Classification of Location Problems

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Abstract

There are several good reasons to introduce classification schemes for optimization problems including, for instance, the ability for concise problem statement opposed to verbal, often ambiguous, descriptions or simple data encoding and information retrieval in bibliographical information systems or software libraries. In some branches like scheduling and queuing theory classification is therefore a widely accepted and appreciated tool.

The aim of this paper is to propose a 5-position classification which can be used to cover all location problems. We will provide a list of currently available symbols and indicate its usefulness in a - necessarily non-comprehensive - list of "classical" location problems. The classification scheme is in use since 1992 and has since proved to be useful in research, software development, classroom, and for overview articles.

1 Introduction

In several branches of optimization classification schemes have been successfully introduced and are used by every author publishing in the respective

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area. Well known examples include the 3-position schemes in scheduling [GLLK79] and in queuing theory [Ken51].

Like any other formalization such classification schemes should be concise and allow a precise description of the problem class at hand. As such it contributes to more transparency in the scientific discussions and avoids misunderstandings in — often ambiguous — verbal problem descriptions. As a tool in data encoding and information retrieval it is absolutely necessary in bibliographical information systems or software libraries.

Proposals for classification schemes for location problems exist since 1979 when Handler and Mirchandani [HM79] suggested a 4-position scheme which is applicable to network location problems with objective functions of the center type. Eiselt et al 1993 [ELT93] used a 5-position scheme which classifies competitive location problems, i.e. models which are based on a game theoretic approach. Carrizosa et al [CCMP95] presented a 6-position scheme for classifying planar problems where both demand and service are given by a probability distribution.

The 5-position classification scheme proposed in this paper is designed in such a way that not only classes of specific location models are covered but all of them in a single scheme.

It is in use since 1992 when it was first implemented in a course on planar location theory [Ham92], [Ham95]. It was presented to the research community since then in conferences and publications see, e.g. ([HN93], [Nic94], [Nic95], [HN96], [NP96], [HNL96], [RCPNF96]) and was received with positive feedback. The software library LOLA (Library of Location Algorithms, [HKNS96]) is based on this classification scheme to provide a comprehensive concise and precise software library.

In the next section the classification scheme will formally be introduced. After a general description of the five positions, specific symbols are proposed for continuous, network and discrete location problems. The last section shows the usefulness of the proposed scheme by referring to some location literature and indicating the corresponding classification.

The authors of this paper and colleagues who have already used the scheme found it useful. It is our hope that the location community will take this paper as starting point of a discussion which will lead to a commonly accepted classification of location problems.

2 A classification scheme for location problems

In this section we will describe a classification scheme for location problems. This scheme has been used in our group since 1992 and (after some modifications) proved to be a useful tool for structuring lectures as well as research papers (see references in the introduction).
First we will give a general description of the structure of this classification scheme. Then we will in three subsections, devoted to continuous, network and discrete location problems describe in more detail the usage of the scheme. Finally we will give a summary of the used symbols.

2.1 General structure of the scheme

The classification scheme has 5 positions written as

$$Pos1/Pos2/Pos3/Pos4/Pos5\,,$$

where the meaning of each position is described in the following.

Pos1 This position contains information about the number and the type of the new facilities.

Pos2 The type of the location problem with respect to the decision space. This information should at least differentiate between continuous, network and discrete problems.

Pos3 In this position is room for describing particularities of the specific location problem. We should, for example, be able to include information about the feasible solutions or about capacity restrictions.

Pos4 This position is devoted to the relation of new and existing facilities. This relation may be expressed by some distance function or simply by assigned costs.

Pos5 The last position contains a description of the objective function.

If we do not make any special assumptions in a position this is indicated by a $\bullet$. For example, a $\bullet$ is Position 5 means that we look at any objective function. The $\bullet$ in Position 3 means that the standard assumptions for the problem described in the remaining four positions hold. For example in planar location problems a $\bullet$ in Position 3 means that we have (as usual)

positive weights for the existing facilities. In general we also assume by default that we minimize the objective function.
In the next subsections we will describe the usage of the classification in the three main areas of location theory: continuous location, network location and discrete location. We will introduce specific symbols to express the information described in this section.

