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Vorwort

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In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe werden sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Residual Demand Modeling and Application to Electricity Pricing

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9th March 2012

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Abstract

Worldwide the installed capacity of renewable technologies for electricity production is rising tremendously. The German market is particularly progressive and its regulatory rules imply that production from renewables is decoupled from market prices and electricity demand. Conventional generation technologies are to cover the residual demand (defined as total demand minus production from renewables) but set the price at the exchange. Existing electricity price models do not account for the new risks introduced by the volatile production of renewables and their effects on the conventional demand curve. A model for residual demand is proposed, which is used as an extension of supply/demand electricity price models to account for renewable infeed in the market. Infeed from wind and solar (photovoltaics) is modeled explicitly and withdrawn from total demand. The methodology separates the impact of weather and capacity. Efficiency is transformed on the real line using the logit-transformation and modeled as a stochastic process. Installed capacity is assumed a deterministic function of time. In a case study the residual demand model is applied to the German day-ahead market using a supply/demand model with a deterministic supply-side representation. Price trajectories are simulated and the results are compared to market future and option prices. The trajectories show typical features seen in market prices in recent years and the model is able to closely reproduce the structure and magnitude of market prices. Using the simulated prices it is found that renewable infeed increases the volatility of forward prices in times of low demand, but can reduce volatility in peak hours. Prices for different scenarios of installed wind and solar capacity are compared and the merit-order effect of increased wind and solar capacity is calculated. It is found that wind has a stronger overall effect than solar, but both are even in peak hours.

Keywords: residual demand modeling, renewable infeed, wind infeed, solar infeed, electricity demand, German power market, merit-order effect

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1 Introduction

Modeling electricity demand is important for every utility taking part in the electricity sector, either as producer, retailer or trader on the financial markets for electricity. In some applications one is interested in an exact load *forecast* on different time scales and spatial resolutions. The short-term load forecast models are mainly based on weather forecasts, whereas long-term models include growth of the economy, progression in energy-saving technologies, population growth, etc. On the other hand financial modeling of electricity prices (for derivative pricing, risk-management or investment planning) requires that the *distribution* of the demand model is close to reality. In this paper we are concerned with the role of demand modeling in electricity price models, which implies that we look at demand prevailing at the whole market in question. Electricity price models, which use demand as a state variable are known as *supply/demand models* (or hybrid models or equilibrium models). In this class of models, the supply side (electricity production) and the demand side (electricity consumption) are described separately. The market price is determined by the marginal production unit in the merit order needed to match demand. All models in the literature use demand as a state variable, which is the only one in the early work by Barlow (2002). More recent approaches further include capacity (cf. Cartea & Villaplana 2008), fuel prices (cf. Pirrong & Jermakyan 2008, Coulon & Howison 2009, Smeers & de Maere 2010, Carmona, Coulon & Schwarz 2011)[1], or both (cf. Aïd, Campi, Huu & Touzi 2009, Aïd, Campi & Langrené 2012) as state variables. Burger, Klar, Müller & Schindlmayr (2004) also have a load dependent component in their model.

In recent years there has been a rapid growth in installed capacity of renewable energy sources (hydro, wind, solar[2], biomass, geothermal), which is expected to continue for decades. Half of the worldwide newly added capacity in 2010 has been of renewable technologies (including hydro). Excluding hydro, the globally installed renewable capacity grew 25% over 2009[3]. In the European Union, renewables account for more than 40% of yearly capacity additions since 2005[4]. Due to strong political support, especially Germany already has a considerable high share of wind and solar power plants in its electricity system. By the end of 2011, Germany had 28 GW installed capacity of wind power plants and 20 GW installed capacity of solar power plants, which together accounts for about 30% of total installed capacity[5]. In 2010, 17% of German electricity consumption has been produced by renewable sources[6], going up from 16.3% the year before despite a rise in demand of 4.3%[7]. Scenarios[8] for the year 2022 see renewable installed capacity in Germany between 93 GW and 150 GW, which is (much) more than the German yearly peak demand (in 2010 at about 83 GW). In all those scenarios, conventional installed capacity is predicted to decrease to between 82 GW and 92 GW.

This considerable amount of renewables sources in the system and the regulatory circumstances (see below) heavily influence the electricity price for Germany, which is traded at the EEX and EPEX. For example, the infeed from solar power plants changes the structure of the conventional demand profile significantly on a sunny day. This implies a change in the (intra-day) seasonality of market prices for electricity, as the seasonality in prices is mainly caused by the seasonality in demand.

In figure 1 the hourly production stack and the corresponding market prices are displayed. Supply is split in production from wind, production from solar and residual load. The prices follow the profile of residual load, which is covered by technologies other than wind and solar, i.e. mainly

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[1] Pirrong & Jermakyan (2008) and Smeers & de Maere (2010) use a reflected load process in order to capture capacity constraints.

[2] In this paper we use the term *solar* for electricity production from solar energy. This technique is known as *photovoltaics*.

[3] Renewable Energy Policy Network for the 21st Century (2011)

[4] Figures available until 2010, Renewable Energy Policy Network for the 21st Century (2011).

[5] Own calculations based on data from European Energy Exchange AG (2012).

[6] Bundesministerium für Umwelt Naturschutz und Reaktorsicherheit (2011)

[7] Renewable Energy Policy Network for the 21st Century (2011)

[8] 50hertz, Amprion, EnBW Transportnetze AG & TenneT TSO (2011)

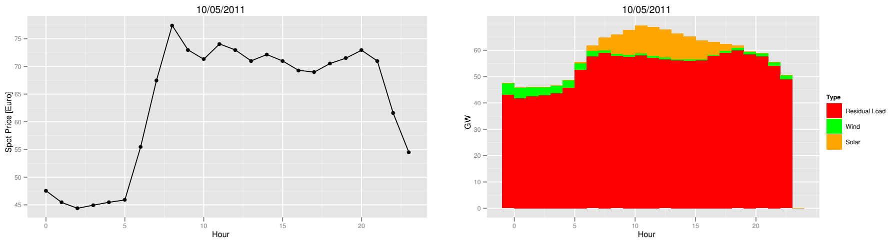

Figure 1: EPEX spot market prices and production profiles for Germany on a day with high renewable infeed.

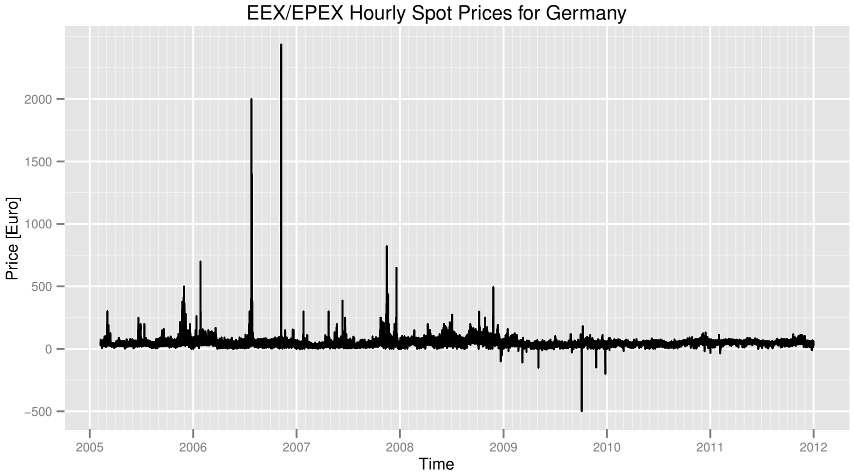

Figure 2: EEX/EPEX hourly spot prices from the year 2005 through 2011.

conventional generation units like nuclear, lignite, coal, and gas. We conclude that the intra-day price shape heavily depends on the amount of renewable infeed. This is not accounted for in existing demand side models in the literature.

The effect of renewable infeed is also seen in time-series of hourly spot prices. Figure 2 shows the hourly spot price at the EEX (since 2009 EPEX) from 2005 to 2011. There are less positive spikes since 2009 and they also reduced in magnitude. On the other hand, negative prices started to occur and especially in 2009 negative spikes are much more present than positive ones. In 2010 and 2011, the size of negative spikes reduced, indicating that producers are learning and trying to avoid negative prices (e.g. by assembling a more flexible power plant portfolio). The time series for 2011 in figure 3 reveals that there are still some moderate spikes in the market, both positive and negative. The typical situation in case of negative spikes is a very high renewable infeed (usually for a few hours only, caused by fluctuating wind infeed) on days with generally low demand (public holiday, weekend). The electricity oversupply during those hours is caused by baseload plants, which cannot be economically switched off for a few hours only (or are considered must-run plants for system stability). They are prepared to accept negative prices for a short time period in order to be able to continue with their production. Positive spikes occur usually in times of high conventional demand (so very expensive plants in the merit order have to be called to meet generation needs). As Germany has enough conventional capacity to meet peak demand even in times of zero renewable infeed (Bundesnetzagentur 2011*b*) the conventional capacity is near its limit only during times of low renewable infeed and high demand. An example is a cold winter day with low wind infeed and many snow covered solar panels. Further causes of positive spikes

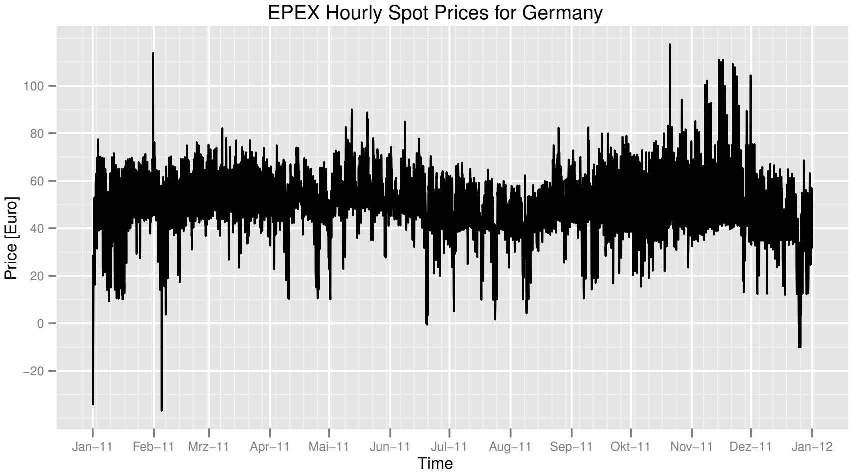

Figure 3: EPEX hourly spot prices for the year 2011.

are unexpected outages. In such a situation, the non-available plant has to meet its obligations by buying additional electricity from the exchange, which obviously drives the market price. If the non-available plant is part of a larger portfolio of plants, the owner is often able to balance the missing production within her portfolio, so the price peaks for a few hours only. The increasing share of solar infeed is reducing the risk for positive spikes. Solar produces mainly during hours of peak demand, so it reduces the conventional peakload. This is the so-called *peak-shaving effect* of solar, which is also visible in figure 1. A third characteristic of spot-prices seen in the figures is the seasonality on different time-scales. There is intra-day seasonality (figure 1), weekly seasonality (figure 3) and a yearly seasonality (figure 3 and 2).

