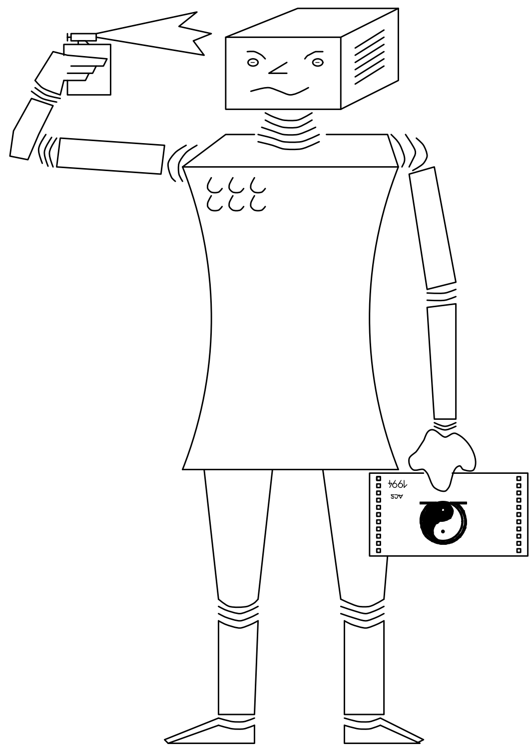

Analogical Reasoning with a Hybrid Knowledge Base

Manfred Kerber   Erica Melis
Jörg H. Siekmann

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Analogical Reasoning with a Hybrid Knowledge Base

Manfred Kerber* Erica Melis† Jörg H. Siekmann‡

Universität des Saarlandes

Im Stadtwald

66041 Saarbrücken

Germany

1 Introduction

The traditional paradigm of knowledge representation and processing in artificial intelligence (AI), dating back at least to McCarthy's Advice Taker [17], is to represent knowledge by a collection of sentences in a formal language. The advantage of such a logic-based approach is the formal logical framework with its precise semantics. On the other hand, there are problems with this traditional approach: For instance, the logically formalized knowledge is not always available and to infer information is sometimes inefficient. Moreover, a purely logical approach is in sharp contrast to many forms of human reasoning which rely on cases, diagrams or typical examples which are not necessarily propositionally represented.

The discussion about the potential advantages of a non-propositional representation has a long tradition in AI [24, 25] and has spawned analogical representations [7, 9, 10] and the development of hybrid knowledge representation systems which combine propositional with a diagrammatical representation. Such hybrid approaches have been suggested among others by Myers and Konolige [22] and by Latecki and Pribbenow [16]. In a similar spirit some authors suggest more semantic oriented forms of reasoning such as model checking [13]. Of course, the very idea of propositionally and non-propositionally represented knowledge in reasoning has a much longer tradition dating back to Plato's rule-based theory of reasoning and education and to Aristotle's [28] reasoning by "exposition".

Just as well-known is for example the switch from proof theoretic arguments to model theoretic ones in formal logic: a theory can be given by a set of first order

*Fachbereich Informatik, e-mail: `kerber@cs.uni-sb.de`

†Fachbereich Informatik, e-mail: `melis@cs.uni-sb.de`

‡DFKI, German Research Center for Artificial Intelligence, e-mail: `siekmann@dfki.uni-sb.de`

formulae and inference rules, or semantically by a class of intended models [4]. By the completeness theorem it is possible to choose between syntactic inferences and model-checking methods depending on which information is *available* and which of these ways is more *efficient*[1].

In this paper, we address the questions of the availability of certain forms of knowledge and of the efficiency of inferences by suggesting an approach to justified analogical reasoning. Starting from a logically oriented view of standard analogical reasoning, we review some empirical results on human concept representation in order to show how to use hybridly represented knowledge for mechanical analogical reasoning as well.

The essence of our approach is to *augment* the propositional knowledge representation system by a non-propositional part consisting of *concept structures* which may have directly represented instances as elements. The necessary information for analogical reasoning is then extracted from either part of the knowledge representation system. In other words, the general idea is to incorporate the particular non-propositional part of a hybrid knowledge representation system into a traditional reasoning system such that it can be extended by inference rules using information extracted from both knowledge representation subsystems. Thus the general framework and foundational importance of logic will remain unquestioned.

Justified Analogical Reasoning

An important concept in analogical reasoning is the so-called *aspect*, which we define as follows:

An aspect is a mapping from the individuals (instances c of a concept C) to formulae with at most one free variable. That is, the value $A(c)$, of the *aspect* A for an individual c, is a formula with the free variable x the c-instance of which describes the aspect A for c. In particular, $A(c)[x/c]$ is true.[2].