2.2 Continuous Location Problems

Since continuous location problems are the oldest location problems and deal with geometrical representations of reality a broad range of different location problem types has to be taken into account. Standard assumptions are positive weights and convex objective functions.
We will now describe the possible symbols for each position.

- Position 1: We have an expression which consists of a number

$$\boldsymbol{n \in \{1, \ldots, N\}}$$

and a string specifying the type of the new locations. This string may be, for instance,

	an empty string stands for location of n points.
l	n lines have to be located.
p	n paths consisting of one or several line segments have to be located.
A	n general areas have to be located. We can also have circles (**C**) or rectangles (**R**)

If several types of new facilities have to be placed we can have in Position 1 several of the above described expressions separated by commas.

- Position 2:

$\mathbb{R}^d$	The problem has to be solved in the d-dimensional space.
P	A problem in the plane ($d = 2$).
H	A problem in a general Hilbert-Space.

- Position 3:

$\boldsymbol{\mathcal{F}}$	A feasibility region is introduced, i.e. $x \in \mathcal{F}$ is required.

$\boldsymbol{\mathcal{R}}$	A forbidden region is introduced, i.e. $x \notin int(\mathcal{R})$ is required. If the shape of the forbidden regions is important, further specification include, for instance, $\mathcal{R}$ convex or $\mathcal{R}$ circle.
$\boldsymbol{\mathcal{B}}$	A barrier is introduced, where neither placement of new facilities nor trespassing is allowed.
$\boldsymbol{w_m = 1}$	An unweighted problem.
$\boldsymbol{w_m \lesseqgtr 0}$	Positive and negative weights are allowed.
$\boldsymbol{w_m}$ **: distribution**	The weights satisfy a specified distribution, for example: $w_m \ : \ P(\lambda)$ means that the weights are Poisson distributed with respect to parameter λ. If we only want to express that the w_m are random variables we write $\boldsymbol{w_m \ : \ RV}$.
$\boldsymbol{w_m \ : \ f(\cdot)}$	The weights are generated by a function f.
mc	Mutual communication between the new facilities. This is the standard assumption for continuous location problems and may be omitted.
alloc	The allocation of existing to new facilities is part of the problem.
queue	The service of the new facilities is combined with a queue. If further specification is needed the 3-position classification for the queue (see [Ken51]) can be included.

- Position 4:
  In continuous location we usually give information about the distance function used. We allow either to specify a distance function or a norm or gauge inducing a distance function by $dist(x, y) := norm(y - x)$. To each symbol an index m can be added to express that every existing facility defines its own distance.

$\boldsymbol{l_p}$	The distance is defined by an l_p-norm, where for example, l_2 is the Euclidean norm.
$\boldsymbol{\gamma}$	A general gauge.
$\boldsymbol{\gamma_{pol}}$	A polyhedral gauge.
$\boldsymbol{\gamma_{mix}}$	A mixed gauge.
$\lVert \cdot \rVert$	A general norm.
$\boldsymbol{d_{Haus}}$	The Hausdorff distance.
$\boldsymbol{d_{inhom}}$	Inhomogeneous distance. The distance function is not everywhere the same in the decision space.

- Position 5:
  Remember that we by default always minimize.

$\sum$	The classical Weber or sum objective function.
$\sum_{ord}$	The ordered Weber or sum objective function.
$\mathbf{max}$	The maximum objective function.
$\sum_{obnox}$ **or** $\mathbf{max}_{obnox}$	Same as above but we maximize the objective function (we have an obnoxious location problem).
CD	Cent-Dian objective function.
$\int_{d}$	We have continuous demand satisfying the distribution d.
$\int_{d_1}\int_{d_2}$	We have continuous demand (d_1) and also the new facilities are distributed with respect to some distribution d_2.
$\sum_{prob}$	A sum objective with some probabilistic influences, like, for example, different scenarios or weights which are random variables. Analogous $\mathbf{max}_{prob}$.
$Q - \sum_{par}$	Multicriteria Weber Problem, where we are looking for Pareto locations. Analogous for other objective functions.
$Q - \sum_{lex}$	Multicriteria Weber Problem, where we are looking for lexicographic minimal locations. Analogous for other objective functions.
$Q - \sum_{MO}$	Multicriteria Weber Problem, where we are looking for max ordering locations. Analogous for other objective functions.
Q-$(\sum, \mathbf{max})_{par}$	Multicriteria Problem, where we are looking for Pareto locations. The objective functions are either of the max or the sum type. Analogous for other objective functions and other criteria.
φ **: property**	A general objective function with some property, e.g. φ : increasing. To indicate properties like increasing or decreasing we can also write $\nearrow$ or $\searrow$, respectively.