Production from renewables introduces new uncertainty (=risk) to the market due to their volatile production profile. Therefore in order to apply supply/demand models on the German power market (or any other market with a high share of renewables in its system), the infeed from wind and solar power plants should be considered, especially as the seasonality in electricity prices is mainly generated by the seasonality in demand. In this work we propose an approach to explicitly include wind and solar infeed in a demand side model, which can be used to refine existing supply/demand models for electricity. Our work is based on the German power market, which is, as outlined above, very progressive in the integration of renewable power production.

The regulatory circumstances in Germany are based on the EEG[9]. It implies that all production from renewables must be fed into the electricity grid. Conventional generation units (nuclear, lignite, hard coal, gas, oil, pumped storage) are to cover the remaining demand only. As an incentive to investors, a guaranteed tariff for the produced electricity is paid for 20 years after the installation of the plant (*feed-in tariff*). We will model wind and solar only, as they have a particularly uncertain and fluctuating infeed profile (unlike hydro or biomass). As most of the wind and solar power plants have been build in recent years and therefore are eligible for the feed-in tariff and infeed priority, it is natural to consider the production from renewables as a demand reduction, i.e.

$$\text{residual demand} = \text{(stochastic) total demand} - \text{(stochastic) infeed from renewables.} \qquad (1)$$

Residual (or conventional) demand is the electricity to be generated by conventional technologies. This is the demand which should be used in the merit order of conventional generation units to determine the market price (assuming a competitive market).

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[9] EEG is short for *Erneuerbare-Energien-Gesetz* (Renewable Energy Sources Act), see `http://www.erneuerbare-energien.de` for more information such as laws, statistics, etc.

A common approach in demand modeling in the literature (i.e. the supply/demand models cited above) is to choose a deterministic seasonal component plus a stochastic process modeling random deviations from the seasonal level, i.e.

$$\text{total demand} = \text{deterministic seasonal demand} + \text{stochastic process}. \tag{2}$$

Deviations occur mainly due to weather conditions, i.e. an unusual cold spell in spring will cause a rise in demand for a few days due to electric heating. As those deviations are temporarily only the stochastic process is chosen to be mean-reverting to level zero. Any trend or seasonality is contained in the deterministic component. As an example we introduce a model for total system load with the desired properties using an Ornstein-Uhlenbeck process, which is used in Aïd et al. (2009), Coulon & Howison (2009), and Lyle & Elliott (2009). We measure time continuously in years and denote it by $t \in [0, T]$, where $T > 0$ is some finite time horizon. All processes are defined on a probability space $(\Omega, \mathbb{P}, \mathcal{F})$ supporting Brownian motion with the filtration $\{\mathcal{F}_t\} = \{\mathcal{F}_t\}_{t\in[0,T]}$ generated by all the Brownian motions W_t used in this paper.

Model 1.1 (Model for total system load)**.** *Denote total system load at time $t \in [0, T]$ by L_t and assume*

$$L_t = \psi_t + l_t,$$

where

ψ_t *is a time-dependent deterministic load forecast, and*

l_t *is an Ornstein-Uhlenbeck process, i.e.*

$$dl_t = -\theta^{load} l_t \; dt + \sigma^{load} \; dW_t^{load}, \quad l_0^{load} = l_0. \tag{3}$$

Model 1.1 is formulated rather general. It can also be applied to log-system load[10] to ensure that total system load is always positive (Coulon & Howison 2009, Smeers & de Maere 2010). However, as the size of the seasonal component usually overweighs the stochastic fluctuations by far, the probability of negative values is negligible. An extension with time-dependent parameters is possible (e.g. seasonal volatility as in Cartea & Villaplana 2008).

The intra-day load pattern is very strong and possible deviations from the seasonal mean are usually present all day. Therefore in applications model 1.1 is evaluated once for each day only and the daily load curve is derived deterministically from this evaluation. This approach is taken in Smeers & de Maere (2010) and also in section 6 of this work. Aïd et al. (2009) and Coulon & Howison (2009) apply their models only to a single hour of the day and therefore there is no need for intra-day demand in their case studies.

Combining equ. (1) and (2) we can write the approach in this paper as follows:

$$\begin{aligned} \text{residual demand} = {} & \text{deterministic seasonal demand} \\ & + \text{stochastic process} \\ & - \text{stochastic infeed from renewables.} \end{aligned}$$

When applied to markets with renewables, existing demand models as expressed in equ. (2) base the calibration of demand on total demand minus infeed from renewables. Such a model does, in contrast to our approach, not distinguish between the risk from demand shocks and the uncertainty in renewable infeed.

The remainder of this paper is structured as follows. In section 2 we introduce our approach separating infeed from installed capacity. We apply this to wind and solar power infeed. The model for wind power infeed is introduced in section 3, the model for solar power infeed in section 4. We combine both to a model for residual demand in section 5, which we use in a case study on the German market in section 6. Concluding remarks are in section 7.

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[10] When modeling log-load, the OU-process in equ. (3) usually has a non-zero mean-reversion level, i.e. it reads $dl_t = \theta^{load}(\mu^{load} - l_t) \; dt + \sigma^{load} \; dW_t^{load}$.

2 General modeling approach

As outlined in the introduction, installed capacity of renewables has seen very strong growth in recent years, which is expected to continue in the future. This causes a trend in renewable infeed which has to be accounted for. We model the infeed from renewables independent of the installed capacity by using *efficiency* (also known as *load factor*) rather than absolute infeed. With this approach we remove the trend in infeed data caused by changes in installed capacity.

In the following, we use the placeholder src for the renewable energy source, i.e. wind or solar.

Definition 2.1 (Efficiency)**.** We define the *efficiency* E_t^{src} of the energy source src at time t by

$$E_t^{src} = \frac{AI_t^{src}}{IC_t^{src}},$$

where

$IC_t^{src} > 0$ denotes installed capacity, and
$AI_t^{src} \in [0, IC_t^{src}]$ denotes absolute infeed

of the energy source src at time t. By construction we have $E_t^{src} \in [0,1]$.

In order to extend the range of possible models for E_t we propose to map efficiency on the whole real line. There are different functions available, i.e. the inverse cdf of any distribution with support on the whole real line can be used. In our work, we use the *logit*-transformation, as we found that it transforms the data approximately to a normal distribution.

Definition 2.2 (Logit-transformation)**.** The *logit transformation*

$$logit : (0,1) \to \mathbb{R}$$

of a variable $x \in (0,1)$ is defined as (Fahrmeir & Tutz 2001)

$$logit(x) = \log\left(\frac{x}{1-x}\right).$$

Corollary 2.3 (Inverse of logit-transformation)**.** *The logit-transformation is strictly monotonic increasing, continuously differentiable, and the inverse logit-transformation is*

$$x = logit^{-1}(y) = \frac{e^y}{1+e^y} = \frac{1}{1+e^{-y}}, \tag{4}$$

where $y = logit(x)$.

Due to the open interval of allowed values in the transformation, we must assume that we have neither zero efficiency (i.e. no infeed at all) nor full efficiency (i.e. all installed plants are fully utilized).

Assumption 2.4 (Neither zero nor full efficiency)**.** For each energy source src and every (relevant[11]) time $t \in [0,T]$ we assume the efficiency $E_t^{src} \in (0,1)$, i.e. we have neither zero ($E_t^{src} = 0$) nor full ($E_t^{src} = 1$) efficiency.

In later sections we find that this assumption is always fulfilled in our dataset. In the following we look at a random variable, whose logit-transformation is normally distributed.

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[11] For solar efficiency we will require only the daily maximum to fulfill the assumption.

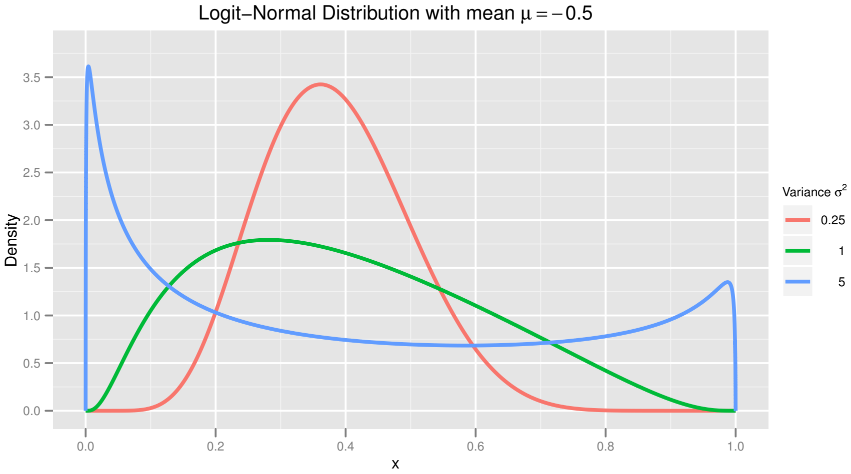

Figure 4: The logit-normal density with mean $\mu = -0.5$ and variance $\sigma^2 = 0.25, 1, 5$.

Definition 2.5 (Logit-normal distribution)**.** Assume a random variable $X \in (0,1)$ and denote its logit-transformation

$$Y := logit(X).$$

Suppose Y is normally distributed with expectation μ and standard deviation σ, i.e.

$$Y \sim N\left(\mu, \sigma^2\right).$$

We call X *logitnormally distributed with mean* μ *and variance* σ^2 and write $X \sim logitN\left(\mu, \sigma^2\right)$.

The logit-normal distribution has not found much attention in the literature. It is discussed in an early work by Johnson (1949) and there is recent work of Frederic & Lad (2008). The latter give some characteristics of the logitnormal distribution, compare it to the family of beta-distributions, and analyze the bivariate logitnormal distribution. Moreover, a generalized logitnormal-distribution using the transformation $logit\left(\frac{x}{A}\right)^{\theta}$ is analyzed in Mead (1965). Johnson (1949) shows some properties of the logitnormal distribution, in particular it is shown that the density f_X of a logit-normally distributed random variable X is unimodal if

$$\sigma > \sqrt{2}. \tag{5}$$

In figure 4 we plot the logitnormal distribution for different parameter sets. Frederic & Lad (2008) note that in the unimodal case, the logitnormal distribution looks similar to a beta-distribution on $(0,1)$. In the bimodal case, however, the beta distribution is unbounded at the boundaries, whereas the logit-normal density is not. For a discussion of the bivariate logit-normal distribution see Frederic & Lad (2008).

We do not expect our distribution to be bimodal, as this reflects either very high infeed or rather low infeed. This is a realistic scenario in neither wind nor solar power infeed. Therefore we use equ. (5) to check calibration results.

