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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

A proof of convergence of a finite volume scheme for modified steady Richards' equation describing transport processes in the pressing section of a paper machine

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1 Introduction

A number of water flow problems in porous media are modelled by Richards' equation [1]. There exist a lot of different applications of this model. We are concerned with the simulation of the pressing section of a paper machine. This part of the industrial process provides the dewatering of the paper layer by the use of clothings, i.e. press felts, which absorb the water during pressing [2]. A system of nips are formed in the simplest case by rolls, which increase sheet dryness by pressing against each other (see Figure 1).

A lot of theoretical studies were done for Richards' equation (see [3], [4] and references therein). Most articles consider the case of x-independent coefficients. This simplifies the system considerably since, after Kirchhoff's transformation of the problem, the elliptic operator becomes linear. In our case this condition is not satisfied and we have to consider nonlinear operator of second order.

Moreover, all these articles are concerned with the nonstationary problem, while we are interested in the stationary case. Due to complexity of the physical process our problem has a specific feature. An additional convective term appears in our model because the porous media moves with the constant velocity through the pressing rolls. This term is zero in immobile porous media. We are not aware of papers, which deal with such kind of modified steady Richards' problem.

The goal of this paper is to obtain the stability results, to show the existence of a solution to the discrete problem, to prove the convergence of the approximate solution to the weak solution of the modified steady Richards' equation, which describes the transport processes in the pressing section. In Section 2 we present the model which we consider. In Section 3 a numerical scheme obtained by the finite volume method is given. The main part of this paper is theoretical studies, which are given in Section 4. Section 5 presents a numerical experiment. The conclusion of this work is given in Section 6.

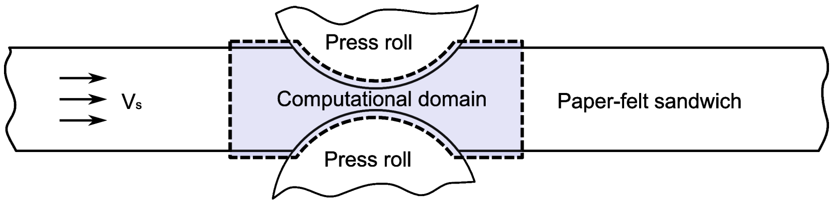

Figure 1: Schematic representation of the pressing section of a paper machine

2 The Model

To model the pressing section, Richards' equation, the Darcy's law in the case of moving porous media and neglected gravity term and a capillary pressure-saturation relation are used (for more detailed explanations see [5]):

$$\frac{\partial(\phi S \rho)}{\partial t} + \operatorname{div}(\phi S \rho V) = 0, \tag{1}$$

$$\phi S(V - V_s) = -\frac{k_r}{\mu} K \operatorname{grad} p, \tag{2}$$

$$p = -p_c(S), \tag{3}$$

where ϕ ($[-]$) is the porosity, S ($[-]$) is the saturation of water, ρ is the density of water measured in $[kg/m^3]$, t is the time in $[s]$, V is the velocity of water in $[m/s]$, V_s is the velocity of solid in $[m/s]$, k_r ($[-]$) is the relative permeability, μ is the viscosity of the water in $[Pa \cdot s]$, K is the intrinsic permeability tensor in $[m^2]$, p is the pressure of water in $[Pa]$.

We assume that water is incompressible and the partial derivative with respect to time is equal to zero since we are interested in a steady state solution. Then, we obtain:

$$-\operatorname{div}\left(\frac{k_r}{\mu} K \operatorname{grad} p\right) + \operatorname{div}(\phi S V_s) = 0 \tag{4}$$

together with equation (3).

In one dimensional case we obtain the model by averaging of the two-dimensional one. Therefore, a thickness of the layer $d(x)$ is included in the model [5], [6]. This function is non-constant (see Figure 1). If we assume that function (3) is invertible, then problem (3), (4) yields:

$$-\frac{\partial}{\partial x}\left(d(x)\frac{k_r(S(p))}{\mu}K(\phi(x))\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(d(x)\phi(x)V_s S(p)\right) = 0, \tag{5}$$

where $d(x)$ is the thickness of the layer measured in $[m]$.

Let us define $b(x) = d(x)K(\phi(x))/\mu$ and $q(x) = d(x)\phi(x)V_s$. Then, the nonlinear convection-diffusion problem (5) yields:

$$-\frac{\partial}{\partial x}\left(b(x)k(S(p))\frac{\partial p}{\partial x}\right) + \frac{\partial(q(x)S(p))}{\partial x} = 0, \quad x \in (0,1) \tag{6}$$

with boundary conditions:

$$p(0) = P_0, \quad \left.\frac{\partial p}{\partial x}\right|_{x=1} = 0 \tag{7}$$

and given constitutive relations:

$$S = S(p), \tag{8}$$

$$k = k(S), \tag{9}$$

where

Assumption 1.