After having described the positions separately we will now give a list of well-known continuous location problems and their classification.

- $1/\mathrm{P}/\bullet/l_2/\sum$
  This is the classical Weber problem with Euclidean distance.
- $1/\mathrm{P}/\bullet/\bullet/\sum$
  This is the class of all one facility Weber problems in the plane.
- $N/\mathrm{P}/(\mathrm{mc})/\bullet/\sum$
  This is the class of all N-facility problems in the plane, where the interaction between the new facilities is given (As explained earlier the symbol "mc" may be omitted).
- $N/\mathrm{P}/\mathrm{alloc}/\bullet/\sum$
  This is the class of all planar multi-Weber problems, where allocation is part of the problem.
- $1l/\mathbb{R}^d/\bullet/\|\cdot\|/\max$
  This is the problem of finding a centre line in the d-dimensional space with respect to some norm.

By using this scheme we can easily describe problems which are not of the classical type. For example, $N/\mathrm{P}/\mathcal{B},\mathcal{R}/\gamma/Q-\sum_{par}$, the problem of finding the set of all Pareto locations with respect to Q objective functions (each of which is of the Weber type) for a N-facility planar location problem under a general gauge with barriers and forbidden regions is not solved yet. In this example we can also see that the verbal description is much clumsier and more ambiguous than the 5-position string.

2.3 Network location problems

- Position 1: Like in the continuous case this position consists of a number

  $$\boldsymbol{n \in \{1,\dots,N\}}$$

  and a string specifying the type of the new locations, e.g.

	An empty string stands for location of n points.
p	n paths have to be located in the network consisting of one or several edges.
T	n trees have to be located in the network.
G	n subgraphs have to be located in the network.

- Position 2:

$\mathcal{G}$	The problem is defined on a network, where the underlying graph is a general undirected graph.
$\mathcal{G}_D$	The problem is defined on a network, where the underlying graph is a general directed graph.
$\mathcal{T}$	The problem is defined on a network, where the underlying graph is a tree.

- Position 3: The same possibilities as in the continuous case. Default is "alloc" and non-mutual communication.

- Position 4: Since on a network the distance is always measured with respect to the shortest-path distance we only have to specify from where to where we are allowed to measure. We therefore have always an expression $d(\cdot,\cdot)$ where the first arguments determines the possibilities for the existing and the second argument the possibilities of the new facilities.

$d(\mathcal{V},\mathcal{V})$	The new and existing facilities have to be nodes of the graph.
$d(\mathcal{V},\mathcal{G})$	The existing facilities are in the nodes of the graph and the new facilities can be any point on the graph. Analogous $d(\mathcal{V},\mathcal{T})$.
$d(\mathcal{G},\mathcal{V})$	The new facilities are in the nodes of the graph and the existing facilities can be any point on the graph. Analogous $d(\mathcal{T},\mathcal{V})$.
$d(\mathcal{G},\mathcal{G})$	The existing and the new facilities are allowed to be any point on the graph. Analogous $d(\mathcal{T},\mathcal{T})$.

- Position 5: Any meaningful continuous objective function can also be used in the network case.

Again we will give some well-known problems as illustration:

- $1/\mathcal{G}/\bullet/d(\mathcal{V},\mathcal{G})/\sum$
  This is the absolute 1-Median problem.

- $2/\mathcal{G}/(\text{alloc}),w_m = 1/d(\mathcal{V},\mathcal{G})/\max$
  This is the unweighted 2-centre problem (with allocation).

