Interner Bericht

Visualization of Unstable Surface Regions

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Abstract. *Computer processing of free form surfaces forms the basis of a closed construction process starting with surface design and up to NC-production. Numerical simulation and visualization allow quality analysis before manufacture. A new aspect in surface analysis is described, the stability of surfaces versus infinitesimal bendings. The stability concept is derived from the kinetic meaning of a special vector field which is given by the deformation. Algorithms to calculate this vector field together with an appropriate visualization method give a tool able to analyze surface stability.*

1. Introduction

During the CAD/CAM-process the analysis of the shape is an important step which amounts the efficiency. Many well known surface interrogation methods like generalized focal surfaces, reflection lines or k-orthotomics [1] are used to analyze continuity or curvature behaviour and to detect aesthetically unwanted behaviour of the shape.

A new aspect of shape control is the stability of surfaces with respect to infinitesimal bendings. The meaning of stability in this paper is only related to the shape of the surface, i.e. its geometry, and not to strength or material aspects.

The derivation of theoretical basics shows that there exists a special vector field for each infinitesimal bending. The stability concept is derived from the kinetic meaning of this so called rotation vector field.

The topic of this paper is to present a visualization method which is adapted to the specific structure of the rotation vector field. It will give us a tool for analizing surface stability by indicating regions on the surface which are more likely to bend than other.

The paper is divided into three parts

- first, a short review of the most important aspects of infinitesimal bendings,
- then, the stability concept is developed by interpretation of the meaning of the rotation vectors,
- finally, the visualization method adapted to the specific structure of this field is presented in order to visualize unstable surface regions.

2. Fundamentals of infinitesimal bendings

In this chapter a short introduction to the theory of infinitesimal bendings is given. A general survey can be found in [2].

Parametric surfaces are represented as vector valued functions $X : G \to \mathbb{R}^3$ of class C^2, where G is a connected domain of $\mathbb{R}^2$.

Definition (2.1): *A one-parameter family $\{X_t\}$ of mappings $X_t : G \to \mathbb{R}^3$ with $t \in I := [0, a)$, $a > 0$ and $X_0 = X$, is called* **deformation** *of X.*

Definition (2.2): *Given a deformation*

$$X^*(u,w) := X_\varepsilon(u,w) := X(u,w) + \varepsilon Z(u,w)\,, \quad \varepsilon \in \mathbb{R} \tag{2.3}$$

where Z is a vector field of class $C^2(G)$. Z is called **deformation vector field.**

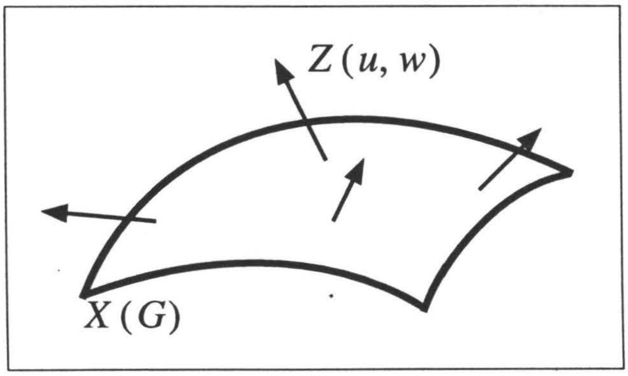

Figure 1: Deformation of the surface X

The vector field Z can be seen as the velocity field of the surface points in the beginning of the deformation.

The basic concept of an infinitesimal theory is to neglect all infinitesimal small quantities of the surface X_e of higher order in ε. This leads to the following definition.

Definition (2.4): *The deformation X^* (2.3) is called* **infinitesimal bending of first order** *of the surface X, if the length L of any arbitrary smooth curve c on the surface keeps unchanged in first order in ε for $\varepsilon \to 0$, i.e. $L(c_\varepsilon) = L(c) + o(\varepsilon)$.*

The following theorem characterizes the infinitesimal bendings by the coefficients of the first fundamental form.

Theorem (2.5): *The deformation $X^* = X + \varepsilon Z$ is an infinitesimal bending of first order of X if and only if*

$$\delta g^*_{ij} := \frac{\partial g^*_{ij}}{\partial \varepsilon}|_{\varepsilon=0} = 0 \quad \Longleftrightarrow \quad \begin{array}{l} < X_u, Z_u >= 0 \\ < X_w, Z_w >= 0 \\ < X_u, Z_w > + < X_w, Z_u >= 0 \end{array} \tag{2.6}$$

where g^*_{ij} are the first fundamental forms of X^*. $< , >$ denotes the dot product.

