

# BERICHTE DER ARBEITSGRUPPE TECHNOMATHEMATIK

FORSCHUNG - AUSBILDUNG - WEITERBILDUNG

Bericht Nr. 119

DOMAIN DECOMPOSITION FOR KINETIC PROBLEMS  
WITH STRONGLY CONTRASTED KNUDSEN NUMBERS

Axel Klar



## FACHBEREICH MATHEMATIK

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# Domain Decomposition for Kinetic Problems with Strongly Contrasted Knudsen Numbers

by AXEL KLAR

## Abstract

A nonequilibrium situation governed by kinetic equations with strongly contrasted Knudsen numbers in different subdomains is discussed. We consider a domain decomposition problem for Boltzmann- and Euler equations, establish the correct coupling conditions and prove the validity of the obtained coupled solution. Moreover numerical examples comparing different types of coupling conditions are presented.

## 1 Introduction

Domain decomposition methods involving kinetic and aerodynamic equations have become of considerable interest, e.g. in space flight problems. The Boltzmann equation is in general considered to give the correct results for rarefied gas flows, but when the mean free path of the molecules becomes small the usual numerical methods to solve the Boltzmann equation are very expensive in computing time. Therefore in regions where Euler- or Navier-Stokes equations are valid, they should be solved.

One of the main problems is to find the correct coupling conditions at the interface between the two regions. The approach usually employed in numerical procedures, see Bourgat et al [3] or Lukschin et al [14], is the following: The boundary conditions at the interface for the aerodynamic equation are determined from the Boltzmann distribution function by equalizing the moments or fluxes. The boundary condition for the Boltzmann equation at the interface, i.e. the ingoing function for the Boltzmann region is given by a Maxwellian distribution with the aerodynamic quantities as parameters. For an equilibrium situation at the interface these coupling conditions are the appropriate ones.

If we consider instead of such a situation strongly contrasted Knudsen numbers in the aerodynamic and kinetic regions and nonequilibrium states then the above coupling conditions will not lead to the correct results. Here the matching requires a more exact analysis. It can be done by modelling the interface region by a transition layer. An asymptotic analysis leads to a kinetic linear half space problem. The asymptotic values of the solution of this problem determine the aerodynamic boundary conditions at the interface. The outgoing flux of the half space problem gives the ingoing distribution function for the Boltzmann region at the interface. Kinetic half space problems have been

widely considered. A mathematical investigation of these problems is among others done e.g. in Arthur and Cercignani [2], Bardos et al. [4], Bensoussan et al. [5], Coron et al. [7]. Many numerical investigations for various different situations have been performed by Sone and coworkers, see e.g. [1], [15]. Obviously the direct solution of the half space problem would be much too expensive. We could as well solve the Boltzmann equation also in the aerodynamic region. Moreover from the above we see that only the asymptotic states and the outgoing fluxes are really required. In Golse/Klar [9] we developed a fast numerical scheme which computes approximately these two things by a Chapman Enskog type expansion procedure. This makes the approach reasonable from a numerical point of view.

For a different approach to the coupling problem we refer to Illner/Neunzert [11].

The paper is organized as follows:

In Section 2 we consider a simple model problem and introduce the coupling conditions given by the analysis of the kinetic layer. We investigate whether the coupled solution obtained with these coupling conditions is the correct one, i.e. whether it is near to the kinetic solution in the whole domain. It is proved that, if the Knudsen number  $\epsilon$  in the aerodynamic region tends to 0, the analysis of the kinetic layer gives the correct coupling conditions up to order  $\epsilon$ .

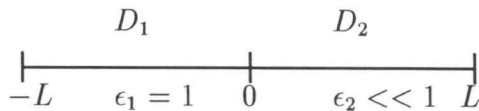
In Section 3 we extend the analysis to the linearized Boltzmann equation coupled with the linearized Euler equations and point out the correct conditions in this case.

In Section 4 we present some numerical examples for the model problem in section 2 comparing the above coupling conditions with the ones obtained by equalizing moments or fluxes. Moreover numerical results for the 3-dimensional BGK model are shown comparing again the different types of coupling conditions.

## 2 Coupling Conditions and Physical Correctness of the Coupled Solution for a Simple Model Problem

We consider for simplicity a one dimensional geometry  $x \in [-L, L]$  with an interface at  $x = 0$ . The physical situation is described by kinetic equations with different mean free paths in the domains  $D_1 = [-L, 0]$  and  $D_2 = [0, L]$ .

We denote the mean free paths in  $D_1$  and  $D_2$  by  $\epsilon_1$  respectively  $\epsilon_2$ . We assume  $\epsilon_2$  in  $D_2$  to be small such that an aerodynamic approximation of the kinetic equation is valid,  $\epsilon_2 := \epsilon$  with  $\epsilon$  tending to 0. In  $D_1$  the mean free path  $\epsilon_1$  is assumed to be large compared to  $\epsilon_2$ ; we put  $\epsilon_1 := 1$ .