- $1/\mathcal{T}/\bullet/d(\mathcal{V},\mathcal{T})/Q-\sum_{par}$
  Is the multicriteria 1-median problem on a tree.

2.4 Discrete location problems

- Position 1:

$\boldsymbol{n \in \{1, \ldots, N\}}$	The number of new facilities.
$\sharp$	The number of new facilities is not known in advance and its determination is part of the problem.
$\sharp, \sharp$	Two different kinds of new locations have to be found. Analogous p-different types.

- Position 2: Always **D**.

- Position 3:

cap	Capacity restrictions.
bdg	Budget restrictions.
$\boldsymbol{d_{max}}$	A maximal distance is given up to which clients can be served. Analogous d_{min}.
price	Denotations for a specific pricing policies, e.g.

  - $\mathbf{price}_{\boldsymbol{M}}$ mill-pricing. The client has to pay the transportation cost.
  - $\mathbf{price}_{\boldsymbol{U}}$ uniform delivery pricing. The client has not to pay the transportation cost.
  - $\mathbf{price}_{\boldsymbol{D}}$ spatial discriminatory pricing. The prices of the product are depending on the distance to the client.

- Position 4: Here any restrictions and specifications of the costs c_{ij} can be given.

- Position 5: Any of the objective functions of the continuous or network case can be used. Additionally specific discrete location type objective such as

$\sum_{\boldsymbol{comp}}$	Competitive location model.
QAP	Quadratic assignment objective function.
$\sum_{\boldsymbol{uncov}}$	Coverage objective function.
$\sum_{\boldsymbol{cov}} + \sum_{\boldsymbol{uncov}}$	Covering objective function.
$\sum_{\boldsymbol{hub}}$	Hub location objective function.

  may be used.

We give again some examples as illustration:

- $N/\mathrm{D}/\bullet/\bullet/\sum$
  This is the discrete N-median problem.
- $\sharp/\mathrm{D}/\bullet/\bullet/\sum$
  This is the uncapacitated facility location problem (UFL).
- $\sharp/\mathrm{D}/d_{max}, \mathrm{bdg}/\bullet/\sum_{uncov}$
  This is the coverage problem.

2.5 Summary of the used symbols

In this section we will give an overview of the introduced so far, symbols to make it easier to compose a specific instance of the problem classification. Of course not all combinations of symbols make sense. The reader should also keep in mind that several symbols of one column may be concatenated by commas if applicable. Obviously, it will be necessary to extend the number of available symbols in the future. We plan to make LaTeX-macros available for generating the classification strings. This will be announced on the LOLA homepage (see [HKNS96]).

Position 1	Position 2	Position 3	Position 4	Position 5
$n \in \{1, \dots, N\}$	$\mathbb{R}^d$	$\mathcal{R}$	l_p	$\sum$
l	P	$\mathcal{F}$	γ	max
p	H	$\mathcal{B}$	γ_{pol}	CD
A	$\mathcal{G}$	$w_m = 1$	γ_{mix}	$\int_d$
C	$\mathcal{G}_D$	$w_m \gtreqless 0$	$\\|\cdot\\|$	$\int_{d_1} \int_{d_2}$
R	$\mathcal{T}$	w_m : distribution	d_{Haus}	$Q - \sum_{par}$
T	D	w_m : RV	d_{inhom}	$Q - \sum_{lex}$
G		w_m : $f(\cdot)$	$d(\mathcal{V}, \mathcal{V})$	$Q - \sum_{MO}$
$\sharp$		mc	$d(\mathcal{V}, \mathcal{G})$	$Q - (\sum, \max)_{par}$
$\sharp, \sharp$		alloc	$d(\mathcal{V}, \mathcal{T})$	$\sum_{comp}$
		cap	$d(\mathcal{G}, \mathcal{V})$	$\sum_{uncov}$
		bdg	$(\mathcal{T}, \mathcal{V})$	$\sum_{cov} + \sum_{uncov}$
		d_{max}	$d(\mathcal{G}, \mathcal{G})$	$\sum_{cov}$
		price	$d(\mathcal{T}, \mathcal{T})$	QAP
		queue		$\sum_{ord}$
				$\sum_{prob}$
				$\max_{prob}$
				$\sum_{hub}$
				φ : property

The current version of LOLA (see [HKNS96]) uses these symbols to guide users of the software library to define and find the respective location model (see Figure 1).

