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Showing, Designating, Paraphrasing – the Three Alternatives of Identification

Motivation

Programs are linguistic structures which contain identifications of individuals: memory locations, data types, classes, objects, relations, functions etc. must be identified selectively or definingly.[1)]

The first part of the essay which deals with identification by showing and designating is rather short, whereas the remaining part dealing with paraphrasing is rather long. The reason is that for an identification by showing or designating no linguistic compositions are needed, in contrast to the case of identification by paraphrasing.

The different types of functional paraphrasing are covered here in great detail because the concept of functional paraphrasing is the foundation of functional programming. The author had to decide whether to cover this subject here or in his essay ”Purpose versus Form of Programs” where the concept of functional programming is presented. Finally, the author came to the conclusion that this essay on identification is the more appropriate place.

Individuals and their identification

Each person is born into a continuum of stimuli and is inseparably connected to this continuum through his sensory organs. In the early years of life, a person goes through the process of creating his view of a structured world. This view may also be called a model. In this structured world, things can be counted, measured and decided. Individuals are *countable*, qualities of individuals are *measurable*, and relations between the individuals are *decidable*.

Only the concrete individuals may be substantiated from the continuum of stimuli; in addition, people create for themselves a world of abstract individuals. In this essay it is assumed that people have completed the individualization of their world, and the issue is now to identify the single individuals.

> Identification in this context is a process, in which a person directs his attention or the attention of his partner in communication to a certain individual.

For example, a person was attacked and the police hope that the victim will be able to identify the offender. In this example, the three different types of identification may be explained.

If the police put together a lineup, they ask the victim to view the lineup and select (show) the offender if he is present. (Show is the first type of identification.) Identification by showing is only possible under two conditions: First, the individual to be identified must be a concretely perceptible individual. Second, the identifying person and the individual to be identified must be close enough so that a perception and definite selection are possible. These two conditions do not apply to the other two types of identification.

1) These concepts will be introduced and explained in detail in this essay.

The second type of identification is designation. In the example given above, it is possible that the victim recognized the offender as someone he knows. Then the victim will be able to tell (designate) the police the name of his attacker.

Identification by designation requires that a so–called symbol is substituted for the individual to be identified . A symbol is an easily reproduced pattern, which is associated with an individual through agreement as a means of identification. For example, the elementary symbol ”5” is associated to the mathematical individual ”natural number five.” The composed symbol ”Beethoven” is generally interpreted as the individual who was born in Bonn and died as a famous composer in Vienna. The most well–known symbols are the elements of natural languages – words. Words have a spoken as well as a written form. But there are also many purely graphic symbols, for example the symbols ♀ and ♂ for the planets Venus and Mars or — in another context — for the terms ”female” and ”male”.

The symbols usually used for interpersonal communication are optical or acoustical patterns. In technical systems there are numerous other patterns for the transfer or processing of information. It is in the nature of a symbol that it only has meaning in its totality (i.e. its parts may not be interpreted separately). Therefore, it would not make sense to ask for the meaning of the letter ”t” in the name ”Beethoven.”

If the victim of an attack cannot identify the offender by showing or designating, the victim would have to resort to the third method of identification — paraphrasing. In paraphrasing ambiguity is possible, while in showing or designating, identification is unequivocal if the condition for an unequivocal symbol agreement has been fulfilled. If the victim of the attack tells the police that the offender was wearing a blue pair of pants and black shoes, that he was at least 6 ft. tall and weighed approximately 180 lb., this paraphrase will certainly exclude some individuals, however it could fit many people. Therefore, a paraphrase is only an attempt at identification.

Paraphrases

Paraphrases may be divided into *selective paraphrases* and *defining paraphrases*.

Concerning selective paraphrases, a predicate P is given with the intention of identifying a one–element set. If, however, the predicate P specifies an empty or a multi–element set (i.e. the number of individuals to which the predicate P applies is not one), the attempt at identification fails. Formally, this may be written as follows:

$$\text{Identified\_by\_selection\_using (P)} = \begin{cases} i & \text{if} \quad P(i) \text{ AND } (\left| \{ x \mid P(x) \} \right| = 1) \\ \text{undefined} & \text{else} \end{cases}$$

Please note that here the individual ”i” is identified and not the set {i}.

