processing arose from the use of Gaussian filter in multiscale image analysis. It can be founded by a system of several axioms like linearity, translational and rotational invariance, and average grey value preservation, and marks the beginning of the scale-space concept [14, 15, 32, 19, 20, 29]. Convolving an image with a Gaussian filter K_σ,

$$K_\sigma := \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|x|^2}{2\sigma^2}\right)$$

with standard deviation σ, is equivalent to the solution of the linear homogeneous diffusion equation

$$\partial_t u \;=\; \Delta u \tag{3.5}$$

where the given image f is used as initial condition $u(0,\cdot) = f$, and we assume homogeneous Neumann boundary conditions $\partial_\nu u = 0$. Here, ν denotes the outer normal of the boundary of our image domain Ω. The stopping time t has to be chosen $t = \sigma^2/2$ for equivalence.

Isotropic nonlinear diffusion. The major drawback of linear diffusion is the delocalisation and blurring of image edges. To circumvent this problem, Perona and Malik [25] introduced the nonlinear diffusion equation

$$\partial_t u = \operatorname{div}(g(|\nabla u|^2)\nabla u) \tag{3.6}$$

The diffusivity g is chosen as a decreasing function of the edge detector $|\nabla u|$. Examples for diffusivity functions can be found in [25, 7, 3, 17]. Catté et al. [6] introduced a regularisation of the gradient of u to make the process well-posed. They use the equation

$$\partial_t u = \operatorname{div}(g(|\nabla u_\sigma|^2)\nabla u) \tag{3.7}$$

with $\nabla u_\sigma := \nabla(K_\sigma * u)$. A review of this filter class can be found in [27], for example.

Anisotropic nonlinear diffusion. Nonlinear isotropic diffusion often shows problems to remove noise close to image edges. It can be helpful to use an anisotropic diffusion filter

$$\partial_t u \;=\; \operatorname{div}(D(u)\nabla u) \tag{3.8}$$

as proposed by Weickert [28] in such cases. The scalar diffusivity function g has been replaced by a matrix $D(u)$ here. Depending on the choice of D this allows for smoothing along edges while smoothing across edges is avoided: the so-called *edge-enhancing diffusion (EED)*. Another classical choice of D

depending on the structure tensor [10, 18] makes enhancement of coherent flow-like structures possible. This process is known as *coherence-enhancing diffusion (CED).* Details on these filters and their numerical implementation can be found in [28, 31], for example.

4 Nonlinear Diffusion with the Sharp Operator

It is clear from the definition of the sharp function that for a pixel z where f has almost uniform grey level region in an image, $f^{\#}(z)$ will be of very small value. However, for the contrast region we get large values for $f^{\#}(z)$. The idea is to accrue more diffusion in the regions of lower oscillation whereas to preserve the regions of higher oscillation.

We have tried to evaluate the performance of sharp operator as a new diffusion entity, defined as

$$g(u^{\#}(x)) = \frac{1}{1 + u^{\#}(x)/\lambda}, \tag{4.9}$$

where λ is a contrast parameter and can be adjusted.

After regularization of model with the Gaussian, we have used the sharp function to calculate the diffusivity. Our model can be written as:

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(g\left(u_\sigma^{\#}\right)\nabla u\right), \tag{4.10}$$

where $u_\sigma := K_\sigma * u$.

Let Ω denotes the open set $(0,1) \times (0,1)$ of $\mathbb{R}^2$, with boundary Γ. We denote $H^k(\Omega), k$ a positive integer, the set of all functions $u(x)$ defined in Ω such that u and its distributional derivatives $D^s(u)$ of order $|s| = \sum_{j=1}^n s_j \leq k$ all belong to $L^2(\Omega)$. $H^k(\Omega)$ is a Hilbert space for the norm

$$\|u\|_{H^k(\Omega)} = \left(\sum_{|s|\leq k} \int_\Omega |D^s u(x)|^2 dx\right)^{1/2}.$$

We denote $L^p(0,T,H^k(\Omega))$, the set of all functions u, such that for almost every t in $(0,T)$, $u(t)$ belong to $H^k(\Omega)$. $L^p(0,T,H^k(\Omega))$ is a normed space for the norm

$$\|u\|_{L^p(0,T,H^k(\Omega))} = \left(\int_0^T \|u(t)\|^p_{H^k(\Omega)}\right)^{1/p},$$

$p > 1$ and k a positive integer.

