

A. Klar, J. Maringer, R. Wegener

A 3d model for fiber lay-down in  
nonwoven production processes

© Fraunhofer-Institut für Techno- und Wirtschaftsmathematik ITWM 2010

ISSN 1434-9973

Bericht 198 (2010)

Alle Rechte vorbehalten. Ohne ausdrückliche schriftliche Genehmigung des Herausgebers ist es nicht gestattet, das Buch oder Teile daraus in irgendeiner Form durch Fotokopie, Mikrofilm oder andere Verfahren zu reproduzieren oder in eine für Maschinen, insbesondere Datenverarbeitungsanlagen, verwendbare Sprache zu übertragen. Dasselbe gilt für das Recht der öffentlichen Wiedergabe.

Warennamen werden ohne Gewährleistung der freien Verwendbarkeit benutzt.

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und  
Wirtschaftsmathematik ITWM  
Fraunhofer-Platz 1

67663 Kaiserslautern  
Germany

Telefon: +49(0)631/3 1600-0  
Telefax: +49(0)631/3 1600-1099  
E-Mail: [info@itwm.fraunhofer.de](mailto:info@itwm.fraunhofer.de)  
Internet: [www.itwm.fraunhofer.de](http://www.itwm.fraunhofer.de)

# Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



# A 3D MODEL FOR FIBER LAY-DOWN IN NONWOVEN PRODUCTION PROCESSES

A. KLAR <sup>†‡</sup>, J. MARINGER <sup>‡</sup>, AND R. WEGENER<sup>‡</sup>

**Abstract.** In this paper a three dimensional stochastic model for the lay-down of fibers on a moving conveyor belt in the production process of nonwoven materials is derived. The model is based on stochastic differential equations describing the resulting position of the fiber on the belt under the influence of turbulent air flows. The model presented here is an extension of an existing surrogate model, see [6, 3].

**Key words.** fiber dynamics, Fokker-Planck equations, diffusion limits

**AMS subject classifications.** 37H10, 60H30, 41A60, 65C05

**1. Introduction.** Nonwoven materials are produced in melt-spinning operations: hundreds of individual endless fibers are obtained by the continuous extrusion of a molten polymer through narrow nozzles that are densely and equidistantly placed in a row at a spinning beam. The viscous / viscoelastic fibers are stretched and spun until they solidify due to cooling air streams. Before the elastic fibers lay down on a moving conveyor belt to form a web, they become entangled and form loops due to the highly turbulent air flows. In [12] a general mathematical model for the fiber dynamics is presented which enables the full simulation of the process. Due to the huge amount of physical details these simulations of the fiber spinning and lay-down usually require an extremely large computational effort and high memory storage, see [13]. Thus, a simplified two-dimensional stochastic model for the fiber lay-down process is introduced in [6]. This model describes the position of the fiber on the transport belt by a stochastic differential system containing parameters that characterize the process. These parameters have to be identified from a few representative fibers simulated by the detailed model. Then, the surrogate model can be used to calculate fast and efficiently the behavior of hundreds of long fibers for fleece production. In [6, 7] an analytic investigation of the corresponding Fokker-Planck equation has been performed, asymptotic properties and ergodicity of the process have been proven and explicit rates for the convergence to the stationary solution have been obtained.

In the present paper the above 2D model is extended to three dimensions. This is crucial, if one wants to describe further properties of the resulting web, like permeability. In a first step we revisit the 2D model and rewrite it in a coordinate free form, see section 2. In section 3 an isotropic 3D model is obtained by a suitable transformation of the deterministic and stochastic processes. Since the resulting fleece is usually rather thin and has an anisotropic orientation of the fibers, we modify the isotropic 3D model accordingly, see section 4. Further, we investigate the connections between the models by looking at the associated Fokker-Planck equations and their limiting behavior. The limit of large turbulence is considered in section 5, as well as a large coiling force limit. Finally, we show numerical results and state a possible strategy for parameter estimation of the modified 3D model in section 6.

---

<sup>†</sup>Fachbereich Mathematik, Technische Universität Kaiserslautern, Germany  
{klar@mathematik.uni-kl.de}

<sup>‡</sup>Fraunhofer ITWM, Kaiserslautern, Germany  
{maringer@itwm.fhg.de, wegenger@itwm.fhg.de}

**2. Revisiting the 2D Model.** Consider a slender, elastic, non-extensible and endless fiber in a lay-down regime. Let the fiber be produced with a certain spinning speed, excited into motion by a surrounding highly turbulent air flow and laid down on a conveyor belt which is for the time being assumed to be non-moving. For the case of a moving belt see Remark 1. Due to its slenderness, the fiber on the two-dimensional transport belt is modeled as an arc-length parametrized curve  $\xi : \mathbb{R}_0^+ \rightarrow \mathbb{R}^2$ , defined by the solution of a dynamical system for  $\xi_t$  with the arc-length  $t$ . This dynamical system is given by the following stochastic differential equations, see [6]

$$\begin{aligned} d\xi_t &= \tau(\alpha_t) dt \\ d\alpha_t &= -\nabla V(\xi_t) \cdot \tau^\perp(\alpha_t) dt + A dW_t, \end{aligned} \quad (2.1)$$

equipped with appropriate initial conditions  $\xi_0, \alpha_0$ . The normalized tangent is  $\tau(\alpha) = (\cos \alpha, \sin \alpha)^T$ . Introducing the corresponding orthonormal polar unit vector  $\tau(\alpha)^\perp = (-\sin \alpha, \cos \alpha)^T$ , the deterministic drift term in the second equation ensures that the fiber tends back to the origin as a consequence of the coiling behavior of the fiber. The throwing ranges of the fiber can be controlled with help of the potential  $V$ . One may, for example, choose  $V(\xi) = (\xi_1^2/\sigma_1^2 + \xi_2^2/\sigma_2^2)/2$  with the throwing ranges  $\sigma_1$  and  $\sigma_2$ , see Remark 2. All stochastic effects occurring in the production process are summarized in the one-dimensional Wiener process  $(W_t)_{t \geq 0}$  with diffusion constant  $A$ . This model is referred to as the *original 2D model*.

We rewrite (2.1) in a coordinate free form using Ito's or Stratonovich's calculus, respectively,

$$\begin{aligned} d\xi_t &= \tau_t dt \\ d\tau_t &= -(\nabla V(\xi_t) \cdot \tau_t^\perp) \tau_t^\perp dt - \frac{1}{2} A^2 \tau_t dt + \tau_t^\perp A dW_t \\ &= -(\nabla V(\xi_t) \cdot \tau_t^\perp) \tau_t^\perp dt + \tau_t^\perp \circ A dW_t \\ \|\tau_t\| &= 1 \end{aligned}$$

or in more compact form

$$\begin{aligned} d\xi_t &= \tau_t dt \\ d\tau_t &= (I - \tau_t \otimes \tau_t) (-\nabla V(\xi_t) dt + A dW_t) - \frac{1}{2} A^2 \tau_t dt \\ &= (I - \tau_t \otimes \tau_t) \circ (-\nabla V(\xi_t) dt + A dW_t) \\ \|\tau_t\| &= 1. \end{aligned} \quad (2.2)$$

Thereby,  $I$  denotes the identity matrix,  $\circ$  denotes the usage of Stratonovich integrals and  $(W_t)_{t \geq 0}$  is a two-dimensional Wiener process. The algebraic constraint takes into account the arc-length parametrization for the inextensible fibers. We note that the stochastic part in (2.2) is a Brownian motion on the unit circle, see for example [14].

**REMARK 1.** *Let the fiber be produced with the spinning speed  $v_{spin}$  and laid down on a conveyor belt moving with the velocity  $v_{belt}$ . The fiber curve is now denoted as  $\eta : \mathbb{R}_0^+ \rightarrow \mathbb{R}^2$ . The arc-length parametrization and in-extensibility gives again*

$$d\eta_t = \begin{pmatrix} \cos \alpha_t \\ \sin \alpha_t \end{pmatrix} dt,$$

where  $\alpha$  denotes the angle of the fiber relative to the direction of motion  $\mathbf{e}_1$  of the transport belt. The reference point of the spinning process determined by the position of the nozzle moves in the coordinate system of the transport belt in the direction  $-\mathbf{e}_1$ . Thus,

$$\boldsymbol{\xi}(t) = \boldsymbol{\eta}(t) - (-\kappa t \mathbf{e}_1)$$

describes the deviation of the fiber from the reference point as a function of the arc-length parameter  $t$ , where  $\kappa = v_{\text{belt}}/v_{\text{spin}} \in [0, 1]$  is the ratio between the belt and spinning speeds. Following [6], we model  $(\boldsymbol{\xi}, \alpha)$  by the stochastic differential system

$$\begin{aligned} d\boldsymbol{\xi}_t &= \boldsymbol{\tau}(\alpha_t) dt + \kappa \mathbf{e}_1 dt \\ d\alpha_t &= -\nabla V(\boldsymbol{\xi}_t) \cdot \boldsymbol{\tau}_t^\perp dt + A dW_t . \end{aligned} \quad (2.3)$$

We might generalize the last model with help of the concept of reference curves allowing the consideration of further specific production processes, see [10]. Introducing a reference curve  $\gamma$  the generalized model reads

$$\begin{aligned} d\boldsymbol{\xi}_t &= \boldsymbol{\tau}(\alpha_t) dt + d\gamma_t \\ d\alpha_t &= -\nabla V(\boldsymbol{\xi}_t) \cdot \boldsymbol{\tau}_t^\perp dt + A dW_t . \end{aligned} \quad (2.4)$$

REMARK 2. The Fokker-Planck equation associated to (2.1) reads

$$\partial_t P = -\boldsymbol{\tau} \cdot \nabla_{\boldsymbol{\xi}} P + \nabla V(\boldsymbol{\xi}) \cdot \partial_\alpha (\boldsymbol{\tau}^\perp P) + \frac{1}{2} A^2 \partial_{\alpha\alpha} P \quad (2.5)$$

and the stationary density is given by

$$P_{\text{stat}}(\boldsymbol{\xi}) = C \exp(-V(\boldsymbol{\xi})) , \quad (2.6)$$

where  $C$  is a normalization constant. Using the potential  $V(\boldsymbol{\xi}) = (\xi_1^2/\sigma_1^2 + \xi_2^2/\sigma_2^2)/2$  the throwing ranges  $\sigma_1, \sigma_2$  are interpreted as standard deviations  $\sigma_1, \sigma_2$  of the normal distribution.

REMARK 3. The limit for large values of the diffusion coefficient  $A$  is

$$d\boldsymbol{\xi}_t = -\frac{1}{A^2} \nabla V(\boldsymbol{\xi}_t) dt + \sqrt{2} \frac{1}{A} d\mathbf{W}_t , \quad (2.7)$$

see [3]. It will be called the reduced 2D model in the following.

