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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

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Optimally Robust Kalman Filtering

Peter Ruckdeschel

May 6, 2010

Abstract

We present some optimality results for robust Kalman filtering.
To this end, we introduce the general setup of state space models which will not be limited to a Euclidean or time-discrete framework. We pose the problem of state reconstruction and repeat the classical existing algorithms in this context. We then extend the ideal-model setup allowing for outliers which in this context may be system-endogenous or -exogenous, inducing the somewhat conflicting goals of tracking and attenuation.
In quite a general framework, we solve corresponding minimax MSE-problems for both types of outliers separately, resulting in saddle-points consisting of an optimally-robust procedure and a corresponding least favorable outlier situation.
Still insisting on recursivity, we obtain an operational solution, the `rLS` filter and variants of it. Exactly robust-optimal filters would need knowledge of certain hard-to-compute conditional means in the ideal model; things would be much easier if these conditional means were linear. Hence, it is important to quantify the deviation of the exact conditional mean from linearity. We obtain a somewhat surprising characterization of linearity for the conditional expectation in this setting.
Combining both optimal filter types (for system-endogenous and -exogenous situation) we come up with a delayed hybrid filter which is able to treat both types of outliers simultaneously.
Keywords: robustness, Kalman Filter, innovation outlier, additive outlier

1 Introduction

State space models are an extremely flexible model class for dynamic phenomena, and even more so if we understand them to also comprise discrete state spaces as used in Hidden Markov Models.

Their applications range from Engineering Sciences, with Aeronautics, Electrical Engineering, speech recognition, over automatic monitoring/surveillance systems with important applications in intensive care medicine, to Genetics, with applications in gene sequencing, evolutional biology, and to Environmetrics and Geo-Statistics, with applications e.g. in hydrology and over to econometrics and finance with applications in prediction of stock prices, option pricing and portfolio optimization.

A survey on applications in econometrics is given in Harvey (1987), for the other domains a short search on the web will produce an abundance of references.

A comprehensive overview of the mathematical methods used in this subject may be found in Chen (1996).

Historically, after pioneering work by Kolmogorov (1941a,b), Wiener (1949), still limited to stationary situations, in two seminal papers, Kalman (1960) and Kalman and Bucy (1961), achieved a breakthrough, finding recursive, orthogonally optimal procedures which also covered non stationary situations, now known as Kalman filter (in the time-discrete setting) and Kalman-Bucy filter (in the continuous-time setting).

1.1 Review of the literature on Robust Kalman filtering

Soon in the history of Robust Statistics people became aware of the robustness problem inherent to Kalman filtering, with first (non-verified) hits on a quick search for "robust Kalman filter" on `scholar.google.com` as early 1962 and 1967, i.e.; the former even before the seminal Huber (1964) paper, often referred to as birthday of Robust Statistics.

In the meantime there is an ever growing amount of literature on this topic —Kassam and Poor (1985) have already compiled as many as 209 references to that subject in 1985... Excellent surveys are given in Ershov and Lipster (1978), Kassam and Poor (1985), Stockinger and Dutter (1987), Martin and Raftery (1987), Schick and Mitter (1994), Künsch (2001).

On the other hand, the mere notion of robustness itself is not understood unanimously in the literature. The notion that we will use in this paper will focus qualitatively on **bounded risk on neighborhoods about an ideal model** as specified in subsection 2.2, which in Problems (3.16), (3.17) will be made quantitative optimizing corresponding risks.

We also emphasize that working with "small" neighborhoods, the minimax formulation of Problem (3.16) will not result in overly pessimistic procedures, or to take up a formulation by C. Rogers, contrary to other minimax settings, you *will* leave the house —even in the presence of ubiquitous dangers, simply because you only look at "realistic" dangers lying "close" to your intended way.

The litmus test for our notion of robustness in this context will be whether a corresponding filter will be bounded in the observations, as otherwise the respective risk will be unbounded on an arbitrarily small neighborhood.

This qualitative notion of robustness should be compared to *"Qualitative Robustness"* as introduced by Hampel (1968), referring to equicontinuity of the distributions of procedure in weak topology with respect to the sample size; our notion is also related, but not identical to a positive *breakdown point* for the procedure on this neighborhood: Not identical, because there is no asymptotics involved and the sample size is 1! Hence, if we defined breakdown point as the infimal radius r such that the procedure becomes unbounded on the respective neighborhood, our procedures would attain breakdown points arbitrarily close to 1, which is not in the spirit of Hampel's original definition, confer Hampel (1968).

In the sequel, we present some of the existing approaches (and distinguish them from ours), and review certain ideas which we will exemplify with corresponding references.

Control Theory has found its own way to robustness, somewhat different from the notion used in statistics; instead of formulating deviations from distributional assumptions, this approach rather only allows for bounded controls —c.f. $\mathcal{H}^\infty/\mathcal{H}^2$— in order to cope with an incompletely specified transfer function. Survey articles are Başar and Bernhard (1991) and Rotea and Khargonekar (1995).

Other authors rather understand robustness as **stability w.r.t. disturbances in the parameters**, cf. Chen and Patton (1996). Judged from our perspective of Robustness, this is awkward: For instance only changing the parameters of a normal distribution will not lead us out of the class of linear filters, hence w.r.t. the unboundedness of linear filters, the robustness problem persists— in general, parametric neighborhoods are simply too small to lead to robust procedures.

Early approaches considered **hard rejection** schemes, cf. Meyr and Spies (1984) which however from the point of view of Theorem 3.3 are clearly suboptimal.

A large stream of articles replaces normality assumptions by corresponding **fat-tailed distributions**, notably t-distributions, cf. Meinhold and Singpurwalla (1989) but also ranges from **Bayesian approaches** such as West (1981, 1984, 1985), and also covers **posterior-mode approaches** by Fahrmeir and Kaufmann (1991), Fahrmeir and Künstler (1999).
The replacement of the ideal / central distribution could be seen as somewhat heuristical, replacing only one distribution (the Gaussian one) by another one. Still, the resulting filters are highly robust, as they yield bounded (even redescending) filters. Theorem 3.3 indicates however, that these distributions might lead to overly pessimistic procedures, if the majority of the data is nearly normally distributed; the argument of course also applies if the majority stems from another non-t- central distribution.
Another set of papers starting with Alspach and Sorenson (1972), works with mixtures, notably of normal distributions, in this case giving the so-called *Gaussian sum filters*. Originally designed to cover non-Gaussian resp. nonlinear situations, this idea has also been applied to tackle robustness

issues in Ershov (1978), Ershov and Lipster (1978), Kitagawa (1987), Peña and Guttman (1988) As one may easily show in case of Gaussian mixtures however, the resulting filters are not bounded, hence not robust in our sense.

Analogy of the state space model to **linear regression models** as noted by Duncan and Horn (1972) has led to approaches where people apply robust regression techniques to the filtering problem, confer Boncelet and Dickinson (1983, 1987), Boncelet (1985), Cipra and Romera (1991). The same approach led to the rIC, mIC filter initiated by H. Rieder and worked out in Ruckdeschel (2001, ch. 3,4). Admittedly, the asymptotics under which the corresponding robust regression estimators are derived is not available in our context; nevertheless these procedures compete well with other robustification approaches, compare Ruckdeschel (2001, ch. 5).

Although not bound to the structure of an SSM, the application of **non-parametric median-type filters** has a long success story, in particular for signal extraction, starting with the 3R-smoother of Tukey (1977) —a running median— and much improved upon by the Dortmund group, using several variants of repeated medians, confer Fried et al. (2006), Fried et al. (2007), and Schettlinger et al. (2006), in particular with applications in intensive care medicine, confer Fried et al. (2000). These filters however do not use the state space model character of the data and have certain weaknesses in higher dimensions, where corresponding medians are more difficult to define and even harder to implement if you want to go beyond coordinate-wise application of the repeated medians; see Fried et al. (2002), though.

With the ever becoming faster computers, and with the refined sampling techniques meanwhile available, the use of many filters running in parallel has become increasingly attractive. Some approaches in this setting do not use sampling but try to **adaptively select** the "optimal" filter in each time step t among a set N_t filters considered at this time, confer, e.g. Pupeikis (1998). As to operability of these filters, particular care must be spent on N_t, confer in this respect the filters proposed by Schick (1989) and Birmiwal and Shen (1993). **Sampling Techniques** in our context are very promising as they allow to assess not only single aspects like posterior mean or posterior mode of our filters but also the whole posterior distribution. Some of these techniques proceed non-recursively, using Markov Chain Monte Carlo or the Gibbs Sampler as in Carlin et al. (1992) and in Carter and Kohn (1994, 1996), while the Particle Filter approach is recursive; in particular the Particle Filter, compare Frühwirth-Schnatter (1994), Godsill and Rayner (1998), Hürzeler and Künsch (1998), Hürzeler (1998), Künsch (2005), seems promising to get hand on exact ideal posterior mean needed in Theorem 3.3. [MORE COMMENTS]

Nearest to our approach are several articles concerned with **minimax robustness** in various specifications. We do not discuss parametric minimax approaches here. References may be found in Ruckdeschel (2001, Sec. 1.5).
In the frequency domain there are papers by Kassam and Lim (1977), Franke and Poor (1984) and Franke (1985). One disadvantage of this approach is that you have to impose a uniform bound on the variance as a bound for the corresponding mass of the spectral measures in a neighborhood. According to the theory of Wiener and Kolmogorov, the optimal filters found in this context are bound to be linear, hence not robust in our sense.
In the time domain, the filter by Masreliez and Martin (1977), later termed *ACM filter* in Martin (1979), appeals to a minimax robustness which uses the asymptotic variance and hence builds up

on Huber (1964, 1981)[1]. This is somewhat problematic as the asymptotics in this non-stationary setting will never "kick in". We will instead use the SO-approach already used by Birmiwal and Shen (1993) and Birmiwal and Papantoni-Kazakos (1994), who obtain similar results as ours although in a more restricted setting and who, when passing back from the "one-step-solution" to the dynamic model setting, proceed differently.

1.2 Organization of the rest of the paper

In section 2 we present the general setting, introducing the necessary notation. Passing from the most simple, linear, time-discrete Euclidean state space model over to more general Hidden Markov Models and Dynamic Bayesian Models, we also introduce a continuous-time setup as it is relevant for Mathematical Finance, and finally even allow for user-specified controls. All these increasingly more complicated models presented in subsection 2.1 are covered by the optimality results we present, as long as mean squared error makes for a reasonable risk. In subsection 2.2, we then present different types of outlier models relevant for this setting and discuss their implications. After an introductory example in subsection 2.3 introducing our reference model, we finally review the classical Kalman filter with its optimality among all linear filters in subsection 2.4, as this (recursive) property will be the starting point for our robustification.

This robustification, the rLS filter, is introduced in section 3. After its definition in subsection 3.1, extending a corresponding result from Ruckdeschel (2001, ch. 8), we preliminarily drop all the dynamics of our model in subsection 3.2 and reduce it to a "Bayesian" type model. In this setting, we are able to show our central result, Theorem 3.3, which yields minimax-optimal solutions on SO neighborhoods in this quite general framework. Translating this result back into our dynamic model context is crucial and follows in subsection 3.3. In this setting, we disprove normality of our filter in Proposition 3.5 and characterize linearity of the corresponding ideal conditional mean in Proposition 3.7. With these results optimality of our rLS filter seems out of reach. Extending the SO neighborhoods a little, however, as done in subsection 3.4, we nevertheless obtain a certain optimality for the rLS in Theorem 3.11 and Proposition 3.12. Finally, as to efficiency in computational aspects we briefly mention stationarity properties of the rLS in subsection 3.5.

Sections 4 and 5 contain recent results extending the setup of Ruckdeschel (2001, ch. 8) to the IO situation and situations where both IO's and AO's are present. The key idea is to specialize our "Bayesian" model from subsection 3.2 to the additive model $Y = X + \varepsilon$ and to use the symmetry of X and ε present in this model: We achieve a translation of the optimality result of Theorem 3.3 to a situation with system-endogenous outliers where tracking is the main goal. Section 5 then presents a delayed hybrid filter which switches between AO- and IO-robust behavior according to the history of window length w of the discrepancies of predicted and realized observations, hence giving a filter that is simultaneously AO- and IO-robust.

Section 6 illustrates our findings with simulations at which we evaluate the classical Kalman filter, the rLS variants rLS.AO from section 3, rLS.IO from Section 4, and rLS.IOAO from section 5 together with the competitors ACM from Masreliez and Martin (1977) and $\text{hybr}_{\text{PRMH}}$ from Fried and

[1] The latter reference compiles some generalization of the former, which were already available to Martin and Masreliez.

Schettlinger (2008), resp. Fried et al. (2006).

Section 7 sketches open ends and starting points for further research, and section 8 describes the state of affairs as to an implementation of our proposals to an R package.

The proofs to the assertions made in sections 3–4 are compiled in section 9.

Finally, in section 10 we summarize the findings of this article.

2 General setup

2.1 Ideal model

To fix ideas, let us start with some definitions and assumptions. We are working in the context of state space models (SSM's) as to be found in many textbooks, confer Anderson and Moore (1979), Harvey (1991), Hamilton (1993), and Durbin and Koopman (2001).

Time Discrete, linear Euclidean Setup: The most prominent setting in this context is the linear, time–discrete, Euclidean Setup where the unobservable p-dimensional state X_t evolves according to a possibly time-inhomogeneous VAR(1) model with innovations v_t and transition matrices F_t.

$$X_t = F_t X_{t-1} + v_t \tag{2.1}$$

The statistician observes a q-dimensional linear transformation Y_t of X_t where we incur an additional observation error ε_t,

$$Y_t = Z_t X_t + \varepsilon_t \tag{2.2}$$

In the ideal model we work in a Gaussian context, that is we assume

$$v_t \overset{\text{indep.}}{\sim} \mathcal{N}_p(0, Q_t), \tag{2.3}$$

$$\varepsilon_t \overset{\text{indep.}}{\sim} \mathcal{N}_q(0, V_t), \tag{2.4}$$

$$X_0 \sim \mathcal{N}_p(a_0, Q_0), \tag{2.5}$$

$$\{v_t\}, \{\varepsilon_t\}, X_0 \text{ indep. as processes} \tag{2.6}$$

As usual, normality assumptions may be relaxed to working only with specified first and second moments, if we restrict ourselves to linear unbiased procedures as in the Gauss-Markov setting. For this paper, we assume the hyper–parameters F_t, Z_t, Q_t, V_t, a_0 to be known.