Remark 2.6 (Parameter estimation for a logitnormally distributed random variable). Given a set of logitnormally distributed observations $A = \{x_1, x_2, \ldots, x_n\}$ the parameters of the logitnormal distribution are estimated in a two-step procedure:

(1) Apply the logit-transformation to the observations, i.e. calculate $\bar{A} = \{logit(x_i), i = 1, 2, \ldots, n\}$.

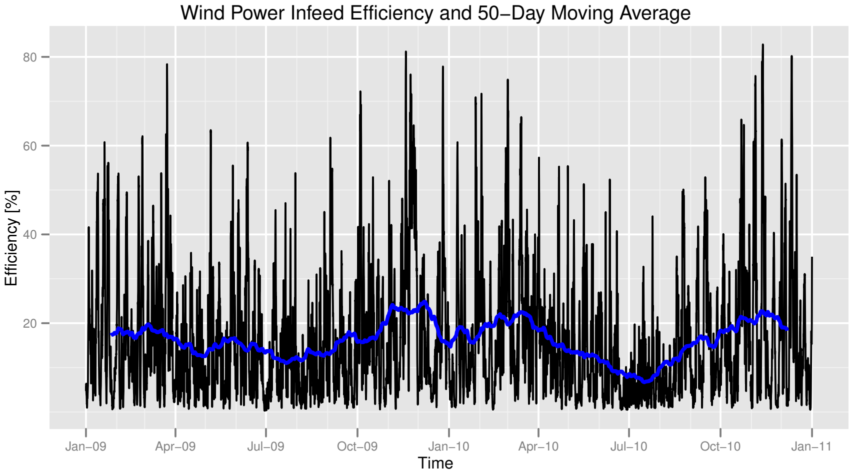

Figure 5: Hourly data of wind power infeed efficiency for Germany in 2009 and 2010, and a centered 50-day moving average.

(2) Use standard methods for the fitting of the normal distribution to $\bar{A}$ like the method of moments or maximum likelihood estimation to obtain the estimated parameters $\hat{\mu}, \hat{\sigma}^2$.

We use the following notation in the next sections. Under assumption 2.4 we denote the logit-transformed efficiency

$$\tilde{E}_t^{src} = logit(E_t^{src}),$$

where $\tilde{E}_t^{src} \in \mathbb{R}$. It splits into a deterministic seasonality $\eta_t^{src} \in \mathbb{R}$ and a (random) deseasonalized efficiency $\bar{E}_t^{src} \in \mathbb{R}$, i.e.

$$\tilde{E}_t^{src} = \eta_t^{scr} + \bar{E}_t^{src} \; \forall \; t \in [0, T]. \tag{6}$$

We assume a yearly seasonality η_t^{scr}, so it must satisfy the condition

$$\eta_t^{scr} = \eta_{t-1}^{scr} \quad \forall \; t \geq 1, \tag{7}$$

in particular this means that η_t^{scr} contains no trend.

3 Model for wind power infeed

In the following we aim for a model of wind power efficiency. There already is literature on wind speed models and a good overview on the various distributions proposed is in Carta, Ramirez & Velazquez (2009). Their results indicate that the Weibull distribution is the preferred choice in wind speed modeling. There is also literature on the generation of wind speed time series, i.e. Aksoy (2004) with an overview of the different techniques proposed.

Wind is transformed to power in an WECS (wind energy conversion system). The power generated depends on the current wind speed and the power curve of the WECS. As illustrated in Carta et al. (2009), the power curve is a non-linear function of wind speed and air density[12]. Therefore we cannot conclude that wind infeed also follows a Weibull distribution.

To our knowledge, the wind power infeed of a large system (like a whole country) has not yet been described in the literature. We propose a model for this in the following and motivate our

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[12] The dependence of the power curve on air density is usually neglected in the literature and very often not even published by the manufacturer (Carta, Ramirez & Velazquez 2008).

Statistic	No Transformation	Logit	Deseasonalized Logit
mean	0.164	-2.008	0.000
median	0.121	-1.980	0.031
standard deviation	0.143	1.144	1.123
skewness	1.512	-0.045	-0.106
kurtosis	5.394	2.854	2.828
minimum	0.003	-5.770	-3.627
maximum	0.828	1.572	3.366

Table 1: Statistical figures for the wind power efficiency data without transformation, with a logit transformation, and after deseasonalization.

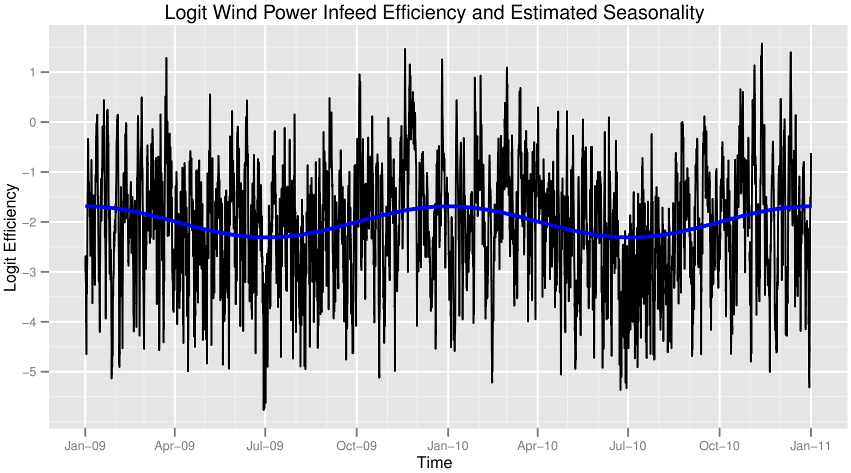

Figure 6: Logit wind power infeed efficiency for Germany in 2009 and 2010, and the seasonality estimate (blue).

choice with data on actual wind power infeed in Germany from 01/01/2009 to 31/07/2011[13]. Instead of modeling wind speeds and converting it using a power curve we model wind power infeed directly.

In figure 5 we plot the data in terms of efficiency, i.e. the figure shows total infeed divided by installed capacity[14]. The figure includes a 50-day moving average, which motivates a seasonality with lower infeed in summer compared to the rest of the year. Moreover, the graph shows strong mean reversion and spikes, i.e. upward jumps followed by a downward jump shortly after. In table 1 we give some statistical figures of the data. In particular it can be seen that assumption 2.4 is fulfilled.

Figure 6 shows wind power infeed efficiency data after a logit-transformation. To account for seasonality we fit a function of the form

$$\eta_t^{wind} = a\cos(2\pi t + b) + c \tag{8}$$

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[13] Data for 2009 has been taken from the four TSOs' websites (`http://www.tennettso.de`, `http://www.50hertz-transmission.net`, `http://www.amprion.net`, and `http://www.enbw.com`) in October 2010. Data for 2010 and 2011 is provided by European Energy Exchange AG (2012). This data is also generated by the TSOs, but compiled into one file for each hour by the EEX.

[14] For 2009, we assume an installed capacity of 25.777 GW (Bundesministerium für Umwelt Naturschutz und Reaktorsicherheit 2010), for 2010 and 2011 we have 25.961 GW and 27.547 GW, respectively (European Energy Exchange AG 2012).

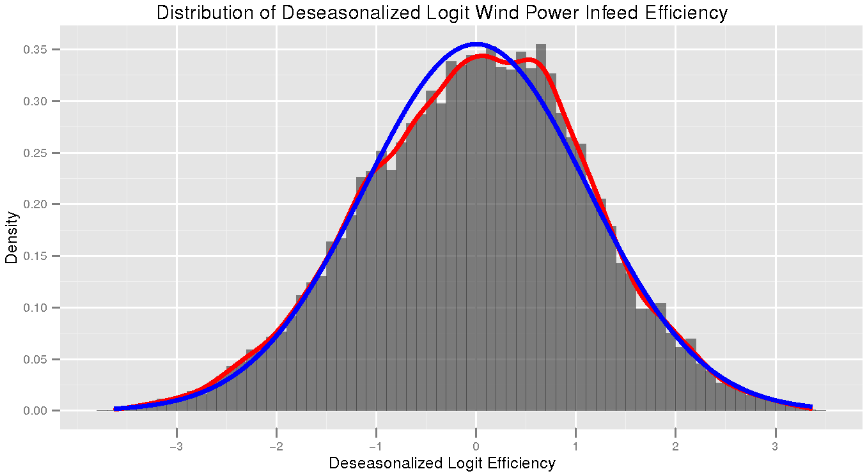

Figure 7: Histogram, empirical distribution (red line), and a fitted normal distribution (blue line) of deseasonalized logit wind power infeed efficiency.

to the data. This function fulfills the condition in equ. (7). The estimated seasonality is also plotted in figure 6.

In the following we argue for a model describing deseasonalized efficiency $\bar{E}_t^{wind}$. The histogram of the deseasonalized data in figure 7 shows that the empirical distribution of the transformed data is very close to normal. This is also supported by the statistical figures in table 1, which show a skewness close to zero and a kurtosis close to three for the deseasonalized data. Mean and median are close, which supports assuming a symmetric distribution such as normal distribution. Further support for the normality assumption is given by a quantile-quantile-plot (not printed), which shows slight deviations from the normality assumptions in the tails of the data only. As we aim for a model of wind infeed over time, the results above suggest to model deseasonalized logit wind power efficiency as an Ornstein-Uhlenbeck process, which is stationary normally distributed.

Model 3.1 (Model for wind power efficiency)**.** *The model for deseasonalized logit wind power efficiency* $\bar{E}_t^{wind}$ *is*

$$d\bar{E}_t^{wind} = -\theta^{wind}\bar{E}_t^{wind}\,dt + \sigma^{wind}\,dW_t^{wind}, \quad \bar{E}_0^{wind} = e_0, \tag{9}$$

where

e_0 *is the initial value,*

θ^{wind} *is the mean reversion speed, and*

σ^{wind} *is the volatility.*

The Ornstein-Uhlenbeck process also fits an autocorrelation. The partial autocorrelation function shows clear autocorrelation in the dataset, which is 0.990 at lag one. There is also significant partial autocorrelation at lag two and three, which the Ornstein-Uhlenbeck process cannot capture. We conclude that it is important to incorporate autocorrelation in the model and that a autoregressive time-series model with higher order (e.g. three) could also be considered as an alternative. As we aim for a continuous model, we focus on model 3.1 in the following.