**3. The Isotropic 3D Model.** In this and the following section the above described two-dimensional model will be extended to three dimensions. Again, we model the motion of the fiber as an arc-length parametrized curve  $\boldsymbol{\xi} : \mathbb{R}_0^+ \rightarrow \mathbb{R}^3$  with normalized tangent  $\boldsymbol{\tau}$ , which is in spherical coordinates given by

$$\boldsymbol{\tau}(\alpha, \theta) = \begin{pmatrix} \cos \alpha \sin \theta \\ \sin \alpha \sin \theta \\ \cos \theta \end{pmatrix} .$$

We introduce the orthonormal spherical unit vectors

$$\mathbf{n}_1(\alpha) = \begin{pmatrix} -\sin \alpha \\ \cos \alpha \\ 0 \end{pmatrix}, \quad \mathbf{n}_2(\alpha, \theta) = \begin{pmatrix} \cos \alpha \cos \theta \\ \sin \alpha \cos \theta \\ -\sin \theta \end{pmatrix}$$

and follow the procedure for the 2D case to get the stochastic differential equations in local coordinates in the 3D case. We start by translating the Stratonovich coordinate free formulation (2.2) to three dimensions introducing an additional factor  $\frac{1}{2}$ :

$$\begin{aligned} d\xi_t &= \tau_t dt \\ d\tau_t &= (I - \tau_t \otimes \tau_t) \circ \left( -\frac{1}{2} \nabla V(\xi_t) dt + A dW_t \right) \\ \|\tau_t\| &= 1. \end{aligned} \tag{3.1}$$

Considering the deterministic part separately gives

$$\begin{aligned} d\tau_t &= -\frac{1}{2} (I - \tau_t \otimes \tau_t) \nabla V(\xi_t) dt \\ &= -\frac{1}{2} (\mathbf{n}_{1t} \otimes \mathbf{n}_{1t} + \mathbf{n}_{2t} \otimes \mathbf{n}_{2t}) \nabla V(\xi_t) dt \\ &= -\frac{1}{2} (\nabla V(\xi_t) \cdot \mathbf{n}_{1t}) \mathbf{n}_{1t} dt - \frac{1}{2} (\nabla V(\xi_t) \cdot \mathbf{n}_{2t}) \mathbf{n}_{2t} dt. \end{aligned} \tag{3.2}$$

Applying the chain rule ( $\tau = \tau(\alpha, \theta)$ ) and comparison of coefficients leads to the deterministic equations in local coordinates

$$\begin{aligned} \sin \theta_t d\alpha_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_1(\alpha_t) dt \\ d\theta_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_2(\alpha_t, \theta_t) dt. \end{aligned}$$

Next, we rewrite the stochastic part in Ito's formulation and compute

$$\begin{aligned} d\tau_t &= (I - \tau_t \otimes \tau_t) A dW_t - A^2 \tau_t dt \\ &= (\mathbf{n}_{1t} \otimes \mathbf{n}_{1t} + \mathbf{n}_{2t} \otimes \mathbf{n}_{2t}) A dW_t - A^2 \tau_t dt \\ &= \mathbf{n}_{1t} A dW_t^{(1)} + \mathbf{n}_{2t} A dW_t^{(2)} - A^2 \tau_t dt, \end{aligned} \tag{3.3}$$

which is a Brownian motion on the unit sphere, see [14] or [15].  $(W_t^{(1)})_{t \geq 0}$  and  $(W_t^{(2)})_{t \geq 0}$  are one-dimensional Wiener processes.

Using Ito's formula for  $\tau(\alpha, \theta)$  the stochastic part in local coordinates is given by

$$\begin{aligned} \sin \theta_t d\alpha_t &= A dW_t^{(1)} \\ d\theta_t &= \frac{1}{2} A^2 \cot \theta_t dt + A dW_t^{(2)}. \end{aligned}$$

Alltogether the three dimensional model equations in spherical polar coordinates are

$$\begin{aligned} d\xi_t &= \tau(\alpha_t, \theta_t) dt \\ \sin \theta_t d\alpha_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_1(\alpha_t) dt + A dW_t^{(1)} \\ d\theta_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_2(\alpha_t, \theta_t) dt + \frac{1}{2} A^2 \cot \theta_t dt + A dW_t^{(2)}. \end{aligned} \tag{3.4}$$



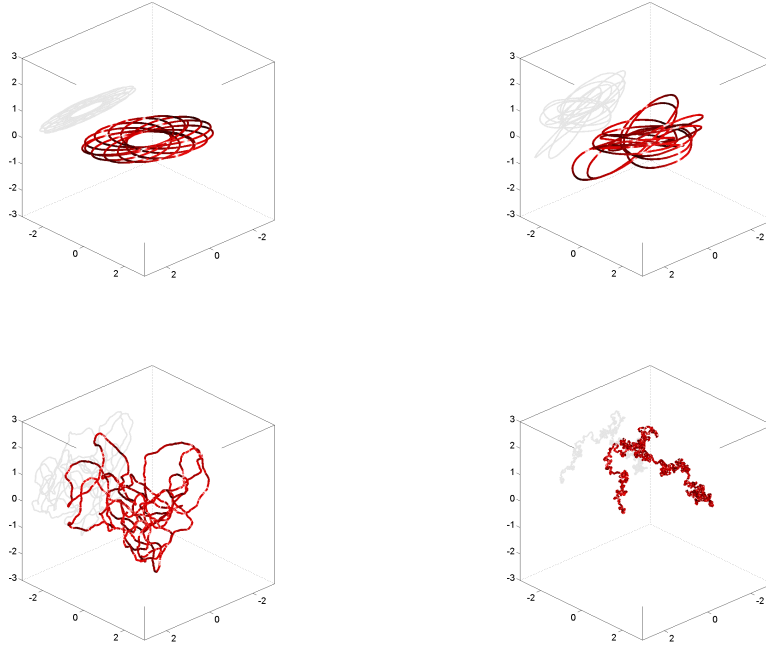


Fig. 3.1: Influence of noise on fiber trajectories. From left to right, top to bottom  $A = 0; 0.1; 1; 5$ .

In the following we refer to this system as the *isotropic 3D model*. Choosing a symmetric potential  $V = V(|\xi|)$  the model is invariant under rotations.

Representative fiber scenarios for varying noise amplitude  $A$  are illustrated in Figure 3.1 computed by (3.4) after nondimensionalization using an isotropic potential  $V(\xi) = |\xi|^2/2$ .

However in typical fiber lay-down processes the resulting nonwovens have usually an anisotropic orientation of the fibers as consequence of physical properties and limitations given for example by the impenetrable conveyor belt. For example, the orientation of most of the fibers will concentrate in a direction parallel to the belt. Thus, it is necessary to modify the model such that the orientation of the fibers can be controlled and adapted to measurements. This will be done in the next section.

REMARK 4. *The generalization of the model using arbitrary reference curves  $\gamma$  is obviously*

$$\begin{aligned}
 d\xi_t &= \tau(\alpha_t, \theta_t) dt + d\gamma_t \\
 \sin \theta_t d\alpha_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_1(\alpha_t) dt + A dW_t^{(1)} \\
 d\theta_t &= -\frac{1}{2} \nabla V(\xi_t) \cdot \mathbf{n}_2(\alpha_t, \theta_t) dt + \frac{1}{2} A^2 \cot \theta_t dt + A dW_t^{(2)}.
 \end{aligned}
 \tag{3.5}$$

REMARK 5. *The Fokker-Planck equation associated to the isotropic model with non-moving belt is*

$$\begin{aligned} \partial_t P = & -\boldsymbol{\tau} \cdot \nabla_{\boldsymbol{\xi}} P + \frac{1}{\sin \theta} \frac{1}{2} \nabla V(\boldsymbol{\xi}) \cdot \partial_{\alpha}(\mathbf{n}_1 P) + \frac{1}{2} \nabla V(\boldsymbol{\xi}) \cdot \partial_{\theta}(\mathbf{n}_2 P) \\ & - \frac{1}{2} A^2 \partial_{\theta}(\cot \theta P) + \frac{1}{2} \frac{A^2}{\sin^2 \theta} \partial_{\alpha\alpha} P + \frac{1}{2} A^2 \partial_{\theta\theta} P \end{aligned} \quad (3.6)$$

and the stationary density is given by

$$P_{stat}(\boldsymbol{\xi}, \theta) = C \sin \theta \exp(-V(\boldsymbol{\xi})) . \quad (3.7)$$

**4. The Modified 3D Model.** The idea of the modified model is that by introducing an additional parameter  $B$  we obtain a 3D model which can be adapted to the distributions of the  $\theta$ -angle in a realistic fleece. With the help of this parameter we are able to weight the directions of the spherical unit-vectors  $\mathbf{n}_1$  and  $\mathbf{n}_2$  differently, such that it is possible to capture the anisotropic orientation of the fibers. We suppose that the belt lies in the  $(\mathbf{e}_1, \mathbf{e}_2)$ -plane and that the spherical coordinates are determined in a standard way, that means  $\theta$  is the angle between the direction  $\mathbf{e}_3$  and the tangent  $\boldsymbol{\tau}$  on the fiber, whereas  $\alpha$  is the angle between the direction  $\mathbf{e}_1$  and the projection of  $\boldsymbol{\tau}$  on the reference plane.

Let  $B \in [0, 1]$ , then we replace in (3.2) the factor  $\frac{1}{2}$  as follows

$$d\boldsymbol{\tau}_t = -\frac{1}{B+1} \left( (\nabla V(\boldsymbol{\xi}_t) \cdot \mathbf{n}_{1t}) \mathbf{n}_{1t} + B (\nabla V(\boldsymbol{\xi}_t) \cdot \mathbf{n}_{2t}) \mathbf{n}_{2t} \right) dt . \quad (4.1)$$

We note, that if  $B = 0$  then we recover the deterministic part in the 2D case observing that  $\mathbf{n}_1$  is the 3D analogue of  $\boldsymbol{\tau}^{\perp}$ . If  $B = 1$  the isotropic 3-D model is recovered. Furthermore, we change the stochastic part (3.3) redefining

$$d\boldsymbol{\tau}_t = \mathbf{n}_{1t} A dW_t^{(1)} + \sqrt{B} \mathbf{n}_{2t} A dW_t^{(2)} - \frac{1}{2} (1+B) A^2 \boldsymbol{\tau}_t dt . \quad (4.2)$$

This ensures again, that for  $B = 0$  we are back to the 2D case and for  $B = 1$  to the isotropic 3D case. The modified 3D model ranges between the isotropic 3D model and the 2D model describing fibers with an orientation ranging between a random orientation and an orientation in the  $(\mathbf{e}_1, \mathbf{e}_2)$ -plane. Using Ito's formula, we obtain the modified 3D model in local coordinates

$$\begin{aligned} d\boldsymbol{\xi}_t &= \boldsymbol{\tau}(\alpha_t, \theta_t) dt \\ \sin \theta_t d\alpha_t &= -\frac{1}{B+1} \nabla V(\boldsymbol{\xi}_t) \cdot \mathbf{n}_1(\alpha_t) dt + A dW_t^{(1)} \\ d\theta_t &= -\frac{B}{B+1} \nabla V(\boldsymbol{\xi}_t) \cdot \mathbf{n}_2(\alpha_t, \theta_t) dt + \frac{1}{2} A^2 \cot \theta_t dt + \sqrt{B} A dW_t^{(2)} . \end{aligned} \quad (4.3)$$

The effect of varying parameter  $B$  in (4.3) for an isotropic potential  $V(\boldsymbol{\xi}) = |\boldsymbol{\xi}|^2/2$  is shown in Figure 4.1. The noise amplitude is chosen as  $A = 1$ . One observes that for decreasing  $B$  the orientation of the fibers is getting more anisotropic. This corresponds to a distribution of the angle  $\theta$  concentrating around  $\theta = \pi/2$ .