For example: The value of the aspect *safety* of an instance c of the concept car might be $airbag(x) \wedge antiblock(x) \wedge max\_speed(x) < 100$ where $airbag(c) \wedge antiblock(c) \wedge max\_speed(c) < 100$ is $\texttt{true}$. For an instance b of the concept bicycle the value of this aspect might be $frame\_diameter(x, 3) \wedge age(x) < 10$.

The transfer of the value of an aspect A_2 from a source case s to a target case t based on the similarity of s and t with respect to another aspect A_1 is the *standard form of reasoning by analogy*. We agree with Davies and Russell [6] that justifications are necessary for such analogical inferences. They are given by *connections* between the aspects A_1 and A_2. These justifications of analogical inferences have usually been represented propositionally e.g., as determinations [6], schemata [12], connections

[1]For example, the standard procedure for showing the consistency of a set of formulae semantically is to find a model rather than to show that no contradiction is inferable. On the other hand, the standard procedure for proving inconsistency of a set of formulae is to try to infer a syntactic contradiction from this set rather than to show that there is no model for the formula set.

[2]$A(c)[x/c]$ denotes the formula $A(c)$ in which x is substituted by c.

[19], or similarity transforms [5].

Examples of such connections are "if two cars are built in the same year and have the same make, then their price will probably be the same", and "if two cities have the same number of inhabitants, then probably the same number of cars is registered in the cities". A corresponding analogical inference that yields the value of the aspect *number_of_cars* for the target *Rome*, e.g., presumes the similarity of *Rome* and another city, say *Madrid*, with respect to the number of inhabitants and the stated connection as inputs. It infers the correspondence of *Rome* and *Madrid* with respect to the aspect *number_of_cars*. Using the additional information of the actual value of this aspect for *Madrid* the value of *number_of_cars(Rome)* can then be inferred. We shall come back to this example later on.

But, what happens, if such explicit connections are not available? And, what about the complexity of reasoning that is required to get the similarity information out of a set of propositions? [3] Our approach allows for justified analogical reasoning that is not necessarily based on propositionally represented connections.

2 Psychological Findings

The approach of this paper is to some extend supported by psychological findings about human concept representation and reasoning by instances, the main results are summarized in the following.

Explicit Rules vs. Instances

Do people reason by applying explicit rules to sententially represented knowledge or do they reason by referring to instances? This problem has been widely investigated by cognitive psychologists, and it seems that psychological experiments have provided evidence that support both modes of reasoning.

On the one hand, some psychological findings point toward an explicit rule application of facts (see [26] for an overview). For example, Braine, Reiser, and Rumain [2] have shown that the more rules are required in order to determine the validity of an argument, the longer the reaction time and the lower the accuracy of the final response to questions.

On the other hand, Cherniak [3] and Medin and Ross [18] found support for their thesis that people reason by using information directly extracted from instances. Their experiments clearly exhibited the retrieval of instances and they were subsequently used for analogical reasoning.

[3] Although the field of analogical reasoning is concerned with reasoning based on examples, surprisingly the importance of reasoning by *typical* instances, as for example, investigated by Rosch [21] and Lakoff [15], has not been elaborated. Only an attempt of Winston [29] was influenced by statistic prototypicality.

However, even for analogical reasoning, where one might expect that it is based on information from instances only, Gick and Holyoak [12] found schemata as a kind of explicit rules. Contrary, empirical evidence was found by Smith, Lopez, and Osherson [27] that there are no explicit rules underlying analogical reasoning.

These conflicting results point to the existence of different modes of analogical reasoning [19]. Whatever the the final outcome may be, one mode of analogical reasoning *based on typical instances* can be distinguished.

Structured Concepts

The typicality of instances of concepts is a well investigated phenomenon in empirical psychology [15, 20, 23], where the existence of a reproducible typicality rating that distinguishes typical instances is one of the main results.

Some of the important experimental methods [15] for the extraction of this typicality rating are the direct rating of representativity, the examination of the reaction time to decide whether an instance belongs to a category, the test of the reproduction of instances, and the use of instances in generalizations and in analogical reasoning.

All these investigations suggest that concepts are structured by typicality relations in a human mind. These relations (which we denote by $\subset_c$) encode the typicality degrees of instances of a concept C: If $a \subset_c b$, then $degree\_of\_typicality(a) < degree\_of\_typicality(b)$[4]. In addition the typicality relations encode some similarity of the instances.[5].