The kinetic equations in  $D_i, i = 1, 2$  are here

$$\partial_t \varphi^{(i)} + (v_1 + u) \partial_x \varphi^{(i)} + \frac{1}{\epsilon_i} (I - K) \varphi^{(i)} = 0, \quad (2.1)$$

where  $x \in D_i, v = (v_1, v_2, v_3) \in S \subseteq \mathbb{R}^N, t \in [0, \infty), N = 1, 2, 3$ .  $S$  is assumed to be a ball or sphere in  $\mathbb{R}^N$ .  $u \in \mathbb{R} \setminus \{0\}$  is a constant and  $K$  an integral operator

$$K\varphi(x, v, t) = \int_S \bar{k}(v, v')\varphi(x, v', t)dv',$$

$\bar{k}$  symmetric in  $v$  and  $v', 0 < k_1 < \bar{k}(v, v') < k_2, k_1, k_2$  some constants and  $\int_S \bar{k}(v, v')dv' = 1$ .

In particular  $K$  is an operator in  $\mathcal{L}^\infty(S)$  with  $\|K\|_\infty \leq 1$ . Moreover the collision operator  $I - K$  has as collision invariants only constants. The aerodynamic equation in  $D_2$  associated to (2.1) is a simple linear advection equation

$$\partial_t \Theta + u \partial_x \Theta = 0, u \in \mathbb{R} \setminus \{0\}. \quad (2.2)$$

Coupling the solution of (2.1) with  $\epsilon_1 = 1$  in  $D_1$  and the solution of (2.2) in  $D_2$  one tries to obtain a coupled solution that approximates the global kinetic one. I.e. the reference solution to which the coupled solution is compared will be the solution of the kinetic equation in the whole domain with different mean free paths  $\epsilon_1 = 1$  in  $D_1$  and  $\epsilon_2 = \epsilon$  tending to 0 in  $D_2$ .

We describe now more precisely the coupled solution.

Let  $\varphi^{(1)}$  be the solution of (2.1) in  $D_1$  with  $\epsilon_1 = 1$  and

$$\begin{aligned} \varphi^{(1)}(-L, v, t) &= f_+(v, t), v_1 + u > 0 \\ \varphi^{(1)}(x, v, 0) &= h(x, v), \end{aligned}$$

Let  $\Theta$  be the solution of (2.2) in  $D_2$  with

$$\Theta(x, 0) = g(x)$$

and

$$\Theta(L, t) = f_-(t)$$

if  $u < 0$ .

$f_+$  is assumed to be uniformly bounded in  $v$  and  $t$  with  $v \in S$  and  $t \in [0, T], T$  fixed but arbitrary.  $f_-, h, g$  are also assumed to be uniformly bounded in  $x, v, t$  with  $t \in [0, T]$ , resp.  $x \in D_1, v \in S$ , resp.  $x \in D_2$ . We assume all data to be as smooth as required and the necessary compatibility conditions for initial and boundary conditions to be satisfied in order to avoid problems connected with nonsmoothness.

It remains to fix the coupling conditions. Taking the usual ones, i.e. equality of moments or fluxes as described in Section 4, will in general lead to wrong results. See Section 4 for numerical examples. One has to make a more exact analysis of the situation near the interface. We neglect the boundary layer at  $x = L$  and proceed for the transition layer similar to the usual boundary layer expansions, see e.g. Cercignani [6]: We assume the distribution function in the aerodynamic region to be equal up to order  $\epsilon$  to the solution of

the macroscopic equation plus a kinetic layer term concentrated in a region in  $D_2$  around the interface. The size of this region is of order  $\epsilon$ . This corresponds to a scaling of the space coordinate  $x$  in the layer like  $\frac{x}{\epsilon}$ . This means that we have to find a solution  $\Phi(x, v, t)$  of the kinetic equation to order  $\epsilon$  in the domain  $D_2$  in the form

$$\begin{aligned}\Phi(x, v, t) &:= \Theta(x, t) + \chi\left(\frac{x}{\epsilon}, v, t\right) \\ &+ \epsilon \hat{W}\left(\frac{x}{\epsilon}, v, t\right) + \epsilon W(x, v, t).\end{aligned}$$

$\Phi$  must fulfill

$$\partial_t \Phi + (v_1 + u) \partial_x \Phi + \frac{1}{\epsilon} (I - K)(\Phi) = 0(\epsilon).$$