We denote $L^\infty(0,T,C^\infty(\Omega))$, the set of all functions such that, for almost every t in $(0,T)$, $u(t)$ belong to $C^\infty(\Omega)$. $L^\infty(0,T,C^\infty(\Omega))$ is a normed space for the norm

$$\|u\|_{L^\infty(0,T,C^\infty(\Omega))} = \inf\{C; \|u(t)\|_{C^\infty(\Omega)} \leq C, \text{a.e. on } (0,T)\}.$$

Let $g : R^+ \to R^+$ be a decreasing function such that $g(0) = 1$ and $\lim_{t\to\infty} g(t) = 0$.

Theorem 4.9 *Let $u_0 \in L^2(\Omega)$, then we have a unique function $u(x,t)$ such that $u \in C([0,T]; L^2(\Omega)) \cap L^2(0,T,H^1(\Omega))$, and verifying*

$$\begin{cases} \frac{\partial u}{\partial t} - div(g(u_\sigma^\#)\nabla u) = 0 \quad on\ (0,T)\times\Omega, \\ \frac{\partial u}{\partial n} = 0 \quad on\ \ \Gamma\times(0,T], \\ u(0) = u_0, \end{cases} \tag{4.11}$$

where $u_\sigma = K_\sigma * u$ and the system is verified in the distributional sense. Moreover, this unique solution is in $C^\infty((0,T)\times\overline{\Omega})$.

Proof. Existence of solution. Here, we show the existence of a weak solution of (4.11) by a classical fixed point theorem of Schauder [11]. We take the space

$$S(0,T) = \left\{ s \in L^2(0,T,H^1(\Omega)),\ \frac{ds}{dt} \in L^2(0,T,(H^1(\Omega))') \right\}.$$

This space is a Hilbert space for the graph norm [21]. Let $s \in S(0,T) \cap L^\infty(0,T,L^2(\Omega))$ such that

$$\|s\|_{L^\infty(0,T,L^2(\Omega))} \leq \|u_0\|_{L^2(\Omega)} \tag{$*$}$$

and consider the following problem (X_s):

$$\left\langle \frac{\partial u}{\partial t}, v \right\rangle + \int_\Omega g(u_\sigma^\#)\nabla u(t)\nabla v = 0, \quad \forall\ v \in H^1(\Omega) \text{ a.e. in } [0,T],$$

$$u(0) = u_0.$$

Since $s \in L^\infty(0,T,L^2(\Omega))$ and g, K_σ are C^∞, one can easily deduce that $g(u_\sigma^\#) \in L^\infty(0,T,C^\infty(\Omega))$. Thus, since g is a decreasing, there exists a constant $\nu > 0$ such that

$$g(u_\sigma^{\#})) \geq \nu \quad \text{a.e. in } (0,T)\times\Omega,$$

where ν depends only on g, K_σ and $\|u_0\|_{L^2(\Omega)}$.

By classical results on parabolic equations [1, 4, 5], we prove that the problem (X_s) has a unique solution $P(s)$ in $S(0,T)$. The rest of the proof can easily be followed from [6].

Discretisation. The simplest time-explicit finite difference scheme for the model

$$\frac{\partial u}{\partial t} = \operatorname{div}(g(u_\sigma^{\#})\nabla u)$$

is given by

$$\frac{\partial u}{\partial t} = \partial x_1(g(u_\sigma^{\#})\partial x_1 u) + \partial x_2(g(u_\sigma^{\#})\partial x_2 u).$$

