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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Wave Based Method: New Applicability Areas

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Abstract

This paper discusses the possibility to use and apply the ideas of the Wave Based Method, which has been developed especially for the steady–state acoustic areas, i.e. to solve the Helmholtz type boundary value problems in a bounded domain, in non–acoustics areas such as steady–state temperature propagation, calculation of the velocity potential function of a liquid flux, calculation of the light irradience in a liver tissue/tumor, etc.

1 Introduction

In the end of 90ies of the twentieth century, a new numerical method has been developed for steady–state acoustic analysis in bounded domains. This novel deterministic numerical technique is based on the indirect Trefftz approach, cf. [18], and has been designed especially for mid–frequency range cases. The reason is simple: neither Finite Element Methods (FEM), cf. [10], nor Statistical Energy Analysis (SEA), cf. [9], cannot be applied exactly in this frequency range, cf. [3], [14], [15], [16], [19], [20], [6], [7]. Some similar ideas have been used in 80ies by Prof. Dr. Willi Freeden et al to solve exterior Dirichlet problems for the homogeneous Helmholtz equations. The main idea was to use an interpolation method using metaharmonic splines to find solution of above mentioned problem in 3D case from given discrete data, [4], [12].

The Finite Element Method usually is applicable in low–frequency cases, where the frequency limit of the FEM is related to the growing number of finite elements required to describe the short wavelength behavior at increasing frequencies. The SEA is a prediction method designed for the high frequency range providing the averaged results which are based on the power balance relations. The classical Wave Based Method (WBM) utilizes complex valued wave functions which have been used to expand the dynamic pressure function and which *a priori* satisfy the homogeneous Helmholtz equation, [3], [14]. Hence, no discretization of the domain is required and the sizes of appropriate governed matrices are rather small. This, obviously, gives the possibility to touch the mid–frequency range. Moreover, by using the natural basis functions, namely wave functions, we reduce certain ”numerical stress” of the numerical approximation of the problem. This was exactly the ideology of Trefftz, who applied basis functions which *a priori* exactly solve one or another differential equation, [18].

Nevertheless we would like to emphasize that the same Trefftz ideas can be applied to the Helmholtz, Poisson, Laplace or even to more general elliptic problems, where the so-called *maximum principle* can be valid[1]. Moreover, in classical wave based approach one assumes that the Helmholtz equation is either homogeneous or has point source as the right hand side function. In this paper, we will consider also non–homogeneous elliptic differential equations what allow to apply the wave based technique in a much broad way.

[1]Let us note that generally in the classical steady–state acoustics maximum principle is not valid, however, this does not play a big role here. We use this property here to separate so–called ”bad” elliptic problems, where so far WBM was used, from ”good” ones.

Main idea of the Wave Based Method

The Wave Based Method, as we mentioned above, has been designed especially for the steady–state acoustic problems. Let us consider some bounded domain $\Omega \subset \mathbb{R}^d,\ d \in \{2,3\}$. We assume that the steady–state acoustic pressure function $u(\mathbf{x})$ is defined inside the domain Ω and is governed by the homogeneous Helmholtz equation

$$\Delta_{\mathbf{x}} u + \kappa^2 u = 0 \ \text{ in } \Omega, \tag{1}$$

where κ, [1/m], is a given acoustic wave number which corresponds to certain given frequency f, [Hz]. In order to find unique solution $u(\mathbf{x})$ we have to prescribe the boundary condition on the boundary $\partial\Omega$ of the domain Ω. We consider the case when the boundary condition, for example, of the Neumann type is given, i.e.

$$\frac{\partial u}{\partial \mathbf{n}} = v(\mathbf{s}), \ \ \mathbf{s} \in \partial\Omega, \tag{2}$$

where, the function $v(\mathbf{s})$ is defined.

