Bicriteria Cost Versus Service Analysis of the Distribution Network of a Chemical Company

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Abstract

In order to improve the distribution system for the Nordic countries the BASF AG considered 13 alternative scenarios to the existing system. These involved the construction of warehouses at various locations. For every scenario the transportation, storage, and handling cost incurred was to be as low as possible, where restrictions on the delivery time were given. The scenarios were evaluated according to (minimal) total cost and weighted average delivery time. The results led to a restriction to only three cases, involving only one new warehouse each. For these a more accurate model for the cost was developed and evaluated, yielding results similar to a simple linear model. Since there were no clear preferences between cost and delivery time, the final decision was chosen to represent a compromise between the two criteria.

1 Introduction

In this paper we will present a case study concerned with the analysis and improvement of the distribution system of the BASF AG for the Nordic countries. The design of a distribution system in industry involves two main aspects. First, the cost incurred by the organization of transports, which includes cost for storage, shipment and handling. The natural aim is to keep cost as low as possible. However, minimizing cost will usually be an aim in conflict with short delivery times. The latter are desired in order to obtain customer satisfaction and can therefore be considered a measure for service quality. The system on hand involves different types of warehouses, various types of products to be shipped from manufacturing plants to customers and many clusters of customers. We will address two main issues: first, the development of an appropriate model, including a realistic representation of the transportation cost, second the analysis of different scenarios for the distribution system (with options to build new warehouses) with respect to total

cost and service time. Finally, a solution will be recommended which appears to be robust with respect to different models of the cost function.
The paper is organized along the lines of the organization of the case study. In Section 2 we will describe the problem in detail. Section 3 is devoted to the development of an appropriate general model that deals with the minimization of the transportation, storage, and handling cost. In Section 4 we will describe the determination of the delivery time, the second criterion for evaluating a distribution network. In Section 5 we shall present a first analysis, using a linear programming version of the cost model of Section 3. Minimal cost and average delivery time are used to evaluate the various scenarios. This evaluation, together with other considerations of the management taken into account will lead to the elimination of most of the scenarios. In Section 6 we will develop a more detailed (nonlinear) approximation of the transportation cost, which will be used in the final analysis of the remaining scenarios based on the general model of Section 3. We will present the results for the remaining scenarios and present the final decision. In an Appendix (Section 8) we will discuss how a compromise can be chosen in general.
As this paper is a case study we do not claim to present any new mathematical theory. At the appropriate places we will, however, refer to literature which is related to the particular problem discussed. We also note that it was a requirement that existing software be used, which led to drawbacks in computing results.

2 The Problem in Industry

The distribution system can be represented as a network involving locations (manufacturing plants, warehouses, and customers) and their interconnections according to the available means of transport between them. Four types of locations are distinguished (abbreviations used throughout the paper are given in parentheses):

1. manufacturing plants (PLA),
2. distribution centers in plant-vicinity (DC1),
3. distribution centers in the vicinity of demand markets (DC2), and
4. demand markets, i.e. customers (DMD).

We notice that single customers are not considered, rather customers are clustered according to ZIP-code regions.

Due to the four types of locations, we have six different types of commodity streams between the locations, namely between:

1. plants and distribution centers in plant-vicinity (PLA DC1),
2. distribution centers in plant- and market-vicinity (DC1 DC2),

3. distribution centers in market-vicinity and demand markets (DC2 DMD),
4. distribution centers in plant-vicinity and demand markets (DC1 DMD),
5. plants and distribution centers in market-vicinity (PLA DC2),
6. plants and demand markets (PLA DMD), also called direct transports.

Thus, schematically, the distribution network can be portrayed as a graph as shown in Figure 1. The existing network considered in this paper consists of 35 plants, 37 DC1, 7 DC2 distribution centers, and 52 demand markets. The distribution network involves transportation of 98 product groups. The DC2 distribution centers were located in Denmark (Arhus, Kolding, Copenhagen), Finland (Turku), Norway (Oslo), and Sweden (Gothenburg, Malmo). Plants and DC1 warehouses considered in this study were in Germany, Belgium, the Netherlands, Spain, and England. The demand regions were distributed as follows: 6 in Denmark, 13 in Finland, 9 in Norway, and 24 in Sweden.
The total amount transported was about 100.000 tons, representing 4% of worldwide demand. Concerning direct shipments from plants to demand markets we have to remark that these orders had to be considered as constant throughout the study.

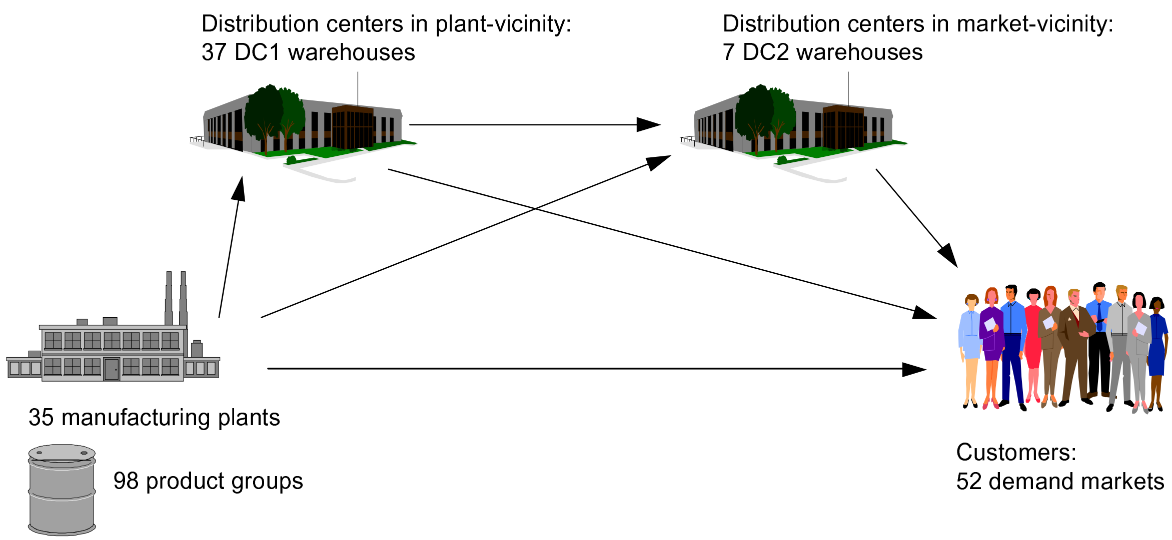

Figure 1: Schematic distribution system

More formally, we have a network $N = (V, L)$ with a set of nodes V (i.e. the locations), and a set of (directed) links L, which represent the transportation routes. There is one link in L for each origin and each destination in the two location classes involved in the respective transport type.
Management was concerned about possible improvement of the distribution system: can transportation cost be reduced and service (i.e. delivery times) improved? In order to do