The distribution function  $\varphi^{(1)}$  in  $D_1$  and the distribution function  $\Phi$  in  $D_2$  are moreover assumed to be continuous at the interface  $x = 0$ , i.e.  $\Phi(0, v, t) = \varphi^{(1)}(0, v, t)$  up to order  $\epsilon$ :

$$\Phi(0, v, t) = \varphi^{(1)}(0, v, t) + 0(\epsilon).$$

Computing  $\partial_t \Phi + (v_1 + u) \partial_x \Phi$  one obtains

$$\begin{aligned}\partial_t \left( \Theta + \chi\left(\frac{x}{\epsilon}\right) + \epsilon \hat{W}\left(\frac{x}{\epsilon}\right) + \epsilon W \right) &+ (v_1 + u) \partial_x \left( \Theta + \epsilon W \right) \\ &+ (v_1 + u) \frac{1}{\epsilon} \partial_x \chi\left(\frac{x}{\epsilon}\right) + (v_1 + u) \partial_x \hat{W}\left(\frac{x}{\epsilon}\right).\end{aligned}$$

To order  $\epsilon$  this must be equal to

$$-\frac{1}{\epsilon} (I - K) \left( \Theta + \chi\left(\frac{x}{\epsilon}\right) + \epsilon \hat{W}\left(\frac{x}{\epsilon}\right) + \epsilon W \right).$$

Comparing the terms of order  $\epsilon^{-1}$  yields the half space problem

$$(v_1 + u) \partial_x \chi + (I - K)(\chi) = 0$$

with  $\chi(\infty, v, t) = 0$ , because the influence of the layer term must be concentrated near the interface. Assuming an ingoing flux of the form

$$\chi(0, v, t) = \varphi^{(1)}(0, v, t) - a(t), v_1 + u > 0$$

with  $a(t)$  arbitrary, there is a unique solution  $\chi$  of this problem for  $u > 0$ , see e.g. Greenberg et al. [9]. In particular  $a$  can not be prescribed, it is determined by the solution. For  $u < 0$  the equation has a unique solution, if  $a(t)$  is fixed in advance.

Terms of order 0 cancel if

$$\partial_t \Theta + (v_1 + u) \partial_x \Theta + (I - K)(W) = 0$$

is satisfied and if a halfspace problem for  $\hat{W}$  is fulfilled. This equation is uniquely solvable, if  $\Theta$  fulfills the macroscopic equation.

Considering the boundary values at  $x = 0$  we obtain

$$\Phi(0, v, t) = \Theta(0, t) + \chi(0, v, t) + 0(\epsilon).$$

Now for  $u > 0$  it is easily seen that  $\Theta(0, t)$ , the boundary condition for (2.2), has to be chosen equal to  $a(t)$  and  $\varphi^{(1)}(0, v, t)$  equal to  $\chi(0, v, t) + \Theta(0, t)$  for  $v_1 + u < 0$  to achieve to order  $\epsilon$

$$\Phi(0, v, t) = \varphi^{(1)}(0, v, t).$$

For  $u < 0$  there is no need of a boundary condition for (2.2) at  $x = 0$ . This corresponds to the solvability of the halfspace equation, if  $a(t)$  is prescribed.  $a(t)$  must be defined by  $\Theta(0, t)$  and  $\varphi^{(1)}(0, v, t), v_1 + u < 0$  has to be chosen as before. Then again the same result is obtained.

Due to this analysis the coupling conditions are found in the following way: Let  $\chi_H$  be the solution of the kinetic linear half space problem

$$\begin{aligned} (v_1 + u)\partial_x \chi_H + (I - K)\chi_H &= 0, \quad x \in [0, \infty) \\ \chi_H(0, v, t) &= \varphi^{(1)}(0, v, t), \quad v_1 + u > 0. \end{aligned} \tag{2.3}$$

For  $u > 0$  the solution  $\chi_H$  is unique. Solving (2.3) one obtains  $\chi_H(\infty, t)$  and  $\chi_H(0, v, t), v_1 + u < 0$ . These values will give us the coupling conditions. The condition at the interface for the aerodynamic equation is given by

$$\Theta(0, t) = \chi_H(\infty, t).$$

The condition for the kinetic equation in  $[-L, 0]$  at  $x = 0$  is

$$\varphi^{(1)}(0, v, t) = \chi_H(0, v, t), \quad v_1 + u < 0.$$

For  $u < 0$  one needs a constraint to obtain again a unique solution. It will be given by the solution of the aerodynamic equation  $\Theta(x, t)$  with boundary condition  $f_-(t)$  at  $x = L$ :  $\chi_H(\infty, v, t) = \Theta(0, t)$ . Solving (2.3) for  $u < 0$  with this constraint gives  $\chi_H(0, v, t), v_1 + u < 0$  and  $\varphi^{(1)}(0, v, t), v_1 + u < 0$  is obtained as before.