<u>Examples:</u>

(1) $P(x) = (x^2 - 7x + 12 = 0)$

The set specified with this predicate is { 3, 4 }. Since this set contains two elements, the selective paraphrase with this predicate does not provide a result.

(2) $P(x) = (x^3 – 15x^2 + 75x – 125 = 0)$

The set specified with this predicate is { 5 }. Since this set contains only one element, the selective paraphrase has a defined result, which is 5.

A selective paraphrase must be interpreted as an attempt at identification, and is subject to fail. In contrast, a defining paraphrase always identifies a certain individual unequivocally.

While it is possible to provide a general form for selective paraphrases which matches the paraphrase with a set–specifying predicate, the author did not find a general form which matches all defining paraphrases.

<u>Examples:</u>

(3) With the paraphrase of the form $M = \{ x \mid P(x) \}$
a set M is defined and identified at the same time.

(4) With the paraphrase of the form $P = P(x)$
a predicate P is defined and identified at the same time.

As an example, a prime number predicate may be definingly paraphrased as follows:

PRIME NUMBER $(x) = [\exists w \in \mathbb{N}: w+1 = x] \cdot [\neg \exists (y, z) \in \mathbb{N}^2: (y+1) * (z+1) = x]$

Since one cannot see by looking at a predicate P(x) which identification purpose it is supposed to serve, a context must always be provided showing whether a selective or a defining paraphrase was intended and whether in case of definition, a set or the predicate itself is to be defined.

(5) The Peano Axioms, which are found in the essay ”The Concept of Formalism”, represent a defining paraphrase of the abstract structure ”Linearly ordered infinite set of discrete elements with a first element. ”

Functional Paraphrases

The remaining part of this essay deals with an important subclass of selective paraphrases.

> A selective paraphrase is called a *functional paraphrase*, if the predicate
> P has the form $x = f$ (argument tuple) hat.

In this case, the set specified with the predicate contains one single element. This is because the argument tuple indicated in a functional paraphrase must belong to the domain of the function f.

<u>Examples:</u>

(6) $P(x) = (x = 3 \cdot SQRT(2) \cdot COS(\pi/4))$

The selective paraphrase with this predicate is functional and identifies the number 3.

The function is

$$f(a, b, c, d) = a \cdot SQRT(b) \cdot COS(c/d),$$

and the argument tuple is

$$(a, b, c, d) = (3, 2, \pi, 4).$$

(7) The presentation of a natural number as a decimal number is a functional paraphrase, which explicitly includes only the argument tuple. The function to be associated herewith is

$$f(d_m, d_{m-1}, \ \dots \ d_2, d_1, d_0) = \sum_{i=0}^{m} d_i * 10^i$$

(8) $$P(x_1, x_2) = ((x_1 + x_2 = 7) \text{ AND } (2x_1 - x_2 = 2))$$

The selective paraphrase with this predicate identifies the two–dimensional vector (3, 4). Although this paraphrase identifies an individual unequivocally, it must not necessarily be seen as a functional paraphrase because no function f is indicated – at least not explicitly.
But nevertheless, it is possible to associate a function f with this paraphrase since the predicate may also be written in the following form:

$$P(x_1, x_2) = \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}^{-1} \bullet \begin{pmatrix} 7 \\ 2 \end{pmatrix} \right)$$

This includes the function

$$f(a, b, c, d, e, f, g) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{e} \bullet \begin{pmatrix} f \\ g \end{pmatrix}$$

and the argument tuple (a, b, c, d, e, f, g) = (1, 1, 2, –1, –1, 7, 2).

If the original form of the predicate is associated with the matrix inversion as a computable function, the paraphrase may be seen as functional. But anyone who does not see this function must classify the paraphrase as non–functional.

A single symbol, too, can be seen as a functional paraphrase, and there are two separate ways for such a view:

– The function f is identified by the symbol itself. It is a function with zero arguments, therefore, an explicit indication of the empty argument tuple is not needed.