Proposition 3.2 (Strong solution of $\bar{E}_t^{wind}$)**.** *The SDE in equ.* (9) *has the strong solution*

$$\bar{E}_t = e_0 e^{-\theta t} + \sigma \int_0^t e^{-\theta(t-s)} dW_s, \tag{10}$$

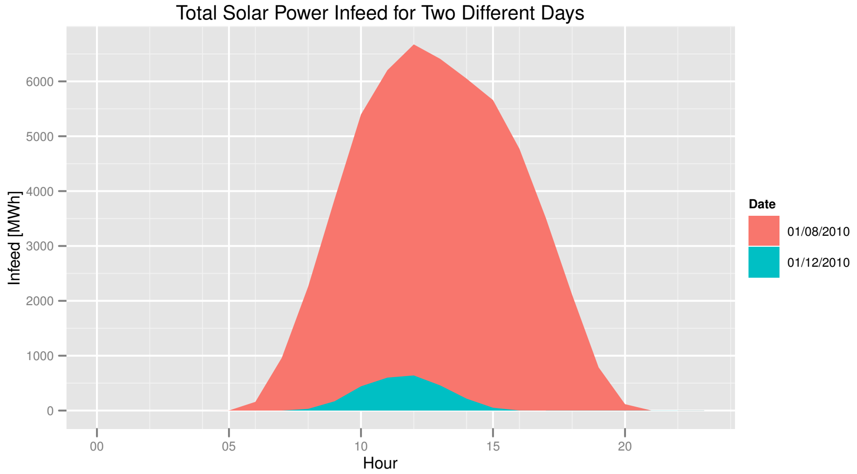

Figure 8: Solar power infeed in Germany for two days in 2010.

where we omit the superscript wind *in the parameters for readability.*
Its first two moments are

$$\mathbb{E}\left[\bar{E}_t|\bar{E}_0 = e_0\right] = e_0 e^{-\theta t},$$
$$\text{Var}\left[\bar{E}_t\right] = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta t}),$$

and its stationary distribution is $N(0, \frac{\sigma^2}{2\theta})$.

Using the notation from definition 2.5 we say that wind infeed efficiency is modeled logitnormal.

To avoid systematic deviations from the seasonal level in future simulations (i.e. simulations not starting from an observation) we suggest to start the process in $e_0 = 0$. Moreover we assume independence of W_t^{load} and W_t^{wind}, which is equivalent to assuming independence in the evolution of wind infeed and electricity demand. This is sensible, as there is no economic or meteorological reason to assume that wind and demand fluctuations should be correlated.

4 Model for solar power infeed

Production from solar power plants is gaining influence on the development of power prices in Germany due to a tremendous increase in installed capacity in recent years and the very strong daily pattern of its infeed.

The daily infeed pattern (intra-day seasonality) is illustrated in figure 8 for a summer and winter day in 2010. The shape of the infeed curve is similar and both days have a certain period of zero infeed. In more detail, we find that infeed is zero at night and, depending on the season, starts to rise at some time in the morning. It reaches its peak around noon, when solar radiation is at its highest level for the day, and decreases afterwards until there is again no infeed in the late evening and during the night. However, length and amount of infeed differs remarkably. Infeed starts earlier and lasts longer on the summer day and it has a much higher peak infeed in comparison to the winter day. Looking at time series of solar power infeed we find that the shape of the curve is resembled every day. We plot the August 2010 infeed in figure 9.

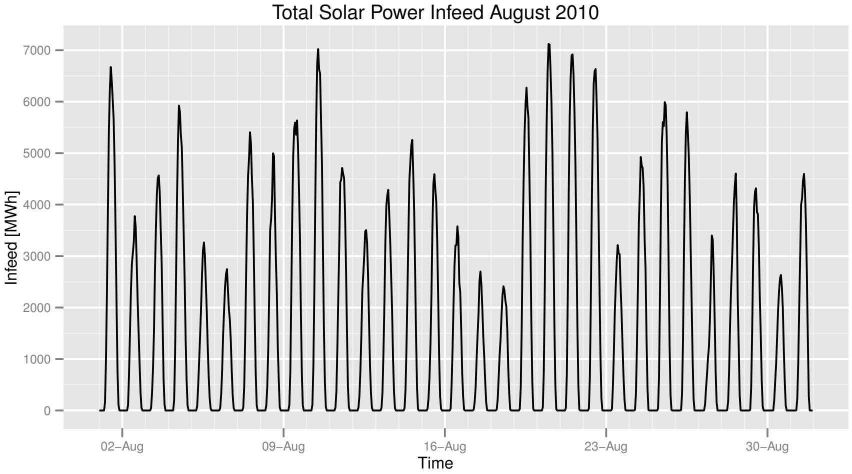

Figure 9: Solar infeed for Germany in August 2010.

This very strong pattern gives rise to a model with a deterministic daily pattern and a stochastic model with daily resolution. We propose to model the daily maximum and deduce the distribution of infeed over the day from historical data.

In the following, we formalize those ideas.

Definition 4.1 (Day-count function)**.** The *day-count function*

$$d : [0, T] \to \mathbb{N}_0$$

is defined by

$$(d_t = n) \equiv (t \text{ is on the n-th day}),$$

starting with 0, i.e. $d_0 = 0$.

In other words, d enumerates the days in $[0, T]$. For each day, we define the daily maximum process $\tilde{M}$.

Definition 4.2 (Daily maximum process)**.** The *daily maximum process* of solar efficiency is defined as

$$\tilde{M}_i^{solar} = logit\left(\max_{(t:d_t=i)}\left(E_t^{solar}\right)\right), i = 0, 1, \ldots, d_T.$$

In figure 10 we plot the daily maximum process for our dataset. Data ranges from 01/08/2010 to 31/07/2011[15]. A 50-day moving average indicates that there is clear seasonality in the data. Similar to the wind power model, we deseasonalize the daily maximum process using

$$\eta_t^{solar} = a_1 \cos(2\pi t + b_1) + a_2 \cos(4\pi t + b_2) + c, \tag{11}$$

which is a seasonality able to capture two peaks per year. It fulfills the condition in equ. (7). The moving average and the seasonality estimate are plotted in figure 10. The figure shows

---

[15] Actual infeed data is provided by European Energy Exchange AG (2012). For installed capacity we assume 10.644 GW and 17.320 GW for 2010 and 2011, respectively. In 2011 there was, especially in the second half of the year, a very strong rise in installed solar capacity, so we use the installed capacity which has been published by European Energy Exchange AG (2012) on 01/07/2011. The daily maximum in the data fulfills assumption 2.4. This year there was a lot of snow covering solar panels in winter (December, January), a very sunny spring (April, May) and a rainy summer (June, July). This year might be considered untypical, but due to a lack of more data we work with this dataset.

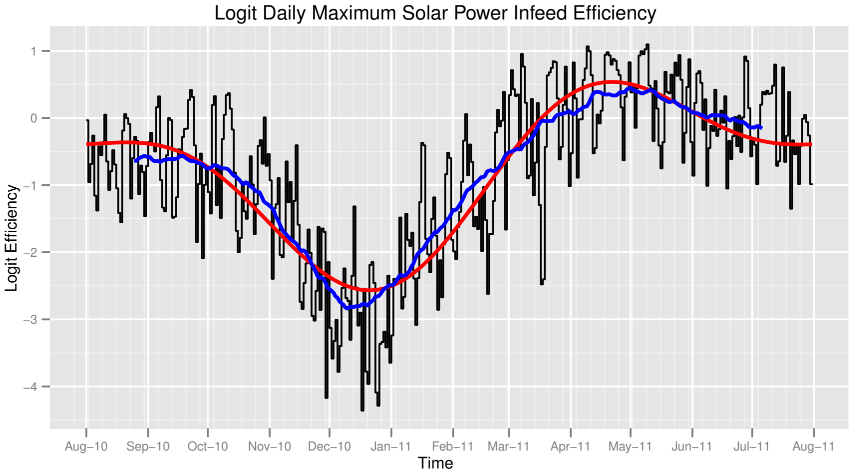

Figure 10: Solar infeed daily maximum process, 50-day moving average (blue) and estimated seasonality (red).

that the daily maximum process tends to have similar levels for a few consecutive days. This comes from weather conditions, which also tend to be similar for a few days before a change in weather implies that the process moves to another level. The process is fluctuating around zero, which motivates the choice of a mean-reverting process. Looking at the process from a time-series point of view we can analyze its partial autocorrelation function. There is strong evidence that an autoregressive model of order one is appropriate. As we have been working with continuous processes throughout this work, we choose an Ornstein-Uhlenbeck process to model the daily maximum.

Model 4.3 (Model for solar efficiency daily maximum)**.** *The model for the* deseasonalized solar efficiency daily maximum $\bar{M}_t^{solar}$ *is an Ornstein-Uhlenbeck process, i.e.*

$$d\bar{M}_t^{solar} = -\theta^{solar}\bar{M}_t^{solar}\ dt + \sigma^{solar}\ dW_t^{solar}, \quad \bar{M}_0^{solar} = m_0,$$

where

m_0 *is the initial value,*

θ^{solar} *is the mean reversion speed, and*

σ^{solar} *is the volatility.*

As for wind efficiency, we plot a histogram of the deseasonalized daily maximum data in figure 11. It shows that the normality assumption, which is implicit in model 4.3, is reasonable. This is also supported by the statistical figures in table 2.

We should point at a little inconsistency in notation. In definition 4.2 we formulated the daily maximum process as a discrete process, but model 4.3 is in continuous time. This will not pose any problems on our work, as in simulations we choose the time-interval to be one day for this process. For the sake of a clean notation, one could define $\bar{M}_i = \max_{(t:d_t=i)}\left(\bar{M}_t^{solar}\right)$.

For the transformation from the daily maximum to the single hours over the day we use a family of deterministic functions based on historical data. This can be interpreted as the inverse transformation to the daily maximum process (see definition 4.2) and is called *daily pattern transformation.*

Definition 4.4 (Daily pattern transformation)**.** The *daily pattern transformation* is a family of functions $\{\delta_i, i = 0, \dots, d_T\}$ with

$$\delta_i : \{t : d_t = i\} \times (0,1) \to [0,1].$$

Statistic	No Transformation	Logit	Deseasonalized Logit
mean	0.358	-0.798	0.000
median	0.364	-0.396	0.026
standard deviation	0.209	0.973	0.703
skewness	0.069	-0.496	-0.262
kurtosis	1.855	1.980	3.140
minimum	0.013	-2.567	-2.470
maximum	0.750	0.538	1.844

Table 2: Statistical figures for the solar power daily maximum efficiency data without transformation and with a logit transformation.