$\varphi^{(1)}$  and  $\Theta$  fulfilling the coupling and boundary conditions will be called the solution of the coupling problem. The reference solution in  $[-L, L]$  to which the coupled solution is compared is the solution of (2.1) in  $D_1$  and  $D_2$  with  $\epsilon_1 = 1$  respectively  $\epsilon_2 = \epsilon$ , the boundary conditions

$$\begin{aligned} \varphi_\epsilon^{(1)}(-L, v, t) &= f_+(v, t), \quad v_1 + u > 0 \\ \varphi_\epsilon^{(2)}(L, v, t) &= f_-(v, t), \quad v_1 + u < 0 \end{aligned}$$

and the coupling conditions

$$\varphi_\epsilon^{(1)}(0, v, t) = \varphi_\epsilon^{(2)}(0, v, t).$$

The initial conditions are

$$\begin{aligned}\varphi_\epsilon^{(1)}(x, v, 0) &= h(x, v) \quad \forall x \in D_1 \\ \varphi_\epsilon^{(2)}(x, v, 0) &= g(x) \quad \forall x \in D_2.\end{aligned}$$

$\varphi^{(1)}$  is also indexed since it depends on  $\epsilon$  by the coupling conditions.

The transition and boundary layer terms are modelled by functions, that are solutions of stationary kinetic linear half space problems. The value of these layer functions at infinity is zero, in order to restrict their influence to the interface and boundary regions. The layer functions are, at  $x = 0$ , the function  $\chi_1$

$$\chi_1(x, v, t) := \chi_H(x, v, t) - \chi_H(\infty, t)$$

and, at  $x = L$ , the function  $\chi_2$ , where  $\chi_2(x, v, t)$  is for  $u > 0$  the solution of

$$\begin{aligned}(v_1 + u)\partial_x \chi_2 + (I - K)\chi_2 &= 0, & x \in (-\infty, 0] \\ \chi_2(0, v, t) &= f_-(t) - \Theta(L, t), & v_1 + u < 0\end{aligned}\tag{2.4}$$

with the condition

$$\chi_2(-\infty, v, t) = 0.$$

For  $u < 0$  we define

$$\chi_2 = 0.$$

Using a perturbation expansion, like e.g. in Bensoussan et al. [5] or Bardos et al. [4], and the above mentioned results on the linear half space problem one can then prove the following theorem, see Klar [12]



## Theorem 2.1

1. For  $T$  fixed but arbitrary there is a unique coupled solution

$$(\varphi^{(1)}, \Theta) \text{ with } \varphi^{(1)} \in \mathcal{L}^\infty(D_1 \times S \times [0, T]), \Theta \in \mathcal{L}^\infty(D_2 \times [0, T])$$

fulfilling the above conditions and a unique reference solution

$$(\varphi_\epsilon^{(1)}, \varphi_\epsilon^{(2)}) \text{ with } \varphi_\epsilon^{(i)} \in \mathcal{L}^\infty(D_i \times S \times [0, T]), i = 1, 2 \quad \forall \epsilon > 0.$$

2. In  $D_1$  we have

$$\|\varphi_\epsilon^{(1)} - \varphi^{(1)}\|_\infty \leq \epsilon C,$$

$C$  some constant.

3. In  $D_2$  we get

$$\begin{aligned} \|\varphi_\epsilon^{(2)} - [\Theta(x, t) + \chi_1\left(\frac{x}{\epsilon}, v, t\right) \\ + \chi_2\left(\frac{x-L}{\epsilon}, v, t\right)]\|_\infty \leq \epsilon C, \end{aligned}$$

where  $\Theta, \chi_1$  and  $\chi_2$  do not depend on  $\epsilon$ .

**Remark:** We mention that in general - in contrast to the global kinetic reference solution - there will be a jump in the macroscopic quantities of the coupled solution at the interface. The coupled solution is a correct approximation for small  $\epsilon$  of the global kinetic solution only outside of boundary and interface layers in  $D_2$ .

## 3 Coupling Conditions for Linearized Boltzmann and Euler Equations

We describe here the extension of the coupling conditions in the preceding section to the full Boltzmann and Euler equations linearized around a constant state  $\bar{\rho}, \bar{u}, \bar{T}$  with  $\bar{u} = (\bar{u}_1, \bar{u}_2, \bar{u}_3)$ .

We restrict ourselves again to a 1-dimensional geometry  $x \in [-L, L]$  and refer to the remark at the end of this section for possible extensions to the multidimensional case.

The kinetic equations in  $D_i, i = 1, 2$ , with  $D_1 = [-L, 0]$  and  $D_2 = [0, L]$  as before, are the Boltzmann equations with shifted velocities  $v \rightarrow v + \bar{u}$

$$\partial_t \varphi^{(i)} + (v_1 + \bar{u}_1) \partial_x \varphi^{(i)} + \frac{1}{\epsilon_i} 2Q(\bar{M}, \varphi^{(i)}) = 0, \quad (3.1)$$

$i=1,2$ , where  $Q$  is the Boltzmann collision operator with a suitable collision law,

$$\bar{M} = \frac{\bar{\rho}}{(2\pi\bar{T})^{3/2}} \exp\left(-\frac{|v|^2}{2\bar{T}}\right),$$

$\bar{u}_1 \in \mathbb{R}, x \in D_i$  and  $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ .

We assume again that  $\epsilon_2 = \epsilon$  in  $D_2$  is small and  $\epsilon_1$  in  $D_1$  is large compared to  $\epsilon_2$ . We set  $\epsilon_1 := 1$ .

The aerodynamic equation in  $D_2$  is the compressible linearized Euler equation

$$\partial_t \Theta + A \partial_x \Theta = 0, \quad (3.2)$$

with  $x \in [0, L], \Theta = (\rho, u_1, u_2, u_3, T)$ . Let  $c > 0$  defined by  $c^2 = \frac{5}{3} \bar{T}$  be the speed of sound.  $A$  is the following matrix

$$A = \begin{bmatrix} \bar{u}_1 & \bar{\rho} & 0 & 0 & 0 \\ \frac{\bar{T}}{\bar{\rho}} & \bar{u}_1 & 0 & 0 & 1 \\ 0 & 0 & \bar{u}_1 & 0 & 0 \\ 0 & 0 & 0 & \bar{\rho}_1 & 0 \\ 0 & \frac{2}{5} c^2 & 0 & 0 & \bar{u}_1 \end{bmatrix}.$$

Now the coupling of (3.1) in  $D_1 = [-L, 0]$  and (3.2) in  $D_2 = [0, L]$  is considered and compared to the solution of the Boltzmann equation in the whole domain  $[-L, L]$ . We concentrate in the following on the coupling conditions at  $x = 0$  disregarding the boundary conditions at  $x = L$  and  $x = -L$ .

The system (3.2) is diagonalizable with Eigenvalues  $\lambda_i = \bar{u}_1, i = 1, 2, 3, \lambda_4 = \bar{u}_1 + c, \lambda_5 = \bar{u}_1 - c$ . One needs at  $x = 0$  for (3.2) 0, 1, 4 or 5 boundary conditions for the characteristic variables according to the value of  $\bar{u}_1$ , i.e.  $\bar{u}_1 < -c, -c < \bar{u}_1 < 0, 0 < \bar{u}_1 < c$  or  $\bar{u}_1 > c$ . Here and in the following we assume  $\bar{u}_1 \neq 0, c, -c$ .

Simply using the equality of moments or fluxes at  $x = 0$  will - as in Section 2 - not lead to the correct results, as we shall see from the simulations in Section 4. An analysis like in section 2 leads to the consideration of

$$\begin{aligned} (v_1 + \bar{u}_1) \partial_x \chi_H + 2Q(\bar{M}, \chi_H) &= 0, \quad x \in [0, \infty) \\ \chi_H(0, v, t) &= \varphi^{(1)}(0, v, t), \quad v_1 + \bar{u}_1 > 0 \end{aligned} \quad (3.3)$$

where  $\varphi^{(1)}(0, v, t)$  is the distribution function in  $D_1$  at  $x = 0$ . This equation has a unique solution, see Coron et al [7] or Greenberg et al. [10], if - according to the values of  $\bar{u}_1$  - a number of constraints is imposed. One needs 5, 4, 1 resp. 0 constraints if  $\bar{u}_1 < -c, -c < \bar{u}_1 < 0, 0 < \bar{u}_1 < c$  resp.  $\bar{u}_1 > c$  with  $c^2 = \frac{5}{3} \bar{T}$  as before. Solving the half space problem gives the asymptotic value

$$\chi_H(\infty, v, t) = \left( \frac{a_0(t)}{\bar{\rho}} + \sum_{i=1}^3 \frac{a_i(t)}{\sqrt{\bar{T}}} \frac{v_i}{\sqrt{\bar{T}}} + \frac{a_4(t)}{\bar{T}} \frac{|v|^2 - 3\bar{T}}{2\bar{T}} \right) \bar{M}.$$

According to the number of constraints one already has 5, 4, 1 resp. 0 equations for  $a_0, \dots, a_4$ . This means that for  $\bar{u}_1 < -c, -c < \bar{u}_1 < 0, 0 < \bar{u}_1 < c$  resp.  $\bar{u}_1 > c$  we obtain 0, 1, 4 resp. 5 new conditions on the asymptotic values  $a_0, \dots, a_4$ . This fits exactly to what is needed for Euler equations as already mentioned by Golse [8].

We restrict ourselves here to supersonic flow  $|\bar{u}_1| > c$ .

For  $\bar{u}_1 > c$  the half space problem is solved for prescribed incoming fluxes without any constraints. This gives the asymptotic values  $a_0(t), a_i(t), a_4(t), i = 1, 2, 3$ .