  As an example, consider the symbol 5. Instead of saying that the symbol identifies the mathematical individual ”natural number 5”, one can say that the symbol identifies a function which, when applied to the empty argument tuple, supplies the mathematical individual ”natural number 5” as the result.

– The function f is the identity function, which as the result always supplies its argument. This function may be implied and does not have to be indicated explicitly.

Functional paraphrases may be classified as *primitive functional* paraphrases and *composed functional* paraphrases.

> A functional paraphrase is *primitive functional* if all function identifications contained therein are primitive (i.e. symbolic).

To each symbol identifying a function belongs the implicit knowledge about the number of arguments and the purpose connected with the respective position of an argument in the tuple.

There are only a few functions where it is irrelevant for the evaluation how the arguments in the tuple are ordered. For example, the paraphrase ”ADDITION(8, 5)” provides the same result 13 as the paraphrase ”ADDITION(5, 8)”. This does not apply to subtractions (i.e. it is important to know where the minuend and where the subtrahend stand in the argument tuple).

Figure 1 shows an example for a primitive functional paraphrase; the structure of this paraphrase, consisting of individual parts, is presented in the form of a tree.

If a functional paraphrase is not primitive functional, but composed functional, it has at least one function which is not identified by indicating the function symbol.

If a function is not identified by its function symbol, a defining paraphrase of this function must be given. In this case, it is not possible to imply the number of its arguments and the purpose of the different positions in the argument tuple. This knowledge must be conveyed explicitly within the defining paraphrase.

