

On the Characters of the Sylow 2-Subgroup of $F_4(2^n)$ and Decomposition Numbers

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Preface

Introduction

The field of research from which the topic of this thesis stems is the **representation theory of finite groups**. This theory deals with the study of **representations** of finite groups, that is, group homomorphisms

$$R : G \rightarrow GL_n(K)$$

from a finite G into the group of invertible $n \times n$ -matrices over a field K , for an $n \in \mathbb{N}$. Representations of a group G encode many properties of the group itself and may be easier to handle than G if G is an abstract group defined by its properties. The **irreducible representations** are of particular interest as they can be viewed as the building blocks of all representations. As it happens, a lot of information about the group is already encoded in the trace map of a representation, called the **character**, which is a class function on G .

In the case the ground field are the complex numbers, the irreducible characters form an orthonormal basis of the space of class functions for an appropriate scalar product and representations are characterized completely by their characters (up to an appropriate notion of isomorphism).

For fields K of positive characteristic $\text{char}(K) = l$, there is a one-to-one correspondence between the isomorphism classes of irreducible K -representations and the set of **irreducible Brauer characters**. Moreover, it turns out that the restriction of an ordinary irreducible character (thus a character associated to a representation over the complex numbers) to the l -regular classes of G is a Brauer character as well and is a nonnegative integral linear combination of irreducible Brauer characters. The multiplicities of the irreducible Brauer characters in this linear combination are called the **l -decomposition numbers** of G .

Which leads to the second part of the title of this thesis:

$$\text{The } l\text{-decomposition numbers of the group } F_4(2^n).$$

The group $F_4(2^n)$, also called the **Chevalley group** of type F_4 , belongs to an important class of simple groups, called the **finite groups of Lie type**. As can be seen from the classification of finite simple groups [GLS], they form (in a sense) most of the nonabelian finite simple groups. The finite groups of Lie type can be viewed as the finite fixed-point subgroups \mathbf{G}^F of simple linear algebraic groups \mathbf{G} under a suitable endomorphism $F : \mathbf{G} \rightarrow \mathbf{G}$ and can be classified by the type of the root system of the corresponding algebraic group and the order of a graph automorphism of the Dynkin diagram induced by F .

When dealing with a finite group of Lie type $G(q)$ with root system of type r defined over the field \mathbb{F}_q with $q = p^n$ a power of a prime p , one generally has to distinguish two cases: p a good prime for $G(q)$ of type r and p a bad prime. Usually, for p bad, many general theories will not apply.

For p and l good primes for $G(q)$ of type F_4 (i.e. $p, l \neq 2, 3$), [Köhler] has computed most of the l -decomposition numbers of unipotent blocks of $F_4(q)$, using generalized Gelfand-Graev characters arising from unipotent subgroups of $F_4(q)$ and Harish-Chandra induction of characters of proper Levi subgroups of $F_4(q)$.

The l -decomposition numbers of the unipotent blocks of all proper Levi subgroups of $F_4(q)$ are known for all primes p and all primes $l \neq p$ for the Levi subgroups of $F_4(q)$ of type A_n, B_2, C_2 and all odd primes $l \neq p$ for the Levi subgroups of $F_4(q)$ of type B_3, C_3 by results of [FS82], [FS90], [White90a], [White90b], [White92], [White95], [White00], [HN], [GH97b], [Puig], [OW].

In this thesis, we deal with the finite group of Lie type $F_4(2^n)$. Here $p = 2$ is a bad prime for type F_4 . The aim is to find information on the characters of the Sylow 2-subgroup of $F_4(2^n)$ and on the l -decomposition numbers of $F_4(2^n)$ on unipotent blocks for $l \neq 2$ and $n \in \mathbb{N}$ arbitrary. We largely follow the approach in [Köhler], however, for p in bad characteristic (i.e. $p \in \{2, 3\}$) and particularly for p very bad (i.e. $p = 2$) far less theory is known than for the case of good characteristic [Köhler] dealt with.

The general strategy we have applied to obtain information about the l -decomposition numbers of unipotent blocks is to induce characters of the unipotent subgroup U of $F_4(q)$, a Sylow 2-subgroup of $F_4(q)$, and Harish-Chandra induce projective characters of proper Levi subgroups of $F_4(q)$ to obtain projective characters of $F_4(q)$. Via Brauer reciprocity, the multiplicities of the ordinary irreducible unipotent characters in these projective characters give us information on the l -decomposition numbers of the unipotent characters of $F_4(q)$. The unipotent characters of $F_4(q)$ have been classified by [Lusztig] and their values on the unipotent conjugacy classes of $F_4(2^n)$ have been computed by [MS]. However, for the prime $p = 2$, there exists no analogue of generalized Gelfand-Graev characters so far. Moreover, [GLMP] exclude the case p very bad in their parametrization of the irreducible characters of the unipotent subgroup U of $F_4(q)$ as well.

So one of the main obstacles to this approach was to obtain irreducible characters of the unipotent subgroup U of $F_4(q)$. Which leads to the first part of the title of this thesis:

Characters of the Sylow 2-Subgroup of $F_4(2^n)$.

We managed to adapt the methods used in [GLMP] for the case $p = 2$ for the group $F_4(q)$. This gives a nearly complete parametrization of the irreducible characters of the unipotent subgroup U of $F_4(q)$, namely of all irreducible characters of U arising from so-called **abelian cores** (Theorem 9.10). The remaining irreducible characters of U arise from so-called **nonabelian cores**. In this case one would have to analyse the nonabelian cores in more detail to obtain a parametrization of the corresponding characters. This should be the topic of further research.

Another obstacle in inducing characters of the unipotent subgroup U to $F_4(q)$ was that we had to compute the (parametrized) conjugacy classes of U first (Theorem 7.1) as well as their fusions into the conjugacy classes of $F_4(q)$ (Theorem 10.1). We present algorithms for both of these tasks in this thesis.

Finally, inducing characters of U to $F_4(q)$ can be quite involved depending on the parameter sets of the (parametrized) representatives of the (parametrized) conjugacy classes of U (Theorem 11.4).

As only the values of the unipotent characters of $F_4(q)$ on the unipotent conjugacy classes of $F_4(q)$ are known, we could not use the same methods [Köhler] presents to test whether a projective character of $F_4(q)$ is in fact indecomposable, i.e. a PIM. Therefore the multiplicities of the unipotent characters of $F_4(q)$ in the projective characters we obtained are only upper bounds for the actual decomposition numbers. Sadly, the projective characters of $F_4(q)$ we obtained were not sufficient to give the shape of the entire decomposition matrix. However, the results we have indicate a lower unitriangular shape of the decomposition matrix for an appropriate ordering of the characters (Theorem 16.1).

This thesis is structured as follows:

In Part I of the thesis, the group $F_4(q)$ is introduced and the method for explicit computations in the group we use is presented.

In Part II, characters of the unipotent subgroup of $F_4(q)$ are constructed. For this, the conjugacy classes of the unipotent subgroup are calculated, then different methods for obtaining characters are presented. In particular, we adapt a method to obtain the irreducible characters of the unipotent subgroup of a finite group of Lie type G presented in [GLMP] for the case p not very bad for G to the case p is very bad, i.e. $p = 2$ for $G = F_4(p^n)$.

In Part III, the characters of the unipotent subgroup are induced to $F_4(q)$. This requires determining the fusions of the conjugacy classes of U into $F_4(q)$, for which we present an algorithm.

Finally, in Part IV, we present our results on the l -decomposition numbers of $F_4(q)$ on unipotent blocks for a prime $l \neq 2$.

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Part I

The Group $F_4(q)$

To define the group $F_4(q)$, we introduce the root system of type F_4 and the corresponding reductive linear algebraic group $\mathbf{G} := F_4(\overline{\mathbb{F}}_q)$ over the field $\overline{\mathbb{F}}_q$ first. The Chevalley group $F_4(q)$ of type F_4 will then occur as a fixed-point subgroup of the reductive linear algebraic group $F_4(\overline{\mathbb{F}}_q)$ under a certain Frobenius morphism. We will also give a presentation of the reductive linear algebraic group and define a normal form for its elements. Moreover, an algorithm to compute the normal form of a product of two elements of \mathbf{G} will be presented and we will describe how this algorithm can be used to do generic computations in all groups of a given type simultaneously. Finally, the finite Chevalley group $F_4(q)$ will be defined and it will be shown that the normal form for the elements of the algebraic group can be defined for the finite group as well and that the algorithm presented for the algebraic group can be employed for the finite group too.

Unless otherwise stated, let $p = 2$ and $q = p^n$ for some $n \in \mathbb{N}$. Moreover, let $k = \overline{\mathbb{F}}_2$. This part is essentially based on [Köhler].

1 The Root System of Type F_4

The aim of this section is to present the root system of type F_4 . The notation is the same as in [Bourbaki, chapter 6, section 4.9]. Let V be a 4-dimensional euclidian real vector space with orthonormal basis $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$ with respect to a scalar product (\cdot, \cdot) . Then

$$\Phi = \{\pm\epsilon_i, (1 \leq i \leq 4), \pm\epsilon_i \pm \epsilon_j, (1 \leq i, j \leq 4, i \neq j), \frac{1}{2}(\pm\epsilon_1 \pm \epsilon_2 \pm \epsilon_3 \pm \epsilon_4)\}$$

is a root system of type F_4 in the sense of [Bourbaki, chapter 6] (the signs are to be chosen independently of one another) with 48 elements. Observe that for all $\alpha \in \Phi$, $(\alpha, \alpha) \in \{1, 2\}$. Set

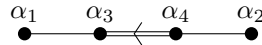
$$\alpha_1 := \frac{1}{2}(\epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4), \alpha_2 := \epsilon_2 - \epsilon_3, \alpha_3 := \epsilon_4, \alpha_4 := \epsilon_3 - \epsilon_4.$$

Then $\Pi = \{\alpha_1, \dots, \alpha_4\}$ forms a fundamental system of Φ ([Köhler, (II.1)]). I.e. any root can be written as an integral linear combination of the roots in the fundamental system with either only positive or only negative coefficients ([MT, (9.3)]). Call the roots in the fundamental system **simple roots**.

The corresponding 24 positive roots are given by

$$\Phi^+ = \{\epsilon_i, (1 \leq i \leq 4), \epsilon_i \pm \epsilon_j, (1 \leq i < j \leq 4), \frac{1}{2}(\epsilon_1 \pm \epsilon_2 \pm \epsilon_3 \pm \epsilon_4)\}.$$

Here α_1 and α_3 are the short simple roots and α_2, α_4 the long simple roots. In the Dynkin diagram of type F_4 they are arranged as follows:



Fix an ordering of the positive roots such that α precedes β if $\beta - \alpha$ is a sum of positive roots (in general, any ordering would do, but this kind of ordering will turn out to be especially nice) and denote them by $\alpha_1, \dots, \alpha_{24}$ (see Appendix A.2 for the ordering chosen here; it is the same ordering as used in the software package **Unipot** [Haller]). Call $\Phi^- := \{-\alpha_i \mid 1 \leq i \leq 24\}$ the negative roots of Φ . By [MT, (9.3)], $\Phi = \Phi^+ \cup \Phi^-$.

Define the coroot α^\vee as $\alpha^\vee := 2\frac{\alpha}{(\alpha, \alpha)}$ for $\alpha \in \Phi$. Then $\Phi^\vee := \{\alpha^\vee \mid \alpha \in \Phi\}$ is a root system of type F_4

as well, with a set of fundamental roots given by $\Pi^\vee := \{\alpha_1^\vee, \dots, \alpha_4^\vee\}$ ([MT, (8.19)]). Define the **Cartan matrix**

$$C := ((\alpha_i, \alpha_j^\vee))_{i,j=1,\dots,4}.$$

For the chosen ordering of the simple roots, this gives the following Cartan matrix:

$$C = \begin{pmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -2 & 2 \end{pmatrix}.$$

To each root, one can associate a reflection in the hyperplane perpendicular to this root; the reflections associated to the fundamental roots generate the Weyl group W of the root system. Let $s_i : \alpha \mapsto \alpha - 2 \frac{(\alpha_i, \alpha)}{(\alpha_i, \alpha_i)} \alpha$ denote the reflection associated to α_i . The Weyl group acts on Φ by permuting roots (from left and right, with the actions defined as usual for permutation groups); this realization as permutation group will also be used later on. Define the length of an element w of the Weyl group W as $l(w) = |\{\alpha \in \Phi^+ \mid \alpha w \in \Phi^-\}|$. Observe that the action of W on Φ has two orbits; one containing the long roots and one containing the short roots. Define the height of a root $\alpha \in \Phi^+$ as $ht(\alpha) = \sum n_i$ for $\alpha = \sum_{i=1}^4 n_i \alpha_i$.

2 The Algebraic Group $F_4(\overline{\mathbb{F}_2})$

In the following, we wish to give a presentation of the algebraic group of type F_4 over the algebraically closed field $k := \overline{\mathbb{F}_2}$ of characteristic 2, of which the Chevalley group $F_4(q)$ is a fixed-point subgroup under a Frobenius morphism F .

For more information on algebraic groups, see also [Carter85, Chapter 1] or [MT, Part I].

Definition 2.1. [MT, (1.1),(3.3),(6.14)] A **linear algebraic group** over the algebraically closed field k is an affine algebraic variety over the field k equipped with a group structure such that the group operations (multiplication and inversion) are morphisms of varieties.

A linear algebraic group is called a **torus** if it is isomorphic to a direct product $(k^\times) \times \dots \times (k^\times)$ of copies of the multiplicative group k^\times of the field k .

A linear algebraic group \mathbf{G} is called **reductive** if $R_u(\mathbf{G}) = 1$, where $R_u(\mathbf{G})$ is the maximal closed connected normal unipotent subgroup of \mathbf{G} , also called the **unipotent radical** (for the definitions of **connected**, **unipotent**, see also [MT, (1.11),(2.1),(2.6)]).

It turns out that one can associate a set of roots to a linear algebraic group:

Definition 2.2. [MT, (7.2),(7.12),(8.1)] Let \mathbf{G} be a linear algebraic group over an algebraically closed field k . The **Lie algebra** of \mathbf{G} is the subspace

$$\text{Lie}(\mathbf{G}) := \{D \in \text{Der}_k(k[\mathbf{G}]) \mid D\lambda_x = \lambda_x D \forall x \in \mathbf{G}\}$$

of **left invariant derivations** of $k[\mathbf{G}]$, where $\lambda_x : k[\mathbf{G}] \rightarrow k[\mathbf{G}]$ is defined by $(\lambda_x.f)(g) := f(x^{-1}g)$ for $f \in k[\mathbf{G}]$ and $g \in \mathbf{G}$, and $k[\mathbf{G}]$ is the coordinate ring of \mathbf{G} .

One can define a representation $\text{Ad} : \mathbf{G} \rightarrow \text{GL}(\text{Lie}(\mathbf{G}))$, called the **adjoint representation**.

Then for a fixed maximal torus \mathbf{T} of the linear algebraic group \mathbf{G} the set $\Phi(\mathbf{G}) := \{\chi \in X(\mathbf{T}) \mid \chi \neq 0, \text{Lie}(\mathbf{G})_\chi \neq 0\}$ is called the **set of roots** of \mathbf{G} with respect to the maximal torus \mathbf{T} of \mathbf{G} , where $X(\mathbf{T}) := \text{Hom}(\mathbf{T}, k^\times)$ and $\text{Lie}(\mathbf{G})_\chi := \{v \in \text{Lie}(\mathbf{G}) \mid (\text{Ad } t)(v) = \chi(t)v \forall t \in \mathbf{T}\}$.

In particular, the algebraic group of type F_4 is defined as the canonical reductive algebraic group ([MT, Chevalley Classification and Existence Theorem, (9.13)]) with root system of type F_4 as defined in the previous Section 1. Now let $\mathbf{G} := F_4(\overline{\mathbb{F}}_2)$ be the linear algebraic group of type F_4 over $\overline{\mathbb{F}}_2$ and fix a maximal torus $\mathbf{T} \subset \mathbf{G}$.

Definition 2.3. [DM, (3.1)] An algebraic variety V over $\overline{\mathbb{F}}_q$ is **defined over** \mathbb{F}_q , if there exists a variety V_0 over \mathbb{F}_q such that $V = V_0 \otimes_{\mathbb{F}_q} \overline{\mathbb{F}}_q$. The **geometric Frobenius endomorphism** $F : V \rightarrow V$ associated to this \mathbb{F}_q -structure is then defined as the endomorphism $F_0 \otimes \text{Id}$, where F_0 is the endomorphism of V_0 that raises the functions on V_0 to the q -th power.

So a connected reductive linear algebraic group \mathbf{G} over $\overline{\mathbb{F}}_q$ is defined over \mathbb{F}_q if \mathbf{G} can be described as the vanishing set of an ideal generated by polynomials with coefficients in \mathbb{F}_q . The Frobenius morphism associated to this \mathbb{F}_q -structure is the map $F : \mathbf{G} \rightarrow \mathbf{G}$ raising the elements of \mathbf{G} , viewed as coordinate vectors in the affine variety, coordinatewise to their q -th power.

Definition 2.4. [MT, (9.10),(9.11)] Let $\Phi, \Phi^\vee := \{\alpha^\vee \mid \alpha \in \Phi\}$ be the root systems of type F_4 as defined in the previous section and set $X(\mathbf{T}) := \text{Hom}(\mathbf{T}, k^\times)$, $Y(\mathbf{T}) := \text{Hom}(k^\times, \mathbf{T})$ for $k = \overline{\mathbb{F}}_2$ and \mathbf{T} a fixed maximal torus of \mathbf{G} . Then $(X(\mathbf{T}), \Phi, Y(\mathbf{T}), \Phi^\vee)$ is called the **root datum** of \mathbf{G} associated to the torus \mathbf{T} .

Lemma 2.5. [Springer, (7.5.4),(8.1.1),(8.1.4)], [MT, (8.1),(8.20)] Let $(X(\mathbf{T}), \Phi, Y(\mathbf{T}), \Phi^\vee)$ be the root datum of \mathbf{G} associated to the torus \mathbf{T} of \mathbf{G} . For each $\alpha \in \Phi$ one can choose an isomorphism x_α from k (viewed as additive group) to a closed subgroup $\mathbf{X}_\alpha \subset \mathbf{G}$, such that

- \mathbf{G} is generated by \mathbf{T} and the \mathbf{X}_α , $\alpha \in \Phi$
- $h^{-1}x_\alpha(x)h = x_\alpha(\alpha(h)x)$ for all $h \in \mathbf{T}$, $x \in k$, $\alpha \in \Phi$
- $n_\alpha := x_\alpha(1)x_{-\alpha}(1)x_\alpha(1) \in N_{\mathbf{G}}(\mathbf{T})$ and its image under the canonical projection to $W = N_{\mathbf{G}}(\mathbf{T})/\mathbf{T}$ is s_α , the reflection associated to α .
- $x_\alpha(x)x_{-\alpha}(-x^{-1})x_\alpha(x) = n_\alpha \alpha^\vee(x) \forall x \in k^\times$

Call \mathbf{X}_α the **root subgroup** of \mathbf{G} (with respect to \mathbf{T}) associated to α .

Theorem 2.6. [Springer, (8.2.3)] Let $(X(\mathbf{T}), \Phi, Y(\mathbf{T}), \Phi^\vee)$ be the root datum of \mathbf{G} associated to the torus \mathbf{T} of \mathbf{G} and $\{x_\alpha\}_{\alpha \in \Phi}$ as in Lemma 2.5. Moreover, let Φ be totally ordered (any order). Then there exist uniquely determined constants $c_{\alpha,\beta,i,j} \in k$ for $\alpha, \beta \in \Phi$, $\alpha \neq \pm\beta$, $i, j \in \mathbb{N}$ with $i\alpha + j\beta \in \Phi$, such that for all $t, s \in k$,

$$[x_\beta(t), x_\alpha(s)] := x_\beta(t)^{-1}x_\alpha(s)^{-1}x_\beta(t)x_\alpha(s) = \prod_{i,j>0, i\alpha+j\beta \in \Phi} x_{i\alpha+j\beta}(c_{\alpha,\beta,i,j}s^i t^j).$$

Here the product on the right side is formed in ascending order with respect to the chosen total ordering.