As we already mentioned above, the Wave Based Method is based on the indirect Trefftz approach, cf. [18], i.e. the approximation $\tilde{u}(\mathbf{x})$ of the dynamic pressure function $u(\mathbf{x})$ satisfies the differential equation (1) exactly. The solution $u(\mathbf{x})$ is approximated in 2D case by the following expression

$$u(x_1,x_2) \approx \tilde{u}(x_1,x_2) = \sum_{j=0}^{N} \left[c_j^{(1)} \Phi_j^{+}(x_1,x_2) + c_j^{(2)} \Phi_j^{-}(x_1,x_2) + c_j^{(3)} \Phi_j^{+}(x_2,x_1) + c_j^{(4)} \Phi_j^{-}(x_2,x_1) \right], \tag{3}$$

where $N \in \mathbb{N}_0$. The wave functions $\Phi_j^{\pm}(x,y)$ satisfy the equation (1) exactly and are given by

$$\Phi_j^{\pm}(x,y) = \cos\frac{j\pi}{L_x} x \ \mathrm{e}^{\pm \mathrm{i}\sqrt{\kappa^2 - \left(\frac{j\pi}{L_x}\right)^2} y}, \tag{4}$$

where the parameter L_x is the size of the smallest box which circumscribes the domain Ω in x coordinate direction. The parameters $c_j^{(1)}$, $c_j^{(2)}$, $c_j^{(3)}$ and $c_j^{(4)}$ are unknown contributions[2]. The approximation $\tilde{u}(\mathbf{x})$ converges towards the exact solution $u(\mathbf{x})$ for N tending towards infinity under sufficient condition that the computational domain Ω is convex, cf. [3], [14], [15], [16], or if $\Omega \subset \mathbb{R}^2$ satisfies the condition of the proposition presented in [8] when Ω is non–convex.

Obviously, the approximation $\tilde{u}(\mathbf{x})$, (3), of the solution $u(\mathbf{x})$ satisfies the homogeneous Helmholtz equation exactly independent of the constants $c_j^{(i)},\ i \in \{1,2,3,4\}$. These constants can be determined by the boundary condition (2). To find these unknown constants the weak formulation of the boundary condition (2) has been considered, i.e.

$$\int_{\partial\Omega} \phi \left(\frac{\partial u}{\partial \mathbf{n}} - v \right) \, dS = 0. \tag{5}$$

This is so–called residual formulation, where ϕ is a "test" function. In order to determine the unknowns we insert the expansion (3) into (5) and choose ϕ alternately equal to one of the wave functions $\Phi_j^{(i)}$, $i \in \{1,2,3,4\},\ j \in \mathbb{N}_0$. Hence, we are able to construct the system of linear equations

$$\mathbf{A}\mathbf{c} = \mathbf{b}, \tag{6}$$

where the components of the matrix $\mathbf{A}$ and of the vector $\mathbf{b}$ are equal to appropriate surface integrals

$$\mathbf{A} : \int_{\partial\Omega} \Phi_j^{(i_1)} \frac{\partial \Phi_n^{(i_2)}}{\partial \mathbf{n}} \, dS, \qquad \mathbf{b} : \int_{\partial\Omega} \Phi_j^{(i_1)} v \, dS, \tag{7}$$

[2]The 3D case is analogous.

where $i_1, i_2 \in \{1,2,3,4\}$, $j,n \in \{0,1,\ldots,N\}$ and the vector $\mathbf{c}$ consists of unknowns constants. $\mathbf{A}$ is symmetric, fully populated, frequency depended, in general, complex matrix, which has rather high condition number when N increases. This can affect final results. Usually, the system (6) is solved by simple Gaussian elimination technique, and for large N values such a strategy can give wrong solution, and no matter how good the integrals (7) have been found. This is, let say, one of the main drawbacks of this methodology.

In this paper, we propose to use real valued wave functions only.

2 Classical Wave Based Method applicability extension

Let us assume that the function $u(\mathbf{x})$ is defined inside the bounded, convex domain $\Omega \subset \mathbb{R}^d$, $d \in \{2,3\}$ and is governed by more general elliptic equation

$$Lu + \kappa u = f(\mathbf{x}) \ \text{ in } \Omega, \tag{8}$$

where κ is some given real number and $Lu := a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^2 u}{\partial y^2}$ in 2D case and $Lu := a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^2 u}{\partial y^2} + c\dfrac{\partial^2 u}{\partial z^2}$ in 3D case, where a, b and c are some real positive numbers. Additionally, supplying boundary conditions on the boundary $\partial\Omega$ of the domain Ω can be defined in the following way

$$\alpha\frac{\partial u}{\partial \mathbf{n}}(\mathbf{s}) = \beta u(\mathbf{s}) - v(\mathbf{s}), \ \ \mathbf{s} \in \partial\Omega. \tag{9}$$

Here, the function $v(\mathbf{s})$ is given, $\mathbf{n}$ is normal to the boundary $\partial\Omega$ vector and the constants α and β have to be chosen according to the type of the boundary condition.