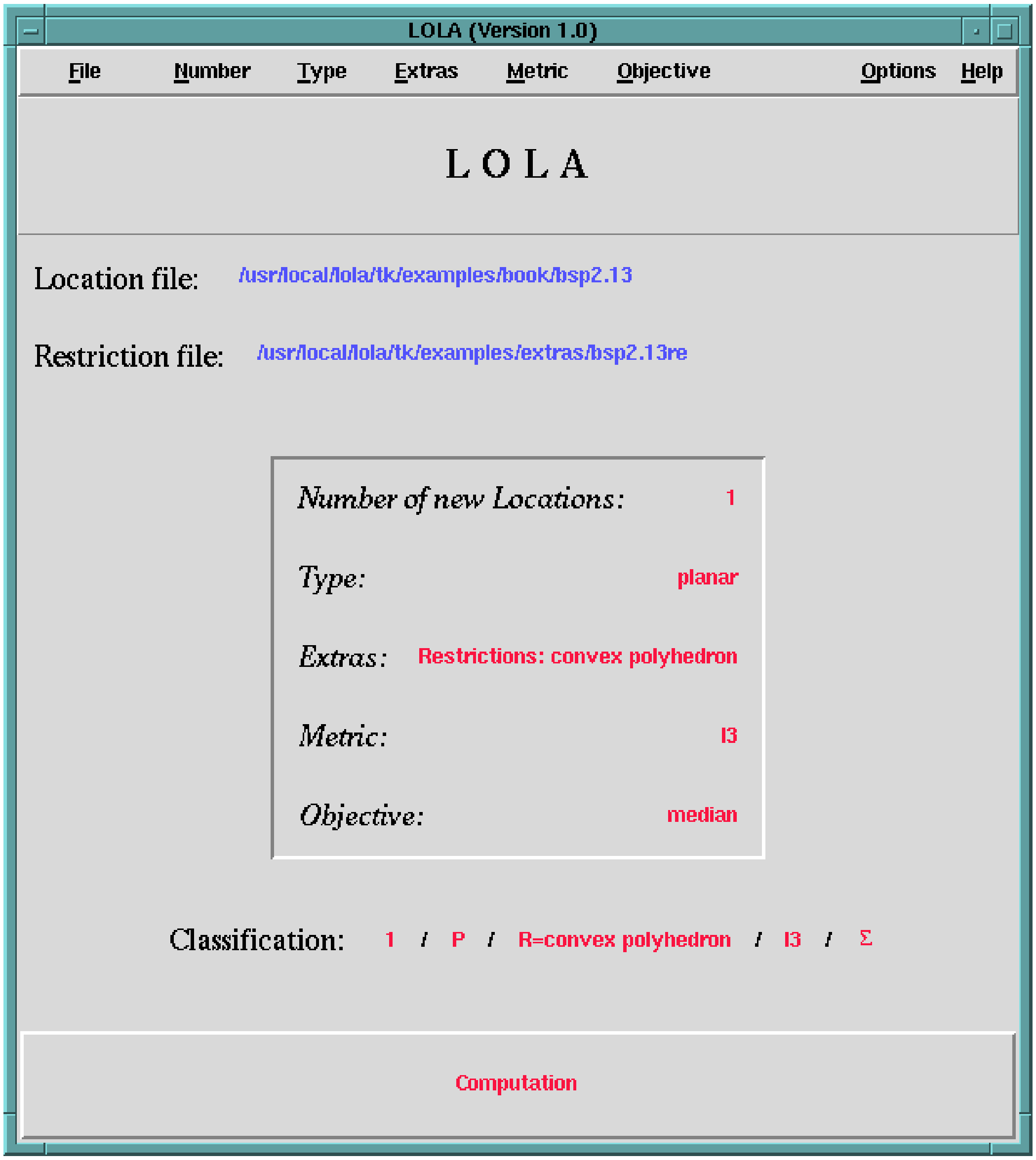

Figure 1: A screenshot of the LOLA frontend.