See appendix A.3 for the constants $c_{\alpha,\beta,i,j}$ fixed by the choices of isomorphisms x_α and total ordering as in [Haller].

These isomorphisms can be chosen such that the following relations hold (for other choices, one would possibly have to multiply some terms by suitable constants):

Theorem 2.7. [Springer, (9.4.2)] [MT, (4.4),(6.3),(12.6)] Let $(X(\mathbf{T}), \Phi, Y(\mathbf{T}), \Phi^\vee)$ be the root datum of \mathbf{G} with respect to the maximal torus \mathbf{T} and let Φ be totally ordered. Let \mathbf{B} be a Borel subgroup of \mathbf{G} containing \mathbf{T} and let $\mathbf{U} := R_u(\mathbf{B})$ be the unipotent radical of $\mathbf{B} = \mathbf{T} \ltimes \mathbf{U}$ (for the definition of a Borel subgroup, see [MT, (6.3)]). Let Φ^+ be the set of positive roots and Π the simple roots with respect to \mathbf{B} .

Moreover, let $\{x_\alpha\}_{\alpha \in \Phi}$ be a choice of isomorphisms as in Lemma 2.5. For every $\chi \in X(\mathbf{T})$ and $\psi \in Y(\mathbf{T})$ define homomorphisms

$$\bar{\chi} : \text{Hom}(X(\mathbf{T}), k^\times) \rightarrow k^\times, \zeta \mapsto \zeta(\chi)$$

resp.

$$\bar{\psi} : k^\times \rightarrow \text{Hom}(X(\mathbf{T}), k^\times), \lambda \mapsto (\chi \mapsto \lambda^{\langle \chi, \psi \rangle})$$

and for every $\alpha \in \Phi$ define an injective map \tilde{x}_α from k into a set of symbols S and an injective map \tilde{h} from $\text{Hom}(X(\mathbf{T}), k^\times)$ into a set of symbols S such that

$$S := \{\tilde{x}_\alpha(t) \mid \alpha \in \Phi, t \in k\} \cup \{\tilde{h}(\chi) \mid \chi \in \text{Hom}(X(\mathbf{T}), k^\times)\}.$$

Write \tilde{n}_α for the element $\tilde{x}_\alpha(1)\tilde{x}_{-\alpha}(1)\tilde{x}_\alpha(1)$ in the free group on the set S and $\tilde{x}_i := \tilde{x}_{\alpha_i}$ for $1 \leq i \leq 24$.

Denote the following set of relations by R :

- $\tilde{n}_\beta^{-1}\tilde{x}_\alpha(t)\tilde{n}_\beta = \tilde{x}_{s_\beta(\alpha)}(t)$.
- $\tilde{x}_i(t_1)\tilde{x}_i(t_2) = \tilde{x}_i(t_1 + t_2)$, $\tilde{x}_i(0) = 1$.
- $\tilde{h}^{-1}(\lambda)\tilde{x}_i(t)\tilde{h}(\lambda) = \tilde{x}_i(\alpha_i(\lambda)t)$ where $\alpha_i(\lambda)$ is given in Appendix A.4, $\lambda \in \text{Hom}(X(T), k^\times)$.
- $[\tilde{x}_\beta(t), \tilde{x}_\alpha(s)] := \tilde{x}_\beta(t)^{-1}\tilde{x}_\alpha(s)^{-1}\tilde{x}_\beta(t)\tilde{x}_\alpha(s) = \prod_{i,j>0, i\alpha+j\beta \in \Phi} \tilde{x}_{i\alpha+j\beta}(c_{\alpha,\beta,i,j}s^i t^j)$.
- $\tilde{h}(\lambda)\tilde{h}(\mu) = \tilde{h}(\lambda\mu)$. $\lambda, \mu \in \text{Hom}(X(T), k^\times)$.
- $\tilde{n}_\alpha^2 = 1$, $\tilde{n}_\alpha = \tilde{n}_{-\alpha}$
- $\tilde{n}_\alpha^{-1}\tilde{h}(\zeta)\tilde{n}_\alpha = \tilde{h}(\xi)$ for all $\zeta \in \text{Hom}(X(T), k^\times)$, $\alpha \in \Phi$, where $\xi \in \text{Hom}(X(T), k^\times)$ is given by $\xi(\chi) := \zeta(\chi^{s_\alpha})$ for all $\chi \in X(T)$.
- $\tilde{x}_\alpha(u)\tilde{x}_{-\alpha}(-u^{-1})\tilde{x}_\alpha(u) = \tilde{n}_\alpha \overline{\alpha^\vee}(u)$ for $\alpha \in \Phi$ and $u \in k^\times$.
- $\underbrace{\tilde{n}_{\pi_1}\tilde{n}_{\pi_2}\tilde{n}_{\pi_1}\dots}_{m(\pi_1, \pi_2)} = \underbrace{\tilde{n}_{\pi_2}\tilde{n}_{\pi_1}\tilde{n}_{\pi_2}\dots}_{m(\pi_1, \pi_2)}$ for $\pi_1 \neq \pi_2 \in \Pi$ and $m(\pi_1, \pi_2)$ the order of $s_{\pi_1}s_{\pi_2}$ in W .

Then \mathbf{G} is isomorphic to the factor group of the free group on the set S modulo the normal subgroup generated by the relations R . This isomorphism can be chosen such that $x_i(t)$ is mapped to $\tilde{x}_i(t)$. Then we have

- $\mathbf{T} \cong \langle \tilde{h}(\zeta) \mid \zeta \in \text{Hom}(X(\mathbf{T}), k^\times) \rangle$,
- $\mathbf{X}_\alpha \cong \langle \tilde{x}_\alpha(t) \mid t \in k \rangle \forall \alpha \in \Phi$,
- $\mathbf{U} \cong \langle \tilde{x}_\alpha(t) \mid \alpha \in \Phi^+, t \in k \rangle$,
- $\mathbf{B} \cong \langle \tilde{h}(\zeta), \tilde{x}_\alpha(t) \mid \zeta \in \text{Hom}(X(\mathbf{T}), k^\times), \alpha \in \Phi^+, t \in k \rangle$.

Then $\mathbf{U} \cong \prod_{\alpha \in \Phi^+} \mathbf{X}_\alpha$ ([MT, (11.5)]).

Remark 1. Set $\mathbf{U}_i := \langle \mathbf{X}_\alpha \mid ht(\alpha) \geq i \rangle$; then $1 \triangleleft \mathbf{U}_{11} \triangleleft \dots \triangleleft \mathbf{U}_2 \triangleleft \mathbf{U}_1 = \mathbf{U}$ by Theorem 2.6 (see also [Carter72, (5.3.3)]).

Write $x_i(t)$ as a shorthand for $x_{\alpha_i}(t)$.

Lemma 2.8. [Springer, (8.2.1)] Let Φ^+ be totally ordered. Then elements of \mathbf{U} can be written uniquely as $u = x_{i_1}(t_1) \cdot \dots \cdot x_{i_{24}}(t_{24})$ with $t_i \in k$ and $i_1 < \dots < i_{24}$ for the fixed ordering of Φ^+ . This form shall be called the **normal form of u** .

In the following, a normal form for the elements of \mathbf{G} shall be introduced. This will prove useful for explicit calculations in the group.

Lemma 2.9. [Köhler, (I.4.1)] For $w \in W$, define $R(w) := \{\alpha \in \Phi^+ \mid \alpha w \in \Phi^-\}$ for each $w \in W$ (recall that the Weyl group acts on Φ by permuting the roots). Furthermore, let \mathbf{U}_w be the subgroup of \mathbf{U} generated by the \mathbf{X}_α for $\alpha \in R(w)$. Then $|R(w)| = l(w)$ (and $l(w)$ is the length of the Weyl group element w) and $\mathbf{U}_w = \prod_{\alpha \in R(w)} \mathbf{X}_\alpha$, where the latter is independent of the order of the factors in the product.

This is used to define the desired normal form:

Theorem 2.10. [Springer, (8.3.9)] Let $\{\widehat{w}\}_{w \in W}$ be a system of representatives of $W = N_{\mathbf{G}}(\mathbf{T})/\mathbf{T}$ in $N_{\mathbf{G}}(\mathbf{T})$ for the fixed maximal torus \mathbf{T} of \mathbf{G} and \mathbf{B} a Borel subgroup of \mathbf{G} containing \mathbf{T} . Then

1.

$$\mathbf{G} = \dot{\bigcup}_{w \in W} \mathbf{B}\widehat{w}\mathbf{B}$$

is called the **Bruhat decomposition** of \mathbf{G} . A double coset $\mathbf{B}\widehat{w}\mathbf{B}$ is also called a **Bruhat cell**.

2. Each $g \in \mathbf{G}$ can be uniquely written as a product $g = u'\widehat{w}hu$ with $u \in \mathbf{U}, h \in \mathbf{T}, w \in W$ and $u' \in \mathbf{U}_w$. This expression for g is called the **normal form** or **Bruhat normal form** of g .

In particular, the Bruhat decomposition is compatible with Frobenius morphisms in the following sense:

Theorem 2.11. [Carter85, (2.9)] Let $F : \mathbf{G} \rightarrow \mathbf{G}$ be a Frobenius morphism of \mathbf{G} and \mathbf{B} an F -stable Borel subgroup containing the fixed F -stable maximal torus \mathbf{T} of \mathbf{G} . Let $\{\widehat{w}\}_{w \in W}$ be a system of representatives of $W = N_{\mathbf{G}}(\mathbf{T})/\mathbf{T}$ in $N_{\mathbf{G}}(\mathbf{T})$ for the fixed maximal torus \mathbf{T} such that $\widehat{w} \in \mathbf{G}^F$ for $w \in W^F$. Then

1. Each $g \in \mathbf{G}^F$ can be written uniquely as a product $g = u'\widehat{w}hu$ with $u \in \mathbf{U}^F, h \in \mathbf{T}^F, w \in W^F$ and $u' \in \mathbf{U}_w^F$. This expression for g is called the **normal form** or **Bruhat normal form** of g .

2. For $g \in \mathbf{G}^F$ and $w \in W^F$, the following are equivalent:

(a) $g \in \mathbf{B}\widehat{w}\mathbf{B}$

(b) $g \in \mathbf{B}^F\widehat{w}\mathbf{B}^F$

Furthermore a parametrization for the elements of the torus will be needed, which is given as follows:

Lemma 2.12. [Carter85, (3.1.2)] Keep the notation from Theorem 2.7.

1. As seen in Theorem 2.7, the groups \mathbf{T} and $\text{Hom}(X(\mathbf{T}), k^\times)$ are isomorphic as abelian groups. Under this isomorphism, an element $\zeta \in \text{Hom}(X(\mathbf{T}), k^\times)$ is mapped to the torus element $h(\zeta)$.

2. As abelian groups, $Y(\mathbf{T}) \otimes k^\times$ is isomorphic to $\text{Hom}(X(\mathbf{T}), k^\times)$ via $Y(\mathbf{T}) \otimes k^\times \rightarrow \text{Hom}(X(\mathbf{T}), k^\times), \gamma \otimes \lambda \mapsto (\chi \mapsto \lambda^{\langle \chi, \gamma \rangle})$, where the tensor products are considered over \mathbb{Z} .

3. Each element of $Y(\mathbf{T}) \otimes k^\times$ can be written uniquely in the form $\sum_{i=1}^4 \gamma_i \otimes \lambda_i$ for $\{\gamma_1, \dots, \gamma_4\}$ a basis of $Y(\mathbf{T})$ and $\lambda_i \in k^\times$. Combining this with the above gives an isomorphism of abelian groups

$$\theta : (k^\times)^4 \rightarrow \mathbf{T}, (\lambda_1, \dots, \lambda_4) \mapsto h(\chi \mapsto \prod_{i=1}^4 \lambda_i^{\langle \chi, \gamma_i \rangle}).$$

3 An Algorithm for Explicit Computations in \mathbf{G} and \mathbf{G}^F

Köhler [Köhler, (I.5.8)] introduces an algorithm for explicit computations in a general reductive algebraic group \mathbf{G} , which will be useful for explicit computations in the finite fixed-point subgroup $F_4(\overline{\mathbb{F}_2})^F$ of $F_4(\overline{\mathbb{F}_2})$ under a suitable Frobenius morphism $F : F_4(\overline{\mathbb{F}_2}) \rightarrow F_4(\overline{\mathbb{F}_2})$ as well. We shall only present the version for a root system of type F_4 and for $k = \overline{\mathbb{F}_2}$.

For this, one has to choose a system of representatives of $W = N_{\mathbf{G}}(\mathbf{T})/\mathbf{T}$ in $N_{\mathbf{G}}(\mathbf{T})$; Köhler chooses the following, which is particularly convenient for explicit calculations.

Lemma 3.1. [Köhler, (I.5.1)] *Let $w \in W$ be an element of the Weyl group. Then w can be written as reduced expression in the reflections associated to the simple roots, i.e., $w = s_{\pi_1} \cdots s_{\pi_{l(w)}}$ with $\pi_i \in \Pi$ for $1 \leq i \leq l(w)$. Define $\widehat{w} := n_{\pi_1} \cdots n_{\pi_{l(w)}} \in N_G(\mathbf{T})$. This definition of \widehat{w} is independent of the choice of reduced expression for w and $\{\widehat{w}\}_{w \in W}$ forms a system of representatives of $W = N_{\mathbf{G}}(\mathbf{T})/\mathbf{T}$ in \mathbf{G} .*

Next, a data structure to store elements of \mathbf{G} is introduced and the required notation is summarized:

Remark 2. [Köhler] Let \mathbf{T} be the fixed maximal torus of the reductive linear algebraic group \mathbf{G} and let $(X(\mathbf{T}), \Phi, Y(\mathbf{T}), \Phi^\vee)$ be the associated root datum. Moreover, let $\mathbf{U} \cong \prod_{\alpha \in \Phi^+} \mathbf{X}_\alpha$ be generated by the root subgroups and let $\mathbf{B} = \mathbf{T} \ltimes \mathbf{U}$ be a Borel subgroup containing \mathbf{T} . Elements $g = u'\widehat{w}hu \in \mathbf{G}$ are stored as quadruples (u', w, h, u) , where the entries of the quadruple are stored as follows:

1. $u' \in \mathbf{U}_w$ and u are elements of \mathbf{U} . An element $u \in \mathbf{U}$ can be written uniquely as $u = x_{i_1}(t_{i_1}) \cdots x_{i_l}(t_{i_l})$ with $i_1 < \cdots < i_l$ and $t_{i_j} \in k^\times$ (see Lemma 2.8) and will be stored as tuple

$$[[i_1, \dots, i_l], [t_{i_1}, \dots, t_{i_l}]].$$

Generally, u can be written uniquely as $u = x_1(t_1) \cdots x_{24}(t_{24})$ with $t_i \in k$, but there is no need to store the $x_{\alpha_i}(t_i)$ with $t_i = 0$ since $x_\alpha(0) = 1$ for all $\alpha \in \Phi$. So in particular, the element $u = 1$ is stored as tuple $[[], []]$.

2. The element $w \in W$ will be stored as permutation of the roots in Φ .
3. An element $h \in \mathbf{T}$ can be parametrized by a uniquely determined 4-tuple $(\lambda_1, \dots, \lambda_4) \in (k^\times)^4$ (see Lemma 2.12); thus h will be stored through this 4-tuple.
4. In particular, since this parametrization is unique, it allows testing two elements of \mathbf{G} for equality in a very efficient way. Two elements are equal if and only if their Bruhat decompositions are equal, which is the case if and only if the quadruples are equal.

Fix and recall the following notation:

1. Let $\{\chi_1, \dots, \chi_4\}$ be a basis of the free \mathbb{Z} -module $X(\mathbf{T})$, and let the simple roots in Π be given as \mathbb{Z} -linear combinations of these.
2. Let $\{\gamma_1, \dots, \gamma_4\}$ be the corresponding dual basis of $Y(\mathbf{T})$ and let the simple coroots $\{\pi_1^\vee, \dots, \pi_4^\vee\}$ be given as \mathbb{Z} -linear combination of these.
3. The roots in Φ and the coroots in Φ^\vee are given as linear combination of the simple roots respectively coroots.
4. The reflections s_α for $\alpha \in \Phi$ are given as permutations on the roots in Φ .

5. Identify the positive roots $\alpha \in \Phi^+$ in increasing order with the numbers $1, \dots, 24$ according to the chosen order of the roots, and identify the negative roots $\alpha \in \Phi^-$ with the numbers $25, \dots, 48$ such that $\alpha_i \in \Phi^+$ is identified with i and $-\alpha_j$ is identified with $j + 24$ for $\alpha_j \in \Phi^+$ identified with the number j . Thus the Weyl group is a permutation group acting on 48 points.
6. $R(w) := \{\alpha \in \Phi^+ \mid \alpha w \in \Phi^-\}$ for $w \in W$.
7. Write x_i for x_{α_i} for $\alpha_i \in \Phi^+$ identified with i .
8. Choose a system of representatives $\{\widehat{w}\}_{w \in W} \subset N_{\mathbf{G}}(\mathbf{T})$ of W in $N_{\mathbf{G}}(\mathbf{T})$ as in Lemma 3.1.

[Köhler] presents the following strategy for computations in \mathbf{G} :

For the given ordering of the positive roots (see Appendix A.2) and the resulting normal form of any unipotent element $u \in \mathbf{U}$, the product of two unipotent elements of \mathbf{U} in normal form can again be brought into normal form using the commutator relations in Appendix A.3. This can be done very efficiently via “**collecting from the left**”, as described in [Sims, pp. 40]. Similarly, the inverse of an element $u \in \mathbf{U}$ in normal form $u = x_{i_1}(t_1) \cdots x_{i_l}(t_l)$ is given as $u^{-1} = x_{i_l}(t_l) \cdots x_{i_1}(t_1)$ and only needs to be normalized again. It remains to determine how to bring a given element of \mathbf{G} into its Bruhat normal form and how to find the Bruhat normal form of the product of two elements $g_1, g_2 \in \mathbf{G}$ in Bruhat normal form and of the inverse of an element $g_1 \in \mathbf{G}$ in Bruhat normal form. Consider the case of determining the Bruhat normal form of a product and an inverse first; the other case can be deduced from this one.

Köhler considers the following special cases first. The general case can then be reduced to these.

Remark 3. [Köhler, (I.5.5)] Let $g_1, g_2 \in \mathbf{G}$ and keep the notation from Remark 2.

1. If $g_1, g_2 \in \mathbf{U}$, then the Bruhat decomposition of the product $g_1 g_2$ and of the inverse of g_1 is obtained via “**collecting from the left**” as described in [Sims, pp. 40].
2. If $g_1, g_2 \in \mathbf{T}$, then so is the product $g_1 g_2$. The corresponding 4-tuple is given by componentwise multiplication of the 4-tuples of g_1 and g_2 . Similarly, the 4-tuple of the inverse of g_1 is given by inverting the 4-tuple of g_1 componentwise.
3. If $g_1 = \widehat{w}$ for some $w \in W$ and $g_2 = n_\alpha$ for some $\alpha \in \Pi$, then the Bruhat decomposition of the product is given by

$$\widehat{w}n_\alpha = \begin{cases} \widehat{ws}_\alpha & \text{if } \alpha w^{-1} \in \Phi^+ \\ (\widehat{ws}_\alpha)h_\alpha & \text{if } \alpha w^{-1} \in \Phi^- \end{cases},$$

with the torus element $h_\alpha \in \mathbf{T}$ given by the 4-tuple $((-1)^{a'_1}, \dots, (-1)^{a'_4})$ and $\alpha^\vee = \sum_{i=1}^4 a'_i \gamma_i$. Since $\widehat{w} = n_{\pi_1} \cdots n_{\pi_l} \in N_{\mathbf{G}}(\mathbf{T})$ for $w = s_{\pi_1} \cdots s_{\pi_l}$ a reduced expression with $\pi_i \in \Pi$, this allows one to determine the Bruhat decomposition of a product $\widehat{w}_1 \widehat{w}_2$ for arbitrary $w_i \in W$.

4. Let $w \in W$ and let $\{\alpha_{i_1}, \dots, \alpha_{i_m}\} = R(w)$ be the set of positive roots that are mapped to negative roots under the action of w . Then $\widehat{w}^{-1} = \widehat{w}^{-1} \prod_{j=1}^m h_{\alpha_{i_j}}$.