The scheme can also be written in the following form:

$$\begin{aligned}\frac{u_{i,j}^{k+1} - u_{i,j}^{k}}{\tau} &= g(u_{i+1,j}^{\#k})(u_{i+1,j}^{k} - u_{i,j}^{k}) - g(u_{i,j}^{\#k})(u_{i,j}^{k} - u_{i-1,j}^{k}) \\ &\quad + g(u_{i,j+1}^{\#k})(u_{i,j+1}^{k} - u_{i,j}^{k}) - g(u_{i,j}^{\#k})(u_{i,j}^{k} - u_{i,j-1}^{k}),\end{aligned}$$

where $u_{i,j}^k$ denotes the sampled values of u^k, i.e., $u_{i,j}^k = u^k(i,j)$ for a suitable scaled image and τ is the time step size. The initial condition reads $u_i^0 = f_i$ for all i. Nevertheless, explicit schemes suffer from limited stability: There are severe restrictions for the time step size in order to achieve stable results. We can also use semi-implicit schemes that are unconditionally stable in time. Further, there are highly efficient *additive-operator splittings (AOS)* available that reduce high-dimensional diffusion processes to several one-dimensional processes [22, 30].

5 Extensions to the Anisotropic Setting

Since structures in images often have a highly anisotropic features, for example, lines or corners, we propose some generalization of the presented method to the anisotropic setting. We start with an anisotropic generalization of the sharp operator.

5.1 Anisotropic sharp operator

So far, we have only used isotropic nonlinear diffusion filters. In the definition (2.4) of the sharp operator, all integration domains Q are cubes.

Therefore, the sharp function only provides information about local variations of the function, but not about the direction of these local variations. In order to allow for a quantitative description of local variations in a certain direction, we propose to use non-symmetric sets instead of cubes. With this concept, an anisotropic extension of the sharp function can be defined as follows:

$$f^{\#}_{\text{aniso}}(x,\varphi) := \sup_{Q(\varphi):x\in Q(\varphi)} \left(\frac{1}{\lambda(Q(\varphi))} \int_{Q(\varphi)} |f(y) - f_{Q(\varphi)}|^p \; dy \right)^{\frac{1}{p}} \; . \quad (5.12)$$

The most important in this definition is the set $Q(\varphi)$. We propose to use ellipses to measure the variation in several directions. So, one could alternatively define

$$f^{\#}_{\text{aniso}}(x) := \sup_{\varphi} \sup_{Q(\varphi):x\in Q(\varphi)} \left(\frac{1}{\lambda(Q(\varphi))} \int_{Q(\varphi)} |f(y) - f_{Q(\varphi)}|^p \; dy \right)^{\frac{1}{p}} \; . \quad (5.13)$$

In this definition, we take the supremum over all angles φ. Therefore, with this measure one is not only be able to find out the direction of the variation, but can also find the largest variation in any existing direction. For our later experiments, we start with the model (5.12) since we want to find the angle of the largest variation in an image.

5.2 Modifications of this basic model

For practical calculations, depending on the number of directions φ used, this measure is computationally very expensive. Thus we propose two simplifications in order to keep the motivation of the sharp operator while obtaining a fast measure of local variations:

Analogously to definition (2.4), the value $f_{Q(\varphi)}$ is defined as the local mean value of f inside the integration domain $Q(\varphi)$:

$$f_{Q(\varphi)} \; := \; \frac{1}{\lambda(Q(\varphi))} \int_{Q(\varphi)} f(x) \; dx.$$

Instead of taking this mean value as function of $Q(\varphi)$, we use a pre-smoothed version of the function f:

$$f_{Q(\varphi)}(x) \; = \; \int_{Q(\varphi)} f(y) \; dy \; . \quad (5.14)$$

This changes the definition (5.12) to

$$f^{\#}_{\text{aniso}}(x,\varphi) := \sup_{Q(\varphi):x\in Q(\varphi)} \left(\frac{1}{\lambda(Q(\varphi))} \int_{Q(\varphi)} |f(y) - f_{Q(\varphi)}(y)|^p \; dy \right)^{\frac{1}{p}} . \tag{5.15}$$

We notice that in this definition, the difference in the integral is a difference between two functions. This offers the possibility to calculate the second function $f_{Q(\varphi)}$ in one step for the whole domain instead of calculating mean values for each set $Q(\varphi)$ independently. We notice that with this change, we do not use the same set $Q(\varphi)$ for both integrations.