Time Discrete, Hidden Markov Models: Our results will in parts be valid in an even more general time-discrete setting which also covers Hidden Markov Models: we start with

$$P(X_0 \in A) = \int_A p_0^{X_0}(x)\,\mu_0(dx) \tag{2.7}$$

and assume that the unobservable state evolves according to a Markov transition:

$$\begin{aligned} P(X_t \in A | X_{t-1} = x_{t-1}, \ldots, X_0 = x_0) &= P(X_t \in A | X_{t-1} = x_{t-1}) = \\ = \textstyle\int_A p_t^{X_t|X_{t-1}=x_{t-1}}(x)\,\mu_t(dx), & \end{aligned} \tag{2.8}$$

Again we only have a transformation Y_t of X_t available which in this case is distributed according to

$$P(Y_t \in B | X_t = x_t) = \int_B q_t^{Y_t|X_t=x_t}(y)\,\nu_t(dy) \tag{2.9}$$

In this setting, we assume known (and existing) [conditional] densities $p_0^{X_0}$, $p_t^{X_t|X_{t-1}=x_{t-1}}$, $q_t^{Y_t|X_t=x_t}$.

Somewhere in-between the model formulation of this paragraph and the Euclidean SSM you may range the dynamic (generalized) linear models as discussed in West et al. (1985) and West and Harrison (1989). These are also covered by Theorem 3.3 as soon as in the state space a squared error makes sense.

Continuous setting: In applications of Mathematical Finance we also need to cover continuous time settings as given by an unobservable state evolving according to an SDE

$$dX_t = f(t, X_t)\,dt + q(t, X_t)\,dW_t \tag{2.10}$$

and where for consistency, we observe Y_t according to

$$dY_t = z(t, X_t)\,dt + v(t)\,dW'_t, \qquad Y_0 = 0 \tag{2.11}$$

For X_0 we assume (2.7), while W_t, W'_t, are independent Wiener processes, and f, q, z, v are suitably measurable, known functions.

This formulation with a time-continuous observation process as in (2.11) may be found in Tang (1998) and James (2005).

More often, however, observations will be made discretely, so that a formulation like the one of Nielsen et al. (2000) and Singer (2002) is more adequate, i.e.; for discrete times $t_1 < \ldots < t_N$ we have observations

$$Y_{t_k} = z_{t_k}(X_{t_k}) + \varepsilon_{t_k} \tag{2.12}$$

In this context, a straightforward approach linearizes the corresponding functions f and z to give the *(continuous-discrete) Extended Kalman Filter (EKF)*, or, improved to second order moment fitting in the *second order nonlinear filter (SNF)* introduced in Jazwinski (1970), also confer Singer (2002, sec. 4.3.1). After this linearization we are again in the context of a (time-inhomogeneous) linear SSM, hence the methodology we develop in the sequel applies to this setting as well.

More recently, approaches to improve on this simple linearization have been introduced, notably the *unscented Kalman filter (UKF)* (Julier et al., 2000) and Hermite expansions as in Aït-Sahalia (2002). We do not cover them here, though. For a survey of these methods, confer Singer (2002, sec. 4.3). For techniques to deal with non-linear time-discrete situations, see Tanizaki (1996).

Control: Going one more step ahead, to cover applications such as optimal portfolio selection, we may allow for controls U_t to be set or determined by the statistician, and which are fed back in the state equations. In the context of the continuous time model from (2.10) and (2.12), this is also known as SDEX, confer Nielsen et al. (2000).
In this setting, the controls U_t are assumed measurable w.r.t. $\sigma(Y_s|s<t)$ or usually even measurable w.r.t. $\sigma(Y_{t-})$.
To integrate these controls into our setting, we just have to generalize functions f, z, q and densities $p_t^{\cdot|\cdot}$, $q_t^{\cdot|\cdot}$ to $f=f(t,X_t,U_t)$ (and z,q likewise) and modify $p_t^{\cdot|\cdot}=p_t^{X_t|X_{t-1}=x_{t-1},U_{t-1}=u_{t-1}}(x)$, and $q_t^{\cdot|\cdot}=q_t^{Y_t|X_t=x_t,U_{t-1}=u_{t-1}}(y)$.
For the application of stochastic control to portfolio optimization, confer Korn (1997).

2.2 Deviations from the ideal model

As usual with Robust Statistics we do not confine ourselves to ideal model assumptions but rather allow for (small) deviations from these assumptions, most prominently generated by outliers.

In our notation, sub/superscript id denotes the *id*eal setting, di the *di*storting (contaminating) situation, re the *re*alistic, contaminated situation.

Contrary to the independent setting, outlier may occur in quite different manors: Following the terminology of Fox (1972), we distinguish *innovation outliers* (or IO's) and *additive outliers* (or AO's). Historically, AO's denote gross errors affecting the observation errors, i.e.,

$$\mathrm{AO} \quad :: \quad \varepsilon_t^{\mathrm{re}} \sim (1-r_{\mathrm{AO}})\mathcal{L}(\varepsilon_t^{\mathrm{id}}) + r_{\mathrm{AO}}\mathcal{L}(\varepsilon_t^{\mathrm{di}}) \tag{2.13}$$

where $\mathcal{L}(\varepsilon_t^{\mathrm{di}})$ is arbitrary, unknown and uncontrollable and $0 \le r_{\mathrm{AO}} \le 1$ is the AO-contamination radius, i.e.; the probability for an AO.

IO's on the other hand are usually defined as outliers which affect the innovations,

$$\mathrm{IO} \quad :: \quad v_t^{\mathrm{re}} \sim (1-r_{\mathrm{IO}})\mathcal{L}(v_t^{\mathrm{id}}) + r_{\mathrm{IO}}\mathcal{L}(v_t^{\mathrm{di}}) \tag{2.14}$$

where again $\mathcal{L}(v_t^{\rm di})$ is arbitrary, unknown and uncontrollable and $0 \le r_{\rm IO} \le 1$ is the corresponding IO-contamination radius.

We stick to this distinction for consistency with the literature, although we will rather use these terms in the following sense: *IO*'s denote endogenous outliers affecting the state equation in general, hence distorting several subsequent states. This also covers level shifts or linear trends; if $|F_t| < 1$ these are not included in the classical definition, as then IO's would then decay geometrically in t. We also extend the meaning of *AO*'s to denote general exogenous outliers which enter the observation equation only and thus only cause distortions at single time points. This also covers substitutive outliers or *SO*'s defined as

$$\mathrm{SO} \quad :: \quad Y_t^{\rm re} \sim (1 - r_{\rm SO})\mathcal{L}(Y_t^{\rm id}) + r_{\rm SO}\mathcal{L}(Y_t^{\rm di}) \tag{2.15}$$

where again $\mathcal{L}(Y_t^{\rm di})$ is arbitrary, unknown and uncontrollable and $0 \le r_{\rm SO} \le 1$ is the corresponding SO-contamination radius.

Apparently, the SO-ball of radius r consisting of all $\mathcal{L}(Y_t^{\rm re})$ according to (2.15) contains the corresponding AO-ball of the same radius when $Y_t^{\rm re} = Z_t X_t + \varepsilon_t^{\rm re}$. However, for technical reasons, we make the additional assumption that

$$Y_t^{\rm id}, Y_t^{\rm di} \quad \text{stochastically independent} \tag{2.16}$$

and then the "contains"-relation no longer holds.

The more general definition of AO's and IO's in the sequel will be labeled "wide-sense" to distinguish it from the "narrow-sense" definitions (2.13) and (2.14).

Remark 2.1. *Whether (narrow-sense) AO's or SO's are better suited to capture model deviations will depend on the actual application; seen from mathematical operability, clearly SO's are easier to treat, compare Remark 3.4(b). They will also lead to different least favorable situations, compare Remark 3.4(d).*

Different and competing goals are induced by endogenous and exogenous outliers: In the presence of (wide-sense) AO's we would like to attenuate their effect to avoid "false alarms", while when there are (wide-sense) IO's the usual goal in online applications would be tracking, i.e.; detect structural changes as fast as possible and/or react on the changed situation.

Obviously we are faced with an identification problem here:
Immediately after a suspicious observation we cannot tell (wide-sense) AO's from (wide-sense) IO's. Such a simultaneous treatment will only be possible with a certain delay —see section 5.

In other, more off-line situations, such as spectral analysis of low flow estimation or inter-individual heart frequency spectra, one would like to recover the situation without structural changes and hence a cleaning from both (wide-sense) IO's and AO's is required; after this cleaning the powerful instruments of spectral analysis will be available; for this and other issues in robust density estimation, confer Kleiner et al. (1979) and Spangl (2008). We will not pursue this goal in this paper, however.

2.3 Example: Steady State Model

Our running example will be a one-dimensional steady state model with hyper-parameters

$$p = q = 1, \quad F_t = Z_t = 1, \qquad \text{in the ideal model: } v_t, \varepsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1) \tag{2.17}$$

In Figure 1, we display a typical realization of an SSM in model (2.17), where outliers are generated according to $r_{\rm IO} = r_{\rm AO} = 0.1$, $v_t^{\rm di}, \varepsilon_t^{\rm di} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(10, 0.1)$.

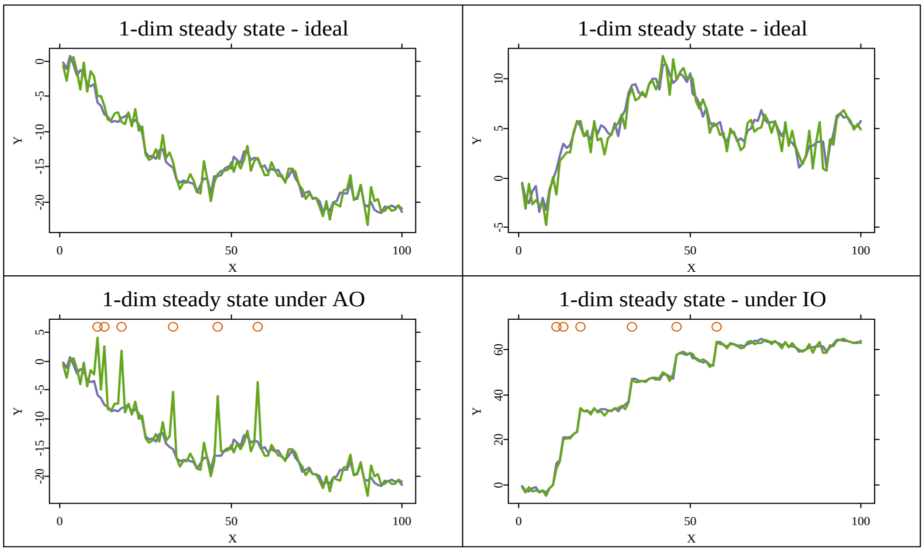

Figure 1:Model (2.17) in the ideal model and under (narrow-sense) AO's and IO's; while AO's only affect single observations, under IO's we never return to the original level. Instances of outliers are marked with red circles.

2.4 Classical Method: Kalman–Filter

Filter Problem The most important problem in SSM formulation is to somehow reconstruct the unobservable states X_t based on the observations Y_t. For abbreviation let us denote

$$Y_{1:t} = (Y_1, \ldots, Y_t), \quad Y_{1:0} := \emptyset \tag{2.18}$$

Then using mean squared error (MSE) risk, the reconstruction problem becomes

$$\mathrm{E}\left|X_t - f_t(Y_{1:s})\right|^2 = \min\nolimits_{f_t} \tag{2.19}$$

Depending on the horizon s of the observations used to reconstruct X_t, we speak of a prediction problem for $s < t$, of a filtering problem if $s = t$ and of a smoothing problem if $s > t$. In the sequel we will confine ourselves to the filtering problem.

Kalman–Filter It is well-known that the general solution to (2.19) is the corresponding conditional expectation $\mathrm{E}[X_t|Y_{1:s}]$. Except for the Gaussian case, this exact conditional expectation however is rather expensive to to compute. Hence similar to the Gauss-Markov setting it is a natural restriction to confine oneself to linear filters. In this context, the seminal work of Kalman (1960) (discrete-time setting) and Kalman and Bucy (1961) (continuous-time setting) introduced a recursive scheme to compute this optimal linear filter:

$$\text{Initialization:} \quad X_{0|0} = a_0, \qquad \Sigma_{0|0} = Q_0 \tag{2.20}$$

$$\text{Prediction:} \quad X_{t|t-1} = F_t X_{t-1|t-1}, \qquad \Sigma_{t|t-1} = F_t \Sigma_{t-1|t-1} F_t^\tau + Q_t \tag{2.21}$$

$$\begin{aligned} \text{Correction:} \quad X_{t|t} &= X_{t|t-1} + M_t^0 \Delta Y_t, & \Delta Y_t &= Y_t - Z_t x_{t|t-1}, \\ M_t^0 &= \Sigma_{t|t-1} Z_t^\tau \Delta_t^{-1}, & \Sigma_{t|t} &= (\mathbb{I}_p - M_t^0 Z_t)\Sigma_{t|t-1}, \\ \Delta_t &= Z_t \Sigma_{t|t-1} Z_t^\tau + V_t \end{aligned} \tag{2.22}$$

where $\Sigma_{t|t} = \mathrm{Cov}(X_t - X_{t|t})$, $\Sigma_{t|t-1} = \mathrm{Cov}(X_t - X_{t|t-1})$, and M_t^0 is the so-called *Kalman gain*. Using orthogonality of $\{\Delta Y_t\}_t$ we may setup similar recursions for the corresponding best linear smoother; see, e.g. Anderson and Moore (1979), Durbin and Koopman (2001).

Optimality of the Kalman–Filter To see that the (classical) Kalman filter solves problem (2.19) (for $s = t$) among all linear filters, let us write

$$\mathrm{lin}(X) := \text{closed linear space generated by } X \tag{2.23}$$

$$\mathrm{oP}(\cdot|X) := \text{orthogonal projection onto } \mathrm{lin}(X) \tag{2.24}$$

and define (recursively)

$$\Delta Y_t = Y_t - \mathrm{oP}(Y_t|Y_{1:t-1}) \tag{2.25}$$

Hence the ΔY_t are mutually orthogonal and

$$X_{t|t-1} = \mathrm{oP}(X_t|Y_{1:t-1}) = F_t\,\mathrm{oP}(X_{t-1}|Y_{1:t-1}) = F_t X_{t-1|t-1} \tag{2.26}$$

$$\begin{aligned} X_{t|t} &= \mathrm{oP}(X_t|Y_{1:t}) = \mathrm{oP}(X_t|Y_{1:t-1}) + \mathrm{oP}(X_t|\Delta Y_t) = \\ &= X_{t|t-1} + \mathrm{oP}(X_t - X_{t|t-1}|\Delta Y_t) = X_{t|t-1} + M_t^0 \Delta Y_t \end{aligned} \tag{2.27}$$

For later purposes, we also introduce a symbol for the prediction error

$$\Delta X_t = X_t - X_{t|t-1}. \tag{2.28}$$

Similar to the Gauss-Markov Theorem, under normality, i.e.; assuming (2.3), (2.4), (2.5), this optimality extends as follows: $X_{t|t[-1]} = \mathrm{E}[X_t|Y_{1:t[-1]}]$, i.e. the Kalman filter is optimal among all $Y_{1:t[-1]}$-measurable filters. It also is the posterior mode of $\mathcal{L}(X_t|Y_{1:t})$ and $X_{t|t}$ can also be seen to be the ML estimator for a regression model with random parameter; for the last property, compare Duncan and Horn (1972).