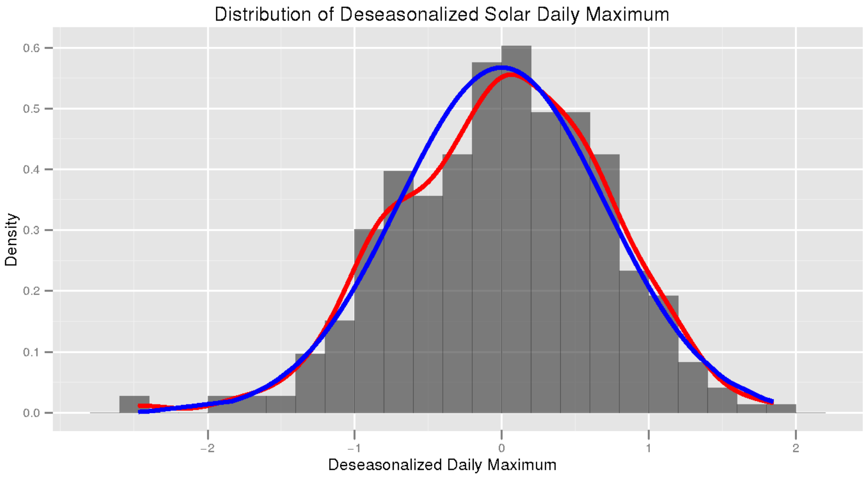

Figure 11: Histogram, empirical distribution (red line), and a fitted normal distribution (blue line) of deseasonalized daily maximum of solar power infeed efficiency.

Depending on the application, the functions δ_i can for example be step-functions with one step for each hour of the day or appropriately scaled Gaussian functions.

Now we have all components to formulate a model for solar power infeed at any time $t \in [0, T]$.

Model 4.5 (Model for solar power efficiency)**.** *The model for solar power efficiency E_t^{solar} at time $t \in [0, T]$ is*

$$E_t^{solar} = \delta_{d_t}\left(t, logit^{-1}\left(\eta_t^{solar} + \bar{M}_t^{solar}\right)\right), \tag{12}$$

where

$\bar{M}_t^{solar}$ is the deseasonalized daily maximum efficiency following model 4.3,

η_t^{solar} is a deterministic seasonality in the daily maximum efficiency, and

δ_{d_t} is the daily pattern transformation for the day d_t.

A reasonable daily pattern transformation is

$$\delta_i(t, x) = x \sum_{k=1}^{24} c_k \mathbf{1}_{t \in (\text{k-th hour of the day})}, \; c_k \in [0, 1] \; \forall \; k, \tag{13}$$

which uses a constant proportion of the daily maximum for each hour of the day. We use equ. (13) in the case study in section 6.

5 Residual Demand Model

We combine the approaches for wind and solar power infeed to model residual demand.

Model 5.1 (Model for conventional demand)**.** *Suppose a model for total system load* L_t*, model 3.1 for wind power efficiency* E_t^{wind} *and model 4.5 for solar power efficiency* E_t^{solar}*. Assume deterministic functions for installed capacity of wind and solar,* IC_t^{wind} *and* IC_t^{solar}*, respectively. The conventional demand* R_t*, i.e. the amount of electricity to be produced by plants other than wind and solar, is modeled as*

$$R_t := L_t - IC_t^{wind} \cdot E_t^{wind} - IC_t^{solar} \cdot E_t^{solar}.$$

Proposition 5.2. *Suppose model 5.1 with model 1.1 for total system load and assume independence of* W_t^{load}*,* W_t^{wind}*, and* W_t^{solar}*. Then the distribution of* R_t *is the convolution*

$$\begin{aligned} f_{R_t}(r) = \int_0^1 \int_0^1 \; & \phi_{\mu^L,\sigma^L}\left(r + IC^{wind}e + IC^{solar}\delta_{d_t}(t,m)\right) \cdot \\ & \frac{1}{e(1-e)}\; \phi_{\mu^W,\sigma^W}\left(logit(e)\right) \cdot \\ & \frac{1}{m(1-m)}\; \phi_{\mu^M,\sigma_t^M}\left(logit(m)\right) de\; dm, \end{aligned} \tag{14}$$

where $\phi_{\mu,\sigma}$ *is the normal density with mean* μ *and variance* σ^2*, and*

$$\begin{aligned} \mu^L &= \psi_t + l_0 e^{-\theta^{load} t}, \\ \mu^W &= \eta_t^{wind} + e_0 e^{-\theta^{wind} t}, \\ \mu^M &= \eta_t + m_0 e^{-\theta^{solar} t}, \\ \left(\sigma^L\right)^2 &= \frac{(\sigma^{load})^2}{2\theta^{load}}(1 - e^{-2\theta^{load} t}), \\ \left(\sigma^W\right)^2 &= \frac{(\sigma^{wind})^2}{2\theta^{wind}}(1 - e^{-2\theta^{wind} t}), \\ \left(\sigma^M\right)^2 &= \frac{(\sigma^{solar})^2}{2\theta^{solar}}\left(1 - e^{-2\theta^{solar} t}\right). \end{aligned}$$

6 Case Study

In this section we apply the residual demand model to a simple supply/demand model and simulate hourly price paths of the German spot market for the year 2012 based on a calibration using data up to 31/07/2011.

Demand-Side

We use model 1.1 for total system load prevailing at one representative hour of the day. For the yearly seasonality we use

$$y_t := a\cos(2\pi t + b) + c, \tag{15}$$

which we assume is the yearly seasonality in one representative hour of the day on business days[16]. The representative hour must not necessarily be the hour with maximum demand, but it should preserve the weekly pattern. For the German market we propose using the time between 11am and noon. Demand on non-business days is lower due to less consumption in industry and offices (weekly seasonality). We use a constant proportion of y_t for those days, i.e. demand at the representative hour on non-business days is

$$(1-w)y_t, \; w \in (0,1).$$

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[16] *Business days* are all days of the year except for weekends and public holidays.

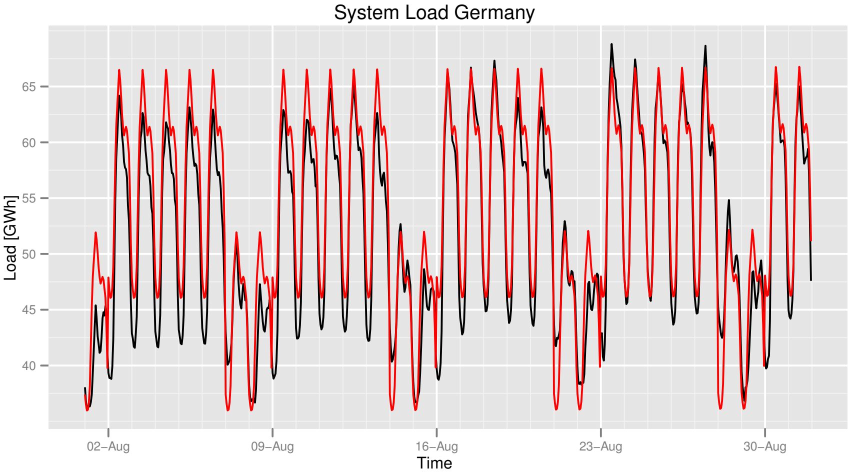

Figure 12: Actual load and seasonality estimate (red) for August 2010.

Therefore the seasonality on a daily scale is

$$\psi_t = (1 - w\mathbf{1}_{\{t \text{ is weekend/public holiday}\}})y_t.$$

One could refine this by using different proportions for Saturdays, Sundays, and public holidays. This leaves us with the intra-day seasonality. We adopt the ideas of Smeers & de Maere (2010) and use a time-dependent intra-day load curve $\delta^L_{d_t}$ transforming the demand of the representative hour to the single hours of the day. Using the notation from model 1.1 we model load at time t as

$$L_t = \delta^L_{d_t}\left(t, \psi_t + l_t\right).$$

Calibration of the Demand Model

We use hourly load data for Germany from 01/08/2010 through 31/07/2011 provided by ENTSOE[17] for the calibration of the load seasonality and process parameters. We do not use a longer time-series, as the economic crisis in 2009 and the beginning of 2010 lead to a downshift in electricity demand, which is not continuing in 2011. We found that the downshift influences the calibration procedure and leads to an underestimated load seasonality.

At first we calibrate the yearly seasonality y_t using least squares on each day except for weekends and public holiday. In a second step we estimate the non-business day correction w as the mean deviation on non-business days from the estimated yearly seasonality.

In a final step we specify the intra-day load curves δ^L_t. Similar to the daily pattern transformation in the solar model, we use a step-function with 24 steps different for each month of the year. The function therefore takes the form

$$\delta^L_t(x) = x\sum_{k=1}^{24} c^L_k \mathbf{1}_{t\in(\text{k-th hour of the day})},\ c_k > 0\ \forall\ k.$$

For calibration of the step weights c^L_k we calculate the ratio between actual load and the reference hour (11th hour) and use the mean value as an estimate. We show in figure 12 actual load data and our seasonality estimate including the yearly, weekly and daily model.

In a final step we calibrate an Ornstein-Uhlenbeck process to the deviations between seasonality estimate and actual load at the reference hour. The estimated parameters are given in table 3.

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[17] `https://www.entsoe.eu`

Seasonality		Stochastic Process	
Parameter	Estimate	Parameter	Estimate
Load			
a	2.173	θ^{load}	336.609
b	-0.521	σ^{load}	89.265
c	68.669		
w	14.563		
Wind			
a	0.311	θ^{wind}	91.151
b	0.002	σ^{wind}	15.155
c	-1.999		
Solar			
a_1	-1.230	θ^{solar}	261.817
b_1	0.476	σ^{solar}	16.087
a_2	-0.614		
b_2	0.093		
c	-0.798		

Table 3: Calibrated parameters of the residual demand model for Germany.

Calibration of Wind Model

For the calibration of the wind model we use the same data as in section 3. As the efficiency process is stationary this does not contradict to the shorter time-series used in load and solar calibration. The estimate parameters are in table 3. Using the calibrated parameters we find that the distribution of wind infeed efficiency is unimodal at any time $t > 0$.

Calibration of Solar Model

The solar model is calibrated on the data used in section 4. The deseasonalized daily maximum process is fitted to an Ornstein-Uhlenbeck process following model 4.3. The parameters are also in table 3 and we find that the distribution of the daily maximum efficiency process is unimodal. For the daily pattern transformation (see definition 4.4) we use the functional form from equ. (13). The calibration is similar to the hourly load shape function. Separately for each month we calculate the mean of

$$\frac{\text{actual infeed for that hour}}{\text{daily maximum infeed}}$$

to determine $c_k, k = 1, \ldots, 24$.

Installed Capacity

By construction of our model we need to specify values for installed capacity of wind and solar. As we aim for a simulation of the year 2012 we need to predict sensible values for IC^{wind} and IC^{solar}. We base our estimates on scenarios for installed capacity from 50hertz et al. (2011). There are three scenarios for installed capacity, separated by technology. They distinguish on- and offshore wind power, which we sum up to obtain one figure for installed wind power capacity. Scenario A is based on plans of the German government, whereas Scenario C is based on regional forecasts of the German Federal States. The basis for Scenario B is also formed by plans of the German government (i.e. Scenario A), but assumes a higher share of renewables in the system.

