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# Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.



Prof. Dr. Dieter Prätzel-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



# Calibrating and Completing the Volatility Cube in the SABR Model

Georgi Dimitroff  
Johan de Kock

This report describes the calibration and completion of the volatility cube in the SABR model. The description is based on a project done for Assenagon GmbH in Munich. However, we use fictitious market data which resembles realistic market data. The problem posed by our client is formulated in section 1. Here we also motivate why this is a relevant problem. The SABR model is briefly reviewed in section 2. Section 3 discusses the calibration and completion of the volatility cube. An example is presented in section 4. We conclude by suggesting possible future research in section 5.

## 1 Formulation of the Problem

We first review the interest rate market setup before we formulate the problem solved. **European options (calls and puts)**, being the most basic and liquidly traded contracts, are the building blocks in the **equity world**. Central concepts like **implied volatility** are defined with respect to equity options and the Black-Scholes formula for pricing these options. The prices of European puts and calls are frequently quoted as implied volatilities, obtained by inverting the Black-Scholes formula. Due to their liquidity, the prices of these options are available on the market. This is the reason why pricing models for exotic (illiquid) products are usually calibrated in such a way that these liquid calls and puts are priced consistently.

The **interest rate world** (or **fixed income world**), on the other hand, is characterized by a large number of diverse products. However, **interest rate caps**, **interest rate floors** and **swaptions** can (to a certain extent) be regarded as the equivalent of Europeans options in the interest rate world (see Brigo and Mercurio [2]). These are the most liquid and nonlinear products in the interest rate world and interest rate models are calibrated to consistently price these instruments. In this report we focus on the swaption market.

We now consider swaptions more closely. Let  $T_m$  be the **option maturity** and  $T_{m+1} < T_{m+2} < \dots < T_n$  the payment dates of the swaption, where  $m, n \in \mathbb{N}$ . Thus,  $T_n - T_m$  is the **swaption tenor** and  $T_n$  the **total maturity**. A swaption with maturity  $T_m$  and tenor  $T_n - T_m$  is a contract allowing the holder to enter an interest rate swap with maturity  $T_n$  at time  $T_m$ . Such a swaption is usually referred to as a  $T_m$  ( $T_n - T_m$ ) swaption. The fixed leg is equal to the **strike**  $K$ . The swaption is essentially a European call on the underlying interest rate swap. We assume that the payment dates are (taking the day count convention into account) equidistant, *i.e.*  $T_i - T_{i-1} = \Delta t$  for  $i = m + 1, \dots, n$ .

The payoff of the swaption at time  $T_m$  is (see Filipović [3])

$$N[r(T_m) - K]^+ \Delta t \sum_{i=m+1}^n P(T_m, T_i),$$

where  $N$  is the notional,  $r(T_m)$  is the swap rate at time  $T_m$  and  $P(t, T)$  is the value (at time  $t$ ) of a zero-coupon bond with maturity  $T$ . It is evident that the price of the swaption is the expectation of  $[r(T_m) - K]^+$ . Therefore, the swaption can be interpreted as a European call on the underlying swap rate  $r(T_m)$ . The fair **forward swap rate** at time  $t$  is (see Brigo and Mercurio [2], for example)

$$S_{mn}(t) = \frac{P(t, T_m) - P(t, T_n)}{\Delta t \sum_{i=m+1}^n P(t, T_i)}.$$

Swaptions are usually priced with the interest rate equivalent of the Black-Scholes formula, namely the Black<sup>1</sup> formula (see Black [1]). The **swaption volatility**, defined with respect to the Black formula, is a central quantity in the interest rate world. Similar to the implied volatility in the equity world, the price of a swaption is frequently quoted in terms of the implied swaption volatility for the underlying swap rate.

