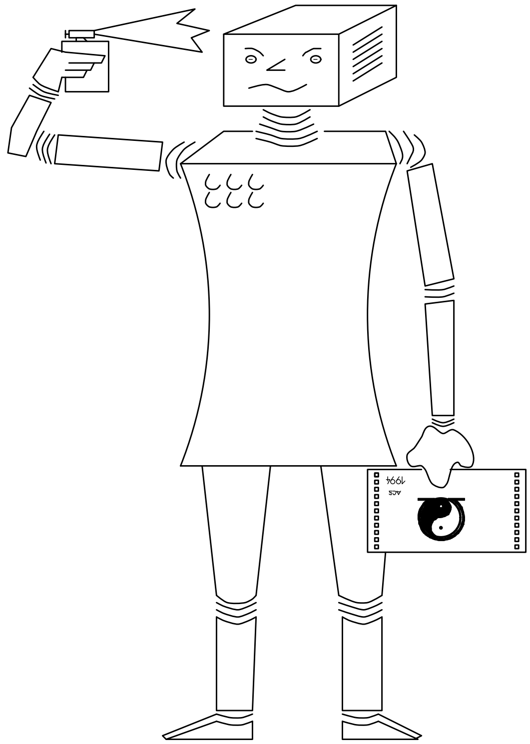

Using Exemplary Knowledge for Justified Analogical Reasoning

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Using Exemplary Knowledge for Justified Analogical Reasoning

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Begriffe ohne Anschauungen sind leer
Anschauungen ohne Begriffe blind.

Immanuel Kant

Abstract

Typical instances, that is, instances that are representative for a particular situation or concept, play an important rôle in human knowledge representation and reasoning, in particular in analogical reasoning. This well-known observation has been a motivation for investigations in cognitive psychology which provide a basis for our characterization of typical instances within concept structures and for a new inference rule for justified analogical reasoning with typical instances. In a nutshell this paper suggests to *augment* the propositional knowledge representation system by a non-propositional part consisting of *concept structures* which may have directly represented instances as elements. The traditional reasoning system is extended by a rule for justified analogical inference with typical instances using information extracted from both knowledge representation subsystems.

Keywords: analogical reasoning, typical instance, hybrid knowledge representation

1 Introduction

The traditional paradigm of knowledge representation and processing in artificial intelligence (AI), dating back at least to McCarthy's Advice Taker [16], is to represent knowledge by a collection of sentences in a formal language. The advantage of such a logic-based approach is the formal logical framework with its precise semantics. On the other hand, there are problems with this traditional approach: For instance, the logically formalized knowledge is not always available and to infer information is sometimes inefficient. Moreover, a purely logical approach is in sharp contrast to

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many forms of human reasoning which rely on cases, diagrams, or typical instances which are not necessarily propositionally represented.
The discussion about the potential advantages of a non-propositional representation has a long tradition in AI [23, 24] and has spawned analogical representations [7, 9, 10] and the development of hybrid knowledge representation systems which combine a propositional with a diagrammatical representation. Such hybrid approaches have been suggested among others by Myers and Konolige [21] and by Latecki and Pribbenow [15]. In a similar spirit some authors suggest more semantic oriented forms of reasoning such as model checking [13]. Of course, the very idea of propositionally and non-propositionally represented knowledge in reasoning has a much longer tradition dating back to Plato's rule-based theory of reasoning and education and to Aristotle's [27] reasoning by "exposition".
Just as well-known is, for example, the switch from proof theoretic arguments to model theoretic ones in formal logic: a theory can be given by a set of first-order formulae and inference rules, or semantically by a class of intended models [4]. By the soundness and completeness theorem it is possible to choose between syntactic inferences and model-checking methods depending on which information is *available* and which of these ways is more *efficient*[1].
Our approach adds to the discussion of hybrid knowledge representation and reasoning. It is to some extend based on the following psychological findings about human concept representation and reasoning:

- On the one hand, there is psychological evidence for an explicit rule application (see [25] for an overview). On the other hand, Cherniak [3] and Medin and Ross [17] found support for their thesis that people reason by using information directly extracted from instances. Corresponding results have been found for analogical reasoning. Gick and Holyoak [12] found schemata as a kind of explicit rules. Contrary, empirical evidence was found by Smith, Lopez, and Osherson [26] that there are no explicit rules underlying analogical reasoning. These conflicting results point to the existence of different modes of analogical reasoning [18].