- *$b(x) \in C([0,1])$, $b(x) > 0$;*
- *$q(x) \in C([0,1])$, $q(x) \geq 0$;*
- *$S \in C(\mathbb{R})$, $S : \mathbb{R} \to [S_*, 1]$, where $S_* \in \mathbb{R}$ and $S_* > 0$,*
- *$k \in C([S_*, 1])$, $k : [S_*, 1] \to [k_*, 1]$ is an increasing function, where $k_* \in \mathbb{R}$ and $k_* > 0$.*

For simplicity we apply variable transformation $p = y + P_0$, then we obtain:

$$-\frac{\partial}{\partial x}\left(b(x)k(S(y+P_0))\frac{\partial y}{\partial x}\right) + \frac{\partial(q(x)S(y+P_0))}{\partial x} = 0, \quad x \in (0,1), \tag{10}$$

$$y(0) = 0, \quad \left.\frac{\partial y}{\partial x}\right|_{x=1} = 0. \tag{11}$$

Let us introduce a subspace of $H^1((0,1))$ denoted by $H^1_{0-}((0,1))$ such that:

$$H^1_{0-}((0,1)) := \{\, f \in H^1((0,1)) \big| \, f(0) = 0\}. \tag{12}$$

Then, the weak formulation of problem (10),(11) with $y \in H^1_{0-}((0,1))$ yields:

$$\int_0^1 b(x)k(S(y+P_0))\frac{\partial y}{\partial x}\frac{\partial \varphi}{\partial x}dx - \int_0^1 q(x)S(y+P_0)\frac{\partial \varphi}{\partial x}dx + q(1)S(y(1)+P_0)\varphi(1) = 0, \tag{13}$$

which is satisfied for all $\varphi \in C^\infty((0,1))$ such that $\varphi(0) = 0$.

3 Discretization

To obtain second-order discretization scheme let us introduce a mesh on [0,1].

Definition 1. *The mesh on $[0,1]$ denoted by $\mathcal{T}$ is given by a family $(K_i)_{i=\overline{0,N}}$, $N \in \mathbb{N}^+$ such that:*

$$K_0 = (x_0, x_{\frac{1}{2}}), \tag{14}$$

$$K_i = (x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}), \quad i = \overline{1, N-1}, \tag{15}$$

$$K_N = (x_{N-\frac{1}{2}}, x_N) \tag{16}$$

and families:

$$\bar{\omega}_1 = \{x_i = ih, \; i = \overline{0,N}, \; h = 1/N\},$$

$$\bar{\omega}_2 = \{x_{i+\frac{1}{2}} = \left(i + \frac{1}{2}\right)h, \; i = \overline{0, N-1}\},$$

where h is the size of the mesh.

Discretizing equation (10) by finite volumes we obtain:

$$
\begin{aligned}
- b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1} - y_i}{h} + b_{i-\frac{1}{2}} k_{i-\frac{1}{2}} \frac{y_i - y_{i-1}}{h} \\
+ (q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} - q_{i-\frac{1}{2}} S_{i-\frac{1}{2}}) = 0, i = 1, ..., N-1. \quad (17)
\end{aligned}
$$

Integrating equation (10) over K_N it yields:

$$
\begin{aligned}
&\int\limits_{x_{N-\frac{1}{2}}}^{x_N} \left[-\frac{\partial}{\partial x} \left(b(x) k(S(y+P_0)) \frac{\partial y}{\partial x} \right) + \frac{\partial}{\partial x} (q(x) S(y+P_0)) \right] dx \\
&\qquad = - \left[b(x) k(S(y+P_0)) \frac{\partial y}{\partial x} \right] \Bigg|_{x_{N-\frac{1}{2}}}^{x_N} + [q(x) S(y+P_0)]|_{x_{N-\frac{1}{2}}}^{x_N} .
\end{aligned}
$$

Using the boundary conditions (11) and the central differences we obtain the following approximation:

$$
y_0 = 0, \tag{18}
$$

$$
b_{N-\frac{1}{2}} k_{N-\frac{1}{2}} \frac{y_N - y_{N-1}}{h} + (q_N S_N - q_{N-\frac{1}{2}} S_{N-\frac{1}{2}}) = 0, \tag{19}
$$

where

$$
k_{i+\frac{1}{2}} = k(S_{i+\frac{1}{2}}), \tag{20}
$$

$$
b_{i+\frac{1}{2}} = \left(\frac{1}{h} \int_{x_i}^{x_{i+1}} \frac{dx}{b(x)} \right)^{-1}, \tag{21}
$$

$$
q_{i+\frac{1}{2}} = \left(\frac{1}{h} \int_{x_i}^{x_{i+1}} \frac{dx}{q(x)} \right)^{-1} \tag{22}
$$

and for the approximation of (8) different choices are possible, for example:

$$
S_{i+\frac{1}{2}} = \frac{1}{2} (S(y_i + P_0) + S(y_{i+1} + P_0)), \quad i = \overline{0, N-1}, \tag{23a}
$$

$$
S_{i+\frac{1}{2}} = S \left(\frac{y_i + y_{i+1}}{2} + P_0 \right), \quad i = \overline{0, N-1}, \tag{23b}
$$

$$
S_N = S(y_N + P_0). \tag{24}
$$

4 Proof of Convergence

In order to obtain a convergence for this scheme (Theorem 1), we should prove the existence and convergence for $h \to 0$ of the solution of (17)-(24) (Lemma 2 and 3). To achieve these results, first, we should obtain an estimate (Lemma 1).

