# Keeping Track of Position and Orientation of Moving Indoor Systems by Correlation of Range-Finder Scans 

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One of the problems of autonomous mobile systems is the continuous tracking of position and orientation. In most cases, this problem is solved by dead reckoning, based on measurement of wheel rotations or step counts and step width. Unfortunately dead reckoning leads to accumulation of drift errors and is very sensitive against slippery. In this paper an algorithm for tracking position and orientation is presented being nearly independent from odometry and its problems with slippery. To achieve this results, a rotating range-finder is used, delivering scans of the environmental structure. The properties of this structure are used to match the scans from different locations in order to find their translational and rotational displacement. For this purpose derivatives of range-finder scans are calculated which can be used to find position and orientation by crosscorrelation.

## 1. Motivation

While navigating through an unknown or partial unknown environment, tracking of position and orientation is of importance. Without an imagination of the actual position and orientation the exploration of the environment is useless and the chance to find a way back to the starting point is small.

### 1.1. State of the Art

Many authors use dead reckoning for position and orientation estimation. This solution is easy to implement on wheeled vehicles and with some more effort also usable on legged systems. Nevertheless,
in both cases, errors will sum up, so that the reliability of position and orientation estimation decreases over distance[5].
Other solutions for position and orientation estimation try to locate the mobile system by means of artificial landmarks, so that position and orientation can be measured relative to these landmarks. Although in most cases artificial landmarks are easy to detect, this approach has disadvantages. There must be at any time enough landmarks visible to the mobile system. It is impossible for a mobile system to work in areas, where no artificial landmarks are preinstalled.
The requirement of artificial landmarks can be overcomed by the usage of natural landmarks. Here are two approaches possible: if a map of the environment is previously known, a range-finder scan can be matched with this map ([1], [2], [4]). Also if the environment is previously unknown, natural landmarks can be extracted from a range-finder scan, so that a map can be built ([3], [7]). Prerequisite for the success of that method is, that the current scan of the range-finder of the moving system includes parts of the surrounding walls. This is necessary to identify these walls as the "surrounding box"[6], because these approaches rely on the assumption, that for every scan such a surrounding box can be found.

### 1.2. The Need of a New Approach

The prerequisite of the presence of walls in the scan range is in most cases too rigid. Whenever a moving system is in a typical industrial environment, it is quite common that walls are not inside the scanned range. In most cases other structures, like some in-
stalled machines or shelves can be seen by the range-finder. Although the surfaces of these structures are not represented by straight lines like walls are, two scans recorded at different places can easily be matched by a human being. The displacement of position and orientation of the two scans is given by this matching. But what is easy for humans is not simple for machines: what is needed are recognizable properties of the scan as a matching criteria, that is computable and allow the derivation of position and orientation.

## 2. Definitions

First of all, we need to define some terms for further use:

## Range-Finder Scan (Scan)

- (Def 1): A range-finder scan (or simply scan) is a finite sequence of numbers, where each element is a number which represents the distance to the nearest obstacle in the direction that is referenced by this element. The assignment of angles to elements of this sequence is in a consecutive manner and evenly spaced.

figure 1 : A single range-finder scan
- (Def 2): A scan can be a circular scan, i.e. the scan represents a full circle of $360^{\circ}$.

Circular scans have the property, that the sequence describes a closed loop, i.e. the first element in the sequence can be interpreted as a successor of the last element. Such a circular scan is shown in fig. 1 , with the moving system in the centre at $(0,0)$.

## Crosscorrelation

- (Def 3): A crosscorrelation is a function given through

$$
c(y)=\lim _{X \rightarrow \infty} \frac{1}{2 X} \int_{-x}^{x} f(x) g(x+y) d x
$$

and is a measurement for the correlation between two stochastic functions regarding the phase-shift y. The crosscorrelation $c(y)$ will have an absolute maximum at s , if $\mathrm{f}(\mathrm{x})$ is equal to $\mathrm{g}(\mathrm{x}+\mathrm{s})$.

## 3. Finding Invariants in Range-Finder Scans for Correlation

In order to match two scans for the calculation of rotation and translation, properties of scans or its derivations must be found, which are invariant concerning the recording place and orientation.

### 3.1. The Simple Thing - No Translation

We easily can see in fig.2, that circular scans show invariants regarding rotation. If two scans are taken at the same position with different orientation, the difference between the two (circular) sequences, is the phase shift.

figure 2 : Real scan and scan shifted $+135^{\circ}$

This phase shift can be estimated by searching for the maximum of the crosscorrelation of both scans. As the scan is given as a discrete sequence, the crosscorrelation formula needs to be transformed into a discrete form. The formula may then be written as

$$
\mathrm{k}(\mathrm{j})=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~s}_{1}(\mathrm{i}) \mathrm{s}_{2}(\mathrm{i}+\mathrm{j})
$$

where $s_{1}$ and $s_{2}$ are the two circular scans. Because circular scans describe closed sequences, there is no need to approximate the integral for more than one full sequence. Since the number of possible values for $j$ is finite, the maximum of $k$ and the corresponding $j$ can easily be found. The phase-shift so found between the two scans can be interpreted as the rotation between them.