Denote the implied swaption volatility<sup>2</sup> for a  $T_m$  ( $T_n - T_m$ ) swaption with strike  $K$  by  $\hat{\sigma}_{mn}(K)$ . Thus the volatility is a function of the option maturity, tenor and strike. It has become common practice to order the implied swaption volatilities in a three-dimensional structure known as a **volatility cube**. The  $x$ -,  $y$ - and  $z$ -dimension of the volatility cube are option maturity, tenor and strike respectively. Each point of the cube is the implied volatility for these three values, *ie*  $\hat{\sigma}_{mn}(K)$  for  $T_m$ ,  $T_n - T_m$  and  $K$ . One does not generally obtain a complete cube from the market. Typically one would have all the volatilities for the at-the-money (ATM) strike values. Therefore, the plane  $\hat{\sigma}_{mn}(K_{\text{ATM}})$  would be complete for all typical values of  $T_m$  and  $T_n - T_m$ , where  $K_{\text{ATM}}$  is the ATM strike. However, for other strike values  $K$  only some market quotes of  $\hat{\sigma}_{mn}(K)$  would be available. **Completing the volatility cube** entails “determining” the missing values of  $\hat{\sigma}_{mn}(K)$ . An intuitive approach would be to interpolate between the known values. This would not work well in cases where few values are available, which is frequently the case (see the discussion in section 3 and the example in section 4).

The aim of this project is to complete the volatility cube. The first decision to be made is how to model the forward swap rate. The most important requirement on the model is that it captures the market smile and the market skew. This is particularly important in the case of cube completion for two reasons. The first reason is that the available volatility data contains smiles and skews. The second reason is that the (missing) values needed to complete the cube must also exhibit smiles and skews in order to be consistent with the given data, *ie* the neighboring points.

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<sup>1</sup>Additional to its use in the interest rate world, the Black formula is also used extensively in the foreign exchange (FX) and commodity worlds.

<sup>2</sup>Note that although the implied swaption volatility is defined in the context of the Black model, an implied volatility can also be extracted when a different model is used (as in the equity world). Consequently, the implied swaption volatility is the volatility which, when substituted into the Black formula yields the swaption price given by the chosen model.

Given the above-mentioned reasons, a stochastic volatility model seems to be best-suited for modeling the forward swap rate. Such a model takes account of smiles and skews. Furthermore, there is empirical evidence that the swaption volatility is stochastic (see Brigo and Mercurio [2], for example). An important example of a stochastic volatility model is the stochastic volatility enhancement of the popular BGM model<sup>3</sup> (see Wu and Zhang [10], for example). Brigo and Mercurio [2] list and cover several stochastic volatility models for the interest rate world. However, our client, Assenagon GmbH, wanted the SABR stochastic volatility model. The next section discusses this model.

Before we proceed to present the SABR model, we shortly discuss the practical relevance of the cube completion problem. Why should traders be that interested in having a complete volatility cube? There are two main answers to this question. The first answer is that a complete cube contains valuable information on the current state of the market. A completed cube enables traders to consistently quote swaptions different from the liquidly traded ones. The second answer is of a practical nature: most commercial interest rate software and modeling tools require a complete cube as input data. Using this cube allows us to price more exotic interest rate derivatives.

## 2 The SABR Model

The **SABR**<sup>4</sup> model by Hagan, Kumar, Lesniewski and Woodward [5] (see also Hagan and Lesniewski [6] and Hagan, Lesniewski and Woodward [7]) involves the modeling of the **forward swap rate**  $S_{mn}(t)$ .

The forward swap rate in the SABR model is assumed to evolve according to the stochastic differential equation (SDE)

$$dS_{mn}(t) = V_{mn}(t) (S_{mn}(t))^{\beta_{mn}} dW_{mn}(t), \quad (1)$$

where the stochastic volatility is modeled by the driftless SDE

$$dV_{mn}(t) = \sigma_{mn} V_{mn}(t) dZ_{mn}(t). \quad (2)$$

The initial volatility is

$$V_{mn}(0) = v_{mn}^0. \quad (3)$$

The correlation between the Wiener processes  $W_{mn}(t)$  and  $Z_{mn}(t)$  is denoted by  $\rho_{mn}$ . The correlations between Wiener processes corresponding to different maturity-tenor pairs are not relevant for the completion of the cube. Therefore, we can consider  $S_{mn}(t)$  and  $V_{mn}(t)$  in isolation.