The following lemmas and theorem are proved using a technique which was used in [7] for a semilinear elliptic problem:

$$
\begin{aligned}
-u_{xx}(x) &= f(x, u(x)), \quad x \in (0, 1); \\
u(0) &= u(1) = 0.
\end{aligned}
$$

Remark 1. *Due to complexity of the presented nonlinear problem there are difficulties for obtaining uniqeness results for both continuous and discrete problems. In this work we are not concerned with this aspect of the problem.*

Lemma 1. (Stability result) (see [7], page 28, Lemma 2.3) *Let Assumption 1 be satisfied and let $\mathcal{T}$ be the mesh on $[0,1]$ (see Definition 1). If there exists $(y_0, y_1, ..., y_N) \in \mathbb{R}^{N+1}$ a solution of (17)-(24), then it satisfies:*

$$\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h} \le C. \tag{25}$$

Proof. Multiplying (17) by y_i and (19) by y_N and summing over $i = \overline{0,N}$ this yields:

$$\sum_{i=1}^{N-1} \left(-b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1}-y_i}{h} + b_{i-\frac{1}{2}} k_{i-\frac{1}{2}} \frac{y_i - y_{i-1}}{h} \right) y_i + \sum_{i=1}^{N-1} (q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} - q_{i-\frac{1}{2}} S_{i-\frac{1}{2}}) y_i + \left(b_{N-\frac{1}{2}} k_{N-\frac{1}{2}} \frac{y_N - y_{N-1}}{h} \right) y_N + (q_N S_N - q_{N-\frac{1}{2}} S_{N-\frac{1}{2}}) y_N = 0.$$

Reordering summation and taking into account that $y_0 = 0$, we have:

$$\sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{(y_{i+1}-y_i)^2}{h} - \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} (y_{i+1}-y_i) + q_N S_N y_N = 0$$

and consequently:

$$\sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{(y_{i+1}-y_i)^2}{h} \le \left| \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} (y_{i+1}-y_i) \right| + |q_N S_N y_N|. \tag{26}$$

By the extreme value theorem [8] and Assumption 1 there exist b_* and b^* such that $0 < b_* \le b(x) \le b^*$ for all $x \in [0,1]$ and consequently $b_* \le b_{i+\frac{1}{2}} \le b^*$ for all $i = \overline{0, N-1}$ and there exist q_* and q^* such that $0 \le q_* \le q(x) \le q^*$ for all $x \in [0,1]$ and consequently $q_* \le q_{i+\frac{1}{2}} \le q^*$ for all $i = \overline{0, N-1}$. Let us remark that $0 < k_* \le k_{i+\frac{1}{2}} \le 1$, for all $i = \overline{0, N-1}$. Now we consider the first term of (26):

$$\sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{(y_{i+1}-y_i)^2}{h} \ge b_* k_* \sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h}. \tag{27}$$

From the second part of (26) and from the Cauchy-Schwarz inequality we obtain:

$$\left| \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} (y_{i+1}-y_i) \right| \le \left(\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h} \sum_{i=0}^{N-1} (q_{i+\frac{1}{2}} S_{i+\frac{1}{2}})^2 h \right)^{\frac{1}{2}}.$$

Then, using the inequality for $q(x)$ and the facts that $S_{i+\frac{1}{2}} \in [S_*, 1]$ for all $i = \overline{0, N-1}$ and $\sum_{i=0}^{N-1} h = 1$, we have:

$$\left| \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} (y_{i+1}-y_i) \right| \le q^* \left(\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h} \right)^{\frac{1}{2}}. \tag{28}$$

The third term yields:

$$|q_N S_N y_N| \le q^*|y_N| = q^*|y_N - y_0| \le q^* \sum_{i=0}^{N-1} |y_{i+1} - y_i|$$

and from the Cauchy-Schwarz inequality we obtain:

$$|q_N S_N y_N| \le q^* \left(\sum_{i=0}^{N-1} \frac{(y_{i+1} - y_i)^2}{h} \right)^{\frac{1}{2}}. \tag{29}$$

Then (26), (27), (28) and (29) yield, with $C = \left(\frac{2q^*}{b_* k_*}\right)^2$:

$$\sum_{i=0}^{N-1} \frac{(y_{i+1} - y_i)^2}{h} \le C. \tag{30}$$

□

Lemma 2. (Existence of solution) (see [7], page 28, Lemma 2.3) *Let Assumption 1 be satisfied and let $\mathcal{T}$ be the mesh on $[0,1]$ (see Definition 1). Then there exists $\mathbf{y} = (y_0, y_1, ..., y_N) \in \mathbb{R}^{N+1}$, a solution of (17)-(24).*

Proof. Let $\mathbf{v} = (v_0, v_1, ..., v_N) \in \mathbb{R}^{N+1}$, then it is easy to show that there exists a unique $\mathbf{y} = (y_0, y_1, ..., y_N) \in \mathbb{R}^{N+1}$, the solution of (17)-(19) with (20)-(22) and instead of (23a)-(24) we will use:

$$S_i = S(v_i), \ \ i = \overline{0, N}, \tag{31}$$

and an appropriate approximation for $S_{i+\frac{1}{2}}$, $i = \overline{0, N-1}$.

It means that there exists a continuous application F from $\mathbb{R}^{N+1}$ to $\mathbb{R}^{N+1}$ such that $\mathbf{y} = F(\mathbf{v})$ and $(y_0, y_1, ..., y_N)$ is a solution of (17)-(24) if and only if $\mathbf{y} = (y_0, y_1, ..., y_N)$ is a fixed point of F.