Proof: see [3]

Up to now we have seen the definition of infinitesimal bendings as special C^2-continuous deformations. Equation (2.6) can also be written in differential forms: $< dX, dZ >= 0$. It is called *differential equation of the infinitesimal bendings* and represents a sufficient and necessary condition for a deformation vector field Z to be an infinitesimal bending.

The following theorem introduces a further vector field related to an infinitesimal bending. It has a special geometric significance which is given in the next chapter. The proof is omitted here, but it can be done directly by application of theorem (2.5).

Theorem (2.7): **- Existence Theorem -** *If the deformation vector field Z verifies the three equations (2.6), then there exists an unique vector field Y of class $C^1(G)$ with the following properties:*

$$[Y, X_u] = Z_u \quad \text{and} \quad [Y, X_w] = Z_w \, , \tag{2.8}$$

where $[,]$ denotes the vector product.
The vector field $Y(u, w)$ is called **rotation vector field.**

3. The stability concept

Deforming a surface means that it can either be moved as a rigid body or it can change its shape, i.e. it can bend. Surfaces which don't allow bendings are called rigid surfaces. Now we will see how the rotation vector field is related to the notion of rigid surfaces.

The differential equation $< dX, dZ >= 0$ (2.6) has always the trivial solution $Z = [C, X] + D$, where C, D are arbitrary constant vectors. These deformations don't cause inner deformations of the surface, because they define a rigid (infinitesimal) motion of the surface.

Definition (3.1): *If $X^* = X + \varepsilon Z$ is an infinitesimal bending with $Z = [C, X] + D$, i.e. $Y = C = const$, then X^* is called* **trivial infinitesimal bending** *or* **infinitesimal motion** *of the surface X.*

Definition (3.2): *A surface which allows only trivial infinitesimal bendings is called* **infinitesimal rigid.**

A classical theorem of infinitesimal bendings says that all closed convex analytic surfaces which are connected are rigid [4]. Open surfaces are generally not rigid. They are nevertheless more or less stable depending on their shape. (3.1) and (3.2) show that the notion of rigid surfaces is defined by the rotation vectors, more precisely by constant rotation vector fields. It will be shown now how *stability* will be defined in this context.
To record quantitatively, how a surface is likely to bend (i.e. how it is stable), is also possible by using the rotation vector field. The next theorem gives the kinetic meaning of this vector field and shows what happens geometrically during the deformation.

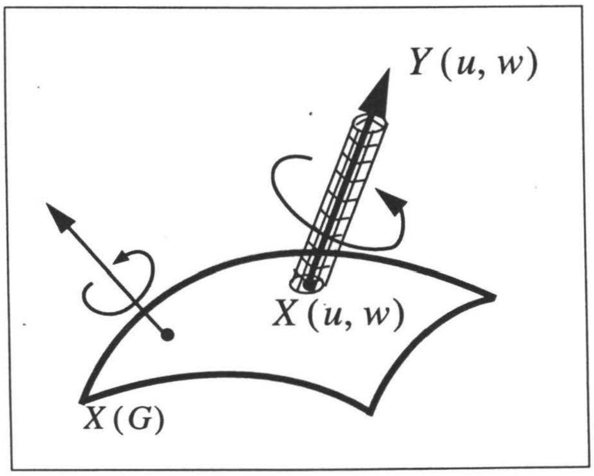

Figure 2: Rotation vector field

Proposition (3.3): *$\varepsilon Y(u,w)$ is the rotation vector field of the infinitesimal bending of the surface $X(u,w)$ into the surface $X_\varepsilon(u,w)$.*
The direction of the rotation vector is the axis of the rotation of the surface element in the point (u,w) during the bending.
The norm and the orientation of the rotation vector determine the angle of rotation, except for quantities of higher order.

Proof: see [5]

Now we can state that the more the rotation vectors of a surface region vary in their directions and lengths, the more the surface is likely to bend in this region, i.e. the less the surface there is *stable*. On the other hand, if the rotation vectors are nearly constant, the surface behaves as a rigid one in this region. This statement is basic for the stability concept and underlines the importance of the rotation vectors in this concept. The aim of the next chapter is to make this concept usable in practice.

Remark:
The calculation of the vector fields of a surface is not an easy task, if we don't want to calculate the corresponding infinitesimal bendings, too. A solution can be found by a least-square fitting with a B-Spline representation for Y [6]. We don't want to go in this detail here.