In order to complete the volatility cube, we need an expression for the implied swaption volatility  $\widehat{\sigma}_{mn}(K)$  in the SABR model. The remainder of this section is devoted to the derivation of an expression for  $\widehat{\sigma}_{mn}(K)$ . The first step in doing this is the derivation of an expression for the option price in the SABR model.

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<sup>3</sup>This model is also known as the LIBOR market model.

<sup>4</sup>SABR stands for **S**tochastic, **A**lpha, **B**eta and **R**ho (see Brigo and Mercurio [2]). Note that the value  $v_{mn}^0$  in (3) is frequently denoted by  $\alpha$ .

The second step is setting this option price equal to the option price in the Black model and thus obtaining the implied volatility. We only perform the first step and refer the reader to Hagan, Kumar, Lesniewski and Woodward [5] for the second step. The first step can again be divided into two parts. The first part is the derivation of a partial differential equation (PDE) for the call price. We do this in somewhat more detail than in [5]. However, the second part of the first step (solving the resulting PDE) is quite technical and not needed for the remainder of our analysis. Thus, we also refer the interested reader to Hagan, Kumar, Lesniewski and Woodward [5] for the solution of this PDE (8).

We now derive an expression for the price (at time  $t$ ) of a European call  $C(t, T, S, V)$  with maturity  $T$ .  $S$  is the forward rate at time  $t$  and  $V$  is the volatility at time  $t$ . This is a call on the forward swap rate. As mentioned in section 1, it is market practice to price a swaption with a formula similar to Black's formula (see Brigo and Mercurio [2]). Therefore, the price of a call on the forward swap rate corresponds to the price of a payer swaption. The derivation is based on perturbation theory and our approach is similar to that of Hagan, Kumar, Lesniewski and Woodward [5]. We omit the indices  $m$  and  $n$  to simplify notation and write the SABR SDEs as

$$dS_t = V_t S_t^\beta dW_t$$

and

$$dV_t = \sigma V_t dZ_t.$$

The correlation between the Wiener processes  $W_t$  and  $Z_t$  is denoted by  $\rho$ . We use the perturbation parameter  $\varepsilon$  and define

$$\tilde{\sigma} := \frac{\sigma}{\varepsilon}$$

and

$$\tilde{V}_t := \frac{V_t}{\varepsilon}.$$

The SDEs now become

$$dS_t = \varepsilon \tilde{V}_t S_t^\beta dW_t$$

and

$$d\tilde{V}_t = \varepsilon \tilde{\sigma} \tilde{V}_t dZ_t.$$

We rewrite these SDEs in terms of independent Wiener processes  $\widehat{W}_t$  and  $\widehat{Z}_t$  as

$$dS_t = \varepsilon \tilde{V}_t S_t^\beta d\widehat{W}_t \tag{4}$$

and

$$d\tilde{V}_t = \varepsilon \tilde{\sigma} \tilde{V}_t \left( \rho d\widehat{W}_t + \sqrt{1 - \rho^2} d\widehat{Z}_t \right). \tag{5}$$

Let the transition probability density function be  $\gamma$ , then

$$\gamma(u, w, x_1, x_2, y_1, y_2) dy_1 dy_2 =$$

$$P \left( y_1 < S_w < y_1 + dy_1, y_2 < \tilde{V}_w < y_2 + dy_2 \mid S_u = x_1, \tilde{V}_u = x_2 \right),$$

where  $u \leq w$ .  $\gamma$  satisfies the forward Kolmogorov equation (see Karatzas and Shreve [8], for example)