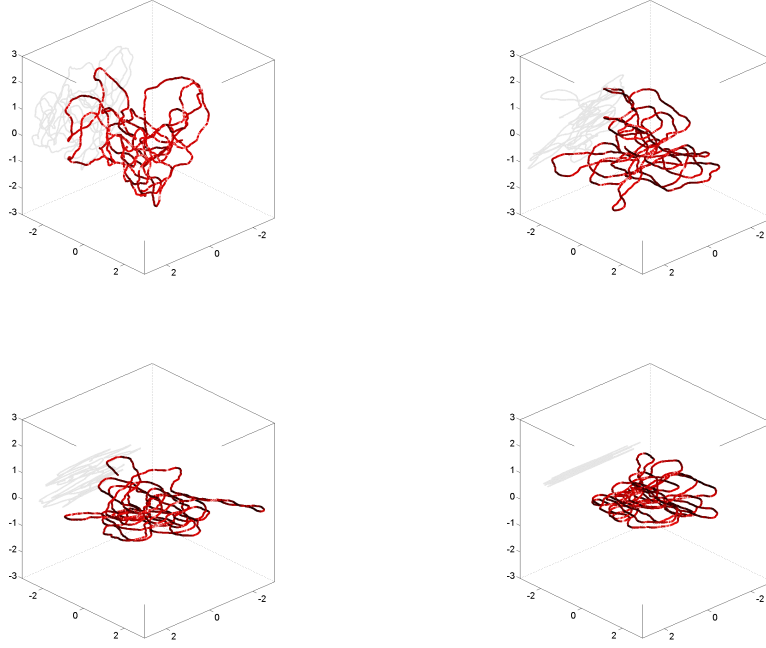


Fig. 4.1: Influence of the parameter  $B$  on fiber trajectories. From left to right, top to bottom  $B = 1; 0.1; 0.01; 0.0001$ .

**5. Asymptotic Limits and Connections between the Models.** In the previous chapters we have distinguished between the 2D model (2.1), the isotropic 3D model (3.4) and the modified 3D model (4.3). In the following we consider the modified 3D model for different limits of the parameters  $B$  and  $A$ .

**5.1. Small and Large B Limit.** We investigate the Fokker-Planck equation associated to (4.3)

$$\begin{aligned} \partial_t P = & -\boldsymbol{\tau} \cdot \nabla_{\boldsymbol{\xi}} P + \frac{1}{\sin \theta} \frac{1}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_{\alpha}(\mathbf{n}_1 P) + \frac{B}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_{\theta}(\mathbf{n}_2 P) \\ & - \frac{1}{2} A^2 \partial_{\theta}(\cot \theta P) + \frac{1}{2} \frac{A^2}{\sin^2 \theta} \partial_{\alpha\alpha} P + \frac{1}{2} B A^2 \partial_{\theta\theta} P. \end{aligned} \quad (5.1)$$

The stationary density is given by

$$P_{stat}(\boldsymbol{\xi}, \theta) = C \exp(-V(\boldsymbol{\xi})) (\sin \theta)^{\frac{1}{B}}. \quad (5.2)$$

We note that in the case  $B = 1$  this is the Fokker-Planck equation and the stationary solution corresponding to the isotropic model (3.6). Next, we turn to the case  $B$  tending to 0. For  $B = 0$  the system of stochastic differential equations (4.3) decouples

into a closed system of ordinary differential equations in  $(\boldsymbol{\xi}_3, \theta)$

$$\begin{aligned} d\boldsymbol{\xi}_{3t} &= \cos \theta_t dt \\ d\theta_t &= \frac{1}{2} A^2 \cot \theta_t dt \end{aligned} \quad (5.3)$$

and a remaining system similar to the 2D model (2.1),

$$\begin{aligned} d\boldsymbol{\xi}_{1t} &= \cos \alpha_t \sin \theta_t dt \\ d\boldsymbol{\xi}_{2t} &= \sin \alpha_t \sin \theta_t dt \\ \sin \theta_t d\alpha_t &= -\nabla V(\boldsymbol{\xi}_t) \cdot \mathbf{n}_1(\alpha_t) dt + A dW_t^{(1)}. \end{aligned} \quad (5.4)$$

The solution of (5.3) with initial values  $\boldsymbol{\xi}_3(0) = \boldsymbol{\xi}_{3_0}$  and  $\theta(0) = \theta_0$  is

$$\begin{aligned} \theta(t) &= \arccos \left( \exp \left( -\frac{1}{2} A^2 t \right) \cos \theta_0 \right) \\ \boldsymbol{\xi}_3(t) &= \boldsymbol{\xi}_{3_0} + \frac{2}{A^2} \cos \theta_0 - \frac{2}{A^2} \exp \left( -\frac{1}{2} A^2 t \right) \cos \theta_0. \end{aligned}$$

In the large time limit  $t \rightarrow \infty$  one obtains  $(\boldsymbol{\xi}_3, \theta) = (\boldsymbol{\xi}_{3_0} + \frac{2}{A^2} \cos \theta_0, \frac{\pi}{2})$ . Plugging this into the remaining system (5.4) we recover the 2D model (2.1). This means that after a transition time or if the initial values are suitably chosen, the 2-D model is recovered. Moreover, one directly observes that the stationary solution of the modified model (5.2) tends towards the stationary solution of the 2D model (2.6) if  $B$  tends to zero:

$$C(\sin \theta)^{\frac{1}{B}} \xrightarrow{B \rightarrow 0} \delta_{\pi/2}.$$

**5.2. Large Diffusion Limit and Reduced Model.** In this section we investigate the large turbulence case with the limit  $A \rightarrow \infty$ . We start from the Fokker-Planck equation (5.1) associated to the modified model and scale the equation using  $t' = \varepsilon t$  and  $A' = \sqrt{\varepsilon} A$ . This yields

$$\begin{aligned} \varepsilon \partial_t P^\varepsilon &= -\boldsymbol{\tau} \cdot \nabla_\xi P^\varepsilon + \frac{1}{\sin \theta} \frac{1}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\alpha (\mathbf{n}_1 P^\varepsilon) + \frac{B}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\theta (\mathbf{n}_2 P^\varepsilon) \\ &\quad - \frac{1}{2} \frac{A^2}{\varepsilon} \partial_\theta (\cot \theta P^\varepsilon) + \frac{1}{2} \frac{A^2}{\varepsilon \sin^2 \theta} \partial_{\alpha\alpha} P^\varepsilon + \frac{1}{2} B \frac{A^2}{\varepsilon} \partial_{\theta\theta} P^\varepsilon. \end{aligned}$$

Plugging in the ansatz  $P^\varepsilon = P^0 + \varepsilon P^1 + \dots$  the leading order problem is

$$\frac{1}{2} A^2 \left( -\partial_\theta (\cot \theta P^0) + \frac{1}{\sin^2 \theta} \partial_{\alpha\alpha} P^0 + B \partial_{\theta\theta} P^0 \right) = 0$$

with solution

$$P^0(\boldsymbol{\xi}, \theta, t) = C(\sin \theta)^{\frac{1}{B}} m^0(\boldsymbol{\xi}, t),$$

where  $C$  is a normalization constant depending on the parameter  $B$ . To next order we have

$$\begin{aligned} 0 &= -\boldsymbol{\tau} \cdot \nabla_\xi P^0 + \frac{1}{\sin \theta} \frac{1}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\alpha (\mathbf{n}_1 P^0) + \frac{B}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\theta (\mathbf{n}_2 P^0) \\ &\quad + \frac{1}{2} A^2 \left( -\partial_\theta (\cot \theta P^1) + \frac{1}{\sin^2 \theta} \partial_{\alpha\alpha} P^1 + B \partial_{\theta\theta} P^1 \right). \end{aligned}$$

Inserting  $P^0$  yields

$$-\partial_\theta(\cot \theta P^1) + \frac{1}{\sin^2 \theta} \partial_{\alpha\alpha} P^1 + B \partial_{\theta\theta} P^1 = C \frac{2}{A^2} (\nabla_\xi m^0 + \nabla V(\boldsymbol{\xi}) m^0) \cdot \boldsymbol{\tau}(\sin \theta)^{\frac{1}{B}} .$$

The solution of this equation is given by

$$P^1(\boldsymbol{\xi}, \theta, \alpha, t) = -\frac{1}{B+1} \frac{2}{A^2} C(\sin \theta)^{\frac{1}{B}} \boldsymbol{\tau} \cdot (\nabla_\xi m^0 + \nabla V(\boldsymbol{\xi}) m^0) .$$

In a next step we integrate the scaled Fokker-Planck equation over the angles  $\alpha$  and  $\theta$ . Defining  $m^\varepsilon := \int_0^{2\pi} \int_0^\pi P^\varepsilon d\theta d\alpha$  we obtain

$$\varepsilon \partial_t m^\varepsilon + \int_0^{2\pi} \int_0^\pi \boldsymbol{\tau} \cdot \nabla_\xi P^\varepsilon d\theta d\alpha = 0 .$$

To first order this is

$$\partial_t m^0 + \int_0^{2\pi} \int_0^\pi \boldsymbol{\tau} \cdot \nabla_\xi P^1 d\theta d\alpha = \mathcal{O}(\varepsilon) .$$

Applying the divergence relation  $\nabla_\xi \cdot (\boldsymbol{\tau} P^1) = \boldsymbol{\tau} \cdot \nabla_\xi P^1 + P^1 \nabla_\xi \cdot \boldsymbol{\tau} = \boldsymbol{\tau} \cdot \nabla_\xi P^1$  we have

$$\partial_t m^0 + \nabla_\xi \cdot \int_0^{2\pi} \int_0^\pi \boldsymbol{\tau} P^1 d\theta d\alpha = \mathcal{O}(\varepsilon)$$

and after replacing  $P^1$  we get

$$\partial_t m^0 - \nabla_\xi \cdot \int_0^{2\pi} \int_0^\pi \boldsymbol{\tau} \frac{1}{B+1} \frac{2}{A^2} C(\sin \theta)^{\frac{1}{B}} \boldsymbol{\tau} \cdot (\nabla_\xi m^0 + \nabla V(\boldsymbol{\xi}) m^0) d\theta d\alpha = \mathcal{O}(\varepsilon) .$$

After evaluating the integral and taking the limit  $\varepsilon \rightarrow 0$  we obtain the reduced model

$$\partial_t m^0 - \frac{1}{A^2(1+2B)} \nabla_\xi \cdot D(\nabla_\xi m^0 + \nabla V(\boldsymbol{\xi}) m^0) = 0$$

with

$$D = \text{diag} \left\{ 1, 1, \frac{2B}{(1+B)} \right\}$$

or the system of stochastic differential equations

$$d\boldsymbol{\xi}_t = -\frac{1}{1+2B} \frac{1}{A^2} D \nabla V(\boldsymbol{\xi}_t) dt + \sqrt{\frac{2}{1+2B}} \frac{1}{A} \sqrt{D} d\mathbf{W}_t \quad (5.5)$$

We note, that for  $B = 0$  we recover the reduced 2D model (2.7).

REMARK 6. *The limiting process for  $A \rightarrow \infty$  in the case of a moving conveyor belt is given by*

$$d\boldsymbol{\xi}_t = -\left( \frac{1}{1+2B} \frac{1}{A^2} D \nabla V(\boldsymbol{\xi}_t) - \kappa \mathbf{e}_1 \right) dt + \sqrt{\frac{2}{1+2B}} \frac{1}{A} \sqrt{D} d\mathbf{W}_t , \quad (5.6)$$

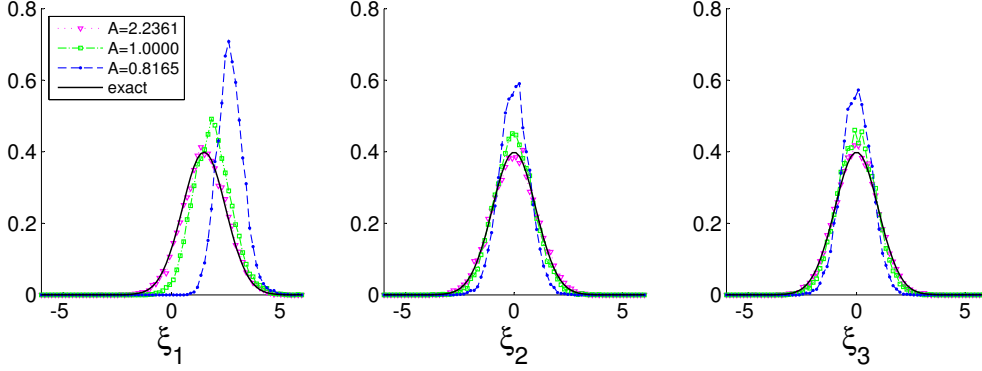


Fig. 5.1: Stationary marginal densities of  $\xi$ -components for different values of  $A$  and fixed  $q = 0.5$

compare [3] for the two-dimensional case.