Features of the Kalman–Filter The Kalman filter stands out for its easy and understandable structure:
We have an initialization, a prediction, and a correction step, all steps are linear, hence easy evaluable and interpretable. Due to the strict recursivity / Markovian structure of the state equation, all information from the past useful for the future may be captured in the value of $X_{t|t-1}$, so there is only very limited memory needed.
From a Robustness point of view, this linearity at the same time is a weakness of this filter — y enters unbounded into the correction step which hence is prone to outliers.
A good robustification of this approach would try to retain as much as possible from these positive properties of the Kalman filter while revising the unboundedness in the correction step.

3 The rLS as optimally robust filter

3.1 Definition

***r*obustifying *r*ecursive *L*east *S*quares: rLS** In a first step we limit ourselves to (wide-sense) AO's. Notationally, where clear from the context, we suppress the time index t.

As no (new) observations enter the initialization and prediction steps, these steps may be left unchanged. In the correction step, we will have to modify the orthogonal projection $\mathrm{oP}(\Delta X|\Delta Y)$ present in (2.27). Suggested by H. Rieder and worked out in Ruckdeschel (2001, ch. 2), the following robustification of the correction step is straightforward: Instead of $M^0\Delta Y$ we use a Huberization of this correction

$$H_b(M^0\Delta Y) = M^0\Delta Y \min\{1, b/\big|M^0\Delta Y\big|\} \tag{3.1}$$

for some suitably chosen clipping height b. Apparently, this proposal removes the unboundedness problem of the classical Kalman filter while still remaining reasonably simple, in particular this modification is non iterative, hence especially useful for online-purposes.
However it should be noted that, departing from the Kalman filter and at the same time insisting on strict recursivity, we possibly exclude "better" non-recursive procedures, compare Remark 3.6. These procedures on the other hand would be much more expensive to compute.

Remark 3.1. *$\big|\cdot\big|$ in expression $\big|M^0\Delta Y\big|$ denotes the Euclidean norm of $\mathbb{R}^q$; instead, however you could also use other norms like a Mahalanobis type norm. With respect to Theorem 3.3, optimality is preserved when instead of the Euclidean norm used in the MSE, you use the corresponding alternative norm.*

Choice of the clipping height b As to the choice of the clipping height b, we make the simplifying assumption that the conditional expectation $\mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y]$ is linear, which will turn out to only be approximately right. In this setting, we have two proposals:

The first one is an Anscombe insurance criterium. To given "insurance premium" δ to be paid in terms of loss of efficiency in the ideal model compared to the optimal procedure in this (ideal) setting, i.e.; the classical Kalman filter, we choose $b = b(\delta)$ such that

$$\mathrm{E}_{\mathrm{id}} \left|\Delta X - H_b(M^0 \Delta Y)\right|^2 \stackrel{!}{=} (1+\delta)\,\mathrm{E}_{\mathrm{id}} \left|\Delta X - M^0 \Delta Y\right|^2 \tag{3.2}$$

The other possibility will become clearer in the next section: To a given size of the (SO-) neighborhood $\mathcal{U}^{\mathrm{SO}}(r)$ specified by a radius $r \in [0,1]$, we determine $b = b(r)$ such that

$$(1-r)\,\mathrm{E}_{\mathrm{id}}(|M^0 \Delta Y| - b)_+ \stackrel{!}{=} rb \tag{3.3}$$

If this radius is unknown, we could follow the idea worked out in Rieder et al. (2008), that is, distinguish a least favorable radius r_0 defined in the following expressions

$$r_0 \;=\; \mathrm{argmin}_{s\in[0,1]} \rho_0(s), \qquad \rho_0(s) = \max_{r\in[0,1]} \rho(r,s), \tag{3.4}$$

$$\rho(r,s) \;=\; \frac{\max_{\mathcal{U}^{\mathrm{SO}}(r)} \mathrm{MSE}(\mathrm{rLS}(b(s)))}{\max_{\mathcal{U}^{\mathrm{SO}}(r)} \mathrm{MSE}(\mathrm{rLS}(b(r)))} \tag{3.5}$$

and use the corresponding $b(r_0)$.

If we have limited knowledge about r, say $r \in [r_l, r_u]$, $0 < r_l < r_u < 1$, we would restrict the variation range of s and r in the respective optimization problems correspondingly.

To this end, define

$$A_r \;=\; \mathrm{E}_{\mathrm{id}}\left[\mathrm{tr}\,\mathrm{Cov}_{\mathrm{id}}[\Delta X|\Delta Y^{\mathrm{id}}] + (|M^0 \Delta Y^{\mathrm{id}}| - b(r))_+^2\right] \tag{3.6}$$

$$B_r \;=\; \mathrm{E}_{\mathrm{id}}\left[|M^0 \Delta Y^{\mathrm{id}}|^2 - (|M^0 \Delta Y^{\mathrm{id}}| - b(r))_+^2\right] + b(r)^2 \tag{3.7}$$

Then we can show the following variant of Kohl (2005, Lemma 2.2.3):

Lemma 3.2. *In equations (3.4) and (3.5), let r, s vary in $[r_l, r_u]$ with $0 \le r_l < r_u \le 1$. Then*

$$\rho_0(r) = \max\{A_r/A_{r_l}, B_r/B_{r_u}\} \tag{3.8}$$

and there exists some $\tilde r_0 \in [r_l, r_u]$ such that

$$A_{\tilde r_0}/A_{r_l} = B_{\tilde r_0}/B_{r_u} \tag{3.9}$$

and it holds

$$\min_{r\in[r_l,r_u]} \rho_0(r) = \rho_0(\tilde r_0), \qquad \textit{i.e.;}\;\; r_0 = \tilde r_0 \tag{3.10}$$

Moreover, if $r_u = 1$, $r_0 = r_u$.

In particular, the last equality shows that one should restrict r_u to be strictly smaller than 1 to get a sensible procedure.

3.2 (One-Step)-Optimality of the rLS

The seemingly ad-hoc robustification proposed in the rLS filter has some remarkable optimality property, though. To see this, let us first forget about the time structure and instead consider the following simplified, but general "Bayesian" model:

We have an unobservable but interesting signal $X \sim P^X(dx)$, where for technical reasons we assume that in the ideal model $\mathrm{E}\,|X|^2 < \infty$.
Instead of X we rather observe a random variable Y of which we know the ideal transition probabilities; more specifically, we assume that these transition probabilities are dominated, again in the ideal model, hence have densities w.r.t. some measure μ,

$$P^{Y|X=x}(dy) = \pi(y,x)\,\mu(dy) \tag{3.11}$$

Our approach relies on the MSE — so we assume that the range of X be such that MSE makes sense, — which essentially amounts to saying that the range of X be a subset of some Hilbert space.

As (wide-sense) AO model, we consider an SO outlier model, i.e.;

$$Y^{\mathrm{re}} = (1-U)Y^{\mathrm{id}} + UY^{\mathrm{di}}, \qquad U \sim \mathrm{Bin}(1,r) \tag{3.12}$$

for U independent of $(X, Y^{\mathrm{id}}, Y^{\mathrm{di}})$ and some distorting random variable Y^{di} for which, in a slight variation of condition (2.16) we assume

$$Y^{\mathrm{di}},\ X \quad \text{independent} \tag{3.13}$$

and the law of which is arbitrary, unknown and uncontrollable. As a first step consider the set $\partial\mathcal{U}^{\mathrm{SO}}(r)$ defined as

$$\partial\mathcal{U}^{\mathrm{SO}}(r) = \Big\{\mathcal{L}(X, Y^{\mathrm{re}})\,|\,Y^{\mathrm{re}} \text{ acc. to (3.12) and (3.13)}\Big\} \tag{3.14}$$

Because of condition (3.13), in the sequel we refer to the random variables Y^{re} and Y^{di} instead of their respective (marginal) distributions only, while in the common gross error model, reference to the respective distributions would suffice. Condition (3.13) also entails that in general, contrary to the gross error model, $\mathcal{L}(X, Y^{\mathrm{id}})$ is not element of $\partial\mathcal{U}^{\mathrm{SO}}(r)$, i.e.; not representable itself as some $\mathcal{L}(X, Y^{\mathrm{re}})$ in this neighborhood.

As corresponding (convex) neighborhood we define

$$\mathcal{U}^{\mathrm{SO}}(r) = \bigcup_{0\le s\le r} \partial\mathcal{U}^{\mathrm{SO}}(s) \tag{3.15}$$

hence the symbol "∂" in $\partial\mathcal{U}^{\mathrm{SO}}$, as the latter can be interpreted as the corresponding surface of this ball. Of course, $\mathcal{U}^{\mathrm{SO}}(r)$ contains $\mathcal{L}(X, Y^{\mathrm{id}})$.

In the sequel where clear from the context we drop the superscript SO and the argument r.

With this setting we may formulate two typical robust optimization problems:

Minimax-SO problem Minimize the maximal MSE on an SO-neighborhood, i.e.; find a measurable reconstruction f_0 for X s.t.

$$\max_{\mathcal{U}} \mathrm{E}_{\mathrm{re}} |X - f(Y^{\mathrm{re}})|^2 = \min_f ! \tag{3.16}$$

Lemma5-SO problem Alluding to Hampel's famous Lemma 5, confer Hampel (1968), minimize the MSE in the ideal model but subject to a side condition on the bias to be fulfilled on the whole neighborhood, i.e.; find a measurable reconstruction f_0 for X s.t.

$$\mathrm{E}_{\mathrm{id}} |X - f(Y^{\mathrm{id}})|^2 = \min_f ! \quad \text{s.t. } \sup_{\mathcal{U}} \big| \mathrm{E}_{\mathrm{re}} f(Y^{\mathrm{re}}) - \mathrm{E}\, X \big| \leq b \tag{3.17}$$

The solution to both problems can be summarized as

Theorem 3.3 (Minimax-SO, Lemma5-S0).

(1) *In this situation, there is a* **saddle-point** $(f_0, P_0^{Y^{\mathrm{di}}})$ *for Problem* (3.16)

$$f_0(y) \;:=\; \mathrm{E}\, X + D(y) \min\{1, \rho/\big|D(y)\big|\} \tag{3.18}$$

$$P_0^{Y^{\mathrm{di}}}(dy) \;:=\; \tfrac{1-r}{r}(\big|D(y)\big|/\rho - 1)_+ \; P^{Y^{\mathrm{id}}}(dy) \tag{3.19}$$

where $\rho > 0$ *ensures that* $\int P_0^{Y^{\mathrm{di}}}(dy) = 1$ *and*

$$D(y) = \mathrm{E}_{\mathrm{id}}[X|Y = y] - \mathrm{E}\, X \tag{3.20}$$

(2) f_0 *from* (3.18) *also is the solution to Problem* (3.17) *for* $b = \rho/r$*.*

(3) *If* $\mathrm{E}_{\mathrm{id}}[X|Y]$ *is linear in* Y*, i.e.;* $\mathrm{E}_{\mathrm{id}}[X|Y] = MY$ *for some matrix* M*, then necessarily*

$$M = M^0 = \mathrm{Cov}(X, Y)\, \mathrm{Var}\, Y^- \tag{3.21}$$

— or in SSM formulation: M^0 *is just the classical Kalman gain and* f_0 *the (one-step) rLS.*

Identifications for the SSM context Our "Bayesian" Model (3.11) already covers one step in our state space model context: we only have to identify X in model (3.11) with ΔX_t and $\pi(y, x)\, \mu(dy)$ with $\mathcal{N}(Z_t \Delta X_t, V_t)(dy)$.

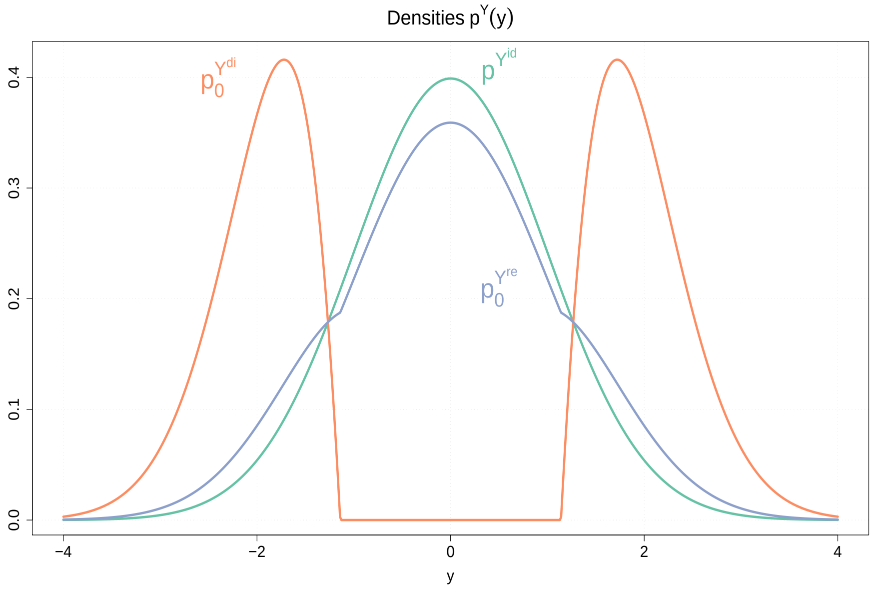

Figure 2: Densities of $P^Y = P^{Y^{\rm id}}$, $\hat P^Y = P_0^{Y^{\rm re}}$, $\tilde P^Y = P_0^{Y^{\rm di}}$ for $P^X = P^\varepsilon = \mathcal{N}(0,1)$, $r = 0.1$; note the "thin" tails.

Example for SO-least favorable densities To illustrate the result of Theorem 3.3, we have plotted the ideal density of $P^{Y^{\rm id}}$, the (least favorable) contaminated density of $P_0^{Y^{\rm re}}$, and the (least favorable) contaminating density of $P_0^{Y^{\rm di}}$ in Figure 2.

Remark 3.4. *(i) SO neighborhoods (without using this name) have already been used by Birmiwal and Shen (1993) and Birmiwal and Papantoni-Kazakos (1994), although in a somewhat less general (one-dimensional) model and without recognizing the explicit connection to the ideal conditional expectation.*

(ii) The use of SO neighborhoods in this (finite sample) context allows for remarkably general optimality results —remarkable, because explicit solutions to robust optimization problems in a finite sample setting are rare. Usually one argues asymptotically instead. Important exceptions are Huber (1968), Huber and Strassen (1973), and even there, in the former case one uses a special (unusual) loss function and is limited to one dimension.

(iii) Although similar as to the model (you could interpret X as a random location parameter) and type of result, the saddle-point differs from the one obtained in the one-dimensional location model in Huber (1964). This becomes obvious when studying the tails of the least favorable $P_0^{Y^{\rm re}}$: while in the Gaussian model in the location setting the tails decay as $ce^{-k|x|}$ for some $c, k > 0$, in our setting they decay as $c'|x|e^{-x^2/2}$ so appear even "less harmful" than in the location case.

