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Vorwort

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Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Constitutive models for static granular systems and focus to the Jiang-Liu hyperelastic law

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Abstract

Granular systems in solid-like state exhibit properties like stiffness dependence on stress, dilatancy, yield or incremental non-linearity that can be described within the continuum mechanical framework. Different constitutive models have been proposed in the literature either based on relations between some components of the stress tensor or on a quasi-elastic description. After a brief description of these models, the hyperelastic law recently proposed by Jiang and Liu [1] will be investigated. In this framework, the stress-strain relation is derived from an elastic strain energy density where the stable properties are linked to a Drucker-Prager yield criteria. Further, a numerical method based on the finite element discretization and Newton-Raphson iterations is presented to solve the force balance equation. The 2D numerical examples presented in this work show that the stress distributions can be computed not only for triangular domains, as previoulsy done in the literature, but also for more complex geometries. If the slope of the heap is greater than a critical value, numerical instabilities appear and no elastic solution can be found, as predicted by the theory. As main result, the dependence of the material parameter ξ on the maximum angle of repose is established.

Keywords: Granular elasticity, Constitutive modelling, Non-linear finite element method.

Contents

1 Introduction

Static stress distributions in granular systems are important in mechanical, geophysical and chemical engineering applications and also to geotechnical engineers for the design and safe of fundations [2]. Particular attention should be paid to design silos [3] because of the problems associated with the formation of steady arches or the apparition of failures during the emptying phase, that can have dramatic consequences.

Sand is known to exhibit a diverse set of behaviours like Reynolds dilatancy, incremental non-linearity, stiffness dependence on stress or history dependence that makes it difficult to model. One of the most surprising property is probably that the stress distribution on a sand pile depend on how the grains were filled into the system [4]. A lot of constitutive models have been proposed and investigated, leading to a complex set of data [5], but there is no general theory to describe the granular solid behavior. Solid mechanics have been successfully employed for more than an hundred years, starting with the pioneering work of Coulomb [6] to determine the stability of sand systems. Analytical and numerical methods have been developped to design industrial equipments. For instance the Janssen model [7] is still used to design silos. Recently, the statics of the granular matter has also been investigated by physicists [8, 9]. The behavior of granular solids is howver still poorly understood and often subjected to intensive debates [9].

The elastic description have been motivated by the experimental observation of reversible deformations (of the order of 10^{-4}) in compacted granular systems [10]. Several non-linear stress-strain relations have been proposed based on theoretical, experimental and numerical investigations [11, 12]. Sound propagation experiments [13, 14] or triaxial tests [10] enable to obtain informations about the dependence of the shear and bulk moduli with the stress state. Interesting results were also obtained from assumptions made at the grain-scale, using contact mechanics [15, 16] and Effective Medium Theory (EMT) [17]. Discrete element simulations were also performed to calculate macroscopic elastic material parameters [13]. These non-linear elastic laws are not valid for large strains but can be extended within the elasto-plastic framework.

Recently, physicists have developed constitutive models motivated by observing inhomogeneous spatial stress distributions in granular systems [18]. Photoelastic experiments [19, 20] have shown the existence of stress chains in granular systems or contact networks and these models are able to reproduce stress transmission phenomena. These laws, generally formulated in 2D, are based on relations between the different components of the stress tensor. Although they have been sucessful, their initial assumptions are questionable [21, 22] and their generalization in 3D remains difficult.

In a recent work, Liu et al. [23, 24, 25, 1, 14] have proposed a non-linear elastic model derived from thermodynamical considerations. The stress-strain relation is determined from the elastic energy density which becomes unstable at yield. This model was successfully compared with experiments and used to calculate stress distributions in silos, sand piles or granular systems under point loads.

In this report, the preliminary results of the granular solid state are first recalled. The constitutive models proposed in the literature are discussed on the second part. The granular elasticity framework is then investigated and the results of the numerical simulation are presented in the final section.

2 Some basic properties of the granular solid

2.1 Reynolds dilatancy

First defined by Reynolds in 1885 [26], dilatancy refers to the volume change associated with shear distorsion of an element in a granular system. It can be qualitatively understood by considering layers of compact spheres as in the Figure 1. To sustain shear, grains must roll or slide over each other, what leads to an expansion of the overall volume. Reynolds dilatancy is a common phenomena observed in soil mechanics and a deailed review can be found in [27]. This phenomena depends strongly on the initial condition of the

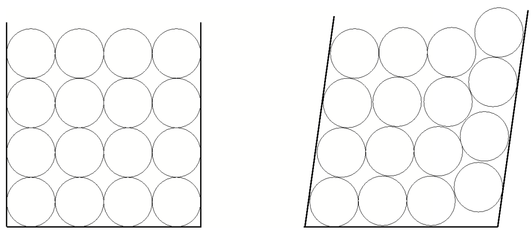

Figure 1: Layers of compact spheres under shear forces.

grains: if the system have been set up in a loose state of packing, a reduction of the volume can be observed. This property is called contractancy.

2.2 Granular Random packings

Random packings have been intensively studied in the last few years and are still a major challenge of the condensed matter physics. They not only provide reliable information about the microscopic structure of granular systems but also other materials like glasses, simple liquids or amorphous solids. Random compaction can be studied by shaking spheres vertically in a box. After the initial work of Scott [28], experimental and numerical investigations enabled the calculation of relevant parameters for random spherical packings. Finney [29], inspired by the work of Bernal [30] about the statistics of random packings, performed computer simulations for packings containing particles of different geometries. He showed that a random packing of spheres cannot have a volume fraction greater than 0.64. The maximal volume fraction that can be obtained when particles are randomly packed is called the random-close-packing, $\hat{\rho}_{cp}$. For hexagonal packings, it reaches a value of 0.74, and is the greatest possible packing that can be obtained for spherical particles. The definition of the random loose packing $\hat{\rho}_{lp}$ is more difficult, it is often referred as the loosest possible packing that is mechanically stable. Onoda et al. [31] identified it as the lowest possible density than can be obtained in a vanishing gravitation field. He carried out experiments by immersing glass spheres in a liquid, wherein the density of the liquid was modified to minimize the effect of gravity, and found the value of $\hat{\rho}_{lp}$ to be 0.56.

2.3 Static stress distribution in sand piles

The pressure distribution below a sand pile does not always exhibit a maximum at the apex. This counterintuitive result has been intensively debated in the literature, and it is now accepted that stress distributions depend on the filling process. Vanel et al [4] showed that by filling sand in successive layers with a sieve a maximum of pressure is measured at the centre. When a hopper is used, a local minimum appears. This is illustrated in Figure 2. A lot of explanations have been proposed (see part 3.2 for instance) in the literature but there is no consensus about it.

2.4 Contact mechanics and effective elastic moduli

Granular Matter is composed of many grains which are in contact and can be deformed. An interesting knowledge about the stresses and deformations which occur when solids are in contact is given by the contact mechanics. The pioneering work of Hertz [32] about frictionless contacts of two elastic

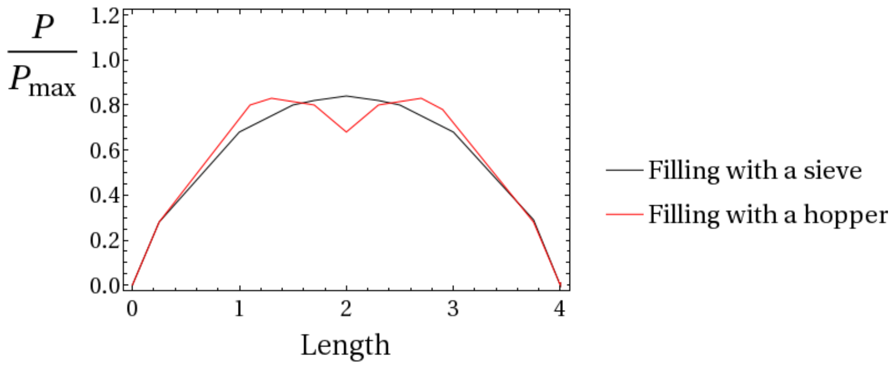

Figure 2: Typical pressure profiles at the bottom of sand pile.