To investigate the practical relevance of the reduced model numerically we compare the isotropic 3D model process with moving belt (3.5) with the reduced model (5.6) with  $B = 1$ . The potential is chosen as  $V(\xi) = |\xi|^2/2$ . The stationary density of (5.6) reads

$$P_{stat}(\xi) = C \exp \left( -\frac{(\xi_1 - (2B+1)\kappa A^2)^2}{2} - \frac{\xi_2^2}{2} - \frac{\xi_3^2}{2} \right),$$

which is independent of the scaling for  $\kappa A^2 = q$ ,  $q \in \mathbb{R}$ . Figure 5.1 shows the stationary marginal densities for the components  $\xi_1, \xi_2$  and  $\xi_3$  for (3.5) with different values of  $A$  and fixed  $q = 0.5$  and the above stationary density of (5.6) for  $B = 1$ . Whereas the marginal densities differ for  $A < 1$ , the approximation is qualitatively better for increasing  $A$ . Good agreement can be observed for  $A > 2$ . Similar results can be shown for the modified 3D model ( $B < 1$ ). Thus the reduced models might be seen as alternatives to the original models even for moderate values of  $A$ . However, we note that in the limit we loose regularity of the fiber path. Moreover, the inextensibility of the fiber is not anymore preserved.

REMARK 7. In [6] also the small diffusion limit  $A \rightarrow 0$  has been considered for the 2D case using a representation of the equations as a stochastic Hamiltonian system and the method of stochastic averaging, see also [1, 2]. Similarly, the 3D system can be rewritten in Hamiltonian form. However, in contrast to the 2D case where the reduced Hamiltonian system has been a 2-dimensional system, it is in the present case a 4-dimensional equation allowing for 3 invariants, which are not as straightforwardly determined.

**5.3. The Large Coiling Force Limit.** In this section we investigate a hyperbolic scaling, the large coiling force and large diffusion limit with  $A' = A/\sqrt{\epsilon}$  and

$V' = V/\epsilon$ . We start from the scaled Fokker-Planck equation

$$\begin{aligned} \partial_t P^\epsilon = & -\boldsymbol{\tau} \cdot \nabla_\xi P^\epsilon + \frac{1}{\epsilon} \frac{1}{\sin \theta} \frac{1}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\alpha(\mathbf{n}_1 P^\epsilon) + \frac{1}{\epsilon} \frac{B}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\theta(\mathbf{n}_2 P^\epsilon) \\ & - \frac{1}{2} \frac{A^2}{\epsilon} \partial_\theta(\cot \theta P^\epsilon) + \frac{1}{2} \frac{A^2}{\epsilon \sin^2 \theta} \partial_{\alpha\alpha} P^\epsilon + \frac{1}{2} B \frac{A^2}{\epsilon} \partial_{\theta\theta} P^\epsilon. \end{aligned}$$

To zeroth order this is

$$\begin{aligned} & \frac{1}{\sin \theta} \frac{1}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\alpha(\mathbf{n}_1 P^0) + \frac{B}{B+1} \nabla V(\boldsymbol{\xi}) \cdot \partial_\theta(\mathbf{n}_2 P^0) \\ & - \frac{1}{2} A^2 \partial_\theta(\cot \theta P^0) + \frac{1}{2} \frac{A^2}{\sin^2 \theta} \partial_{\alpha\alpha} P^0 + \frac{1}{2} B A^2 \partial_{\theta\theta} P^0 = 0 \end{aligned}$$

with the solution

$$P^0(\boldsymbol{\xi}, \alpha, \theta, t) = \frac{\rho(\boldsymbol{\xi}, t)}{N(\boldsymbol{\xi})} (\sin \theta)^{\frac{1}{B}} \exp\left(-\frac{1}{A^2} \frac{2}{B+1} \boldsymbol{\tau} \cdot \nabla V\right)$$

with

$$N(\boldsymbol{\xi}) = \int (\sin \theta)^{\frac{1}{B}} \exp\left(-\frac{1}{A^2} \frac{2}{B+1} \boldsymbol{\tau} \cdot \nabla V\right) d\alpha d\theta.$$

Integrating the scaled Fokker-Planck equation over  $\alpha$  and  $\theta$  yields up to order  $\epsilon$

$$\partial_t \rho + \nabla_\xi \cdot \int \boldsymbol{\tau} P^0 d\alpha d\theta = 0.$$

This can be rewritten as

$$\partial_t \rho + \nabla_\xi \cdot (\rho \mathbf{U}) = 0$$

with

$$\mathbf{U} = \mathbf{U}(\boldsymbol{\xi}) = \frac{1}{N(\boldsymbol{\xi})} \int \boldsymbol{\tau} (\sin \theta)^{\frac{1}{B}} \exp\left(-\frac{1}{A^2} \frac{2}{B+1} \boldsymbol{\tau} \cdot \nabla V\right) d\theta d\alpha.$$

Under suitable assumptions concerning the symmetry of the process, for example, in the isotropic case  $B = 1$  and assuming that  $V(\boldsymbol{\xi}) = V(|\boldsymbol{\xi}|)$  we have  $N(\boldsymbol{\xi}) = N(|\boldsymbol{\xi}|)$  and we can rewrite  $\mathbf{U}$  as

$$\mathbf{U}(\boldsymbol{\xi}) = \lambda(|\boldsymbol{\xi}|) \boldsymbol{\xi}$$

with

$$\lambda(|\boldsymbol{\xi}|) = \frac{1}{V'(|\boldsymbol{\xi}|) |\boldsymbol{\xi}| N(|\boldsymbol{\xi}|)} \int \nabla V \cdot \boldsymbol{\tau} \sin \theta \exp\left(-\frac{1}{A^2} \boldsymbol{\tau} \cdot \nabla V\right) d\theta d\alpha.$$

Thus, the limit equation is in such a case given by

$$\partial_t \rho + \nabla_\xi \cdot (\rho \lambda(|\boldsymbol{\xi}|) \boldsymbol{\xi}) = 0. \quad (5.7)$$

In this case one shows that  $\lambda$  is negative, i.e. the vector field  $\mathbf{U}$  points towards the origin. For  $B = 1$  and the potential  $V(\boldsymbol{\xi}) = |\boldsymbol{\xi}|^2/2$  we plot  $|\boldsymbol{\xi}| \lambda(|\boldsymbol{\xi}|)$  in Figure 5.2. For general  $B < 1$ , one can show that the vector field  $\mathbf{U}$  is symmetric with respect to the  $\mathbf{e}_3$ -axis, compare Figure 5.2.

REMARK 8. *An equivalent result can be proven with the same method in the 2D case.*

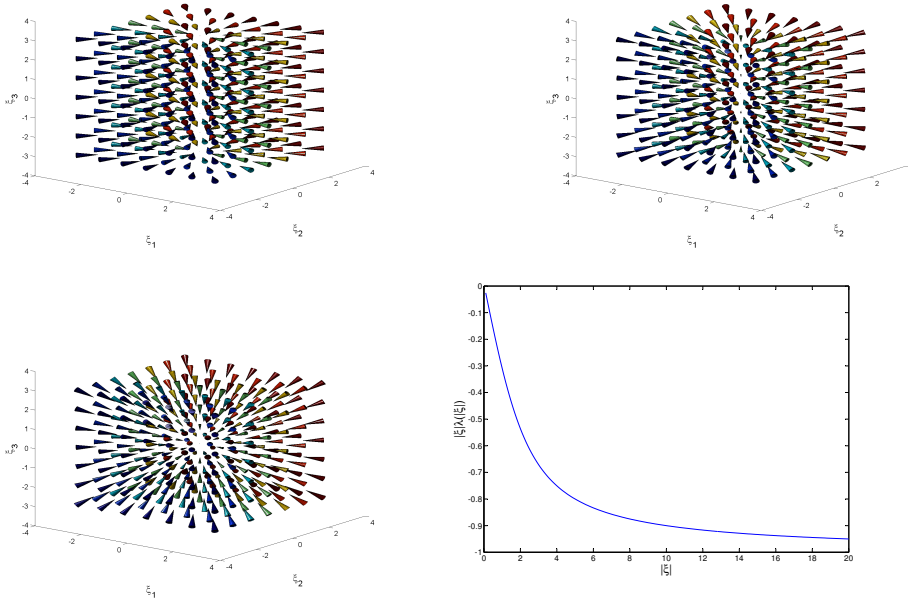


Fig. 5.2: Plots of the vector field  $\mathbf{U}$  for different parameter  $B = 0; 0.1; 1$  and plot of  $|\xi|\lambda(|\xi|)$  on the bottom right for the isotropic case ( $B = 1$ ) and fixed  $A$

**5.4. Relations between the Models.** Figure 5.3 shows the scaling limits of the modified 3D model (4.3).

**6. Numerical Simulations and Identification of the Parameters.** In the 2D case the parameter  $A$  and the shape of the potential  $V$  have been estimated in [10] for different production processes. There, full simulations of a single representative fiber have been performed with the software tool FIDYST<sup>1</sup>, see [12, 8] for a description of the algorithms. The noise amplitude  $A$  and the potential  $V$  in the 2D model (2.4) have been identified on the basis of the full simulation. Figure 6.1 shows as an example the fiber trajectories computed by the calibrated 2D model and, in comparison, an underlying full FIDYST simulation. The fiber mass distribution is captured qualitatively well in the surrogate model. The potential in  $\xi_3$  might be assumed as confining potential which models the impenetrable conveyor belt.

It remains to give a method to determine the distribution of the  $\theta$ -angle in a real material and translate this into the corresponding parameter  $B$ . Therefore we assume that the measurement of the  $\theta$ -angle in a real nonwoven leads to an axisymmetric distribution in  $\theta$  with mean  $m_\theta = \pi/2$  and variance  $var_\theta = \sigma_\theta^2$ . Then the parameter  $B$  in the stationary  $\theta$ -distribution  $P_B(\theta) = C \sin \theta^{\frac{1}{B}}$  is chosen such that the variance of this distribution coincides with  $var_\theta$ .

In Figure 6.2 an example of a virtual fleece generated by the modified 3D model is shown, where we simulate 10 fibers on a moving conveyor belt with assumed speed ratio  $\kappa = 0.0238$ . The distance between two spinning nozzles is chosen as  $d = 2.5 \cdot$

<sup>1</sup>FIDYST:Fiber Dynamics Simulation Tool developed at Fraunhofer ITWM, Kaiserslautern



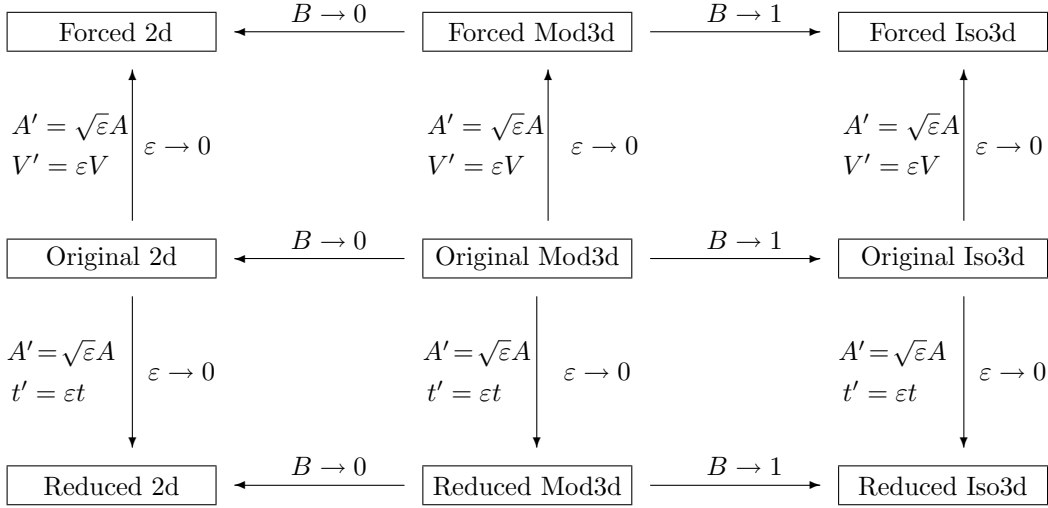


Fig. 5.3: Asymptotic limits of the 3D fiber model for different scalings.