- The typicality of instances of concepts is a phenomenon investigated in empirical psychology [14, 19, 22]. Reproducible typicality ratings that distinguish typical instances have been found. Some experimental methods [14] for the extraction of this typicality rating are the direct rating of representativity, the examination of the reaction time to decide whether an instance belongs to a category, the test of the reproduction of instances, and the use of instances in generalizations and in analogical reasoning. These results lead to the notation *structured concept* in the psychological literature that refers to a set of

[1] For example, the standard procedure for showing the consistency of a set of formulae semantically is to find a model rather than to show that no contradiction is inferable. On the other hand, the standard procedure for proving inconsistency of a set of formulae tries to infer a syntactic contradiction from this set rather than to show that there is no model for the formula set.

instances with a typicality relation, which we denote by $\sqsubseteq$. The typicality relation compares the degrees of typicality of instances of a concept[2]. For example, a hammer is commonly considered a more typical tool than a compass saw.

The psychological findings are a motivation and a legitimation for our representation of concepts. We consider concepts C to be represented by a set of instances with a partial order $\sqsubseteq_c$. The elements of these concept structures represent concept instances. A concept structure is displayed as a directed acyclic graph in figure 2. Some machine learning systems such as PROTOS and COBWEB [1, 8] also use representations of concepts with typical instances. They use another representation of concepts as clusters around typical instance[3].

In this paper, we propose an architecture for a hybrid system in which two kinds of knowledge can be processed: assertionally represented knowledge and exemplary represented knowledge. We show how to use this knowledge for analogical reasoning and discuss the advantages and drawbacks compared with well-known justified rule-based analogical reasoning. To this end we will first recall the basic notions of standard rule-based analogical reasoning and then introduce analogical reasoning based on typical instances, which make use of an exemplary representation. The latter reasoning mechanism can be seen as a special form of inheritance, which is, however, not inheritance between different concepts in the sense of KL-ONE, but inheritance within one concept. Furthermore this inheritance is based on semantically represented typicalities rather than on sets of sentences. We will show how the justification mechanism of rule-based analogical reasoning can be transferred to instance-based analogical reasoning.

2 Justified Rule-Based Analogical Reasoning

An important notion in analogical reasoning is the so-called aspect. An *aspect* A is a partial function mapping the individuals (instances c of a concept C) to non-tautological formulae with at most one free variable x such that $A\langle c\rangle[x/c]$ is $\texttt{true}$[4]. Informally spoken, if c is an instance and A an aspect then $A\langle c\rangle$ is a formula describing A of c. For example: The value of $\mathit{safety}\langle c\rangle$ of a car c might be $\mathit{airbag}(x) \wedge \mathit{antiblock}(x) \wedge (\mathit{max\_speed}(x) < 100)$. It is assumed that $\mathit{airbag}(c) \wedge \mathit{antiblock}(c) \wedge (\mathit{max\_speed}(c) < 100)$ is $\texttt{true}$. For a bicycle b, $\mathit{safety}\langle b\rangle$ might be $\mathit{frame\_diameter}(x, 3) \wedge (\mathit{age}(x) < 10)$.

The transfer of the value of an aspect A_2 from a source case s to a target case t based on the similarity of s and t with respect to another aspect A_1 is the *standard form of justified reasoning by analogy* investigated, for instance, in [6]. Justifications

[2] Here the notion "concept structure" denotes the structure of one single concept and not the relationship between different concepts like in KL-ONE.