The second step is now to write this as a convolution. Instead of an elliptical set $Q(\varphi)$ we prefer to use an anisotropic Gaussian kernel here and write:

$$f^{\tilde{\#}}_{\text{aniso}}(x,\varphi) \;:=\; \sup_{Q(\varphi):x\in Q(\varphi)} \left(\frac{1}{\lambda(Q(\varphi))} \int_{Q(\varphi)} |f(y) - (G_\varphi * f)(y)|^p \; dy \right)^{\frac{1}{p}} . \tag{5.16}$$

And lastly we replace also the outer integral with a convolution with an anisotropic Gaussian.

$$f^{\#,\text{fast}}_{\text{aniso}}(x,\varphi) \;:=\; \left(G_\varphi * |f - G_\varphi * f|^p \right)^{\frac{1}{p}} \tag{5.17}$$

This measure can be evaluated in a very efficient way using the methods of Geusebroek et al. [12] for fast anisotropic Gaussian convolution. This makes it possible to incorporate it in an image processing tool as described in the following section.

5.3 Anisotropic diffusion with the fast sharp operator

Now we want to use the anisotropic variant of the sharp operator to steer an anisotropic diffusion process

$$\partial_t u \;=\; \operatorname{div}(D(u)\nabla u)$$

as it has been proposed by Weickert [28]. In order to use this concepts for anisotropic diffusion in this formulation, we have to define a diffusion tensor based on the anisotropic sharp operator to obtain a process of the form

$$\partial_t u \;=\; \operatorname{div}(D(u^{\#,\text{fast}}_{\text{aniso}})\nabla u)$$

We define the diffusion tensor as follows: Let a point $x \in \Omega$ in the image domain be given, then we search for the direction

$$\varphi_{\max} := \text{argmax}_{\varphi} |f^{\#,\text{fast}}_{\text{aniso}}(x, \varphi)|$$

where the absolute value of the anisotropic sharp operator is maximal. The eigenvectors of the diffusion tensor are then the unit vectors pointing in this direction and the orthogonal one, i. e.

$$v_1 \ := \ \begin{pmatrix} \cos(\varphi_{\max}) \\ \sin(\varphi_{\max}) \end{pmatrix} \quad \text{and} \quad v_2 \ := \ \begin{pmatrix} -\sin(\varphi_{\max}) \\ \cos(\varphi_{\max}) \end{pmatrix} \ .$$

The eigenvalues are defined analogously as for edge-enhancing diffusion:

$$\lambda_1 \ := \ (1 + \frac{m^2}{\alpha^2})^{-1} \, , \quad \lambda_2 \ := \ 1 \ ,$$

where

$$m := \max_{\varphi} f^{\#,\text{fast}}_{\text{aniso}}(x, \varphi)$$

is the maximal sharp value in the point x.

Having these definitions for the diffusion tensor at hand, we can use classical discretisation for anisotropic diffusion filters as described in [28], for example.

6 Computational Experiments

To compare the sharp operator based diffusion approach with classical derivative based methods, we show filtering examples in Fig. 1. It is clear that the parameters of the anisotropic diffusion process have to be specified in practical situations: The time t is an inherent parameter in each diffusion process that controls the amount of simplification applied to the data. The variance of evolving image decreases monotonically to zero in time. The contrast parameter λ allows to steer the edge preservation properties by distinguishing between important edges that should be preserved and smaller edges that are removed. For our discrete sharp operator, there are number of directions as an artificial parameter. Fig. 2 shows that the process is somehow sensitive with respect to the number of directions used in this discretisation of the sharp operator. In practice, using between 10 and 20 directions was enough to approximate rotational invariance for visual enhancement of images.