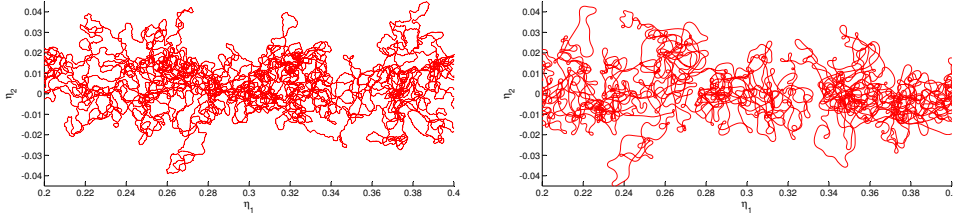


Fig. 6.1: Comparison of fiber path associated to the adapted 2D surrogate model (left) and the FIDYST simulation (right)

$10^{-3}$  and the variance of the  $\theta$ -distribution as  $\sigma_\theta^2 = 0.0169$  which corresponds to a parameter  $B = 0.0171$ . The throwing ranges  $\sigma_1, \sigma_2$  in the standard buckling potential  $V(\boldsymbol{\xi}) = (\xi_1^2/\sigma_1^2 + \xi_2^2/\sigma_2^2)/2$  and the noise amplitude  $A$  are determined by the parameter identification from FIDYST data. The throwing range in  $z$ -direction or the fleece thickness is supposed to be  $d_{fleece} = 0.01$ .

**7. Conclusion and Outlook .** In this paper we derived a new 3D model for the fiber lay down process in technical textile production taking into account the anisotropic orientation of the fibers in the resulting fleece. In future work we plan to apply the model to real fiber process problems, where the distribution of the angle  $\theta$  can be obtained by a computer tomography of the non-woven and associated image analysis. Moreover, from the theoretical point of view the convergence to equilibrium of the 3D models will be discussed with methods developed in [4]. See [5] for the 2-D

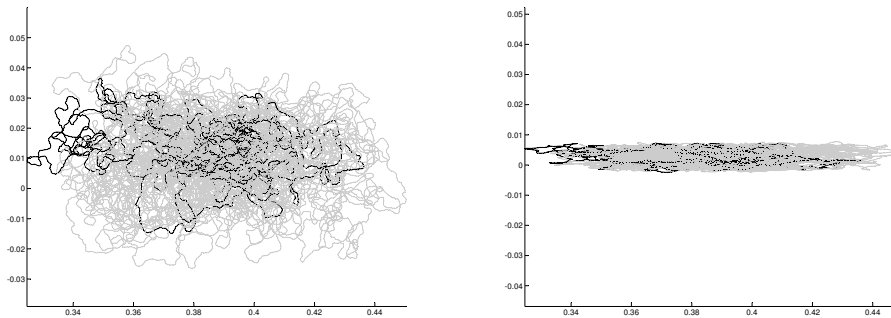


Fig. 6.2: *Example of a virtual fleece (10 fibers) with top view (left) and side view (right). A representative single fiber is emphasized as darker curve.*

case. The drawback of non-differentiable fiber path can be overcome by replacing the Wiener Process by an Ornstein-Uhlenbeck process leading to a more realistic smooth model [11] analogous to the 2-D case, see [8].

**Acknowledgments.** This work has been supported by Deutsche Forschungsgemeinschaft (DFG), WE 2003/3-1, KL 1105/18-1 and by Bundesministerium für Bildung und Forschung (BMBF), Verbundprojekt ProFil, 03MS606

#### REFERENCES

- [1] S. Albeverio and A. Klar, *Longtime behaviour of nonlinear stochastic oscillators*, J. Math. Phys. 35(8) 4005-4027, 1994.
- [2] S. Albeverio and A. Klar, *Longtime behaviour of stochastic Hamiltonian systems*, Potential Analysis 12, 281-297, 2000.
- [3] L. Bonilla and T. Götz and A. Klar and N. Marheineke and R. Wegener, *Hydrodynamic limit of a Fokker-Planck equation describing fiber lay-down processes*, SIAM J. Appl. Math. 68(3), 648-665, 2007
- [4] J. Dolbeault and C. Mouhot and C. Schmeiser *Hypoocoercivity for linear equations conserving mass*, preprint
- [5] J. Dolbeault and A. Klar and C. Mouhot and C. Schmeiser *Hypoocoercivity and a Fokker-Planck equation for fiber lay-down*, preprint
- [6] T. Götz and A. Klar and N. Marheineke and R. Wegener, *A stochastic model and associated Fokker-Planck equation for the fiber lay-down process in nonwoven production processes*, SIAM J. Appl. Math. 67(6), 1704-1717, 2007
- [7] M. Grothaus and A. Klar, *Ergodicity and rate of convergence for a non-sectorial fiber lay-down process*, SIAM J. Math. Anal. 40(3), 968-983, 2008
- [8] M. Herty and A. Klar and S. Motsch and F. Olawsky, *A smooth model for fibre lay-down processes and its diffusion approximation*, KRM 2 (3), 480-502, 2009
- [9] A. Klar and P. Reuterswärd and M. Seaïd *A semi-Lagrangian method for a Fokker-Planck equation describing fiber dynamics*, J.Sci.Comp. 38(3), 349-367, 2009
- [10] A. Klar and N. Marheineke, and R. Wegener, *Hierarchy of mathematical models for production processes of technical textiles*, ZAMM 89(12), 941-961, 2009
- [11] A. Klar and J. Maringer, and R. Wegener *A Smooth 3D Model for Fiber Lay-down in Nonwoven Production Processes*, preprint
- [12] N. Marheineke and R. Wegener, *Fiber dynamics in turbulent flows: General modeling framework*, SIAM J. Appl. Math. 66(5), 1703-1726, 2006
- [13] N. Marheineke, and R. Wegener, *Fiber dynamics in turbulent flows: Specific Taylor drag*, SIAM J. Appl. Math. 68(1), 1-23, 2007
- [14] Bernt Øksendal, *Stochastic differential equations*, Springer, 2007

- [15] D.W. Stroock, *On the growth of stochastic integrals*, Z.Wahr. verw. Geb. 18, 340-344

# Published reports of the Fraunhofer ITWM

The PDF-files of the following reports are available under:

[www.itwm.fraunhofer.de/de/zentral\\_\\_berichte/berichte](http://www.itwm.fraunhofer.de/de/zentral__berichte/berichte)

1. D. Hietel, K. Steiner, J. Struckmeier  
**A Finite - Volume Particle Method for Compressible Flows**  
(19 pages, 1998)
2. M. Feldmann, S. Seibold  
**Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing**  
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics  
(23 pages, 1998)
3. Y. Ben-Haim, S. Seibold  
**Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery**  
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis  
(24 pages, 1998)
4. F.-Th. Lentens, N. Siedow  
**Three-dimensional Radiative Heat Transfer in Glass Cooling Processes**  
(23 pages, 1998)
5. A. Klar, R. Wegener  
**A hierarchy of models for multilane vehicular traffic**  
**Part I: Modeling**  
(23 pages, 1998)  
**Part II: Numerical and stochastic investigations**  
(17 pages, 1998)
6. A. Klar, N. Siedow  
**Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes**  
(24 pages, 1998)
7. I. Choquet  
**Heterogeneous catalysis modelling and numerical simulation in rarified gas flows**  
**Part I: Coverage locally at equilibrium**  
(24 pages, 1998)
8. J. Ohser, B. Steinbach, C. Lang  
**Efficient Texture Analysis of Binary Images**  
(17 pages, 1998)
9. J. Orlik  
**Homogenization for viscoelasticity of the integral type with aging and shrinkage**  
(20 pages, 1998)
10. J. Mohring  
**Helmholtz Resonators with Large Aperture**  
(21 pages, 1998)
11. H. W. Hamacher, A. Schöbel  
**On Center Cycles in Grid Graphs**  
(15 pages, 1998)
12. H. W. Hamacher, K.-H. Küfer  
**Inverse radiation therapy planning - a multiple objective optimisation approach**  
(14 pages, 1999)
13. C. Lang, J. Ohser, R. Hilfer  
**On the Analysis of Spatial Binary Images**  
(20 pages, 1999)
14. M. Junk  
**On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes**  
(24 pages, 1999)
15. M. Junk, S. V. Raghurame Rao  
**A new discrete velocity method for Navier-Stokes equations**  
(20 pages, 1999)
16. H. Neunzert  
**Mathematics as a Key to Key Technologies**  
(39 pages, 1999)
17. J. Ohser, K. Sandau  
**Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem**  
(18 pages, 1999)
18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel  
**Solving nonconvex planar location problems by finite dominating sets**  
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm  
(19 pages, 2000)
19. A. Becker  
**A Review on Image Distortion Measures**  
Keywords: Distortion measure, human visual system  
(26 pages, 2000)
20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn  
**Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem**  
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut  
(21 pages, 2000)
21. H. W. Hamacher, A. Schöbel  
**Design of Zone Tariff Systems in Public Transportation**  
(30 pages, 2001)
22. D. Hietel, M. Junk, R. Keck, D. Teleaga  
**The Finite-Volume-Particle Method for Conservation Laws**  
(16 pages, 2001)
23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel  
**Location Software and Interface with GIS and Supply Chain Management**  
Keywords: facility location, software development, geographical information systems, supply chain management  
(48 pages, 2001)
24. H. W. Hamacher, S. A. Tjandra  
**Mathematical Modelling of Evacuation Problems: A State of Art**  
(44 pages, 2001)
25. J. Kuhnert, S. Tiwari  
**Grid free method for solving the Poisson equation**  
Keywords: Poisson equation, Least squares method, Grid free method  
(19 pages, 2001)
26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier  
**Simulation of the fiber spinning process**  
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD  
(19 pages, 2001)
27. A. Zemitis  
**On interaction of a liquid film with an obstacle**  
Keywords: impinging jets, liquid film, models, numerical solution, shape  
(22 pages, 2001)
28. I. Ginzburg, K. Steiner  
**Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids**  
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models  
(22 pages, 2001)
29. H. Neunzert  
**»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«**  
**Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001**  
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalalanalyse, Strömungsmechanik  
(18 pages, 2001)
30. J. Kuhnert, S. Tiwari  
**Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations**  
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation  
AMS subject classification: 76D05, 76M28  
(25 pages, 2001)
31. R. Korn, M. Krekel  
**Optimal Portfolios with Fixed Consumption or Income Streams**  
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems  
(23 pages, 2002)
32. M. Krekel  
**Optimal portfolios with a loan dependent credit spread**  
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics  
(25 pages, 2002)
33. J. Ohser, W. Nagel, K. Schladitz  
**The Euler number of discretized sets – on the choice of adjacency in homogeneous lattices**  
Keywords: image analysis, Euler number, neighborhood relationships, cuboidal lattice  
(32 pages, 2002)