The 'macroscopic density function' in  $D_2$  is a linearized Maxwellian with parameters given by the solution of Eulers equations:

$$\varphi_{macro}(x, v, t) := \bar{M}_{(\rho(x,t), u(x,t), T(x,t))}^{lin}(v)$$

with

$$\bar{M}_{(\rho, u, T)}^{lin}(v) := \left( \frac{\rho}{\bar{\rho}} + \sum_{i=1}^3 \frac{u_i}{\sqrt{\bar{T}}} \frac{v_i}{\sqrt{\bar{T}}} + \frac{T}{\bar{T}} \frac{|v|^2 - 3\bar{T}}{2\bar{T}} \right) \bar{M}.$$

At  $x = 0$  this is

$$\bar{M}_{(\rho(0,t), u(0,t), T(0,t))}^{lin}(v).$$

Comparing it to  $\chi_H(\infty, v, t)$  one obtains

$$\begin{aligned} \rho(0, t) &= a_0(t) \\ u_i(0, t) &= a_i(t), \quad i = 1, 2, 3 \\ T(0, t) &= a_4(t) \end{aligned} \tag{3.4}$$

Thus the solution of the half space problem gives us the boundary conditions required for Eulers equations with  $\bar{u}_1 > c$  at  $x = 0$ , i.e.  $\rho(0, t), u(0, t), T(0, t)$ .

Moreover the outgoing flux  $\chi_H(0, v, t), v_1 + \bar{u}_1 < 0$  gives  $\varphi^{(1)}(0, v, t), v_1 + \bar{u}_1 < 0$ , i.e. the boundary condition at  $x = 0$  for the domain  $D_1$ .

**Remark:** If  $\varphi^{(1)}(0, v, t), v_1 + \bar{u}_1 > 0$  is a linearized Maxwellian

$$\varphi^{(1)}(0, v, t) = \bar{M}_{(\rho^{(1)}(t), u^{(1)}(t), T^{(1)}(t))}^{lin}(v), \quad v_1 + \bar{u}_1 > 0,$$

we get

$$\chi_H(\infty, v, t) = \varphi^{(1)}(0, v, t) = \bar{M}_{(\rho^{(1)}(t), u^{(1)}(t), T^{(1)}(t))}^{lin}(v), \forall v \in \mathbb{R}^3.$$

This yields equality of the macroscopic quantities  $\rho(0, t) = \rho^{(1)}(t), u(0, t) = u^{(1)}(t), T(0, t) = T^{(1)}(t)$ . For general functions  $\varphi^{(1)}(0, v, t), v_1 + \bar{u}_1 > 0$  this is usually not the case. The moments of  $\varphi^{(1)}(0, v, t)$  do not coincide with  $\rho(0, t), u(0, t), T(0, t)$ . One obtains a jump in the macroscopic quantities.

For  $\bar{u}_1 < -c$  the situation is the following: To solve the half space problem 5 constraints on the solution are necessary. Comparing the macroscopic density function, with parameters given by the solutions of Eulers equations, with the asymptotic value  $\chi_H(\infty, v, t)$ , we get the necessary number of constraints. Remark that for  $\bar{u}_1 < -c$  we do not need any boundary condition at  $x = 0$  for Eulers equation in  $D_2$ . We can then solve the half space problem, which yields  $\varphi^{(1)}(0, v, t) = \chi_H(0, v, t), v_1 + \bar{u}_1 < 0$ .

**Remark:** In the multidimensional case one can proceed in the same way. Suppose that the interface  $\Sigma$  divides the computational domain  $\Omega$  into subdomains  $\Omega_1$  and  $\Omega_2$ . At each point  $x \in \Sigma$  one has to solve a one dimensional half space problem with coordinate axis along the unit normal  $n(x)$  to  $\Sigma$  at the point  $x$ . This will lead for each  $x \in \Sigma$  to the correct boundary conditions.

## 4 Numerical Results

In this section we investigate the coupling procedure proposed in Sections 2 and 3 numerically. The coupling conditions described there are compared with the ones obtained by equalizing moments or fluxes. This means that they are determined by the following procedures:

For the model equations in Section 2 and  $u > 0$  the following equations are used to determine  $\Theta(0, t)$  from  $\varphi^{(1)}(0, v, t)$

$$\int \varphi^{(1)}(0, v, t) dv = \int \Theta(0, t) dv$$

resp.

$$\int_{v_1+u>0} (v_1 + u) \varphi^{(1)}(0, v, t) dv = \int_{v_1+u>0} (v_1 + u) \Theta(0, t) dv$$

The ingoing function for the Boltzmann region  $\varphi^{(1)}(0, v, t)$ ,  $v_1 + u < 0$  is determined from  $\Theta(0, t)$  by

$$\varphi^{(1)}(0, v, t) = \Theta(0, t), \quad v_1 + u < 0.$$

$\Theta(0, t)$  must not be prescribed for  $u < 0$ .  $\varphi^{(1)}(0, v, t)$ ,  $v_1 + u < 0$  is given by a function  $\tilde{\Theta}(t)$  independent of  $v$  s.t. the equality of moments is fulfilled. The equality of fluxes does in this case not give any conditions on the ingoing function. We simply take  $\varphi^{(1)}(0, v, t) = \Theta(0, t)$ ,  $v_1 + u < 0$ .