[3] In order to define analogical reasoning based on such a kind of representation, the definition of relevancy in section 3.2 would only have to be adjusted.

[4] $A\langle c\rangle[x/c]$ denotes the formula $A\langle c\rangle$ in which the free variable x is substituted by c.

for such analogical inferences are usually represented explicitly and propositionally, as determinations [6], as schemata [12], as connections [18], or as similarity transforms [5].
In the following we will use the notion of a connection. A *connection* is a pair of aspects $[A_1, A_2]$. An example of such connections is $[population, cars]$ expressing "if two cities have the same number of inhabitants, then probably the same number of cars is registered in the cities". A corresponding analogical inference that yields a value of the aspect *cars* for the target instance *Rome*, presumes the similarity of *Rome* and another city, say *Madrid*, with respect to the number of inhabitants and the stated connection as inputs. It infers the correspondence of *Rome* and *Madrid* with respect to the aspect *cars*. Using the additional information of the actual value of $cars\langle Madrid\rangle$ the value of $cars\langle Rome\rangle$ can be inferred. We shall come back to this example later on.
But, what happens, if such an explicit connection is not available? Our account allows for justified analogical reasoning that is not necessarily based on explicit connections.[5] It employs a hybrid framework.

3 The Hybrid Framework

The framework consists of three parts: a hybrid knowledge base, a reasoner, and procedures which deliver information from the knowledge base to the reasoner. The knowledge base itself is partitioned into two parts: a collection of propositions and non-propositional representations of concepts. The reasoner consists of inference methods that operate on the propositional part of the knowledge base and of methods that use information contained in the conceptual part of the knowledge base. Figure 1 displays this framework.

3.1 The Hybrid Knowledge Base

The Propositional Subsystem

The propositional subsystem consists, as usual, of a set Γ of (sorted first-order) formulae. Aspects, as explained above, can be defined in this subsystem. The propositionally represented connections of aspects – as far as they are available – belong to this subsystem as well.

The Conceptual Subsystem

We extend the knowledge base by a *conceptual* part consisting of *concept structures* which are non-propositional representations of concepts. A concept structure is a directed acyclic graph consisting of a set of instances and the typicality relation $\sqsubseteq_C$.

[5]Although the field of analogical reasoning is concerned with reasoning based on examples, surprisingly the importance of reasoning by *typical* instances, as for example, investigated by Rosch [20] and Lakoff [14], has not been elaborated. Only an attempt of Winston [29] was influenced by statistic prototypicality.