Next Köhler shows the following:

Remark 4. [Köhler, (I.5.6)] Keep the notation of Remark 2. Let $\pi \in \Pi, \alpha \in \Phi^+, \beta \in \Phi$ and $t \in k^\times$. Furthermore, let $h \in \mathbf{T}$ be a torus element given by the 4-tuple $(\lambda_1, \dots, \lambda_4) \in (k^\times)^4$ and let $x_\alpha(t) \in \mathbf{U}$ be a unipotent element.

1. $x_\alpha(t)^h = x_\alpha(t \prod_{i=1}^4 \lambda_i^{a_i})$, where the a_i are given as coefficients in the linear combination $\alpha = \sum_{i=1}^4 a_i \chi_i$. Similarly ${}^h x_\alpha(t) = x_\alpha(t \prod_{i=1}^4 \lambda_i^{-a_i})$.
2. $x_\alpha(t)^{n_\beta} = x_{\alpha s_\beta}(t)$ and ${}^{n_\beta} x_\alpha(t) = x_{s_\beta \alpha}(t)$.
3. $h^{n_\pi} = {}^{n_\pi} h = h'$, where h' is given by the 4-tuple (μ_1, \dots, μ_4) with $\mu_i = \prod_{j=1}^4 \lambda_j^{s_{ji}}$ for h given by the 4-tuple $(\lambda_1, \dots, \lambda_4)$ and s_{ji} defined as the entries of a matrix $S \in \mathbb{Z}^{4 \times 4}$ representing the action of the Weyl group on $Y(\mathbf{T})$ with respect to the chosen basis $\{\gamma_1, \dots, \gamma_4\}$.

Finally, Köhler formulates the following lemma:

Lemma 3.2. *[Köhler, (I.5.7)] Again, keep the notation of Remark 2. Let $g_1 \in \mathbf{G}$ be given in its Bruhat normal form $g_1 = u' \widehat{w} h u$ and stored as a tuple of parameters as described above in Remark 2 and let $g_2 \in \mathbf{G}$ be one of the following:*

1. $g_2 = h' \in \mathbf{T}$ stored as 4-tuple $(\mu_1, \dots, \mu_4) \in (k^\times)^4$,
2. $g_2 = x_\alpha(t)$ with $\alpha \in \Phi^+$ and $t \in k^\times$, stored as a list $[[\alpha], [t]]$,
3. $g_2 = n_\pi$ with $\pi \in \Pi$ stored as the permutation on the roots induced by s_π .

Then the Bruhat normal form of the product $g_1 g_2$ is given as follows:

1. In this case

$$g_1 g_2 = (u' \widehat{w} h u) h' = u' \widehat{w} (h h') (u^{h'})$$

and this is stored as described above in Remark 3 (2) and Remark 4 (1).

2. In this case

$$g_1 g_2 = (u' \widehat{w} h u) x_\alpha(t) = u' \widehat{w} h (u x_\alpha(t))$$

is the Bruhat normal form of the product $g_1 g_2$ and it is stored as described in Remark 3 (1).

3. This is the most involved case. Write $u = x_\pi(t') u''$ with $u'' = x_{\alpha_{i_1}}(t_1) \cdots x_{\alpha_{i_m}}(t_m) \in \mathbf{U}$ with $\alpha_{i_1}, \dots, \alpha_{i_m} \in \Phi^+ \setminus \{\pi\}$ and $t_1, \dots, t_m \in k^\times$. The following cases for t' are considered separately:

- (a) $t' = 0$,
- (b) $t' \neq 0$, $\pi w^{-1} \in \Phi^+$,
- (c) $t' \neq 0$, $\pi w^{-1} \notin \Phi^+$.

The Bruhat normal form of the product $g_1 g_2$ in these three cases is given as follows:

- (a) In this case

$$g_1 g_2 = (u' \widehat{w} h u) n_\pi = u' \widehat{w} h u'' n_\pi = u' (\widehat{w} n_\pi) (h^{n_\pi}) ((u'')^{n_\pi}).$$

As described in Remark 3 (3), $\widehat{w} n_\pi (h^{n_\pi}) = \widehat{w}' h'$ with $w' \in W$ and $h' \in \mathbf{T}$. Now write $u' = u'_1 u'_2$ with $u'_1 \in \mathbf{U}_{w'}$ and $u'_2 \in \mathbf{U}$. Then

$$g_1 g_2 = u'_1 \widehat{w}' h' ((u'_2)^{\widehat{w}' h'} ((u'')^{n_\pi})).$$

Finally, by Theorem 2.7, this is the desired Bruhat normal form of the product $g_1 g_2$.

(b) In this case

$$g_1 g_2 = (u' \widehat{w} h u) n_\pi = u' \widehat{w} h x_\pi(t') u'' n_\pi = (u' (\widehat{w}^h(x_\pi(t')))) (\widehat{w} n_\pi) h^{n_\pi} ((u'')^{n_\pi}).$$

By Theorem 2.7, $\widehat{w}^h(x_\pi(t')) \in \mathbf{U}$ and so this Bruhat normal form can be computed using the Remarks 3 and 4, analogously to the previous case.

(c) In this case $\widehat{w}^h(x_\pi(t')) \notin \mathbf{U}$ by Theorem 2.7 (relation $\tilde{n}_\beta^{-1} \tilde{x}_\alpha(t) \tilde{n}_\beta = \tilde{x}_{s_\beta(\alpha)}(t)$). By applying relation $\tilde{x}_\alpha(u) \tilde{x}_{-\alpha}(-u^{-1}) \tilde{x}_\alpha(u) = \tilde{n}_\alpha \overline{\alpha^\vee}(u)$ for $\alpha \in \Phi$ and $u \in k^\times$ of Theorem 2.7, get

$$g_1 g_2 = (u' (\widehat{w}^h(x_{-\pi}(t'^{-1})))) \widehat{w} (h \overline{\pi^\vee}(-t'^{-1})) (x_\pi(-t')^{\overline{\pi^\vee}(-t'^{-1})}) (u'')^{n_\pi}.$$

$\overline{\pi^\vee}(-t'^{-1})$ is represented by the 4-tuple $((-t')^{-\pi_1^\vee}, \dots, (-t')^{-\pi_4^\vee})$ with $\pi^\vee = \sum_{i=1}^4 \pi_i^\vee \gamma_i$. So again the Bruhat normal form can be computed analogously to the first case. Observe that in this case it may be necessary to invert elements of the field k .

[Köhler, (I.5.8)] gives an explicit algorithm to bring the product of two elements in Bruhat normal form again into Bruhat normal form.

Algorithm 1. Bruhat Normal Form

INPUT:

- $g_1 = u'_1 \widehat{w}_1 h_1 u_1 \in \mathbf{G}$, stored as described in Remark 2.
- $g_2 = u'_2 \widehat{w}_2 h_2 u_2 \in \mathbf{G}$, stored as described in Remark 2.

VARIABLES: `erg`, which stores the intermediate results of the algorithm after each step.

INITIALIZATION: `erg:=g1`.

ITERATION:

1. Set `erg:=erg·u'2` according to Lemma 3.2 (2) and go to Step 2.
 2. Write $\widehat{w}_2 = n_{\pi_1} \cdots n_{\pi_r}$ with $\pi_i \in \Pi$ for $1 \leq i \leq r$ and go to Step 3.
 3. For all n_{π_i} with $1 \leq i \leq r$ do:
 For `erg:=u' \widehat{w} h u` write $u = x_{\pi_i}(t') u''$ as in Lemma 3.2 (3) and consider the following cases
 - (a) If $t' = 0$, then set `erg:=erg·nπi` according to Lemma 3.2 (3a)
 - (b) If $t' \neq 0$, $\pi_i w^{-1} \in \Phi^+$, then set `erg:=erg·nπi` according to Lemma 3.2 (3b)
 - (c) If $t' \neq 0$, $\pi_i w^{-1} \notin \Phi^+$, then set `erg:=erg·nπi` according to Lemma 3.2 (3c)
- Go to Step 4
4. Set `erg:=erg·h2` according to Lemma 3.2 (1) and go to Step 5.
 5. Set `erg:=erg·u2` according to Lemma 3.2 (2) and go to Step 6.
 6. Return `erg`.

OUTPUT: $g_1 g_2 = u' \widehat{w} h u \in \mathbf{G}$, saved as described in Remark 2.

Remark 5. [Köhler, (I.5.9, I.5.10)]

- Algorithm 1 can also be used to compute the Bruhat normal form of the inverse of a given element and to compute the Bruhat normal form of an element of \mathbf{G} given as product in the generators of \mathbf{G} (recall that \mathbf{G} is generated by $\{h, x_\alpha(t) \mid h \in \mathbf{T}, \alpha \in \Phi, t \in k\}$, see Lemma 2.5):

1. Let $g = u'\widehat{w}hu \in \mathbf{G}$, then the inverse of g is given by first inverting u' , \widehat{w} , h , and u separately and then calculating the Bruhat normal form of $u^{-1}h^{-1}\widehat{w}^{-1}u'^{-1}$ with the help of Algorithm 1.
 2. \mathbf{G} is generated by $\{h, x_\alpha(t) \mid h \in \mathbf{T}, \alpha \in \Phi, t \in k\}$. Now let $g = g_1 \cdots g_m$ be given as a word in these generators. If $g_i = x_\alpha(t)$ with $\alpha \in \Phi^-$, then there exists a $\pi \in \Pi$ and a $w \in W$ with $\pi w = \alpha$ ([MT, (9.1),(9.4)]). From the relations in Theorem 2.7 it follows that $x_\alpha(t) = \widehat{w}^{-1}x_\pi(t)\widehat{w}$, and this expression can be transformed into its Bruhat normal form via Algorithm 1. Replace g_i by this expression. Then the Bruhat normal form of g can be calculated via Algorithm 1.
- In Step 3 of Algorithm 1, elements of the field k may have to be inverted. When computing with parametrized elements, this will have to be taken into account. In all the other steps of the algorithm, only the ring structure of k is used.
 - Let F be a Frobenius morphism of \mathbf{G} and let the root datum of \mathbf{G} be given with respect to an F -stable maximal torus \mathbf{T} . If F acts trivially on the roots of \mathbf{G} , and if the corresponding finite Chevalley group is generated by the elements $\{h, x_\alpha(t) \mid h \in \mathbf{T}^F, \alpha \in \Phi, t \in k^F\}$, then Algorithm 1 describes explicit computing in \mathbf{G}^F if the representatives $\{\widehat{w}\}_{w \in W}$ of W in \mathbf{G} are chosen in \mathbf{G}^F .

4 Computations using Parametrized Elements

Keep the notation from the previous section, in particular from Remark 2.

It is one of the aims of this thesis to obtain information on the decomposition numbers of $F_4(q)$ for q an arbitrary power of 2. So in particular, it will be necessary to run computations generically for infinitely many groups at once. To achieve this, the computations will be conducted using parametrized elements. Inserting an allowable value for the parameters for this parametrized element will then give an element of the group itself.

By the Bruhat decomposition Theorem 2.10 and Remark 2, each element $g \in \mathbf{G}$ lies in precisely one double coset $\mathbf{B}\widehat{w}\mathbf{B}$ for some $w \in W$ and \mathbf{B} a Borel subgroup as in Remark 2, and each element of such a coset may be uniquely written as

$$x_{\beta_1}(s_1) \cdots x_{\beta_{l(w)}}(s_{l(w)}) \widehat{w} \theta(\lambda_1, \dots, \lambda_4) x_{\alpha_1}(t_1) \cdots x_{\alpha_{24}}(t_{24}) \quad (1)$$

with $\beta_j \in R(w)$, $\alpha_i \in \Phi^+$ and $s_j, t_i \in k, \lambda_1, \dots, \lambda_4 \in k^\times$ for $1 \leq j \leq l(w)$ and $1 \leq i \leq 24$. Rather than looking at all the single elements of this coset, one can consider λ_i, s_i, t_i as indeterminates instead and do the computations over the field of rational functions of the form as in (1). Computations with Algorithm 1 and checks of equality will install restrictions on the allowable specializations of the parameters. In this context, the possible inverting of elements in Step 3 of Algorithm 1 becomes relevant. This will lead to some case distinctions and in particular, this means that the computations cannot be done over the multivariate polynomial ring in the indeterminates λ_i, s_i, t_i , but do indeed have to be conducted over the field of fractions.

Köhler introduces a few notions from commutative algebra to facilitate formulating an algorithm to run computations with parametrized elements.

Definition 4.1. [Kunz, (I.§1),(III.§1)] Let $s \in \mathbb{N}$, let $k[\underline{Y}] := k[Y_1, \dots, Y_s]$ be the multivariate polynomial ring in the indeterminates Y_1, \dots, Y_s over a field k , and let $k(\underline{Y})$ be the associated field of fractions.

1. For an ideal I of $k[\underline{Y}]$, call the ideal $\sqrt{I} := \{f \in k[\underline{Y}] \mid \exists n \in \mathbb{N} \text{ s.t. } f^n \in I\}$ of $k[\underline{Y}]$ the **radical** of I . If $I = \sqrt{I}$, call I a **radical ideal**.

2. For $f \in k[\underline{Y}]$, let

$$D_f := \{t \in k^s \mid f(t) \neq 0\} \subseteq k^s$$

be an **elementary open subset** of k^s .

3. Let $F \subseteq k[\underline{Y}]$. Then

$$V(F) := \{t \in k^s \mid f(t) = 0 \forall f \in F\} \subseteq k^s$$

is called a **closed subset** of k^s .

4. Let $f \in k(\underline{Y})$. There exist polynomials $g, h \in k[\underline{Y}]$ with $\gcd(g, h) = 1$ such that $f = \frac{g}{h}$. Then $\mathfrak{D}_f := D_h \subseteq k^s$ is called the **domain of definition** of f , and $f(t) := \frac{g(t)}{h(t)}$ is well defined for all $t \in \mathfrak{D}_f$.

5. For a finite subset $F \subseteq k(\underline{Y})$ of the field of fractions,

$$\mathfrak{D}_F := \bigcap_{f \in F} \mathfrak{D}_f \subseteq k^s$$

is called the **domain of definition** of the set of polynomials F , and $f(t)$ is well defined for all $f \in F, t \in \mathfrak{D}_F$.

The following is a brief summary of some well known properties of closed subsets of an affine space that will be relevant for what is to follow.

Lemma 4.2. [Kunz, (I.§1)] Let $f, g \in k[\underline{Y}]$ with $k[\underline{Y}]$ as in Definition 4.1 and $I, J \trianglelefteq k[\underline{Y}]$. Then

1. $D_f \cap D_g = D_{fg}$,
2. $V(I) \cap V(J) = V(I + J)$,
3. $k^s = V(\langle f \rangle) \dot{\cup} D_f$,
4. $V(I) \subseteq V(J)$ if and only if $\sqrt{J} \subseteq \sqrt{I}$, and
5. $\sqrt{\langle f \rangle} \subseteq \sqrt{I}$ if and only if $f \in \sqrt{I}$.

Lemma 4.3. [Köhler, (I.6.3)] Let $f, g \in k[\underline{Y}]$ with $k[\underline{Y}]$ as in Definition 4.1, $I \trianglelefteq k[\underline{Y}]$ an ideal and $P := V(I) \cap D_f$, $P_1 := \{t \in P \mid g(t) = 0\}$, $P_2 := \{t \in P \mid g(t) \neq 0\}$. Then

1. $P = \emptyset$ if and only if $f \in \sqrt{I}$,
2. $P_1 = V(\langle g \rangle + I) \cap D_f$, and
3. $P_2 = V(I) \cap D_{fg}$.

Lemma 4.4. [CLO, (4.2.8)] (*Rabinovitch-trick*)

Let $I \trianglelefteq k[\underline{Y}]$ be an ideal in the polynomial ring and $f \in k[\underline{Y}]$ a polynomial. Then $f \in \sqrt{I}$ if and only if $1 \in I + \langle 1 - Zf \rangle \subseteq k[\underline{Y}, Z]$, where $k[\underline{Y}, Z]$ is the polynomial ring in the indeterminates Y_1, \dots, Y_s, Z .

Remark 6. [Köhler, (I.6.5)] For a polynomial $f \in \mathbb{Z}(\underline{Y})$ with integral coefficients, the reduction of f modulo a prime p can be viewed as $f \in k(\underline{Y})$ for k a field of characteristic p . Consequently, one can define $\mathfrak{D}_f \subseteq k^s$ for $f \in \mathbb{Z}(\underline{Y})$ and $f(t) \in k$ is then well-defined for all $t \in \mathfrak{D}_f$.

Having introduced these notions from commutative algebra, Köhler finally defines parametrized elements and a parametrization as such.

Definition 4.5. [Köhler, (I.6.6)] As before, let $k[\underline{Y}] := k[Y_1, \dots, Y_s]$ be the multivariate polynomial ring in the indeterminates Y_1, \dots, Y_s for an $s \in \mathbb{N}$ over a field k , and let $k(\underline{Y})$ be the associated field of fractions. Let $\mathbf{G}(k(\underline{Y}))$ be the reductive group over the field $k(\underline{Y})$ given by the root datum of \mathbf{G} .

1. A triple $\mathcal{A} := (A, A', P)$ is called a **parametrized basic element** of \mathbf{G} with **parameter set** P if $A \in \mathbf{G}(k(\underline{Y}))$, A' is a presentation of A as word in the generators of $\mathbf{G}(k(\underline{Y}))$ of the form

$$x_{\beta_1}(f_1) \cdots x_{\beta_{l(w)}}(f_{l(w)}) \widehat{w} \theta(g_1, \dots, g_4) x_{\alpha_1}(h_1) \cdots x_{\alpha_{24}}(h_{24})$$

with $w \in W$, $\beta_i \in R(w)$, $\alpha_r \in \Phi^+$ and $f_i, g_j, h_r \in k(\underline{Y})$ for $1 \leq i \leq l(w)$, $1 \leq j \leq 4$, $1 \leq r \leq 24$ and α_i, β_i ordered according to the fixed ordering of the roots, and

$$P \subseteq P(\mathcal{A}) := \mathfrak{D}_{\{f_i, g_j, h_r\}_{1 \leq i \leq l(w), 1 \leq j \leq 4, 1 \leq r \leq 24}} \cap D_{g'_1 \cdots g'_4}.$$

Here $\mathfrak{D}_{\{f_i, g_j, h_r\}_{1 \leq i \leq l(w), 1 \leq j \leq 4, 1 \leq r \leq 24}}$ is the intersection of the domain of definition of the rational functions f_i, g_j, h_r and let $g_i = \frac{g'_i}{g''_i}$ with $g'_i, g''_i \in k[\underline{Y}]$ as in Definition 4.1. If $f_i, g_j, h_r \in \mathbb{Z}[\underline{Y}]$, \mathcal{A} is called an **integral parametrized basic element** with parameter set P (recall Remark 6). Inserting a parameter $t \in P$ into A' will give an element of the group $\mathbf{G} = \mathbf{G}(k)$ (specializing the indeterminates of a word

$$x_{\beta_1}(f_1) \cdots x_{\beta_{l(w)}}(f_{l(w)}) \widehat{w} \theta(g_1, \dots, g_4) x_{\alpha_1}(h_1) \cdots x_{\alpha_{24}}(h_{24})$$

to elements in k respectively k^\times will precisely give an element of \mathbf{G} of the form as described in Theorem 2.10 and Lemma 2.12), which shall be denoted by $\mathcal{A}(t)$. $P(\mathcal{A})$ is precisely the maximal set of allowable parameters t such that the specialization $\mathcal{A}(t)$ is well defined. As none of the parameters affect the \widehat{w} in the presentation A' of A , all specializations $\mathcal{A}(t)$ for $t \in P(\mathcal{A})$ lie in the same Bruhat cell $\mathbf{B}\widehat{w}\mathbf{B}$ of \mathbf{G} . This double coset shall be called the **Bruhat cell** associated to \mathcal{A} .