Let us introduce the discrete L_2-norm:

$$||\mathbf{v}||_{L_2((0,1))} = \left(\sum_{i=0}^{N} v_i^2 h \right)^{\frac{1}{2}} \ \text{for } \mathbf{v} = (v_0, v_1, ..., v_N) \in \mathbb{R}^{N+1}, \ v_0 = 0. \tag{32}$$

Now we are going to prove the next inequality:

$$||\mathbf{v}||_{L_2((0,1))} \le \left(\sum_{i=0}^{N-1} \frac{(v_{i+1} - v_i)^2}{h} \right)^{\frac{1}{2}}. \tag{33}$$

For $|v_i|$ using the triangle inequality and the Cauchy-Schwarz inequality we have:

$$|v_i| \le \sum_{j=0}^{i-1} |v_{j+1} - v_j| \le \sum_{j=0}^{N-1} |v_{j+1} - v_j| \le \left(\sum_{j=0}^{N-1} \frac{(v_{j+1} - v_j)^2}{h} \right)^{\frac{1}{2}}, \ \text{for all } i = \overline{0, N};$$

then:

$$||\mathbf{v}||_{L_2((0,1))} = \left(\sum_{i=0}^{N} v_i^2 h\right)^{\frac{1}{2}} \underset{(v_0=0)}{=} \left(\sum_{i=1}^{N} v_i^2 h\right)^{\frac{1}{2}} \leq \left(\sum_{i=1}^{N} h \sum_{j=0}^{N-1} \frac{(v_{j+1}-v_j)^2}{h}\right)^{\frac{1}{2}}. \tag{34}$$

Thereby, (33) is proved.

Note, that inequality (25) is also true for (17)-(19) with (20), (22) and (31). Then (33) together with (25) gives

$$||F(\mathbf{v})||_{L_2((0,1))} = ||\mathbf{y}||_{L_2((0,1))} \leq \hat{C} \text{ for all } ||\mathbf{v}||_{L_2((0,1))} \leq \hat{C},$$

where $\hat{C} = C^{\frac{1}{2}}$. This means $F(B_{\hat{C}}) \subset B_{\hat{C}}$, where $B_{\hat{C}}$ is a closed ball of radius $\hat{C}$ and center 0 in $\mathbb{R}^{N+1}$. Then thanks to the Brouwer's fixed point theorem [9], F has a fixed point in $B_{\hat{C}}$. This fixed point is a solution to (17)-(24). Thereby, existence is proved. □

Lemma 3. (Compactness) (see [7], page 29, Lemma 2.4) *Let Assumption 1 be satisfied and let $\mathcal{T}$ be a mesh on $[0,1]$ (see Definition 1). Let $(y_0, y_1, ..., y_N) \in \mathbb{R}^{N+1}$ be a solution of (17)-(24) and let $y_{\mathcal{T}} : (0,1) \to \mathbb{R}$ by $y_{\mathcal{T}}(x) = y_i$ if $x \in K_i$, $i = \overline{0, N}$. Then the set $y_{\mathcal{T}}$ for all $\mathcal{T}$ is relatively compact in $L^2((0,1))$. Furthermore, if $y_{\mathcal{T}_n} \to y$ in $L^2((0,1))$ and $h_n \to 0$, as $n \to \infty$, then, $y \in H^1_{0-}((0,1))$.*

Proof. By the Kolmogorov compactness theorem (see [7], page 93, Theorem 3.9), it is sufficient to show that $y_{\mathcal{T}}$ is relatively compact in $L^2((0,1))$:

- the set $y_{\mathcal{T}}$ is bounded in $L^2(\mathbb{R})$ for all $\mathcal{T}$,
- $||y_{\mathcal{T}}(\cdot + \nu) - y_{\mathcal{T}}||_{L^2(\mathbb{R})} \to 0$ as $\nu \to 0$ uniformly.

Step 1. Function $y_{\mathcal{T}}(x)$ can be redefined as $y_{\mathcal{T}}(x) = y_i$ if $x \in K_i$, $i = \overline{0, N}$ otherwise $y_{\mathcal{T}}(x) = 0$. Using the facts that $y_0 = 0$, the Cauchy-Schwarz inequality and estimate (25), for all $x \in \mathbb{R}$ we have:

$$|y_{\mathcal{T}}(x)| \leq \sum_{i=0}^{N-1} |y_{i+1} - y_i| \leq \left(\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h}\right)^{\frac{1}{2}} \leq \hat{C}. \tag{35}$$

This means that the set $y_{\mathcal{T}}(x)$ for all $\mathcal{T}$ is bounded in $L^2(\mathbb{R})$.

Step 2. Let $0 < \nu < 1$. We define $\chi_{i+\frac{1}{2}} : \mathbb{R} \to \mathbb{R}$ for $i = \overline{0, N-1}$ such that $\chi_{i+\frac{1}{2}}(x) = 1$ if $x_{i+\frac{1}{2}} \in [x, x+\nu]$ and $\chi_{i+\frac{1}{2}}(x) = 0$ if $x_{i+\frac{1}{2}} \notin [x, x+\nu]$ and $\chi_{N+\frac{1}{2}}(x) = 1$ if $x_N \in [x, x+\nu]$ and $\chi_{N+\frac{1}{2}}(x) = 0$, otherwise.