The psychological findings are a motivation and a legitimation for our representation of concepts. We consider concepts C to be represented by models which are partially ordered by the relation $\subset_c$. The elements of these concept structures represent concept instances. In the following we assume the models to be given[6]. A concept structure is displayed as a directed acyclic graph in the next section.

3 The Hybrid Framework

The framework consists of three parts: a hybrid knowledge base (KB), a reasoner, and procedures which connect the KB with the reasoner. The KB itself is partitioned into two part, a collection of propositional representations of knowledge and some non-propositional representations of concepts. The reasoner consists of inference methods combined with their control that operate on the propositional part of the KB and of methods that use information contained in the conceptual part of the KB. In addition, there are procedures which supply information extracted from the conceptual subsystem of the KB to the reasoner.

[4]For example, a hammer is commonly considered a more typical tool than a compass saw

[5]If the typicality relations encode degrees of typicality only the analogical inference rule AT given below has to be changed slightly

[6]Such concept representations are used in some learning systems such as PROTOS and COBWEB [1, 8].

3.1 The Hybrid Knowledge Base

The Propositional Subsystem

The propositional subsystem consists as usual of a set of (sorted first order) formulae Γ. *Aspects*, as explained above, can be defined in this subsystem. The propositionally represented connections of aspects – as far as they are available – belong to this subsystem as well.

The Conceptual Subsystem

We extend the KB by a *conceptual* part consisting of *concept structures* which are non-propositional representations of (human) concepts. A concept structure is a model consisting of a set of instances and the typicality relation $\subseteq_C$. Note that the structure may vary between individuals. The information about the similarity of instances is explicitly given in the concept structure.

The maximal elements, w.r.t. the typicality relation, that are not isolated are called *typical instances*[7].

We assume the instances are for example represented by neural nets, analogous repesentations such as diagrams or some other means including symbolic representations. The particular way of these non-propositional representations is of no concern for the rest of the paper, however, in the following example that demonstrates our main idea, we view the representation of *city*-instances as maps (see figure 1).

[7]For example, a hammer is commonly considered a typical tool and a violin a typical musical instrument.