bodies is still a reference for mechanical engineers working on processes involving elastic contacts. The Hertz model has been extended to complicated geometries and used to derive constitutive laws. The main result of the Hertz theory is presented here, and a detailed review can be found in [15]. Consider two elastic spheres of radius R which are in contact, as shown in Figure 3. If these two bodies are pressed together with a force F, deformations occur near the point of contact. Hertz showed that the radius of the circular contact area a, where deformation occur, depends on the displacement δ which is given as

$$\delta = \frac{a^2}{R}. \tag{2.1}$$

The relation between force and displacement is non-linear

$$F \sim \sqrt{R}\delta^{3/2}. \tag{2.2}$$

When the gains deforms, an elastic energy W_e is accumulated, which can be evaluated by integrating the product of the force and the displacement over the volume of the sphere

$$W_e \sim \sqrt{R}\delta^{5/2}. \tag{2.3}$$

Although the two spheres are elastic, the force does not linearly depend on the displacement. When the force increases, the contact area also increases and so does the stiffness of the material. The case of spheres subject to tangential forces is more complicated but have been studied by Mindlin [33]. He established that, in this case the system is path dependent: the values of work depends on the loading steps. Different behaviors are observed depending on rather the system is first sheared and then compressed or vice-versa. Effective Medium Theory (EMT) is based on the Hertz-Mindlin description and

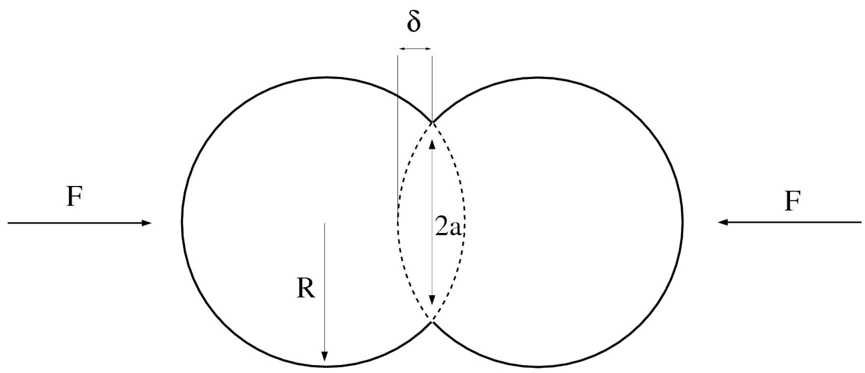

Figure 3: Two elastic spheres of radius R in contact.

enables to obtain the value of effective and bulk modulus of a system compressed with pressure P. These coefficients were shown to have typical dependence on $P^{1/3}$. Some disagreements are observed with experiments where dependences in $P^{1/2}$ are generally measured. Several explanations based on micro-mechanical considerations have been proposed to explain that, like for instance the presence of shell oxide layers between the grains [16]. This issue has not been resolved and is currently under study.

2.5 Characteristics angles of granular materials

2.5.1 Angle of repose of a cohesionless material

Simple macroscopic observations show that granular matter in a static equilibrium can exhibit a conical pile structure. The angle between the free surface of the pile and the horizontal plane is called the angle of repose. The critical angle of repose refers to the greatest angle that can be found for a given material. There are actually two angles of repose, namely the static and the dynamic ones. The static angle is defined as the angle at which an initially static grain assembly begins to flow. The dynamic angle is the angle at which a given quantity of sand becomes at rest. Both static and dynamic angles depend on material parameters like the grain size distribution or the grain shape. There are a lot of measurement techniques to determine these angles and results are very sensitive on the procedure of pouring employed. What is the best design or size of equipment to perform this measurement is still an open issue [8]. Typical values of angles of repose are between 25° and 55°.

2.5.2 Angles of friction

According to Coulomb's law of friction [6] a solid at rest on an inclined plane making an angle ϕ with the ground horizonal plane begins to slide when the tangential force F_t reaches a certain value, proportional to the normal force F_n:

- Solid at rest:
$$F_t < \mu_f F_n, \tag{2.4}$$

- Solid in movement:
$$F_t = \mu_d F_n, \tag{2.5}$$

where μ_s and μ_f are respectively the static and dynamical friction coefficients. Both are empirical parameters that have to be determined experimentally. Because the solid is subjected to gravity, the normal force F_n and the radial one can F_t be expressed as:

$$F_n = \rho g \cos\phi, \tag{2.6}$$

$$F_t = \rho g \sin\phi, \tag{2.7}$$

so that the ratio F_t/F_n is equal to $\tan\phi$. The angle ϕ is called friction angle. In the case of granular systems, frictional contacts appear between the grains and are often modelled with the Coulomb friction law applied to the internal stress tensor (see part 3.4.3). In this report, the notation μ and ϕ refer respectively to the internal friction coefficient and the internal angle of friction. This parameter is employed in civil engineering applications and usually measured with the triaxial test. Although it is possible to apply the Coulomb rule in the case of one grain rolling or being at rest on an inclined plane, generalizing it to an amount of grain is not self-evident. It is important to keep in mind that the angle is defined by assuming an analogy between a rigid solid body and sand. It also does not provide any information about the influence of nature of the grains on the friction coefficients. In practical applications, the angle of friction may differ by a few degree from the angle of repose.

3 Review of continuum models

In this section the constitutive laws that were presented in the literature to account for the solid behaviour of granular systems are presented. The static momentum equation is first recalled here. In a body subjected to a volume force $f_{i,v}$, the local force equilibrium equation is given as

$$\frac{\partial \sigma_{ij}}{\partial x_j} = f_{i,v} \tag{3.1}$$

where σ_{ij} is the symmetric stress tensor. This conservation equation is a system of 3 equations in 6 unknowns that can only be solved if some empirical relations are added, for example between the stress components or between the stress and strain tensors.

3.1 Janssen model for silo

The knowledge about the stress within a silo packed with granular matter is crucial for designers to avoid structure failure. Reliable data about the interaction of the granular media and the silo structure can also help in optimizing processes such as filling or emptying. This topic has been intensively studied by engineering [22] and physicists [34]. Consider a silo filled with a certain mass of grains. Experiments show that the weight measured at the bottom plate fo the silo is only a fraction of the total mass of the silo. In his pioneering work in 1895, Janssen [35] proposed a model to describe the vertical stress in silos, which reproduces the pressure saturation observed, see Figure 4. Although some quantitative differences between the Janssen theory and experiments have been observed, this model is still used. The first assumption proposed by Janssen is that a fraction of vertical stress is transferred to horizontal stress

$$\sigma_{rr} = k_j \sigma_{zz}. \tag{3.2}$$

The shear stress on the walls is then assumed to have reached the maximal value given by the Coulomb failure criterion

$$\sigma_{rz} = \mu_j \sigma_{rr} = \mu_f k_j \sigma_{zz}. \tag{3.3}$$

where μ_j is the internal friction coefficient between the grains and the wall. Consider a horizontal slice of diameter $2R$ and height dz. A the surface of the silo, where $z = 0$, the force balance is

$$d\sigma_{zz} + \frac{2}{R}\sigma_{rz}|_{r=R} = \rho g dz. \tag{3.4}$$

Inserting this condition into the equilibrium condition leads to

$$\frac{d\sigma_{zz}}{dz} + \frac{2\mu_j k_j}{R}\sigma_{zz} = \rho g. \tag{3.5}$$