In order to solve the problem (8), (9), which is linear, using the Wave Based Technique we first of all have to apply superposition principle, i.e. we separate the solution $u(\mathbf{x})$ into two parts: homogeneous one $u_{\mathrm{h}}(\mathbf{x})$ and particular one $u_{\mathrm{p}}(\mathbf{x})$. We assume that the particular solution $u_{\mathrm{p}}(\mathbf{x})$ satisfies the equation (8) and the homogeneous solution $u_{\mathrm{h}}(\mathbf{x})$ solves the homogeneous equation

$$Lu_{\mathrm{h}} + \kappa u_{\mathrm{h}} = 0 \ \text{ in } \Omega, \tag{10}$$

under modified boundary condition

$$\alpha\frac{\partial u_{\mathrm{h}}}{\partial \mathbf{n}}(\mathbf{s}) = \beta u_{\mathrm{h}}(\mathbf{s}) - \alpha\frac{\partial u_{\mathrm{p}}}{\partial \mathbf{n}}(\mathbf{s}) + \beta u_{\mathrm{p}}(\mathbf{s}) - v(\mathbf{s}), \ \ \mathbf{s} \in \partial\Omega. \tag{11}$$

Assume we know the particular $u_{\mathrm{p}}(\mathbf{x})$ part and would like to find the homogeneous part $u_{\mathrm{h}}(\mathbf{x})$. As in the case of the homogeneous Helmholtz equation (1) we approximate the solution $u_{\mathrm{h}}(\mathbf{x})$ of (10) in 2D case using the same expression (3). The wave functions are defined as follows

$$\Phi_j^+(x,y) \ = \ \begin{cases} \cos\frac{j\pi}{aL_x}x \ \cos\sqrt{\frac{a}{b}\left(\frac{\kappa}{a} - \frac{j^2\pi^2}{a^2L_x^2}\right)}\, y, & \text{if } \kappa > \dfrac{j^2\pi^2}{aL_x^2} \\ \cos\frac{j\pi}{aL_x}x \ \mathrm{e}^{\sqrt{\frac{a}{b}\left(\frac{j^2\pi^2}{a^2L_x^2} - \frac{\kappa}{a}\right)}\,(y-L_y)}, & \text{otherwise,} \end{cases} \tag{12}$$

$$\Phi_j^-(x,y) \ = \ \begin{cases} \cos\frac{j\pi}{aL_x}x \ \sin\sqrt{\frac{a}{b}\left(\frac{\kappa}{a} - \frac{j^2\pi^2}{a^2L_x^2}\right)}\, y, & \text{if } \kappa > \dfrac{j^2\pi^2}{aL_x^2} \\ \cos\frac{j\pi}{aL_x}x \ \mathrm{e}^{-\sqrt{\frac{a}{b}\left(\frac{j^2\pi^2}{a^2L_x^2} - \frac{\kappa}{a}\right)}\, y}, & \text{otherwise.} \end{cases} \tag{13}$$

Here, the parameters L_x and L_y are again the sizes of the smallest rectangle circumscribing the domain Ω in corresponding coordinate directions. As one can see all Φ functions are real valued and satisfy the differential equation (10). Using the boundary condition (11) written in the weak form, i.e.

$$\int\limits_{\partial\Omega} \phi\left(\alpha\frac{\partial u_{\mathrm{h}}}{\partial \mathbf{n}} + \alpha\frac{\partial u_{\mathrm{p}}}{\partial \mathbf{n}} - \beta u_{\mathrm{h}} - \beta u_{\mathrm{p}} + v\right) dS = 0, \tag{14}$$

one is able to construct the system of linear equations in order to find unknown contributions $c_j^{(1,2,3,4)}$ for $\forall j \in \{1, 2, \ldots, N\}$, cf. (5) – (7). In this case the matrix $\mathbf{A}$ is real valued. The only task we have to carry about is to find particular solution $u_{\mathrm{p}}(\mathbf{x})$.