2. Let $\mathcal{A}_1 := (A_1, A'_1, P_1)$ and $\mathcal{A}_2 := (A_2, A'_2, P_2)$ be two parametrized basic elements. Define

$$\mathcal{A}_1 \cdot \mathcal{A}_2 := (A_1 A_2, A'_1 A'_2, P_1 \cap P_2)$$

where $A_1 A_2$ is the product of A_1 and A_2 in $\mathbf{G}(k(\underline{Y}))$ and $A'_1 A'_2$ is the concatenation of the words A'_1 and A'_2 (of course the concatenation of two parametrized basic elements will in general not be a parametrized basic element again). Call a product

$$\mathcal{A} := \mathcal{A}_1 \cdots \mathcal{A}_l = (A, A', P)$$

of (integral) parametrized basic elements \mathcal{A}_i , $1 \leq i \leq l$ a **(integral) parametrized element** of \mathbf{G} with **parameter set** P . As for the parametrized basic elements, $\mathcal{A}(t)$ is an element of the group \mathbf{G} for all $t \in P$. Write $\mathcal{A}(P) := \{\mathcal{A}(t) \mid t \in P\}$ for the sets of specializations of \mathcal{A} .

A priori, the definition of parametrized element $\mathcal{A} = (A, A', P)$ seems redundant, since it would suffice to define the parametrized element as (A', P) . However, a fixed element $A \in \mathbf{G}(k(\underline{Y}))$ may have many different presentations A' as word in the generators (although the specialization of a parametrized element has a unique normal form, the parametrized element itself can have different presentations as word in the generators, since if for example $P \subset V(f)$, then $x_\alpha(f)$ and $x_\alpha(0)$ would specialize to the same element but $x_\alpha(f) \neq x_\alpha(0)$ as elements of $\mathbf{G}(k(\underline{Y}))$). To emphasize this difference between the element A and its presentation A' , Köhler defines the parametrized element as triple (A, A', P) . However, occasionally a parametrized element $\mathcal{A} = (A, A', P)$ will be abbreviated as $\mathcal{A} := A'$ with parameter set P .

Definition 4.6. Let $M \subseteq \mathbf{G}$ be a subset of \mathbf{G} . Then a set $\{\mathcal{A}_i\}_{1 \leq i \leq l}$ of (integral) parametrized elements with parameter sets P_i is called a **(integral) parametrization** of M if

- the specialization map $t \mapsto \mathcal{A}_i(t)$, $t \in P_i$ is injective for all $1 \leq i \leq l$ and
- $M = \dot{\bigcup}_{1 \leq i \leq l} \mathcal{A}_i(P_i)$.

For $l = 1$, also write \mathcal{A}_1 instead of $\{\mathcal{A}_i\}_{1 \leq i \leq 1}$.

Example 1. Let $k[\lambda_1, \dots, \lambda_4, t_1, \dots, t_{24}, s_1, \dots, s_{24}]$ be the polynomial ring in $4 + 2 \cdot 24 = 52$ indeterminates over k .

1. Let $w \in W$, $\{\beta_1, \dots, \beta_{l(w)}\} = R(w)$ and $\{\alpha_1, \dots, \alpha_{24}\} = \Phi^+$. Then

$$\mathcal{B}w\mathcal{B} := x_{\beta_1}(s_1) \cdots x_{\beta_{l(w)}}(s_{l(w)}) \widehat{w}\theta(\lambda_1, \dots, \lambda_4) x_{\alpha_1}(t_1) \cdots x_{\alpha_{24}}(t_{24})$$

with parameter set

$$P(\mathcal{B}w\mathcal{B}) = V(\{\{s_i\}_{l(w) < i \leq 24}\}) \cap D_{\lambda_1 \cdots \lambda_4} \cong (k^\times)^4 \times k^{24+l(w)}$$

is an integral parametrization of the double coset $\mathbf{B}\widehat{w}\mathbf{B}$. Recall that Definition 4.6 required the specialization map to be injective. Therefore the indeterminates $\{s_i\}_{l(w) < i \leq 24}$ need to be fixed in the parameter set $P(\mathcal{B}w\mathcal{B})$ although they do not appear in the presentation $\mathcal{B}w\mathcal{B}$. Similar considerations apply to the other examples.

2. If $w = 1$, this gives a parametrization of the Borel subgroup \mathbf{B} ; call this parametrization \mathcal{B} .
3. $\mathcal{G} := \{\mathcal{B}w\mathcal{B}\}_{w \in W}$ is an integral parametrization of \mathbf{G} .
- 4.

$$\mathcal{U} := x_{\alpha_1}(t_1) \cdots x_{\alpha_{24}}(t_{24})$$

with parameter set

$$P(\mathcal{U}) = V(\{\{s_i\}_{1 \leq i \leq 24}\}) \cap V(\{\{\lambda_i - 1\}_{1 \leq i \leq 4}\}) \cong k^{24}$$

is an integral parametrization of \mathbf{U} .

- 5.

$$\mathcal{T} := \theta(\lambda_1, \dots, \lambda_4)$$

with parameter set

$$P(\mathcal{T}) = V(\{\{s_i\}_{1 \leq i \leq 24}\}) \cap V(\{\{t_i\}_{1 \leq i \leq 24}\}) \cap D_{\lambda_1 \cdots \lambda_4} \cong (k^\times)^4$$

is an integral parametrization of \mathbf{T} .

6. For $w \in W$ and $\{\beta_1, \dots, \beta_{l(w)}\} = R(w)$,

$$\mathcal{U}_w := x_{\beta_1}(s_1) \cdots x_{\beta_{l(w)}}(s_{l(w)})$$

with parameter set

$$P(\mathcal{B}w\mathcal{B}) = V(\{\{s_i\}_{l(w) < i \leq 24}\}) \cap V(\{\{t_i\}_{1 \leq i \leq 24}\}) \cap V(\{\{\lambda_i - 1\}_{1 \leq i \leq 4}\}) \cong k^{l(w)}$$

in an integral parametrization of \mathbf{U}_w .

Remark 7. [Köhler, (I.6.10),(I.6.11)] Let \mathcal{A}, \mathcal{B} be parametrized elements of \mathbf{G} with parameter sets $P(\mathcal{A})$ respectively $P(\mathcal{B})$. Then

1. $(\mathcal{A} \cdot \mathcal{B})(t) = \mathcal{A}(t)\mathcal{B}(t)$ for all $t \in P(\mathcal{A}) \cap P(\mathcal{B})$, i.e. specialization is compatible with multiplication.
2. If \mathcal{B} is obtained from \mathcal{A} by applying relations from Theorem 2.7, then $\mathcal{A}(t) = \mathcal{B}(t)$ for all $t \in P(\mathcal{A}) \cap P(\mathcal{B})$. Therefore any manipulations of parametrized elements with relations from Theorem 2.7 will still be valid after specializing to an allowable parameter.
3. If \mathcal{A} is an integral parametrized element and \mathcal{B} is obtained from \mathcal{A} by applying relations from Theorem 2.7, then \mathcal{B} is an integral parametrized element as well.

Köhler defines a normal form for parametrized elements analogous to Theorem 2.10. Moreover, this normal form can be computed algorithmically as well.

Definition 4.7. [Köhler, I.6.12] Let \mathcal{A} be a parametrized element with parameter set P . A set $\{\mathcal{A}_i\}_{1 \leq i \leq l}$ of (integral) parametrized basic elements with parameter sets P_i is called **(integral) normal form** or **(integral) Bruhat normal form** of \mathcal{A} , if

$$P = \bigcup_{i \in \{1, \dots, l\}} P_i$$

and if $\mathcal{A}_i(t)$ is the normal form of $\mathcal{A}(t) \in \mathbf{G}$ according to Theorem 2.10 for all $i \in \{1, \dots, l\}$ and $t \in P_i$.

Remark 8. [Köhler, (I.6.13),(I.6.14)]

- A priori, it is not clear that such a normal form always exists, i.e. if there always exists a finite disjoint union of the parameter sets as required in the definition. However, Köhler presents an algorithm to compute the normal form for any parametrized element, thus showing that such a form does indeed always exist. This normal form is not unique, since e.g. a refinement of the decomposition of the parameter set leads to a different normal form. This is not a problem however, since the normal form of the specialization is unique. The parametrized elements are only used because they allow one to conduct computations for many elements simultaneously, which is required for generic computations. The Bruhat cell of a specialized element moreover can be discerned from the normal form of the parametrized element.
- By Remark 7 and a close study of Algorithm 1, it becomes evident that this algorithm can be applied to parametrized elements as well, if the parameter set is taken account of. All steps except Step 3(c) can be computed for the entire parameter set of an element. In Step 3(c), it may be necessary to invert an element, which may not be possible for all elements of the parameter set. Therefore:
- Let erg be a parametrized element in Step 3(c) of Algorithm 1 with parameter set P_{erg} , then $\text{erg} = u' \widehat{w} h u$ with parametrized elements u', h, u . The element u has the form $x_\pi(f) u''$, where f is a rational function and $\pi w^{-1} \notin \Phi^+$. Set

$$P_1 := \{t \in P_{\text{erg}} \mid f(t) = 0\} = P_{\text{erg}} \cap V(\langle f \rangle)$$

and

$$P_2 := \{t \in P_{\text{erg}} \mid f(t) \neq 0\} = P_{\text{erg}} \cap D_f.$$

This gives a parametrized element (erg, P_1) of type (a) in Step 3 of Algorithm 1 and a parametrized element (erg, P_2) of type (c) such that all specializations with parameters of the corresponding parameter set result in valid equalities in \mathbf{G} .

Finally, [Köhler, (I.6.15)] obtains the following algorithm to compute the Bruhat normal form of a parametrized element.

Algorithm 2. Bruhat Normal Form of Parametrized Elements

INPUT: A parametrized element \mathcal{A} with parameter set P .

VARIABLES:

- list, a list that stores the different cases of \mathcal{A}, P for a case distinction as in Remark 8,
- erg, which stores the intermediate results of the algorithm after each step,
- i , which counts the number of different cases that have been dealt with already for a case distinction as in Remark 8.

INITIALIZATION: Use Remark 5 to write \mathcal{A} as parametrized element in which only expressions of the form $x_\alpha(f)$, n_π and $\theta(g_1, \dots, g_4)$ occur. This does not change the parameter set.

- list:=[[\mathcal{A}, P]],
- erg:=[],
- $i:=1$.

ITERATION:

1. If $i > \text{Length}(\text{list})$, go to Step 3, else set $[X, P_X] := \text{list}[i]$ and go to Step 2.
2. Apply Algorithm 1 to $[X, P_X]$ until Step 3(c) of the algorithm requires a case distinction or until $[X, P_X]$ is in normal form.
 - (a) If $[X, P_X]$ is a normal form as defined in Definition 4.7, set $\text{Add}(\text{erg}, [X, P_X])$ and $i := i + 1$ and go to Step 1.
 - (b) If Step 3(c) of Algorithm 1 should be applied next and a case distinction as in Remark 8 is required, one obtains two parametrized elements X_1, X_2 with parameter sets P_1 resp. P_2 . Set $[X, P_X] := [X_1, P_1]$, $\text{Add}(\text{list}, [X_2, P_2])$ and go to Step 1.
3. Return erg.

OUTPUT: A normal form of \mathcal{A} as defined in Definition 4.7.

Remark 9. [Köhler, (I.6.16)]

1. If the parameter set P in the input of the algorithm is of the form $P = V(I) \cap D_f$ for an ideal I and a polynomial f , then the same holds for the parameter sets of the output of the algorithm. In particular, the Rabinovitch-trick 4.4 can be used to test whether such a parameter set is empty.
2. If \mathcal{A} is an integral parametrized element with parameter set $P = V(I) \cap D_f$ for an ideal I and a polynomial f from the polynomial ring over \mathbb{Z} , then the same holds for the parameter sets of the output of the Algorithm 2.

Definition 4.8. [Köhler, (I.6.17)] A parameter set P is called **integral** if $P = V(I) \cap D_f$ for an ideal I and a polynomial f from the corresponding polynomial ring over \mathbb{Z} .

Remark 10. [Köhler, (I.7),(I.8)]

- Let \mathcal{A} be a parametrized element. Save \mathcal{A} as pair $(A', P_{\mathcal{A}})$. Save A' as in Remark 2, except replacing the elements $t_i \in k, \lambda_i \in k^\times$ of the field k by rational functions from the corresponding field of rational functions $k(\underline{Y})$.
- It will turn out that for all parametrized elements \mathcal{A} with parameter set P occurring in the computations, $P = V(I) \cap D_f$ for an ideal I and a polynomial f from the corresponding polynomial ring.
- Generic computations: Let \mathbf{G} be defined over \mathbb{F}_q for $q = p^n$ a prime power and F the corresponding Frobenius morphism and \mathbf{T} an F -stable maximal torus. Then $G(q) = \mathbf{G}^F$ and generic computations are computations using the root datum, but not the prime power q . In particular, the following holds:
- Let $G(q) = \mathbf{G}^F$ be a Chevalley group and \mathcal{A} a parametrization of a set $M \subseteq \mathbf{G}$ with parameter set P . Then \mathcal{A} with parameter set $P^F := \{t \in P \mid t^F = t\}$ is a parametrization of $M^F \subseteq \mathbf{G}^F$.
- Let $P = V(I) \cap D_f \neq \emptyset$. By Lemma 4.2,

$$V(I) = V(I) \cap (V(\langle f \rangle) \dot{\cup} D_f) = V(I + \langle f \rangle) \dot{\cup} P.$$

Then

$$|P^F| = |V(I)^F| - |V(I + \langle f \rangle)^F|.$$

- Finding $|V(I)^F|$: Let $I \trianglelefteq k[\underline{Y}]$ be an ideal generated by polynomials with integer coefficients. Sadly, no algorithm solving the polynomial equations occurring in the computation of $|V(I)^F|$ automatically in all cases is known. Thus these computations have to be done by hand. Gröbner bases can help with this. See also [Köhler, (I.7.2)].
- **Conjugacy Test:** Let \mathbf{H} be an F -stable subgroup of \mathbf{G} and $u, u_1 \in \mathbf{H}^F$ unipotent elements. Let $\mathcal{H} := \{\mathcal{A}_1, \dots, \mathcal{A}_l\}$ be an integral parametrization of \mathbf{H} , with parameter set P_i associated to the parametrized element \mathcal{A}_i , and let u, u_1 be given as integral parametrized elements. The aim is to test whether u and u_1 are conjugate in \mathbf{H}^F . For this, define

$$C_i := \{t \in P_i \mid \mathcal{A}_i(t)u = u_1\mathcal{A}_i(t)\}$$

for $i \in \{1, \dots, l\}$. Then the set of elements of \mathbf{H} conjugating u to u_1 is given by

$$Z := \bigcup_{i \in \{1, \dots, l\}} \mathcal{A}_i(C_i) \subseteq \mathbf{H}$$

and the set of such elements lying in \mathbf{H}^F is given by

$$Z^F := \bigcup_{i \in \{1, \dots, l\}} \mathcal{A}_i(C_i^F) \subseteq \mathbf{H}^F.$$

Thus it remains to compute the sets C_i . For this, compute the normal form of $\mathcal{A}_i \cdot u$ and $u_1 \cdot \mathcal{A}_i$ via Algorithm 2. By the uniqueness of the Bruhat normal form, this leads to a system of equations with rational functions for the parameters $t \in P_i$ lying in C_i . Multiplying these equations by the denominators leads to a system of polynomial equations that have to be solved by hand again.

Then u and u_1 are not conjugate in \mathbf{H}^F if and only if $Z^F = \emptyset$; if $u = u_1$, Z resp. Z^F is the centralizer of u in \mathbf{H} resp. \mathbf{H}^F and $|Z^F|$ is the cardinality of this centralizer.

- **Representatives for Conjugacy Classes of \mathbf{U}^F :** To determine the conjugacy classes of a subgroup of \mathbf{U}^F , Köhler uses an algorithm to compute conjugacy classes in solvable groups presented

by Mecky and Neubüser in [MN, (3.1)].

Let G be a p -group and N an elementary abelian central normal subgroup of G . Moreover, let $h \in G$ and let C be the full preimage of $C_{G/N}(hN)$, the centralizer of hN in G/N . Then C acts on hN and a system of representatives R for the orbits of this action is given as follows:

if $[h, C] \neq 1$ then $R = hK$, for K a complement of $[h, C]$ in N .

This can be used to determine the conjugacy classes of \mathbf{U}^F : Let $i \in \{1, \dots, h\}$ and assume that $\{h_1^{(i)}\mathbf{U}_i^F, \dots, h_k^{(i)}\mathbf{U}_i^F\}$ is a system of representatives for the conjugacy classes of $\mathbf{U}^F/\mathbf{U}_i^F$. Let \overline{C}_j be the centralizer of $h_j^{(i)}\mathbf{U}_i^F$ in $\mathbf{U}^F/\mathbf{U}_i^F$ and C_j the full preimage in $\mathbf{U}^F/\mathbf{U}_{i+1}^F$. Then C_j acts on $h_j^{(i)}\mathbf{U}_i^F$ and the algorithm of Mecky-Neubüser [MN] described above can be applied. The union of these representatives of the conjugacy classes for all $h_j^{(i)}$, $1 \leq j \leq k$ gives a set of representatives for the conjugacy classes of $\mathbf{U}^F/\mathbf{U}_{i+1}^F$. Finally, this gives a system of representatives for the conjugacy classes of \mathbf{U}^F .

- **Fusions of Conjugacy Classes:** Let $\mathbf{H}_1 \leq \mathbf{H}_2$ be subgroups of \mathbf{G} and let the unipotent conjugacy classes of $\mathbf{H}_1^F, \mathbf{H}_2^F$ be known. To determine the fusions of the unipotent classes of \mathbf{H}_1^F into \mathbf{H}_2^F , it suffices to conduct conjugacy tests as described above.

5 Computations in the Groups $F_4(q)$

Finally, consider the finite group $F_4(q)$. Recall that so far we had considered computations in the algebraic group $F_4(k)$ for k an algebraic closure of \mathbb{F}_q .

For k an algebraic closure of \mathbb{F}_q , Chevalley gives an embedding of $F_4(k)$ into $GL_{52}(k)$, see [Chevalley55, (§2,§3)] and [Chevalley58, (exposée 3)]. such that $F_4(k)$ is defined over \mathbb{F}_q (see Lemma 2.3) and one can let the corresponding Frobenius morphism $F : F_4(k) \rightarrow F_4(k)$ act on $F_4(k)$ and consider the finite fixed-point subgroup $F_4(k)^F := \{g \in F_4(k) \mid F(g) = g\}$. This is called the **Chevalley group** of type F_4 and is denoted by $F_4(q)$.

In Section 2, the torus \mathbf{T} of \mathbf{G} and also $\mathbf{B}, \mathbf{U}, N_{\mathbf{G}}(\mathbf{T})$ can be chosen to be F -stable (i.e. \mathbf{T}^F is a torus in \mathbf{G}^F , and analogously for $\mathbf{B}^F, \mathbf{U}^F, N_{\mathbf{G}}(\mathbf{T})^F$), and in particular $W \cong N_{\mathbf{G}}(\mathbf{T})^F/\mathbf{T}^F$, see [Steinberg1, 11.6] and [Carter85, (1.17)].

Moreover, recall that \mathbf{G} is generated by $\{x_\alpha(t) \mid t \in k\} \cup \{h(\lambda) \in \mathbf{T} \mid \lambda \in (k^\times)^4\}$ (Lemma 2.5) and the Frobenius morphism $F : \mathbf{G} \rightarrow \mathbf{G}$ acts on the generators as follows (see [Steinberg2, Lemma 34]):

$$F(x_\alpha(t)) = x_\alpha(t^q), F(h(\lambda_1, \lambda_2, \lambda_3, \lambda_4)) = h(\lambda_1^q, \lambda_2^q, \lambda_3^q, \lambda_4^q).$$

Thus $\mathbf{G}^F = F_4(q)$ is generated by $\{x_\alpha(t) \mid t \in \mathbb{F}_q\} \cup \{h(\lambda) \mid \lambda \in (\mathbb{F}_q^\times)^4\}$ and the relations are as in $F_4(k)$. Recall that $W \cong N_{\mathbf{G}}(\mathbf{T})^F/\mathbf{T}^F$. For each $w \in W$, choose a preimage $\widehat{w} \in N_{\mathbf{G}^F}(\mathbf{T}^F)$. Then by Theorem 2.11 there is a Bruhat decomposition $\mathbf{G}^F = \prod_{w \in W} \mathbf{B}^F \widehat{w} \mathbf{B}^F$, and all elements of a Bruhat cell can be written uniquely as $u' \widehat{w} h u$ with $u' \in \mathbf{U}_w^F = \langle X_\alpha \mid \alpha w \in \Phi^- \rangle^F$, $h \in \mathbf{T}^F$, $u \in \mathbf{U}^F$. Thus elements of \mathbf{G}^F can be written uniquely as $u' \widehat{w} h u$ with $u' \in \mathbf{U}_w^F$, $w \in W$, $h \in \mathbf{T}^F$, $u \in \mathbf{U}^F$.