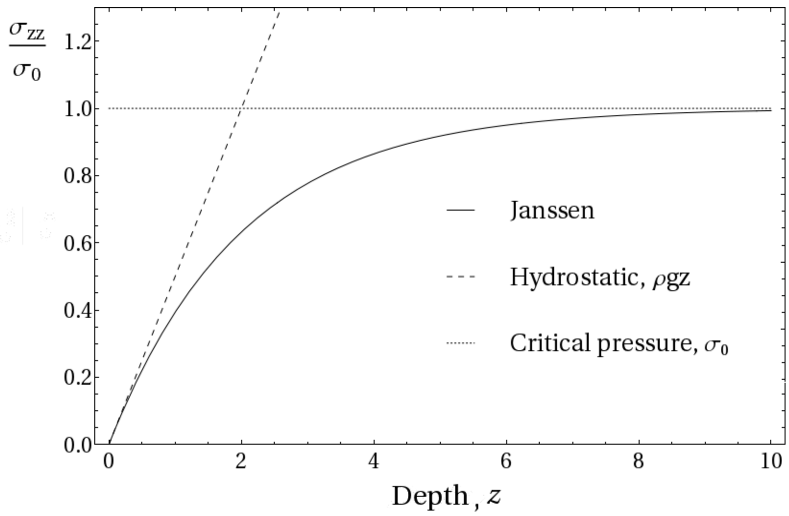

Figure 4: Pressure profile in silos

Introducing the characteristic length $L_0 = R/2\mu_f k_j$ and the pressure $\sigma_0 = \rho g L_0$, the solution can be expressed as

$$\sigma_{zz} = \sigma_0 \left(1 - \exp\left(-\frac{z}{L_0}\right)\right). \tag{3.6}$$

This result can be easily interpreted by looking at Figure 4. Near the free surface of the silo, for $z \ll L_0$, the pressure has the same profile as the hydrostatic one, and for larger depth $z \gg L_0$, it reaches the critical value σ_0. In many industrial applications, the dependence of the Janssen coefficient with the friction angle is expressed via the empirical relation [22]

$$k_j = \sin\phi. \tag{3.7}$$

Janssen model shows quantitative disagreement with experimental data, and more elaborated descriptions have been proposed [36]. The main questionable point is the assumption in equation (3.2), also called assumption of incipient failure, which supposes that forces of friction have reached their maximal values. Due to the absence of reliable experimental data to measure internal stresses, the invalidity of this hypothesis couldn't be clearly proven [37]. However numerical simulations using discrete element simulation using three dimensional packings showed that this criterion wasn't justified.

3.2 Stress-based laws

Fully motivated by providing an explanation to the pressure minimum sometimes observed in sand piles (see part 2.3), constitutive models based on relations between some components of the stress tensor have been proposed in the literature. The simplest one is the incipient failure everywhere theory [22] which supposes that the granular media is at slip failure everywhere. It is however generally accepted that this condition is actually just valid at the free surface of the sand pile. Bouchaud et al. [18] proposed a proportionality relation between the horizontal and principal stresses. The main consequence of this assumption is that, in two dimensions, the force balance equation becomes a hyperbolic equation so that a stress transmission phenomena appears trough certain preferred directions. Because it couldn't explain the stress dip on sand pile, more general models called "Oriented Stress Lineary" [38] have been formulated. On the other hand Edwards [39] performed a description based on the presence of arches on the sand heap which only supports their own weight. A consistent implementation of Edward's picture has then been provided by the "Fixed principal axes model" which can be seen as a direct model for stress propagation and was shown to provide history dependence. A detailed review and comparison of these models can be found in [39]. Although pertinent in 2D, these models seem to be difficult to generalize in 3D [21].

3.3 Elastic modeling

Contrary to the previous models, elastic models consider a displacement field on granular matter from which an elastic strain field can be calculated. It starts from the simple idea that the grains deform elastically. It is crucial to keep in mind that when sand behaves like a solid, it is like a very special solid and therefore its global behaviour is still very different from that of an elastic body. The solid state is only stable under specific conditions. In soil mechanics, it is common to use yield criterion, which specify when plastic flows happen. In this part, the linear elastic framework is reviewed and its applicability to granular systems is discussed.

3.3.1 Elastic strain

The linear elasticity theory [28] describes the deformation of elastic bodies subject to different loading conditions. When an elastic body is loaded, deformations appear, which can be usually described by a displacement field u_i. Here, the term "elastic" means that these perturbations are reversible i.e if the loading is released, the body comes back to its initial state; the elastic displacement leads to a reversible energetic change. Assuming small strains($\frac{\partial u_i}{\partial x_j} << 1$) and rotations, the elastic deformation can be expressed as a function of the elastic displacement through the following kinematic relation

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{3.8}$$

The trace of the strain tensor refers to volume change, its opposite, noted $\Delta = -\epsilon_{ll}$ is positive when the body is in a compressed state. The deviatoric strain $\varepsilon^D_{ij} = \varepsilon_{ij} - \varepsilon_{ll}/3\delta_{ij}$ accounts for the distortion. In the same way, the stress tensor σ_{ij} can also be decomposed into a pressure part $P = \sigma_{ll}/3$ and the a deviatoric stress $\sigma^D_{ij} = \sigma_{ij} - P\delta_{ij}$ which quantifies shearing efforts due to a state of stress.

3.3.2 Isotropic linear elastic material law

In the case of an isotropic linear elastic material, the Hooke law postulates a linear relation between stress and strain tensors :

$$\sigma_{ij} = K\Delta\delta_{ij} - 2G\varepsilon^D_{ij}. \tag{3.9}$$

where K and G are the bulk and shear modulus respectively. The bulk modulus quantifies the resistance of the solid to volume changes and the shear modulus is the resistance to volume preserving shear deformations. The stress-strain relations can be expressed in term of the fourth order stiffness tensor C_{ijkl}

$$\sigma_{ij}\left(\varepsilon_{kl}\right) = C_{ijkl}\varepsilon_{kl}. \tag{3.10}$$

The symmetry of the stress tensor and the existence of a elastic strain energy (see part 3.3.3) implies symmetric properties which reduces it to 21 independent values

$$C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}. \tag{3.11}$$

It is also current to define the compliance tensor S_{ijkl} as

$$\varepsilon_{ij}\left(\sigma_{kl}\right) = S_{ijkl}\sigma_{kl}. \tag{3.12}$$

3.3.3 Strain energy density

When an elastic material is deformed, it accumulates an elastic energy density. In this work W denotes the elastic strain energy density. The strain energy increment is linked to the strain increment through the following relation

$$\partial W = -\sigma_{ij}\partial\varepsilon_{ij}. \tag{3.13}$$

In the case of linear isotropic material, the energy is thus given by

$$W = \frac{1}{2} K \Delta^2 + G \varepsilon_s^2. \tag{3.14}$$

The first term accounts for the energy due to compression, and the second one for shearing. Notice that the stiffness tensor can also be expressed as a function of the elastic energy (which justifies the first relation of (3.11))

$$C_{ijkl} = -\frac{\partial W}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}. \tag{3.15}$$

The values of the stiffness is determined by the Hooke law

$$C_{ijkl} = \left(\frac{2}{3} G - K \right) \delta_{ij} \delta_{kl} - G \left(\delta_{ik} \delta_{jl} + \delta_{jk} \delta_{il} \right). \tag{3.16}$$

The stability of the elastic energy requires that the strain energy density to be a convex function of strain [40]. Being a quadratic function of the compression and the shear, the linear isotropic elastic energy satisfies this condition.

3.3.4 Shortcomings of linear elasticity to model granular matter in solid state

The stress-strain relation of the Hooke law enables to close the force balance and stress distributions can be calculated in this way. The linear elasticity theory can model the elastic behaviour of a wide range of materials (metals, glass, polymers,...) submitted to small deformations. But linear isotropic laws are not appropriate to describe granular materials. The stress-strain relations (3.9) are linear and thus doesn't account for the stiffness dependence on stress currently observed on the granular Matter [41] and for dilatancy and contractancy. Although the behaviour of each grain can be modelled as linear elastic, the mechanical behaviour of an amount of grain is not linear elastic. Sand does not always exhibit a solid behaviour: there exist a state from which the granular matter starts to flow, and soil mechanicians use elastoplastic laws to account for that. In plasticity theories, a yield criterion must be formulated to determine when plastic flows appear. Finally, if the Hooke law could model sand, it would mean that sand has the same behaviour during loading or unloading, what is not the case. This property is sometimes called incremental non-linearity and any relevant continuum model should include it.

3.4 Plasticity

3.4.1 Plastic strain

Plastic models describe the non-reversible deformation of a material submitted to applied forces. Generally, in the beginning of a loading process, a solid body exhibit an elastic behaviour. After a certain loading time, irreversible effects occur and plastic contribution must be taken into account in the total deformation field : $u_i = u_i^e + u_i^p$. To specify when plastic deformations appear, plasticity theories formulate yield criterion. Plastic models have been adapted for granular systems to model the transition from the solid state to the fluid one. Different yield surface and flow rules have been proposed in the literature [22]. Some of them will be introduced here.