Then, for all $x \in \mathbb{R}$ we have:

$$\begin{aligned}
(y_{\mathcal{T}}(x+\nu) - y_{\mathcal{T}}(x))^2 &\leq \left(\sum_{i=0}^{N-1} |y_{i+1} - y_i| \chi_{i+\frac{1}{2}}(x) + y_N \chi_{N+\frac{1}{2}}(x)\right)^2 \\
&\leq 2\left(\sum_{i=0}^{N-1} |y_{i+1} - y_i| \chi_{i+\frac{1}{2}}(x)\right)^2 + 2y_N^2 \chi_{N+\frac{1}{2}}(x) \\
&\leq 2\left(\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{h} \chi_{i+\frac{1}{2}}(x)\right)\left(\sum_{i=0}^{N-1} h \chi_{i+\frac{1}{2}}(x)\right) + 2y_N^2 \chi_{N+\frac{1}{2}}(x).
\end{aligned} \tag{36}$$

Integrating (36) over $\mathbb{R}$ we obtain:

$$\begin{aligned}\|y_{\mathcal{T}}(\cdot+\nu)-y_{\mathcal{T}}\|^2_{L^2(\mathbb{R})} &\le \int_{\mathbb{R}} 2\left(\sum_{i=0}^{N-1}\frac{(y_{i+1}-y_i)^2}{h}\chi_{i+\frac{1}{2}}(x)\right)\left(\sum_{i=0}^{N-1}h\chi_{i+\frac{1}{2}}(x)\right)dx\\
&\quad+\int_{\mathbb{R}}2y_N^2\chi_{N+\frac{1}{2}}(x)dx\\
&\le 2(\nu+2h)\int_{\mathbb{R}}\left(\sum_{i=0}^{N-1}\frac{(y_{i+1}-y_i)^2}{h}\chi_{i+\frac{1}{2}}(x)\right)dx\\
&\quad+2\hat{C}^2\int_{\mathbb{R}}\chi_{N+\frac{1}{2}}(x)dx\\
&=2(\nu+2h)\sum_{i=0}^{N-1}\frac{(y_{i+1}-y_i)^2}{h}\int_{\mathbb{R}}\chi_{i+\frac{1}{2}}(x)dx+2C\nu\\
&\le 2C(\nu+2h)\int_{\mathbb{R}}\chi_{i+\frac{1}{2}}(x)dx+2C\nu=2C\nu(\nu+2h+1).
\end{aligned}\tag{37}$$

Since that $h<1$ and $\nu<1$, we conclude:

$$\|y_{\mathcal{T}}(\cdot+\nu)-y_{\mathcal{T}}\|^2_{L^2(\mathbb{R})}\le 8C\nu. \tag{38}$$

Thereby, the second condition in the Kolmogorov compactness theorem is proved.

Step 3. Let us prove that $(y_{\mathcal{T}_n}(x+h_n)-y_{\mathcal{T}_n}(x))/h_n$ converges to $\partial y/\partial x$ for all $x\in(-\infty,1)$ in a weak sense and $h_n\to 0$, as $n\to\infty$. Let $\psi\in C_0^\infty((-\infty,1))$ and $\operatorname{supp}\psi\subset(0,1)$. The discrete function $\psi_{\mathcal{T}}$ is defined in the following way:

$$\psi_{\mathcal{T}}(x)=\begin{cases}\psi_i=\psi(x_i),\ \text{if } x\in K_i,\ i=\overline{0,N};\\ 0,\ \text{otherwise.}\end{cases}$$

Let us redefine the function $y_{\mathcal{T}}(x)$ such that if $x\in[x_N,x_{N+\frac{1}{2}}]$ than $y_{\mathcal{T}}(x)=y_N$ and if $x\in(x_{N+\frac{1}{2}},x_{N+\frac{3}{2}}]$ than $y_{\mathcal{T}}(x)=y_{N+1}=y_N$, then we obtain:

$$\begin{aligned}\left(\frac{y_{\mathcal{T}_n}(\cdot+h_n)-y_{\mathcal{T}_n}}{h_n},\psi_{\mathcal{T}_n}\right)_{L_2((-\infty,1))}&=\int_{-\infty}^{1}\frac{y_{\mathcal{T}_n}(x+h_n)-y_{\mathcal{T}_n}(x)}{h_n}\psi_{\mathcal{T}_n}dx\\
&=\sum_{i=0}^{N}\frac{y_{i+1}-y_i}{h_n}\psi_ih_n\\
&=-\sum_{i=0}^{N-1}y_{i+1}\frac{\psi_{i+1}-\psi_i}{h_n}h_n\\
&=-\sum_{i=0}^{N}y_i\frac{\psi_i-\psi_{i-1}}{h_n}h_n\\
&=-\int_{-\infty}^{1}y_{\mathcal{T}_n}(x)\frac{\psi_{\mathcal{T}_n}(x)-\psi_{\mathcal{T}_n}(x-h_n)}{h_n}dx.
\end{aligned}\tag{39}$$

The function $\frac{y_{\mathcal{T}_n}(\cdot+h_n)-y_{\mathcal{T}_n}}{h_n}$ is bounded in $L_2(\mathbb{R})$ (see (25)). Then, for any sequence of meshes $(\mathcal{T}_n)_{n\in\mathbb{N}}$ such that $h_n\to 0$, as $n\to\infty$, there exists a subsequence, still denoted by $(\mathcal{T}_n)_{n\in\mathbb{N}}$, such that function $\frac{y_{\mathcal{T}_n}(\cdot+h_n)-y_{\mathcal{T}_n}}{h_n}$ weakly converges to some function $g(x)$. We also know that $y_{\mathcal{T}_n} \to y$ in $L^2((-\infty,1))$ and $h_n \to 0$, as $n \to \infty$.