$$\begin{aligned} \frac{\partial}{\partial w} \gamma(u, w, x_1, x_2, y_1, y_2) &= \frac{1}{2} \frac{\partial^2}{\partial y_1^2} \left[ \varepsilon^2 y_1^{2\beta} y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right] + \\ &\quad \frac{1}{2} \frac{\partial^2}{\partial y_2^2} \left[ \varepsilon^2 \tilde{\sigma}^2 y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right] + \\ &\quad \frac{\partial^2}{\partial y_1 \partial y_2} \left[ \varepsilon^2 \tilde{\sigma} y_1^\beta y_2^2 \rho \gamma(u, w, x_1, x_2, y_1, y_2) \right] \\ &= \frac{1}{2} \varepsilon^2 y_2^2 \frac{\partial^2}{\partial y_1^2} \left[ y_1^{2\beta} \gamma(u, w, x_1, x_2, y_1, y_2) \right] + \\ &\quad \frac{1}{2} \varepsilon^2 \tilde{\sigma}^2 \frac{\partial^2}{\partial y_2^2} \left[ y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right] + \\ &\quad \varepsilon^2 \tilde{\sigma} \rho \frac{\partial^2}{\partial y_1 \partial y_2} \left[ y_1^\beta y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right], \end{aligned} \quad (6)$$

where we have used (4) and (5). We now turn to the call price, which is given by

$$\begin{aligned} C(t, T, S, \tilde{V}) &= \mathcal{E} \left( [S_T - K]^+ \mid S_t = S, \tilde{V}_t = \tilde{V} \right) \\ &= \int_{-\infty}^{\infty} \int_K^{\infty} (y_1 - K) \gamma(t, T, S, \tilde{V}, y_1, y_2) dy_1 dy_2 \\ &= [S - K]^+ + \\ &\quad \int_{-\infty}^{\infty} \int_K^{\infty} \int_t^T (y_1 - K) \frac{\partial}{\partial w} \gamma(t, w, S, \tilde{V}, y_1, y_2) dw dy_1 dy_2, \end{aligned}$$

where we have used the fact that<sup>5</sup>

$$\gamma(u, w, x_1, x_2, y_1, y_2) = \delta(y_1 - x_1) \delta(y_2 - x_2) + \int_u^w \frac{\partial}{\partial s} \gamma(u, s, x_1, x_2, y_1, y_2) ds.$$

However, since

$$0 = \int_{-\infty}^{\infty} \frac{\partial^2}{\partial y_2^2} \left[ y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right] dy_2$$

and

$$0 = \int_{-\infty}^{\infty} \frac{\partial^2}{\partial y_1 \partial y_2} \left[ y_1^\beta y_2^2 \gamma(u, w, x_1, x_2, y_1, y_2) \right] dy_2$$

(see Wu [11] or note that  $\gamma$  vanishes at infinity) we obtain

$$C(t, T, S, \tilde{V}) = [S - K]^+ + \frac{1}{2} \varepsilon^2 \int_{-\infty}^{\infty} \int_K^{\infty} \int_t^T (y_1 - K) y_2^2 \frac{\partial^2}{\partial y_1^2} \left[ y_1^{2\beta} \gamma(t, w, S, \tilde{V}, y_1, y_2) \right] dw dy_1 dy_2$$

by substituting (6) into the equation for the call price. Furthermore, integration by parts yields

$$y_2^2 K^{2\beta} \gamma(u, w, x_1, x_2, K, y_2) = \int_K^{\infty} y_2^2 (y_1 - K) \frac{\partial^2}{\partial y_1^2} \left[ y_1^{2\beta} \gamma(u, w, x_1, x_2, y_1, y_2) \right] dy_1.$$

Therefore, the call price is given by

$$C(t, T, S, \tilde{V}) = [S - K]^+ + \frac{1}{2} \varepsilon^2 K^{2\beta} \int_{-\infty}^{\infty} \int_t^T y_2^2 \gamma(t, w, S, \tilde{V}, K, y_2) dw dy_2,$$

where we have changed the order of integration. Let

$$\begin{aligned} \Gamma(u, w, x_1, x_2) &:= \int_{-\infty}^{\infty} y_2^2 \gamma(u, w, x_1, x_2, K, y_2) dy_2 \\ &= \mathcal{E} \left( \tilde{V}_w^2 \mid S_u = x_1, \dots \right), \end{aligned}$$