3.4.2 Invariants to describe yield surfaces

Principal stresses and invariants are often used to formulate yield criterion or energy density. In a stressed body, it is always possible to find three planes where the stress vector is normal to the plane and does not have shear components. These three stresses are called the principal stresses σ_i^p. The strain and stresses have respective invariants I_i, whose values do not depend on the coordinate system. It can be the three principal stresses but it is also useful to express it in term of the stress

$$I_1 = \frac{\sigma_{ii}}{3} = P, \tag{3.17}$$

$$I_2 = \frac{1}{2}\left(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}\right), \tag{3.18}$$

$$I_3 = \det\left(\sigma_{ij}\right). \tag{3.19}$$

The deviatoric stress tensor also has a set of invariants. The principal directions of the deviatoric tensor are the same as the principal directions of the stress tensor and the invariants are given as

$$J_1 = 0, \tag{3.20}$$

$$J_2 = \sigma_{ij}^D \sigma_{ij}^D = \frac{1}{3} I_1^2 - I_2 \equiv \frac{\sigma_s^2}{2}, \tag{3.21}$$

$$J_3 = \det\left(\sigma_{ij}^D\right). \tag{3.22}$$

These invariants are often used in the formulation of yield criterion. It is also useful to use the strain invariants, the shear strain $\epsilon_s = \sqrt{\varepsilon_{ij}^D \varepsilon_{ij}^D}$ is defined as the second invariant of deviatoric strain tensor.

3.4.3 Mohr Coulomb failure criterion

Coulomb studied the stability of granular matter submitted to forces. He found that plastic failure appears if the shear stress τ_n on a plane exceed a constant fraction of the normal stress σ_n [22]:

$$|\tau_n| > \mu\sigma_n + c. \tag{3.23}$$

where μ is the coefficient of friction and c the cohesion. In the case of cohesionless material c is equal to 0. Assuming the analogy with solid friction developed in (2.5.2), the angle of friction is defined such that $\tan\phi = \mu$. It depends on the nature of the material and ranges from 20 and 50°. This criteria can also be expressed in term of the principal stresses, according to the Mohr-Coulomb failure analysis [22]:

$$\frac{\sigma_1^p}{\sigma_3^p} < \frac{1+\sin(\phi)}{1-\sin(\phi)}. \tag{3.24}$$

3.4.4 Drucker-Prager yield function

Generally used in the soil mechanics, the Drucker-Prager [23] yield surface is an approximation of the Mohr-Coulomb criteria that accounts for the hydrostatic pressure component of the stress. Plastic effects appear if :

$$\frac{\sigma_s}{P} > \alpha, \tag{3.25}$$

where α is a material parameter which must be determined experimentally. Because α is often difficult to measure, it is preferable to express it as a function of the angle of friction. In many soil mechanics applications, it is assumed that the Drucker-Prager yield surface circumscribes that of Mohr-Coulomb and thus the following relation holds [22]

$$\alpha = \frac{2\sin\phi}{\sqrt{3}\,(3-\sin\phi)}. \tag{3.26}$$

In the case of strain plane deformation it reduces to [22]

$$\alpha = \frac{\sin\phi}{\sqrt{2}}. \tag{3.27}$$

4 Jiang-Liu hyperelastic constitutive model

4.1 Hyperelastic constitutive law

Hyperelastic models are generally used to describe materials displaying non-linear stress-strain behaviour, even for small deformations [42]. The terms "hyper" means that the stress-strain relation is derived from an energy potential, in opposition to "hypo". Once formulated, the expression of strain energy density determines the stress-strain relation. The modelling effort is then reduced to finding a relevant scalar expression for the strain energy density and it ensures that the deformations of the material are reversible [42]. For an isotropic material, the energy does not depend on the loading direction of the material, which means that it must be function of the strain tensor invariants.

4.2 Granular energy density

First attempts to express non-linear stress-strain relations can be found in the early work of Boussinesq [12] who formulates stresses-dependent bulk and shear moduli. In an early work, Goddard [11] developed a hyperelastic approach and proposed nonlinear strain energy functional to describe the sand at solid state. Liu and Jiang, in a theory called granular elasticity [23], starts from a strain energy density which is a special form from those formulated by Goddard:

$$\begin{cases} \text{If} \quad \Delta \geqslant 0 \quad W(\Delta, \epsilon_s) = \sqrt{\Delta}\left(\frac{2}{5}B\Delta^2 + A\epsilon_s^2\right) = B\sqrt{\Delta}\left(\frac{2}{5}\Delta^2 + \xi\epsilon_s^2\right) \\ \text{If} \quad \Delta < 0 \quad W(\Delta, \epsilon_s) = 0 \end{cases} \tag{4.1}$$

with $\Delta = -\varepsilon_{ll}$, $\varepsilon_s^2 = \varepsilon_{ij}^D \varepsilon_{ij}^D$, $\xi = B/A$ and A, B are two material parameters. The elastic energy is a function of the first invariant of the strain tensor and of the second invariant of deviatoric strain. It could be also possible to add a dependence on the third invariant of strain, as initially proposed by Goddard [11], but this extension is not considered here as it is assumed that the no linear behaviour of sand can be described only by taking account for shear and compression deformations. This energy is only defined for compaction states ($\Delta \geq 0$) and its expression makes it consistent with the Hertz contact (cf equation (2.3)). Liu and Jiang [14] demonstrated that the elastic energy is convex if the following conditions are satisfied

$$\frac{\partial W}{\Delta} > 0, \quad \frac{\partial^2 W}{\partial \Delta^2} > 0, \tag{4.2}$$

$$\frac{\partial^2 W}{\partial \varepsilon_s^2} > 0, \quad \frac{\partial^2 W}{\partial \varepsilon_s^2}\frac{\partial^2 W}{\partial \Delta^2} > \left(\frac{\partial^2 W}{\partial \varepsilon_s \partial \Delta_s}\right)^2. \tag{4.3}$$

The three first conditions leads to positivity of the elastic parameters A and B. The fourth condition can be calculated from the expression of the energy [24]

$$\frac{\varepsilon_s}{\Delta} \leq \sqrt{2\xi}. \tag{4.4}$$

This inequality can also be expressed with the stress tensor invariants

$$\frac{\sigma_s}{P} \leq \sqrt{\frac{1}{\xi}}. \tag{4.5}$$

Equation (4.5) is the Drucker-Prager variant of the Coulomb condition, a yield function presented in the part (3.4.4). It can be observed from Figures 5 and 6 that the elastic energy is composed of a convex and a concave part, Figure 5 shows the remarkable property that the stability properties of the elastic energy are fixed by the Drucker-Prager criteria. The angle of friction is determined by the parameter ξ which is the ratio of the two elastic constants A and B. In contrary, the elastic strain function from which the Hooke law cenbe derived, is a convex function of the shear and of compression according to (3.14) and doesn't exhibit any instabilities.