(iv) Attempts to solve corresponding robust optimization problems in a (narrow-sense) AO neigh-

borhood are much more difficult and only partial results in this context have been obtained in Donoho (1978), Bickel (1981), and Bickel and Collins (1983); in particular one knows, that in the setup of our example the corresponding least favorable $\tilde{P}^{\varepsilon} = P_0^{\varepsilon^{\rm di}}$ must be discrete with only possible accumulation points $\pm\infty$. In addition, existence of a saddle-point may be shown using abstract compactness and continuity arguments, but in order to obtain specific solutions one has to recur to numeric approximation techniques as worked out in Ruckdeschel (2001, sec. 8.3); in particular, one obtains redescending optimal filters; this redescending in filtering context is not a problem as it is in robust estimation, because we do not iterate the filter.

(v) *The approach by Masreliez and Martin (1977) to translate the Huber (1964) minimax variance result to this dynamic setting uses redescenders in the corresponding ACM filter, too.*
It should be noted that the corresponding least-favorable (SO-)situation is not in the tails but rather where the corresponding ψ function takes its maximum in absolute value. An SO outlier could easily place contaminating mass on this maximum, while this is much harder if not impossible to achieve in a (narrow-sense) AO situation. Hence in simulations where we produce "large" outliers, the ACM filter tends to outperform the rLS filter, as these "large" outliers are least favorable for the rLS but not for the ACM. The "inliers" producing the least favorable situation for the ACM on the other hand will be much harder to detect on naïve data inspection than "large" outliers, in particular in higher dimensions.

3.3 Back in the ΔX Model for $t > 1$

So far we have ignored the fact that our X in model (3.11) resp. ΔX_t in the state space model context will stem from a past which has already used our robustified version of the Kalman filter. In particular, the law of ΔX_t (even in the ideal model) is not straightforward and hence (ideal) conditional expectation appearing in the optimal solution f_0 in Theorem 3.3 in practice are not so easily computable.

Approaches to go back — lots of "BUT's" The issue to assess the law ΔX_t is common for any (non-linear) robustification of the Kalman filter, and hence there already exist a couple of approaches to deal with it:

Masreliez and Martin (1977) and Martin (1979) assume $\mathcal{L}(\Delta X_t)$ *normal* and propose using robust location estimators (with redescending ψ-function) as alternatives to the linear correction step. Contradicting this assumption, we have the following proposition

Proposition 3.5. *Whenever in one correction step in the ΔX_t past one has used a bounded correction step then $\{\Delta X_t\}$ (as a process) cannot be normally distributed; this assertion cannot even hold asymptotically, as long as for the clipping heights b_t we can say*

$$0 < \liminf_t b_t \le \limsup_t b_t < \infty \tag{3.22}$$

Schick (1989) and Schick and Mitter (1994) use Taylor-expansions for non-normal $\mathcal{L}(\Delta X_t)$; doing so they end up with stochastic error terms but do not give an indication as to uniform integrability. Hence it is not clear whether the approximation stays valid after integration. More importantly, at time instance t, they come up with a bank of (at least t) Kalman–filters which is not very operational.

Birmiwal and Shen (1993) work with the exact $\mathcal{L}(\Delta X_t)$ and hence have to split up the integration according to the the history of outlier occurrences which yields 2^t different terms — which is not very operational either.

Remark 3.6. *One of the features of the ideal Gaussian model is that* $\mathrm{E}_{\mathrm{id}}[\Delta X_t|Y_{1:t}]$ *is Markovian in the sense that* $\mathrm{E}_{\mathrm{id}}[\Delta X_t|Y_{1:t}] = \mathrm{E}_{\mathrm{id}}[\Delta X_t|\Delta Y_t]$ *hence only depends on the one value of* ΔY_t. *When using bounded correction steps, however, this property gets lost, hence the restriction to strictly recursive procedures as is the rLS filter is a real restriction.*

Theorem 3.3 does not make any normality assumptions, but in assertion (3), we have seen that the rLS would result optimal once we can show that $\mathrm{E}_{\mathrm{id}}[\Delta X_t|\Delta Y_t]$ for ΔX stemming from an rLS past is *linear*. This leads to the question:

When is $\mathbf{E}_{\mathbf{id}}[\boldsymbol{\Delta X}|\boldsymbol{\Delta Y}]$ linear?

As to this question we have (omitting time indices t)

Proposition 3.7. *Assume*

$$\mathcal{L}_{\mathrm{id}}(\varepsilon) = \mathcal{N}_q(0, V) \tag{3.23}$$

Then $\mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y]$ *is linear*

$$\iff \quad \mathcal{L}(\Delta X) \quad \text{is normal} \tag{3.24}$$

$$\iff \quad M_3(e) := \mathrm{E}\Big[\Big(e^\tau(\Delta X - \mathrm{E}[\Delta X|\Delta Y])\Big)^3 \,\Big|\, \Delta Y = y\Big] = 0 \quad \forall\, e \in \mathbb{R}^p \tag{3.25}$$

Remark 3.8. *(i) The first equivalence (together with Proposition 3.5) shows that, stemming from an rLS-past, we will never be SO-optimal with the rLS except for the very first time step.*

(ii) Simulations however show that rLS gives very reasonable results. So in fact we could/should be close to an ideal linear conditional expectation.

(iii) "Closeness" to linearity could be operationalized by the second derivative $\partial^2/\partial y^2\, \mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y = y]$, *which in fact leads us to expression (3.25).*

(iv) *The second equivalence (conditional unskewedness of ΔX) is somewhat surprising, as it seems much weaker than normality of the prediction error.*

A test for linearity In particle filter context where you simulate many stochstically independent filters in parallel, Proposition 3.7 suggests the following test for linearity/normality:

Proposition 3.9. *Let $\Delta X_i^\natural$, $i = 1, \ldots, n$ be an i.i.d. sample from $\mathcal{L}(\Delta X_t)$, the law of the prediction errors of some filter at time t; let $\Sigma = \mathrm{Cov}(\Delta X_t)$, σ^2 its maximal eigen value and e a corresponding eigen vector (of norm 1); let $\hat{\Sigma}_n$, $\hat{\sigma}_n^2$, and $\hat{e}_n$ the corresponding empirical counter parts (all assumed consistent). Define the test statistic*

$$T_n = \frac{1}{n}\sum_{i=1}^{n}(\hat{e}_n^\tau \Delta X_i^\natural)^3 \tag{3.26}$$

Then under normality of $\mathcal{L}(\Delta X_t)$,

$$\sqrt{n}\, T_n \underset{\mathrm{w}}{\longrightarrow} \mathcal{N}(0, 15\sigma^6) \tag{3.27}$$

and the test

$$\mathrm{I}(|T_n| > \sqrt{15/n}\, \hat{\sigma}_n^3 u_{\alpha/2}) \tag{3.28}$$

for u_α the upper α-quantile of $\mathcal{N}(0,1)$ is asymptotically most powerful among all unbiased level-α-tests for testing

$$H_0\colon \quad \sup_{|e|=1} M_3(e) = 0 \qquad \textit{vs.} \qquad H_1\colon \quad \sup_{|e|=1} |M_3(e)| > 0 \tag{3.29}$$

3.4 Way out: eSO-Neighborhoods

Another approach to explain the good empirical findings for the rLS is to once again extend the original SO-neighborhoods. To this end, consider the following outlier model —the extended *SO* or eSO–*model*: In this model, we also allow for model deviations in X, i.e.; we assume a realistic $(X^{\mathrm{re}}, Y^{\mathrm{re}})$ according to

$$(X^{\mathrm{re}}, Y^{\mathrm{re}}) := (1-U)(X^{\mathrm{id}}, Y^{\mathrm{id}}) + U(X^{\mathrm{di}}, Y^{\mathrm{di}}) \tag{3.30}$$

for $X^{\mathrm{id}} \sim P^{X^{\mathrm{id}}}$, Y^{id} according to equation (3.11), $X^{\mathrm{di}} \sim P^{X^{\mathrm{di}}}$, $Y^{\mathrm{di}} \sim P^{Y^{\mathrm{di}}}$, $U \sim \mathrm{Bin}(1, r_{\mathrm{eSO}})$, where

$$U \text{ and } (X^{\mathrm{id}}, Y^{\mathrm{id}}) \text{ independent as well as (mutually) } U, X^{\mathrm{di}}, Y^{\mathrm{di}} \tag{3.31}$$

and the joint law $P^{X^{\mathrm{id}}, Y^{\mathrm{id}}}$ and the radius $r = r_{\mathrm{eSO}}$ are known, while $P^{X^{\mathrm{di}}}, P^{Y^{\mathrm{di}}}$ are arbitrary, unknown and uncontrollable; however, we assume that

$$\mathrm{E}_{\mathrm{di}}\, X^{\mathrm{di}} = \mathrm{E}_{\mathrm{id}}\, X^{\mathrm{id}}, \qquad \mathrm{E}_{\mathrm{di}}\, |X^{\mathrm{di}}|^2 \le G \tag{3.32}$$

for some known $0 < G < \infty$, and accordingly define

$$\mathcal{U}^{\mathrm{eSO}}(r) := \bigcup_{0 \le s \le r} \partial\mathcal{U}^{\mathrm{eSO}}(s), \qquad \partial\mathcal{U}^{\mathrm{eSO}}(r) := \{\, \mathcal{L}(X^{\mathrm{re}}, Y^{\mathrm{re}}) \text{ acc. to (3.30)–(3.32)} \,\} \tag{3.33}$$

Remark 3.10. *At first glance, moment condition (3.32) seems to be in conflict with the spirit of Robustness; however, this condition has not been introduced to induce a higher degree of robustness, but rather to extend the applicability of Theorem 3.3.*

Theorem 3.11 (minimax-eSO). *The pair $(f_0, P_0^{Y^{\mathrm{di}}})$, optimal in the Minimax-SO-problem to radius $r_{\mathrm{SO}} = r$ from Theorem 3.3, extended to $\left(f_0, P_0^{Y^{\mathrm{di}}} \otimes P_0^{X^{\mathrm{di}}}\right)$ for any $P_0^{X^{\mathrm{di}}}$ such that $\mathrm{E}_{\mathrm{di}}\, |X^{\mathrm{di}}|^2 = G$, remains a saddle-point in the corresponding Minimax-Problem on the eSO-neighborhood $\mathcal{U}^{\mathrm{eSO}}$ to the same radius r —no matter what bound G in equation (3.32) holds. The minimax risk depends on G, though.*

Consequences of Theorem 3.11 In the Gaussian setup, i.e.; we assume (2.3), (2.4), and (2.5), we no longer regard the (SO–) saddle-point solution to an $\mathcal{U}(r)$-neighborhood around $\mathcal{L}(\Delta X)$ stemming from an rLS-past, but use Theorem 3.11 as follows:

Proposition 3.12. *Assume that for each time t there is a* (fictive) *random variable $\Delta X^{\mathcal{N}} \sim \mathcal{N}_p(0, \Sigma)$ such that $\Delta X_t^{\mathrm{rLS}}$ stemming from an rLS-past can be considered an X^{di} in the corresponding eSO-neighborhood around $\Delta X^{\mathcal{N}}$ with radius r.*

Then, in this setup the rLS is exactly minimax for each time t

Remark 3.13. *(i) Proposition 3.12 gives an explanation for the good empirical results obtained with the rLS filter, compare [BSPANGL-REF].*

(ii) The existence of $\Delta X^{\mathcal{N}} \sim \mathcal{N}_p(0, \Sigma)$ in a general setting is not yet proved. To this end one has to show moment condition (3.32) and that

$$\sup\nolimits_\lambda \left(\log p^{\Delta X_t^{\mathcal{N}}} - \log p^{\Delta X_t} \right) \ge \log(1 - r) \tag{3.34}$$

where $p^{\Delta X_t^{\mathcal{N}}}$, $p^{\Delta X_t}$ are the corresponding Lebesgue densities and $\sup_\lambda$ is the corresponding essential supremum w.r.t. Lebesgue measure in the respective dimension. Moment condition (3.32) is not hard to fulfill — we only need to check that $\mathrm{E}_{\mathrm{id}}\, \Delta X_t = 0$, which for the rLS follows from symmetry of the distributions in the ideal model, and that the second moment is bounded — which also clearly holds. So (3.34) is the more difficult point to show.

(iii) *As to the choice of the covariance Σ for $\Delta X_t^{\mathcal{N}}$ two candidates suggest themselves: $\Sigma = \operatorname{Cov}\Delta X_t^{\rm rLS}$ and $\Sigma = \Sigma_{t|t-1}$ from the classical Kalman filter. While the former takes up the actual error covariances, the latter is much easier to compute. In our numerical examples in Ruckdeschel (2001), we could not find any significant advantages for the former in terms of precision and hence propose the latter for computational reasons.*

(iv) *For $p = 1$, condition (3.34) could be checked numerically in a number of models, confer Ruckdeschel (2001, Table 8.1)*

(v) *For $p > 1$, particle filter techniques should be helpful.*

3.5 Stationarity Aspects

One can show that in a time invariant (linear, time discrete, Euclidean) state space model (i.e., hyper-parameters F_t, Z_t, Q_t and V_t are constant in t), whenever the corresponding Kalman filter gets asymptotically stationary, the same also goes for the rLS when we use a sufficiently small time-invariant insurance premium δ in (3.2) or a sufficiently small time-invariant radius in (3.3); confer Ruckdeschel (2001, chap. 7).
Asymptotic stationarity for the Kalman filter holds whenever the state space model is *completely detectable* in the sense of Anderson and Moore (1979).
This stationarity is in particular useful as then also the Kalman-gains M_t^0 converge in t as well as the error covariances $\Sigma_{t|t[-1]}$, and hence also the corresponding determining equations (3.2) and (3.3) for the clipping height b of the rLS; i.e.; as this convergence is geometrically fast, we only need to calculate b for a small number of t's (until M_t^0, $\Sigma_{t|t[-1]}$ "stabilize"), which, if the hyper-parameters are known, can be done offline, before having made any observation.

4 IO-optimality

So far we have only considered (wide-sense) AO-Robustness. In the presence of IO's, we have already noted that instead of attenuating (the influence of) a dubious observation we would rather want to follow an IO outlier as fast as possible. In this context, it is well-known that the Kalman filter tends to be too inert and that we need a faster tracking filter. To do so, let us go back to our "Bayesian" model (3.11) but now assume an additive structure, i.e.; we specify the transition densities $\pi(y, x)$ to come from an observation Y which is built up as

$$Y = X + \varepsilon \tag{4.1}$$

Equation (4.1) reveals a remarkable symmetry of X and ε which we are going to exploit now: Apparently

$$\mathrm{E}[X|Y] = Y - \mathrm{E}[\varepsilon|Y] \tag{4.2}$$

This is helpful if we are now assuming that ε will always be ideally distributed, and instead the states X_t get corrupted. To this end, we retain the SO-model from the preceding sections, i.e., Y^{id} will be replaced from time to time by Y^{di}. Contrary to the AO formulation however, we now assume that this replacement by Y^{di} reflects a corresponding change in X, as we now want to track the distorted signal. As a consequence this gives the following IO-version of the minimax problem (where the only visible difference is the superscript re for X).

$$\max_{\mathcal{U}} \mathrm{E}_{\mathrm{re}} |X^{\mathrm{re}} - f(Y^{\mathrm{re}})|^2 = \min_f ! \tag{4.3}$$

But, using $X^{\mathrm{re}} = Y^{\mathrm{re}} - \varepsilon$, and setting $\tilde{f}(y) = y - f(y)$ we obtain the equivalent formulation

$$\max_{\mathcal{U}} \mathrm{E}_{\mathrm{re}} |\varepsilon - \tilde{f}(Y^{\mathrm{re}})|^2 = \min_{\tilde{f}} ! \tag{4.4}$$

and we are back in the situation of subsection (3.2) with the respective rôles of X and ε interchanged. That is; the corresponding theorems translate word by word and give

Theorem 4.1 (Minimax-IO).