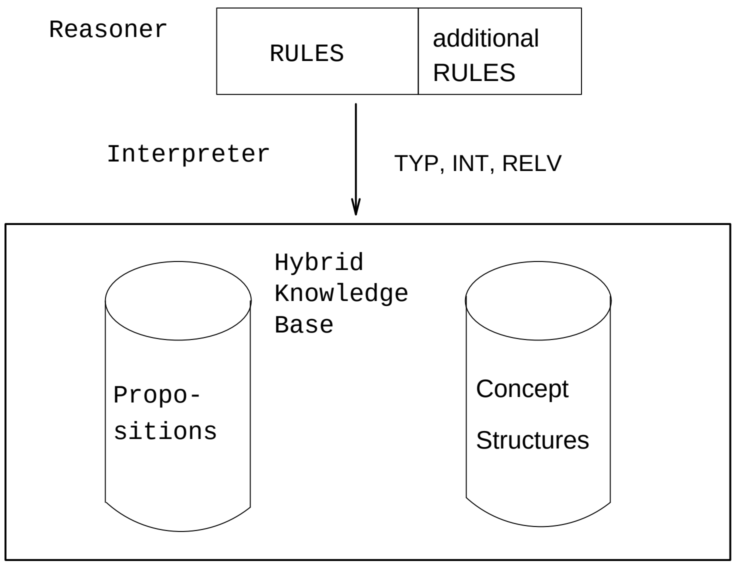

Figure 1: System structure

Within such concept structures we define typical instances:
An instance $c \in C$ is called a *typical instance* if it is maximal (that is, there is no $c' \neq c$ which $c \sqsubseteq_C c'$), written as typex(c). For example, a hammer is commonly considered a typical tool and a violin a typical musical instrument.
The instances themselves might be represented by neural nets, maps, diagrams or some other means including symbolic representations. The particular type of these representations is of no concern for the rest of the paper. Figure 2 shows a concept representation, where the instances are represented as maps. In this case, a procedure extracting information from the conceptual part of the knowledge base has to be a visual inspection. Actually we deal with another instance representation which is similar to that employed by Barwise and Etchemendy [2] and which encodes instances as tables as shown in figure 3. Here we do not investigate the procedure extracting information from instance representations. The conceptual part of the knowledge base is still non-propositional because of the concept structure.
A partial interpretation $\mathcal{I}$ of formulae potentially assigns to each pair (instance, formula with one variable) one of the values `true`,`false`,`unknown` and can be thought of as the result of an inspection of the instances.
The concept structure together with the instance interpretation $\mathcal{I}$ forms a Kripke-like model with a 3-valued assignment. This model connects two levels of representation, the instance level and the concept level. In this respect the model agrees with the models of Halpern and Vardi [13].

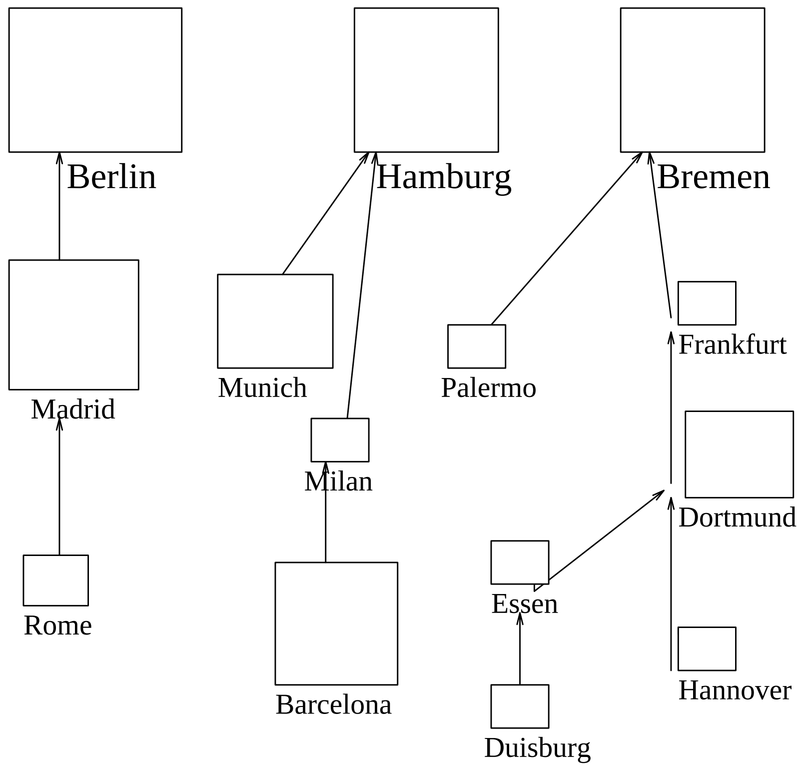

Figure 2: Concept structure with maps of cities*

3.2 Procedures on the Conceptual Subsystem

In order to integrate the conceptual subsystem into the framework, its informational content has to be accessible. Several inspection procedures work on the conceptual subsystem and provide the information that is needed by the rules of the reasoner:

- A TYP-procedure provides access to the structural content of the concept structures in that it finds a typical instance s with $t \sqsubseteq_C s$ for any instance t of a concept C. For example, TYP yields for the instance *Rome* the typical instance *Berlin* by looking up the concept structure *city*.[6]

- An interpreting *partial* procedure, called ASP, computes the values of aspects A for the *typical* instances c out of the representation of the instances. If there is a value it is required to be a formula $A\langle c\rangle$ with $\mathcal{I}(A\langle c\rangle) = \texttt{true}$. For example, ASP yields a value of the aspect *public_transportation* for the typical instance *Berlin*: $subway(x) \wedge bus(x) \wedge taxi(x) \wedge airport(x)$ by looking up the representation of *Berlin*.