Since elements of the torus \mathbf{T}^F of the finite group \mathbf{G}^F can be parametrized by tuples $\lambda \in (\mathbb{F}_q^\times)^4$ and elements of \mathbf{U}^F can be written uniquely as $u = x_{i_1}(t_1) \dots x_{i_{24}}(t_{24})$ with $i_1 < \dots < i_{24}$ and $t_i \in \mathbb{F}_q$ (see Lemma 2.8) and \mathbf{U}_w^F is a subgroup of \mathbf{U}^F , it follows that elements of $F_4(q)$ can be parametrized uniquely by tuples $(\mathbb{F}_q)^{|R(w)|} \times \{w\} \times (\mathbb{F}_q^\times)^4 \times (\mathbb{F}_q)^{24}$ for $w \in W$. Recall that $w \in W$ can be viewed as permutation of the roots. Observe that the Weyl group W can be viewed as a subgroup of $F_4(q)$ parametrized by tuples $(0)^{|R(w)|} \times \{w\} \times (1)^4 \times (0)^{24}$ for $w \in W$.

By Remark 5, Algorithm 1 describes the computations in $F_4(q)$. The data structures to handle elements

of the Chevalley group will be the same as those presented for the reductive group in Remark 2. Moreover, generic computations as described in Remark 10 can be conducted and Algorithm 2 is applicable.

Part II

Characters of U in Characteristic 0

From now on, write U for the fixed unipotent subgroup U^F of $F_4(q)$ and also write G for $\mathbf{G}^F = F_4(q)$. In the following we shall present two methods for finding characters of the unipotent subgroup U in characteristic 0. The aim will be to induce characters of U to the entire group G and decompose these induced characters into irreducible ones. For this, first of all we recall some basic notions of (ordinary) representation theory of finite groups.

6 Some Background on Representations and Characters of Finite Groups

For now, let G be an arbitrary finite group and H a subgroup of G .

Definition 6.1. [Isaacs, (2.1)] A **K -representation** of G is a group homomorphism $R : G \rightarrow GL_n(K)$ for some integer $n \in \mathbb{N}$ and a field K . Call n the **degree** of the representation.

Instead of the group G itself, one can also study the group algebra KG whose elements are of the form $\sum_{g \in G} a_g g$ with $a_g \in K$ for all $g \in G$ and with multiplication defined by linear extension of the multiplication in the group itself and addition componentwise. Then G can be viewed as a basis of this algebra (by identifying $g \in G$ with $1g \in KG$) and any representation of G can also be viewed as an algebra representation of KG by linear extension, i.e. an algebra homomorphism $KG \rightarrow \text{End}_K(K^n)$. Since G can be viewed as lying in the group algebra by identifying $g \in G$ with $1g \in KG$ and elements of G are invertible in KG under this identification, an algebra homomorphism $R : KG \rightarrow \text{End}_K(V)$ with V a K -vectorspace can be restricted to G and the restriction will again be a group homomorphism $R|_G : G \rightarrow GL_K(V) \cong GL_n(K)$ if the dimension of V is n . Given a representation $R : KG \rightarrow \text{End}_K(V)$, define a KG -module structure on V by $g.v := R(g)(v)$ for $g \in KG, v \in V$. Vice versa, given a KG -module V , define a representation $R : KG \rightarrow \text{End}_K(V)$ by $R(g)(v) := g.v$ for $g \in G, v \in V$ and restrict it to G to obtain a representation of the group G .

This defines a bijection between the set of (finite dimensional) representations of G and the set of (finite dimensional) KG -modules. In some cases, it might be more useful to view a representation as a map from G into a matrix group, and in other cases, one might prefer to view a representation as a KG -module and use what is known on module theory in general (see also [Isaacs] for an introduction to representation and character theory).

Definition 6.2. [Isaacs, Chapter 1]

- A representation $R : G \rightarrow GL_n(K)$ of the group G over the field K is called **indecomposable** if, viewed as a module over the group algebra KG , it cannot be decomposed further into a direct sum of non-zero submodules.
- A module is called **irreducible** or **simple** if the only submodules are 0 and the module itself and the module is itself not 0. In particular every simple module is indecomposable ([Isaacs, (1.4)]).

In the case the ground field K is the field of complex numbers \mathbb{C} , any indecomposable module is simple. So in particular any representation can be decomposed into a direct sum of simple modules. More pre-

cisely:

Theorem 6.3. *Maschke's Theorem [Navarro, Theorem 1.21]: If G is a finite group and K is a field such that $\text{char}(K)$ does not divide $|G|$, then KG is semisimple.*

From now on, let $K = \mathbb{C}$ be the field of complex numbers.

Definition 6.4. [Isaacs]

- Let $R : G \rightarrow GL_n(\mathbb{C})$ be a \mathbb{C} -representation of G . Then the **\mathbb{C} -character** χ of G afforded by R is the function given by $\chi(g) = \text{Tr}(R(g))$ for $g \in G$ ([Isaacs, (2.2)]). We will also write character respectively representation instead of \mathbb{C} -character respectively \mathbb{C} -representation. Call $\chi(1) = n$ the **degree** of the character.
- Call the character χ afforded by the representation $R : G \rightarrow GL_n(\mathbb{C})$ **irreducible** if the representation R is irreducible. Denote the set of irreducible characters of G by $\text{Irr}(G)$. This is well defined as two representations R, R' affording the same character χ are similar, in particular they are either both irreducible or both not irreducible ([Isaacs, (2.9)]).
- Call a character of degree one **linear** and denote the **trivial** or **principal** character of G by 1_G (i.e. the character defined by $1_G(g) := 1$ for all $g \in G$).
- Denote by $\ker(\chi) = \{g \in G \mid \chi(g) = \chi(1)\}$ the **kernel** of a character $\chi \in \text{Irr}(G)$ ([Isaacs, (2.26)]).

Lemma 6.5. [Isaacs, (2.3),(2.6),(2.7)]

- Let $\chi : G \rightarrow \mathbb{C}$ be a character of the group G and let $g_1, g_2, h \in G$ be elements of G such that $g_2 = h^{-1}g_1h$. Then $\chi(g_1) = \chi(g_2)$. In particular, χ is constant on conjugacy classes of G .
- The number of irreducible characters of G is equal to the number of conjugacy classes of G .
- The group G is abelian if and only if every irreducible character of G is linear.

Lemma 6.6. [Isaacs, (2.16)] There is an inner product on the \mathbb{Z} -span of the set of characters of G defined by $(\chi, \phi) := \frac{1}{|G|} \sum_{g \in G} \chi(g)\phi(g^{-1})$ for two (generalized) characters ϕ, χ of G .

Theorem 6.7. [Isaacs, (2.14)] Let G be a finite group and $\chi, \phi \in \text{Irr}(G)$. Then

$$(\chi, \phi) = \frac{1}{|G|} \sum_{g \in G} \chi(g)\phi(g^{-1}) = \delta_{\chi, \phi}.$$

Definition 6.8. • Given an arbitrary subgroup H of G and a character $\phi \in \text{Irr}(H)$, the **induced character** $\text{Ind}_H^G(\phi)$ of G is defined by

$$\text{Ind}_H^G \phi(x) := \frac{1}{|H|} \sum_{g \in G} \hat{\phi}(x^g)$$

for $x \in G$, where $\hat{\phi}(x) := \phi(x)$ for $x \in H$ and zero elsewhere. Write $\phi^G := \text{Ind}_H^G \phi$ as a shorthand for $\text{Ind}_H^G \phi$ for a $\phi \in \text{Irr}(H)$ ([Isaacs, (5.1)]).

- For $\phi \in \text{Irr}(H)$, define $\text{Irr}(G \mid \phi) := \{\chi \in \text{Irr}(G) \mid (\chi, \phi^G) \neq 0\}$ ([GLMP, Chapter 2]).

Definition 6.9. For a normal subgroup N of G , there is an **inflation map** from $\text{Irr}(G/N)$ to $\text{Irr}(G)$ taking $\chi \in \text{Irr}(G/N)$ to $\tilde{\chi} := \text{Inf}_{G/N}^G \chi \in \text{Irr}(G)$ with $\tilde{\chi}(g) := \chi(gN)$ for $g \in G$ ([Isaacs, (2.22)]).

Given $g \in G, x \in N$ and $\psi \in \text{Irr}(N)$, write x^g for $g^{-1}xg$ and ${}^g\psi : N \rightarrow \mathbb{C}$ for the character of N defined by ${}^g\psi(x) = \psi(x^g)$ ([Isaacs, (Chapter 6)]).

Remark 11. [GLMP, Chapter 2]

- The following elementary and well known commutativity property between induction and inflation holds:

For $N \leq H \leq G$ with $N \trianglelefteq G$ and $\psi \in \text{Irr}(H/N)$,

$$\text{Inf}_{G/N}^G \text{Ind}_{H/N}^{G/N} \psi = \text{Ind}_H^G \text{Inf}_{H/N}^H \psi.$$

- Another elementary result will be used later on as well: Let Z_1 and Z_2 be subgroups of $Z(G)$ such that $Z_1 \cap Z_2 = 1$ is the trivial subgroup. Thus Z_1 can be identified with a subgroup of G/Z_2 . Now let $\lambda \in \text{Irr}(Z_1)$ and let $\tilde{\lambda}$ denote its inflation to $Z_1 Z_2$. Then it is straightforward to show that there is a bijection $\text{Irr}(G \mid \tilde{\lambda}) \leftrightarrow \text{Irr}(G/Z_2 \mid \lambda)$.

7 Conjugacy Classes in U

From now on, let G denote the Chevalley group $F_4(q)$ as defined in Section 5 and U the unipotent subgroup of G for the fixed maximal torus T and the fixed Borel subgroup B of G .

In order to find representations of the unipotent subgroup, first of all the (parametrized) conjugacy classes of elements of U need to be determined (recall that we consider parametrized elements for the groups $F_4(q)$ in order to do generic computations for all $q = 2^n$ simultaneously, see Definition 4.5).

With the help of the **GAP**-package **Unipot** (see [Haller]), the commutators of all elements $x_i(v)$, $v \in \mathbb{F}_q$, $i \in \{1, \dots, 24\}$ with one another are determined. They are included in Appendix A.3.

Now the (parametrized) conjugacy classes are determined inductively, similar to the approach presented in [Köhler] (see Remark 10).

Let U_i denote the normal subgroup of U given as $U_i := \langle x_j(u) \mid u \in \mathbb{F}_q, ht(\alpha_j) \geq i \rangle$ (U_i is indeed normal, since $[x_\beta(t), x_\alpha(s)] = \prod_{i,j>0, i\alpha+j\beta \in \Phi} x_{i\alpha+j\beta}(s^i t^j)$ by Theorem 2.7) and $1 \triangleleft U_{11} \triangleleft \dots \triangleleft U_2 \triangleleft U_1 = U$ (see Remark 1).

The following is a pseudo-code form of the algorithm to determine the (parametrized) conjugacy classes of U :

Algorithm 3. Determine the Conjugacy Classes of U

VARIABLES:

- i keeps record of the iterations of the algorithm,
- \mathcal{C} is the set of (parametrized) conjugacy classes of U/U_{i-1} ,
- \mathcal{D} is the set of (parametrized) conjugacy classes of U/U_i .

INITIALIZATION:

- $i := 3$,
- $\mathcal{C} := \{ \{ x_1(u)x_2(v)x_3(w)x_4(t) \}_{u,v,w,t \in \mathbb{F}_q} \}$,
- $\mathcal{D} := \emptyset$.

ITERATION:

1. Set $u = \prod_{j, ht(\alpha_j)=i-1} x_j(t_j)$, $u' = \prod_{j, ht(\alpha_j)=i-1} x_j(t'_j)$ with t_j, t'_j indeterminates for each j such that $ht(\alpha_j) = i - 1$. Go to Step 2.

2. Pick $c \in \mathcal{C}$. Analyse the commutator relations in U/U_i for elements $x_\alpha(t), x_\beta(s), t, s$ parameters, such that there are $l, j \in \mathbb{N}$ with $l\alpha + j\beta \in \Phi$ and $ht(x_{l\alpha+j\beta}(t^l s^j)) = i - 1$ and find out for which restrictions on the parameters $\{t'_j \mid ht(\alpha_j) = i - 1\}$ the element $c \cdot u$ is conjugate to $c \cdot u'$ in U/U_i , i.e. do a conjugacy test for the elements $c \cdot u$ and $c \cdot u'$ (see Remark 10).

By this, obtain the (parametrized) conjugacy classes of cu in U/U_i and add them to \mathcal{D} .

Remove c from \mathcal{C} .

Go to Step 3.

3. If $\mathcal{C} = \emptyset$, go to Step 4. Else go to Step 2.

4. If $i = 12$, return \mathcal{D} and finish. Else $\mathcal{C} := \mathcal{D}$; $\mathcal{D} := \{\}$; and $i := i + 1$; Go to Step 1.

OUTPUT: \mathcal{D} , the set of (parametrized) conjugacy classes of U .

Let us illustrate the second step of one iteration of the algorithm in an example:

Example 2. $\{x_1(t)x_5(a) \mid a \in \mathbb{F}_q\}$ with $t \in \mathbb{F}_q^\times$ is a conjugacy class in U/U_3 . The preimage in U/U_4 is given by $\{x_1(t)x_5(a)x_8(b)x_9(c)x_{10}(d) \mid a, b, c, d \in \mathbb{F}_q\}$. In U/U_4 , $x_7(t)x_1(s) = x_1(s)x_7(t)x_8(ts)$ and $x_5(t)x_4(s) = x_4(s)x_5(t)x_8(ts)$ for $s, t \in \mathbb{F}_q$, and all other relations follow from these. This gives that the conjugacy classes of U/U_4 contained in the preimage of $\{x_1(t)x_5(a) \mid a \in \mathbb{F}_q\}$ under the projection $U/U_4 \rightarrow U/U_3$ are given by $\{x_1(t)x_5(a)x_8(b)x_9(s)x_{10}(t) \mid a, b \in \mathbb{F}_q\}_{s, t \in \mathbb{F}_q}$.

Theorem 7.1. *The conjugacy classes in U can be found in Appendix A.5. We have not included the conjugacy classes of U/U_i since they can be easily discerned from those of U .*

Proof: The proof is by induction on i , $2 \leq i \leq 11$:

The group U/U_2 is abelian, since for roots α, β of height $ht(\alpha) = ht(\beta) = 1$, the height of a root α_i such that $i\alpha + j\beta = \alpha_i$, $i, j > 0$ of an element $x_i(r)$, $r \in \mathbb{F}_q$ that occurs in the product $[x_\beta(t), x_\alpha(s)] = \prod_{i, j > 0, i\alpha + j\beta \in \Phi} x_{i\alpha + j\beta}(s^i t^j)$ is $ht(\alpha_i) = ht(i\alpha + j\beta) = i \cdot ht(\alpha) + j \cdot ht(\beta) = i + j \geq 2$. So all the commutators of elements of U vanish in U/U_2 . Therefore every (parametrized) conjugacy class in U/U_2 contains precisely one element.

Given the (parametrized) conjugacy classes of U/U_i , the (parametrized) conjugacy classes of U/U_{i+1} are determined as follows:

If two elements $u, u' \in U/U_{i+1}$ of U/U_{i+1} are conjugate in U/U_{i+1} , then their images under the projection map $U/U_{i+1} \rightarrow U/U_i$ must be conjugate in U/U_i as well.

On the other hand, let C be a (parametrized) conjugacy class of U/U_i . Then the preimage of C in U/U_{i+1} under the projection map $U/U_{i+1} \rightarrow U/U_i$ is given by $\{u \cdot \prod_{j, ht(\alpha_j)=i+1} x_j(t_j) \mid u \in C, t_j \in \mathbb{F}_q \text{ for } j \text{ s.t. } ht(\alpha_j) = i + 1\}$, where the factors in the product $\prod_{j, ht(\alpha_j)=i+1} x_j(t_j)$ are ordered according to the normal form presented in Definition 4.7. So it is clear that the preimage of a (parametrized) conjugacy class of U/U_i under the projection map $U/U_{i+1} \rightarrow U/U_i$ in U/U_{i+1} must be a union of (parametrized) conjugacy classes of U/U_{i+1} again. However, the union may be a proper one as $u \cdot \prod_{j, ht(\alpha_j)=i+1} x_j(t_j)$ for $u \in C$, $t_j \in \mathbb{F}_q$ for j such that $ht(\alpha_j) = i + 1$ need not be conjugate to u in U/U_{i+1} for every choice of the $t_j \in \mathbb{F}_q$, j such that $ht(\alpha_j) = i + 1$.

U_i/U_{i+1} is central in U/U_{i+1} (by Theorem 2.7), thus conjugation of an element u in U/U_{i+1} with elements of U commutes with the multiplication of u with elements of U_i/U_{i+1} . Therefore all conjugacy classes in the preimage of a fixed conjugacy class C of U/U_i in U/U_{i+1} under the projection $U/U_{i+1} \rightarrow U/U_i$ have the same number of elements.

In order to find the individual (parametrized) conjugacy classes of U/U_{i+1} in this union, let U act on the preimage of C in U/U_{i+1} under the projection map $U/U_{i+1} \rightarrow U/U_i$ for a conjugacy class C of U/U_i and do a conjugacy test (see Remark 10). That is, check all the commutators (see Appendix A.3) to see

whether and if so, which elements $u \cdot \prod_{j, ht(\alpha_j)=i+1} x_j(t_j)$ for $u \in C$, $t_j \in \mathbb{F}_q$, j such that $ht(\alpha_j) = i + 1$, are conjugate to one another. In some cases, this will require a case distinction. This way, obtain all the (parametrized) conjugacy classes C' of U/U_{i+1} whose image in U/U_i under the projection $U/U_{i+1} \rightarrow U/U_i$ is C . \square

In several cases, elements will be conjugate if a certain parameter is of a certain form. To make this more precise, consider the map $f' : \mathbb{F}_q \rightarrow \mathbb{F}_q$, $t \mapsto t^2 + t$. The kernel of this map is given by $\ker(f') = \{0, 1\}$, so in particular the map cannot be surjective. Let $\eta \in \mathbb{F}_q \setminus f'(\mathbb{F}_q)$ be such an element not in the image of f' . As $(m+n)^2 + m+n = m^2 + n^2 + m+n$ for $m, n \in \mathbb{F}_q$, f' is a group homomorphism in the additive group of \mathbb{F}_q and precisely half the elements in \mathbb{F}_q are in the image of f' . The other half of \mathbb{F}_q can be written as $m^2 + m + \eta$ for some $m \in \mathbb{F}_q$ since the sum of an element from the image of f' and η cannot be in the image of f' again, or η would be in the image as well. Similarly, consider the map $f'' : \mathbb{F}_q \rightarrow \mathbb{F}_q$, $t \mapsto t^3 + t$. Again f'' is not surjective as $f''(1) = f''(0) = 0$ and f'' is a map between finite sets. Now, in some cases elements of U will be conjugate in U if a certain parameter is either in the image of this f' respectively f'' for all elements involved, or is not in the image for all elements of U involved. I.e. one conjugacy class of U will consist of elements of the form $ux_i(t)$, $u \in \mathfrak{U}$, $t \in \text{im}(f')$ resp. $t \in \text{im}(f'')$ and another conjugacy class of U will consist of the elements of the form $ux_i(t)$, $u \in \mathfrak{U}$, $t \in \mathbb{F}_q \setminus \text{im}(f')$ resp. $t \in \mathbb{F}_q \setminus \text{im}(f'')$ for suitable parametrized elements $\mathfrak{U} \subset U \setminus \{x_i(t') \mid t' \in \mathbb{F}_q\}$.

The parameter set for a parametrized conjugacy class of U will generally be of the form $V(I) \cap D_f$ as described in Remark 10. When inducing characters of U to G later on, one major obstacle was to determine the cardinality of $V(I) \cap D_f$. In particular, we did not find a way to have this done automatically. Instead we calculated the values by hand.