On the other hand, thanks to the regularity of the function $\psi(x)$ we have that $\psi_{\mathcal{T}_n}$ strongly converges to ψ and $\frac{\psi_{\mathcal{T}_n}(x)-\psi_{\mathcal{T}_n}(x-h_n)}{h_n}$ strongly converges to $\frac{\partial\psi}{\partial x}$. Then, passing to the limit in (39), we obtain:

$$\int_{-\infty}^{1} g(x)\psi(x)dx = -\int_{-\infty}^{1} y(x)\frac{\partial\psi(x)}{\partial x}dx. \tag{40}$$

This equality proves that $g(x) = \frac{\partial y}{\partial x}$. Using (25) we have:

$$\left\|\frac{\partial y}{\partial x}\right\|^2_{L_2((-\infty,1))} \le C.$$

We also have that $\frac{\partial y}{\partial x} = 0$ if $x \in (-\infty,0)$. Hence, the restriction of y to $(0,1)$ is in $H^1_{0-}((0,1))$. ☐

Theorem 1. (Convergence) *Let Assumption 1 be satisfied. For the mesh $\mathcal{T}$ on $[0,1]$ (see Definition 1) let $(y_0, y_1, ..., y_N) \in \mathbb{R}^{N+1}$ be a solution of (17)-(24) and let $y_{\mathcal{T}} : (0,1) \to \mathbb{R}$ by $y_{\mathcal{T}}(x) = y_i$ if $x \in K_i$, $i = \overline{0,N}$.*

Then, for any sequence of meshes $(\mathcal{T}_n)_{n\in\mathbb{N}}$ such that $h_n \to 0$, as $n \to \infty$, there exists a subsequence, still denoted by $(\mathcal{T}_n)_{n\in\mathbb{N}}$, such that $y_{\mathcal{T}_n} \to y$ in $L^2((0,1))$, as $n \to \infty$, where $y \in H^1_{0-}((0,1))$ is a solution to (13) with given functions (8), (9).

Proof. Let $(\mathcal{T}_n)_{n\in\mathbb{N}}$ be a sequence of meshes on $[0,1]$ such that $h_n \to 0$, as $n \to \infty$. Lemma 2 gives us the existence of solution to the problem (17)-(24) for any mesh $\mathcal{T}_n$ from sequence $(\mathcal{T}_n)_{n\in\mathbb{N}}$. By Lemma 3 there exists a subsequence, still denoted by $(\mathcal{T}_n)_{n\in\mathbb{N}}$, such that $y_{\mathcal{T}_n} \to y$ in $L^2((0,1))$ as $n \to \infty$. In order to conclude the proof, we have to prove that $y \in H^1_{0-}((0,1))$ is a solution of (13).

Let $\varphi \in C^\infty((0,1))$ such that $\varphi(0) = 0$. Then, the weak formulation (13) can be rewritten in the following way:

$$T_1 + T_2 - T_3 = 0, \tag{41}$$

where:

$$T_1 = \int_0^1 b(x)k(S(y+P_0))\frac{\partial y}{\partial x}\frac{\partial\varphi}{\partial x}dx, \tag{42}$$

$$T_2 = q(1)S(y(1)+P_0)\varphi(1), \tag{43}$$

$$T_3 = \int_0^1 q(x)S(y+P_0)\frac{\partial\varphi}{\partial x}dx. \tag{44}$$

Let $\mathcal{T}_n$ be a mesh on $[0,1]$ (see Definition 1) which is one of the meshes of the extracted subsequence $(\mathcal{T}_n)_{n\in\mathbb{N}}$, and $\varphi_i = \varphi(x_i)$, $i = \overline{1,N}$ and $\varphi_0 = 0$. If $(y_0, y_1, ..., y_N)$ is a solution to (17)-(24) on the mesh $\mathcal{T}_n$, multiplying (17), (19) by φ_i and summing over $i = \overline{1, N}$ yields:

$$\sum_{i=1}^{N-1} \left(-b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1} - y_i}{h_n} + b_{i-\frac{1}{2}} k_{i-\frac{1}{2}} \frac{y_i - y_{i-1}}{h_n} \right) \varphi_i + \sum_{i=1}^{N-1} (q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} - q_{i-\frac{1}{2}} S_{i-\frac{1}{2}}) \varphi_i + b_{N-\frac{1}{2}} k_{N-\frac{1}{2}} \frac{y_N - y_{N-1}}{h_n} \varphi_N + (q_N S_N - q_{N-\frac{1}{2}} S_{N-\frac{1}{2}}) \varphi_N = 0. \quad (45)$$