so, it would probably be necessary to open one or several new warehouses, closer to the customers in the Nordic countries. A list of several changes of the distribution network was provided, and the task was to evaluate the resulting scenarios according to two criteria. For the cost criterion we had to find optimal (in terms of cost) transportation plans based on data from the past (i.e. December 1994 to November 1995). The second consisted in determination of service quality for the optimal transportation plans.
The possible changes included the construction of new DC2 warehouses in Copenhagen, Jonkoping, and Hamburg. The various designs differ in the DC2 warehouses to be used, restrictions to serve some markets from specified warehouses and restrictions on maximal delivery time.

The following 14 scenarios were considered:

1. R_{00}: This is the distribution network prior to optimization, also called the Base-Case.

2. R_{01}: All DC2 warehouses except Turku are allowed. Swedish and Finnish markets have to be delivered by Swedish warehouses and Denmark and Norway by their national warehouses. The markets are to be delivered within eighteen hours, if possible.

3. R_{02}: Only the DC2-warehouses in Jonkoping, Copenhagen (only the new one) and Turku are permitted. The delivery time restriction is set at twenty-four hours.

4. R_{02A}: This case is identical to R_{02}, except that the demand markets are to be delivered within eighteen hours, where possible.

5. R_{03}: Only the new DC2 warehouse in Copenhagen is permitted here. Again, the delivery time restriction was set at eighteen hours.

6. R_{04}: This case is identical to R_{03}, except that Jonkoping is used instead of Copenhagen.

7. R_{05}: Only the DC2 warehouses Jonkoping, Hamburg and Copenhagen are permitted here. The demand markets are to be delivered within eighteen hours, where possible.

8. R_{06}: Here all Swedish warehouses except Malmo and the new warehouse in Copenhagen are allowed as DC2-warehouses. Again, the delivery time restriction was set at eighteen hours.

9. R_{07}: This case is identical to R_{03}, except that Hamburg is allowed instead of Copenhagen.

10. R_{08}: Here, all DC2 warehouses are permitted and a time restriction of eighteen hours is imposed.

11. R_{09}: This scenario is the same as R_{04}, yet Hamburg can be used as well.

12. R_{10}: Finnish demands are to be fulfilled from Turku and Scandinavia from the new warehouse in Copenhagen, both within eighteen hours.

13. R_{11}: This is identical to R_{10}, yet Jonkoping is used instead of Copenhagen.

14. R_{12}: Again, this case is the same as R_{10}, except that Hamburg is permitted in place of Copenhagen.

Note: In scenarios R_{01} to R_{12} the DC1 warehouses in Belgium, the Netherlands and one DC1 warehouse in Ludwigshafen were used to store goods for non Nordic demands that were fulfilled from the Nordic warehouses in the Base-Case due to shortages. No other DC1 warehouses were allowed. A delivery time restriction of H hours means that each demand market is to be delivered from its corresponding warehouse(s) within H hours. If no such warehouse exists for a demand market the nearest warehouse must be used, see also Section 4.

3 A General Cost Model

One idea was to consider a transportation model with multiple objectives, a problem that has been discussed relatively intensively in the literature from the point of view of network design (see the reviews [Current and Min, 1986] and [Current and Min, 1993]) and the actual organisation of transports. Literature, however, mainly focuses on the linear transportation problem (e.g. [Isermann, 1979], [Gallagher and Saleh, 1994], and [Aneja and Nair, 1979]), where sometimes one of the objectives considered is of the bottleneck type ([Rajendra Prasad et al., 1993] and [Srinivasan and Thompson, 1973]). These models were not appropriate for our problem, which is in fact a two-stage problem.
In order to minimize cost for a given design of the distribution system, we have to take into account all shipment cost as well as storage and handling cost at warehouses of types DC1 and DC2. This is done with the model described below.
We first list the variables, sets, and constants appearing in the model, that have not already been mentioned above. Dimensions are given in parentheses.

- sets
  - PRD set of product groups
  - PTY set of hazardous product groups, a subset of PRD
- variables
  - C_k portion of the storage area used in warehouse k (%)
  - Y_k portion of the storage area for hazardous goods used in warehouse k (%)
  - R_{ik} throughput of product group i through warehouse k (t)
  - T_{ijk} amount of product group i transported from j to k (t)

- constants

$[AM]_{ij}$	amount of product group i produced in plant j (t)
$[MX]_{ijm}$	fixed amount of product group i transported directly from plant j to market m (t)
$[DP]_{im}$	demand for product group i in market m (t)
$[DC]_k$	storage area of warehouse k (m^2)
$[DT]_k$	storage area of warehouse k for hazardous goods (m^2)
$[AR]_{ik}$	area required for the storage of product group i in warehouse k (m^2/t)
$[PT]_{ik}$	area required for the storage of hazardous product group i in warehouse k (m^2/t)

- other parameters

$[TR]_{jk}$	transportation cost from location j to location k (DM/t)
$[HA]_{ik}$	handling cost of product group i in warehouse k (DM/t)
$[ST]_{ik}$	storage cost of product group i in warehouse k per year (DM/t)
$[TU]_{ik}$	turns of product group i in warehouse k