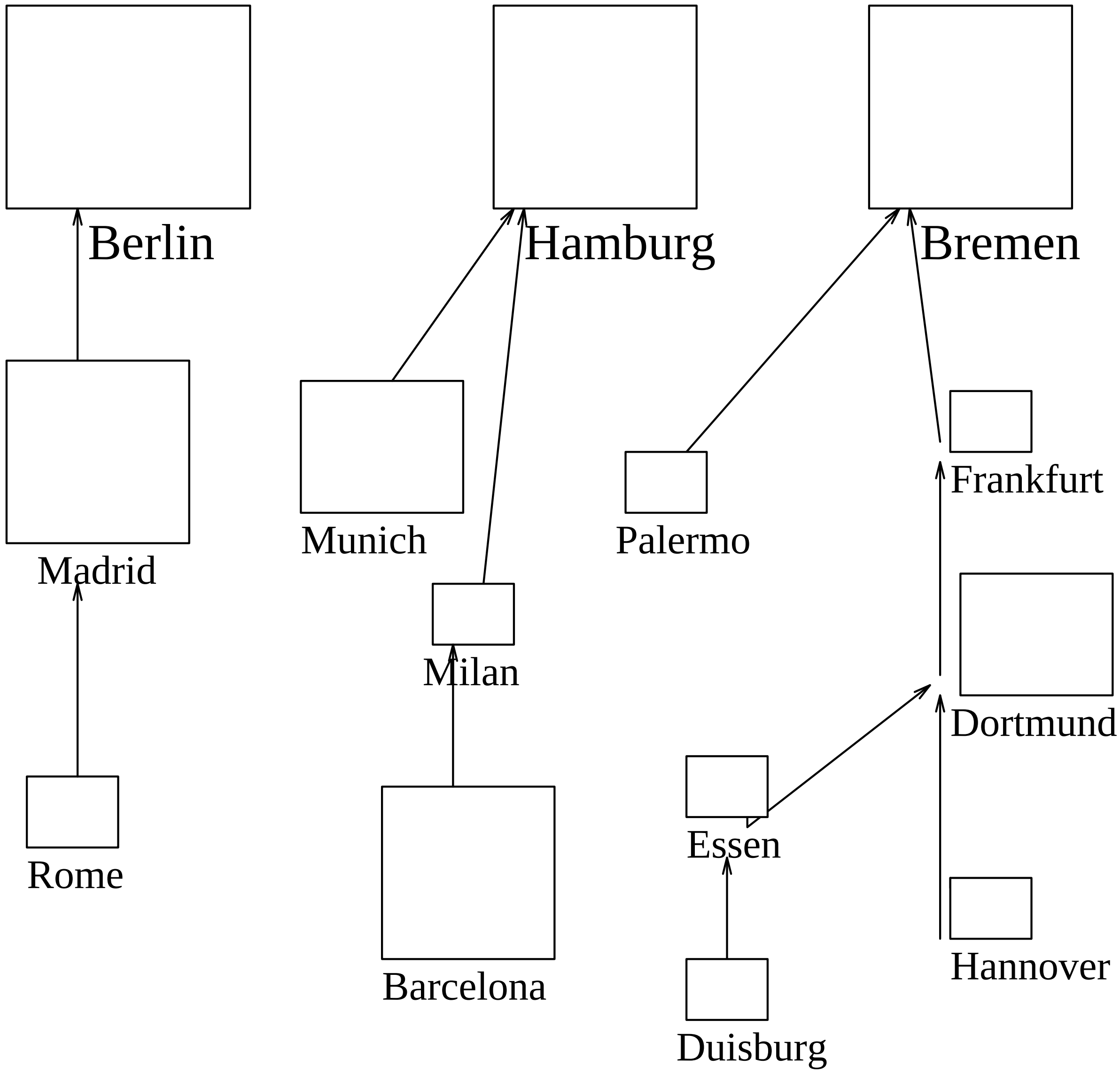

Figure 1: Concept structure of CITY

A partial interpretation $\mathcal{I}$ of formulae is attached to the instance representation, which potentially assigns to each pair (instance, formula with one variable) one of the values `true`,`false`,`unknown` and can be thought of as the result of an inspection of the instances.

The concept structure together with the instance interpretations $\mathcal{I}$ form a Kripke-like model with a 3-valued assignment. It connects two levels of representation, the instance level and the concept level. In this respect the model agrees with the models of Halpern and Vardi [13]. As we shall see later a suggestion of these authors concerning the complexity of model checking is fulfilled by our particular models.

3.2 Procedures on the Conceptual Subsystem

We wish to have access to the information in the concept structures and in the instance representation on an as needed basis. Therefore several inspection procedures work on the conceptual subsystem and they provide the information that is needed by the rules of the reasoner:

- A TYP-procedure provides access to the structural content of the concept structures in that it finds for an instance t of a concept C a typical instance s with $t \subseteq_C s$ (s and t are names of instances). For example, TYP yields for the instance *Rome* the typical instance *Berlin* by looking up the concept *city*.

- An interpreting INT-procedure computes the values of aspects A for the *typical* instances c out of the representation of the instances. The values are required to be formulae $A(c)$ with $\mathcal{I}\ (A(c)) = \texttt{true}$. INT can also be used to transfer certain information from the conceptual part to the propositional part of the KB. INT yields, for example, for the aspect *public_transportation* of the typical instance *Berlin*: $underground(x) \wedge bus(x) \wedge taxi(x)$ by looking up the map of Berlin.

- A RELV-procedure provides true/false-information about the relevancy of aspects for the typical instances that are encoded in the concept representation. An aspect A is relevant for an instance c iff $A(c)$ is true for c under $\mathcal{I}$ and for all instances i, rated under $c, A(i)$ is true or undefined. This notion of relevancy corresponds to a kind of modal operator, if a 3-valued assignment is presumed.

3.3 Analogical Inference Rules of the Reasoner

A reasoner that works on the hybrid knowledge representation system has at least two rules, namely **AR** for rule-based analogical reasoning and **AT** for analogical reasoning with typical instances. These rules use information extracted from the hybrid knowledge representation system by the above processes. **AR** and **AT** draw analogical inferences in a justified manner that are supposed to be similar to those observed in human reasoning [19].

The rule **AR** takes as input

- the similarity of a source case s and the target case t expressed via equality of an aspect A_1. For example, let the source s be *Madrid* and let the target t be *Rome*. Now suppose we want to infer the number_of_registered_cars in *Rome* . Assume that $A_1(Madrid) = A_1(Rome)$, e.g., both have the value $number\_of\_inhabitants(x, 3million)$. This information can be obtained from the propositional part of the KB.

- a rule that expresses the connection between the aspect A_1 and the aspect A_2, both of which belong to the propositional part of the KB. For example, if the number of inhabitants of two cities agree, then probably the number of cars registered in these cities agrees too.