[6]This example shows that typical instances are not independent of the culture background. For Italians in general, *Rome* would be the typical instance and not *Berlin*.

* In the original version of the paper you can find maps of the cities. These are unfortunately not available in the PostScript format.

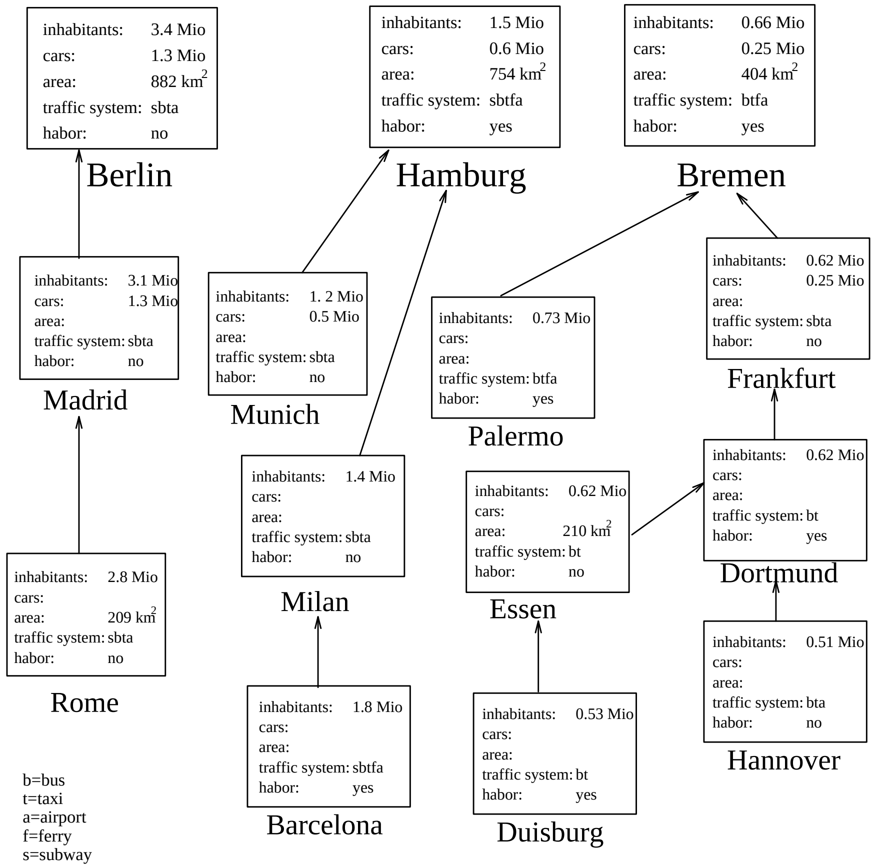

Figure 3: Concept structure with tables of cities

- A RELV-procedure provides `true`/`false`-information about the relevancy of aspects for the typical instances that is encoded in the concept representation. An aspect A is *relevant* for an instance c iff $\mathcal{I}(A\langle c\rangle[x/c]) = \texttt{true}$ and for all instances $c' \sqsubseteq_C c$ holds $\mathcal{I}(A\langle c\rangle[x/c']) \in \{\texttt{true}, \texttt{undefined}\}$. This notion of relevancy corresponds to a kind of modal operator. We assume that every typical instances has at least one relevant aspect.