The "other parameters" above are not explicitly defined. They may be constants or, in principle, functions of any kind. We will assume $[HA]_{ik}$, $[ST]_{ik}$, and $[TU]_{ik}$ to be constant throughout. For the transportation cost $[TR]_{jk}$, however, we will later consider the model where these parameters are constants (see Section 5) as well as where they are, more realistically, piecewise linear functions, see Section 6. The problem of minimizing total cost of a given configuration of the distribution system can be written as the following mathematical program. We presume that goods are not stored at plants and that all storage and handling cost that arises at demand markets is covered by the customer. All handling cost at plants is presumed constant, as it is (or should be) independent of where products are transported to afterwards. Therefore neither handling nor storage cost at plants or demand markets is considered in our model.

$$\min Z \; := \; \sum_{i \in PRD} \sum_{(j,k) \in L} [TR]_{jk} T_{ijk} \tag{1}$$

$$+ \; \sum_{i \in PRD} \sum_{k \in DC1 \cup DC2} \left([HA]_{ik} + \frac{[ST]_{ik}}{[TU]_{ik}} \right) R_{ik} \tag{2}$$

Here (1) is the transportation cost for the various types of commodity streams and (2) is storage and handling cost for DC1 and DC2 warehouses, respectively.
The fraction $\frac{[ST]_{ik}}{[TU]_{ik}}$ can be explained as follows: $[ST]_{ik}$ symbolizes the storage cost per year and $[TU]_{ik}$ the turns per year. Thus, $\frac{1}{[TU]_{ik}}$ is the average storage time in years of the corresponding product group. Therefore the above fraction is equal to the variable storage cost per ton at the average storage time of the product group.

The restrictions involve conditions that guarantee that all transported goods are available and that demand can be satisfied. Furthermore the throughput of a warehouse must be equal to the total amount transported there, capacity of warehouses must not be violated and enough space for hazardous material must be allocated. All constraints are listed below.

$$[AM]_{ij} - \sum_{k \in DC1} T_{ijk} - \sum_{l \in DC2} T_{ijl} - \sum_{m \in DMD} T_{ijm} = 0 \quad \forall i \in PRD, j \in PLA \tag{3}$$

The amount transported from a plant must have been produced there.

$$\sum_{j \in PLA} T_{ijk} - \sum_{l \in DC2} T_{ikl} - \sum_{m \in DMD} T_{ikm} = 0 \quad \forall i \in PRD, k \in DC1 \tag{4}$$

The amount transported from a warehouse in plant-vicinity must be equal to that transported to that warehouse.

$$\sum_{j \in PLA} T_{ijl} + \sum_{k \in DC1} T_{ikl} - \sum_{m \in DMD} T_{ilm} = 0 \quad \forall i \in PRD, l \in DC2 \tag{5}$$

Analogous to (4) the warehouses in market-vicinity are considered.

$$\sum_{j \in PLA} T_{ijm} + \sum_{k \in DC1} T_{ikm} + \sum_{l \in DC2} T_{ilm} - [DP]_{im} = 0 \quad \forall i \in PRD, m \in DMD \tag{6}$$

The demand must be satisfied.

$$\sum_{l \in DC2} T_{ikl} + \sum_{m \in DMD} T_{ikm} - R_{ik} = 0 \quad \forall i \in PRD, k \in DC1 \tag{7}$$

For all warehouses in plant-vicinity the throughput must be equal to the amount transported from the warehouse.

$$\sum_{m \in DMD} T_{ilm} - R_{il} = 0 \quad \forall i \in PRD, l \in DC2 \tag{8}$$

Analogous to (7) the warehouses in market-vicinity are considered.

$$\sum_{i \in PRD} R_{ik}[AR]_{ik} - C_k[DC]_k \leq 0 \quad \forall k \in DC1 \cup DC2 \tag{9}$$

For all warehouses the area needed for the stored goods has to be equal to or less than the used area.

$$\sum_{i \in PTY} R_{ik}[PT]_{ik} - Y_k[DT]_k \leq 0 \quad \forall k \in DC1 \cup DC2 \tag{10}$$

Analogous to (9), but only the hazardous goods are considered.

$$Y_k[DT]_k \leq C_k[DC]_k \quad \forall k \in DC1 \cup DC2 \tag{11}$$

For all warehouses the area used for the storage of hazardous goods must be less than or equal to the area used all together.

$$0 \leq C_k \leq 1 \qquad \forall k \in DC1 \cup DC2 \tag{12}$$
$$0 \leq Y_k \leq 1 \qquad \forall k \in DC1 \cup DC2 \tag{13}$$
$$R_{ik} \geq 0 \qquad \forall i \in PRD, k \in DC1 \cup DC2 \tag{14}$$
$$T_{ijk} \geq 0 \qquad \forall i \in PRD, (j,k) \in L \tag{15}$$
$$T_{ijm} = [MX]_{ijm} \qquad \forall i \in PRD, j \in PLA, m \in DMD \tag{16}$$

The direct transports between plants and demand markets are presumed constant, which is expressed by equation (16).
We also have to allow for the possibility of specifying a maximal delivery time, i.e. the markets are to be served within the time H (a constant), if this is possible. If there is a demand market for which a warehouse within a radius of H hours does not exist, then the nearest warehouse must be used. All links from warehouses to markets that do not meet this requirement are excluded from the network.

4 The Delivery Times

All transports take place via truck. The truck drivers are required by law to take breaks. In some cases ferries have to be used. In addition border-crossing times also play a role in the calculation of the delivery times. Therefore delivery times do not solely depend upon the distance. For each source-to-destination combination (i.e. each link) the delivery time h_{lm} in minutes is calculated as follows:

$$h_{lm} \quad := \quad dist(l,m) * 0.9 + \text{additional times}$$

where the additional time consist of breaks, border-crossing time, and ferry time.
Note:

- Distance actually means the part of the route from l to m that is on land. Distance via water is considered in the ferry time.
- An average truck speed of 66.6 km/h on land is presumed, which is equal to 0.9 minutes per kilometer.
- The time for breaks results from the driving time. The truck driver is required by law to follow a certain pattern of breaks.
- The border-crossing time depends on the number and type of borders (European Union or Non-European Union) which are crossed.
- If a ferry is used, then the average ferry time between the corresponding countries is included in the delivery time.