Using this data we obtain constant growth rates for each scenario and technology and calculate probable values for installed capacity in 2012 (see table 4). As the scenarios are based on actual data from 2010, the divergence for 2012 is not particularly big, but still there are uncertainties of

Technology	Scenario A	Scenario B	Scenario C
wind	29.548	30.769	33.073
solar	19.000	20.510	20.027

Table 4: Estimated installed capacity by technology for the year 2012 in GW based on scenarios from 50hertz et al. (2011) and assuming a constant growth rate for each technology.

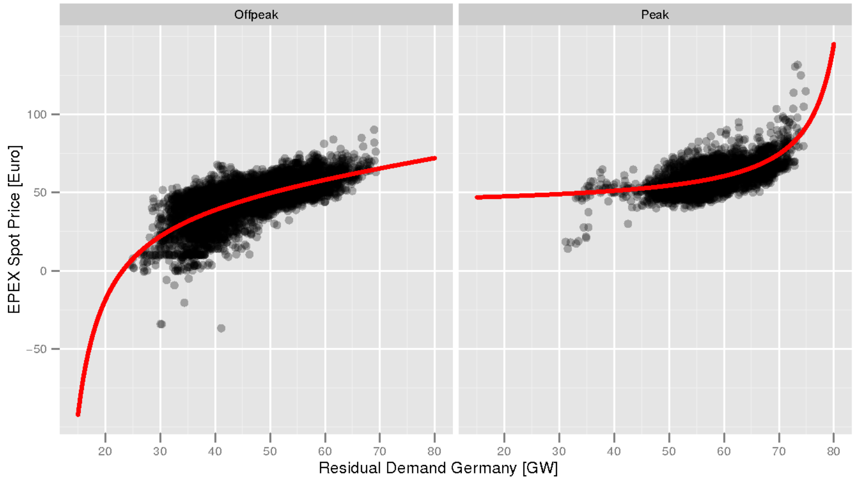

Figure 13: Residual demand against spot price and estimated supply functions (red).

up to 10%. In the following analysis we will focus on Scenario A, but we will also show the effects of the different scenarios on the price development.

An alternative method to predict installed capacity is to look at power plants under construction and their expected completion date. This is especially suitable for short time horizons and wind power. Solar panels, however, are mounted mainly on private houses and have a very short time-to-build, so data on units planned or under construction can hardly be obtained.

Supply-Side

Motivated by Barlow (2002) we model the supply-side as a deterministic function $f : \mathbb{R} \to \mathbb{R}$ of residual demand, i.e.

$$S_t = f(R_t),$$

where S_t denotes the spot price of power and R_t residual demand at time t. The desired properties of the function $f(\,\cdot\,)$ are motivated by figure 13, which shows historical data[18] on residual demand against spot price. The figure shows a sharp increase in price at a residual demand at about 75 GWh, and on the other hand some negative prices at low residual demand. Between those extremes, the plot suggest a fairly linear relation.

We split the data in peak- and offpeak hours, as on the EEX all products are traded for base (this is all hours of the day), peak (weekdays from 8am to 8pm), and offpeak (all but peak) hours, respectively. Therefore such a distinction does enlarge the database which can be used to calibrate the model. Moreover, it simplifies calibration of the supply function a lot. We found that the following functions provide a reasonable fit to the data with only three (two) parameters for peak (offpeak) hours.

---

[18] Data ranges from 01/08/2010 to 31/07/2011. We are using this short time frame for calibration, as we have data on solar infeed for this period only. We give some statistics for the spot price in table 6.

Model 6.1 (Supply functions for peak and offpeak)**.** *The supply-function*

$$f : \mathbb{R} \to [p_{floor}, p_{cap}]$$

maps residual demand to price. Define

$$h(x, g(x)) = \begin{cases} p_{floor} & x \leq x_{floor} \\ \min\left(p_{cap}, \max\left(p_{floor}, g(x)\right)\right) & x \in (x_{floor}, x_{cap}) \\ p_{cap} & x \geq x_{cap}. \end{cases}$$

For offpeak hours the supply-function is

$$f(x) = h(x, g^{offpeak}(x))$$

where

$$g^{offpeak}(x) = a + \tfrac{b}{x - x_{floor}} + cx.$$

For peak hours it is

$$f(x) = h(x, g^{peak}(x))$$

where

$$g^{peak}(x) = a + \tfrac{b}{x_{cap} - x}.$$

The values for price cap and floor, p_{floor}, p_{cap}, respectively, and load cap and floor, x_{floor}, x_{cap}, respectively, are not calibrated to data but fixed to market-specific values.

In case of the German market, the price boundaries at the EPEX are −3000 EUR and +3000 EUR, so we choose $p_{floor} = -3000$, $p_{cap} = 3000$. The load boundaries are less obvious, but we found $x_{floor} = 10$ and $x_{cap} = 85$ to be reasonable. Note that this are boundaries on the available conventional generation power. The choice is motivated by the fact that 10 GW of online conventional generation seems to be the minimum to ensure system stability[19]. The load cap at 85 GW is justified by guaranteed capacity arguments. According to Bundesministerium für Wirtschaft und Technologie (2011) the guaranteed capacity of Germany in the year 2010 (based on a system adequacy forecast, UCTE 2009) has been 89.9 GW. Taking the nuclear moratorium[20] into account we find that 85 GW of remaining guaranteed capacity is sensible.

Using least-squares, we calibrate model 6.1 on the historical spot price data and residual demand. The resulting functions are plotted in red in figure 13. This is a calibration under the historical measure. We also calibrated the supply-functions under the risk-neutral or market measure. For this purpose we used 1000 simulations of the residual demand model under Scenario A (see before) and, using least-squares, calibrated the supply-functions on quotes of the quarterly futures from 1st of August 2011 (bold-printed data in table 7). The estimated parameters for both calibrations are in table 5.

Results

We simulate 5000 paths of residual demand for the year 2012 in order to assess the proposed model. Using the scenarios and supply functions outlined above we calculate hourly spot prices from residual demand simulations. We start with the trajectorial properties of the model. Such an analysis has already been suggested in Geman & Roncoroni (2006) for a judgement of the quality of an electricity price model. A good trajectorial fit is also important for the valuation of path-dependent derivatives, which are found a lot in the electricity OTC-trading. Moreover, many complex risk-management tools require price trajectories as an input.

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[19] For example, the installed nuclear capacity in Germany after the moratorium in 2011 is at about 12 GW, and as nuclear is usually considered a must-run plant, this also supports the choice for x_{floor}.

[20] After the moratorium 8.4 GW of installed nuclear capacity (Bundesnetzagentur 2011*a*) have been finally switched off, including roughly 2 GW which were already offline for a couple of years due to technical problems. As nuclear generation is generally considered very reliable, a decrease in guaranteed capacity of 5 GW is a reasonable assumption. This is also supported by Bundesnetzagentur (2011*a*).

Parameter	Spot Calibration	Future Calibration
Offpeak		
a	43.000	57.569
b	-713.804	-712.733
c	0.491	0.318
Peak		
a	39.425	23.827
b	528.343	1226.018

Table 5: Parameters for peak and offpeak supply function from a calibration on EPEX day-ahead (=spot) prices from 01/08/2010 to 31/07/2011 (spot calibration) and a calibration on EEX future prices from 01/08/2011 (future calibration).

Statistic	Value
mean	50.219
median	50.150
standard deviation	14.028
skewness	-0.527
kurtosis	4.902
minimum	-36.820
maximum	131.790

Table 6: Statistics for hourly EPEX day-ahead price data from 01/08/2010 to 31/07/2011. For a plot of the data see figure 13.

For an analysis of historical prices see section 1. In figure 14 we show sample price paths of our model. The intra-day and weekly shapes of simulated prices are displayed in figure 15, which shows the details of one trajectory over two months. The observed spot trajectory in figure 3 is less regular, but due to our simple supply side representation this model naturally cannot capture all the risks inherent in the market. However, the occurrence of both positive and negative spikes, as well as their magnitude is nicely reproduced. Moreover the model is able to reproduce the stochastic change of the daily residual load profile depending on the magnitude of renewable infeed during that day, which is visualized in figure 16. A model only based on load generates the same intraday profile for both days, whereas the residual demand model is able to produce different profiles. Therefore we believe that our model is well suited to generate trajectories with the stylized facts of observed spot prices in recent years.

Using the simulated paths we calculate forward prices. We compare our results to future price quotes from the EEX from 01/08/2011. However, we note that the simple model set up in this case study is stationary and therefore in general not well suited for the pricing of derivatives like futures or options. This is also pointed out in Barlow (2002): *Since the price arises from a model for each of the supply and demand curves for power, it is easy in principle to incorporate additional factors to account for long-term effects, or changes in market structure. Such extensions of our model would be essential before it could satisfactorily deal with options and futures prices.* On the other hand the calculation of forward and option prices can help us to assess the fitting quality of our model with respect to seasonal behaviour and the reproduction of extreme events. Therefore we continue with a study on forward prices, keeping in mind that the model captures only the risk inherent in load, wind, and solar fluctuations. It does in particular ignore changes in the conventional supply structure like fluctuating fuel and emission prices, power plant outages, and others.

Electricity futures have a delivery period and fulfillment can be instantaneously (which is common for exchange traded products) or at maturity. For instantaneous fulfillment we have the following