34. I. Ginzburg, K. Steiner  
**Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting**  
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; up-wind-schemes (54 pages, 2002)
35. M. Günther, A. Klar, T. Materne, R. Wegener  
**Multivalued fundamental diagrams and stop and go waves for continuum traffic equations**  
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions (25 pages, 2002)
36. S. Feldmann, P. Lang, D. Prätzel-Wolters  
**Parameter influence on the zeros of network determinants**  
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory (30 pages, 2002)
37. K. Koch, J. Ohser, K. Schladitz  
**Spectral theory for random closed sets and estimating the covariance via frequency space**  
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum (28 pages, 2002)
38. D. d'Humières, I. Ginzburg  
**Multi-reflection boundary conditions for lattice Boltzmann models**  
Keywords: lattice Boltzmann equation, boundary conditions, bounce-back rule, Navier-Stokes equation (72 pages, 2002)
39. R. Korn  
**Elementare Finanzmathematik**  
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht (98 pages, 2002)
40. J. Kallrath, M. C. Müller, S. Nickel  
**Batch Presorting Problems: Models and Complexity Results**  
Keywords: Complexity theory, Integer programming, Assignment, Logistics (19 pages, 2002)
41. J. Linn  
**On the frame-invariant description of the phase space of the Folgar-Tucker equation**  
Key words: fiber orientation, Folgar-Tucker equation, injection molding (5 pages, 2003)
42. T. Hanne, S. Nickel  
**A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects**  
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set (29 pages, 2003)
43. T. Bortfeld, K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus  
**Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem**  
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy (31 pages, 2003)
44. T. Halfmann, T. Wichmann  
**Overview of Symbolic Methods in Industrial Analog Circuit Design**  
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index (17 pages, 2003)
45. S. E. Mikhailov, J. Orlik  
**Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites**  
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions (14 pages, 2003)
46. P. Domínguez-Marín, P. Hansen, N. Mladenovic, S. Nickel  
**Heuristic Procedures for Solving the Discrete Ordered Median Problem**  
Keywords: genetic algorithms, variable neighborhood search, discrete facility location (31 pages, 2003)
47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto  
**Exact Procedures for Solving the Discrete Ordered Median Problem**  
Keywords: discrete location, Integer programming (41 pages, 2003)
48. S. Feldmann, P. Lang  
**Padé-like reduction of stable discrete linear systems preserving their stability**  
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation (16 pages, 2003)
49. J. Kallrath, S. Nickel  
**A Polynomial Case of the Batch Presorting Problem**  
Keywords: batch presorting problem, online optimization, competitive analysis, polynomial algorithms, logistics (17 pages, 2003)
50. T. Hanne, H. L. Trinkaus  
**knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making**  
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications. (26 pages, 2003)
51. O. Iliev, V. Laptev  
**On Numerical Simulation of Flow Through Oil Filters**  
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation (8 pages, 2003)
52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva  
**On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media**  
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media (17 pages, 2003)
53. S. Kruse  
**On the Pricing of Forward Starting Options under Stochastic Volatility**  
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options (11 pages, 2003)
54. O. Iliev, D. Stoyanov  
**Multigrid – adaptive local refinement solver for incompressible flows**  
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity (37 pages, 2003)
55. V. Starikovicus  
**The multiphase flow and heat transfer in porous media**  
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation (30 pages, 2003)
56. P. Lang, A. Sarishvili, A. Wirsen  
**Blocked neural networks for knowledge extraction in the software development process**  
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection (21 pages, 2003)
57. H. Knaf, P. Lang, S. Zeiser  
**Diagnosis aiding in Regulation Thermography using Fuzzy Logic**  
Keywords: fuzzy logic, knowledge representation, expert system (22 pages, 2003)
58. M. T. Melo, S. Nickel, F. Saldanha da Gama  
**Largescale models for dynamic multi-commodity capacitated facility location**  
Keywords: supply chain management, strategic planning, dynamic location, modeling (40 pages, 2003)
59. J. Orlik  
**Homogenization for contact problems with periodically rough surfaces**  
Keywords: asymptotic homogenization, contact problems (28 pages, 2004)
60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld  
**IMRT planning on adaptive volume structures – a significant advance of computational complexity**  
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods (24 pages, 2004)
61. D. Kehrwald  
**Parallel lattice Boltzmann simulation of complex flows**  
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding (12 pages, 2004)
62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicus  
**On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations**



Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow (17 pages, 2004)

63. R. Ciegis, O. Iliev, S. Rief, K. Steiner  
**On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding**  
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media (43 pages, 2004)

64. T. Hanne, H. Neu  
**Simulating Human Resources in Software Development Processes**  
Keywords: Human resource modeling, software process, productivity, human factors, learning curve (14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov  
**Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media**  
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements (28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich  
**On numerical solution of 1-D poroelasticity equations in a multilayered domain**  
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid (41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe  
**Diffraction by image processing and its application in materials science**  
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum (13 pages, 2004)

68. H. Neunzert  
**Mathematics as a Technology: Challenges for the next 10 Years**  
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, turbulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology (29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich  
**On convergence of certain finite difference discretizations for 1D poroelasticity interface problems**  
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates (26 pages, 2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva  
**On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver**  
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media (25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder  
**Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration**  
Keywords: territory design, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems (40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinle-Bitzer, A. Wiegmann, J. Ohser  
**Design of acoustic trim based on geometric modeling and flow simulation for non-woven**  
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven (21 pages, 2005)

73. V. Rutka, A. Wiegmann  
**Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials**  
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials (22 pages, 2005)

74. T. Hanne  
**Eine Übersicht zum Scheduling von Baustellen**  
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie (32 pages, 2005)

75. J. Linn  
**The Folgar-Tucker Model as a Differential Algebraic System for Fiber Orientation Calculation**  
Keywords: fiber orientation, Folgar-Tucker model, invariants, algebraic constraints, phase space, trace stability (15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda  
**Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung**  
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration (20 pages, 2005)

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süß, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke  
**Multicriteria optimization in intensity modulated radiotherapy planning**  
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering (51 pages, 2005)

78. S. Amstutz, H. Andrä  
**A new algorithm for topology optimization using a level-set method**  
Keywords: shape optimization, topology optimization, topological sensitivity, level-set (22 pages, 2005)

79. N. Ettrich  
**Generation of surface elevation models for urban drainage simulation**  
Keywords: Flooding, simulation, urban elevation models, laser scanning (22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann  
**OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)**  
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung (77 pages, 2005)

81. N. Marheineke, R. Wegener  
**Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework**  
Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields (20 pages, 2005)

**Part II: Specific Taylor Drag**  
Keywords: flexible fibers;  $k-\epsilon$  turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process (18 pages, 2005)

82. C. H. Lampert, O. Wirjadi  
**An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter**  
Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters (25 pages, 2005)

83. H. Andrä, D. Stoyanov  
**Error indicators in the parallel finite element solver for linear elasticity DDFEM**  
Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decomposition, parallel implementation, a posteriori error estimates (21 pages, 2006)

84. M. Schröder, I. Solchenbach  
**Optimization of Transfer Quality in Regional Public Transit**  
Keywords: public transit, transfer quality, quadratic assignment problem (16 pages, 2006)

85. A. Naumovich, F. J. Gaspar  
**On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains**  
Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation (11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke  
**Slender Body Theory for the Dynamics of Curved Viscous Fibers**  
Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory (14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev  
**Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids**  
Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing (18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener  
**A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures**  
Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling (16 pages, 2006)

89. R. Ciegis, O. Iliev, V. Starikovicius, K. Steiner  
**Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media**  
Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows (16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz  
**On 3D Numerical Simulations of Viscoelastic Fluids**  
Keywords: non-Newtonian fluids, anisotropic viscosity, integral constitutive equation  
(18 pages, 2006)
91. A. Winterfeld  
**Application of general semi-infinite Programming to Lapidary Cutting Problems**  
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering  
(26 pages, 2006)
92. J. Orlik, A. Ostrovska  
**Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems**  
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate  
(24 pages, 2006)
93. V. Rutka, A. Wiegmann, H. Andrä  
**EJIM for Calculation of effective Elastic Moduli in 3D Linear Elasticity**  
Keywords: Elliptic PDE, linear elasticity, irregular domain, finite differences, fast solvers, effective elastic moduli  
(24 pages, 2006)
94. A. Wiegmann, A. Zemitis  
**EJ-HEAT: A Fast Explicit Jump Harmonic Averaging Solver for the Effective Heat Conductivity of Composite Materials**  
Keywords: Stationary heat equation, effective thermal conductivity, explicit jump, discontinuous coefficients, virtual material design, microstructure simulation, EJ-HEAT  
(21 pages, 2006)
95. A. Naumovich  
**On a finite volume discretization of the three-dimensional Biot poroelasticity system in multilayered domains**  
Keywords: Biot poroelasticity system, interface problems, finite volume discretization, finite difference method  
(21 pages, 2006)
96. M. Krekel, J. Wenzel  
**A unified approach to Credit Default Swap-tion and Constant Maturity Credit Default Swap valuation**  
Keywords: LIBOR market model, credit risk, Credit Default Swap-tion, Constant Maturity Credit Default Swap-method  
(43 pages, 2006)
97. A. Dreyer  
**Interval Methods for Analog Circuits**  
Keywords: interval arithmetic, analog circuits, tolerance analysis, parametric linear systems, frequency response, symbolic analysis, CAD, computer algebra  
(36 pages, 2006)
98. N. Weigel, S. Weihe, G. Bitsch, K. Dreßler  
**Usage of Simulation for Design and Optimization of Testing**  
Keywords: Vehicle test rigs, MBS, control, hydraulics, testing philosophy  
(14 pages, 2006)
99. H. Lang, G. Bitsch, K. Dreßler, M. Speckert  
**Comparison of the solutions of the elastic and elastoplastic boundary value problems**  
Keywords: Elastic BVP, elastoplastic BVP, variational inequalities, rate-independency, hysteresis, linear kinematic hardening, stop- and play-operator  
(21 pages, 2006)
100. M. Speckert, K. Dreßler, H. Mauch  
**MBS Simulation of a hexapod based suspension test rig**  
Keywords: Test rig, MBS simulation, suspension, hydraulics, controlling, design optimization  
(12 pages, 2006)
101. S. Azizi Sultan, K.-H. Küfer  
**A dynamic algorithm for beam orientations in multicriteria IMRT planning**  
Keywords: radiotherapy planning, beam orientation optimization, dynamic approach, evolutionary algorithm, global optimization  
(14 pages, 2006)
102. T. Götz, A. Klar, N. Marheineke, R. Wegener  
**A Stochastic Model for the Fiber Lay-down Process in the Nonwoven Production**  
Keywords: fiber dynamics, stochastic Hamiltonian system, stochastic averaging  
(17 pages, 2006)
103. Ph. Süß, K.-H. Küfer  
**Balancing control and simplicity: a variable aggregation method in intensity modulated radiation therapy planning**  
Keywords: IMRT planning, variable aggregation, clustering methods  
(22 pages, 2006)
104. A. Beaudry, G. Laporte, T. Melo, S. Nickel  
**Dynamic transportation of patients in hospitals**  
Keywords: in-house hospital transportation, dial-a-ride, dynamic mode, tabu search  
(37 pages, 2006)
105. Th. Hanne  
**Applying multiobjective evolutionary algorithms in industrial projects**  
Keywords: multiobjective evolutionary algorithms, discrete optimization, continuous optimization, electronic circuit design, semi-infinite programming, scheduling  
(18 pages, 2006)
106. J. Franke, S. Halim  
**Wild bootstrap tests for comparing signals and images**  
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression  
(13 pages, 2007)
107. Z. Drezner, S. Nickel  
**Solving the ordered one-median problem in the plane**  
Keywords: planar location, global optimization, ordered median, big triangle small triangle method, bounds, numerical experiments  
(21 pages, 2007)
108. Th. Götz, A. Klar, A. Unterreiter, R. Wegener  
**Numerical evidence for the non-existing of solutions of the equations describing rotational fiber spinning**  
Keywords: rotational fiber spinning, viscous fibers, boundary value problem, existence of solutions  
(11 pages, 2007)
109. Ph. Süß, K.-H. Küfer  
**Smooth intensity maps and the Bortfeld-Boyer sequencer**  
Keywords: probabilistic analysis, intensity modulated radiotherapy treatment (IMRT), IMRT plan application, step-and-shoot sequencing  
(8 pages, 2007)
110. E. Ivanov, O. Gluchshenko, H. Andrä, A. Kudryavtsev  
**Parallel software tool for decomposing and meshing of 3d structures**  
Keywords: a-priori domain decomposition, unstructured grid, Delaunay mesh generation  
(14 pages, 2007)
111. O. Iliev, R. Lazarov, J. Willems  
**Numerical study of two-grid preconditioners for 1d elliptic problems with highly oscillating discontinuous coefficients**  
Keywords: two-grid algorithm, oscillating coefficients, preconditioner  
(20 pages, 2007)
112. L. Bonilla, T. Götz, A. Klar, N. Marheineke, R. Wegener  
**Hydrodynamic limit of the Fokker-Planck equation describing fiber lay-down processes**  
Keywords: stochastic differential equations, Fokker-Planck equation, asymptotic expansion, Ornstein-Uhlenbeck process  
(17 pages, 2007)
113. S. Rief  
**Modeling and simulation of the pressing section of a paper machine**  
Keywords: paper machine, computational fluid dynamics, porous media  
(41 pages, 2007)
114. R. Ciegis, O. Iliev, Z. Lakdawala  
**On parallel numerical algorithms for simulating industrial filtration problems**  
Keywords: Navier-Stokes-Brinkmann equations, finite volume discretization method, SIMPLE, parallel computing, data decomposition method  
(24 pages, 2007)
115. N. Marheineke, R. Wegener  
**Dynamics of curved viscous fibers with surface tension**  
Keywords: Slender body theory, curved viscous fibers with surface tension, free boundary value problem  
(25 pages, 2007)
116. S. Feth, J. Franke, M. Speckert  
**Resampling-Methoden zur mse-Korrektur und Anwendungen in der Betriebsfestigkeit**  
Keywords: Weibull, Bootstrap, Maximum-Likelihood, Betriebsfestigkeit  
(16 pages, 2007)
117. H. Knaf  
**Kernel Fisher discriminant functions – a concise and rigorous introduction**  
Keywords: wild bootstrap test, texture classification, textile quality control, defect detection, kernel estimate, nonparametric regression  
(30 pages, 2007)
118. O. Iliev, I. Rybak  
**On numerical upscaling for flows in heterogeneous porous media**