where  $u \leq w$ , then the call price can be expressed as

$$C(t, T, S, \tilde{V}) = [S - K]^+ + \frac{1}{2} \varepsilon^2 K^{2\beta} \int_t^T \Gamma(t, w, S, \tilde{V}) dw, \quad (7)$$

where we have again changed the order of integration.  $\Gamma$  satisfies the Feynman-Kac formula (see Karatzas and Shreve [8], for example)

$$\begin{aligned} -\frac{\partial}{\partial u} \Gamma(u, t, x_1, x_2) &= \frac{1}{2} \varepsilon^2 x_1^{2\beta} x_2^2 \frac{\partial^2}{\partial x_1^2} \Gamma(u, t, x_1, x_2) + \\ &\quad \frac{1}{2} \varepsilon^2 \tilde{\sigma}^2 x_2^2 \frac{\partial^2}{\partial x_2^2} \Gamma(u, t, x_1, x_2) + \\ &\quad \varepsilon^2 \tilde{\sigma} \rho x_1^\beta x_2^2 \frac{\partial^2}{\partial x_1 \partial x_2} \Gamma(u, t, x_1, x_2) \end{aligned} \quad (8)$$

with the boundary condition

$$\Gamma(t, t, x_1, x_2) = x_2^2 \delta(x_1 - K).$$

We omit the detail of solving this PDE since the solution steps are quite technical and do not contribute to understanding the model. See Hagan, Kumar, Lesniewski and Woodward [5] for the solution details. Substituting the solution of (8) into (7), one obtains

$$C(t, T, S, \tilde{V}) = [S - K]^+ + \frac{|S - K|}{4\sqrt{\pi}} \int_a^{\infty} q^{-\frac{3}{2}} e^{-q} dq,$$

where

$$a := -\ln \left( \frac{S^{1-\beta} - K^{1-\beta}}{(S-K)(1-\beta)} (KS)^{\frac{\beta}{2}} \right) - \ln b - \frac{1}{4} \varepsilon^2 \rho \tilde{V} \tilde{\sigma} c + \frac{d}{2T}.$$

The expressions for  $b$ ,  $c$  and  $d$  are quite involved and since they are not directly needed in the remainder of this report, we refer the reader to Hagan, Kumar, Lesniewski and Woodward [5].

Using the same technique to derive the option price in the Black setup, one can equate the call prices in the two models (SABR and Black) to obtain an approximate expression for  $\tilde{\sigma}$ . Re-introducing the indices  $m$  and  $n$  and stressing the strike-dependence  $K$ , one obtains the formula

$$\begin{aligned} \hat{\sigma}_{mn}(K) \approx v_{mn}^0 & \left[ 1 + \frac{(1-\beta)^2}{24} \left( \ln \frac{S_{mn}(0)}{K} \right)^2 + \frac{(1-\beta)^4}{1920} \left( \ln \frac{S_{mn}(0)}{K} \right)^4 \right]^{-1}. \\ & (S_{mn}(0)K)^{-\frac{1-\beta}{2}} \frac{z}{x(z)} \left\{ 1 + \left[ \frac{(1-\beta)^2 (v_{mn}^0)^2}{24 (S_{mn}(0)K)^{1-\beta}} + \right. \right. \\ & \left. \left. \frac{\rho_{mn} \beta_{mn} \sigma_{mn} v_{mn}^0}{4 (S_{mn}(0)K)^{\frac{1-\beta}{2}}} + \sigma_{mn}^2 \frac{2 - 3\rho_{mn}^2}{24} \right] T_m \right\}, \end{aligned} \quad (9)$$

where

$$z := \frac{\sigma_{mn}}{v_{mn}^0} (S_{mn}(0)K)^{\frac{1-\beta}{2}} \ln \frac{S_{mn}(0)}{K}$$

and

$$x(z) := \ln \frac{\sqrt{1 - 2\rho_{mn}z + z^2} + z - \rho_{mn}}{1 - \rho_{mn}}.$$

Note that only terms of order  $O(\varepsilon^2)$  have been included. Furthermore,  $\varepsilon$  was set to one to recover  $V_t$  and  $\sigma$  from  $\tilde{V}_t$  and  $\tilde{\sigma}$ . The formula above is the version found in Mercurio and Pallavicini [9].