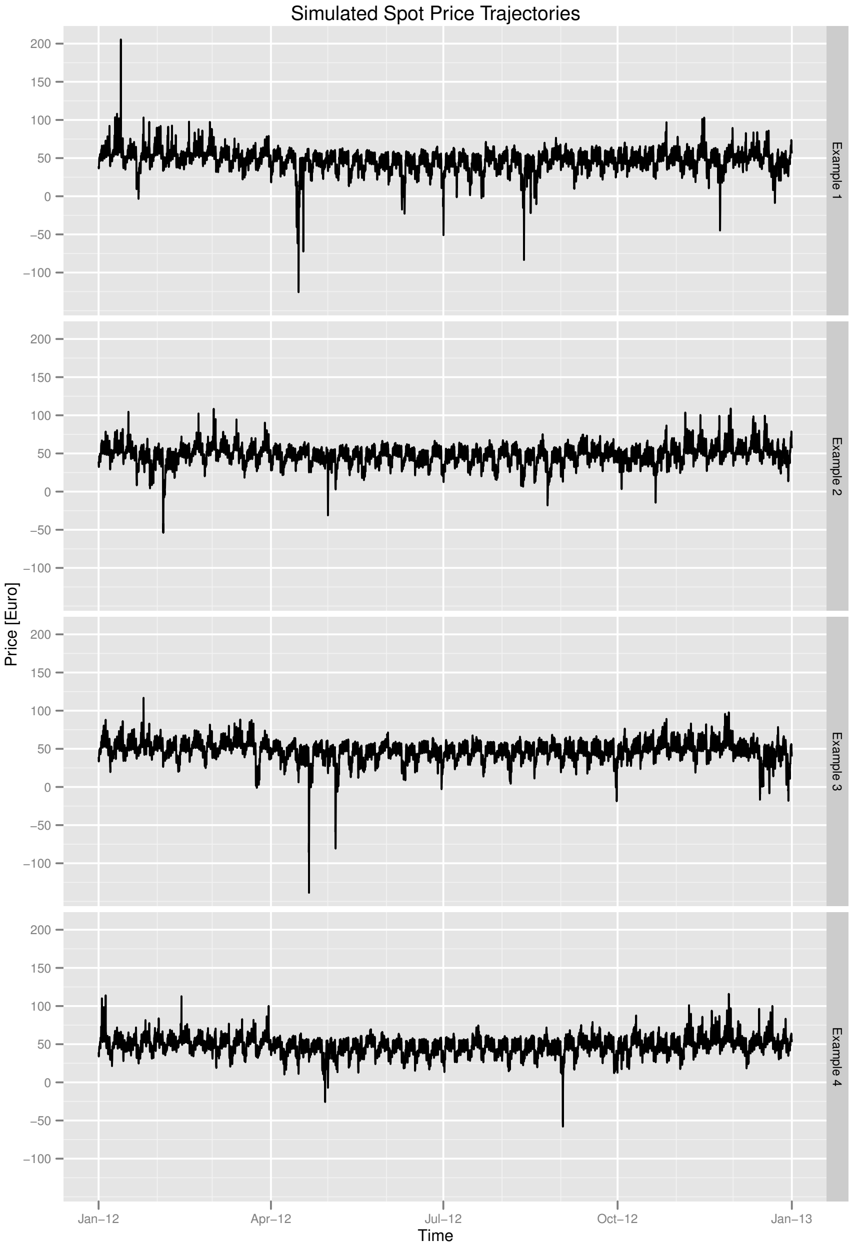

Figure 14: Sample trajectories of hourly simulated spot prices for 2012 under Scenario A.

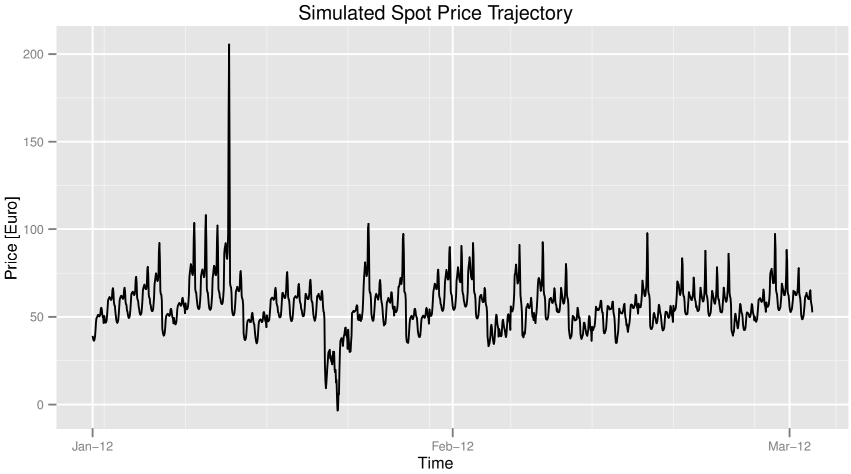

Figure 15: Detail of sample trajectory of hourly simulated spot prices for 2012 under Scenario A.

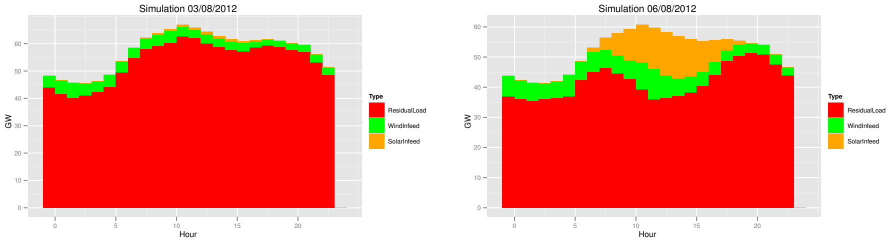

Figure 16: Simulated demand profiles for days with high and low solar infeed, respectively.

result.

Proposition 6.2 (Valuation of a forward with instantaneous fulfillment)**.** *The time-t price of a electricity forward contract with instantaneous fulfillment and delivery period* $[T_1, T_2]$ *is*

$$F_t^{\mathbb{Q}}(T_1, T_2) = \frac{r}{e^{r(T_2 - T_1)}} \int_{T_1}^{T_2} e^{r(T_2 - T)} \mathbb{E}^{\mathbb{Q}}[S_T | \mathcal{F}_t] \, dT,$$

where $\mathbb{Q}$ *is the market measure (i.e. a risk-neutral measure equivalent to the real-world measure* $\mathbb{P}$ *and chosen by the market) and* r *the risk-free interest rate.*

Proof. The forward price (strike price) $K = F_t^{\mathbb{Q}}(T_1, T_2)$ is chosen such that the contract has zero expected value at time t:

$$0 = \mathbb{E}^{\mathbb{Q}} \left[\int_{T_1}^{T_2} e^{r(T_2 - T)} (S_T - K) \, dT | \mathcal{F}_t \right]. \qquad (16)$$

The payoff $S_T - K$ is received at time T (instantaneous fulfillment) and reinvested (or borrowed, depending on the sign of $S_T - K$) at the risk-free rate r until the final settlement at T_2. Solving equ. (16) for K proves the assertion. □

We assume a risk-free rate of 2% in all our calculations, i.e. $r = 0.02$. In table 7 we compare forward prices under Scenario A for the spot and future calibration, i.e. we calculate $F_t^{\mathbb{P}}(T_1, T_2)$

	Market	Spot Calibration				**Future** Calibration			
Product	Price	Price	SD	Forward Premium		Price	SD	Difference	
Base	57.94	49.47	1.06	8.47	(14.61%)	57.95	1.22	-0.01	(-0.02%)
Peak	71.89	60.21	0.92	11.68	(16.25%)	71.92	1.67	-0.03	(-0.04%)
Offpeak	50.21	43.52	1.40	6.69	(13.32%)	50.21	1.32	0.00	(0.00%)
Peak Q1	79.41	63.69	2.50	15.72	(19.80%)	79.83	4.46	-0.42	(-0.52%)
Peak Q2	63.90	56.45	1.01	7.45	(11.65%)	63.39	1.74	0.51	(0.80%)
Peak Q3	64.94	57.42	0.75	7.52	(11.58%)	65.59	1.73	-0.65	(-1.00%)
Peak Q4	79.20	63.22	2.40	15.98	(20.18%)	78.76	4.32	0.44	(0.56%)
Offpeak Q1	53.13	47.51	1.89	5.62	(10.58%)	53.61	1.68	-0.48	(-0.91%)
Offpeak Q2	47.52	40.21	3.34	7.31	(15.39%)	47.37	3.22	0.15	(0.33%)
Offpeak Q3	47.44	40.68	3.32	6.76	(14.25%)	47.77	3.20	-0.33	(-0.70%)
Offpeak Q4	52.76	45.72	2.23	7.04	(13.34%)	52.10	2.02	0.66	(1.26%)
Peak 01	80.95	66.45	5.56	14.50	(17.91%)	85.97	9.47	-5.02	(-6.20%)
Peak 02	81.50	64.04	4.30	17.46	(21.42%)	80.68	7.56	0.82	(1.01%)
Offpeak 01	54.52	48.17	2.90	6.35	(11.65%)	54.19	2.55	0.33	(0.61%)
Offpeak 02	54.15	48.03	3.19	6.12	(11.31%)	54.05	2.80	0.10	(0.18%)

Table 7: Market prices for 2012 future products (01/08/2011, source `www.eex.de`) and simulation results under Scenario A in EUR/MWh. The absolute standard deviation (SD) and the difference to market price (absolute and as percentage) is given for all simulations. The bold printed products have been used in the future calibration. The first block are yearly products, the second quarterly and the third are monthly contracts. The remaining months were not yet traded. The difference between simulated and market prices is calculated and also given as percentage relative to market price. In case of spot calibration, this difference is known as *forward premium.*

and $F_t^{\mathbb{Q}}(T_1, T_2)$, respectively. The forward premium[21] found is positive for all products ranging from 11% to 21% of market price. The premium is the highest for the products *Peak Q1* and *Peak Q4* (first and fourth quarter 2012) as well as *Peak 01* and *Peak 02* (January and February 2012). This supports the assumption that the nuclear moratorium added uncertainty in the market, especially the risk of positive spikes in times of high demand (winter months). However, liquidity in the monthly contracts was very low. The future calibration is able to significantly reduce the deviations between simulated and market prices, especially the yearly products are matched almost exactly. The difference is unusual for the *Peak 01* product, which is due to the demand model not taking the Christmas holidays into account (usually the load is lower in the last and first week of the year as there is reduced industrial production). However, an even more sophisticated demand-side model is not in the scope of this work, but is usually already implemented in any utility taking part in electricity trading. In general, the offpeak results are slightly better (i.e. closer to observed prices) as the peak products. This has been expected as the offpeak supply-function has three parameters, whereas for the peak supply-function only two parameters were to calibrate. However, the good fit of the remaining products and the good representation of the general shape supports our model.

The standard deviations differ remarkably between the analysed products and show seasonal behaviour. In general, a higher standard deviation is expected when demand during the delivery period is within areas of high gradient of the supply functions. Looking at the quarterly contracts, we find that for the peak products there is a lower standard deviation in the second and

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[21] The forward premium is defined as $F_t^{\mathbb{Q}}(T_1, T_2) - F_t^{\mathbb{P}}(T_1, T_2)$, i.e. the difference between the forward price evaluated under the risk-neutral measure (which is the observed market price) and the real-world measure. There is extensive research on the forward premium in electricity markets, e.g. Bessembinder & Lemmon (2002), Longstaff & Wang (2004), Benth & Meyer-Brandis (2009), and Biegler-König, Benth & Kiesel (2011).