4.3 Stress-Strain relation

Starting from (4.1), the first derivate of the energy can be calculated to get the stress tensor

$$\sigma_{ij}\left(\varepsilon_{kl}\right) = -\frac{\partial W}{\partial \varepsilon_{ij}} = K\Delta\delta_{ij} - 2G\varepsilon_{ij}^{D}. \tag{4.6}$$

where the bulk and shear moduli are given as a function of the strain as

$$K = B\sqrt{\Delta}\left(1 + \frac{\varepsilon_s^2}{2\Delta^2\xi}\right), \tag{4.7}$$

$$G = \frac{B\sqrt{\Delta}}{\xi}. \tag{4.8}$$

The pressure P can thus be calculated as

$$P = B\Delta^{\frac{3}{2}} + \frac{1}{2}A\frac{\epsilon_s^2}{\sqrt{\Delta}}. \tag{4.9}$$

and the shear stress σ_s

$$\sigma_s = 2A\sqrt{\Delta}\varepsilon_s. \tag{4.10}$$

The pressure is composed of two terms, the first one, also employed in the Boussinesq model [10], reflects the non linear increase of the pressure under isotropic compression. This is in agreement with the results of the EMT, which also predict a dependence on the pressure in $\Delta^{3/2}$. The physical interpretation of the second is more difficult as it adds a non-trivial dependence on

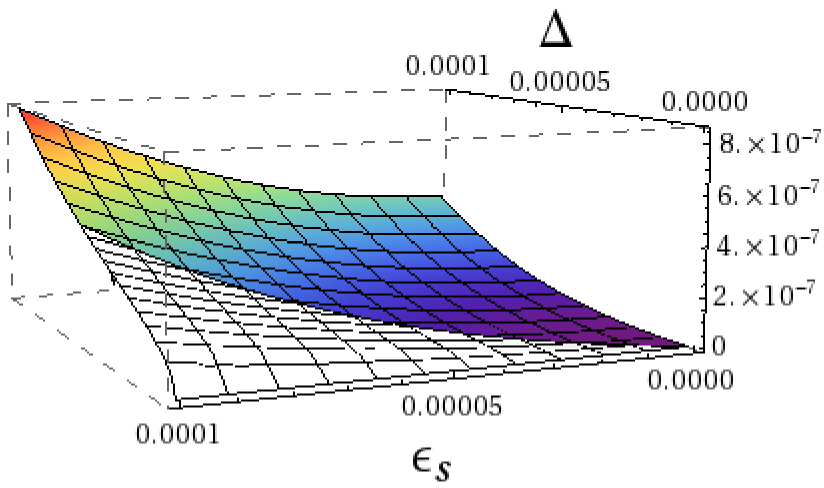

Figure 5: Graph : Plot if the elastic energy W in μJ for Δ and ε_s varying from 0 to 10^{-4}. The colored zone corresponds to the convex part of the elastic energy. The white zone corresponds to the concave part.

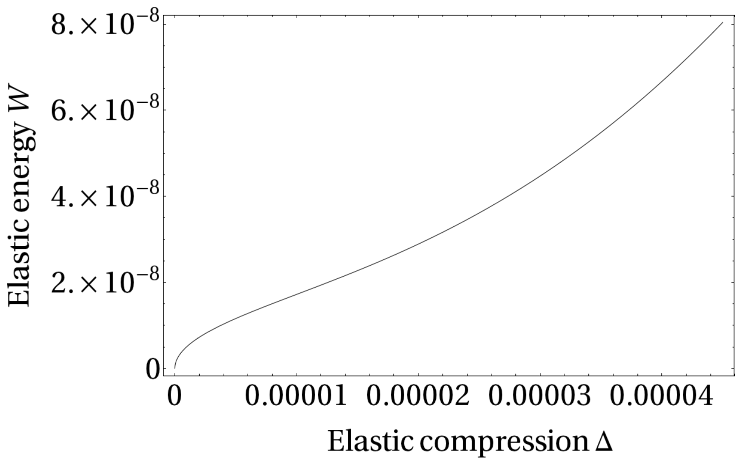

Figure 6: Plot of the values of the elastic energy W in μJ at constant shear, $\varepsilon_s = 5.10^{-4}$.

the shear to the elastic compression, which could be explained by Reynolds dilatancy. But one should keep in mind Reynolds dilatancy may be explained by plastic effects rather than elastic ones, and the contribution to elastic effects on dilatancy seems *a priori* difficult to quantify. This term is however a direct consequence of the stability properties of the energy density. The variation of shear modulus G in $\sqrt{\Delta}$ is also found in EMT. The Figures 7 and 8 show the pressure and the shear stress as function of the compression at constant shear rate. It can be observed that the pressure exhibits a minimum at $\varepsilon_s/\Delta = \sqrt{6\xi}$, which is situated in the concave domain of the elastic strain energy. Beyond the yield region there is qualitatively, no difference with a linear elastic law, the compressibility $\frac{\partial P}{\partial \Delta}|_{\varepsilon_s} = \frac{3B}{2\sqrt{2}} - \frac{A\varepsilon_s^2}{4\Delta^{3/2}}$ is positive. The region where it becomes negatives and then displays a non-linear behaviour, is situated on the concave domain of the elastic strain energy function.

4.4 Stiffness tensor

The calculation of the stiffness matrix leads to the following expression

$$C_{ijkl} = A\sqrt{\Delta}\left(\left(\frac{\varepsilon_s^2}{4\Delta^2} - \frac{3B}{2A} + \frac{2}{3}\right)\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} + \frac{1}{\Delta}\left(\delta_{kl}\varepsilon_{ij}^d + \delta_{ij}\varepsilon_{kl}^d\right)\right). \tag{4.11}$$

Assuming that elastic strain dominate in static stress distribution and that small strain increments lead to a reversible stress transformation, a comparison with experimental data is possible. The values of the shear and bulk modulus of the Liu model were thus successfully compared to the experiments of Kuwano [10, 14].

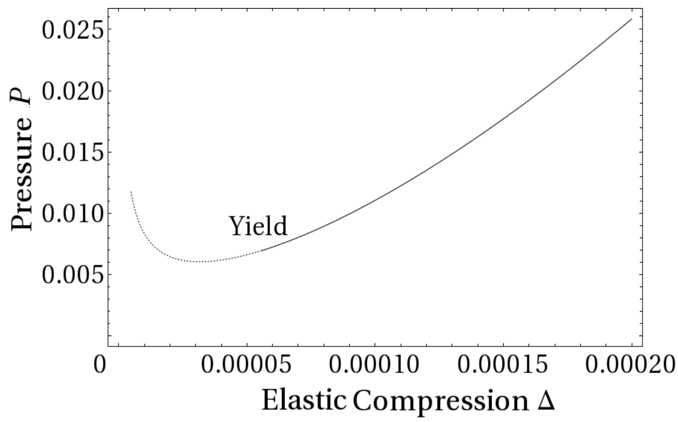

Figure 7: Plot of the pressure at constant shear, $\epsilon_s = 0.002$.

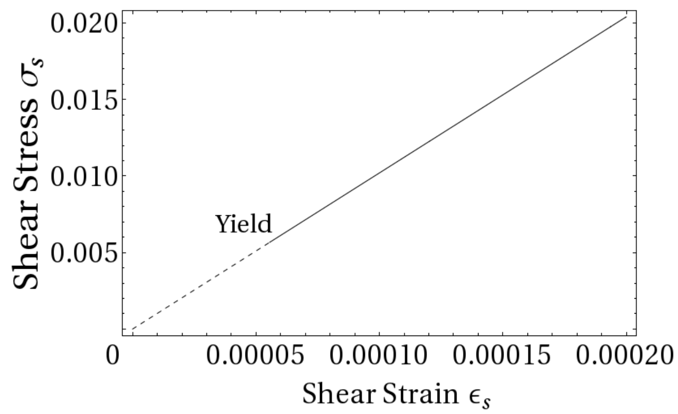

Figure 8: Plot of the shear stress at constant compression $\Delta = 0.0002$.

5 Numerical application: calculation of stress distribution in sand piles with the Jiang-Liu model

Numerical applications concerning static stress distributions in silos, under point loads and in a sand pile using the Jiang-Liu model, introduced in the previous part, can be found in [24, 39]. Good agreements with experimental data were observed. Particularly in the case of silos, it reproduces the stress saturation currently observed. The anisotropic response of a sheared granular layer subjected to a point-load at his surface was calculated [25]. Humrickhouse [39] numerically tested the influence of adding a dependence on the third strain invariant on the strain energy density. Numerical application of the stress distribution in a sand pile is detailed here. The influence of the geometry of the problem on the stability of the solution are investigated. The approach detailed here is based on a discretization of the domain with the finite element method. The Newton-Raphson iterations are used to solve the resulting non-linear equations. The finite element freeware Freefem [39] is employed to discretize and solve the equations.