Although (9) is an approximate formula, we shall ignore the approximate sign and consider the expression on the right-hand side of (9) as the implied volatility. In order to distinguish between the model implied volatility  $\hat{\sigma}_{mn}(K)$  and the market-quoted implied volatility, we shall denote the latter by  $\tilde{\sigma}_{mn}(K)$  (not to be confused with the perturbation volatility  $\tilde{\sigma}$  without indices).

### 3 Calibrating the Model and Completing the Cube

The aim of this project is to complete the volatility cube. As pointed out in section 1, the  $x$ -,  $y$ - and  $z$ -dimension of the volatility cube are option maturity,

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<sup>5</sup> $\delta$  denotes the Dirac delta.

tenor and strike respectively. Each point of the cube is the implied volatility for these three values, *ie*  $\widehat{\sigma}_{mn}(K)$  (from equation (9)) for  $T_m$ ,  $T_n - T_m$  and  $K$  in the case of the SABR model. As pointed out in section 1, one generally only has a complete  $\widehat{\sigma}_{mn}(K_{\text{ATM}})$ -plane for all typical values of  $T_m$  and  $T_n - T_m$ . One reason why one cannot simply interpolate between the given volatility values to find the missing ones, is the sparseness of the given cube. For many strike values  $K$  there are so few points  $\widehat{\sigma}_{mn}(K)$  known that it defeats the concept of interpolation. A second reason is the nonlinear structure. For fixed values of  $m$  and  $n$ , the mapping  $K \mapsto \widehat{\sigma}_{mn}(K)$  is anything but flat. It is rather parabolic. This is known as the volatility smile. One can also view it on a surface level. Each plane  $\widehat{\sigma}_{mn}(K)$  (where  $K$  remains fixed) can be mapped onto a volatility surface<sup>6</sup>. The  $x$ - and  $y$ -dimension of the surface are the option maturity and tenor respectively. The  $z$ -dimension is the value  $\widehat{\sigma}_{mn}(K)$ . The volatility surface is usually far from being flat. The modeling of the volatility surface is an active area of research and there are entire books (see Gatheral [4], for example) devoted to this topic. The volatility surface is basically a “landscape” with mountains and valleys. Therefore, interpolation will (in most cases) deliver bad results. Knowing only a few points of this landscape, it is impossible to deduce exactly where the mountains and valleys start and end or how high the mountains are and how low the valleys are. Consequently, interpolation makes no sense at all. Therefore, we have decided to calibrate an entire SABR model for each pair  $(m, n)$  corresponding to fixed values of  $T_m$  and  $T_n - T_m$  and all values of  $K$ . In the landscape analogy this means that we fix the  $x$ - and  $y$ -coordinates of a point and calibrate its “height” in all the landscapes simultaneously.

It is evident from the SABR model description (1), (2) and (3), that four parameters need to be calibrated. Hagan and Lesniewski [6] suggest fixing one of the parameters  $\rho_{mn}$  (or  $\beta_{mn}$ ) and calibrating the remaining three parameters  $\sigma_{mn}$ ,  $v_{mn}^0$  and  $\beta_{mn}$  (or  $\rho_{mn}$ ). We have experimented with this approach and obtained the best results when  $\beta_{nm}$  was fixed at 0.5.

The first step is to determine the pair  $(m, n)$  with the largest number of market-quoted volatilities. That is the swaption with option maturity  $T_m$  and tenor  $T_n - T_m$  for which  $\widehat{\sigma}_{mn}(K)$  is available for the largest number of different strikes  $K$ . Once this pair  $(m', n')$  has been found,  $\sigma_{m'n'}$ ,  $v_{m'n'}^0$  and  $\rho_{m'n'}$  are calibrated by means of the least-squares method<sup>7</sup>.