For Eulers equation in Section 3 the boundary values  $\rho(0, t)$ ,  $u(0, t)$  and  $T(0, t)$  are found for  $\bar{u}_1 > c$  by

$$\int \left( \frac{1}{|v|^2} \right) \varphi^{(1)}(0, v, t) dv = \int \left( \frac{1}{|v|^2} \right) \bar{M}_{(\rho(0,t), u(0,t), T(0,t))}^{lin}(v) dv$$

resp.

$$\begin{aligned} & \int_{v_1+\bar{u}_1>0} (v_1 + \bar{u}_1) \left( \frac{1}{|v|^2} \right) \varphi^{(1)}(0, v, t) dv \\ &= \int_{v_1+\bar{u}_1>0} (v_1 + \bar{u}_1) \left( \frac{1}{|v|^2} \right) \bar{M}_{(\rho(0,t), u(0,t), T(0,t))}^{lin}(v) dv, \quad i = 1, 2, 3. \end{aligned}$$

The ingoing function is for  $v_1 + \bar{u}_1 < 0$

$$\varphi^{(1)}(0, v, t) = \bar{M}_{(\rho(0,t), u(0,t), T(0,t))}^{lin}(v).$$

For the other values of  $\bar{u}_1$  one can proceed in an analogous way.

The coupling conditions obtained by the analysis of the kinetic half space problem are determined by the first step of the numerical scheme mentioned in the introduction and described in detail in Golse/Klar [9], see also Klar [12]. We remark that for  $\bar{u}_1 = 0$  the first step of the scheme reduces to the so called variational method, see Golse [8] or Loyalka [13].

In Figure 1 we consider the model problem in (2.1) with  $v \in S = [-1, 1]$  and  $K\varphi = \frac{1}{2} \int_{-1}^1 \varphi(v) dv$ . The macroscopic quantity  $\int \varphi^{(1)}(x, v, t) dv$  in  $D_1$  and  $\Theta(x, t)$  the solution of (2.2) in  $D_2$  is shown at a fixed time  $t = T$  so large that a stationary state is obtained. We took  $\epsilon_1 = 1$  and  $u = 0.3 > 0$ . For  $t \in [0, T]$  the ingoing function at  $x = -L$  was chosen as  $f_+(v, t) = v$ , at  $x = L$  we took  $f_-(t) = 1$ . The figure shows the 3 kinds of coupling conditions. Moreover the kinetic solution in the whole domain with the parameters  $\epsilon_1 = 1$  and  $\epsilon_2 = 0.01$  is shown.

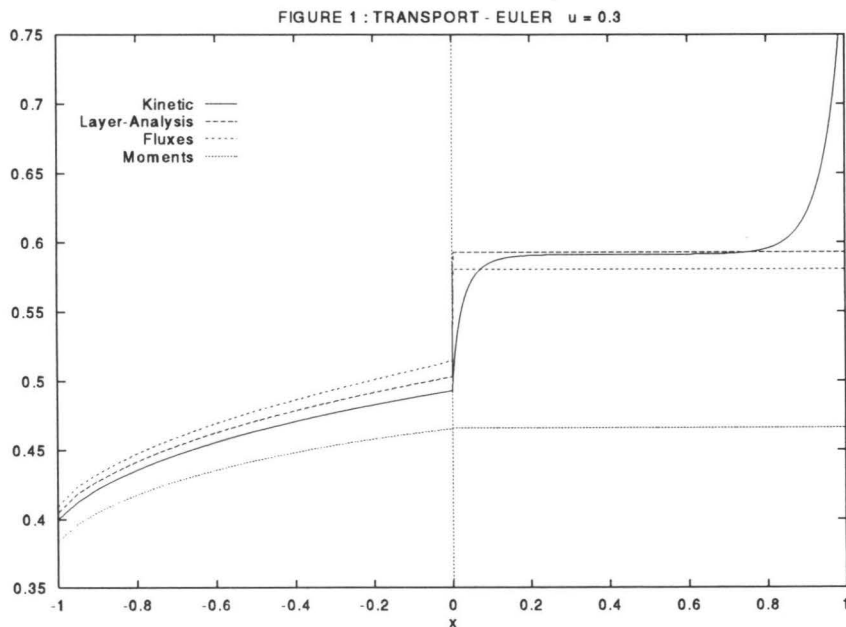
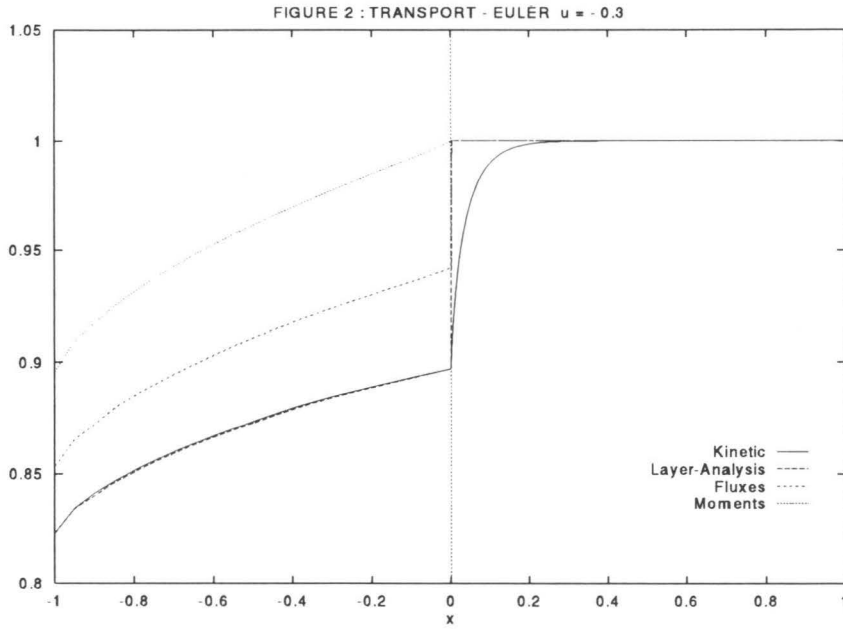