By reordering summation in (45), we obtain:

$$\hat{T}_1^n + \hat{T}_2^n - \hat{T}_3^n = 0, \quad (46)$$

where:

$$\hat{T}_1^n = \sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1} - y_i}{h_n} \frac{\varphi_{i+1} - \varphi_i}{h_n} h_n, \quad (47)$$

$$\hat{T}_2^n = q_N S_N \varphi_N, \quad (48)$$

$$\hat{T}_3^n = \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} \frac{\varphi_{i+1} - \varphi_i}{h_n} h_n. \quad (49)$$

Thanks to the regularity of the function φ, we notice, that:

$$\frac{\varphi_{i+1} - \varphi_i}{h_n} = \left. \frac{\partial \varphi}{\partial x} \right|_{x_{i+\frac{1}{2}}} + R_{i+\frac{1}{2}}, \text{ where } |R_{i+\frac{1}{2}}| < C_1 h_n^2, \quad (50)$$

with some C_1 only depending on φ. Therefore, (47) yields:

$$\hat{T}_1^n = \hat{T}_{1,1}^n + \hat{T}_{1,2}^n, \quad (51)$$

$$\hat{T}_{1,1}^n = \sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1} - y_i}{h_n} h_n \left. \frac{\partial \varphi}{\partial x} \right|_{x_{i+\frac{1}{2}}}, \quad (52)$$

$$\hat{T}_{1,2}^n = \sum_{i=0}^{N-1} b_{i+\frac{1}{2}} k_{i+\frac{1}{2}} \frac{y_{i+1} - y_i}{h_n} R_{i+\frac{1}{2}} h_n. \quad (53)$$

Substituting (50) in (49), we obtain:

$$\hat{T}_3^n = \hat{T}_{3,1}^n + \hat{T}_{3,2}^n, \quad (54)$$

$$\hat{T}_{3,1}^n = \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} h_n \left. \frac{\partial \varphi}{\partial x} \right|_{x_{i+\frac{1}{2}}}, \quad (55)$$

$$\hat{T}_{3,2}^n = \sum_{i=0}^{N-1} q_{i+\frac{1}{2}} S_{i+\frac{1}{2}} R_{i+\frac{1}{2}} h_n. \quad (56)$$

Since the functions $q(x)$ and $S(y)$ are bounded:

$$\hat{T}_{3,2}^n \to 0, \text{ as } n \to \infty. \quad (57)$$

Since the difference $(y_{i+1} - y_i)$ (see (35)) and function $k(S)$ are bounded, then:

$$\hat{T}^n_{1,2} \to 0, \text{ as } n \to \infty. \tag{58}$$

We remark, that:

$$\hat{T}^n_2 \to T_2, \text{ as } n \to \infty. \tag{59}$$

Let $\hat{T}^n_{1,1}$ and $\hat{T}^n_{3,1}$ be presented in the following way:

$$\hat{T}^n_{1,1} = \int_0^1 b_{\mathcal{T}_n} k_{\mathcal{T}_n} \frac{y_{\mathcal{T}_n}(x+h_n) - y_{\mathcal{T}_n}(x)}{h_n} \left(\frac{\partial \varphi}{\partial x}\right)_{\mathcal{T}_n} dx, \tag{60}$$

$$\hat{T}^n_{3,1} = \int_0^1 q_{\mathcal{T}_n} S_{\mathcal{T}_n} \left(\frac{\partial \varphi}{\partial x}\right)_{\mathcal{T}_n} dx, \tag{61}$$

where

$$\begin{aligned} b_{\mathcal{T}_n}(x) &= b_{i+\frac{1}{2}}, \\ q_{\mathcal{T}_n}(x) &= q_{i+\frac{1}{2}}, \\ S_{\mathcal{T}_n}(x) &= S_{i+\frac{1}{2}}, \\ k_{\mathcal{T}_n}(x) &= k_{i+\frac{1}{2}}, \\ \left(\frac{\partial \varphi}{\partial x}\right)_{\mathcal{T}_n} &= \left.\frac{\partial \varphi}{\partial x}\right|_{x_{i+\frac{1}{2}}}, \end{aligned}$$

if $x \in [x_i, x_{i+1}]$ for all $i = \overline{0, N-1}$.

Now let us show that $S_{\mathcal{T}_n}$ converges to S as $n \to \infty$. Let $S_{i+\frac{1}{2}}$ be approximated by (23a), then:

$$S_{\mathcal{T}_n}(x) = \frac{1}{2}(S(y_{\mathcal{T}_n}(x+h_n) + P_0) + S(y_{\mathcal{T}_n}(x) + P_0)). \tag{62}$$

Since $y_{\mathcal{T}_n} \to y$ in $L^2((0,1))$ as $n \to \infty$, $S_{\mathcal{T}_n} \to S$ in $L^2((0,1))$ as $n \to \infty$. It is also clear that $k_{\mathcal{T}_n} \to k$ as $n \to \infty$, $b_{\mathcal{T}_n}(x) \to b(x)$ and $q_{\mathcal{T}_n}(x) \to q(x)$ as $n \to \infty$, $\alpha = 1, 2$. Remembering $(y(\cdot + h_n) - y)/h_n$ converges to $\partial y/\partial x$ in the weak sense of $L_2((-\infty, 1))$ as $n \to \infty$ (see proof of Lemma 3), we obtain:

$$\begin{aligned} \hat{T}^n_{1,1} &\to T_1 \text{ as } n \to \infty, \\ \hat{T}^n_{3,1} &\to T_3 \text{ as } n \to \infty. \end{aligned}$$