The formulation of the objective function for service quality has to be made up from the individual times for each link, described above. Orders from customers are always dealt with at the $DC1$ or $DC2$ warehouses, with exception of the direct transports, which are fixed. Hence in order to judge the quality of service, measured in terms of the time needed to deliver an order to a customer, we have to consider all deliveries from $DC1$ and $DC2$ warehouses to demand regions.

Obviously, one wishes to minimize the delivery times, but in which form? Is the sum of the delivery times to be minimized or is the longest delivery time supposed to be as short as possible? Are large clients more important than those with little demand, which would lead to a weighting by demand? Is another weighting (such as revenue) perhaps better?

Minimizing the maximum of the delivery time is problematic, because faraway regions like Greenland, etc. are incorporated in our model and one would only look at these regions.

In our opinion a weighted sum of the delivery times seems the most appropriate. We will use the amount of goods delivered from a specific warehouse j to a specific demand region k as the weight w_{jk}, because these are the only data available to us. Weights have been normalized such that $0 \leq w_{jk} \leq 1$ and such that all weights sum up to 1.

Let DL denote the set of all links between DC1, respectively DC2 warehouses and demand markets. The following objective function for the delivery times will be used:

$$Z := \sum_{(j,k) \in DL} w_{jk} h_{jk}. \tag{17}$$

With equation (17) we can calculate the corresponding average delivery time for each possible configuration of the distribution network.

5 Bicriteria Evaluation of the Scenarios

We intended to compare the 13 scenarios according to cost and service and identify those, which improve both criteria as compared to the Base-Case. So, theoretically, we had to solve a bicriteria problem with a finite number of alternatives. For ease of notation, we denote by SC the set of scenarios and by R_i we refer to the i-th scenario.

Concerning the cost criterion we used a naive approach (the one that has been used by the company before) for the first evaluation. That is to calculate an average transportation price per ton for each combination of locations. Thus, $[TR]_{jk}$ are constants. The resulting cost model is then a linear program, which can be solved easily. In the following this will be called the LP-model.

The formulation of the objective function for the cost of scenario R_i has been thoroughly discussed in Section 3:

$$Z_1(R_i) \quad := \quad \min \sum_{i \in PRD} \sum_{(j,k) \in L} [TR]_{jk} T_{ijk}$$

$$+\sum_{i\in PRD}\sum_{k\in DC1\cup DC2}\left([HA]_{ik}+\frac{[ST]_{ik}}{[TU]_{ik}}\right)R_{ik} \tag{18}$$

subject to $(3)-(16)$.

The second criterion to be taken into account were the delivery times. Their derivation has been described in Section 4. They were computed for the cost optimal solution of each of the scenarios.

$$Z_2(R_i) := \sum_{(j,k)\in DL} w_{jk}h_{jk}. \tag{19}$$

The bicriteria decision problem is then:

$$\min_{R_i\in SC}(Z_1(R_i), Z_2(R_i)). \tag{20}$$

For the 14 scenarios (including the Base-Case) the average delivery times were plotted against the minimal total cost according to the LP solution, yielding the picture of Figure 2. (Information on the actual cost cannot be published.)

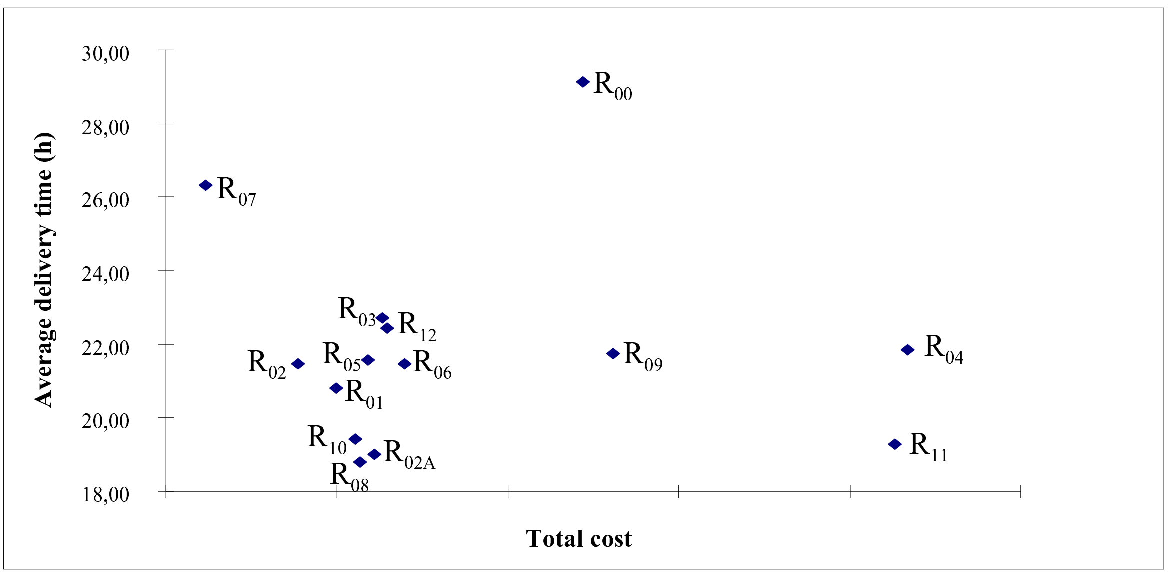

Figure 2: Evaluating cost and service

The results were then normalized, in order to be able to compare the two objectives, using the formula

$$100\times\frac{\max_{R_i\in SC} Z_j(R_i)-Z_j(R_i)}{\max_{R_i\in SC} Z_j(R_i)-\min_{R_i\in SC} Z_j(R_i)}. \tag{21}$$