5.1 Problem setting

2D problem A two dimensional sand heap is considered. The only external forces applied is the gravity force g_i. x and z correspond to the horizontal and vertical axis respectively. The displacement, strain and stress in the y direction are assumed to be zero. The two components of the displacement fields are the two unknowns of the force balance equation, which is a second order system of coupled differential equations in u_x and u_z. As Boundary conditions, grains are glued at the bottom of the sand pile $\partial\Omega_2$ and the normal stress equals to zero on the free surface of the pile $\partial\Omega_1$:

$$\begin{cases} \frac{\partial \sigma_{ij}}{\partial x_j} = \rho g_i & \text{in } \Omega \\ \sigma_{ij} n_j = 0 & \text{in } \partial\Omega_1 \\ u_i = 0 & \text{in } \partial\Omega_2 \\ \sigma_{ij} = B\Delta\sqrt{\Delta}\left(1 + \frac{\varepsilon_s^2}{2\Delta^2\xi}\right)\delta_{ij} - \frac{B\sqrt{\Delta}}{\xi}\varepsilon_{ij}^D & \text{in } \Omega \\ \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) & \text{in } \Omega \end{cases} \tag{5.1}$$

Because of the non-linearity of the strain-stress relation, (5.1) is a system of non linear differential equations in u_i, and therefore requires a numerical method to approximate the solution. The Newton-Raphson will be used here.

Weak formulation In the finite element analysis, the principle of virtual work is currently used to write the force balance equation in an integral form. It consists on finding a displacement field u_i which satisfies

$$\int_\Omega \sigma_{ij}\left(\varepsilon_{kl}\right)\varepsilon^*_{ij}d\Omega - \int_\Omega f^v_i u^*_i d\Omega = 0, \tag{5.2}$$

for all the virtual displacement field u^*_i. In order to simplify the formalism of equation, Voigt notations are introduced, using the symmetrical properties of the strain and stress tensor

$$\varepsilon_i \equiv \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix} = \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix} \; \sigma_i \equiv \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix} = \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}. \tag{5.3}$$

The compression and the traceless part of the deformation can be expressed as

$$\Delta = -\varepsilon_1 - \varepsilon_2 \qquad \begin{bmatrix}\varepsilon^D_1\\ \varepsilon^D_2\\ \varepsilon^D_3\end{bmatrix} = \begin{bmatrix}\frac{\varepsilon_1-\varepsilon_2}{2}\\ \frac{\varepsilon_2-\varepsilon_1}{2}\\ \frac{\varepsilon_3}{2}\end{bmatrix}. \tag{5.4}$$

The stress-strain relation reads

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix} = \begin{bmatrix}K\Delta - 2G\varepsilon^D_{11}\\ K\Delta - 2G\varepsilon^D_{22}\\ -2\mu\varepsilon_{12}\end{bmatrix} = \begin{bmatrix}K\Delta - G(\varepsilon_1-\varepsilon_2)\\ K\Delta - G(\varepsilon_2-\varepsilon_1)\\ -G\varepsilon_3\end{bmatrix}. \tag{5.5}$$

Because $\sigma_{ij}\varepsilon_{ij} = \sigma_i\varepsilon_i$, the weak formulation can now be written in a more simplified form as

$$\int_\Omega \sigma_i\left(\varepsilon_p\right)\varepsilon^*_i d\Omega - \int_\Omega f^v_i u^*_i d\Omega = 0. \tag{5.6}$$

Finite element discretization The displacement field is interpolated at a set of n nodes. The unknown displacement vector at each nodal point is written u^a_i. A finite element space can be generated from given geometries, the canonical basis is built with continuous piecewise quadratic functions $N_a(x,z)$, which enables to interpolate the displacement field

$$u_i(x,z) = \sum_{a=1}^{n} N_a(x,z)u^a_i. \tag{5.7}$$

The virtual velocity field can therefore be interpolated in a similar fashion and finally Eq (5.6) is approximated to a alegraic system of equations, that can be solved using efficient iterative solvers. More details about the way to build the stiffness matrix can be found in [42].

Newton-Raphson iterations Because the stress-strain relation is non-linear in u_i, the Newton-Raphson process is used to approximate the solution, the same procedure is sometimes employed for the solution of static hypoelastic problems [42]. Let suppose that u_p^n is solution of (5.6). At the next integration step the new displacement field u_p^{n+1} is expressed the sum of the old one and a displacement increment δu_p^{n+1} :

$$u_p^{n+1} = u_p^n + \delta u_p^{n+1}. \tag{5.8}$$

By defining ε_p^n as $\varepsilon_p^n = \varepsilon_p^n(u_l^n)$, the term is also expressed as:

$$\varepsilon_p^{n+1} = \varepsilon_p^n + \delta \varepsilon_p^{n+1}. \tag{5.9}$$

The weak formulation is given by:

$$\int_\Omega \sigma_i \left(\varepsilon_p^n + \delta\varepsilon_p^{n+1}\right) \varepsilon_i^* d\Omega - \int_\Omega f_i^v u_i^* d\Omega = 0, \tag{5.10}$$

and the first term can be linearized at the first order of ϵ_p :

$$\int_\Omega \left(\sigma_i \left(\varepsilon_p^n\right) + \frac{\delta\sigma_i(\varepsilon_p^n)}{\delta\varepsilon_p^n}\delta\varepsilon_p^{n+1}\right) \varepsilon_i^* d\Omega - \int_\Omega f_i^v u_i^* d\Omega = 0, \tag{5.11}$$

$$\int_\Omega \left(\sigma_i \left(\varepsilon_p^n\right) + C_{ip}(\varepsilon_p^n)\delta\varepsilon_p^{n+1}\right) \varepsilon_i^* d\Omega - \int_\Omega f_i^v u_i^* d\Omega = 0, \tag{5.12}$$

where $C_{ip} \equiv \frac{\delta\sigma_i}{\delta\varepsilon_p^n}$ is 3-3 symmetric matrix, and (5.12) leads to :

$$\int_\Omega \left(\sigma_i \left(\varepsilon_p^n \left(u_l^n\right)\right) + \delta\sigma_i \left(u_l^n, \delta u_l^{n+1}\right) \varepsilon_i^*\right)\right) d\Omega - \int_\Omega f_i^v u_i^* d\Omega = 0. \tag{5.13}$$

(5.13) is a linear system equations in δu_x^{n+1} and δu_z^{n+1}.

Convergence The convergence of the Newton-Raphson iteration strongly depends on the proximity of the initial guess with the solution. If the guess is sufficiently close to the correct answer, it converges quadratically. Using the non-linear elastic energy for this work, it is also crucial that the guess satisfies the convexity condition: $\frac{\varepsilon_s}{\Delta} \leqslant \sqrt{2\xi}$. A way to obtain a pertinent guess can be to solve the force balance equation for a linear stress-strain relation which approximates the real ones. But for the case of geometries displaying a great angle of inclination, it doesnt ensure that the stability condition (4.4) is respected. It is then reasonable to start from the following guess which assures a compaction state and the energy to be convex:

$$u_i^{(0)} = \begin{bmatrix} u_x^0(x,z) \\ u_z^0(x,z) \end{bmatrix} = \begin{bmatrix} 0 \\ -10^{-5}(z-H)/H \end{bmatrix}. \tag{5.14}$$

Algorithm Iterations are performed until $\left|\dfrac{\delta u_i^{(n+1)}}{u_i^{n+1}}\right|$ becomes smaller than the a tolerance value tol. The algorithm is given by

1. Setting the value of the guess using (5.14)

2. Assuming $u_i^{(n)}$ is known, calculating $\delta u_i^{(n+1)}$ by solving (5.13)

3. $u_i^{(n+1)} = u_i^{(n)} + \delta u_i^{(n+1)}$

4. If $\left|\dfrac{\delta u_i^{(n+1)}}{u_i^{n+1}}\right| \leqslant tol$ then stop calculation. Else go to 2. where tol is the tolerance of the solution.