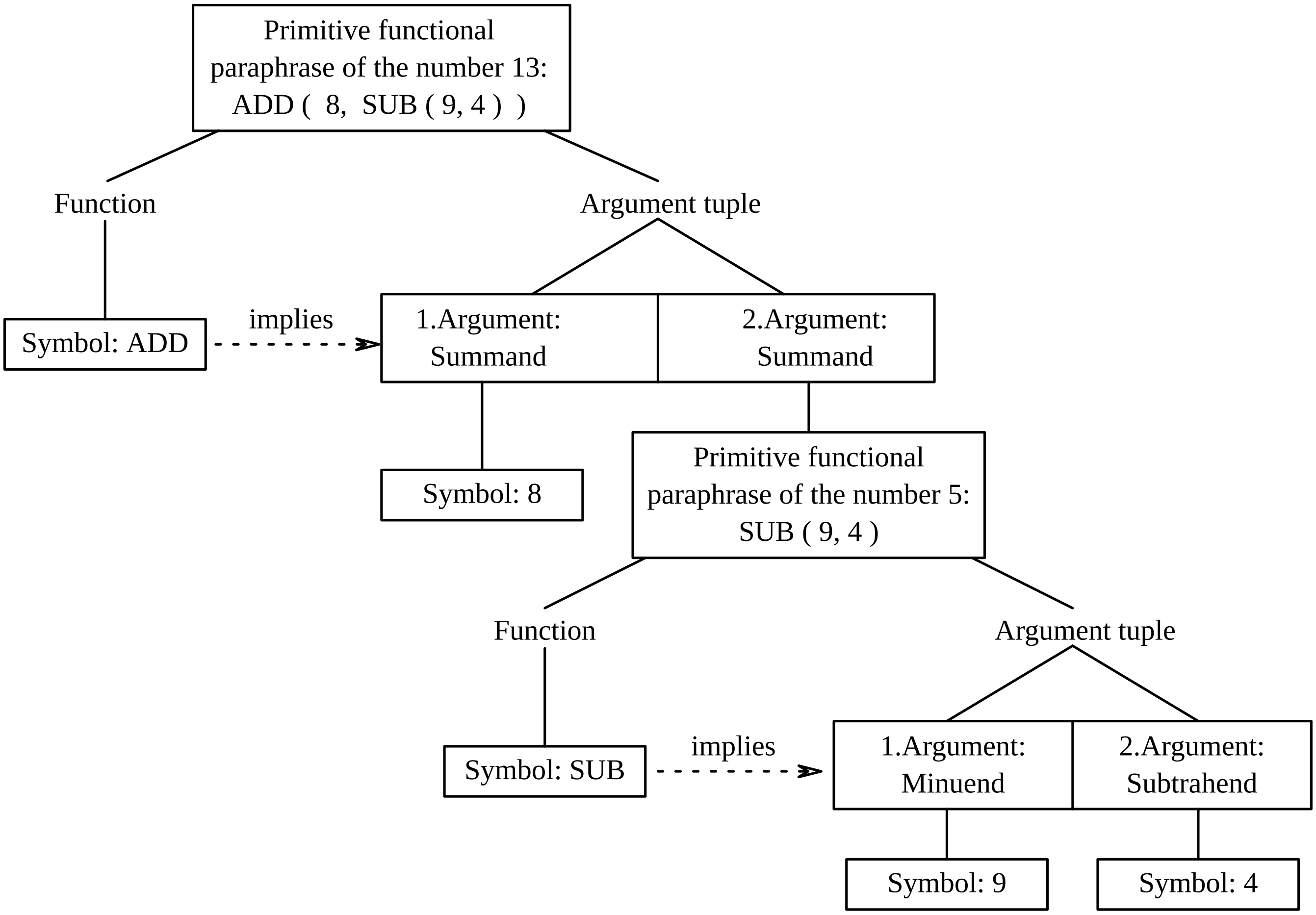

Figure 1 Example of a primitive functional paraphrase, presented as a tree structure

Figure 2 shows an example for a composed functional paraphrase, also presented as a tree structure using the same symbols as in Figure 1.

In Figure 2, the number 13 is paraphrased with the expression $3 + 2 \cdot 5$. This paraphrase could have been formulated as a primitive functional paraphrase. Then its tree would be structurally identical with the tree in Figure 1, because both paraphrases ADD(8, SUB(9, 4)) and ADD(3, MULT(2, 5)) are structurally identical and are only different with reference to the symbols.