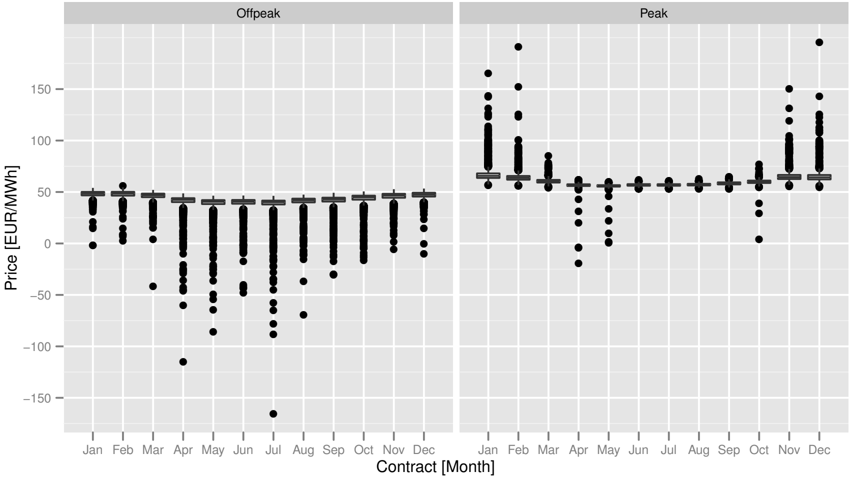

Figure 17: Boxplots of simulated prices for the monthly futures 2012 based on the spot calibration and Scenario A.

third quarter compared to others, whereas for the offpeak products the situation is vice versa. In order to assess this in more detail, we show boxplots for simulated monthly future prices 2012 in figure 17. The box, which contains 50% of the simulations, is small compared to the absolute size of the price. There are outliers for both offpeak and peak simulations. For the offpeak products, most outliers are below the box, meaning that in these paths the average price for the corresponding month is below the mean. Some outliers even have a price below zero. In those paths, the residual demand process hit the lower boundary and produced the system's minimum price of $-3000[EUR]$ for a few hours. For the peak futures, the situation is threefold. There are months with outliers mainly above the box, which correspond to the winter months with usually high demand. The April and May products show similar behavior as the offpeak futures. This is due to the solar seasonality, which has its peak in spring and there is the chance for very high solar infeed (which mainly affects the peak products). The third pattern observed is present in March, but even more pronounced in summer and early autumn. There are hardly any outliers seen in the simulations. The solar infeed for those month is fairly high, so the peak-shaving effect of solar (see also section 1) reduces the spike risk and therefore outliers above the box are rare. As the solar seasonality is lower compared to April and May, there is less chance for very high solar infeed, explaining the absence of outliers below the box.

The described effects have, to our knowledge, not yet been reported in the literature. They could have implications for the hedging strategies of both electricity consuming and electricity producing utilities. Following the results in our model, there is less risk for the consumer in an unhedged offpeak position, as the spot prices are not expected to rise far above the future price. However, there is the chance to buy electricity significantly cheaper on the spot market. On the other hand, the consumer should hedge the winter peak positions in order to reduce her spike risk. For the producer, the reverse arguments apply, i.e. she should try to sell forward her offpeak production, but for her peak production in winter she has the chance to generate much higher returns when selling on the spot. For the summer months (with hardly any outliers) the results indicate that for both, consumer and producer, there is hardly any difference to be expected between the forward sold position and the open position (which is covered on the spot). Again, however, we note that the model does not capture changes on the conventional supply structure.

Despite the stationary nature of our model we calculate the expected payoff for European call and put options on the *Base 2012* future contract from the model simulations. In figure 18

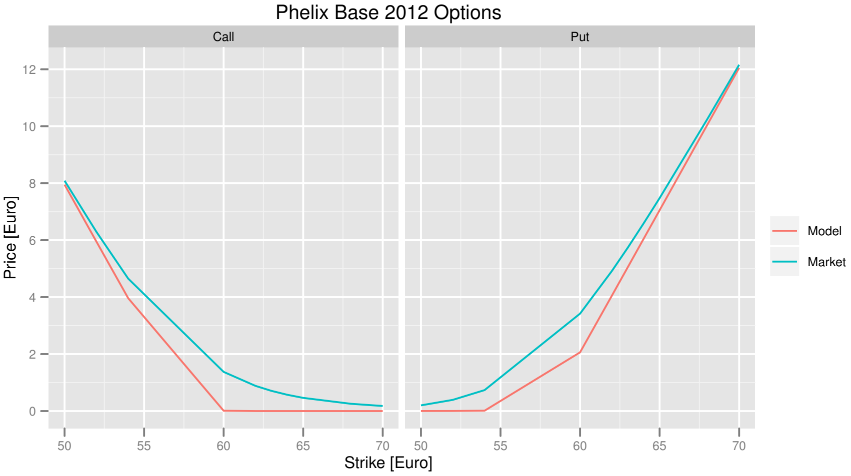

Figure 18: Expected payoff (Scenario A, future calibration) and market prices for European Call and Put Options on the *Base 2012* contract with maturity January 2012.

we compare them to observed market prices. The modeled payoff is always below the observed market prices, but the model is able to capture the structure and magnitude of option prices very well. This indicates that our model performs well also on the tails of the price distribution. Naturally the model price is below the market option price due to the simplicity of the model.

The analysis conducted above was based on Scenario A only. However, as we use deterministic installed capacities in our model we can use the same paths to study prices under a different scenario. We will use the spot-calibrated supply functions in the following (otherwise a recalibration is needed as the future-calibration is obviously based on assumptions on the future installed capacities). We compare the simulated prices in table 8. We find that the prices under Scenario B are lower compared to Scenario A, and Scenario C shows even lower prices than Scenario C for all products. The decrease in price is due to the greater installed capacities for wind and solar in Scenario B and C compared to Scenario A, which, by construction, reduces wholesale electricity prices. However, the picture is less clear when looking at the volatility (standard deviation) in the market. We consider the quarterly peak and offpeak contracts. Volatility increases for all offpeak contracts, because higher renewable infeed brings residual demand more often in the steep region of the supply function. For the peak contracts, however, volatility does not change significantly, expect for the *Peak Q2* product (which is the quarter with the on average highest solar infeed). We conclude that increased capacity of renewables decreases wholesale electricity market prices (known as the *merit-order effect*), but increases volatility in times of low demand. In order to separate the effects of wind and solar we analyse the model's sensitivity with respect to wind and solar installed capacity separately. A common procedure in sensitivity analysis is to change the parameter in question but leaving the remaining parameters unchanged. We base our analysis on the spot calibration and Scenario A and add one GW of installed capacity of wind (scenario *SensWind*) and solar (scenario *SensSolar*), respectively. The results are also displayed in table 8. Considering the *Base* contract we find that the overall price effect of wind is stronger than of solar. In peak hours, however, the effect is balanced. In the sunny quarters *Q2* and *Q3* solar has also a considerable effect on the offpeak prices. In the most expensive products (which are *Peak Q1* and *Peak Q4*), solar shows little price decrease compared to wind. For the offpeak products, additional wind capacity generally leads to a stronger decrease in price as for additional solar capacity. Given the already high share of installed solar capacity the results indicate that new solar capacity does not have a higher merit-order effect than wind. Moreover, the sensitivity

Product	A (reference)		B (difference)		C (difference)		SensWind (difference)		SensSolar (difference)	
	Price	SD	Price	SD	Price	SD	Price	SD	Price	SD
Base	49.47	1.06	- 0.51	0.22	- 1.19	0.66	- 0.27	0.14	- 0.11	0.03
Peak	60.21	0.92	- 0.35	0.03	- 0.58	0.06	- 0.13	0.01	- 0.13	0.01
Offpeak	43.52	1.40	- 0.59	0.37	- 1.53	1.06	- 0.35	0.23	- 0.10	0.04
Peak Q1	63.69	2.50	- 0.39	- 0.05	- 0.74	- 0.09	- 0.18	- 0.02	- 0.12	- 0.01
Peak Q2	56.45	1.01	- 0.40	0.42	- 0.51	0.82	- 0.08	0.17	- 0.19	0.09
Peak Q3	57.42	0.75	- 0.31	- 0.00	- 0.40	0.02	- 0.07	0.01	- 0.14	- 0.01
Peak Q4	63.22	2.40	- 0.31	- 0.02	- 0.67	- 0.03	- 0.18	- 0.00	- 0.07	0.00
Offpeak Q1	47.51	1.89	- 0.42	0.32	- 1.12	1.04	- 0.27	0.20	- 0.05	0.03
Offpeak Q2	40.21	3.34	- 0.80	1.02	- 1.87	2.72	- 0.40	0.58	- 0.18	0.15
Offpeak Q3	40.68	3.32	- 0.70	0.91	- 1.77	2.56	- 0.40	0.58	- 0.12	0.11
Offpeak Q4	45.72	2.23	- 0.44	0.39	- 1.34	1.45	- 0.33	0.29	- 0.02	0.01

Table 8: Simulated future prices and empirical standard deviation (SD) under different scenarios. Scenario A is the reference scenario, for all other cases the differences with respect to the corresponding value of Scenario A are given. For Scenario A, B, and C, respectively, see table 4. Scenario *SensWind* and *SensSolar* give the results for a sensitivity analysis with respect to 1 GW increase in installed capacity of wind and solar, respectively.

analysis supports our assumptions on price volatility. In times of low demand both wind and solar add volatility to the market, whereas volatility can be reduced in peak hours with high demand.

7 Conclusions

We proposed an extension to electricity demand models for markets with a considerable infeed from wind and solar power plants. It can be applied to existing supply/demand models to account for infeed from those renewables. As we model renewable infeed as demand reduction, our model is suitable for markets with a guaranteed infeed and feed-in tariff for production from renewables (like Germany). If there is no such system, production from renewables will bid at price zero (i.e. their marginal production costs) on the exchange. In this case, our approach is only suitable when additionally assuming that no negative prices are allowed at the exchange.

The model has been motivated using data from the German power market and we aggregated all wind (solar) power plants installed in Germany and are looking at it as one big plant. Therefore this model might not be suitable for modeling infeed from a single wind turbine or solar plant, as we expect much more variability on this scale. However, the wind and solar infeed models can also be used in applications other than electricity pricing, i.e. in the optimization of power plant portfolios or power plant dispatching.

The design of the model has many practical advantages. Due to the separation of total demand, wind infeed, and solar infeed a calibration of the model is easy, as data for each component is separately available. Moreover, using our model one can utilize existing supply/demand models to evaluate the impact of wind and solar infeed on electricity prices. The concept of not including installed capacity in the model (i.e. as trend) is motivated by the fact that the development of renewables can hardly be deduced from time series of historical data, as it is much more driven by exogenous factors, especially political decisions. For example, in 2012 the German government decided to cut the feed-in tariff for solar, which is expected to slow the rise of installed solar capacitiy. On the oher hand the German nuclear phaseout until 2022 might boost development in renewables. Effects like this or other political decisions are easy to include in our approach. When there is much uncertainty in the development of installed capacity, one could evaluate the model for different scenarios, possibly weighted by their expected probability of occurrence (those scenarios can be evaluated in a single simulation). This can also be used in risk-management or investment departments of energy utilities to assess their risk-exposure w.r.t. different development schemes.

As we made no assumption on the model for total system load (model 1.1 only served as an example), utilities can continue using their existing model for system load and extend it using our wind and solar infeed model to a more accurate residual demand model. In ongoing research, we apply the more sophisticated supply/demand model of Aïd et al. (2009) on the German market using the Residual Demand Model proposed in this paper for the demand side.

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