3 Examples

The purpose of this section is **not** to provide a comprehensive overview of location literature. But is is intended to indicate to the reader that location problems of various kinds can be described unambiguously sing the proposed 5-position scheme. Of course the classification can not represent the complete contents of a paper, but the 5-position scheme can reflect the major particularities of the problem(s) investigated.

3.1 Continuous Location Problems

A. Aly, D. Kay and D. Litwhiler jr. Location dominance on spherical surfaces. *Operations Research*, 27:972-981,1979

$1/\mathbb{R}^3/\bullet/\rho(P_i, X)$:shortest great circle distance/$\sum$

Y.P. Aneja and M. Parlar. Algorithms for Weber facility location in the presence of forbidden regions and/or Barriers to travel. *Transportation Science*, 28(1):70-76,1994.

$1/P/\mathcal{R},\mathcal{B}/l_p/\sum$

G. M. Appa and I. Giannikos. Is linear programming necessary for single facility location with maximum of rectilinear distance? *J. Oper. Res. Soc*, 45(1):97-101,1994.

$1/P/w_m = 1/l_1/max_{obnox}$

M. L. Brandeau. Characterisation of the stochastic median queue trajectory in a plane with generalized distances. *Operations Research*, 40(2):331-341,1992.

$1/P/queue/l_p/\sum_{prob}$

J. Brimberg and R. F. Love. Global convergence of a generalized iterative procedure for the minisum location problem with l_p distances. *Journal of Oper. Res.*, 41:1153-1163,1993.

$1/P/\bullet/l_p/\sum$

E. Carrizosa and F. Plastria. Locating an Undesirable Facility by Generalized Cutting Planes. submitted to *Mathematics of Operations Research*, 1995.

$1/\mathbb{R}^d/\mathcal{F}/l_2^2/\varphi_{obnox} : \nearrow$

R. Carbone and A. Mehrez. The single facility minimax distance problem under stochastic location of demand. *Management Science*, 26:113-115,1980.

$1/P/w_m : RV/l_1/max_{prob}$

R. Chen and G. Y. Handler. The conditional p-center problem in the plane. *Nav. Res. Logist.*, 40(1):117-127,1993.

$k/P/\bullet/l_2/max$

V.F. Doekmeci. A quantitative model to plan regional health facility systems. *Management Science*, 24:411-419,1977.

$\sharp,\sharp,\sharp,\sharp/P/w_m\ :\ RV/l_2/\sum_{prob}$

Z. Drezner. Discon: A new method for the layout problem. *Operations Research*, 28:1375-1384,1980.

$kC/P/\bullet/l_2/\sum$

Z. Drezner and G. O. Wesolowsky. A trajectory method for the optimisation of the multifacility location problem with l_p distances. *Management Science,* 24:1507-1514,1978a.

$k/P/\bullet/l_p/\sum$

Z. Drezner and D. Simchi-Levi. Asymptotic behaviour of the Weber location problem on the plane. *Ann. Oper. Res.*, 40:163-172,1992.

$1/P/w_m = 1/\bullet/\sum$

Z. Drezner and G. O. Wesolowsky. The Weber problem on the plane with some negative weights. *INFOR*, 29(2):87-99,1991

$1/P/w_m \gtrless 0/l_1/\sum,$
$1/P/w_m \gtrless 0/l_2/\sum,$
$1/P/w_m \gtrless 0/l_2^2/\sum,$

R. Durier and C. Michelot. On the set of optimal points to the Weber Problem: Further results. *Transportation Science*, 28(2):116-149,1994.

$\bullet/P/\bullet/\gamma/\sum$

H.A. Eiselt and G. Charlesworth. A note on p-center problems in the plane. *Transportation Science*, 20(2):130-133,1986.