### 3.2. Angle Histograms, Roughly Invariant Against Rotation and Translation

Unfortunately, the simple scheme above, which finds easily the rotation between two scans taken from the same position, does not properly work when applied to two scans from different positions. This happens, because a translation will produce shifts in a scan, that are not necessarily symmetric, so that the above scheme will misinterpret this as a rotation. Therefore we need a function to represent a scan, which is robust against translation but where rotation appears as a proportional phase shift.
In ([6]) the angle histogram is introduced. If the scan elements are interpreted as vectors, the vector difference of two consecutive vectors can be calculated. If a sequence of vectors represents a scanned flat surface, all these calculated vectors are nearly colinear. A statistic of the distribution of the angles of these vector differences with respect to a symmetry axis of the moving system is then called angle histogram.
One problem, that arises here is noise in the length of the vectors, i.e. in distance measurement. This noise can be reduced, if not two consecutive vectors, but a vector and its n-th successor are used to calculate the vector difference.

figure 3 : Calculating angles for angle-histograms

Such a histogram shows local maxima for the main directions that appear in the scan, e.g. angle histograms of rooms show maxima for the angles under which the walls appear. For industrial environments, where typically the walls can not be seen, the main direction of greater structures, which are mainly flat appear. In addition, a angular histogram is also circular, it repeats every $360^{\circ}$, or if two vectors which differ only in sign are supposed to have the same angle, every $180^{\circ}$. This property of being circular appears also if the underlying scan is not circular.

figure 4 : Angle-histogram of the single scan (maximum at $-24^{\circ}$ )

What happens to such an angle histogram, if the position and orientation where the corresponding scan is taken is altered? If the orientation is altered, only a phase shift will occur, similar to a circular scan. If the position is altered, the distribution of angles will also change, but this new angle histogram is also a representation of the distribution of directions in the new scan. So in the new angle histogram the same directions, which appeared to be the local maxima in the old angle histogram, will reappear.

Some will be higher, since the corresponding flat surfaces are nearer now and therefore more vector differences represent them, some are lower for opposite reasons, but the distribution itself will be very similar to the old one. This property now can be used to find the rotation between two different angle histograms from two different positions and orientations by crosscorrelation. The crosscorrelation function looks similar to the above

$$
k(j)=\sum_{i=1}^{n} h_{1}(i) h_{2}(i+j)
$$

where $h_{1}$ and $h_{2}$ represent here the two angle histograms. For translations, which are small with respect to the range finder scan range, it is sufficient to find the maximum of this crosscorrelation function, since the two histograms differ only slightly. If this is not the case, only a local maximum of the crosscorrelation function may appear. Therefore a rough estimation of the rotation is necessary. This can be done by evaluation of odometry, that, if it is used for this purpose, can have some slippery.

## 3.3. $X$ - and $Y$ - Histograms for Translation Computation

A further step is to find the translational displacement of the two scans after the rotational displacement has been corrected. In ([6]) the $x$-, and $y$-histogram is introduced for the purpose of calculation of the size of the surrounding room. A $x$ - or $y$ - histogram is a statistic about the number of scan points in $x$ - or $y$-direction. In the mentioned approach, the total scan was turned, so that the $x$ - and $y$-axis were in parallel to the walls. In this case, the maxima of the $x$ - and $y$ - histogram will show the position of the walls. In our approach we will not rely on walls, but on the translation of structures in $x$ - and $y$-direction from one histogram to the other.
One could think of crosscorrelating the x - and y histograms of two scans which were already rotational corrected, but two problems arise. First, the xand $y$ - histograms are no circular sequences, so that they can not be easily crosscorrelated. Second, the xand y - histograms show only significance, if the scan points accumulate at distinct $x$ - or $y$-values.

For the first problem, the histograms need to be modified in a way, that the sequence becomes circular and the phase information will not get lost. This is true, if the whole $x$ - or $y$-axis is folded to a finite size through a matching

$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=\mathrm{x} \backslash \text { size } \\
& \mathrm{f}(\mathrm{y})=\mathrm{y} \backslash \text { size }
\end{aligned}
$$

where " $\backslash$ "is the modulo operator. There is a value needed big enough as "size" constant, so that the phase shift between the two $x$ - and $y$ - histograms, respectively the translation, is still ambiguous.
The second problem, that the $x$ - and $y$ - histograms may show no significant data, appear if the $x$ - and $y$ values are distributed very evenly. If the $x$ - and $y$-axis are in parallel to the common directions in the scan, this will not happen. Therefore the scans need to be rotated before the $x$ - and $y$-histograms are calculated, so that the axis are parallel to the common directions. The most common direction for one scan can be detected by finding the maximum of the angle histogram of this scan. The other scan must be in the same direction. It can be assumed, that the other scan has the same most common direction or at least this direction is very common, too.
The crosscorrelation of the x - and y -histograms can now be computed in the same manner as it was done for the angle histogram. Here is also a problem of ambiguity to solve, since the new defined $x$ - and $y$ - histograms are modulo a constant factor. For this purpose, a rough estimation of the translation is necessary too, like for the rotation.