3.3 Analogical Inference Rules of the Reasoner

The reasoner that works on the presented hybrid knowledge representation system has two rules, namely the classical rule **AR** for rule-based analogical reasoning and **AT** for analogical reasoning with typical instances. These rules use information extracted from the hybrid knowledge representation system by the above processes. **AR** and **AT** draw analogical inferences in a justified manner that are supposed to be similar to those observed in human reasoning [18]. **AR** is the standard rule of rule-based justified analogical reasoning described for instance in [6].

$$(\mathbf{AR}) \qquad \frac{A_1\langle t\rangle \Leftrightarrow A_1\langle s\rangle, [A_1, A_2]}{A_2\langle t\rangle := A_2\langle s\rangle}$$

AR which finally provides information about a target case t takes as input:

- The similarity of a source case s and the target case t expressed by the equivalence with respect to an aspect A_1. For example, let be $s = Madrid$, $t = Rome$, and $A_1 = population$. The input is $population\langle Rome\rangle \Leftrightarrow population\langle Madrid\rangle$.
- A connection $[A_1, A_2]$ which belongs to the propositional part of the knowledge base. Such a connection is $[population, cars]$ which means that, if the populations of two cities agree, then probably the number of cars registered in these cities agrees too.

AR yields $A_2\langle t\rangle := A_2\langle s\rangle$. For the example input above: $cars\langle Rome\rangle := cars\langle Madrid\rangle$. Using the additional information about the actual value of $A_2\langle s\rangle$, which is explicitly given by the propositional subsystem, we can infer the value of $A_2\langle t\rangle$.
At a glance: Let the following information be given

$[population, cars]$	
$population\langle Madrid\rangle =$ $approx\_no\_of\_inhabitants(x, 3million)$	$population\langle Rome\rangle =$ $approx\_no\_of\_inhabitants(x, 3million)$
$cars\langle Madrid\rangle =$ $approx\_no\_of\_cars(x, 1million)$	$cars\langle Rome\rangle =$?

AR infers by analogy $cars\langle Rome\rangle = cars\langle Madrid\rangle$.
Hence it is possible to infer $cars\langle Rome\rangle = approx\_no\_of\_cars(x, 1million)$.

Besides this standard form of analogical reasoning we have investigated analogical reasoning with typical instances that is carried out by **AT**.

$$(\mathbf{AT}) \qquad \frac{\text{typex}(s), t \sqsubseteq_C s, relevant(A_2, s)}{A_2\langle t\rangle := A_2\langle s\rangle}$$

The **AT** rule of the reasoner takes as inputs:

- a typical instance s with $t \sqsubseteq_C s$ which is computed by the TYP-procedure, and
- information about the relevancy of A_2 for this s. This information is either extracted by a RELV-procedure or explicitly represented in the propositional part of the knowledge base as, for instance, suggested by Gentner [11].

The **AT** rule then infers $A_2\langle t\rangle := A_2\langle s\rangle$. Using the additional information about the actual value of $A_2\langle s\rangle$, which is computed by ASP, $A_2\langle t\rangle$ can be inferred.

Let us look at an example where the individual concept structure of the concept *city* is given. We want to compute $A_2\langle t\rangle = public\_transportation\langle Rome\rangle$ by analogy to a typical city. If there is no explicit connection for *public_transportation*, then **AR** cannot be applied and we proceed as follows:

- The TYP-procedure computes the typical instance *Berlin* as a typical city that is rated over *Rome*.
- RELV tests whether *public_transportation* is a relevant aspect for *Berlin*. Provided that the result is `true`, **AT** yields $public\_transportation\langle Berlin\rangle = public\_transportation\langle Rome\rangle$.
- With the additional information $public\_transportation\langle Berlin\rangle = (subway(x) \wedge bus(x) \wedge taxi(x) \wedge airport(x))$ which is provided by the ASP-procedure or explicitly given in the propositional subsystem, we infer $public\_transportation\langle Rome\rangle = (subway(x) \wedge bus(x) \wedge taxi(x) \wedge airport(x))$ by analogy.

At a glance: let the following information be given

$relevant(public\_transportation, Berlin)$ typex($Berlin$) and $Rome \sqsubseteq_{\text{city}} Berlin$.	
$public\_transportation\langle Berlin\rangle =$ $(subway(x) \wedge bus(x) \wedge$ $taxi(x) \wedge airport(x))$	$public\_transportation\langle Rome\rangle =$?