$k/P/\bullet/l_2/max$

L. R. Foulds and H. W. Hamacher. Optimal bin location and sequencing in printed circuit board assembly. *European Journal of Operations Research*, 66(3):279-290,1993.

$N/P/\mathcal{R}/l_p/\sum$

R.L. Francis, T.J. Lowe and M.B. Rayco. Row-column aggregation for rectilinear distance p-median problems. *Transportation Science*, 30(2):160-174,1996.

$k/P/\bullet/l_1/\sum$

H.W. Hamacher and S. Nickel. Multicriteria planar location problems. *Europ. J. of Oper. Res.*, 94(1):66-86,1996.

$1/P/\bullet/l_p/Q-(\sum, max)_{lex}$
$1/P/\bullet/l_p/Q-\sum_{lex}$
$1/P/\bullet/l_1/Q-\sum_{lex}$
$1/P/\bullet/l_\infty/Q-max_{lex}$
$1/P/\bullet/l_2^2/Q-\sum_{lex}$
$1/P/\bullet/l_2^2/Q-\sum_{par}$
$1/P/\bullet/l_2^2/Q-\sum_{MO}$
$1/P/\bullet/l_1/2-\sum_{par}$
$1/P/\bullet/l_1/2-\sum_{MO}$
$1/P/\bullet/l_\infty/Q-\max_{par}$
$1/P/\bullet/l_\infty/Q-max_{MO}$

P. Hansen, D. Peeters, D. Richard and J.-F. Thisse.The minisum and minimax location problems revisited. *Operations Research*, 33:1251-1265,1985.

$1/P/\mathcal{F}$ union of convex polygons, $w_m\ :\nearrow/l_p/max$,
$1/P/\mathcal{F}$ union of convex polygons, $w_m\ :\nearrow/l_p/\sum$

P. Hansen, J. Perreur and J.-F. Thisse. Location theory, dominance and convexity: Some further results. *Operations Research*, 28:1241-1250,1980.

$k/P/\bullet/l_p/\sum$
$k/P/\bullet/(l_p)_m/\sum$

D.W. Hearn and J. Vijay. Efficient algorithms for the (weighted) minimum circle problem. *Operations Research*, 30(4):777-795,1982.

$1/P/\bullet/l_2/max$

J. Karkazis and C. Papadimitriu. A branch-and-bound algorithm for the location of facilities causing atmospheric pollution. *European Journal of Operations Research,* 58(3):363-373,1992.

$1/P/\mathcal{R}/l_2/\sum_{obnox}$
$1/P/\mathcal{R}/l_2/\max_{obnox}$

A. Kolen. Equivalence between the direct search algorithm and the cut approach to the rectilinear distance location problem. *Operations Research*, 29(3):616-620,1981.

$k/P/\bullet/l_1/\sum$

L.F. McGinnis and J.A. White. A single facility rectilinear location problem with multiple criteria. *Transportation Science*, 12:217-231,1978.

$1/P/\bullet/l_1/2-(\sum, max)_{par}$

I.D. Moon and L. Papayanopoulus. Minimax location of two facilities with minimum separation: Interactive graphical solution. *J. Oper. Res. Soc.*, 42(8):685-694,1991.

$2/P/w_m=1, d_{min}(x_1,x_2)/l_2/max$

L. Ostresh jr. On the convergence of a class of iterative methods for solving the Weber location problem. *Operations Research*, 26:597-609,1978a.

$1/\mathbb{R}^n/\bullet/l_2/\sum$

J. Picard and H.D. Ratliff. A cut approach for the rectilinear distance facility location problem. *Operations Research*, 26(3):422-433,1978.

$k/P/\bullet/l_1/\sum$

C. ReVelle, D.Marks and J.C. Liebman. An analysis of private and public sector location models. *Management Science*, 16:692-707,1970.

$1/P/\bullet/l_p/\sum$,
$1/\mathcal{G}/\bullet/d(\mathcal{V},\mathcal{G})/\sum$

K.E. Rosing. An optimal method for solving the (generalized) multi-Weber problem. *European Journal of Operations Research*, 58(3):414-426,1992.