- Keywords: numerical upscaling, heterogeneous porous media, single phase flow, Darcy's law, multiscale problem, effective permeability, multipoint flux approximation, anisotropy (17 pages, 2007)
119. O. Iliev, I. Rybak  
**On approximation property of multipoint flux approximation method**  
Keywords: Multipoint flux approximation, finite volume method, elliptic equation, discontinuous tensor coefficients, anisotropy (15 pages, 2007)
120. O. Iliev, I. Rybak, J. Willems  
**On upscaling heat conductivity for a class of industrial problems**  
Keywords: Multiscale problems, effective heat conductivity, numerical upscaling, domain decomposition (21 pages, 2007)
121. R. Ewing, O. Iliev, R. Lazarov, I. Rybak  
**On two-level preconditioners for flow in porous media**  
Keywords: Multiscale problem, Darcy's law, single phase flow, anisotropic heterogeneous porous media, numerical upscaling, multigrid, domain decomposition, efficient preconditioner (18 pages, 2007)
122. M. Brickenstein, A. Dreyer  
**POLYBORI: A Gröbner basis framework for Boolean polynomials**  
Keywords: Gröbner basis, formal verification, Boolean polynomials, algebraic cryptanalysis, satisfiability (23 pages, 2007)
123. O. Wirjadi  
**Survey of 3d image segmentation methods**  
Keywords: image processing, 3d, image segmentation, binarization (20 pages, 2007)
124. S. Zeytun, A. Gupta  
**A Comparative Study of the Vasicek and the CIR Model of the Short Rate**  
Keywords: interest rates, Vasicek model, CIR-model, calibration, parameter estimation (17 pages, 2007)
125. G. Hanselmann, A. Sarishvili  
**Heterogeneous redundancy in software quality prediction using a hybrid Bayesian approach**  
Keywords: reliability prediction, fault prediction, non-homogeneous poisson process, Bayesian model averaging (17 pages, 2007)
126. V. Maag, M. Berger, A. Winterfeld, K.-H. Küfer  
**A novel non-linear approach to minimal area rectangular packing**  
Keywords: rectangular packing, non-overlapping constraints, non-linear optimization, regularization, relaxation (18 pages, 2007)
127. M. Monz, K.-H. Küfer, T. Bortfeld, C. Thieke  
**Pareto navigation – systematic multi-criteria-based IMRT treatment plan determination**  
Keywords: convex, interactive multi-objective optimization, intensity modulated radiotherapy planning (15 pages, 2007)
128. M. Krause, A. Scherrer  
**On the role of modeling parameters in IMRT plan optimization**  
Keywords: intensity-modulated radiotherapy (IMRT), inverse IMRT planning, convex optimization, sensitivity analysis, elasticity, modeling parameters, equivalent uniform dose (EUD) (18 pages, 2007)
129. A. Wiegmann  
**Computation of the permeability of porous materials from their microstructure by FFF-Stokes**  
Keywords: permeability, numerical homogenization, fast Stokes solver (24 pages, 2007)
130. T. Melo, S. Nickel, F. Saldanha da Gama  
**Facility Location and Supply Chain Management – A comprehensive review**  
Keywords: facility location, supply chain management, network design (54 pages, 2007)
131. T. Hanne, T. Melo, S. Nickel  
**Bringing robustness to patient flow management through optimized patient transports in hospitals**  
Keywords: Dial-a-Ride problem, online problem, case study, tabu search, hospital logistics (23 pages, 2007)
132. R. Ewing, O. Iliev, R. Lazarov, I. Rybak, J. Willems  
**An efficient approach for upscaling properties of composite materials with high contrast of coefficients**  
Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams (16 pages, 2008)
133. S. Gelareh, S. Nickel  
**New approaches to hub location problems in public transport planning**  
Keywords: integer programming, hub location, transportation, decomposition, heuristic (25 pages, 2008)
134. G. Thömmes, J. Becker, M. Junk, A. K. Vainkuntam, D. Kehrwald, A. Klar, K. Steiner, A. Wiegmann  
**A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method**  
Keywords: Lattice Boltzmann method, Level Set method, free surface, multiphase flow (28 pages, 2008)
135. J. Orlik  
**Homogenization in elasto-plasticity**  
Keywords: multiscale structures, asymptotic homogenization, nonlinear energy (40 pages, 2008)
136. J. Almqvist, H. Schmidt, P. Lang, J. Deitmer, M. Jirstrand, D. Prätzel-Wolters, H. Becker  
**Determination of interaction between MCT1 and CAII via a mathematical and physiological approach**  
Keywords: mathematical modeling; model reduction; electrophysiology; pH-sensitive microelectrodes; proton antenna (20 pages, 2008)
137. E. Savenkov, H. Andrä, O. Iliev  
**An analysis of one regularization approach for solution of pure Neumann problem**  
Keywords: pure Neumann problem, elasticity, regularization, finite element method, condition number (27 pages, 2008)
138. O. Berman, J. Kalcsics, D. Krass, S. Nickel  
**The ordered gradual covering location problem on a network**  
Keywords: gradual covering, ordered median function, network location (32 pages, 2008)
139. S. Gelareh, S. Nickel  
**Multi-period public transport design: A novel model and solution approaches**  
Keywords: Integer programming, hub location, public transport, multi-period planning, heuristics (31 pages, 2008)
140. T. Melo, S. Nickel, F. Saldanha-da-Gama  
**Network design decisions in supply chain planning**  
Keywords: supply chain design, integer programming models, location models, heuristics (20 pages, 2008)
141. C. Lautensack, A. Särkkä, J. Freitag, K. Schladitz  
**Anisotropy analysis of pressed point processes**  
Keywords: estimation of compression, isotropy test, nearest neighbour distance, orientation analysis, polar ice, Ripley's K function (35 pages, 2008)
142. O. Iliev, R. Lazarov, J. Willems  
**A Graph-Laplacian approach for calculating the effective thermal conductivity of complicated fiber geometries**  
Keywords: graph laplacian, effective heat conductivity, numerical upscaling, fibrous materials (14 pages, 2008)
143. J. Linn, T. Stephan, J. Carlsson, R. Bohlin  
**Fast simulation of quasistatic rod deformations for VR applications**  
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients (7 pages, 2008)
144. J. Linn, T. Stephan  
**Simulation of quasistatic deformations using discrete rod models**  
Keywords: quasistatic deformations, geometrically exact rod models, variational formulation, energy minimization, finite differences, nonlinear conjugate gradients (9 pages, 2008)
145. J. Marburger, N. Marheineke, R. Pinnau  
**Adjoint based optimal control using mesh-less discretizations**  
Keywords: Mesh-less methods, particle methods, Eulerian-Lagrangian formulation, optimization strategies, adjoint method, hyperbolic equations (14 pages, 2008)
146. S. Desmettre, J. Gould, A. Szimayer  
**Own-company stockholding and work effort preferences of an unconstrained executive**  
Keywords: optimal portfolio choice, executive compensation (33 pages, 2008)