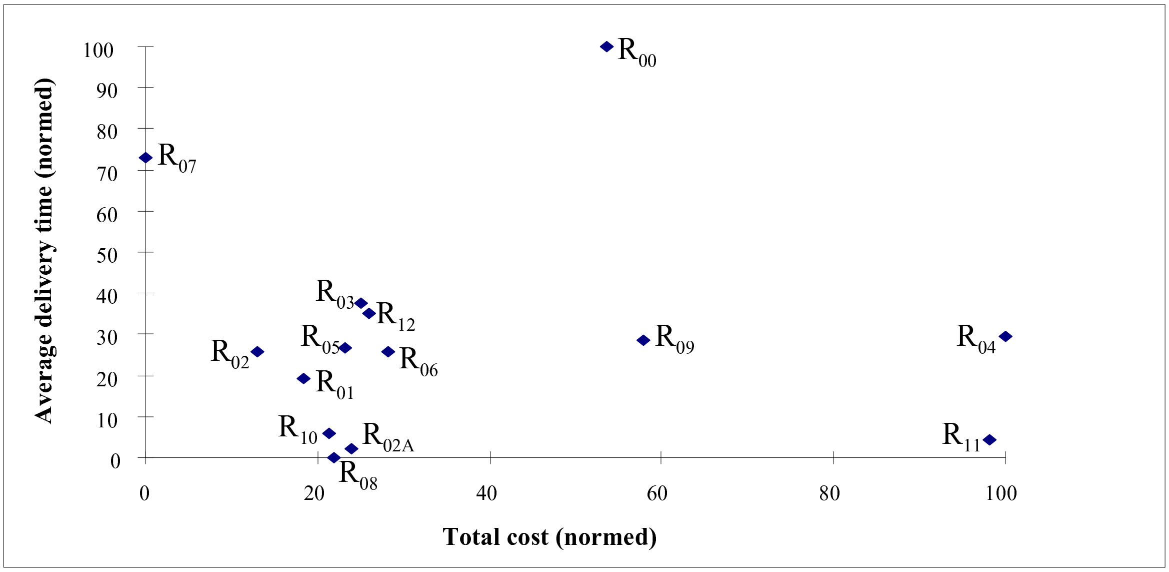

Figure 3: Norming objectives

First of all, it is remarkable that with all scenarios an improvement of service is achieved. The reduction of average delivery times is between 9.6% (R_{07}) and 35.4% (R_{08}). The latter is not surprising, since it allows all DC2 warehouses, including the three new ones. We also note that an average delivery time of 18 hours could not be met, which is due to the fact that deliveries to e.g. Greenland make this goal unattainable. Note that R_{07} is even above the 24 hour limit.

Concerning total cost, all but R_{09}, R_{04}, and R_{11} are cheaper than the Base-Case. The reduction is between 6.0% and 12.6%. The more expensive scenarios (increase between 1.0% and 9.3%) all involve the new warehouse in Jonkoping, indicating that this distribution center is too remote, i.e. implying detours of deliveries, a fact that would account for higher transportation cost.

From Figure 3 the scenario to be chosen should be one of $R_{07}, R_{02}, R_{01}, R_{10}$, or R_{08}. These are Pareto optimal: none of the others is at least as good for both criteria and better with respect to one criterion. These five scenarios illustrate the trade-off between cost and delivery time: better service can be achieved through higher cost. Four out of the five, except R_{07}, are very close together: the reduction in delivery time is between 26.3% and 35.4%, the cost decrease between 7.5% and 9.6%. They all involve at least two warehouses, at least one of which is new. R_{08} and R_{01} use all, respectively all but one, of the DC2 warehouses.

The only relatively small differences between these four scenarios and the big deviation from R_{07} but also from R_{04} indicate that the locations, rather than the number of the warehouses, crucially influence the outcome.

Another observation is that we have one large cluster of 9 points and four single points R_{07}, R_{09}, R_{04} and R_{11}. Comparing the scenarios in the cluster we find that their cost varies by only 4.0%, their delivery time by 20.6%, comparing the best and worst of the cluster, respectively. From the point of view of the cost they are therefore hardly distinguished, if we take the possible inaccuracy of the model into account. Even with respect to service the range is not that big, since the total variation is 40% among all the scenarios except R_{00}.

Before proposing a final decision, other considerations have to be taken into account. In the decision for a distribution system design not only the cost and the delivery time play a role. The resulting network should be as simple as possible. Every additional warehouse increases the expense for administration and coordination. Only approximately 4% of the worldwide amount is sold in the Nordic countries. So from a global point of view, one should try to manage with a very small number of warehouses. In our study we did not consider the fixed cost of the warehouses. The total cost we dealt with is the variable cost to be carried every year. Also, as we have observed above, the number of the warehouses seemed to be less influential than their location.

For these reasons it was decided that a scenario involving only one of the new locations should be chosen. Therefore only three of the thirteen scenarios are considered for the final decision: R_{03} (warehouse in Copenhagen), R_{04} (warehouse in Jonkoping, Sweden), and R_{07} (warehouse in Hamburg). Note that these represent the big cluster (R_{03}) as well as the exceptions (R_{07} and R_{04}). For these remaining options, it was decided that a more detailed analysis of the cost should be made.

6 Nonlinear Transportation Cost

The transportation cost is dependent on many factors that have not incorporated in the LP-model. Most importantly they depend on the weight of the shipment, i.e. in general, the more the shipment weighs, the lower the price per ton, an effect which is due to the following fact: The prices for the delivery of goods are calculated on the basis of prices for full transports (20 tons). The price for a truck to deliver goods from point A to point B is independent of the shipment weight – it is the same, whether it is carrying one ton or twenty. Of course, one tries to use the available capacity as well as possible. Tolls, ferry cost, waiting times and the carrying of hazardous goods also influence the prices. Orders above 2.5 tons are delivered directly from the warehouse to the customer. Below this mark, the products are delivered to a local transportation company, which then delivers them to the customer.