$k/P/\bullet/l_2/\sum$

C.S. Sung and C.M. Joo. Locating an obnoxious facility on an Euclidean network to minimize neighbourhood damage. *Networks*, 24(1):1-9,1994.

$1/P/\mathcal{F}=Network/l_2/\sum_{obnox}$

J.F. Thisse, J. Ward and R. Wendell. Some properties of location problems with block and round norms. *Operations Research*, 32:1309-1327,1984.

$1/P/w_m=1/\|\cdot\|_{block}/max$,
$1/P/w_m=1/l_p, 1<p<\infty/max$,
$1/P/w_m=1/\|\cdot\|_{block}/\sum$,
$1/P/w_m=1/l_p, 1<p<\infty/\sum$

D. Trietsch. Optimal multifacility defensive location on planes with rectilinear distances. *Networks*, 23(6):517-523,1993.

$k/P/w_m = 1/l_1/\sum_{comp}$,
$k/P/w_m = 1/l_1/max_{comp}$

R. Vergin and J. Rogers. An algorithm and computational procedure for locating economic facilities. *Management Science*, 13B:240-254,1967

$1/P/\bullet/l_1/\sum$,
$1/P/\bullet/l_2/\sum$,
$k/P/\bullet/l_2/\sum$,

G.O. Wesolowsky and R.F. Love. The optimal location of new facilities using rectangular distances. *Operations Research*, 19:124-130,1971.

$k/P/\bullet/l_1/\sum$

G.O. Wesolowsky and R.F. Love. A nonlinear approximation method for solving a generalized rectangular distance Weber problem. *Management Science*, 18:656-663,1972.

$k/P/\bullet/l_1/\sum$

3.2 Network Location Problems

O.Berman, R.C Larson and S. Chiu. Optimal server location on a network operating as an M/G/1 queue. *Operations Research*, 33:746-771,1985.

$1/\mathcal{G}/queue/d(\mathcal{V},\mathcal{G})/\sum_{prob}$

O. Berman, D. Simchi-Levi and A. Tamir. The minimax multistop location problem on a tree. *Networks*, 18:39-49,1988

$1/\mathcal{T}/w_m : RV, cap/d(\mathcal{V},\mathcal{G})/max_{prob}$

O. Berman and D. Simchi-Levi. Minisum location of a travelling salesman. *Networks*, 16:239-254,1986.

$1/\mathcal{G}/w_m : RV, cap/d(\mathcal{V},\mathcal{G})/\sum_{prob}$,
$1/\mathcal{T}/w_m : RV, cap/d(\mathcal{V},\mathcal{G})/\sum_{prob}$

A. Billionnet and M.C. Costa. Solving the uncapacitated plant location problem on trees. *Discrete Applied Mathematics*, 49(1-3):51-59,1994.

$1/\mathcal{T}/\bullet/d(\mathcal{V},\mathcal{V})/\sum$

M.L. Brandeau and S.S. Chiu. A unified family of queueing location models. *Operations Research*, 38:1034-1044,1990.

$1/\mathcal{G}/queue/d(\mathcal{V},\mathcal{G})/\sum_{prob}$

M.L. Chen, R.L. Francis, J.F. Lawrence, T.J. Lowe and S. Tufekci. Block vertex duality and the one-median problem. *Networks*, 15:395-412,1985.

$1/\mathcal{G}/\bullet/d(\mathcal{V},\mathcal{V})/\sum$

S.S. Chiu. A dominance theorem for the stochastic queue median problem. *Operations Research*, 34:942-944,1986a.

$1/\mathcal{G}/queue/d(\mathcal{V},\mathcal{G})/\sum_{prob}$

S.S. Chiu, O. Berman and R.C. Larson. Locating a mobile server queueing facility on a tree network. *Management Science*, 31:764-772,1985.

$1//\mathcal{T}/queue/d(\mathcal{V},\mathcal{G})/\sum_{prob}$

R.L. Church and R.S. Garfinkel. Locating an obnoxious facility on a network. *Transportation Science*, 12(2):107-118,1978.

$1/\mathcal{G}/\bullet/d(\mathcal{V},\mathcal{G})/\sum_{obnox}$

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