3 Particular Solution

In order to find a particular solution $u_{\mathrm{p}}(\mathbf{x})$ one can apply different techniques. For example, if some analytical function $f(\mathbf{x})$ is given, then one can try to use certain properties of this function to find corresponding particular solution $u_{\mathrm{p}}(\mathbf{x})$. On the other hand, in practice some discrete values of $f(\mathbf{x})$ are given and in this case an appropriate interpolation can be done. Our aim is to apply a method which finds rather smooth function $u_{\mathrm{p}}(\mathbf{x})$. We assume that the right hand side function $f(\mathbf{x})$ belongs to the space $\mathcal{L}_2(\Omega)$. It means that the function is square integrable over the domain Ω, i.e. $\int\limits_{\Omega} f^2(\mathbf{x})\, dV < \infty$. According to the theory of the elliptic differential equations, [5], the particular solution $u_{\mathrm{p}}(\mathbf{x})$ belongs to Sobolev space $\mathcal{H}^2(\Omega)$, [5], [1].

3.1 Fourier expansion

We assume that the function $f(\mathbf{x}) \in \mathcal{L}_2(\Omega)$ in 2D can be expanded into Fourier series

$$\begin{aligned} f(x_1, x_2) &= \sum_{j,k=0}^{\infty} f_{jk}^{\mathrm{ac}} \cos\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 + f_{jk}^{\mathrm{as}} \sin\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 \\ &+ f_{jk}^{\mathrm{bc}} \cos\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2 + f_{jk}^{\mathrm{bs}} \sin\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2, \end{aligned} \tag{15}$$

where the coefficients f_{jk}^{ac}, f_{jk}^{as}, f_{jk}^{bc} and f_{jk}^{bs} are 2D Fourier coefficients which can easily be found using appropriate well–known formulae, [17]. On the other hand, we assume that the solution $u_{\mathrm{p}} \in \mathcal{H}^2(\Omega)$ in 2D can also be presented in Fourier series form, i.e.

$$\begin{aligned} u_{\mathrm{p}}(x_1, x_2) &= \sum_{j,k=0}^{\infty} a_{jk}^{\mathrm{c}} \cos\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 + a_{jk}^{\mathrm{s}} \sin\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 \\ &+ b_{jk}^{\mathrm{c}} \cos\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2 + b_{jk}^{\mathrm{s}} \sin\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2, \end{aligned} \tag{16}$$

where a_{jk}^{c}, a_{jk}^{s}, b_{jk}^{c} and b_{jk}^{s} are unknown coefficients, yet.
Inserting the above given relations (15) and (16) into the equation

$$Lu_{\mathrm{p}} + \kappa u_{\mathrm{p}} = f, \tag{17}$$

we get relations for unknowns $a_{jk}^{\mathrm{c,s}}$ and $b_{jk}^{\mathrm{c,s}}$, $j, k = 0, 1, \ldots$, i.e.

$$\begin{aligned} \sum_{j,k=0}^{\infty} &\left[\left(\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2\right) a_{jk}^{\mathrm{c}} - f_{jk}^{\mathrm{ac}}\right] \cos\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 \quad + \\ &\left[\left(\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2\right) a_{jk}^{\mathrm{s}} - f_{jk}^{\mathrm{as}}\right] \sin\frac{\pi j}{L_{x_1}} x_1 \cos\frac{\pi k}{L_{x_2}} x_2 \quad + \\ &\left[\left(\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2\right) b_{jk}^{\mathrm{c}} - f_{jk}^{\mathrm{bc}}\right] \cos\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2 \quad + \\ &\left[\left(\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2\right) b_{jk}^{\mathrm{s}} - f_{jk}^{\mathrm{bs}}\right] \sin\frac{\pi j}{L_{x_1}} x_1 \sin\frac{\pi k}{L_{x_2}} x_2 \quad = \quad 0. \end{aligned} \tag{18}$$

Obviously, the relation (18) is true only in the case when all the coefficients vanish. Hence, we find the coefficients

$$a_{jk}^{\mathrm{c,s}} = \frac{f_{jk}^{\mathrm{ac,as}}}{\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2}, \tag{19}$$

$$b_{jk}^{\mathrm{c,s}} = \frac{f_{jk}^{\mathrm{bc,bs}}}{\kappa - a\left(\frac{\pi j}{L_{x_1}}\right)^2 - b\left(\frac{\pi k}{L_{x_2}}\right)^2}, \tag{20}$$