The prices for full transports are dependent on the distance, the infrastructure, the probability of finding goods that have to be transported back, etc. and are calculated from one ZIP-code region to another, i.e. individually for each link. In general, different prices are charged for transports of up to one ton, between 1 and 2.5 tons, 2.5 and 5 tons, 5 and 10 tons, 10 and 15 tons, 15 and 20 tons, and finally for full load transports. Thus, for each combination of source and destination relevant in the network we have a picture like that in Figure 4, where the intervals above are represented by their midpoints.

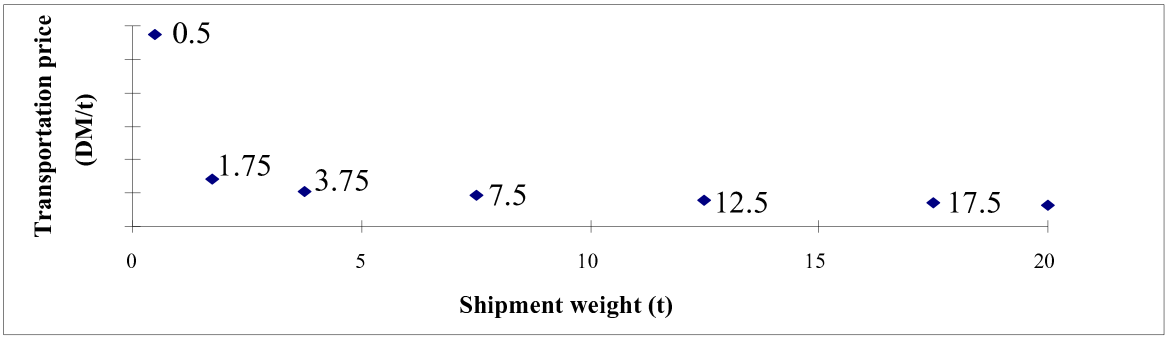

Figure 4: Decreasing price per ton

Although the LP-model of Section 3 ensures the linearity of the objective function, the transportation price per ton is approximated relatively roughly (assuming $[TR]_{jk}$ constant). This is evident from Figure 4 and the fact that transported amounts of various products vary widely. Our objective was to find a better approximation whilst keeping the objective function as simple as possible. Looking at Figure 4, it seems quite obvious that a very good approximation of the cost can be obtained by using a piecewise linear function, which consists of a line through the first two data points, a regression line through the data points three to seven and a connection of these two lines by a third line.

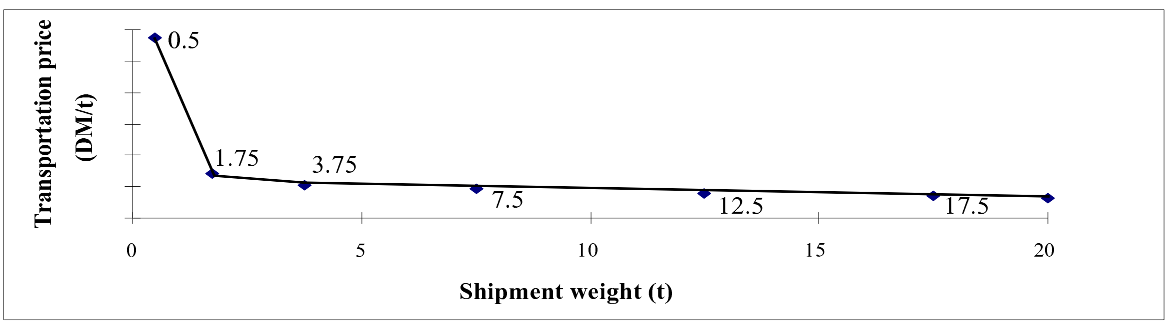

Figure 5: Piecewise linear price per ton function

The formula for the function in Figure 5 is

$$f_{jk}(T) = \begin{cases} 0 & \text{if } \; T = 0 \\ a_{jk}^{(1)} T + b_{jk}^{(1)} & \text{if } \; T \in (0, 1.75] \\ a_{jk}^{(2)} T + b_{jk}^{(2)} & \text{if } \; T \in (1.75, 3.75] \\ a_{jk}^{(3)} T + b_{jk}^{(3)} & \text{if } \; T \in (3.75, \infty] \end{cases} . \tag{22}$$

As the price per ton now also depends on the amount transported (T_{ijk}), the objective function will then be piecewise quadratic and have the form:

$$Z := \sum_{i\in PRD}\sum_{(j,k)\in L}\begin{cases} a_{jk}^{(1)}T_{ijk}^2 + b_{jk}^{(1)}T_{ijk} & \text{if} \quad T_{ijk}\in[0,1.75] \\ a_{jk}^{(2)}T_{ijk}^2 + b_{jk}^{(2)}T_{ijk} & \text{if} \quad T_{ijk}\in(1.75,3.75] \\ a_{jk}^{(3)}T_{ijk}^2 + b_{jk}^{(3)}T_{ijk} & \text{if} \quad T_{ijk}\in(3.75,\infty) \end{cases}$$
$$+ \sum_{i\in PRD}\sum_{k\in DC1\cup DC2}\left([HA]_{ik} + \frac{[ST]_{ik}}{[TU]_{ik}}\right) R_{ik}$$

If this objective function is used, then one very important point is still neglected: time. The amount shipped from one location to another is shipped there spread throughout one year. Thus, if 16 tons are transported from A to B, then we cannot presume that these 16 tons are shipped together, as we would, if we used the above function. Two tons could be shipped on January 5, 3 on February 8, etc. We would not pay the rate for 16 tons once, instead we would pay for all these transports individually. But how can this be incorporated in our model? We decided to presume that the shipment size is equal to the average shipment size to the particular destination. Say the average size was 1.5 tons. Then we would presume that we had 10 shipments with 1.5 tons and one transport with 1 ton and calculate the total cost accordingly. Thus, the following objective function is obtained, where $\bar{T}_k$ stands for the average shipment size to destination k:

$$Z := \sum_{i\in PRD}\sum_{(j,k)\in L}(T_{ijk}\text{div}\bar{T}_k)f_{jk}(\bar{T}_k) + (T_{ijk}\text{mod}\bar{T}_k)f_{jk}(T_{ijk}\text{mod}\bar{T}_k)$$
$$+ \sum_{i\in PRD}\sum_{k\in DC1\cup DC2}\left([HA]_{ik} + \frac{[ST]_{ik}}{[TU]_{ik}}\right) R_{ik}.$$