5.2 Results

Sand Pile with constant slope The force balance equation is solved on a triangular geometry, the mesh system can be seen in Figure 9. The values of elastic constants of the Jiang-Liu constitutive law (4.6) are taken to be $A = 5100$ MPa and $B = 8500$MPa, what gives to $\xi = B/A$ the value 5/3, as suggested by Jiang and Liu [1]. For an inclination angle of 26° calculations were stopped after 6 iterations for a tolerance of 1.10^{-4}.
The values of the elastic strain tensor can be seen in Figures 10-12 and Figures 13-18 illustrate the displacement and stress fields. The profile of the normalized stress on Figure 16 provides qualitatively good agreements with the pressure profile measured by Vanel at al [4] in the case of a sand pile constructed with a sieve. The maximal value of the normalized stress is reached at the point$(-L/2; 0)$. The displacement in the z direction illustrated in Figure 17, is always negative due to the axial compression of the pile due to his own weight. Figure 19 shows the value of the convexity coefficient defined as $C_{vx} = -\varepsilon_s/\Delta + \sqrt{2\xi}$, which must stay positive to ensure that yield is not reached. The resulting deformed geometry can be observed in Figure 20, which is obtained from the initial geometry by making the following transformation (as post processing): any point A(x_a, y_a) is translated to A'$(x_a + u_x|_A, y_a + u_y|_A)$.

Sand Pile with non constant slope A simple observation of dunes and sand piles reveal that they do not have a regular triangle profile and the slope is often inclined. The force balance equation is solved one the geometries of the Figures 23-26. Simulations show that it is possible to find elastic solutions as long as the slope is not higher than a critical value.

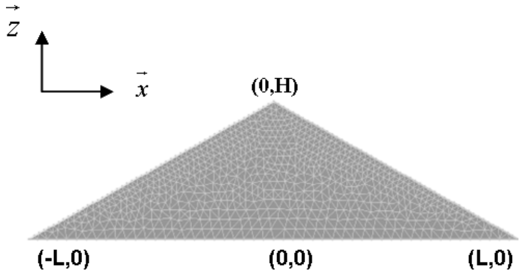

Figure 9: Mesh System.

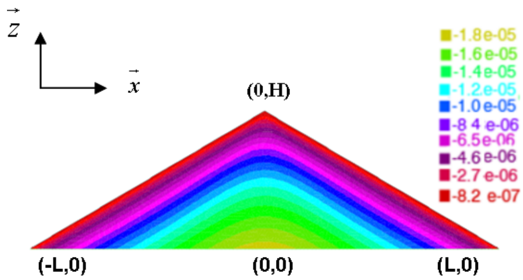

Figure 10: Elastic Strain ε_{zz}.

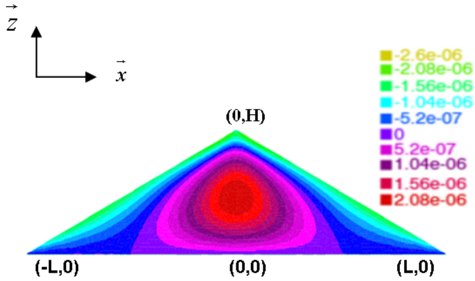

Figure 11: Elastic Strain ε_{xx}.

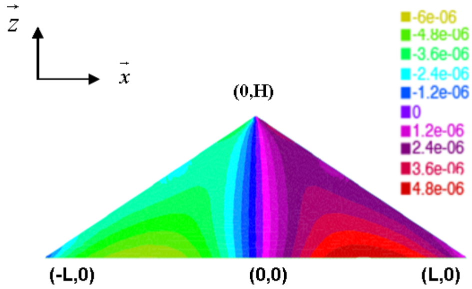

Figure 12: Elastic Strain ε_{xz}.

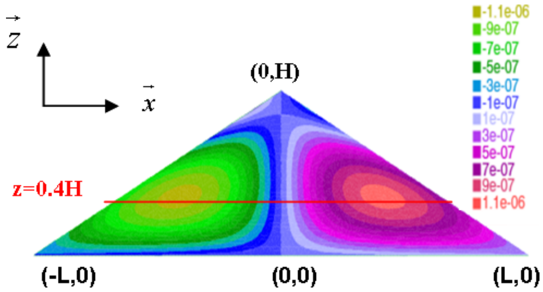

Figure 13: Normalized displacement u_x/H.

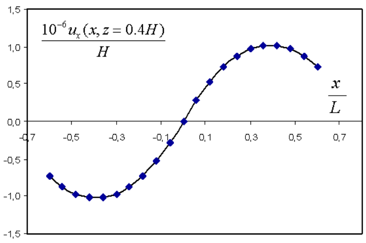

Figure 14: Plot of the normalized displacement u_x/H at $z = 0.4H$.

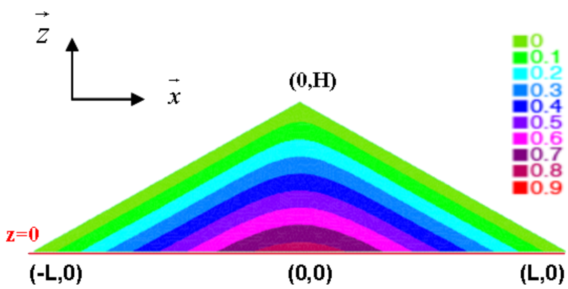

Figure 15: Normalized stress $\sigma_{zz}/\rho g H$.

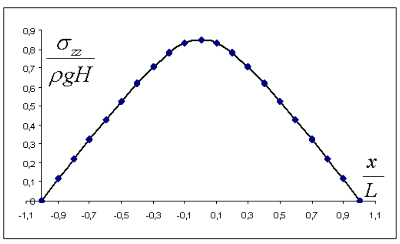

Figure 16: Normalized $\sigma_{zz}/\rho g H$ at the bottom of the pile.

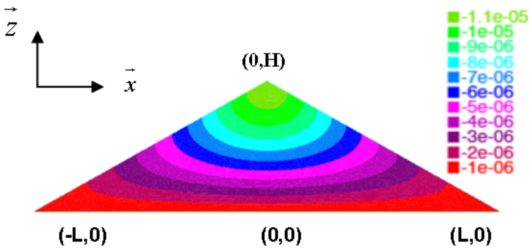

Figure 17: Normalized displacement u_y/H.

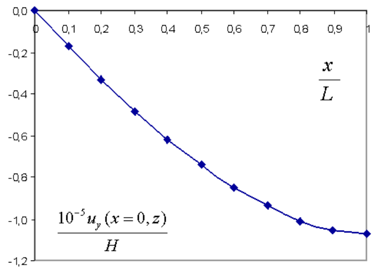

Figure 18: Normalized displacement u_y/h at $x = 0$.

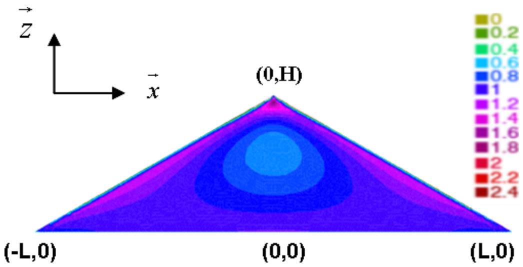

Figure 19: Plot of the convexity coefficient.

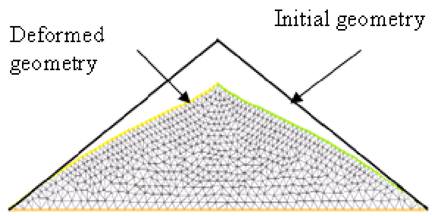

Figure 20: Plot of the deformed geometry ($\times 25000$).

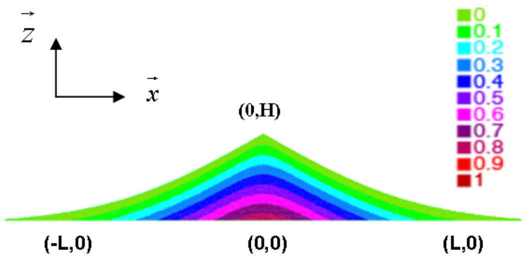

Figure 21: Normalized stress $\sigma_{zz}/\rho g H$ on a pile with strong slope variations.

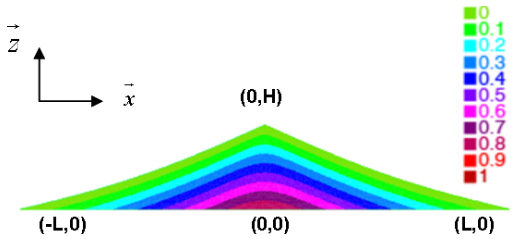

Figure 22: Plot of the normalized stress $\sigma_{zz}/\rho g H$ on a pile with small slope variations.