## 4. Experimental results

We used an infrared range-finder, which measures the phase-shift between a continuously modulated wave emitted by a LED and the returned light. The range-finder does two panorama scans per second with 720 points each. It has therefore an angular resolution of $0.5^{\circ}$. The measurement range is from about 60 cm to 6 m . The accuracy of the sensor is, although corrected by software and hardware, reflection dependent. As long as the reflection is high enough, the absolute distance error is around 5 cm . This error can easily grow up to 30 cm if the return signal is weak.

## Examples

The principle was tested in various examples, where one is showed here. Fig. 5 shows two scans from different points of view as they appear to the range-finder at position $(0,0)$, heading to the $+y$-axis.

figure 5 : Two scans with rotation of $+43^{\circ}$, X-transition of $+14 \mathrm{~cm}, \mathrm{Y}$-transition of +96 cm

We see in fig. 6 the angle histograms, in which the maxima of the two histograms can be found. It looks like there is a shift between the signals of about $40^{\circ}$, which should also be known by means of odometry.

figure 6 : Angle-histograms of the two positions
Although the two histograms are somewhat noisy, we can deduce a relatively smooth crosscorrelation
function, as it can be seen in fig. 7.The local maximum around the previously estimated value leeds to a rotational angel of $43^{\circ}$, while by other means a rotation of $42.9^{\circ} \pm 1^{\circ}$ was measured.

figure 7 : Crosscorrelation of the two anglehistograms (maximum at $43^{\circ}$ )

For the computation of the $x$ - and $y$-histogram the most common directions were sought and found to be $-24^{\circ}\left(+19^{\circ}\right)$, so the scans were turned $+24^{\circ}\left(-19^{\circ}\right)$. The calculation of this most common direction was made by detecting the maximum of the first angle-histogram and defining the common direction in the second histogram to be the same direction as in the first scan plus the rotational displacement between the two scans.

figure 8 : Scans rotated according to the maximum of their angle-histograms $\left(+24^{\circ},-19^{\circ}\right)$

The size of the $x$ - and $y$ - histograms was set to be 4 m , so that a translation up to nearly 2 m is ambiguous. Since the histogram can be only calculated for discrete values for x - or y - distance, this distance was divided in intervals of 5 cm . The translation between the scans was read by a tape measure to be 96 cm in y - and -14 cm in x -direction. The accuracy of this measurement was due to problems in finding a reference point about $\pm 1 \mathrm{~cm}$.

figure 9 : Crosscorrelation of the $x$ - translation (maximum at -35 cm , corresponding with -14 cm in the rotated scan)

What can be seen in fig. 9 is an absolute maximum at -35 cm (or 4.65 m , what is the same). The other local maxima result from the coincident crosscorrelation between two different structures. They are the reasons, why a rough estimation of translation is necessary.

figure 10 : Crosscorrelation of the y - translation (maximum at +90 cm , corresponding with +98 cm in the rotated scan)

After calculating the x - and y - translations with respect to the most common direction in the angle histogram to be -35 cm for the x - and 90 cm for the y direction, this translations need to be transformed into the original coordinates. This transformation results with -14 cm for x - and 98 cm for y - direction
and is surprisingly good. Fig. 11 shows then the transposition of the two scans. In principle there is even a higher accuracy possible: since the crosscorrelations are discrete, there appears a discretisation error, that could be handled by constructing a interpolation polynom from some values in the near vicinity of the local maxima of the crosscorrelation function. The position of the maximum of this interpolation polynom should give then an even more accurate $x$ - or $y$ - translation respectively rotation.

figure 11 : x - translation correction of +14 cm and $y$ - translation correction of -98 cm

## Computing Time

The algorithms shown here are in the order of $\mathrm{O}\left(\mathrm{n}^{3}\right)$, although it should be kept in mind, if the distance is represented in integer values, the atomic operations are really simple integer operations and " n " is a fixed and relative small number. In our example we used a 25 Mhz MC68040, a calculation time below 200 ms was possible.

## 5. Conclusion

An algorithm was shown, that is able to calculate translation and rotation between two different positions and orientations of a mobile system by correla-
tion of range-finder scans made at the different locations, as long as there is enough significant data that can be used to match.
Although in experiments the algorithm seems to be highly stable against noise, there is actual no criteria, if a successful correlation can be expected. Experiments showed that even moving obstacles do not harm, as long as they do not take the majority of matchable data. It should be possible to formulate a criteria that shows the degree of confidence of the correlation. Further investigations need to be done here.

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