4. Visualization of unstable surface regions

In the previous chapter the kinetic meaning of the rotation vector field and its importance for the notion of stability was explained. Now a visualization tool has to be developed which is able to detect unstable surface regions. Based on the concept presented above, unstable surface regions are characterized by rotation vectors which are changing their directions a lot in a small area or by rotation vectors with a relatively high length. The visualization of these properties of the vector field remains the object.

Most of the vector field visualization methods have been developed for applications in special domains, e.g. fluid dynamics [7], [8], [9] or texturing [10], [11], and therefore they are not suitable in the present case. The rotation vector fields also have a special significance as mentioned above.

The vectors could be visualized by a normalized arrow plot as shown in figure 4, but it doesn't give a correct result because the projection of 3D vectors into a 2D domain is ambiguous. Also the information about the length of the vectors gets lost.

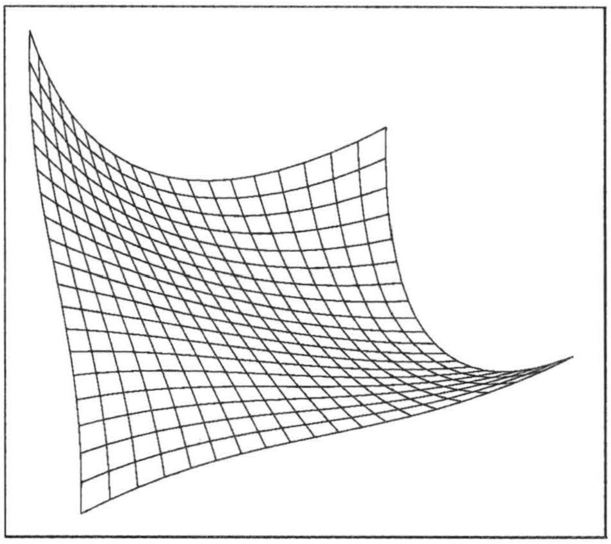

Figure 3: test surface I (bicubic Bézier patch)

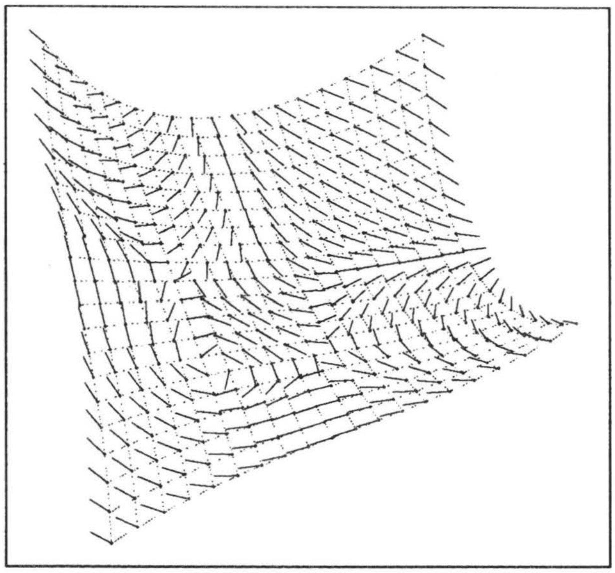

Figure 4: arrow plot

Visualization of the lengths of the vectors can directly be done with iso-lines or a classical color mapping because it's a scalar field. Figure 5 shows a color map of $||Y(u, w)||$ on the surface.

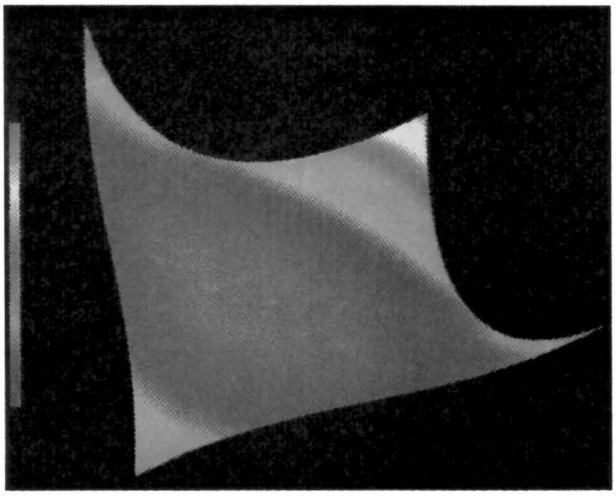

Figure 5: $||Y||$-color mapping of the rotation vector field

A linear color scale was taken in order not to present a distorted picture of the lengths. The highest values are lying in the three corners of the patch, while most of the rotation vectors have small magnitudes. On the other side, the visualization of areas on the surface where the rotation vectors vary a lot in their directions, is needed. Anyway we are not