AT infers by analogy $public\_transportation\langle Rome\rangle := public\_transportation\langle Berlin\rangle$. Hence it is possible to infer $public\_transportation\langle Rome\rangle := (subway(x) \wedge bus(x) \wedge taxi(x) \wedge airport(x))$.
The **AT** rule models common human analogical reasoning with typical instances. Of course, one cannot expect a rule for analogical reasoning to be sound in the logical sense, as this kind of reasoning is tentative in principle and produces hypotheses only.

Relationship between AR and AT

The justification of **AR** by a connection has been shown, for instance, in [6]. **AT** can be heuristically justified in terms of this known justification of **AR**: Let s and t be instances, that is, elements of a concept C, and let A_1 and A_2 be aspects. Assume:

(i) typex(s), $t \sqsubseteq_C s$, $relevant(A_2, s)$.

Since s is a typical instance, from $t \sqsubseteq_C s$ follows heuristically [28] that there is an aspect A_1 with $relevant(A_1, s)$ and $A_1(t) \Leftrightarrow A_1(s)$. Hence, we have

(ii) $A_1(t) \Leftrightarrow A_1(s), relevant(A_1, t), relevant(A_2, t)$, typex($t$).

Because of the definition of "relevant", $relevant(A_1, s)$, $relevant(A_2, s)$, and typex(s) support the connection $[A_1, A_2]$, at least for all instances rated under s. Hence, from (ii) follows

(iii) $A_1\langle t\rangle \Leftrightarrow A_1\langle s\rangle, [A_1, A_2]$.

Because of (iii) and the inference rule **AR** we have

(iv) $A_2\langle t\rangle := A_2\langle s\rangle$.

That is, we have the permission to define A_2 for t by $A_2\langle s\rangle$. Thus if **AR** is justified, then heuristically **AT** is too. In this way we succeeded to give a heuristic justification for **AT**.

4 Conclusion

In this paper, we suggested to *augment* a propositional knowledge representation system by a non-propositional part consisting of *concept structures* which may have directly represented instances as elements. The reasoning system, containing the traditional analogical reasoning rule **AR**, is extended by an inference rule using information extracted from both knowledge representation subsystems.

The reasoner can employ two different rules for justified analogical reasoning, the traditional connection-based **AR** and the instance-based **AT**, which yields a kind of inheritance within a concept and which employs a semantic representation of instances. We have shown how the rule **AR** and the rule **AT** work on a hybrid knowledge base. **AR** implies that *all* instances of the concept have to be checked for similarity to the target instance and the ASP-procedure is called for *all* sufficiently similar instances. If typical instances can be singled out, **AT** is substantially less complex than **AR**. In addition, the extension of the reasoner by **AT** enables the system to cope with justified analogical reasoning in the situation of missing explicit connections. The advantages do not come for free, however, because the system structure is more complex. Both rules are tentative and can be seen as special instances of default reasoning: in the case of **AR** the defaults are given by propositionally represented connections, in the case of **AT** by the typical instances.

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References

[1] E.R. Bareiss, B.W. Porter, and C.C. Wier, Protos: An exemplar-based learning apprentice, *International Journal of Man-Machine Studies*, 1988, 29:549–561.

[2] J. Barwise and J. Etchemendy, *Hyperproof*, CSLI Lecture Notes, University of Chicago press, 1993.

[3] C. Cherniak, Prototypicality and deductive reasoning, *Journal of Verbal Learning and Verbal Behavior*, 1984, 23:625–642.

[4] C.C. Chang and H.J. Keisler, *Model Theory*, North Holland, 1973.

[5] A.M. Collins and R. Michalski, The logic of plausible reasoning: A core theory, *Cognitive Science*, 1989, 13(1):1–50.