Here again, we used the transportation price function f_{jk} of (22).
This will be called the PQ-model (PQ for piecewise quadratic). Here, some literature was found dealing with transshipment or network flow problems with nonlinear functions, see [Boland et al., 1995], [Daenick and Smeers, 1977], or [Rao and T.L., 1980]. But these models did not include handling cost at transshipment nodes. With the additional requirement of using the existing software, we decided to implement a penalty method to solve the PQ-model, taking into account that solution times would be high, because no nonlinear optimization package was available. We had to simplify further and leave all hazardous materials out of the model, to obtain data we could handle with our program. The penalty method was implemented using the Fletcher-Reeves conjugate gradient method with Golden Section line search. We refer to textbooks (e.g. [Bazaara et al., 1993]) for details.
Once the optimization of the remaining scenarios with respect to total cost has been carried out, the decision for one of the options has to be made. The results for the remaining scenarios, based on the PQ-model are presented in Figure 6.
As expected transportation costs were higher for the PQ-model, as we dealt with smaller shipments and thus higher transportation prices. The delivery times, however, were

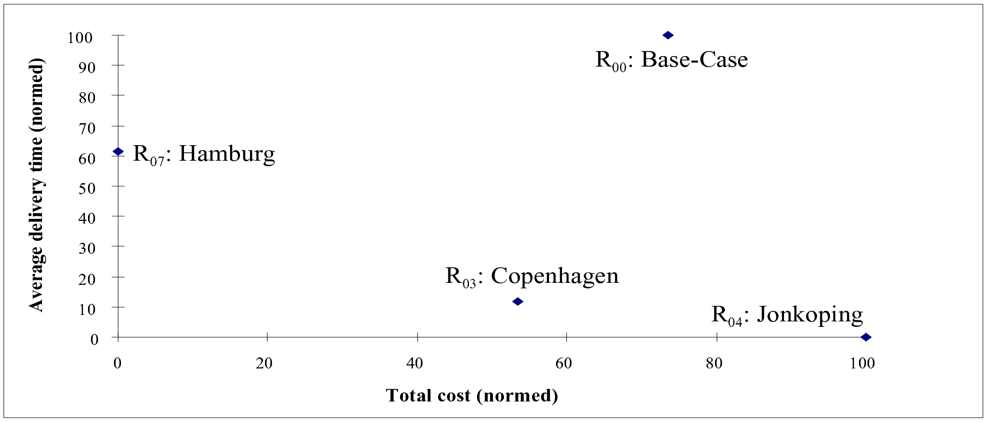

Figure 6: Remaining scenarios, PQ-model

unaffected by the change of the cost model: the chosen transportation routes only varied marginally. Hence, qualitatively, the solution looks very much like the one we obtained for the simple LP-model. This is illustrated by Figure 7, where we compare the results of the PQ-model and the LP-model for the remaining scenarios. Here the numbers have been calculated using the results of the LP-model of the Base-Case as a basis (=100) for both criteria. Recall that by elimination of hazardous material, the number of product groups has been considerably reduced. The lower cost values for the PQ-model are due to this omission. With hazardous goods taken into account, the cost would increase for all scenarios, but the order unchanged.

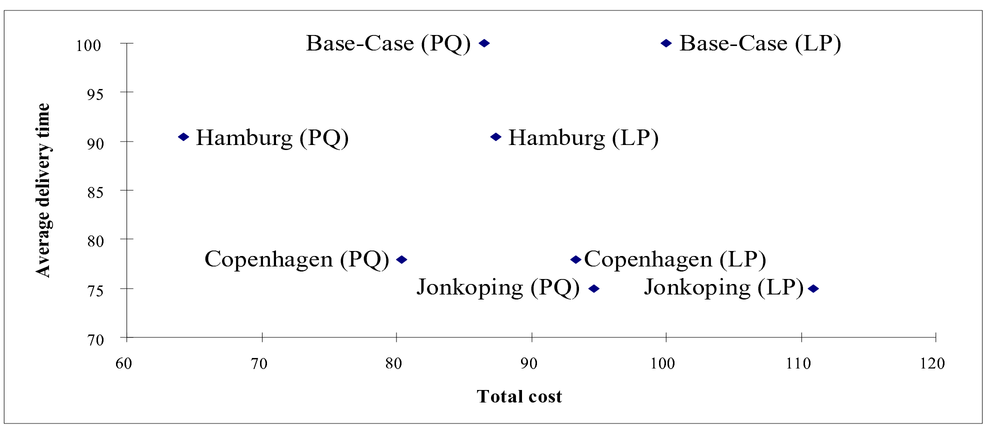

Figure 7: Remaining scenarios, LP-PQ comparison

Although the absolute heights varied slightly, the overall picture for the PQ-model (see Figure 6) was the same as in the LP-model. Again, Hamburg had the worst delivery time and Jonkoping the highest cost, so that the choice of Copenhagen as a compromise could be recommended.

This comparison indicates, that the different choice of the transportation cost had no significant influence on the final results, whilst on the other hand providing a more realistic approximation of the true transportation cost. We remark that the solution of the PQ-model involved much higher computation times, so that we could not run it for the scenarios not considered for the final decision.
Concluding that the results are rather robust with respect to a change of the transportation cost function, the solution R_{03} was finally agreed upon. Clearly, R_{04}, with a 9.3% increase in cost (10.9% for the LP-model), was not acceptable. On the other hand, the cheapest solution R_{07} was not satisfying with an improvement of only 9.6% in delivery times, whereas the other 2 options had 22.1% and 25.0%, respectively. The decision to build a new distribution center in Copenhagen and use it as the only DC2 warehouse, is expected to yield about 22% decrease in the average delivery time, and a cost reduction of about 7%. The choice of Copenhagen may also be justified by some ad hoc arguments: it is located in the region considered, but also not too far away from the DC1 distribution centers and manufacturing plants. In this sense, both transports to the DC2 center and from there to customers can be expected to be in the middle of the range for both cost and service. Furthermore, after completion of the Oresund bridge we can aspect a decrease in delivery times and cost, when ferries need no longer be used. These intuitive arguments support the decision derived by the detailed analysis.