and the solution $u_{\mathrm{p}}(\mathbf{x})$ can be found. Actually, such a representation of the right hand side function $f(\mathbf{x})$, (15), is rather computationally ineffective: to achieve appropriate approximation of $f(\mathbf{x})$ one, probably, has to calculate large amount of Fourier coefficients, i.e. do many 2D integrations which are time consuming operations. On the other hand, we can try to apply *Discrete Cosine Transform* to approximate the right hand side function. This procedure is rather fast. But disadvantage here is that the function $f(\mathbf{x})$ has to be discretized first and then the particular solution of the equation (8) can be found. Here, one has to assume that appropriate discrete cosine transform of the particular solution $u_{\mathrm{p}}(\mathbf{x})$ is done and we search then for corresponding coefficients. Next disadvantage here is the fact that the particular solution is discrete function, hence, probably certain interpolation need to be done. Moreover, some differentiation of $u_{\mathrm{p}}(\mathbf{x})$ difficulties can appear. Therefore, we do not consider any additional details here.

3.2 Uniform B–Spline approximation

From our point of view B–Splines could be an appropriate, elegant approximation of the particular solution $u_{\mathrm{p}}(\mathbf{x})$. For the sake of brevity we omit here definitions and long explanations of the elements commonly used in the theory of B–Splines. All the necessary information our reader can find in [2] or [13]. First of all we assume that the right hand side $f(\mathbf{x})$ in 2D case can be represented as

$$f(\mathbf{x}(s,t)) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,1}(t)N_{j,1}(s)f_{ij}, \tag{21}$$

where $N_{i,1}(t)$ and $N_{j,1}(s)$ are the i–th and j–th B-Spline basis functions of degree 1, respectively, s and t are real parameters and the coefficients f_{ij} are so–called control points of the surface f. These functions are defined as follows

$$N_{i,0}(t) = \begin{cases} 1, & \text{if } T_i \le t < T_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

$$N_{i,p}(t) = \frac{t - T_i}{T_{i+p} - T_i} N_{i,p-1}(t) + \frac{T_{i+p+1} - t}{T_{i+p+1} - T_{i+1}} N_{i+1,p-1}(t). \tag{23}$$

Here the vector $T = \{T_0, T_1, \ldots, T_{n+p+1}\}$ is so–called *knot* vector. The length of the knot vector T one defines taking into account the smoothness order p and the number of the control points n. To construct the surface in 2D case we have to define for two coordinate directions two knot vectors T and S of lengths $p + n + 1$ and $p + m + 1$, respectively. n and m depend on the discretization level of the object in each coordinate direction, [2], [13]. Moreover, we always assume that both knot vectors T and S are equidistant.

In order to find the particular solution $u_{\mathrm{p}}(\mathbf{x})$ of the above presented problem, we assume that this function in 2D has the following representation

$$u_{\mathrm{p}}(\mathbf{x}(s,t)) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,3}(t)N_{j,3}(s)u_{ij}. \tag{24}$$

Our aim is to find the unknown u_{ij} contributions. Taking into account the knot vectors T and S are equidistant, we get the following relations

$$\frac{\partial u_{\mathrm{p}}}{\partial x} = C_x \frac{\partial u_{\mathrm{p}}}{\partial t} = \frac{3C_x}{dT_3} \sum_{i=0}^{n} \sum_{j=0}^{m} \left(N_{i,2}(t) - N_{i+1,2}(t)\right) N_{j,3}(s) u_{ij}, \tag{25}$$

and

$$\frac{\partial u_{\mathrm{p}}}{\partial y} = C_y \frac{\partial u_{\mathrm{p}}}{\partial s} = \frac{3C_y}{dS_3} \sum_{i=0}^{n} \sum_{j=0}^{m} \left(N_{j,2}(s) - N_{j+1,2}(s)\right) N_{i,3}(t) u_{ij}, \tag{26}$$

where $dT_p = T_{i+p} - T_i$ and $dS_p = S_{i+p} - S_i$ for $\forall i$ and obviously $C_x = \left(\frac{\partial x}{\partial t}\right)^{-1} = \mathrm{const}_x$ and $C_y = \left(\frac{\partial y}{\partial s}\right)^{-1} = \mathrm{const}_y$. Moreover, obviously $\frac{\partial x}{\partial s} = 0,\ \forall s$ and $\frac{\partial y}{\partial t} = 0,\ \forall t$. Further we find the second derivatives