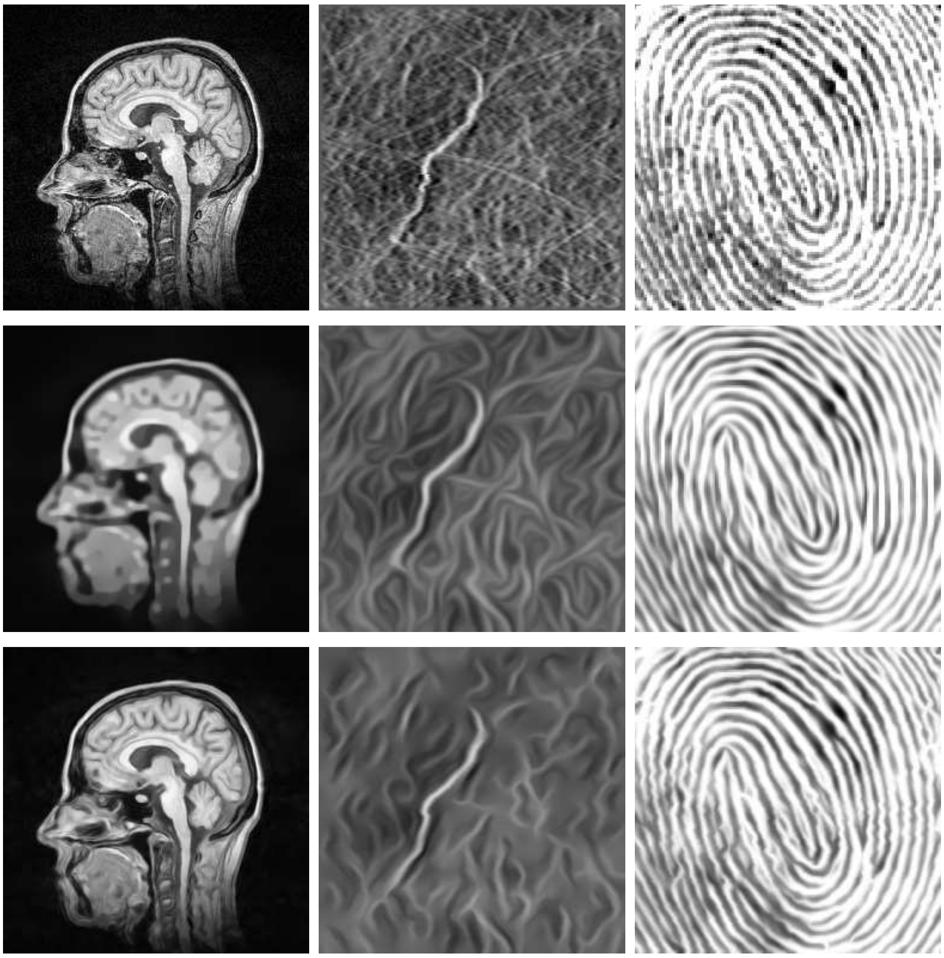

Figure 1: Comparison between classical and sharp operator based anisotropic diffusion. *First row:* Noisy input images, *Second row:* Classical edge-enhancing diffusion (EED), *Third row:* Sharp operator based diffusion.

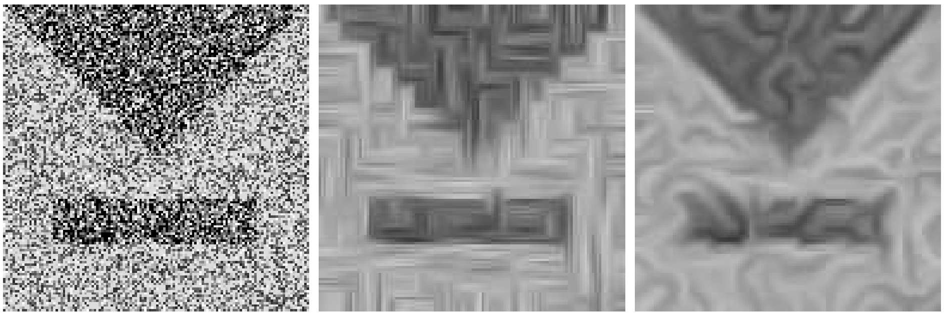

Figure 2: Sensitivity of sharp operator based anisotropic diffusion with respect to the number of directions. *Left:* Noisy input image, *Middle:* Sharp operator based diffusion with 2 directions, *Right:* Same with 4 directions.

7 Summary and Outlook

We have investigated the use of the sharp operator for image processing applications. We have used the sharp operator to steer diffusion filters. With the classical notion it is suitable to be used inside the diffusivity of a Perona-Malik filter. For anisotropic filters, we have used fast anisotropic Gauss filters to extend the sharp operator to a fast directional-dependent measure of variation. With the help of this measure, we could construct an alternative diffusion tensor for an anisotropic diffusion process. The results are quite similar to classical anisotropic diffusion filters. We have seen that the sharp operator is not only of theoretical interest but may also be used in practical applications.

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