The second step is to calibrate the entire maturity-tenor grid, keeping the maturity fixed and calibrating for each tenor. For each maturity we start by calibrating the tenor with the largest number of quoted volatilities. The starting values of  $\sigma_{mn}$ ,  $v_{mn}^0$  and  $\rho_{mn}$  for each pair  $(m, n)$  are the values calibrated for the previous pair. The values  $\sigma_{m'n'}$ ,  $v_{m'n'}^0$  and  $\rho_{m'n'}$  are used for calibrating the very first pair.

A smoothness parameter is applied to the calibration of each pair  $(m, n)$  for which few quotes are available. These are typically the pairs for which we only have ATM market volatilities. The smoothness parameter prevents the calibrated parameters from moving too far away from the initial values.

We have also implemented a second approach which differs from the ap-

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<sup>6</sup>This volatility surface is not the same as the volatility surface in the equity world where the  $x$ - and  $y$ -dimension of the surface are the strike and maturity.

<sup>7</sup>This implies that the volatilities calculated using (9) are fitted to the market volatilities  $\widetilde{\sigma}_{mn}(K)$ .

proach above in the calibration of the pairs  $(m, n)$  for which several market quotes are available. The parameters for such a pair are calibrated several times. Thus, we calibrate the parameters and then calibrate again using the current values as the starting values.

## 4 Example

We now apply the procedure from section 3 to fictitious data resembling actual market data from September 2010. The payment frequency is quarterly, *i.e.*  $\Delta t = 0.25$ . Figure 1 is a plot the ATM (fictitious) market volatility surface. The  $z$ -dimension is the volatility in percentage. We have the complete ATM volatility surface, which is frequently the case.

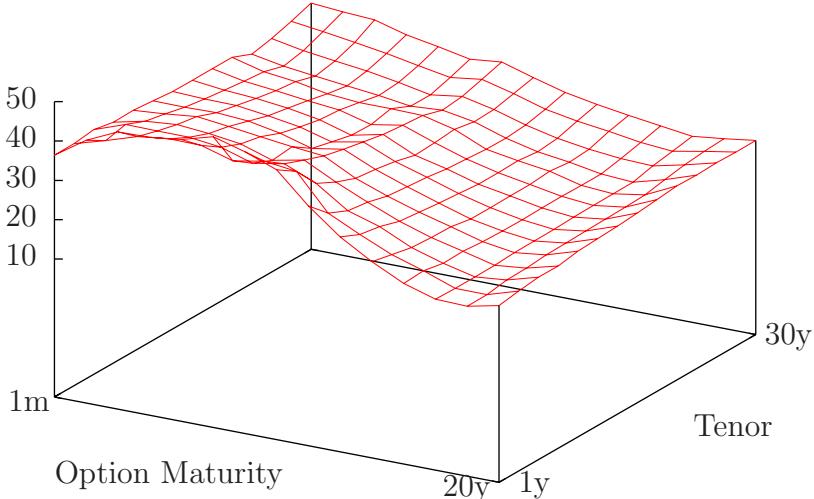


Figure 1: The Market ATM Volatility Surface

We calibrate the data using the second approach described in section 3. Figure 2 is a plot of the ATM volatility surface resulting from our cube completion. As mentioned in section 3, we fix a point  $(m, n)$  and calibrate the SABR model on all strikes simultaneously. Thus, after having considered every point  $(m, n)$  all volatility surfaces have been calibrated. Therefore, the ATM volatility surface (which we also have as input data) is a “by-product” of the calibration procedure. Since we know what it should look like (figure 1), the calibrated ATM surface is a good test of our calibration ansatz. Comparing figures 1 and 2, one observes that the two surfaces are almost identical. We can conclude that our calibration procedure can fit the ATM volatilities very accurately.