$$\frac{\partial^2 u_{\mathrm{p}}}{\partial x^2} = C_x^2 \frac{\partial^2 u_{\mathrm{p}}}{\partial t^2} = \frac{6C_x^2}{dT_3 dT_2} \sum_{i=0}^{n} \sum_{j=0}^{m} \left(N_{i,1}(t) - 2N_{i+1,1}(t) + N_{i+2,1}(t)\right) N_{j,3}(s) u_{ij}, \tag{27}$$

and

$$\frac{\partial^2 u_{\mathrm{p}}}{\partial y^2} = C_y^2 \frac{\partial^2 u_{\mathrm{p}}}{\partial s^2} = \frac{6C_y^2}{dS_3 dS_2} \sum_{i=0}^{n} \sum_{j=0}^{m} \left(N_{j,1}(s) - 2N_{j+1,1}(s) + N_{j+2,1}(s)\right) N_{i,3}(t) u_{ij}. \tag{28}$$

Using the above presented relations (21), (23), (24) and (27), (28) and substituting them into (17), collecting equal B–Splines we can construct a system of linear equations, where the appropriate matrix is symmetric and in 2D case has 5 diagonal structure. Here, we have to note that the knot vectors T and S for both functions $u_{\mathrm{p}}(\mathbf{x})$ and $f(\mathbf{x})$ have to have equal number of knots, i.e. $n+3+1$ and $m+3+1$, respectively. This means that in order to find all necessary control points u_{ij} one has to provide additional control points for f function.

Of course, one can use uniform B–Splines of higher order $p > 1$ to approximate the right hand side function f. But in this case some additional time and computational effort have to be spend to calculate control points f_{ij}. Other difficulty may appear if the computational domain Ω has non–rectangular shape. This means that the function f outside the domain Ω is not defined and has to be extended until the smallest rectangle which is circumscribing Ω. According to the theory of Sobolev spaces this always is possible[3], [1].

4 Numerical examples

To show that our ideas really work we present a couple of simple examples, namely, we consider so–called light diffusion approximation to the transport theory. Assume unknown function $\phi \in \mathcal{H}^2(\Omega)$ is governed by the equation (29) under boundary condition (30), i.e.

$$\nabla \left(D \nabla \phi\right) - \mu_{\mathrm{a}} \phi = -Q \ \text{ in } \Omega, \tag{29}$$

$$\frac{\partial \phi}{\partial \mathbf{n}} = 0 \ \text{ on } \partial\Omega. \tag{30}$$

Here, $\phi,\ [\mathrm{W\ cm}^{-2}]$, is the light irradiance, $\mu_{\mathrm{a}},\ [\mathrm{cm}^{-1}]$, is the absorption coefficient, $Q,\ [\mathrm{W\ cm}^{-3}]$, is the source term, D is the diffusion coefficient defined by $\left(3\left(\mu_{\mathrm{a}} + \mu_{\mathrm{s}}\left(1-g\right)\right)\right)^{-1}$, where $\mu_{\mathrm{s}},\ [\mathrm{cm}^{-1}]$, and $g,\ [–]$, are the scattering coefficient and the anisotropy factor, respectively. The above presented relations coupled with the so–called bio–heat equation, which we will not consider here, are used to model the damage of the

[3] under appropriate circumstances.

tumor tissues in a liver, [11]. The anisotropy factor takes its values for most biological tissues in range 0.7 and 0.99. The absorption and the scattering coefficients in reality depend on the temperature. We assume that these coefficients are constant, i.e. both μ_{a} and μ_{s} coefficients are so–called *native*, [11]. Let us choose, for example, parameters of the porcine liver. In the literature the native absorption coefficient μ_{a} can be equal to 0.195 [cm^{-1}], native scattering coefficient $\mu_{\mathrm{s}} = 43.50$ [cm^{-1}] and the anisotropy factor $g = 0.9$. Moreover, let us assume that the right hand side term

$$Q = 27\mathrm{e}^{-\frac{(x-5)^2+(y-5.5556)^2}{20}}, \text{ [W cm}^{-3}\text{]}. \tag{31}$$

The constant function on the right hand side would make the whole problem rather trivial, i.e., obviously, the particular solution in this case would be equal to a constant. Next, we define computational domain[4] Ω which satisfies the convergence criterion, [3], [14], [15], [16], [8]. The computational domain we have chosen is depicted below, cf. Figure 1, where as $\partial\Omega$ so–called *bean curve* has been used. As one can see the iso–