Figure 2 shows the same but in contrast to Figure 1  $u$  is here less than 0,  $u = -0.3$ .



In Figure 3 the 3-dimensional BGK model is considered, i.e. in  $D_1$  we consider equation (3.1) with  $\epsilon_1 = 1$  and substitute the following expression for the collision operator  $2Q(\bar{M}, \varphi)$  :

$$\varphi - \left( \int \varphi dv + \sum_{i=1}^3 v_i \int v_i \varphi dv + \frac{|v|^2 - 3}{2} \int \frac{|v|^2 - 3}{3} \varphi dv \right) \bar{M}$$

with

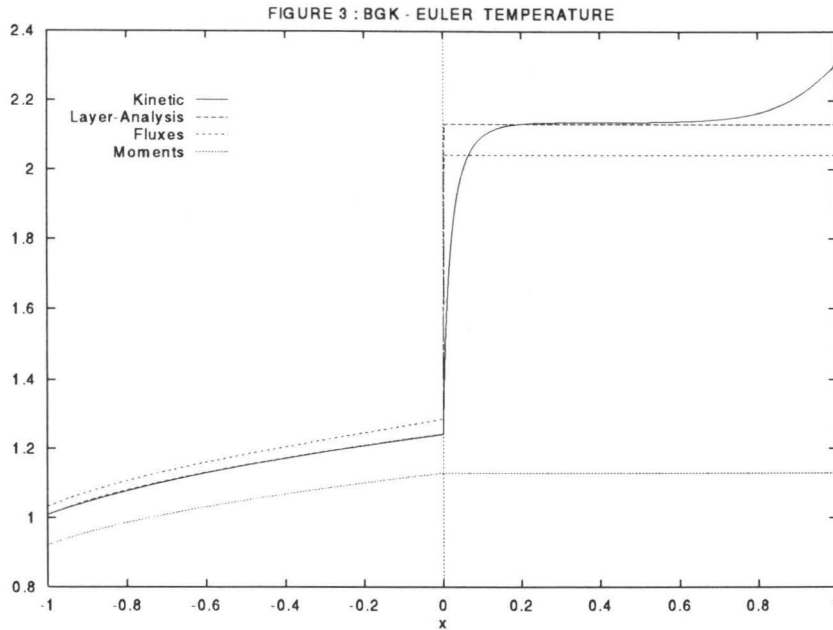
$$\bar{M} = \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{|v|^2}{2}\right).$$

We linearized here around  $\bar{\rho} = \bar{T} = 1$ .

In  $D_2$  Eulers equation (3.2) is solved.  $\bar{u}_1$  is equal to 1.4, i.e. bigger than  $c = \sqrt{\frac{5}{3}}$ . The ingoing function at  $x = -L$  is  $f_+(v, t) = v(|v|^2 - 5)\bar{M}$  for  $t \in [0, T]$ .

The figure shows the temperature  $\int \frac{|v|^2 - 3}{3} \varphi^{(1)}(x, v, t) dv$  on  $D_1$  and  $T(x, t)$  on  $D_2$ .

Again the 3 types of coupling conditions are shown together with the kinetic solution in the whole domain with  $\epsilon_1 = 1$  and  $\epsilon_2 = 0.002$ .



**Remark:**

As can be seen in the figures the usual coupling conditions may in certain cases lead to completely wrong results. The kinetic layer analysis combined with only the first step of the above mentioned numerical scheme for half space problems however leads to a considerable improvement. The coupled solution is a good approximation of the true kinetic solution outside of boundary and interface layers.

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