8 Irreducible Representations of U

To find (parametrized) irreducible representations of U , it is sometimes easier to find (parametrized) irreducible representations of U/U_i for some $2 \leq i \leq 11$ instead and then inflate these to U , where they stay irreducible. Recall that the U_i are normal in U . Since U/U_2 is abelian (as shown in the proof of Theorem 7.1), all (parametrized) irreducible representations of U/U_2 are linear and it is thus comparatively easy to find all these irreducible representations. To find additional (parametrized) irreducible representations of U/U_3 , we let **GAP** [GAP04] calculate the character table of U/U_3 for small q . Then we try to generalize the results for arbitrary q and attempt to find representations matching the character table. Sadly, for larger q the dimensions of the representations quickly grow too large to have the character table calculated. Thus it can sometimes be difficult to generalize representations to arbitrary q if they are only known for very small q (e.g. $q = 2$).

In particular, a (parametrized) representation may perhaps turn up in $q - 2$ varieties (i.e. with $q - 2$ different possibilities for a parameter), thus not turning up at all for $q = 2$. Only by adding up the total number of representations found one will realize that these are less than there are conjugacy classes.

Moreover, U/U_i will eventually grow too large to let **GAP** calculate the character table on our computer at all for any q . This was the case for U/U_6 . In order to let **GAP** calculate the character table of some U/U_i , we wrote a **GAP**-program to implement the group U/U_i in **GAP** (see Appendix A.13 for the code). This program realizes U/V for $V \trianglelefteq U$ as a quotient of the free group on $24q$ elements, where the $x_j(t) \in V$ are factored out, as are identities due to the relations among the remaining x_j .

Of course one can also endeavour to find representations of subgroups of U and then induce these to U .

However, this approach will in general not give irreducible representations of U even if the representation of the subgroup was irreducible. Instead the resulting representations of U would need to be decomposed into irreducible representations.

The results are included in Appendix A.6, as the characters could also be described using the notion of antichains and the reduction method presented in the next section.

9 Reduction Method

Another method to find characters of the unipotent subgroup U is based on an algorithm presented in [GLMP]. The algorithm presented is for primes p not very bad (i.e. for $p \neq 2$). However, in our case, it can be adapted for the prime $p = 2$.

The idea is to find (linear) characters of (abelian) subquotients of U and lift these to U via a series of inflations and inductions. In some cases it will not be possible to deal only with abelian subquotients. These cases will have to be studied separately and characters will have to be found by hand. However, in many cases the final subquotients will be abelian and thus the corresponding characters will be linear and very easy to find.

The key result for this method is the following lemma proved in [HLM, Lemma 2.1], which will be referred to as the reduction lemma.

Lemma 9.1. (*Reduction Lemma*) *Let G be a finite group, let $H \leq G$ and let X be a transversal of H in G . Let Y and Z be subgroups of H , and $\lambda \in \text{Irr}(Z)$. Suppose that*

1. $Z \subseteq Z(G)$,
2. $Y \trianglelefteq H$,
3. $Z \cap Y = 1$,
4. $ZY \trianglelefteq G$,
5. *for the inflation $\tilde{\lambda} \in \text{Irr}(ZY)$ of λ , we have that $x\tilde{\lambda} \neq y\tilde{\lambda}$ for all $x, y \in X$ with $x \neq y$.*

Then we have a bijection

$$\text{Irr}(H/Y \mid \lambda) \rightarrow \text{Irr}(G \mid \lambda) \cap \text{Irr}(G \mid 1_Y), \chi \mapsto \text{Ind}_H^G \text{Inf}_{H/Y}^H \chi.$$

Moreover, if $|X| = |Y|$, then $\text{Irr}(G \mid \lambda) \cap \text{Irr}(G \mid 1_Y) = \text{Irr}(G \mid \lambda)$.

In the following we shall recall some notions and definitions as presented in [GLMP], which are needed for the algorithm to parametrize the irreducible characters of U . Afterwards, we shall present our modified version of the algorithm for the prime 2 for the Chevalley group $F_4(q)$.

For p a prime and $q = p^n$ for some $n \in \mathbb{Z}_{\geq 1}$, denote by $\text{Tr} : \mathbb{F}_q \rightarrow \mathbb{F}_p$ the trace map, and define $\rho : \mathbb{F}_q \rightarrow \mathbb{C}^\times$, $x \mapsto e^{\frac{2\pi i \text{Tr}(x)}{p}}$. Then ρ is a nontrivial character from the additive group of \mathbb{F}_q into the multiplicative group \mathbb{C}^\times . Note that $\ker(\rho) = \ker(\text{Tr})$. For $a \in \mathbb{F}_q$ define $\rho_a \in \text{Irr}(\mathbb{F}_q)$ by $\rho_a(t) = \rho(at)$; then $\text{Irr}(\mathbb{F}_q^+) = \{\rho_a \mid a \in \mathbb{F}_q\}$.

Lemma 9.2. ([GLMP, Lemma 2.2]) *For a fixed $a \in \mathbb{F}_q^\times$, let $\mathbb{T}_a = \{t^p - a^{p-1}t \mid t \in \mathbb{F}_q\}$. Then $a^{-p}\mathbb{T}_a = \ker(\text{Tr})$.*

9.1 Quattern groups

The algorithm to determine the irreducible characters of U uses certain subquotients of U , which shall be referred to as quattern groups.

A subset $P \in \Phi^+$ is **closed** (or a **pattern**) if $\alpha + \beta \in P$ for all $\alpha, \beta \in P$ s.t. $\alpha + \beta \in \Phi^+$. For a closed subset P of Φ^+ , a subset $K \subseteq P$ is said to be **normal** in P (write $K \trianglelefteq P$) if $\delta + \alpha \in K$ for all $\delta \in K$ and $\alpha \in P$ such that $\delta + \alpha \in \Phi^+$. Finally, a subset $S \in \Phi^+$ is called a **quattern** if $S = P \setminus K$, where P is closed and K normal in P . Define $X_P = \prod_{\alpha \in P} X_\alpha$. It follows from the commutator relations that X_P is a subgroup of U ; a subgroup of this form will be called a **pattern group**. A further consequence of the commutator relations is that X_K is a normal subgroup of X_P , so one can define

$$X_S = X_{P \setminus K} := X_P / X_K.$$

From the construction of X_S , it follows that the natural map (between sets) $\prod_{\alpha \in S} X_\alpha \rightarrow X_S$ is a bijection. A subquotient of U of the form X_S will be called a **quattern group**. Up to canonical isomorphism, X_S is independent of the choice of P, K such that $S = P \setminus K$, so the notation X_S is unambiguous. From now on, identify X_α with its image in X_S for $\alpha \in S$ and for a subset $A \subset S$, let X_A be the subgroup of X_S generated by the X_α , $\alpha \in A$.

So let $S \subseteq \Phi^+$ be a quattern and X_S the corresponding quattern group. Define

$$Z(S) := \{\gamma \in S \mid \gamma + \alpha \notin S \forall \alpha \in S\}$$

and

$$D(S) := \{\gamma \in Z(S) \mid \alpha + \beta \neq \gamma \forall \alpha, \beta \in S\}.$$

Using the commutator relations it can be shown that

$$Z(X_S) \supseteq X_{Z(S)},$$

however, unlike for the case that p is not very bad presented in [GLMP], equality does not always hold for $p = 2$: Let for example $S = \{\alpha, \beta, \alpha + \beta, 2\alpha + \beta\}$ where α is short and β is long. Then $Z(S) = \{2\alpha + \beta\}$ but $Z(X_S) = \{x_{\alpha+\beta}(t)x_{2\alpha+\beta}(s) \mid t, s \in \mathbb{F}_q\}$ by the commutator relations (see Appendix A.3). But

$$X_S \cong X_{S \setminus D(S)} \times X_{D(S)}$$

holds more generally for all p (indeed the elements of the root subgroups X_α of roots $\alpha \in D(S)$ are precisely those that do not occur in any commutator of elements of $X_{S \setminus \{\alpha\}}$ (see the commutator relations Appendix A.3)). Observe that $D(S)$ and $S \setminus D(S)$ are again quatterns (If $S = P \setminus K$, then $K \cup D(S)$ and $K \cup S \setminus D(S)$ are both normal in P as well).

Let $Z \subseteq \{\alpha \in \Phi^+ \mid X_\alpha \subseteq Z(X_S)\}$ and define

$$\text{Irr}(X_S)_Z := \{\chi \in \text{Irr}(X_S) \mid X_\alpha \not\subseteq \ker(\chi) \forall \alpha \in Z\}.$$

This is a slight modification of the definition given in [GLMP, Chapter 2] in so far as they only allow $Z \subseteq Z(S)$. These sets of irreducible characters will be essential for the algorithm presented in the next section.

Recall that a subset Σ of Φ^+ is called an **antichain** if for all $\alpha, \beta \in \Sigma$, $\alpha \not\prec \beta$ and $\beta \not\prec \alpha$ in the partial order given by

$$\gamma < \delta \Leftrightarrow \exists \alpha_i \in \Phi^+ : \delta = \gamma + \sum_i \alpha_i,$$

i.e. α and β are incomparable in this partial order.

For an antichain $\Sigma \subset \Phi^+$, the set $K_\Sigma = \{\beta \in \Phi^+ \mid \beta \not\prec \gamma \forall \gamma \in \Sigma\}$ is a normal subset of Φ^+ . Now, given a

normal subset K of Φ^+ , the set Σ_K of maximal elements of $\Phi^+ \setminus K$ certainly is an antichain (in general every set of maximal elements of $\Phi^+ \setminus L$ for $L \subset \Phi^+$ is an antichain, but for each such antichain, there also exists a normal set K giving the same antichain). This gives a bijective correspondence between antichains in Φ^+ and normal subsets of Φ^+ . The previous assertions are standard properties of posets and can for example be found in [CP, Section 4].

For a given antichain Σ , define the quattern $S_\Sigma = \Phi^+ \setminus K_\Sigma$; as an easy consequence of the definitions we have $Z(S_\Sigma) = \Sigma$ ($Z(S_\Sigma)$ are precisely the maximal elements of S_Σ , which by definition of S_Σ are precisely the elements in Σ).

Let $\chi \in \text{Irr}(U)$ and define

$$R(\chi) = \{\alpha \in \Phi^+ \mid X_\alpha \subseteq \ker(\chi), \nexists \beta \in \Phi^+, \beta \geq \alpha \text{ s.t. } X_\beta \not\subseteq \ker(\chi)\}. \quad (2)$$

Note that the definition of $R(\chi)$ differs from the definition in [GLMP, Chapter 2]. This is necessary to take into account that for p a very bad prime, $Z(X_S) \supseteq X_{Z(S)}$, and still retain that $R(\chi)$ is a normal subset of Φ^+ . Using the commutator relations, one can easily see that the set thus defined is normal in Φ^+ . So $\Sigma_{R(\chi)}$ is an antichain in Φ^+ . Now, for an antichain $\Sigma \in \Phi^+$, define $\text{Irr}(U)_\Sigma = \{\chi \in \text{Irr}(U) \mid \Sigma_{R(\chi)} = \Sigma\}$. This gives a partition

$$\text{Irr}(U) = \bigsqcup_{\Sigma} \text{Irr}(U)_\Sigma,$$

where the union is taken over all antichains in Φ^+ . For the case that p is not a very bad prime, [GLMP] have that any character in $\text{Irr}(U)_\Sigma$ is the inflation of an irreducible character in $\text{Irr}(X_{S_\Sigma})_\Sigma$. With the adapted normal sets $R(\chi)$, this holds for the prime $p = 2$ as well:

Theorem 9.3. *Let U be the unipotent subgroup of the Chevalley group $F_4(q)$ for a fixed maximal torus T and $q = 2^n$ for a natural number $n \in \mathbb{N}$. Then any character in $\text{Irr}(U)_\Sigma$ is the inflation of an irreducible character in $\text{Irr}(X_{S_\Sigma})_\Sigma$.*

Proof: As $\Sigma_{R(\chi)}$ is defined as the maximal elements of $\Phi^+ \setminus R(\chi)$ and so $S_{\Sigma_{R(\chi)}} = \Phi^+ \setminus R(\chi)$, $\text{Irr}(U)_\Sigma$ are precisely the characters of U that are obtained from inflating an irreducible character of $\text{Irr}(X_{S_\Sigma})_\Sigma$. \square

[GLMP] introduce some more notation for inflation and induction of characters, which we shall use too: Let $S' = P' \setminus K'$ and $S = P \setminus K$ be quatterns, and let ψ be a character of $X_{S'}$. If $P' = P$ and $K' \supseteq K$, then let $L = K' \setminus K$ and write $\text{Inf}_L(\psi) := \text{Inf}_{X_{S'}}^{X_S} \psi$; in case $L = \{\alpha\}$ has one element only, write $\text{Inf}_\alpha \psi := \text{Inf}_L \psi$. Similarly, if $P' \subseteq P$ and $K' = K$, let $T = P \setminus P'$ and write $\text{Ind}^T \psi$ for $\text{Ind}_{X_{S'}}^{X_S} \psi$; for $T = \{\alpha\}$, write $\text{Ind}^\alpha \psi := \text{Ind}^T \psi$.

9.2 The Algorithm

The following two lemmata are the basis of the reductions performed in the algorithm. The first lemma is a more specific version of Lemma 9.1. It is not exactly the version stated in [GLMP, Lemma 3.1] since we had to adapt it for the prime 2. The following version is specified to work for $F_4(2^n)$.

Lemma 9.4. *Let $S = P \setminus K$ be a quattern, let $Z \subseteq Z(S)$ and let $\gamma \in Z$. Suppose that there exist $\delta, \beta \in S \setminus \{\gamma\}$, with $\delta + \beta = \gamma$, such that for all $\alpha, \alpha' \in S$ we have $\alpha + \alpha' \neq \beta$, and that for all $\alpha \in S \setminus \{\beta\}$ we have $\delta + \alpha \notin S$ and if γ is a long root, then β and δ are long roots too. Let $P' = P \setminus \{\beta\}$ and $K' = K \cup \{\delta\}$. Then we have that $S' = P' \setminus K'$ is a quattern with $X_{S'} = X_{P'}/X_{K'}$, and we have a bijection*

$$\text{Irr}(X_{S'})_Z \rightarrow \text{Irr}(X_{S' \cup \{\delta\}})_Z \rightarrow \text{Irr}(X_S)_Z, \chi \mapsto \text{Ind}^\beta \text{Inf}_\delta \chi^{X_S}$$

by first inflating over X_δ and then inducing to X_S over X_β .

Proof: • S' is a quattern:

For this part of the proof, the argument is exactly as in [GLMP]. Let $\alpha, \alpha' \in P'$. If $\alpha \in K'$ or $\alpha' \in K'$, then it cannot be that $\alpha + \alpha' = \beta$, since in that case one gets $\beta \in K'$, a contradiction to $\beta \in S$. If $\alpha, \alpha' \in S'$, by assumption the equality $\alpha + \alpha' = \beta$ cannot hold either. Since $P' = S' \cup K'$, this proves that P' is closed. Let now $\alpha \in P'$ and $\alpha' \in K'$. If $\alpha' \in K$, then $\alpha + \alpha' \in K'$ whenever $\alpha + \alpha' \in \Phi^+$, since $K \trianglelefteq P$. Otherwise, $\alpha' = \delta$, and by assumption $\alpha + \delta \notin S$ since $\alpha \neq \beta$, therefore if $\alpha + \delta \in \Phi^+$ then $\alpha + \delta \in K'$. Thus $K' \trianglelefteq P'$ and $S' = P'/K'$ is a quattern.

• One can apply the Reduction Lemma 9.1:

As for the case of p not very bad in [GLMP], it is again immediate that conditions (1) – (4) of Lemma 9.1 hold with $G = X_S$, $Z = X_\gamma$, $H = X_{S \setminus \{\beta\}}$, $X = X_\beta$, and $Y = X_\delta$. To see that condition (5) holds as well, let $\lambda \in \text{Irr}(Z)$ and $\tilde{\lambda} = \text{Inf}_Z^{Y/Z} \lambda$. For $s_1, s_2 \in \mathbb{F}_q$, again $x_\beta(s_1) \tilde{\lambda} = x_\beta(s_2) \tilde{\lambda}$ if and only if $\lambda([x_\beta(s_1), x_\delta(t)]) = \lambda([x_\beta(s_2), x_\delta(t)])$ for all $t \in \mathbb{F}_q$. By the commutator relations (see Appendix A.3), condition (5) is thus satisfied. As $|X| = |Y| = q$, the lemma follows from the Reduction Lemma 9.1. \square

Condition (5) of the Reduction Lemma 9.1 is also where the restriction that β and δ are long if γ is, is needed to ensure that $[x_\delta(t), x_\beta(s)] \neq 0$ for $t, s \in \mathbb{F}_q^\times$:

Assume γ is long and one of β, δ is short and the other one is long. Then by the commutator relations (Appendix A.3), $x_\beta(t)$ and $x_\delta(s)$ commute for all $t, s \in \mathbb{F}_q$. Thus condition (5) of the Reduction Lemma 9.1 would not be satisfied.

The following example illustrates why we cannot replace the condition $Z \subseteq Z(S)$ by $Z \subseteq \{\alpha \in S \mid X_\alpha \in Z(X_S)\}$, i.e. why we cannot use all central elements of X_S for the reduction.

Example 3. First assume that $\gamma \in \{\alpha \in S \mid X_\alpha \in Z(X_S)\} \setminus Z(S)$. In this case γ is a short root and there are $\alpha_i, \alpha_j \in S$ such that α_i is a short root and α_j is a long root and such that $\gamma = \alpha_i + \alpha_j$ and $2\alpha_i + \alpha_j \in S$ as well. If one wants to apply Lemma 9.1 with $G = X_S$, $Z = X_\gamma$, $H = X_{S \setminus \{\beta\}}$, $X = X_\beta$ and $Y = X_\delta$ for β, δ chosen as α_i, α_j , then condition 4 of Lemma 9.1 does not hold since $(X_\gamma X_\delta)^{x_{\alpha_i}(t)} \subseteq X_\gamma X_\delta X_{\gamma + \alpha_i}$ for $t \in \mathbb{F}_q^\times$ and in particular ZY is not normal in G . If one wants to apply the Reduction Lemma 9.1 with $Z = X_\gamma X_{\gamma + \alpha_i}$ instead to fix condition (4), condition (5) of the reduction lemma will not necessarily hold anymore if $\lambda \in \text{Irr}(Z)$ acts nontrivially on both X_γ and $X_{\gamma + \alpha_i}$.

For example, let $q = 4$, $S = \{\alpha_i \mid i = 3, 4, 7, 10\}$, $\delta = \alpha_4$, $\gamma = \alpha_7$, $\beta = \alpha_3$; for $\mathbb{F}_4 \cong \mathbb{F}_2[T]/(T^2 + T + 1)$, let λ map $x_7(T), x_7(1+T), x_{10}(T), x_{10}(1+T)$ to -1 , and all other $x_7(t), x_{10}(t)$ to 1. Then $\tilde{\lambda}(x_4(t)^{x_3(1)}) = \tilde{\lambda}(x_4(t))$ for all t (as Z is abelian, λ is linear). So condition (5) of Lemma 9.1 is not satisfied.

The second lemma is an immediate consequence of the definitions and does not have to be adapted for the prime 2, so we state it as in [GLMP, Lemma 3.2].

Lemma 9.5. *Let S be a quattern and let $\alpha \in Z(S)$. Then there is a bijection $\text{Irr}(X_S) \rightarrow \text{Irr}(X_S)_{\{\alpha\}} \sqcup \text{Irr}(X_{S \setminus \{\alpha\}})$. Observe that $S \setminus \{\alpha\}$ is again a quattern as $\alpha \in Z(S)$ implies that $K \cup \{\alpha\}$ is normal if $S = \Phi^+ \setminus K$.*

The algorithm below is a modified version of the algorithm found in [GLMP, Algorithm 3.3]. It is used to calculate the elements of $\text{Irr}(U)_\Sigma$ that are an inflation of an element of $\text{Irr}(X_{S_\Sigma})_\Sigma$, for each antichain $\Sigma \subset \Phi^+$. We state the algorithm in the same form of pseudocode as in [GLMP], with comments in *italics*.