[6] T.R. Davis and S.J. Russell, A logical approach to reasoning by analogy, in *Proceedings of the 10th IJCAI*, Milan, Italy, 1987, pp. 264–270.

[7] C.M. Eastman, Automated space planning, *Artificial Intelligence*, 1973, 4:41–64.

[8] D.H. Fisher, Conceptual clustering, learning from examples, and inference, in *Proceedings of the 4th International Machine Learning Workshop*, 1987, pp. 38–49.

[9] B.V. Funt, Whisper: A problem solving system utilizing diagrams and aparallel processing retina, in *Proceedings of the 5th IJCAI*, 1977, pp. 459–464.

[10] H. Gelernter, Realization of a geometry theorem-proving machine, in *Proceedings of the International Conference on Information Processing, UNESCO*, 1959.

[11] D. Gentner, The mechanisms of analogical learning, in *Similarity and Analogical Reasoning*, S. Vosniadou and A. Ortony (eds.), 1989, Cambridge University Press, pp. 199–241.

[12] M.L. Gick and K.J. Holyoak, Schema induction and analogical transfer. *Cognitive Psychology*, 1983, 15(1):1–38.

[13] J.Y. Halpern and M.Y. Vardi, Model checking vs. theorem proving, in *Proceedings of the Second International Conference on Knowledge Representation*, Cambridge MA, 1991, pp. 325–334.

[14] G. Lakoff, *Women, Fire and Dangerous Things*, The University of Chicago Press, 1987.

[15] L. Latecki and S. Pribbenow, On hybrid reasoning for spatial expressions, *Proceedings of the 10th European Conference on Artificial Intelligence*, 1992, Vienna, pp. 289–393, 1992.

[16] J. McCarthy, Programs with common sense, in *Semantic Information Processing*, M. Minsky (ed.), MIT Press, 1968, pp. 403–418.

[17] D.L. Medin and B.H. Ross, The specific character of abstract thought: Categorization, problem solving, and induction, in *Advances in the Psychology of human intelligence*, volume 5, R.J. Sternberg (ed.), Erlbaum, 1989.

[18] E. Melis, Study of modes of analogical reasoning, TASSO-report 5, Gesellschaft für Mathematik und Datenverarbeitung, Birlinghoven, Germany, 1990.

[19] C. Mervis and E. Rosch, Categorization of Natural Objects, *Annual Review of Psychology*, 1981, 32:89–115.

[20] C.B. Mervis and E. Rosch, Family resemblance: Studies in the internal structure of categories, *Cognitive Psychology*, 1975, 7:573–605.

[21] K.L. Myers and K. Konolige, Reasoning with analogical representations, in *Proceedings of the Third International Conference on Knowledge Representation*, 1992, pp. 189–200.

[22] L. Rips, Inductive judgements about natural categories, *Journal of Verbal Learning and Verbal Behavior*, 1975, 14:665–681.

[23] A. Sloman, Interactions between philosophy and artificial intelligence: The role of intuition and non-logical reasoning in intelligence, *Artificial Intelligence*, 1971, 2:209–225.

[24] A. Sloman, Afterthoughts on analogical representation, in *Proceedings of Theoretical Issues in Natural Language Processing*, 1975.

[25] E.E. Smith, C. Langston, and R.E. Nisbett, The case for rules in reasoning, *Cognitive Science*, 1992, 16(1):1–40.

[26] E.E. Smith, A. Lopez, and D.N. Osherson, Category membership, similarity, and native induction, in *Essays in honor of W.K. Estes*, A. Healy, R. Shiffrin, and S.M. Kosslyn (eds.) Erlbaum, 1992.

[27] M. Wallies, ed., *Alexandri in Aristotelis Analyticorum Priorum Librum I Commentarium*, 1883.

[28] E.J. Weiner, A knowledge representation approach to understanding metaphors, *Computational Linguistics*, 1984, 10(1):1–14.

[29] P. Winston, Learning by creating and justifying transfer frames, *Artificial Intelligence*, 1978, 10(2):147–172.