(1)' *In this situation, there is a* **saddle-point** $(f_1, P_1^{Y^{\mathrm{di}}})$ *for Problem* (4.3)

$$\begin{aligned} f_1(y) &:= y - \tilde{D}(y) \min\{1, \tilde{\rho}/\big|\tilde{D}(y)\big|\} & (4.5)\\ P_1^{Y^{\mathrm{di}}}(dy) &:= \tfrac{1-r}{r}(\big|\tilde{D}(y)\big|/\tilde{\rho} - 1)_+ \, P^{Y^{\mathrm{id}}}(dy) & (4.6) \end{aligned}$$

where $\tilde{\rho} > 0$ *ensures that* $\int P_1^{Y^{\mathrm{di}}}(dy) = 1$ *and*

$$\tilde{D}(y) = y - \mathrm{E}_{\mathrm{id}}[X|Y = y] \tag{4.7}$$

(3)' *If* $\mathrm{E}_{\mathrm{id}}[X|Y]$ *is linear in* Y*, i.e.;* $\mathrm{E}_{\mathrm{id}}[X|Y] = MY$ *for some matrix* M*, then necessarily*

$$M = M^0 = \mathrm{Cov}(X, Y)\,\mathrm{Var}\,Y^- \tag{4.8}$$

—or in the SSM formulation: M^0 *is just the classical Kalman gain and* f_1 *the (one-step) rLS.IO defined below.*

Note that contrary to Theorem 3.3 where $\mathrm{E}\,X$ need not be 0, here $\mathrm{E}\,\varepsilon = 0$, which simplifies the definition of $\tilde{D}$ in (4.7).

rLS.IO: In analogy to the definition of the rLS in equation (3.1), we set up an IO-robust version of the rLS as follows: We retain the initialization and prediction step of the classical Kalman filter and, assuming Z_t invertible for the moment, replace the correction step by

$$X_{t|t} = X_{t|t-1} + Z_t^{-1}[\Delta Y_t - H_b\Big((\mathbb{I}_q - Z_t M_t^0)\Delta Y_t\Big)] \tag{4.9}$$

where the same arguments for the choice of the norm and the clipping height apply as for the AO-robust version of the rLS.

To better distinguish (wide-sense) IO- and AO-robust filters, let us call the IO-robust version *rLS.IO* and (for distinction) the AO-robust filter *rLS.AO* in the sequel.

Invertibility problem Back in the (linear, discrete-time, Euclidean) state space model the approach just described faces the problem that in general matrix Z_t will not be invertible, so we cannot reconstruct X injectively from Y and ε.
Under a certain full-rank condition, this problem can be solved by passing to corresponding rLS-type smoothers. The assumption we need is a version of *complete constructibility*, confer Anderson and Moore (1979, Appendix), adopted to the time-inhomogeneous case which reads:

Denoting the product $F_{t+p}F_{t+p-1} \cdot \ldots \cdot F_t$ by $F_{t+p:t}$ we assume that for each t, $F_{t+p-1:t}(\mathbb{R}^p)$ is contained in $[Z_t^\tau, F_t^\tau Z_{t+1}^\tau, F_{t+1:t}^\tau Z_{t+2}^\tau, \ldots, F_{t+p-1:t}^\tau Z_{t+p-1}^\tau](\mathbb{R}^q)$. Details will be given in a subsequent paper.

Remark 4.2. *(i) It is worth noting that also our IO-robust version is a filter, hence does not use information of observations made after the state to reconstruct; rLS.IO is strictly recursive and non iterative, hence well-suited for online applications.*

(ii) An alias to rLS.IO could be "hysteric filter" as it completely hysterically follows any changes in the Y's.

5 Simultaneous Treatment of AO's and IO's

As already mentioned, simultaneous treatment of (wide-sense) AO's and IO's is only possible with a certain delay. With this delay, we can base our decision of whether there was an AO or an IO on the size of subsequent $|\Delta Y_t|$'s — if there was an AO this should result in only one "large" $|\Delta Y_t|$ in a row, while in case of an IO there should be a whole sequence of $|\Delta Y_t|$'s. So a hybrid filter (called rLS.IOAO for simplicity) could be designed as follows:

To a given delay window width w, we run in parallel rLS.AO and rLS.IO (but only store the last w values of rLS.IO). By default we return the rLS.AO values. Whenever there is a run of w "large"

$|\Delta Y_t^{\rm rLS.AO}|$'s we replace the last w filter values by the corresponding rLS.IO values and use these ones to continue with the rLS.AO.

In the ideal (Gaussian) model, the ΔY_t's should be independent, so a reasonable decision on whether a sequence of $|\Delta Y_t^{\rm rLS.AO}|$'s is "large" could be based on corresponding quantiles of $|\Delta Y_t|$ in the ideal model. Relaxing this condition a little, we already switch to rLS.IO when a high percentage h (default: 80% of the last w instances of $|\Delta Y_t^{\rm rLS.AO}|$) are larger than this given quantile.

This leaves us to determine several tuning parameters: window-width w (proposal: 5 seems to be a good value, but thorough testing still remains to be done), the clipping heights for rLS.IO and rLS.AO (proposal: according to (versions of) (3.2) or (3.3)), the percentage h, and the corresponding quantile (default 99%) assuming that $\Delta Y_t \sim\ N(0, \Delta_t)$.

Remark 5.1. *(i)* *Note that although the decision whether we issue the rLS.IO or the rLS.AO values is made w observations after the state to be reconstructed, we still only use filters, hence the information of Y_{t+j}, $j = 1, \ldots, w-1$ is not used to improve the reconstruction so far, as this would involve corresponding (yet-to-be-robustified) smoothers. Once the corresponding work on robust smoothing will be done (see section 7), we could surely use this additional information.*

(ii) *As noted in the corresponding discussion in subsection 3.5, in general Δ_t will usually converge in t exponentially fast, so these tuning parameters will only have to be determined for a small number of time instances t. In fact, setting them time-invariant will already do a reasonable job.*

6 Simulation Example: Steady State Model

Returning to our reference example, model (2.17), let us see how classical Kalman filter, rLS.AO, rLS.IO, and rLS.IOAO perform in this model and under (wide sense) AO's and IO's. More specifically, we have generated (deterministic) (wide sense) AO's in observations 10,15,23, and (wide-sense) IO's in observations 20–25 (a local linear trend) and 37–42 (level shift).

As competitors, we include the ACM filter by Martin (1979) as implemented by B. Spangl in R package `robKalman`, and a variant hybr$_{\rm PRMH}$ of `robfilter`, confer Fried and Schettlinger (2008) as to its implementation and Fried et al. (2006) as to its definition, which is a non-parametric filter fitting local levels and linear trends.

The results are plotted in Figures 3–6, where in the plots, we confine ourselves to the rLS-variants, which already makes for five curves to be plotted in one panel.

In the ideal situation, all filters perform well, with slight advantages for the classical Kalman filter (which has smallest theoretical MSE), but closely followed (and in the prediction case slightly beaten) by the rLS.IO.

In the IO situation, the "hysteric" rLS.IO filter performs best, beating the classical Kalman filter, and both rLS.IOAO and $\mathrm{hybr}_{\mathrm{PRMH}}$ perform reasonably well, while the (wide-sense) AO-robust filters ACM and rLS.AO are not able to track the IO at all (as they can only perform bounded correction steps) and hence, like a hanging slope, only closely recover the changed situation.

In the AO situation, we have the complementary image; here ACM performs best (see also Remark 6.1(a)), but rLS.AO only performs slightly weaker. rLS.IOAO is a little worse, and with a certain gap, but still reasonably well follows $\mathrm{hybr}_{\mathrm{PRMH}}$, while both classical Kalman filter and rLS.IO (the latter even worse) perform drastically bad.

Finally, in the mixed IO and AO situation, $\mathrm{hybr}_{\mathrm{PRMH}}$ is by far the best solution, then followed with a certain gap by the rLS.IOAO, while all other filters perform unacceptably bad. By construction, rLS.IOAO assumes that at every time instance there only can be either an AO or an IO (both "wide-sense"). Otherwise the corresponding MSE gets unbounded on every neighborhood $\mathcal{U}(r)$ for $r > 0$. Hence the AO in observation 23 really confuses rLS.IOAO completely: it has just switched to "hysteric" IO behavior and hence faithfully follows the AO. $\mathrm{hybr}_{\mathrm{PRMH}}$ based on (repeated) medians does not have this problem, as the median even stays stable under (almost) arbitrary substitutive outliers, hence it is able to keep the local linear trend. Omitting observation 23 results in a much better performance of rLS.IOAO, which then even beats $\mathrm{hybr}_{\mathrm{PRMH}}$, confer Table 2.

Averaging over time in one realization of the state space model, we get the "ergodic" empirical MSEs as displayed in Table 1

empirical MSE							
Situation	Type	Kalman	$\mathrm{rLS}_{\mathrm{IO}}$	$\mathrm{rLS}_{\mathrm{AO}}$	$\mathrm{rLS}_{\mathrm{IOAO}}$	ACM	$\mathrm{hybr}_{\mathrm{PRMH}}$
ideal	filter	**0.59**	0.60	0.75	1.08	0.77	1.41
	pred	1.69	**1.67**	1.96	2.26	2.01	
IO	filter	1.04	**0.83**	6.54	1.36	25.19	1.36
	pred	5.28	**4.71**	12.17	5.42	32.16	
AO	filter	15.25	30.38	0.91	1.16	**0.82**	1.79
	pred	15.15	29.68	2.00	2.25	**2.05**	
IO&AO	filter	17.00	30.52	12.89	**7.78**	28.76	**1.53**
	pred	21.94	34.56	19.23	**13.87**	36.08	

Table 1: "ergodic" estimates for the MSE of the variants of the rLS and the ACM and $\mathrm{hybr}_{\mathrm{PRMH}}$ in the situation described in the text; best results are printed in bold face.

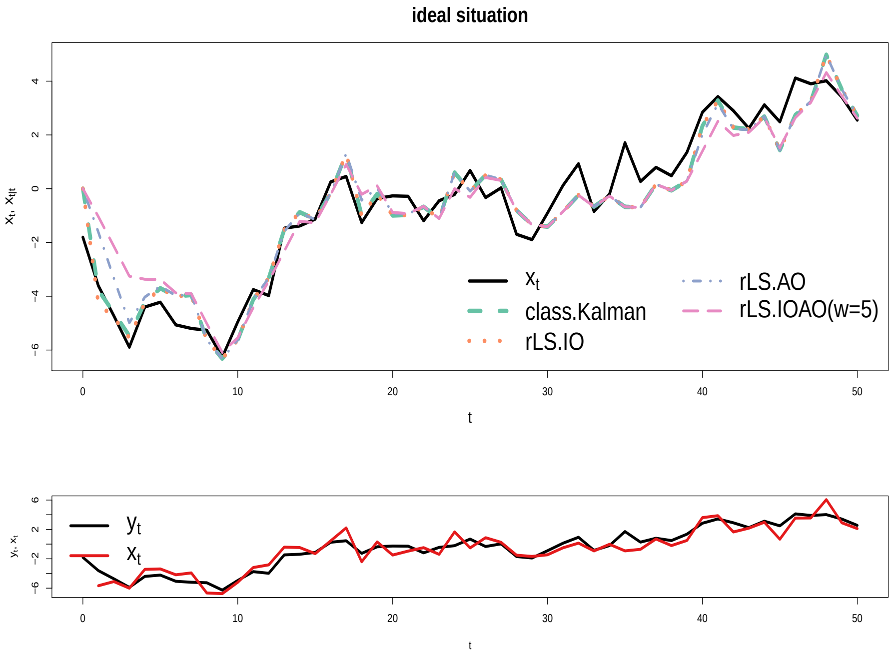

Figure 3: rLS-filter variants in model (2.17) in the ideal model; in the panel below (note the different y-scale) both actual states (black) and observations (red) are plotted.

Remark 6.1. *(i) The fact that the ACM filter beats the rLS may be explained by the fact that the contamination in this study clearly covers the worst-case behavior of the rLS but not of the ACM filter, compare Remark 3.4(e), and also fails for hybr$_{\text{PRMH}}$.*

(ii) rLS.IOAO really has its advantages in higher dimensions where median-based filters are much harder to define and get computationally very expensive. One might even think of combining rLS.IOAO and hybr$_{\text{PRMH}}$ in these settings: first let rLS.IOAO do a preliminary, fast, and dimension-independent cleaning, and then let hybr$_{\text{PRMH}}$ polish this result coordinate-wise.

(iii) It is still an open question whether we can improve on the rLS.IOAO behavior, using the state space model

$$Z_t = (1,t), X_t = (a_t, b_t)^\tau, F_t = \mathbb{I}_2, Q_t = 0.1\mathbb{I}_2, V_t = 1 \tag{6.1}$$

which (upto the specification of error/innovation variance) is essentially the model in the background of hybr$_{\text{PRMH}}$. In this setting Z_t is not invertible, but the model is completely constructible, so passing to smoothers might help.

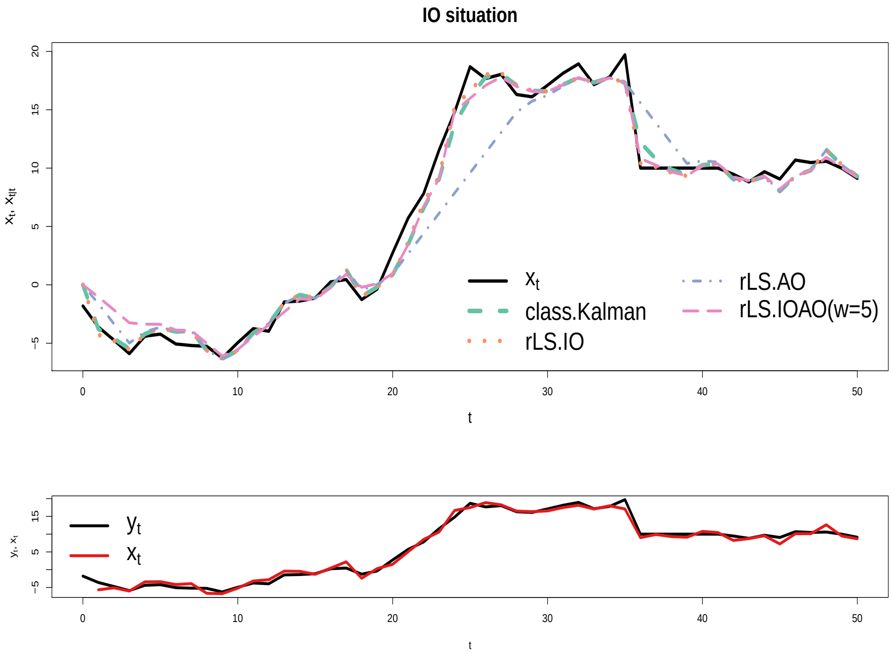

Figure 4: rLS-filter variants in model (2.17) with IO's: a local linear trend at X_{20}–X_{25} and a level shift for states X_{37}–X_{42}; the panel below (note the different y-scale) is as in Figure 3.