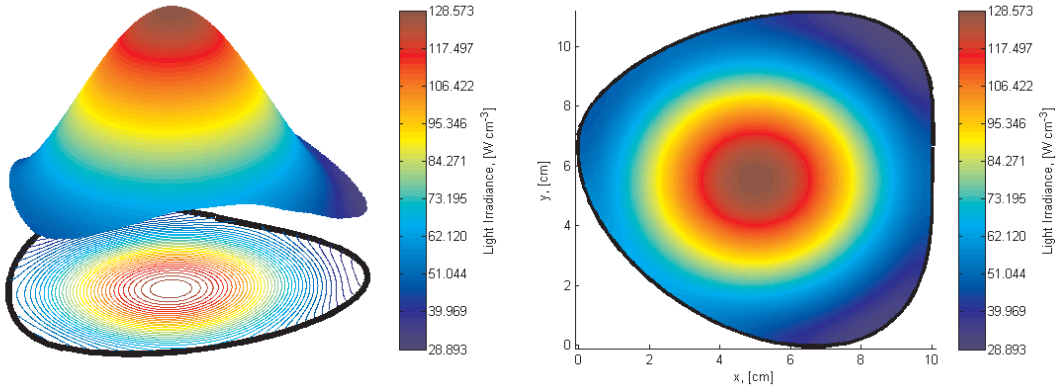

Figure 1: Numerical wave based solution of the problem (29), (30) and (31). The wave functions amount $(N+1)$ is 8 per coordinate direction.

lines of the numerical solution are orthogonal to the boundary of the domain Ω. It means that the boundary condition (30) is fulfilled. To check the correctness of the presented numerical wave based solution we have done the same numerical simulation using the commercial package COMSOL, where the finite element are used. We have chosen 60 points in the domain Ω and compared both numerical solutions, i.e. we measured the relative errors between these tho solutions, cf. Figure 3 on the left. As one can see the maximal relative error between two numerical solutions is less than 2.5%. Hence, the wave based method works and is not worse than finite element based methods[5].

For the second example we choose another right hand side function, namely, we take some non–continuous data which represents kind of chess board picture. In this case the physical interpretation does not make much sense but from the point of view of the numerical methodology it should not be important what exactly we solve. Therefore, let us choose Q as

$$Q = \left| \frac{\mathrm{sign}(5-x)\mathrm{sign}(5.5556-y)+1}{2} - \frac{\mathrm{sign}(7.5-x)\mathrm{sign}(y-2.7778)+1}{2} \right| + 1, \text{ [W cm}^{-3}\text{]}. \tag{32}$$

Obviously, the function Q is discontinuous. As one can see the results also in this case are in "a right way". The iso–lines are orthogonal to the boundary what corresponds to the homogeneous Neumann boundary condition. The solution itself is "diffusive", what belongs to the nature of solutions of "good" elliptic type differential equations. As in the previous example we compared the numerical wave based solution with the numerical finite element based solution. At 60 points we calculated relative errors between two solutions. The maximal error value does not exceed 3%, cf. Figure 3 on the right. We have to note that in the case of

[4] non–rectangular, for example.

[5] at least relative to this example.

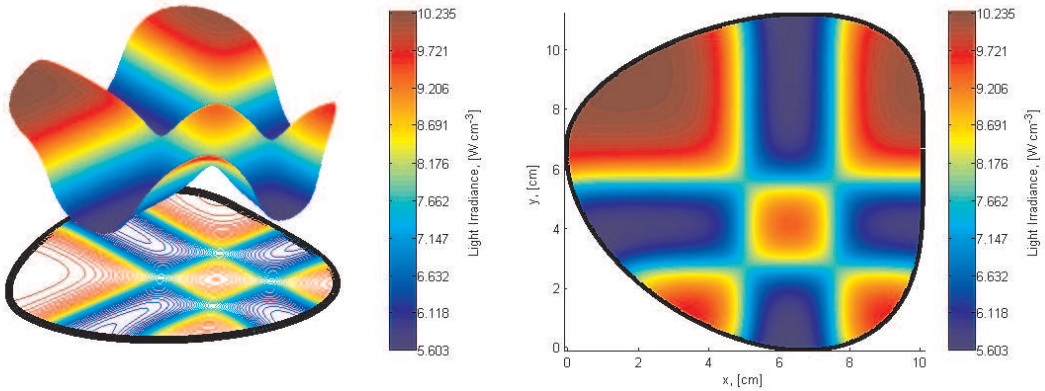

Figure 2: Numerical wave based solution of the problem (29), (30) but with another source function Q, see (32). The wave functions amount $(N+1)$ is 8 per coordinate direction.