Algorithm 4. Algorithm to Construct Irreducible Characters of the Unipotent Subgroup U of $G = F_4(2^n)$

INPUT:

- $\Phi^+ = \{\alpha_1, \dots, \alpha_{24}\}$ the set of positive roots of the root system of type F_4 , with fixed enumeration such that $i \leq j$ whenever $\alpha_i \leq \alpha_j$.

- Σ , an antichain in Φ^+

VARIABLES:

- $S \subseteq \Phi^+$ is a quattern,
- $Z \subseteq Z(S)$,
- $A \subseteq \Phi^+$ keeps record of the roots β used in a TYPE R reduction,
- $L \subseteq \Phi^+$ keeps record of the roots δ used in a TYPE R reduction,
- $K \subseteq \Phi^+$ keeps record of the roots indexing root subgroups in the quotient of the associated quattern subgroup,
- \mathfrak{S} is a stack of tuples of the form (S, Z, A, L, K) as above to be considered later in the algorithm,
- $\mathfrak{D} = (\mathfrak{D}_1, \mathfrak{D}_2)$ is the output. It consists of so-called abelian and nonabelian cores. This will be discussed in more detail in Section 9.3.

INITIALIZATION:

- $K := K_\Sigma$,
- $S := \Phi^+ \setminus K_\Sigma$,
- $Z := \Sigma$,
- $A := \emptyset$,
- $L := \emptyset$,
- $\mathfrak{S} := \emptyset$,
- $\mathfrak{D} := (\emptyset, \emptyset)$.

During the algorithm one considers $\text{Irr}(X_S)_Z$; one goes into four subroutines called “ABELIAN CORE”, “TYPE R”, “TYPE S”, and “NONABELIAN CORE”.

ABELIAN CORE:

if $S = \{\alpha \in \Phi^+ \mid X_\alpha \in Z(X_S)\}$ **then**

Adjoin (S, Z, A, L, K) to \mathfrak{D}_1 .

In this case X_S is abelian and we can parametrize the characters in $\text{Irr}(X_S)_Z$

if $\mathfrak{S} = \emptyset$ **then**

Finish and **output** \mathfrak{D} .

In this case we have no more characters to consider, so we are done.

else

Remove the tuple at the top of the stack \mathfrak{S} and replace (S, Z, A, L, K) with it, and go to ABELIAN CORE.

end if

else

Go to TYPE R.

end if

TYPE R:

Look for pairs $(\beta, \delta) = (\alpha_i, \alpha_j)$ that satisfy the condition of Lemma 9.4 for some $\gamma \in Z$.

if such a pair (α_i, α_j) exists **then**

Choose the pair with j maximal, and update the variables as follows:

- $S := S \setminus \{\alpha_i, \alpha_j\}$,
- $A := A \cup \{\alpha_i\}$,
- $L := L \cup \{\alpha_j\}$,
- $K := K \cup \{\alpha_j\}$.

We are replacing S with S' as in Lemma 9.4, and recording this in A and L .

Go to ABELIAN CORE

else

Go to TYPE S

end if

TYPE S:

if $Z(S) \setminus (Z \cup D(Z)) \neq \emptyset$ **then**

Let i be maximal such that $\alpha_i \in Z(S) \setminus (Z \cup D(Z))$, and update as follows:

- $\mathfrak{S} := \mathfrak{S} \cup \{(S \setminus \{\alpha_i\}, Z, A, L, K \cup \{\alpha_i\})\}$,
- $Z := Z \cup \{\alpha_i\}$.

Here we are using Lemma 9.5. We first add $(S \setminus \{\alpha_i\}, Z)$ to the stack to be considered later, recording that X_{α_i} is in the kernel of these characters by adding α_i to K .

Then we replace (S, Z) with $(S, Z \cup \{\alpha_i\})$ for the current run.

Go to ABELIAN CORE.

else

Go to NONABELIAN CORE

end if

NONABELIAN CORE:

Adjoin (S, Z, A, L, K) to \mathfrak{D}_2 .

We are no longer able to apply reductions of TYPE R or of TYPE S, and X_S is not abelian, so the algorithm gives up, and this case is output as a nonabelian core as discussed further later.

if $\mathfrak{S} = \emptyset$ **then**

Finish and **output** \mathfrak{D} .

In this case we have no more characters to consider, so we are done.

else

Remove the tuple at the top of the stack \mathfrak{S} and replace (S, Z, A, L, K) with it, and go to ABELIAN CORE.

end if

Compared to the version of this algorithm found in [GLMP], the part ABELIAN CORE has been altered.

In the ABELIAN CORE subroutine, $Z(S)$ is replaced by the set of roots of the full center of S .

Example 4. Let $\Sigma = \{11\}$. Then $K_\Sigma = \{\alpha_{10}, \alpha_i, i \geq 12\}$ and $S_\Sigma = \{\alpha_i, i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 11\}$.

Initialize

- $K := K_\Sigma$,
- $S := \Phi^+ \setminus K_\Sigma$,
- $Z := \Sigma$,

- $A := \emptyset$,
- $L := \emptyset$,
- $\mathfrak{S} := \emptyset$,
- $\mathfrak{D} := (\emptyset, \emptyset)$.

Since X_S is not abelian, go to TYPE R reduction and look for a pair $\beta, \delta \in S$ such that $\alpha_{11} = \beta + \delta$, $\beta \neq \alpha + \alpha' \forall \alpha, \alpha' \in S$ and $\delta + \alpha \notin S \forall \alpha \in S \setminus \{\beta\}$.

Choose $\beta = \alpha_1$, $\delta = \alpha_9$ and apply the TYPE R reduction. This gives $K = \{\alpha_9, \alpha_{10}, \alpha_i, i \geq 12\}$, $S = \{\alpha_i, i = 2, 3, 4, 5, 6, 7, 8, 11\}$, $A = \{\alpha_i, i = 1\}$, $L = \{\alpha_i, i = 9\}$.

As X_S still is not abelian, go again to TYPE R reduction and look for a pair β, δ . Choose $\beta = \alpha_2$, $\delta = \alpha_8$ and apply the TYPE R reduction. This gives $K = \{\alpha_8, \alpha_9, \alpha_{10}, \alpha_i, i \geq 12\}$, $S = \{\alpha_i, i = 3, 4, 5, 6, 7, 11\}$, $A = \{\alpha_i, i = 1, 2\}$, $L = \{\alpha_i, i = 8, 9\}$.

Again, S is still not abelian, so go to TYPE R reduction and look for a pair β, δ . Choose $\beta = \alpha_5$, $\delta = \alpha_6$ and apply the TYPE R reduction. This gives $K = \{\alpha_6, \alpha_8, \alpha_9, \alpha_{10}, \alpha_i, i \geq 12\}$, $S = \{\alpha_i, i = 3, 4, 7, 11\}$, $A = \{\alpha_i, i = 1, 2, 5\}$, $L = \{\alpha_i, i = 6, 8, 9\}$.

Now X_S is not abelian, but neither is there a pair $\beta, \delta \in S$ as required for the TYPE R reduction. So go to TYPE S reduction and find $\alpha \in Z(S) = \{\alpha_7, \alpha_{11}\}$ such that $\alpha \notin Z = \{\alpha_{11}\}$ and update as follows:

- $\mathfrak{S} := \mathfrak{S} \cup \{(S \setminus \{\alpha_7\}, Z, A, L, K \cup \{\alpha_7\})\}$,
- $Z := Z \cup \{\alpha_7\}$.

Go to TYPE R reduction.

Choose $\beta = \alpha_3$, $\delta = \alpha_4$. This gives $K = \{\alpha_i, i = 4, 6, 8, 9, 10, \alpha_i, i \geq 12\}$, $S = \{\alpha_i, i = 7, 11\}$, $Z = \{\alpha_7, \alpha_{11}\}$, $A = \{\alpha_i, i = 1, 2, 3, 5\}$, $L = \{\alpha_i, i = 4, 6, 8, 9\}$.

Now X_S is abelian, so adjoin (S, Z, A, L, K) to \mathfrak{D}_1 and consider $(S = \{\alpha_i, i = 3, 4, 11\}, Z = \{\alpha_{11}\}, A = \{\alpha_i, i = 1, 2, 5\}, L = \{\alpha_i, i = 6, 8, 9\})$, $(K = \{\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_i, i \geq 12\})$ next.

As this X_S is abelian too, adjoin it to \mathfrak{D}_1 as well. With this, we are done.

9.3 Interpreting the Output of the Algorithm

The interpretation of the output of the algorithm is not affected by moving to prime 2 for $F_4(q)$ and is consequently as in [GLMP].

Definition 9.6. Suppose that Algorithm 4 has run with input (Φ^+, Σ) , resulting in the output \mathfrak{D} .

- An element (S, Z, A, L, K) of \mathfrak{D}_1 is called an **abelian core** for $\text{Irr}(U)_\Sigma$.
- An element (S, Z, A, L, K) of \mathfrak{D}_2 is called a **nonabelian core** for $\text{Irr}(U)_\Sigma$.

In the following, we shall illustrate how to determine the characters in $\text{Irr}(U)_\Sigma$ corresponding to a core $\mathfrak{C} = (S, Z, A, L, K) \in \mathfrak{D}_1 \cup \mathfrak{D}_2$. When $\mathfrak{C} \in \mathfrak{D}_1$ is an abelian core, we can give a complete description of the irreducible characters in the way described in [GLMP], however for nonabelian cores, more work and a case by case study is required.

[GLMP] introduce some notation to describe what happens in the algorithm, which we shall relate in the following.

One obtains \mathfrak{C} through a sequence of reductions of TYPE R and of TYPE S applied in Algorithm 4; here though it is only necessary to consider the TYPE S reduction in this sequence if a root γ is added to K (rather than to Z). So consider the sequence of reductions where in each one either:

- a pair of roots β, δ is taken from S in a TYPE R reduction, and β is added to A and δ is added to L and K ; or
- a root γ is taken from S and added to K .

Let $l = l_{\mathfrak{C}}$ be the number of these reductions, and define the sequence $T(\mathfrak{C}) = (t_1, \dots, t_l)$, where $t_i = R$ if the i th reduction is of TYPE R, and $t_i = S$ if it is of TYPE S. Let $I(R, \mathfrak{C})$ be the set of i such that $t_i = R$ and $I(S, \mathfrak{C})$ the set of i such that $t_i = S$. For $i \in I(R, \mathfrak{C})$ write (β_i, δ_i) for the pair of roots used in the type R reduction, and for $i \in I(S, \mathfrak{C})$, write γ_i for the root added to K in the TYPE S reduction. Thus $A = \{\beta_i \mid i \in I(R, \mathfrak{C})\}$, $L = \{\delta_i \mid i \in I(R, \mathfrak{C})\}$, and $K \setminus K_{\Sigma} = L \cup \{\gamma_i \mid i \in I(S, \mathfrak{C})\}$. Furthermore, [GLMP] define subsets P^0, P^1, \dots, P^l and K^0, K^1, \dots, K^l of Φ^+ recursively by

$$P^0 = \Phi^+ \qquad K^0 = K_{\Sigma}$$

$$P^i = \begin{cases} P^{i-1} \setminus \{\beta_i\} & \text{if } t_i = R \\ P^{i-1} & \text{if } t_i = S \end{cases} \qquad K^i = \begin{cases} K^{i-1} \cup \{\delta_i\} & \text{if } t_i = R \\ K^{i-1} \cup \{\gamma_i\} & \text{if } t_i = S \end{cases}$$

The following lemma [GLMP, Lemma 3.5] is used to define characters of $\text{Irr}(U)_{\Sigma}$ associated to the given core \mathfrak{C} .

Lemma 9.7. *For each $i, j = 1, \dots, l$ with $i \leq j$, we have that P^j is a closed set, and K^i is normal in P^j . In particular, the $S^{i,j} = P^j \setminus K^i$ are quaterns.*

Now [GLMP] let $\psi \in \text{Irr}(X_S)$ and define characters $\overline{\psi}_i \in \text{Irr}(X_{P^i \setminus K^i})$ for $i = l, l-1, \dots, 1, 0$ recursively by the following sequence of inflations and inductions:

$$\overline{\psi}_l = \psi$$

$$\overline{\psi}_{i-1} = \begin{cases} \text{Ind}^{\beta_i} \text{Inf}_{\delta_i} \overline{\psi}_i & \text{if } t_i = R, \\ \text{Inf}_{\gamma_i} \overline{\psi}_i & \text{if } t_i = S. \end{cases}$$

Set $\overline{\psi} = \text{Inf}_{K_{\Sigma}} \overline{\psi}_0 \in \text{Irr}(U)$.

Now suppose that $\mathfrak{C} = (S, Z, A, L, K) \in \mathfrak{D}_1$ is an abelian core. Let $Z = \{\alpha_{i_1}, \dots, \alpha_{i_m}\}$ and $S \setminus Z = \{\alpha_{j_1}, \dots, \alpha_{j_n}\}$. Then

$$\text{Irr}(X_S)_Z = \{\mu_{\underline{b}}^{\underline{a}} \mid \underline{a} = (a_{i_1}, \dots, a_{i_m}) \in (\mathbb{F}_q^{\times})^m, \underline{b} = (b_{j_1}, \dots, b_{j_n}) \in (\mathbb{F}_q)^n\},$$

where the linear characters $\mu_{\underline{b}}^{\underline{a}}$ are defined by

$$\mu_{\underline{b}}^{\underline{a}}(x_{\alpha_{i_k}}(t)) = \rho(a_{i_k} t) \text{ and } \mu_{\underline{b}}^{\underline{a}}(x_{\alpha_{j_k}}(t)) = \rho(b_{j_k} t).$$

Set $\chi_{\underline{b}}^{\underline{a}} = \overline{\mu_{\underline{b}}^{\underline{a}}}$ and

$$\text{Irr}(U)_{\mathfrak{C}} = \{\chi_{\underline{b}}^{\underline{a}} \mid \underline{a} = (a_{i_1}, \dots, a_{i_m}) \in (\mathbb{F}_q^{\times})^m, \underline{b} = (b_{j_1}, \dots, b_{j_n}) \in (\mathbb{F}_q)^n\}.$$

Through the bijections given by Lemmata 9.4 and 9.5, this is precisely the set of characters in $\text{Irr}(U)_{\Sigma}$ corresponding to \mathfrak{C} .

Now consider $\mathfrak{C} = (S, Z, A, L, K) \in \mathfrak{D}_2$ a nonabelian core. Since X_S is not abelian, a parametrization of $\text{Irr}(X_S)_Z$ has to be determined by hand. Supposing this has been done and

$$\text{Irr}(X_S)_Z = \{\psi_{\underline{c}} \mid \underline{c} \in \mathcal{J}_{\mathfrak{C}}\}$$

for some indexing set $J_{\mathfrak{C}}$, [GLMP] define $\chi_{\underline{c}} = \overline{\psi_{\underline{c}}}$ and

$$\text{Irr}(U)_{\mathfrak{C}} = \{\chi_{\underline{c}} \mid \underline{c} \in J_{\mathfrak{C}}\}.$$

Again, through the bijections given by Lemmata 9.4 and 9.5, this is precisely the set of characters in $\text{Irr}(U)_{\Sigma}$ corresponding to \mathfrak{C} .

The following two theorems from [GLMP, Theorem 3.6, Theorem 3.8] hold for $p = 2$ as well as they do not depend on the prime.

Theorem 9.8. *Suppose that Algorithm 4 has run with input (Φ^+, Σ) and given output $\mathfrak{D} = (\mathfrak{D}_1, \mathfrak{D}_2)$. Then we have*

$$\text{Irr}(U)_{\Sigma} = \bigsqcup_{\mathfrak{C} \in \mathfrak{D}_1} \text{Irr}(U)_{\mathfrak{C}} \sqcup \bigsqcup_{\mathfrak{C} \in \mathfrak{D}_2} \text{Irr}(U)_{\mathfrak{C}}.$$

Theorem 9.9. *1. Let $\mathfrak{C} = (S, Z, A, L, K) \in \mathfrak{D}_1$ be an abelian core, and let $\chi_{\underline{b}}^{\underline{a}} \in \text{Irr}(U)_{\mathfrak{C}}$ be defined as above. Then*

$$\chi_{\underline{b}}^{\underline{a}} = \text{Ind}^A \text{Inf}_K \mu_{\underline{b}}^{\underline{a}}.$$

In particular, $\chi_{\underline{b}}^{\underline{a}}$ is induced from a linear character of $X_{S \cup K}$.

2. Let $\mathfrak{C} = (S, Z, A, L, K) \in \mathfrak{D}_2$ be a nonabelian core, and let $\chi_{\underline{b}} \in \text{Irr}(U)_{\mathfrak{C}}$ be defined as above. Then

$$\chi_{\underline{c}} = \text{Ind}^A \text{Inf}_K \psi_{\underline{c}}.$$

Remark 12. As stated in [GLMP], the enumeration of Φ^+ has a significant effect on how the algorithm runs. Consequently, the resulting parametrization of $\text{Irr}(U)_{\Sigma}$ depends on this choice of enumeration.

Remark 13. As [GLMP] remark, they make a slight abuse in the notation $\chi_{\underline{b}}^{\underline{a}}$. In fact, each a_j and b_i is supposed to record not just a value in \mathbb{F}_q^{\times} and \mathbb{F}_q respectively, but also root indices i and j , so that $\chi_{\underline{b}}^{\underline{a}}$ should read $\chi_{((i_1, b_{i_1}), \dots, (i_r, b_{i_r}))}^{((j_1, a_{j_1}), \dots, (j_s, a_{j_s}))}$, for corresponding choices of i_1, \dots, i_r and j_1, \dots, j_s indexing positive roots.

Theorem 9.10. *The irreducible characters of the unipotent subgroup U of $F_4(q)$, $q = 2^n$ and $n \in \mathbb{N}$, belonging to all the abelian cores can be found in Appendix A.6. The nonabelian cores are listed in Appendix A.7.*

Example 5. Let again $\Sigma = \{11\}$.

- Consider the abelian core $(S = \{\alpha_i, i = 7, 11\}, Z = \{\alpha_7, \alpha_{11}\}, A = \{\alpha_i, i = 1, 2, 3, 5\}, L = \{\alpha_i, i = 4, 6, 8, 9\}, K = \{\alpha_i, i = 4, 6, 8, 9, 10, \alpha_i, i \geq 12\})$ from the previous example.

Then $\chi^{a_7, a_{11}} = \text{Ind}^A \text{Inf}_K \mu^{a_7, a_{11}}$ with $\mu^{a_7, a_{11}}$ a linear character of X_S defined as $\mu^{a_7, a_{11}}(x_{\alpha_7}(t)) = \rho(a_7 t)$ and $\mu^{a_7, a_{11}}(x_{\alpha_{11}}(t)) = \rho(a_{11} t)$ (and $\rho : \mathbb{F}_q \rightarrow \mathbb{C}^*$, $x \mapsto e^{\frac{2\pi i \text{Tr}(x)}{p}}$ where $\text{Tr} : \mathbb{F}_q \rightarrow \mathbb{F}_p$ is the trace map.). By applying inflation and induction, obtain

$$\chi^{a_7, a_{11}}(u) = \begin{cases} 0 & \text{if some of } t_i \neq 0, i \in \{1, 2, 3, 4, 5, 6\} \\ & \text{or if some of } t_8, t_9 \neq 0 \text{ and } t_7 = 0 \\ q^4 \cdot \rho(a_{11} t_{11}) & \text{if } t_i = 0 \forall i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ q^3 \cdot \rho(a_7 t_7) \rho(a_{11} t_{11}) & \text{if } t_i = 0 \forall i \in \{1, 2, 3, 4, 5, 6\} \text{ and } t_7 \neq 0 \end{cases}$$

for $u = x_1(t_1) \cdot \dots \cdot x_{24}(t_{24})$. These characters have degree q^4 .