7 Open ends with **rLS**

Open questions and possible extensions of our rLS-filters concern robust smoothing, where we have already seen the need for in the IO context when Z_t's are not invertible. As the corresponding Kalman Smoother is structurally very similar to the Kalman filter, a rLS-type robustification is straightforward, and we expect the same type of optimality results to hold there.

Robustified Kalman smoothing is also key issue when we want to estimate the hyper-parameters from the data. In the ideal model setup there is a path-breaking application of the EM-algorithm by Shumway and Stoffer (1982) which has been improved upon by Durbin and Koopman (2001). Alternatives to the EM-algorithm have been conceived by [Dempster???][NEykov???]

A robustification using the fact that for filtering the hyper parameters can be seen a nuisance parameters has been proposed in Ruckdeschel (2001, Section 10.5.8) but still needs to be implemented to software.

In both the Shumway-Stoffer approach and the mentioned robustification, starting values for the hyper-parameters are crucial to initialize the EM-algorithm. To this end robust multivariate autoco-

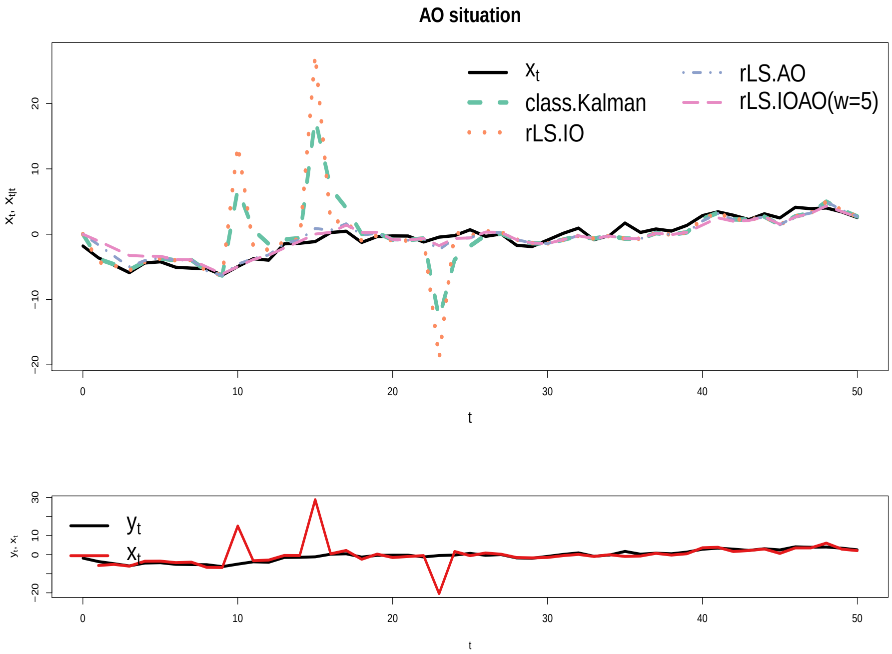

Figure 5:rLS-filter variants in model (2.17) with AO's in observations 10,15,23; the panel below (note the different y-scale) is as in Figure 3.

variances would be extremely helpful.

Also there is a strong need to elaborate the connection to particle filters as they might help to get hand on the exact conditional expectation, needed "desparately" for Theorem 3.3.

Extensions with names. . . [2]

- robust smoothing (with Cezar Chirila (ITWM))
- robust EM-Algorithm to estimate unknown hyper parameters (extending Shumway/Stoffer[82]) (with Irina Ursachi (ITWM))
- interpretation as random coefficient regression
  $\rightsquigarrow$ other approach using robust regression (**rIC**, **mIC**)
  (implementation: with Bernhard Spangl)
- connection to particle filters — theory and interface to `DEBI`
  (with Carlos Prieto (Madrid) and Simon Godsill)
- simultaneous treatment (with delay) of IO's and AO's (with Carlos Prieto, Bernhard)

[2] will not be included in the end version; only for internal use. . .

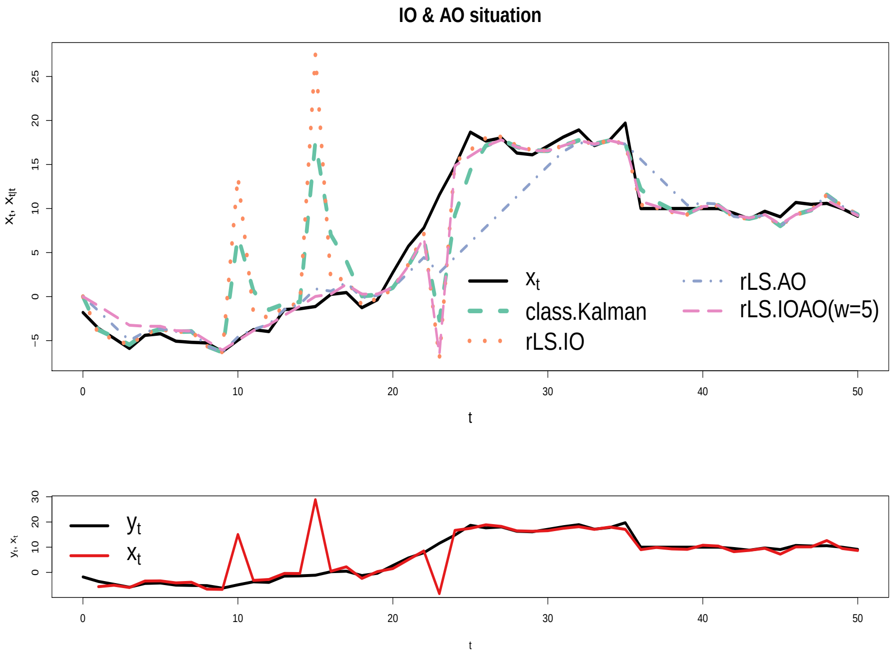

Figure 6:rLS-filter variants in model (2.17) with both IO's as in Figure 4 and AO's as in Figure 5; the panel below (note the different y-scale) is as in Figure 3.

8 Implementation: R-package `robKalman`

In an ongoing project with Bernhard Spangl, BOKU, Vienna, and I. Ursachi and C. Chirila (both ITWM), we are about to implement the rLS filter to R, see R Development Core Team (2009), more specifically to an R-package `robKalman`, the development of which is done under `r-forge` project `https://r-forge.r-project.org/projects/robkalman/`, see also R-Forge Administration and Development Team (2008). Under this address you will also find a preliminary version available for download. Details to the implementation will be discussed in Ruckdeschel and Spangl (2008).

empirical MSE —without obs. 23							
Situation	Type	Kalm	$\mathrm{rLS}_{\mathrm{IO}}$	$\mathrm{rLS}_{\mathrm{AO}}$	$\mathrm{rLS}_{\mathrm{IOAO}}$	ACM	$\mathrm{hybr}_{\mathrm{PRMH}}$
ideal	filter	**0.59**	0.60	0.75	1.10	0.78	1.43
	pred	1.71	**1.69**	1.99	2.29	2.03	
IO	filter	0.94	**0.74**	6.08	1.26	24.48	1.38
	pred	5.59	**4.98**	12.05	5.73	31.66	
AO	filter	12.46	24.07	0.86	1.15	**0.84**	1.83
	pred	12.18	23.05	1.94	2.25	**2.10**	
IO&AO	filter	13.28	24.21	11.58	**1.31**	27.93	1.56
	pred	17.01	26.34	17.80	**5.63**	35.38	

Table 2: results as in Table 1, but excluding the values for observation 23, where we had coincidence of (wide-sense) IO and AO, a situation not covered in the design of rLS.IOAO.

9 Proofs

As we will use Theorem 3.3 to prove Lemma 3.2, we postpone the proof of the latter.
PROOF TO THEOREM 3.3:

(1) We start with solving $\max_{\partial\mathcal{U}}\min_f[\ldots]$. To this end we note that the $\max\min$-Problem amounts to solving $\min_{\partial\mathcal{U}} \mathrm{E}_{\mathrm{re}}[\big|\,\mathrm{E}_{\mathrm{re}}[X|Y^{\mathrm{re}}]\big|^2]$. For fixed element $P^{Y^{\mathrm{di}}}$ assume w.l.o.g. that $\mu \gg P^{Y^{\mathrm{di}}}$ for μ from (3.11) — otherwise we replace μ by $\mu + P^{Y^{\mathrm{di}}}$; this gives us a μ-density $q(y)$ of $P^{Y^{\mathrm{di}}}$. Determining joint (real) law $P^{X,Y^{\mathrm{re}}}(dx,dy)$ as

$$P(X\in A, Y^{\mathrm{re}}\in B) = \int \mathrm{I}_A(x)\,\mathrm{I}_B(y)[(1-r)\pi(y,x) + rq(y)\,P^X(dx)\,\mu(dy) \tag{9.1}$$

we deduce that $\mu(dy)$-a.e.

$$\mathrm{E}_{\mathrm{re}}[X|\hat{Y}=y] = \frac{rq(y)\,\mathrm{E}\,X + (1-r)p^{Y^{\mathrm{id}}}(y)\,\mathrm{E}_{\mathrm{id}}[X|Y]}{rq(y)+(1-r)p^{Y^{\mathrm{id}}}(y)} =: F(q) \tag{9.2}$$

Hence we have to minimize F in $M_0 = \{q\in L_1(\mu)\,|\quad q\ge 0,\quad \int q\,d\mu = 1\}$. To this end, we note that F is convex on the non-void, convex set $M = \{q\in L_1(\mu)\,|\,q\ge 0\}$, so we may consider the Lagrangian

$$L_{\tilde\rho}(q) := F(q) + \tilde\rho\int q\,d\mu \tag{9.3}$$

for some positive Lagrange multiplier $\tilde\rho$. Pointwise (in y) minimization of $L_{\tilde\rho}(q)$ on M gives us the form

$$\hat{q}_s(y) = \tfrac{1-r}{r}(\big|D(y)\big|/s\,-1)_+\;p^Y(y) \tag{9.4}$$

for some constant $s = s(\tilde\rho) = (\big|\,\mathrm{E}\,X\big|^2 + \tilde\rho/r)^{1/2}$.
Considering

$$H(s) = \int \hat{q}_s(y)\,\mu(dy) \tag{9.5}$$

we note that the integrand is isotone and continuous in $s \geq 0$, hence by monotone convergence, H, too, is isotone and continuous. Now $\lim_{s\to\infty} H(s) = \infty$, $H(0) = 0$, so by continuity, there is some $\rho \geq 0$ with $H(\rho) = 1$. On M_0, $\int q\,d\mu = 1$, but $\hat{q}_\rho \in M_0$ and is optimal on $M \supset M_0$ hence it also minimizes F on M_0. In particular, we get representation (3.19) and note that the least favorable $P_0^{Y^{\rm di}}$ is dominated according to $P_0^{Y^{\rm di}} \ll P^{Y^{\rm id}}$.

As next step we return to the minmax problem, i.e.; $\min_f\, \max_{\partial\mathcal{U}}[\ldots]$ and show that

$$\max_{\partial\mathcal{U}} \min_f\, [\ldots] = \min_f\, \max_{\partial\mathcal{U}}[\ldots] \tag{9.6}$$

To this end we first obtain $f_0(y)$ as $f_0(y) = {\rm E}_{\rm re}[X|Y^{\rm re} = y]$ giving (3.18) and determine ${\rm E}_{\rm re}\,\big|X - f_0(Y^{\rm re})\big|^2$ for general $q(y)$: Writing a sub/superscript ${\rm re};q$ for the evaluation under the corresponding situation generated by this $q(y)$ we obtain that

$$\begin{aligned} {\rm MSE}_{{\rm re};\,q}[f_0(Y^{{\rm re};\,q})] &= (1-r)\,{\rm E}_{\rm id}\,\big|X - f_0(Y^{\rm id})\big|^2 + r\,{\rm tr}\,{\rm Cov}\,X + \\ &\quad + r\;{\rm E}_q \min(|D(Y^{{\rm di};,q})|^2, \rho^2) \end{aligned} \tag{9.7}$$

which achieves its maximum (in q) for any q that is concentrated on the set $\big\{\,|D(Y^{{\rm di};,q})| > \rho\,\big\}$, which is true for $\hat{q}_\rho$. Hence for all contaminating densities $q(y)$

$$ {\rm E}_{{\rm re};\,q}\,\big|X - f_0(Y^{{\rm re};\,q})\big|^2 \leq {\rm E}_{{\rm re};\,\hat{q}_\rho}\,\big|X - f_0(Y^{{\rm re};\,\hat{q}_\rho})\big|^2 \tag{9.8}$$

and $\max_{\partial\mathcal{U}} \min_f\, [\ldots] \geq \min_f\, \max_{\partial\mathcal{U}}[\ldots]$, so we have shown (9.6).