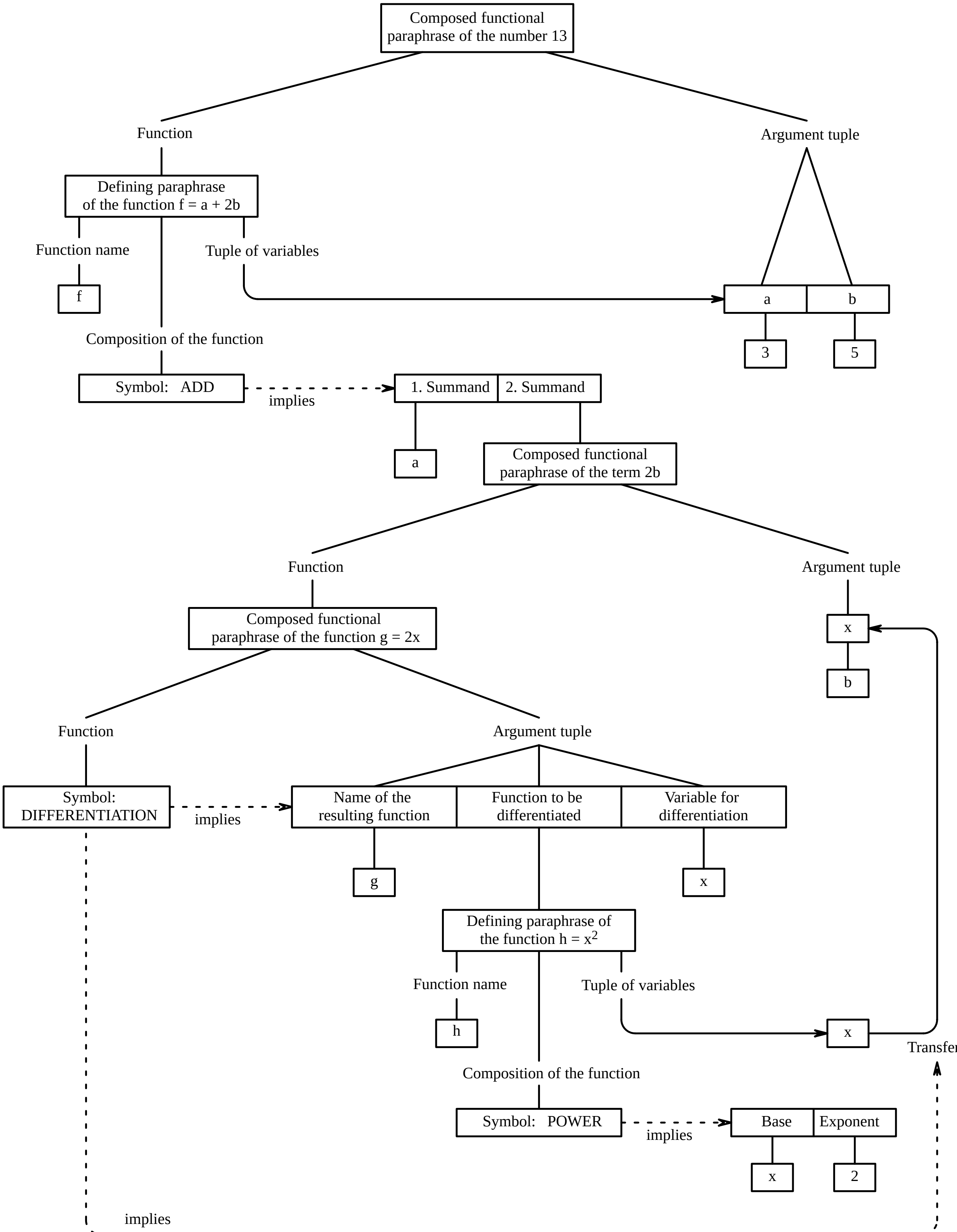

Bild 2 Example of a composed functional paraphrase, presented as tree structure

Comparing the two Figures 1 and 2, which show two very different paraphrases of the number 13, it becomes evident that the individual to be paraphrased does not restrict the form of paraphrase. The person paraphrasing is always free to choose any form of paraphrase. In Figure 2, the following arbitrary decisions are taken:

It was decided to define a function $f(a,b) = a + 2b$ and to preset the argument tuple $(a, b) = (3, 5)$. Furthermore, it was decided to gain the term 2b by defining a function $g(x) = 2x$, to which the argument tuple $(x) = (b)$ is preset. Finally, it was decided to gain the function g(x) by differentiating the function $h(x) = x^2$.

Both functions f and h were paraphrased by definition (i.e. for each of them the tuple of the argument variables and the function composition was indicated). The function g, on the contrary, is paraphrased functionally because it is paraphrased as a result of the differentiation of the function h. The paraphrase of the function g is a composed functional paraphrase because the function h to be differentiated is paraphrased. Since the description of the function g is not a defining paraphrase, the tuple of the argument variable of the function g — that is the tuple (x) — is not determined by definition; rather it is a result of the differentiation being applied to the argument tuple preset for this differentiation function. This argument tuple contains the function h to be differentiated, and for the definition thereof, (x) is preset as its argument tuple. The differentiation function implies the transfer of the argument tuple from the function to be differentiated to the function obtained from the differentiation – in the above case this is a transfer of (x) from the function h to the function g.

In the example observed, there is no need to have names for the three introduced functions a + 2b, 2x and x^2. Only in the case of a recursive function definition, a function name is needed. There is a recursive function definition if, in the function composition, the name of the function to be defined is used as a function symbol. In this case, the determination of a function name is necessary. For the purposes of uniformity, it is convenient to determine a name in all function definitions, even in cases where the function is not defined recursively. Therefore, the function names f, g and h were determined.

Final Remarks

At the end of this discussion concerning alternative possibilities for identification, it shall be pointed out that the only purpose for each type of paraphrase is to refer the unknown back to the known. It is impossible to paraphrase without using symbols, and the use of symbols makes it necessary to imply agreements concerning the symbols’ meaning. A paraphrase cannot be understood unless the implied agreements for the symbols used are known.