7 Conclusions

In this paper we have discussed the bicriteria analysis of (a part of) the distribution system of the BASF AG. We have taken into account the two main aspects of such a system, namely cost and service. In the case study 14 different scenarios have been considered and evaluated according to the two criteria. We discussed in detail the derivation of the appropriate mathematical representations of the two criteria, where two different versions of the cost function were considered. For every scenario the minimal cost solution was determined and this minimal cost and the resulting average delivery time at this particular solution were used as criteria to evaluate the various scenarios.
The analysis according to an LP-model led to the conclusion that the decision could be restricted to the all scenarios involving only one warehouse each. Further analysis for the transportation cost, yielding a nonlinear model, showed that the results are rather robust with respect to changes of the cost model.
Our investigation shows that a bicriteria analysis is certainly superior to a decision based on the cost or the service criterion alone. The respective optimal choices have both been considered unsatisfactory.
Evaluation of the full PQ-model for all scenarios would have been interesting, but was not possible due to limitations on the available time on the computer systems. Another option would have been to use the LP-model results and decide that only one of the points in the cluster should be chosen as the final scenario, and investigate those more closely. However, we did not pursue this approach, as it was not in accordance with the strategic decision

for only one DC2 warehouse.

8 Appendix: Finding a Compromise Solution

This Section presents some general considerations about how to find a compromise solution in a problem involving two (or more) criteria. These thoughts are especially valid in a situation, where the list of alternatives contains more than three elements. It is therefore presented as an appendix, but the general idea also works for the case study presented in this paper. For the description we restrict ourselves to the case of two criteria, and use the results of the case study for illustration. We refer to the situation displayed in Figure 6, the situation where the final decision has to be made.
Which of a set of alternatives is considered a best decision? There are several possibilities to solve (20). First, if there is a ranking among the two criteria, one should use the lexicographic optimization i.e.

$$\text{lexmin}_{R_i \in SC}(Z_1, Z_2)$$

or

$$\text{lexmin}_{R_i \in SC}(Z_2, Z_1)$$

In this case the final decision should be either R_{07} (building a warehouse in Hamburg). This solution involves the greatest reduction of cost, compared to the Base-Case. Or, in the reverse order of importance, R_{04}, the Jonkoping solution, with best delivery times would be chosen. However, both did not seem to be satisfying, as they are also the worst solutions with respect to the other criterion. (R_{04} is much more expensive than the present situation, the delivery time of R_{07} does not improve significantly compared to R_{00}).
If, on the other hand a decision maker is indifferent with respect to the criteria, he may focus on the worst of the objectives, i.e.

$$\min_{R_i \in SC} \max\{Z_1, Z_2\}.$$

This rule, known as minimax optimization or max-ordering optimization, see [Du, 1995, Ehrgott, 1997b, Hamacher and Ruhe, 1994], is appropriate in conservative planning. However, in general, proceeding in this way may yield solutions, which can be improved with respect to the criteria not attaining the maximum value.
The need for a compromise solution is therefore evident. This should be chosen from the set of Pareto optimal, or efficient, solutions. Recall that an efficient solution of a bicriteria problem is one that does not allow for another solution, which is better with respect to both criteria. In the scenarios of our final analysis all three remaining options are efficient. However, in general the set of efficient solutions can be quite large. Two of them (R_{07} and R_{03}) dominate the Base-Case, i.e. they yield lower cost and shorter delivery times.
A compromise solution can be found generalizing both the max-ordering and the Pareto concept. If we assume that the decision maker is indifferentwith repect to the given criteria,

we can apply the normalization (21) to bring all criteria to the same scale. This enables a comparison of the different criteria. Then we propose to make the final decision according to the lexicographic max-ordering rule. Given the objective value vectors, we sort their components in a nondecreasing way and compare the sorted vectors lexicographically. Formally, if sort is a function reordering a vector in a non increasing way, we solve

$$\text{lexmin}_{R_i \in SC} \text{sort}(Z_1, Z_2).$$

In this way, we can assure that

- the worst criterion is minimized, i.e. the goal of conservative planning is satisfied,
- the chosen solution is efficient, which is the minimum requirement for a compromise solution.

Moreover, these solutions have two properties, which are be desirable from the decision makers' point of view. The important fact being that, should a decision maker find these properties acceptable for his solution, then he has to act according to the lexicographic max-ordering principle. The first is a conservativeness assumption. A final solution should be such that the worst criterion is as good as possible. The second is concerned with additional information. Suppose that, for whatever reason, for some criteria the values that an optimal solution attains are known. Then it should be possible to disregard these criteria in optimization considering only the remaining ones, and formulate restrictions for the objectives, for which the values are known. In [Ehrgott, 1997b] these properties are called regularity and reduction property. It has formally been shown that they characterize the lexicographic max-ordering principle. For combinatorial optimization problems the lex-MO optimization has been studied in [Ehrgott, 1996] and [Ehrgott, 1997a]. It is important to emphasize that the minimax or max-ordering optimization does not fulfill the reduction property, the Pareto optimality even fails for both.
Although not widely used, this concept has been discussed in location theory, see for example [Ogryczak, 1997] and [Ehrgott et al., 1998]. According to the lexicographic max-ordering rule, the final decision should be R_{03} involving a new warehouse in Copenhagen. As mentioned above, this appears to be a reasonable compromise, as improvement in delivery times is almost as high as for Jonkoping, with a reduction of cost, which is half that of R_{07}.
Applying this concept to the set of all scenarios (see Figure 3) would yield a recommendation of R_{01}.

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