Finally, we pass over from $\partial\mathcal{U}$ to $\mathcal{U}$. To this end, in this paragraph, we use f_r, $P_r^{Y^{\rm di}}$ to denote the components of the saddle-point for $\partial\mathcal{U}(r)$, as well as $\rho(r)$ for the corresponding Lagrange multiplier and w_r for the corresponding weight, i.e.

$$w_r = w_r(y) = \min(1, \frac{\rho(r)}{|D(y)|}) \tag{9.9}$$

Let $R(f, P, r)$ be the MSE of procedure f at the SO model $\partial\mathcal{U}(r)$ with contaminating $P^{Y^{\rm di}} = P$. As can be seen from (3.19), $\rho(r)$ is antitone in r; in particular, as $P_r^{Y^{\rm di}}$ is concentrated on $\{|D(Y)| \geq \rho(r)\}$ which for $r \leq s$ is a subset of $\{|D(Y)| \geq \rho(s)\}$,

$$R(f_s, s, P_s^{Y^{\rm di}}) = R(f_s, s, P_r^{Y^{\rm di}}) \qquad \text{for } r \leq s$$

But for $r < s$ and for arbitrary $P^{Y^{\rm di}}$, using that

$${\rm tr}\,{\rm Cov}\,X = {\rm E}_{\rm id}\Big[\,{\rm tr}\,{\rm Cov}_{\rm id}[X|Y^{\rm id}] + |D(Y^{\rm id})|^2\Big] \tag{9.10}$$

we obtain

$$
\begin{aligned}
R(f_s,P,r) &= (1-r)\,\mathrm{E}_{\mathrm{id}}\Big[\operatorname{tr}\operatorname{Cov}_{\mathrm{id}}[X|Y^{\mathrm{id}}] + (|D(Y^{\mathrm{id}})|-\rho(s))_+^2\Big] + \\
&\quad + r\ \operatorname{tr}\operatorname{Cov}X + r\ \mathrm{E}_P[\min(|D(Y^{\mathrm{di}})|,\rho(s))^2] \le \\
&\le\ (1-r)\,\mathrm{E}_{\mathrm{id}}\Big[\operatorname{tr}\operatorname{Cov}_{\mathrm{id}}[X|Y^{\mathrm{id}}] + (|D(Y^{\mathrm{id}})|-\rho(s))_+^2\Big] + \\
&\quad + r\ \operatorname{tr}\operatorname{Cov}X + r\,\rho(s)^2 = R(f_s,P_r^{Y^{\mathrm{di}}},r) = \\
&=\ \mathrm{E}_{\mathrm{id}}\Big[\operatorname{tr}\operatorname{Cov}_{\mathrm{id}}[X|Y^{\mathrm{id}}] + (|D(Y^{\mathrm{id}})|-\rho(s))_+^2\Big] + \\
&\quad + r\Big\{\ \mathrm{E}_{\mathrm{id}}\Big[|D(Y^{\mathrm{id}})|^2\Big(1-\big(1-w_s(Y^{\mathrm{id}})\big)^2\Big)\Big] + \rho(s)^2\ \Big\} < \\
&<\ \mathrm{E}_{\mathrm{id}}\Big[\operatorname{tr}\operatorname{Cov}_{\mathrm{id}}[X|Y^{\mathrm{id}}] + (|D(Y^{\mathrm{id}})|-\rho(s))_+^2\Big] + \\
&\quad + s\Big\{\ \mathrm{E}_{\mathrm{id}}\Big[|D(Y^{\mathrm{id}})|^2\Big(1-\big(1-w_s(Y^{\mathrm{id}})\big)^2\Big)\Big] + \rho(s)^2\ \Big\} = \\
&=\ R(f_s,P_r^{Y^{\mathrm{di}}},s) = R(f_s,P_s^{Y^{\mathrm{di}}},s)
\end{aligned}
\tag{9.11}
$$

Hence the saddle-point extends to $\mathcal{U}(r)$, and we have shown (3). In particular the maximal risk is never attained in the interior $\mathcal{U}(r)\setminus\partial\mathcal{U}(r)$.
For later reference, we determine the minimax risk as

$$
R(f_r,P_r^{Y^{\mathrm{di}}},r) = \operatorname{tr}\operatorname{Cov}(X) - (1-r)\,\mathrm{E}_{\mathrm{id}}\big[\,|D(Y^{\mathrm{id}})|^2 w_r(Y^{\mathrm{id}})\big] \tag{9.12}
$$

(2) Denoting $\tilde f(Y) = f(Y) - \mathrm{E}\,X$, and $X^0 = X - \mathrm{E}\,X$, we may restate (3.17) as

$$
\mathrm{E}_{\mathrm{id}}\,|X^0 - \tilde f(Y)|^2 = \min\nolimits_{\tilde f}! \quad \text{s.t. } \sup\nolimits_{\mathcal{U}}\big|\,\mathrm{E}_{\mathrm{re}}\,\tilde f(Y^{\mathrm{re}})\big| \le b \tag{9.13}
$$

Upon noting that $\sup_{\mathcal{U}}|\,\mathrm{E}_{\mathrm{re}}\,\tilde f| = \sup|\tilde f|$ (follows just as in Rieder (1994, chap. 5)) and writing

$$
\mathrm{E}_{\mathrm{id}}\,|X^0 - \tilde f(Y)|^2 = \mathrm{E}_{\mathrm{id}}\Big[\,\mathrm{E}[|X^0 - \tilde f(Y)|^2\,\Big|\,Y]\Big],
$$

pointwise minimization of the inner expectation subject to $\big|\tilde f(Y^{\mathrm{re}})\big| \le b$ gives the result.

(3) If $\mathrm{E}_{\mathrm{id}}[X|Y]$ is linear in Y, the corresponding optimal matrix M^0 is just the respective Fourier coefficient, i.e.; $\operatorname{Cov}(X,Y)\operatorname{Var}Y^-$ where A^- stands for the Moore-Penrose inverse. In subsection 2.4 we have seen that the classical Kalman filter is optimal among all linear filters; hence the corresponding Kalman gain M^0 is then the optimal linear transformation in the state space context.

////

Remark 9.1. *(i) An alternative proof which follows Rieder (1994, Appendix B), showing existence of Lagrange multipliers in (1) by abstract compactness and continuity arguments is given in Ruckdeschel (2001, pp.156–163).*

(ii) *A similar proof to the one given here is given in Birmiwal and Shen (1993). However, they invoke a minimax result by Ferguson (1967) which in our infinite dimensional setting is not applicable. Also their setting is restricted to one dimension, and they assume Lebesgue densities right away (without mentioning this). In particular, they do not realize the connection to the exact conditional mean present in equation* (3.20).

(iii) *The fact that the solutions to Problems* (3.16) *and* (3.17) *coincide parallels the situation in the estimation problem for a one-dimensional location parameter.*

PROOF TO LEMMA 3.2: We start by showing that

$$\rho_0(s) \le \max\{A_s/A_{r_l}, B_s/B_{r_u}\} \tag{9.14}$$

To this end, we use the fact that for $0 \le a, b, c, d$

$$(a+b)/(c+d) \le \max(a/c, b/d) \tag{9.15}$$

Equation (3.3) shows that $b(r)$ is (strictly) decreasing in r (for $r > 0$). Hence A_r is increasing in r, and B_r decreasing. One easily shows by dominated convergence that $b(r)$, and hence A_r and B_r are continuous in r. Thus (3.9) follows from the intermediate value theorem. For $r_u = 1$, one argues letting $r_n \in [0, 1)$ tend to 1.

To show equality in (9.14), as in the proof of Kohl (2005, Lemma 2.2.3), we first show that for $r \ge s$, s fixed, $\rho(r, s)$ is increasing and correspondingly, for $r \le s$, s fixed, decreasing, which will entail equation (3.8): Let $0 \le s < r_1 < r_2 \le 1$. Then by monotony of A_r, B_r, $(A_s B_s^{-1} + r_1)^{-1} \ge (A_{r_1} B_{r_1}^{-1} + r_1)^{-1}$; multiplying this inequality with $(r_2 - r_1)$, we get

$$\frac{(r_2 - r_1)B_s}{A_s + r_1 B_s} \ge \frac{(r_2 - r_1)B_{r_1}}{A_{r_1} + r_1 B_{r_1}} \tag{9.16}$$

Now, due to optimality of $A_r + rB_r$ for radius r, so

$$\begin{aligned} 0 &\le \frac{(r_2 - r_1)B_s}{A_s + r_1 B_s} - \frac{(r_2 - r_1)B_{r_1} + A_{r_2} + r_2 B_{r_2} - A_{r_1} - r_2 B_{r_1}}{A_{r_1} + r_1 B_{r_1}} = \\ &= \frac{(r_2 - r_1)B_s}{A_s + r_1 B_s} - \frac{A_{r_2} + r_2 B_{r_2}}{A_{r_1} + r_1 B_{r_1}} + 1 \end{aligned}$$

Multiplying with $(A_s + r_1 B_s)/(A_{r_2} + r_2 B_{r_2})$, we obtain indeed

$$0 \le \frac{A_s + r_2 B_s}{A_{r_2} + r_2 B_{r_2}} - \frac{A_s + r_1 B_s}{A_{r_1} + r_1 B_{r_1}} = \rho(r_2, s) - \rho(r_1, s)$$

and similarly for $0 \ge s > r_1 > r_2 \ge 1$. Next, for equation (3.10), we show, that for r fixed, and $s \ge r$, $\rho(r, s)$ is increasing and correspondingly, for $s \le r$, decreasing: Let $0 \le r < r_1 < r_2 \le 1$

$$\begin{aligned} &A_{r_2} + rB_{r_2} - A_{r_1} + rB_{r_1} = \\ &= (r_1 - r)(B_{r_1} - B_{r_2}) + A_{r_2} + r_1 B_{r_2} - A_{r_1} - r_1 B_{r_1} \ge 0 \end{aligned} \tag{9.17}$$

and similarly for $0 \geq r > r_1 > r_2 \geq 1$.

For the last assertion, we note that by (3.3), $b(1) = 0$, hence $B_1 = 0$. Hence $\rho_0(s) = \max\left\{A_s/A_{r_l}, B_s/B_1\right\}$ is ∞ for each $s < 1$, while for $s = 1$, we get $\rho_0(1) = \max\{A_1/A_{r_l}, 1\} = 1$. ////

PROOF TO PROPOSITION 3.5: Recall that by the Cramér-Lévy Theorem (confer Feller (1971, Thm. 1, p. 525)) the sum of two independent random variables has Gaussian distribution iff each summand is Gaussian. This can easily be translated into a corresponding asymptotic statement, confer Ruckdeschel (2001, Prop. A.2.4), i.e.; the sum of two independent random variables converges weakly to a Gaussian distribution iff each summand converges weakly to a Gaussian distribution.

We first consider (for fixed t, omitted from notation where clear) the filter error,

$$\widetilde{\Delta X} := X_t - X_{t|t} = \Delta X - H_b(M^0 \Delta Y) \tag{9.18}$$

where we assume ΔX, ε, and v normal. With

$$g := M^0 \Delta Y - H_b(M^0 \Delta Y) = \left(\left|M^0 \Delta Y\right| - b\right)_+ \tag{9.19}$$

Then for the conditional law of $\tilde{\Delta} X$ given ΔY we have

$$\mathcal{L}(\widetilde{\Delta X} | \Delta Y) = \mathcal{N}_p(g, (\mathbb{I}_p - M^0 Z)\Sigma) \tag{9.20}$$

for $\Sigma = \operatorname{Cov} \Delta X$. Hence

$$\mathcal{L}(\widetilde{\Delta X}) = \mathcal{L}(g) * \mathcal{N}_p(0, (\mathbb{I}_p - M^0 Z)\Sigma) \tag{9.21}$$

which by Cramér-Lévy cannot be normal, as g is obviously not normal. Conseqently

$$\Delta X_{t+1} = F_{t+1} \tilde{\Delta} X_t + v_{t+1} \tag{9.22}$$

cannot be normal either. Hence starting with normal ΔX_t and ε_t, ΔX_{t+1} cannot be normal. The same assertion clearly holds if v_t is not normal. As by (3.22), g_t does neither converge to 0 nor to $M^0 \Delta Y$, the asymptotic version of Cramér-Lévy also excludes asymptotic normality.

A similar assertion for the case that v_t is normal but not both ΔX_t and ε_t are, seems plausible and we conjecture that this is true; it may also be proven in particular cases, but in general, it is hard to obtain due to the lack of independence of $\Delta X - g$ and ΔY.

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For the second equivalence in Proposition 3.7 we use the following lemma and a corollary of it:

Lemma 9.2. *Let $\varepsilon \sim \mathcal{N}_q(0, V)$, $X \sim P^X$ and for some measurable function $h\colon \operatorname{range}(X) \to \mathbb{R}^q$ let $Y = h(x) + \varepsilon$. Let $g \in L_1^l(P^X)$, i.e., $g\colon \operatorname{range}(X) \to \mathbb{R}^l$ measurable and $\operatorname{E}_{P^X} |g(X)| < \infty$. Then*

$$\frac{\partial}{\partial y} \operatorname{E}[g(X) | Y = y] = \operatorname{Cov}[g(x), h(x) | Y = y] V^{-1} \tag{9.23}$$

PROOF TO LEMMA 9.3: For simplicity, we only consider the case $\operatorname{rk} V = q$; otherwise we may pass to $\varepsilon = A\tilde{\varepsilon}$ for some $\tilde{\varepsilon} \sim \mathcal{N}_{\tilde{q}}(0, \tilde{V})$ with $\operatorname{rk} \tilde{V} = \tilde{q}$ and use the generalized inverse V^{-} instead of V^{-1} everywhere in the proof.

Let p^{ε} be the Lebesgue density of ε and denote $\Lambda^{\varepsilon}(\varepsilon) := \frac{\partial}{\partial \varepsilon} \log p^{\varepsilon}(\varepsilon)$. Then, no matter wether ε is Gaussian, it holds that

$$\mathrm{E}[g(X)|Y = y] = \frac{\int g(x) p^{\varepsilon}(y - h(x))\, P^X(dx)}{\int p^{\varepsilon}(y - h(x))\, P^X(dx)}$$

Hence, if we may interchange differentiation and integration (which is the case if ε normal), we obtain that

$$\frac{\partial}{\partial y} \mathrm{E}[g(X)|Y = y] = \mathrm{Cov}[g(X), \Lambda^{\varepsilon}(Y - h(X))\, |Y = y]$$

But as $\varepsilon \sim \mathcal{N}_q(0, V)$, it holds that $\Lambda^{\varepsilon}(\varepsilon) = -V^{-1}\varepsilon$, which entails

$$\Lambda^{\varepsilon}(y - h(X)) - \mathrm{E}[\Lambda^{\varepsilon}(Y - h(X))|Y = y] = V^{-1}(h(X) - \mathrm{E}[h(X)|Y = y])$$

and thus (9.23) follows. ////

Corollary 9.3. *In our linear time discrete, Euclidean state space model, ommiting indices t, assume that $\operatorname{rk} V = q$ and let*

$$U := V^{-1} Z \Delta X, \qquad U^0 := U - \mathrm{E}[U|\Delta Y], \qquad \Delta X^0 := \Delta X - \mathrm{E}[\Delta X|\Delta Y] \tag{9.24}$$

Then

$$\frac{\partial}{\partial y} \mathrm{E}[\Delta X|\Delta Y = y] = \mathrm{Cov}(\Delta X, U|\Delta Y = y) \tag{9.25}$$

$$\frac{\partial^2}{\partial y_j \partial y_k} \mathrm{E}[\Delta X_i|\Delta Y = y] = \mathrm{E}(\Delta X_i^0 U_j^0 U_k^0|\Delta Y = y) \tag{9.26}$$

PROOF TO COROLLARY 9.3: During the proof we will omit Δ in notation. Equation (9.25) is just plugging in Lemma 9.2. We note that equivalently to (9.23) we could have written

$$\frac{\partial}{\partial y} \mathrm{E}[X|Y = y] = \mathrm{E}[X(U^0)^{\tau}|Y = y] = \mathrm{E}[XU^{\tau}|Y = y] - \mathrm{E}[X|Y = y]\, \mathrm{E}[U|Y = y]^{\tau}$$