We now turn to one of the volatility surfaces for which we have very few

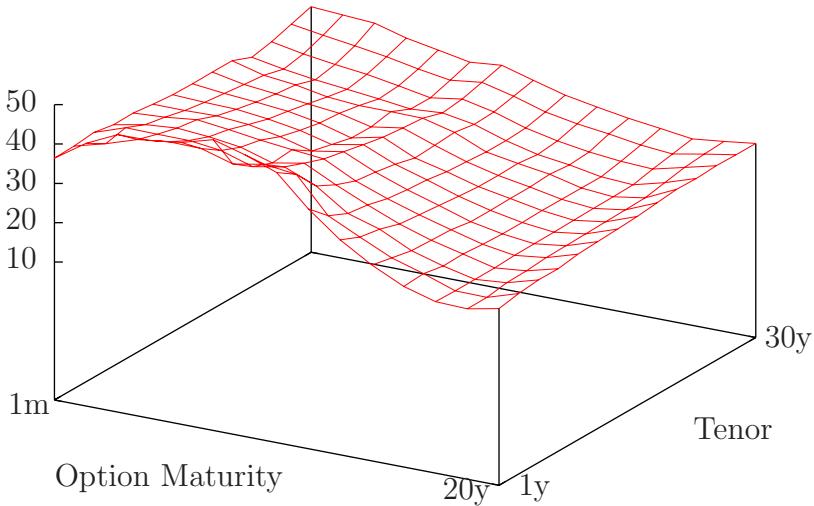


Figure 2: The Calibrated ATM Volatility Surface

points (volatility values). Figure 3 is a plot of the completed volatility surface (produced by our method) for a strike which is 25 basis points from the ATM strike. Although the shape is plausible, this surface is not as smooth as the ATM surface in figure 2. The reason for this is that our client wanted a completed volatility cube to use as the input to existing valuation software. The accuracy of the volatility values is more important than the smoothness of the surface in this case. A smoother surface be obtained by using stricter smoothing parameters.

## 5 Conclusion

The method described in this report has been implemented in MATLAB. A corresponding EXCEL sheet, which communicates with MATLAB via the EXLINK add-in, has also been created. An important trade-off in the implementation is between the smoothness of the resulting volatility surfaces and the accuracy of the calibration. Accuracy in this context refers to the extent to which the known volatilities are matched by the calibrated surfaces. We have mainly stressed the quality in the example of section 4. An interesting area of future research might be determining an optimal trade-off between smoothness and accuracy.

An important advantage of this approach is that the produced volatility surface is free of arbitrage for fixed tenor and maturity. This means that for each volatility curve  $(m, n)$ , the mapping  $K \mapsto \hat{\sigma}_{mn}(K)$  is arbitrage-free and therefore traders can safely quote prices along this curve. The input values we have used in the example were arbitrage-free. However, frequently market data

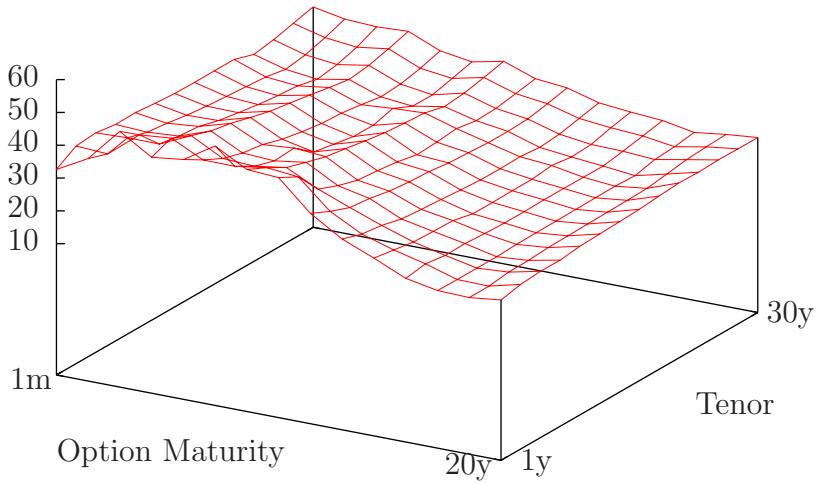


Figure 3: The Calibrated Volatility Surface (Strike = 25 bps)

contains arbitrage.

Another interesting area for future research is exploring arbitrage opportunities across maturities and tenors in order to implement restrictions on the model which result in a completely arbitrage-free cube.

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