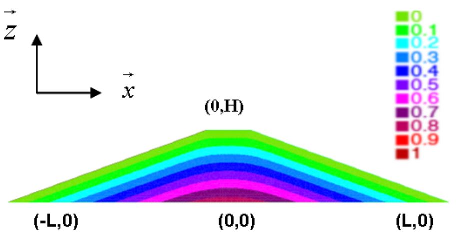

Figure 23: Normalized stress $\sigma_{zz}/\rho g H$ on a pile with a flat apex.

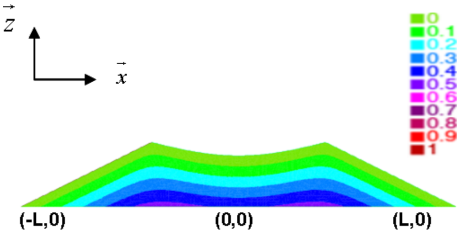

Figure 24: Plot of the normalized stress $\sigma_{zz}/\rho g H$ on a pile with two apexes.

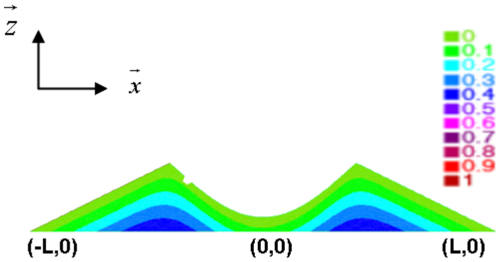

Figure 25: Plot of the normalized stress $\sigma_{zz}/\rho g H$.

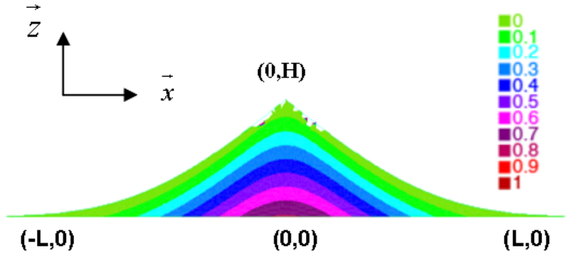

Figure 26: Plot of the normalized stress $\sigma_{zz}/\rho g H$.

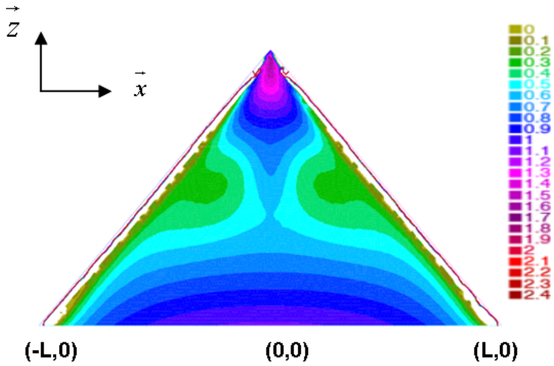

Figure 27: Plot of the convexity coefficient at the fourth iterations of the Newton process.

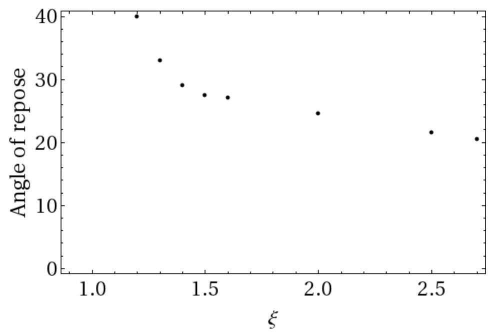

Figure 28: Variation of the maximal angle of repose with the parameter ξ.

Unstable solutions For the same values of the materials parameters, solutions cannot be calculated for an angle of pile greater than 26°. It was observed that if during the Newton-Raphson iterations, the stability condition is violated, then the linear system obtained from (5.13) cannot be solved, this is a natural consequence of lost of convexity of the elastic energy. Figure 27 show the value of the convexity coefficient after 4 iterations in the Newton process, the white zone showing the region where the energy becomes concave. Theses instabilities appear near the free surface of the sand. This can also be observed on the Figures 25 and 26.

Relation between ξ and the angle of repose In granular elasticity, the parameter ξ fixes the yield surface. The dependence on this material parameter with the maximal angle of repose obtained from the computations is investigated here. For each value of ξ, static solutions on the geometry of the Figure 9 are calculating starting from small inclination angles of the pile and increasing it until no static solutions can be calculated. The maximal value of the inclination angle is referred to the angle of repose of the material. This operation is repeated by varying ξ from 0.5 to 2, and by setting the elastic parameter B to 8500 MPa. Notice that the inverse procedure that involve setting the value A also lead to the same results. Figure 28 show that static solution can be obtained for angle of repose varying between 20° and 40,°, and that the maximal angle of repose is inversely proportional to ξ.

6 Summary and conclusion

Although granular matter sometimes behaves like a solid, it displays some typical characteristics like yield, non-linear stress-strain relations, Reynolds dilatancy that any relevant continuum model should be able to reproduce. Some simple constitutive models assuming relations between stress components, were shown to reproduce correctly stress distributions in silos but are based on some questionable hypothesis. Some disagreements with experimental data were also observed and model extensions proposed [36]. On the other hand, the quasi-elastic approach, which postulates non-linear stress strain-relations, is a powerful tool to account for this effects. Coupled with some thermodynamical considerations, the Granular Elasticity approach developed by Liu et al. [1] was shown to capture the main features of the granular solid. One interesting point of this theory is that the elastic strain energy is either a convex or a concave function of the elastic variables and the energy becomes concave when the Drucker-Prager yield surface is violated. The expression of the elastic energy provides a stress-strain relation compatible with Hertz contacts which can be used to close the system of equations and to calculate static stress distributions. Resulting elastic coefficients were compared with experiments [14], providing good agreements. A numerical example detailed in this work showed the relevance of this approach: the force balance equation was solved using the Newton-Raphson method on two dimensional domains discretized with finite elements. Numerical results shows that static stable solutions can be found only when the slope of the pile is smaller than a critical value which can related to the material parameter ξ. Stress distributions were computed not only on triangular domains but also for more complicated geometries.

However there is some limitations on the approach presented here. Vanel at al [4] experiments show that the stress profiles in a sand pile depends on how sand is filled. It suggests that the elastic variables must be a result of all the sand "history", and that the predictive power of static theories is restricted by the lack of information from the past. Although some static models have been built to account for the history dependence, one should consider that a calculation of static stress distributions should be seen as a result of a more general description of granular systems which includes a description of the dilute and dense flows and of the transition between the fluid state and the solid ones.

Acknowledgment This work is supported by the Kaiserslauterer Innovationszentrum für Applied System Modeling. The authors are grateful to Prof. Liu for the fruitful discussion and the comments to our work.

List of Symbols

Acronyms

EMT	Effective Medium Theory

Notations

A	Elastic constant	16
B	Elastic constant	16
C_{vx}	Convexity coefficient	24
C_{ijkl}	Stiffness matrix	19
f_V	Volume force	9
F	Force	6
F_n	Normal force	8
F_t	Tangential force	8
g_i	Gravitation field	8
g	Gravitation field	21
G	Shear Modulus	12
K	Bulk Modulus	12
P	Pressure	17
u_i	Displacement field	12
W	Elastic energy density	12

ε_{ij}	Elastic strain	12
ε_{ij}^D	Deviatoric strain	12
ε_S	Second deviator strain invariant	14
μ	Friction coefficient	8
μ_j	Internal friction coefficient in the Janssen model	9
μ_s	Solid friction coefficient	8
μ_f	Fluid friction coefficient	8
ξ	Elastic constant in the Liu model	16
$\hat{\rho}_{lp}$	Random loose packing	5
$\hat{\rho}_{cp}$	Random close packing	5
σ_{ij}	Cauchy stress tensor	12
σ_{ij}^D	Deviatoric stress tensor	12
σ_s	Second deviator stress invariant	14
τ	Shear stress	14
ϕ	Internal angle of friction	8
Δ	Elastic compression	12

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