Hence applying Lemma 9.2 for $g(X) = X_i U_j$ and $g(X) = U_j$ to the last two terms we obtain

$$\begin{aligned} \frac{\partial^2}{\partial y_j \partial y_k} \mathrm{E}[X_i|Y = y] &= \mathrm{E}[X_i U_j U_k^0|Y = y] - \mathrm{E}\, X_i\, \mathrm{E}[U_j U_k^0|Y = y] = \\ &= \mathrm{E}[X_i^0 U_j U_k^0|Y = y] = \mathrm{E}[X_i^0 U_j^0 U_k^0|Y = y] \end{aligned}$$

////

Proof to Proposition 3.7: Equivalence (3.24):
If $\mathcal{L}(\Delta X)$ is normal, the random variables $\Delta X, \Delta Y$ are jointly normal, hence linearity of conditional expectation is a well-known fact. If $\mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y]$ is linear, after substracting $\int MZxp^{\varepsilon}(y - Zx)\, P^X(dx)$ from both sides, we may write the corresponding Radon-Nikodym equation for the conditional expectation $P^Y(dy)$-a.e. as

$$M \int (y - Zx) p^{\varepsilon}(y - Zx)\, P^X(dx) = (\mathbb{I}_p - MZ) \int x p^{\varepsilon}(y - Zx)\, P^X(dx) \tag{9.27}$$

Let us introduce $q^{\varepsilon}(y) = y p^{\varepsilon}(y)$ and the signed measure $Q^X(dx) = x\, P(dx)$; if we denote the mapping $h\colon \mathbb{R}^q \to \mathbb{R}, y \mapsto h(y) = \int f(y - Zx)\, G(dx)$ as $f *_Z G$, (9.27) becomes

$$M q^{\varepsilon} *_Z P^X = (\mathbb{I}_p - MZ) p^{\varepsilon} *_Z Q^X \tag{9.28}$$

Passing over to the Fourier transforms (denoted with $\hat{\cdot}$) for $s \in \mathbb{R}^p$, $t \in \mathbb{R}^q$

$$\hat{q}^X(s) = \int \exp(is^{\tau}x) Q^X(dx), \qquad \hat{p}^X(s) = \int \exp(is^{\tau}x)\, P^X(dx)$$

$$\hat{q}^{\varepsilon}(t) = \int \exp(it^{\tau}y) q^{\varepsilon}(y)\, dy, \qquad \hat{p}^{\varepsilon}(t) = \int \exp(it^{\tau}y) p^{\varepsilon}(y)\, dy,$$

as usual convolution translates into products in Fourier space, in our case

$$\widehat{f *_Z G}(t) = \hat{f}(t) \hat{G}(Z^{\tau} t), \qquad t \in \mathbb{R}^q \tag{9.29}$$

and hence (9.28) in Fourier space is

$$M \hat{q}^{\varepsilon} \hat{p}^X(Z^{\tau} \cdot) = (\mathbb{I}_p - MZ) \hat{p}^{\varepsilon} \hat{q}^X(Z^{\tau} \cdot) \tag{9.30}$$

Now we obtain for the derivatives $(\hat{p}^X)'(s)$, $(\hat{p}^{\varepsilon})'(t)$ for $s \in \mathbb{R}^p$ and $t \in \mathbb{R}^q$,

$$(\hat{p}^X)'(s) = i(\hat{q}^X)(s), \qquad (\hat{p}^{\varepsilon})'(t) = i(\hat{q}^{\varepsilon})(t) \tag{9.31}$$

Assume $\mathbb{I}_p - MZ$ and V invertible —otherwise pass to the generalized inverses and to some $\tilde{\varepsilon}$ of lower dimension as indicated in the proof to Corollary 9.3; then $\hat{p}^{\varepsilon}(t) = \exp(-t^{\tau} V t/2) > 0$ and together with (9.31), this gives the linear differential equation

$$(\hat{p}^X)'(Z^{\tau} t) = -(\mathbb{I}_p - MZ)^{-1} M V t \hat{p}^X(Z^{\tau} t) \tag{9.32}$$

Fixing any direction t_0 such that $Z^{\tau} t_0 \neq 0$, this becomes an ODE

$$g'(s) = -t_0^{\tau} Z (\mathbb{I}_p - MZ)^{-1} M V t_0 s g(s), \qquad g(0) = 1 \tag{9.33}$$

which has a unique solution given by

$$g(s) = \exp(-t_0^{\tau} Z (\mathbb{I}_p - MZ)^{-1} M V t_0 s^2/2) \tag{9.34}$$

On the other hand we already know from the first part of the proof that $P^X(dx) = \mathcal{N}_p(0, \Sigma)$ solves (9.32). Hence we have shown that only $\mathcal{N}_p(0, \Sigma)$ leads to a linear conditional expectation.

Equivalence (3.25):
If $\mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y]$ is linear, by equivalence (3.24) ΔX and ΔY are jointly normal with expectation 0, so the conditional law of ΔX given ΔY is again normal with expectation 0, hence in particular symmetric so the assertion follows.
Now assume

$$\mathrm{E}\left[\left(e^\tau(\Delta X - \mathrm{E}[\Delta X|\Delta Y])\right)^3 \Big| \Delta Y\right] = 0 \qquad \forall\, e \in \mathbb{R}^p \tag{9.35}$$

Apparently, $\mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y]$ is linear iff

$$\partial^2/\partial y \partial y^\tau\, \mathrm{E}_{\mathrm{id}}[\Delta X|\Delta Y] = 0.$$

But Corollary 9.3 gives (in the notation of (9.24))

$$\frac{\partial^2}{\partial y_j \partial y_k}\, \mathrm{E}[\Delta X_i|\Delta Y = y] = \mathrm{E}(\Delta X_i^0 U_j^0 U_k^0|\Delta Y = y) \tag{9.36}$$

As $\mathrm{E}[\Delta X^0|\Delta Y] = 0$, (9.35) also entails that $\mathrm{E}[\Delta X_i^0 \Delta X_j^0 \Delta X_k^0|Y = y] = 0$ for all $i, j, k \in \{1, \ldots, p\}$. But with $\tilde{Z} = ZV^{-1}$, the RHS of (9.36) is just

$$\sum_{h,l=1}^{p} \tilde{Z}_{j,h} \tilde{Z}_{k,l}\, \mathrm{E}(\Delta X_i^0 \Delta X_h^0 \Delta X_l^0|\Delta Y = y),$$

so the assertion follows. ////

Proof to Theorem 3.11: We proceed as in Theorem 3.3, but note that in the eSO context (9.1) becomes

$$\begin{aligned} P(X \in A, Y^{\mathrm{re}} \in B) \;&=\; (1-r) \int \mathrm{I}_A(x)\, \mathrm{I}_B(y) \pi(y,x)\, P^{X^{\mathrm{id}}}(dx) + \\ &\quad + r \int \mathrm{I}_A(x)\, \mathrm{I}_B(y) q(y)\, P^{X^{\mathrm{di}}}(dx)\, \mu(dy) \end{aligned} \tag{9.37}$$

and hence (9.2) becomes

$$\mathrm{E}_{\mathrm{re}}[X|\hat{Y} = y] = \frac{rq(y)\, \mathrm{E}_{\mathrm{di}}[X^{\mathrm{di}}] + (1-r) p^{Y^{\mathrm{id}}}(y)\, \mathrm{E}_{\mathrm{id}}[X|Y]}{rq(y) + (1-r) p^{Y^{\mathrm{id}}}(y)} \tag{9.38}$$

But by (3.32), the RHS of (9.38) is exactly $F(q)$ from (9.2). Thus, we may jump to the proof of Theorem 3.3 from this point on, replacing $\mathrm{tr}\,\mathrm{Cov}\, X$ by

$$\tilde{G} := \mathrm{tr}\,\mathrm{Cov}_{P_0^{X^{\mathrm{di}}}}\, X^{\mathrm{di}} = G - |\,\mathrm{E}_{\mathrm{id}}\, X^{\mathrm{id}}|^2 \tag{9.39}$$

in equation (9.7). For passing from $\partial\mathcal{U}^{\rm eSO}$ to $\mathcal{U}^{\rm eSO}$, let f_r, $P_r^{Y^{\rm di}} \otimes P_r^{X^{\rm di}}$ be the components of the saddle-point and $R(f, P \otimes Q, r)$ be the MSE of procedure f at the eSO model $\partial\mathcal{U}^{\rm eSO}(r)$ with contaminating $P^{Y^{\rm di}} \otimes P^{X^{\rm di}} = P \otimes Q$. Instead of equation (9.10), we use

$$\Delta G := \tilde G - \operatorname{tr}\operatorname{Cov}_{\rm id} X^{\rm id} = G - \operatorname{E}_{\rm id}|X^{\rm id}|^2 \geq 0 \tag{9.40}$$

$$\tilde G = \Delta G + \operatorname{E}_{\rm id}\Big[\operatorname{tr}\operatorname{Cov}_{\rm id}[X|Y^{\rm id}] + |D(Y^{\rm id})|^2\Big] \tag{9.41}$$

and obtain

$$\begin{aligned}
& R(f_s, P\otimes Q, r) = (1-r)\operatorname{E}_{\rm id}\Big[\operatorname{tr}\operatorname{Cov}_{\rm id}[X|Y^{\rm id}] + (|D(Y^{\rm id})| - \rho(s))_+^2\Big] + \\
& \quad + r\ \operatorname{tr}\operatorname{Cov}_Q X^{\rm di} + r\ \operatorname{E}_P[\min(|D(Y^{\rm di})|, \rho(s))^2] \leq \\
\leq\ & (1-r)\operatorname{E}_{\rm id}\Big[\operatorname{tr}\operatorname{Cov}_{\rm id}[X|Y^{\rm id}] + (|D(Y^{\rm id})| - \rho(s))_+^2\Big] + \\
& \quad + r\,\tilde G + r\,\rho(s)^2 = R(f_s, P_r^{Y^{\rm di}} \otimes P_r^{X^{\rm di}}, r) = \\
=\ & \operatorname{E}_{\rm id}\Big[\operatorname{tr}\operatorname{Cov}_{\rm id}[X|Y^{\rm id}] + (|D(Y^{\rm id})| - \rho(s))_+^2\Big] + \\
& \quad + r\Big\{\Delta G + \operatorname{E}_{\rm id}\Big[|D(Y^{\rm id})|^2\Big(1 - \big(1 - w_s(Y^{\rm id})\big)^2\Big)\Big] + \rho(s)^2\Big\} < \\
<\ & \operatorname{E}_{\rm id}\Big[\operatorname{tr}\operatorname{Cov}_{\rm id}[X|Y^{\rm id}] + (|D(Y^{\rm id})| - \rho(s))_+^2\Big] + \\
& \quad + s\Big\{\Delta G + \operatorname{E}_{\rm id}\Big[|D(Y^{\rm id})|^2\Big(1 - \big(1 - w_s(Y^{\rm id})\big)^2\Big)\Big] + \rho(s)^2\Big\} = \\
=\ & R(f_s, P_r^{Y^{\rm di}} \otimes P_r^{X^{\rm di}}, s) = R(f_s, P_s^{Y^{\rm di}} \otimes P_s^{X^{\rm di}}, s)
\end{aligned} \tag{9.42}$$

Hence the saddle-point extends to $\mathcal{U}^{\rm eSO}(r)$. For later reference, we determine the minimax risk as

$$\begin{aligned}
R(f_r, P_r^{Y^{\rm di}} \otimes P_r^{X^{\rm di}}, r) &= \operatorname{tr}\operatorname{Cov}_{\rm id} X^{\rm id} + r(G - \operatorname{E}_{\rm id}|X^{\rm id}|^2) - \\
& \quad -(1-r)\operatorname{E}_{\rm id}\big[\,|D(Y^{\rm id})|^2 w_r(Y^{\rm id})\big]
\end{aligned} \tag{9.43}$$

////

PROOF TO PROPOSITION 3.9: Under H_0, due to Proposition 3.7, $\Delta X_i^\natural \stackrel{\rm i.i.d.}{\sim} \mathcal{N}_p(0, \Sigma)$. Hence $e^\tau \Delta X_i^\natural \stackrel{\rm i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$. Thus by the Lindeberg-Lévy CLT,

$$\frac{1}{\sqrt{n}} \sum_{i=1}^n (e^\tau \Delta X_i^\natural)^3 \xrightarrow{\rm w} \mathcal{N}(0, \operatorname{E}[(e^\tau \Delta X_t)^6])$$

But the sixth moment of $\mathcal{N}(0, \sigma^2)$ is just $15\sigma^6$. Hence by the assumed consistency of $\hat e_n$ for e, Slutsky's Lemma yields (3.27). Asymptotically, the testing problem is a test for a normal mean μ to be 0 or not, which yields the corresponding optimality for the Gauss test given in (3.28). ////

PROOF TO PROPOSITION 3.12: Let us identify $X \rightsquigarrow \Delta X^{\mathcal{N}}$, $Y \rightsquigarrow \Delta Y^{\mathcal{N}} := Z\Delta X^{\mathcal{N}} + \varepsilon$, and set $P^\varepsilon = \mathcal{N}_q(0, V)$, $P^X = \mathcal{N}_p(0, \Sigma)$, and let p^ε the corresponding Lebesgue density, then $\pi(y, x) =$

$p^\varepsilon(y - Zx)$.
Assertions (1) and (3) of Theorem 3.11 show that the eSO-optimal procedure f_0 in our "Bayesian" model of subsection 3.2 is just $f_0(y) = M^0(y)\min\{1, \rho/|M^0 y|\}$ with ρ according to (3.19) such that $\int dP_0^{Y^{\rm di}} = 1$ and $M^0 = \Sigma Z^\tau (Z\Sigma Z^\tau + V)^{-1}$.
By assumption, $\Delta X^{\rm rLS}$ lies in the corresponding eSO-neighborhood $\mathcal{U}(r)$ about $\Delta X^{\mathcal{N}}$ so the value of the saddle-point from equation (9.12) is also a bound for the MSE of $X^{\rm rLS}_{t|t}$ on $\mathcal{U}(r)$. ////

Remark 9.4. *One should mention, however, that due to assumption* (2.16) *resp.* (3.13), *members of an SO-neighborhood* $\mathcal{U}'(r')$ *about* $\mathcal{L}(\Delta X^{\rm rLS}, \Delta Y^{\rm rLS})$ *need not lie in an* eSO *neighborhood* $\mathcal{U}(r + r')$ *about* $\mathcal{L}(\Delta X^{\mathcal{N}}, \Delta Y^{\mathcal{N}})$.

10 Conclusion

In the extremely flexible class of dynamic models consisting in state space models we were able to obtain optimality results for filtering. In this generality this is a novelty. We could show that contrary to common prejudice a simultaneous treatment of (wide-sense) IO's and AO's is possible in SSM's—albeit with minor delay.
The filters that we propose are model based (in contrast to the non-parametric $\text{hybr}_{\text{PRMH}}$) which means that we need a higher degree of model specification in that we possibly have to estimate the hyper-parameters, but which also could help to get more precise in ideal model.
Our filters are non-iterative, recursive, hence fast, and valid for for higher dimensions.
They are available in R in some devel versions and hopefully on `CRAN` soon.

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