Hence the theorem is proved. □

5 Numerical Experiment

To illustrate the theoretical results obtained in the previous sections we carry out a numerical experiment for a test problem. We consider problem (6)-(9) with input data given in Table 1. Note, that these data satisfy Assumption 1. In general the problem (6)-(9) does not have an analytical solution. But in this particular case it is given by:

$$p(x) = -x^3 + 3x - 1, \quad x \in [0, 1].$$

Input data	Value
$b(x)$	e^x
$q(x)$	$e^x(-3x^2+3)$
$S(p)$	$\frac{1}{2\pi}\arctan p + \frac{1}{2}$
$k(S)$	S
P_0	-1.0

Table 1: Input Data

To solve this nonlinear problem we use the Newton-iteration method. A termination criterion for the iteration process is:

$$\frac{\|p^{k+1}_{\mathcal{T}_n} - p^k_{\mathcal{T}_n}\|_{L_2}}{\|p^0_{\mathcal{T}_n}\|_{L_2}} < \epsilon,$$

where k is the current iteration number, $\mathcal{T}_n$ is the given mesh and $\epsilon = 10^{-4}$. The pressure and saturation are shown in Figures 2 and 3, respectively. Figure 4 represents the dependence of the error E between the discrete solution $p_{\mathcal{T}_n}$ and continuous solution p in L_2-norm of the number of nodes $(N+1)$ (see Defenition 1), which corresponds to the particular mesh $\mathcal{T}_n$. The error E is obtained with the following relation:

$$E(N+1) = \frac{\|p - p_{\mathcal{T}_n}\|_{L_2}}{\|p\|_{L_2}}$$

and it converges with the rate $O(h_n^2)$ as $n \to \infty$.

This numerical experiment illustrates one particular example when the discrete problem has a solution as it was proved in Lemma 2 and this solution converges to the analytical one as $n \to \infty$ (see Theorem 1). Hence, these numerical results and work [6], which are concerned with the numerical investigation of this problem, agree with the theory derived in the previous section.

6 Conclusion

In this work we were concerned with theoretical studies for a mathematical model, which was developed to simulate the pressing section of a paper machine. We were able to prove the stability of the discrete problem. The existence of a solution to the discrete problem and to the continuous problem in the weak formulation was shown. We also presented the proof of convergence of the approximate solution to the weak solution. Let us note that the uniqueness of the solutions to continuous and discrete problems was not discussed in this work since there are certain difficulties for getting these results due to complexity of the problem. As the final result the numerical experiment was developed for the test problem with a known analytical solution. With the help of these numerical results we illustrated the agreement of the developed theory with this particular case.

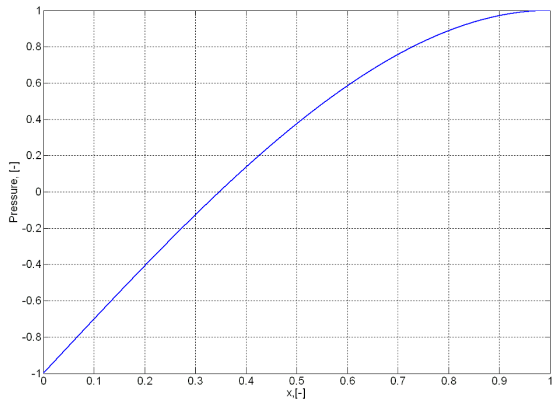

Figure 2: Pressure p

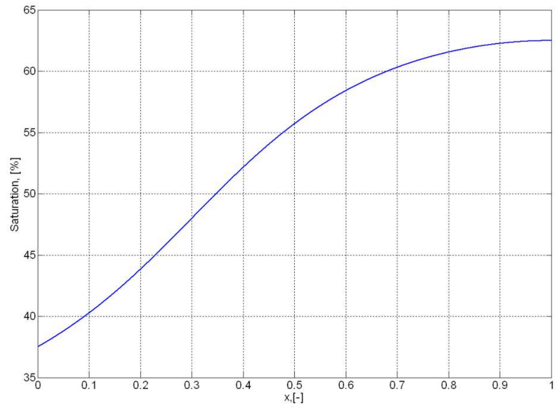

Figure 3: Saturation S

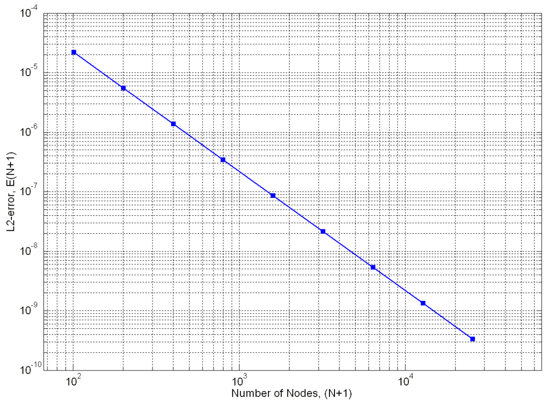

Figure 4: L^2-error between analytical and numerical solutions for different number of discretization nodes

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