147. M. Berger, M. Schröder, K.-H. Küfer  
**A constraint programming approach for the two-dimensional rectangular packing problem with orthogonal orientations**  
Keywords: rectangular packing, orthogonal orientations non-overlapping constraints, constraint propagation (13 pages, 2008)
148. K. Schladitz, C. Redenbach, T. Sych, M. Godehardt  
**Microstructural characterisation of open foams using 3d images**  
Keywords: virtual material design, image analysis, open foams (30 pages, 2008)
149. E. Fernández, J. Kalcsics, S. Nickel, R. Ríos-Mercado  
**A novel territory design model arising in the implementation of the WEEE-Directive**  
Keywords: heuristics, optimization, logistics, recycling (28 pages, 2008)
150. H. Lang, J. Linn  
**Lagrangian field theory in space-time for geometrically exact Cosserat rods**  
Keywords: Cosserat rods, geometrically exact rods, small strain, large deformation, deformable bodies, Lagrangian field theory, variational calculus (19 pages, 2009)
151. K. Dreßler, M. Speckert, R. Müller, Ch. Weber  
**Customer loads correlation in truck engineering**  
Keywords: Customer distribution, safety critical components, quantile estimation, Monte-Carlo methods (11 pages, 2009)
152. H. Lang, K. Dreßler  
**An improved multiaxial stress-strain correction model for elastic FE postprocessing**  
Keywords: Jiang's model of elastoplasticity, stress-strain correction, parameter identification, automatic differentiation, least-squares optimization, Coleman-Li algorithm (6 pages, 2009)
153. J. Kalcsics, S. Nickel, M. Schröder  
**A generic geometric approach to territory design and districting**  
Keywords: Territory design, districting, combinatorial optimization, heuristics, computational geometry (32 pages, 2009)
154. Th. Fütterer, A. Klar, R. Wegener  
**An energy conserving numerical scheme for the dynamics of hyperelastic rods**  
Keywords: Cosserat rod, hyperelastic, energy conservation, finite differences (16 pages, 2009)
155. A. Wiegmann, L. Cheng, E. Glatt, O. Iliev, S. Rief  
**Design of pleated filters by computer simulations**  
Keywords: Solid-gas separation, solid-liquid separation, pleated filter, design, simulation (21 pages, 2009)
156. A. Klar, N. Marheineke, R. Wegener  
**Hierarchy of mathematical models for production processes of technical textiles**  
Keywords: Fiber-fluid interaction, slender-body theory, turbulence modeling, model reduction, stochastic differential equations, Fokker-Planck equation, asymptotic expansions, parameter identification (21 pages, 2009)
157. E. Glatt, S. Rief, A. Wiegmann, M. Knefel, E. Wegenke  
**Structure and pressure drop of real and virtual metal wire meshes**  
Keywords: metal wire mesh, structure simulation, model calibration, CFD simulation, pressure loss (7 pages, 2009)
158. S. Kruse, M. Müller  
**Pricing American call options under the assumption of stochastic dividends – An application of the Korn-Rogers model**  
Keywords: option pricing, American options, dividends, dividend discount model, Black-Scholes model (22 pages, 2009)
159. H. Lang, J. Linn, M. Arnold  
**Multibody dynamics simulation of geometrically exact Cosserat rods**  
Keywords: flexible multibody dynamics, large deformations, finite rotations, constrained mechanical systems, structural dynamics (20 pages, 2009)
160. P. Jung, S. Leyendecker, J. Linn, M. Ortiz  
**Discrete Lagrangian mechanics and geometrically exact Cosserat rods**  
Keywords: special Cosserat rods, Lagrangian mechanics, Noether's theorem, discrete mechanics, frame-indifference, holonomic constraints (14 pages, 2009)
161. M. Burger, K. Dreßler, A. Marquardt, M. Speckert  
**Calculating invariant loads for system simulation in vehicle engineering**  
Keywords: iterative learning control, optimal control theory, differential algebraic equations (DAEs) (18 pages, 2009)
162. M. Speckert, N. Ruf, K. Dreßler  
**Undesired drift of multibody models excited by measured accelerations or forces**  
Keywords: multibody simulation, full vehicle model, force-based simulation, drift due to noise (19 pages, 2009)
163. A. Streit, K. Dreßler, M. Speckert, J. Lichter, T. Zenner, P. Bach  
**Anwendung statistischer Methoden zur Erstellung von Nutzungsprofilen für die Auslegung von Mobilbaggern**  
Keywords: Nutzungsvielfalt, Kundenbeanspruchung, Bemessungsgrundlagen (13 pages, 2009)
164. I. Correia, S. Nickel, F. Saldanha-da-Gama  
**The capacitated single-allocation hub location problem revisited: A note on a classical formulation**  
Keywords: Capacitated Hub Location, MIP formulations (10 pages, 2009)
165. F. Yaneva, T. Grebe, A. Scherrer  
**An alternative view on global radiotherapy optimization problems**  
Keywords: radiotherapy planning, path-connected sub-levelsets, modified gradient projection method, improving and feasible directions (14 pages, 2009)
166. J. I. Serna, M. Monz, K.-H. Küfer, C. Thieke  
**Trade-off bounds and their effect in multi-criteria IMRT planning**  
Keywords: trade-off bounds, multi-criteria optimization, IMRT, Pareto surface (15 pages, 2009)
167. W. Arne, N. Marheineke, A. Meister, R. Wegener  
**Numerical analysis of Cosserat rod and string models for viscous jets in rotational spinning processes**  
Keywords: Rotational spinning process, curved viscous fibers, asymptotic Cosserat models, boundary value problem, existence of numerical solutions (18 pages, 2009)
168. T. Melo, S. Nickel, F. Saldanha-da-Gama  
**An LP-rounding heuristic to solve a multi-period facility relocation problem**  
Keywords: supply chain design, heuristic, linear programming, rounding (37 pages, 2009)
169. I. Correia, S. Nickel, F. Saldanha-da-Gama  
**Single-allocation hub location problems with capacity choices**  
Keywords: hub location, capacity decisions, MILP formulations (27 pages, 2009)
170. S. Acar, K. Natcheva-Acar  
**A guide on the implementation of the Heath-Jarrow-Morton Two-Factor Gaussian Short Rate Model (HJM-G2++)**  
Keywords: short rate model, two factor Gaussian, G2++, option pricing, calibration (30 pages, 2009)
171. A. Szimayer, G. Dimitroff, S. Lorenz  
**A parsimonious multi-asset Heston model: calibration and derivative pricing**  
Keywords: Heston model, multi-asset, option pricing, calibration, correlation (28 pages, 2009)
172. N. Marheineke, R. Wegener  
**Modeling and validation of a stochastic drag for fibers in turbulent flows**  
Keywords: fiber-fluid interactions, long slender fibers, turbulence modelling, aerodynamic drag, dimensional analysis, data interpolation, stochastic partial differential algebraic equation, numerical simulations, experimental validations (19 pages, 2009)
173. S. Nickel, M. Schröder, J. Steeg  
**Planning for home health care services**  
Keywords: home health care, route planning, metaheuristics, constraint programming (23 pages, 2009)
174. G. Dimitroff, A. Szimayer, A. Wagner  
**Quanto option pricing in the parsimonious Heston model**  
Keywords: Heston model, multi asset, quanto options, option pricing (14 pages, 2009)
174. G. Dimitroff, A. Szimayer, A. Wagner  
**Model reduction of nonlinear problems in structural mechanics**  
Keywords: flexible bodies, FEM, nonlinear model reduction, POD (13 pages, 2009)

176. M. K. Ahmad, S. Didas, J. Iqbal  
**Using the Sharp Operator for edge detection and nonlinear diffusion**  
Keywords: maximal function, sharp function, image processing, edge detection, nonlinear diffusion (17 pages, 2009)
177. M. Speckert, N. Ruf, K. Dreßler, R. Müller, C. Weber, S. Weihe  
**Ein neuer Ansatz zur Ermittlung von Erprobungslasten für sicherheitsrelevante Bauteile**  
Keywords: sicherheitsrelevante Bauteile, Kundenbeanspruchung, Festigkeitsverteilung, Ausfallwahrscheinlichkeit, Konfidenz, statistische Unsicherheit, Sicherheitsfaktoren (16 pages, 2009)
178. J. Jegorovs  
**Wave based method: new applicability areas**  
Keywords: Elliptic boundary value problems, inhomogeneous Helmholtz type differential equations in bounded domains, numerical methods, wave based method, uniform B-splines (10 pages, 2009)
179. H. Lang, M. Arnold  
**Numerical aspects in the dynamic simulation of geometrically exact rods**  
Keywords: Kirchhoff and Cosserat rods, geometrically exact rods, deformable bodies, multibody dynamics, partial differential algebraic equations, method of lines, time integration (21 pages, 2009)
180. H. Lang  
**Comparison of quaternionic and rotation-free null space formalisms for multibody dynamics**  
Keywords: Parametrisation of rotations, differential-algebraic equations, multibody dynamics, constrained mechanical systems, Lagrangian mechanics (40 pages, 2010)
181. S. Nickel, F. Saldanha-da-Gama, H.-P. Ziegler  
**Stochastic programming approaches for risk aware supply chain network design problems**  
Keywords: Supply Chain Management, multi-stage stochastic programming, financial decisions, risk (37 pages, 2010)
182. P. Ruckdeschel, N. Horbenko  
**Robustness properties of estimators in generalized Pareto Models**  
Keywords: global robustness, local robustness, finite sample breakdown point, generalized Pareto distribution (58 pages, 2010)
183. P. Jung, S. Leyendecker, J. Linn, M. Ortiz  
**A discrete mechanics approach to Cosserat rod theory – Part 1: static equilibria**  
Keywords: Special Cosserat rods; Lagrangian mechanics; Noether's theorem; discrete mechanics; frame-indifference; holonomic constraints; variational formulation (35 pages, 2010)
184. R. Eymard, G. Printsypar  
**A proof of convergence of a finite volume scheme for modified steady Richards' equation describing transport processes in the pressing section of a paper machine**  
Keywords: flow in porous media, steady Richards' equation, finite volume methods, convergence of approximate solution (14 pages, 2010)
185. P. Ruckdeschel  
**Optimally Robust Kalman Filtering**  
Keywords: robustness, Kalman Filter, innovation outlier, additive outlier (42 pages, 2010)
186. S. Repke, N. Marheineke, R. Pinnau  
**On adjoint-based optimization of a free surface Stokes flow**  
Keywords: film casting process, thin films, free surface Stokes flow, optimal control, Lagrange formalism (13 pages, 2010)
187. O. Iliev, R. Lazarov, J. Willems  
**Variational multiscale Finite Element Method for flows in highly porous media**  
Keywords: numerical upscaling, flow in heterogeneous porous media, Brinkman equations, Darcy's law, subgrid approximation, discontinuous Galerkin mixed FEM (21 pages, 2010)
188. S. Desmettre, A. Szimayer  
**Work effort, consumption, and portfolio selection: When the occupational choice matters**  
Keywords: portfolio choice, work effort, consumption, occupational choice (34 pages, 2010)
189. O. Iliev, Z. Lakdawala, V. Starikovicius  
**On a numerical subgrid upscaling algorithm for Stokes-Brinkman equations**  
Keywords: Stokes-Brinkman equations, subgrid approach, multiscale problems, numerical upscaling (27 pages, 2010)
190. A. Latz, J. Zausch, O. Iliev  
**Modeling of species and charge transport in Li-Ion Batteries based on non-equilibrium thermodynamics**  
Keywords: lithium-ion battery, battery modeling, electrochemical simulation, concentrated electrolyte, ion transport (8 pages, 2010)
191. P. Popov, Y. Vutov, S. Margenov, O. Iliev  
**Finite volume discretization of equations describing nonlinear diffusion in Li-Ion batteries**  
Keywords: nonlinear diffusion, finite volume discretization, Newton method, Li-Ion batteries (9 pages, 2010)
192. W. Arne, N. Marheineke, R. Wegener  
**Asymptotic transition from Cosserat rod to string models for curved viscous inertial jets**  
Keywords: rotational spinning processes; inertial and viscous-inertial fiber regimes; asymptotic limits; slender-body theory; boundary value problems (23 pages, 2010)
193. L. Engelhardt, M. Burger, G. Bitsch  
**Real-time simulation of multibody-systems for on-board applications**  
Keywords: multibody system simulation, real-time simulation, on-board simulation, Rosenbrock methods (10 pages, 2010)
194. M. Burger, M. Speckert, K. Dreßler  
**Optimal control methods for the calculation of invariant excitation signals for multibody systems**  
Keywords: optimal control, optimization, mbs simulation, invariant excitation (9 pages, 2010)
195. A. Latz, J. Zausch  
**Thermodynamic consistent transport theory of Li-Ion batteries**  
Keywords: Li-Ion batteries, nonequilibrium thermodynamics, thermal transport, modeling (18 pages, 2010)
196. S. Desmettre  
**Optimal investment for executive stockholders with exponential utility**  
Keywords: portfolio choice, executive stockholder, work effort, exponential utility (24 pages, 2010)
197. W. Arne, N. Marheineke, J. Schnebele, R. Wegener  
**Fluid-fiber-interactions in rotational spinning process of glass wool production**  
Keywords: Rotational spinning process, viscous thermal jets, fluid-fiber-interactions, two-way coupling, slender-body theory, Cosserat rods, drag models, boundary value problem, continuation method (20 pages, 2010)
198. A. Klar, J. Maringer, R. Wegener  
**A 3d model for fiber lay-down in nonwoven production processes**  
Keywords: fiber dynamics, Fokker-Planck equations, diffusion limits (15 pages, 2010)

Status quo: December 2010