AR yields the equality $A_2(Madrid) = A_2(Rome)$. Using the additional information of the actual value of $A_2(Madrid)$ which is explicitly given by the propositional subsystem as $number\_of\_cars(x, 1million)$ we can deductively infer the value of $A_2(Rome)$.

At a glance: Let the following information be given

connection between A_1 and A_2	
$A_1(Madrid) =$ $number\_of\_inhabitants(x, 3million)$	$A_1(Rome) =$ $number\_of\_inhabitants(x, 3million)$
$A_2(Madrid) =$ $number\_of\_cars(x, 1million)$	$A_2(Rome)=$?

AR infers by analogy $A_2(Madrid) = A_2(Rome)$.

Hence it is possible to infer $A_2(Rome) = number\_of\_cars(x, 1million)$

Besides this standard form of analogical reasoning we have investigated analogical reasoning with typical examples that is carried out by **AT**.

The **AT** rule of the reasoner takes as inputs:

- a typical instance s with $t \subseteq s$ computed by the TYP-procedure,
- the relevancy of A_2 for s which is either extracted by a RELV-procedure or else it is explicitly represented in the propositional part of the KB as, e.g., suggested by Gentner [11].

The **AT** rule then infers $A_2(t) = A_2(s)$ from $relevant(A_2, s)$. The value of $A_2(s)$ is computed by INT and transferred to the target case t.

Let us look at the example where the individual concept structure of the concept *city* is given. We want to compute $A_2(t) = public\_transportation(Rome)$ by analogy, but now based on the concept of a typical city. As long as there is no explicit connection for *public_transportation* **AR** cannot be used. So we proceed as follows:

- The TYP-procedure computes the typical instance *Berlin* as a typical city is rated over *Rome*, since it is in the concept structure of *city*.
- RELV tests whether *public_transportation* is a relevant aspect for *Berlin*. Provided that the result is `true`, **AT** yields $A_2(Berlin) = A_2(Rome)$.

- With the additional information $A_2(Berlin) = (underground(x) \wedge bus(x) \wedge taxi(x))$, which is provided by the INT-procedure or else is explicitly given in the propositional subsystem, we infer $A_2(Rome) = (underground(x) \wedge bus(x) \wedge taxi(x))$ by analogy.

At a glance: let the following information be given

relevant(A_2,*Berlin*) *typex*(*Berlin*) and *Rome*$\subseteq$*Berlin*.	
$A_2(Berlin) =$ $(underground(x) \wedge\ bus(x) \wedge\ taxi(x))$	$A_2(Rome) =$?

AT infers by analogy $A_2(Berlin) = A_2(Rome)$.

Hence it is possible to infer $A_2(Rome)$=$(underground(x) \wedge\ bus(x) \wedge\ taxi(x))$.

The **AT** rule models common human analogical reasoning with typical instances and, as shown in [14], **AT** is *justified provided* **AR** *is justified* in the sense of Russell [6]. The derivation in [14] is based on reformulating conditions of analogical inferences and supported by empirical results. It concludes that if **AR** is justified then **AT** is (less) justified depending on the concept structure. Therefore the rule **AT** is heuristically sound. Anyway, one cannot expect a rule for analogical reasoning to be sound in the logical sense, as this kind of reasoning is tentative in principle and produces hypotheses only.

4 Conclusion

We have shown how the rule **AR** for *standard analogical reasoning* and the rule **AT** for *analogical reasoning with typical instances* work on a hybrid knowledge base.

AR implies that *all* instances of the concept have to be checked for similarity to the target instance and the INT-procedure is called for *all* sufficiently similar instances. By restricting the choice of the base instance to *typical* instances and consequently the calls of the INT-procedure to *typical* instances, **AT** leads to a kind of default reasoning and is substantially less complex than **AR**. The focusing of **AT** on particular instances can be considered as one of the ways to do model checking *efficiently* as mentioned by Halpern and Vardi [13].

Our framework is a bit of both, hybrid knowledge representation as in [22] and model checking as in [13]. [22] deals with diagrams combined with inspection procedures on a non-propositional representation. As diagrams, concept structures are to some extend easier to change than propositional representations: new instances can be built in the concept structure and do not cause problems like the frame problem. In [13] Kripke-like models are constructed to decouple the description of a model from the description of the properties to be proven about the model. The efficiency and usage of the different representations depend on the *available* information and the aim of the reasoning just as in logic.

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