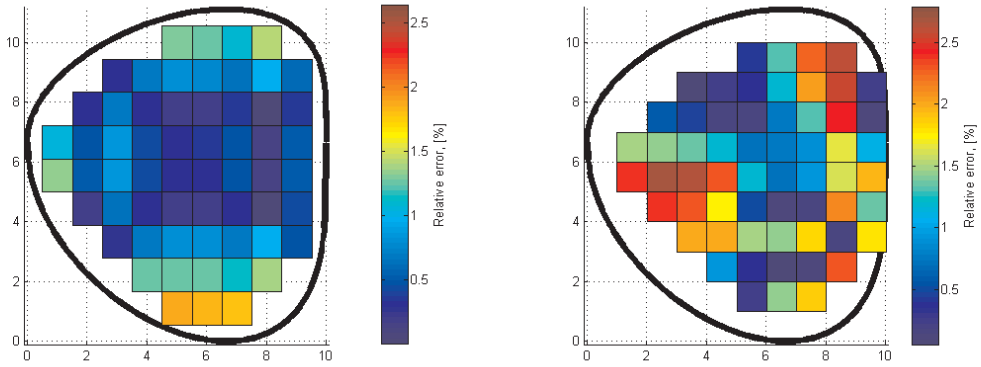

Figure 3: Relative error (right) between the COMSOL results based on the finite elements and the numerical wave based solution of (29), (30) and (31). The wave functions amount $(N+1)$ is 8 per coordinate direction. Relative error (left) between the COMSOL results based on the finite elements and the numerical wave based solution of (29), (30) and (32). The wave functions amount $(N+1)$ is 8 per coordinate direction.

finite elements rather fine discretization of the computational domain has been done, otherwise, because of discontinuous Q, relative rough solution has been found. The amount of the wave and B–Spline functions was the same as in the first example.

5 Conclusion

In this work we present the possibility to extend the applicability of the wave based method, which has been designed especially for acoustic areas, to more general class of elliptic differential equations for which the maximum principle can be valid. This fact gives the possibility to avoid domain discretization what is rather important in 3D cases. Moreover, the sizes of appropriate governing matrices are relative small and, as presented here, are real valued. Thus, some simple direct solver can be applied to find the numerical solution of a problem. We showed how the inhomogeneity can be evaluated. There are several ways to do this, but from our point of view, the uniform B–Splines approach is more preferred. We presented also two examples with different right hand side functions: one of them was sufficiently smooth and another one was discontinuous function. In both cases we found the numerical solution of the light diffusion equation and compared these wave based solutions with corresponding finite element based solutions calculated with the aid of COMSOL package. 60 ”control” points have been chosen and at each such a point the relative error

has been calculated. In both examples the maximal relative error among all relative errors did not exceed 2.5% and 3%, respectively. Both solutions look diffusive what represent the elliptic nature of the solutions.

We considered only the cases when all corresponding coefficients, i.e. a, b and κ, are constants. This fact makes, of course, the whole problem much simpler. Next step of our research to reflect the real nature would be to deal exactly the variable coefficients with. This is, in particular, of big interest in geomathematics, where the waves propagations in rock layers have to be calculated, However, there already exist the theory developed by Prof. Dr. Willi Freeden et al, [4], [12]. Let us also note that the theory of wave based method in 3D case does not differ much from 2D case. One has to modify old and provide additional base functions for third coordinate direction.

The aim of this work was to show that wave based method, developed by Prof. Dr. Wim Desmet et al for acoustics in bounded domains, can be applied also in non–acoustic areas. Finally, we would like to note that this methodology gives numerical solution which exactly satisfies the governed differential equation. Even in cases, if an inhomogeneous right hand side is present, we always get exact particular solution corresponding to the approximation of the source side, because of the natural continuous differentiation of sufficiently smooth B–Spline functions. In other words, we do not change much the differential equation itself, in comparison with the finite element methods, where the numerical solution satisfies the discretized version of the original equation.

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