- Next consider the abelian core ($S = \{\alpha_i, i = 3, 4, 11\}$, $Z = \{\alpha_{11}\}$, $A = \{\alpha_i, i = 1, 2, 5\}$, $L = \{\alpha_i, i = 6, 8, 9\}$), ($K = \{\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_i, i \geq 12\}$).
Then $\chi_{b_3, b_4}^{a_{11}} = \text{Ind}^A \text{Inf}_K \mu_{b_3, b_4}^{a_{11}}$ with $\mu_{b_3, b_4}^{a_{11}}$ a linear character of X_S defined as $\mu_{b_3, b_4}^{a_{11}}(x_{\alpha_i}(t)) = \rho(b_i t)$ for $i = 3, 4$ and $\mu_{b_3, b_4}^{a_{11}}(x_{\alpha_{11}}(t)) = \rho(a_{11} t)$. By applying inflation and induction, obtain

$$\chi_{b_3, b_4}^{a_{11}}(u) = \begin{cases} 0 & \begin{array}{l} \text{if some of } t_i \neq 0, i \in \{1, 2\} \\ \text{or if } t_5 \neq 0 \text{ and } t_3 = 0 \\ \text{or if } t_6 \neq 0 \text{ and } t_4 = 0 \\ \text{or if } t_8 \neq 0 \text{ and } t_4 = t_7 = 0 \\ \text{or if } t_9 \neq 0 \text{ and } t_3 = t_7 = 0 \end{array} \\ q^3 \cdot \rho(a_{11} t_{11}) & \text{if } t_i = 0 \forall i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ q^2 \cdot \rho(b_3 t_3) \rho(a_{11} t_{11}) & \begin{array}{l} \text{if } t_i = 0 \forall i \in \{1, 2, 4, 6, 8\} \\ \text{and } t_3 \neq 0 \end{array} \\ q^2 \cdot \rho(b_4 t_4) \rho(a_{11} t_{11}) & \begin{array}{l} \text{if } t_i = 0 \forall i \in \{1, 2, 3, 5, 9\} \\ \text{and } t_4 \neq 0 \end{array} \\ q^2 \cdot \rho(a_{11} t_{11}) & \begin{array}{l} \text{if } t_i = 0 \forall i \in \{1, 2, 3, 4, 5, 6\} \\ \text{and } t_7 \neq 0 \end{array} \\ q \cdot \rho(b_3 t_3) \rho(b_4 t_4) \rho(a_{11} t_{11}) & \text{if } t_i = 0 \forall i \in \{1, 2\} \text{ and } t_3, t_4 \neq 0 \end{cases}$$

for $u = x_1(t_1) \cdot \dots \cdot x_{24}(t_{24})$. These characters have degree q^3 .

So

$$\text{Irr}(U)_{\alpha_{11}} = \{\chi^{a_7, a_{11}} | a_7, a_{11} \in \mathbb{F}_q^\times\} \sqcup \{\chi_{b_3, b_4}^{a_{11}} | a_{11} \in \mathbb{F}_q^\times, b_3, b_4 \in \mathbb{F}_q\}$$

and $\text{Irr}(U)_{\alpha_{11}}$ consists of $(q-1)^2$ characters of degree q^4 and $q^2(q-1)$ characters of degree q^3 .

Depending on the character, even for abelian cores determining the values of the induction to U can be not entirely trivial and may require a considerable amount of work. This is one reason why in this thesis the inductions of characters of U to G have not been calculated even for all the abelian cores. Another reason is that depending on the parameter sets of (parametrized) representatives of (parametrized) conjugacy classes of U fusing into a given conjugacy class of G , calculating the value of the induction to G of a character on this conjugacy class was not entirely trivial either (see Section 11). Whenever the induction to G of a character corresponding to a core was calculated, the name of the induction of this character to G is included in the rightmost column of the tables in Appendix A.6.

In conclusion, the best way to obtain irreducible characters of U seems to be to use the reduction method to parametrize the characters. Then in case of need, calculate explicit characters of (quotients of) U for small values of q with the help of **GAP** (in fact, X_{S_Σ} would precisely be the quotient for which one should let **GAP** calculate characters for small q if necessary). This might be particularly helpful to deal with nonabelian cores or for calculating the induction of a character to U .

Analysing the nonabelian cores to find the corresponding characters should be a further project for the future.

9.4 Nonabelian cores

In the following, we shall present a method to analyse a type of nonabelian core that is characteristic for the prime $p = 2$ for $F_4(q)$ and which is consequently not presented in [GLMP]. It was used to obtain the character T_{12} in A.6.

Due to the additional restriction on the acceptable pairs (β, δ) for Type R reductions in Algorithm 4, many of the nonabelian cores contain a long root $\gamma \in Z$. In this case, the Reduction Lemma 9.1 is still

applicable with $G = X_S$, $Z = X_\gamma X_{\gamma-\alpha_i}$, $H = X_{S \setminus \{\beta\}}$, $X = X_\beta$ and $Y = X_\delta$ (β, δ) such that $\gamma = 2\alpha_i + \alpha_j$ where α_i is the short root of (β, δ) if $\lambda \in \text{Irr}(Z)$ operates trivially on $X_{\gamma-\alpha_i} = X_{\beta+\delta}$ (see Example 3).

[GLMP] present two further strategies to deal with nonabelian cores, which can be applied to $G = F_4(2^n)$ as well. We did not use them to analyse the nonabelian cores in detail as this requires a considerable amount of work, but wish to present them nevertheless:

Let $\mathfrak{C} = (S, Z, A, L, K)$ be a nonabelian core. [GLMP, Section 4.2] present the following method to analyse this core, which is applicable for $p = 2$ and $G = F_4(p^n)$ as well:

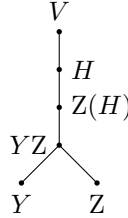
Let $Z = \{\alpha_{i_1}, \dots, \alpha_{i_m}\}$. For each $\underline{a} = (a_{i_1}, \dots, a_{i_m}) \in (\mathbb{F}_q^\times)^m$, define $\mu = \mu^{\underline{a}} : X_Z \rightarrow \mathbb{F}_q$ by $\mu(x_{i_h}(t)) = a_{i_h} t$ for each $h \in \{1, \dots, m\}$. Then $\lambda = \lambda^{\underline{a}} = \rho \circ \mu^{\underline{a}}$ is a linear character of X_Z .

Now the aim is to analyse $\text{Irr}(X_S \mid \lambda)$. Set $V = X_S / \ker \mu$ and $Z = X_Z / \ker \mu$. Since $\ker \mu \subseteq \ker \lambda$, λ factors through Z and they also write λ for this character of Z . Inflation over $\ker \mu$ gives a bijection between $\text{Irr}(V \mid \lambda)$ and $\text{Irr}(X_S \mid \lambda)$, so one can work with $\text{Irr}(V \mid \lambda)$ instead of $\text{Irr}(X_S \mid \lambda)$. Given $\alpha \in S \setminus Z$, identify X_α with its image in V .

[GLMP] aim to find subsets I, J of $S \setminus Z$ such that the following hold:

- $|I| = |J|$,
- $H = X_{S \setminus I} Z$ is a subgroup of V ,
- $Y = X_J \leq Z(H)$,
- $YZ \trianglelefteq V$.

Note that the preceding conditions imply $X = X_I$ is a transversal of H in V . The following diagram illustrates the inclusion relations between the subgroups $Z, Y, YZ, Z(H), H$ of V ; the convention used is that a group lying below another group is included in the upper one:



[GLMP] want to apply the Reduction Lemma 9.1, and conditions 1) – 4) hold, but so far condition 5) may not be satisfied, so [GLMP] adapt the situation slightly.

Consider the inflation $\hat{\mu}$ of μ to YZ and let $\hat{\lambda} = \rho \circ \hat{\mu}$ be the inflation of λ to YZ . For $v \in V$ consider the map $\psi_v : Y \rightarrow \mathbb{F}_q$ given by $\psi_v(y) = \hat{\mu}([v, y])$. Since YZ is abelian and $YZ \trianglelefteq V$, one can deduce from the commutator relations that ψ_v is \mathbb{F}_q -linear. Let

$$Y' = \bigcap_{v \in V} \ker(\psi_v) = \{y \in Y \mid {}^v \hat{\mu}(y) = \hat{\mu}(y) \forall v \in V\}.$$

Then Y' is an \mathbb{F}_q -subspace of $Y \cong \mathbb{F}_q^{|J|}$. Moreover, they define

$$\tilde{H} = \text{Stab}_V(\hat{\mu}) = \{v \in V \mid {}^v \hat{\mu} = \hat{\mu}\}.$$

This \tilde{H} is a subgroup of V and $\tilde{H} = X' H$ for $X' = \{x \in X \mid {}^x \hat{\mu} = \hat{\mu}\}$. By considering the V -orbit of $\hat{\mu}$ in the space of homomorphisms from YZ to \mathbb{F}_q and then applying the orbit-stabilizer theorem they deduce that $|X'| = |Y'|$.

Write $I = \{\alpha_{j_1}, \dots, \alpha_{j_r}\}$ and $J = \{\alpha_{k_1}, \dots, \alpha_{k_r}\}$, where $j_1 \leq \dots \leq j_r$ and $k_1 \leq \dots \leq k_r$. The sets X' and Y' can be determined by the following equation

$$\hat{\mu}([x_{\alpha_{k_1}}(s_{k_1}) \cdots x_{\alpha_{k_r}}(s_{k_r}), x_{\alpha_{j_1}}(t_{j_1}) \cdots x_{\alpha_{j_r}}(t_{j_r})]) = 0. \quad (3)$$

[GLMP] note that as the map ψ_x for $x \in X$ is \mathbb{F}_q -linear the left hand side of Equation 3 is linear in s_{k_1}, \dots, s_{k_r} . Therefore, the solutions of the equation in s_{k_1}, \dots, s_{k_r} for every t_{j_1}, \dots, t_{j_r} form an \mathbb{F}_q -subspace of Y , which determines Y' .

With the help of an additional assumption, [GLMP] can apply Lemma 9.1 in the following lemma: Define \overline{H} to be the preimage of \tilde{H} in X_S .

Lemma 9.11. ([GLMP, Lemma 4.1]) *Suppose that there exists a subgroup \tilde{Y} of Y such that $Y = Y' \times \tilde{Y}$ and $[X, \tilde{Y}] \subseteq \tilde{Y}Z$. Then we have a bijection*

$$\text{Irr}(\tilde{H}/\tilde{Y} \mid \lambda) \rightarrow \text{Irr}(V \mid \lambda), \chi \mapsto \text{Ind}_{\tilde{H}}^V \text{Inf}_{\tilde{H}/\tilde{Y}}^{\tilde{H}} \chi.$$

Consequently we have a bijection

$$\text{Irr}(\tilde{H}/\tilde{Y} \mid \lambda) \rightarrow \text{Irr}(X_S \mid \lambda), \chi \mapsto \text{Ind}_{\tilde{H}}^{X_S} \text{Inf}_{\tilde{H}/\tilde{Y}}^{\tilde{H}} \chi.$$

The proof given in [GLMP] does not rely on p not being a very bad prime.

[GLMP] note that if $[X, Y] \subseteq Z$, then one may take an arbitrary complement \tilde{Y} of Y' in Y , and the assumption $[X, \tilde{Y}] \subseteq \tilde{Y}Z$ is satisfied.

Furthermore, [GLMP] remark that the parametrization of characters resulting from Lemma 9.11 does not actually depend on the choice of \tilde{Y} .

If the condition in Lemma 9.11 is satisfied, [GLMP] observe that Y' is central in \tilde{H}/\tilde{Y} and that in fact \tilde{Y} is centralized by H and $[x', y'] = 1$ for every $x' \in X'$, $y' \in Y'$ because they have taken the quotient by $\ker(\mu)$.

Remark 14. ([GLMP, Remark 4.2]) Suppose that Lemma 9.11 applies and let $\psi \in \text{Irr}(\tilde{H}/\tilde{Y} \mid \lambda)$. Then we have that $\text{Ind}_{\tilde{H}}^{X_S} \text{Inf}_{\tilde{H}/\tilde{Y}}^{\tilde{H}} \psi \in \text{Irr}(X_S)$, and

$$\overline{\psi} = \text{Ind}_{X_{U \cup X}}^U \text{Inf}_{X_S}^{X_{U \cup X}} \text{Ind}_{\tilde{H}}^{X_S} \text{Inf}_{\tilde{H}/\tilde{Y}}^{\tilde{H}} \psi \in \text{Irr}(U)\mathfrak{c}$$

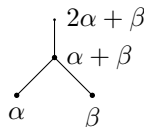
by Theorem 9.9. Since $X_K \trianglelefteq U$, $\overline{H}X_K$ is a subgroup of U , and $X_K \trianglelefteq \overline{H}X_K$. Finally,

$$\overline{\psi} = \text{Ind}_{\overline{H}X_K}^U \text{Inf}_{\tilde{H}/\tilde{Y}}^{\overline{H}X_K} \psi.$$

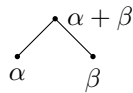
In the case that $Y' = 1$ for all choices of λ and that Y is normal in \overline{H} , we have $\overline{H}/Y = X_{S \setminus (I \cup J)}$. Define $\overline{\psi} = \text{Ind}^{A \cup I} \text{Inf}_{K \cup J} \psi$ for $\psi \in \text{Irr}(X_{S \setminus (I \cup J)})$; this sets up a bijection from $\text{Irr}(X_{S \setminus (I \cup J)})$ to $\text{Irr}(U)\mathfrak{c}$.

Another strategy [GLMP] present to deal with nonabelian cores is the following:

Let $|\text{comrel}|$ denote the number of different nontrivial commutator relations between root subgroups in a given nonabelian core. Then most nonabelian cores with the same number of nontrivial commutator relations are isomorphic up to some abelian part that does not turn up in any of the nontrivial commutator relations of this core. Therefore it might be helpful to analyse the characters associated to a set of roots that lead to a given number of nontrivial commutator relations. The diagram

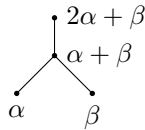


encodes the relation $[x_\alpha(t), x_\beta(s)] = x_{\alpha+\beta}(ts)x_{2\alpha+\beta}(t^2s)$ and



encodes the relation $[x_\alpha(t), x_\beta(s)] = x_{\alpha+\beta}(ts)$.

The smallest number of nontrivial commutator relations that can turn up is 1 and this is associated to the core given as follows:



Similarly, the different possibilities for the nontrivial commutator relations for $|comrel| > 1$ can be analysed and the nonabelian cores can be classified accordingly. In the tables in Appendix A.7, the number $|comrel|$ of different nontrivial commutator relations between root subgroups for each nonabelian core are included in the rightmost column.

Part III

Characters of $F_4(q)$ in Characteristic 0

Before the characters of U obtained in the previous section can be induced to $F_4(q)$, the fusions of the conjugacy classes of U into the conjugacy classes of $F_4(q)$ have to be determined.

10 Fusions of Conjugacy Classes of U into G

A representative for each conjugacy class of $F_4(q)$ containing elements of U and the orders of the centralizers of these representatives in $F_4(q)$ have been determined by [Shinoda], see Appendix A.8. However, for each (parametrized) element of U we had to determine into which of these classes it fuses.

In order to do this, we proceeded inductively as follows:

We start with $U_{11} = \{x_{24}(t) \mid t \in \mathbb{F}_q\}$. Then if we have found the fusions into the conjugacy classes of $F_4(q)$ for all elements of the form $u = x_{i+1}(t_{i+1})x_{i+2}(t_{i+2}) \dots x_{24}(t_{24}) \in U$, $t_l \in \mathbb{F}_q$ for $i+1 < l \leq 24$ and $t_{i+1} \in \mathbb{F}_q^\times$, we attempt to find the fusions into the conjugacy classes of $F_4(q)$ for all elements of the form $v = x_i(t_i)x_{i+1}(t_{i+1})x_{i+2}(t_{i+2}) \dots x_{24}(t_{24}) \in U$ with $t_l \in \mathbb{F}_q$ for $i < l \leq 24$ and $t_i \in \mathbb{F}_q^\times$:

For a given element $v = x_i(t_i)x_{i+1}(t_{i+1}) \dots x_{24}(t_{24}) \in U$ with $t_l \in \mathbb{F}_q$ for $i < l \leq 24$ and $t_i \in \mathbb{F}_q^\times$, we calculate the orbit under the action of the Weyl group with the help of a **GAP**-program we wrote (the code is included in Appendix A.13). Then we check whether this v is conjugate under the Weyl group action to some element $u = x_j(t_j)x_{j+1}(t_{j+1}) \dots x_{24}(t_{24}) \in U$ with $j > i$ and $t_l \in \mathbb{F}_q$ for $j \leq l \leq 24$. If this is the case, we can sort v into the corresponding conjugacy class of u in $F_4(q)$. This is generally much faster than the algorithm presented in [Köhler, (I.5.8)] (see Algorithm 1) for conjugacy tests, as we only consider the conjugation action of the Weyl group on v instead of the conjugation action of the entire group G .

Thus it is not necessary to run Algorithm 1 for every v . Instead, we only have to use it for conjugacy tests between v and representatives of the conjugacy classes of U in $F_4(q)$ if the orbit of v under the Weyl group action does not contain any $u = x_j(t_j)x_{j+1}(t_{j+1}) \dots x_{24}(t_{24})$ with $j > i$ and $t_l \in \mathbb{F}_q$ for $j \leq l \leq 24$. Let us illustrate this with the help of an example:

Example 6. Let $v = x_{17}(t)x_{23}(s) \in U$ with $t, s \in \mathbb{F}_q^\times$. Then the orbit of v under the Weyl group contains the element $u = x_{21}(t)x_{24}(s) \in U$, which is the representative y_3 of the conjugacy class in $F_4(q)$ of the element $y_3 \in U$ (see Appendix A.8). So v must lie in the conjugacy class of y_3 .

Our **GAP**-programm, based on the algorithm described in [Köhler, (I.5.8)], proceeds as follows (see Appendix A.13 for the code):

To check whether two given elements $x, y \in U$ are conjugate in $F_4(q)$, the normal forms of gx and yg are calculated for a chosen element w of the Weyl group and for an arbitrary parametrized element g of the Bruhat cell $B\hat{w}B$ of w , as defined in Definition 4.5, parametrized by variables (recall the Bruhat decomposition of $F_4(q)$, see Theorems 2.10 and 2.11). Next a generating system for the ideal over the \mathbb{F}_2 -polynomial ring in the variables parametrizing g spanned by the expressions in $gx - yg$ in the parameters of the x_i , $1 \leq i \leq 24 + l(w)$, occurring in the normal forms of gx and yg , is determined (i.e. let the x_i , $1 \leq i \leq 24 + l(w)$, occurring in the normal form of gx be parametrized by t_i and let the x_i , $1 \leq i \leq 24 + l(w)$, occurring in the normal form of yg be parametrized by t'_i . Then the ideal is spanned by the $t_i - t'_i$ for all $1 \leq i \leq 24 + l(w)$). Now one has to check, in part by hand, whether there exists any valid choice of values for the variables such that all the generators vanish (recall that the variables parametrizing elements of the torus may only take values in \mathbb{F}_q^\times , see Definition 4.5). If and only if this

is the case for at least one $w \in W$, x and y are conjugate in $F_4(q)$. Since these calculations are to be independent of q , the x, y will in general be parametrized by indeterminates as well. That is, x might for example be of the form $x_i(s)x_j(t)$ for fixed i, j and s, t indeterminates, with $s \neq 0$ perhaps.

The following is a pseudo-code version of the algorithm:

Algorithm 5. Conjugacy Test in $F_4(q)$ for Two Parametrized Elements x, y of U as Defined in Definition 4.5

INPUT: Two parametrized elements x, y of U as defined in Definition 4.5 with parameter sets p_x and p_y .

VARIABLES:

- \mathcal{W} , a list of elements of the Weyl group
- $w \in W$, a Weyl group element,
- g , a parametrized element of the Bruhat cell $B\hat{w}B$ of w , as defined in Definition 4.5,
- l , a list in which the pairs (l_1, l_2) of allowable parameters for x, y are to be stored such that x and y are conjugate,
- $p = (p_1 \times p_2)$ the set of allowable parameters for the tuple (x, y) .

INITIALIZATION:

- $\mathcal{W} := \{w \in W\} \setminus \{1\}$
- $w = 1$,
- $g := tu$ parametrized as usual ($t = (h_1, h_2, h_3, h_4)$, $u = x_1(s_1) \dots x_{24}(s_{24})$ with $h_i \in \mathbb{F}_q^\times$ and $s_i \in \mathbb{F}_q$).
- $l := []$,
- $p_1 := p_x, p_2 := p_y$.

ITERATION:

1. Bring gx and yg into the unique normal $gx = \tilde{u}'\hat{w}t\tilde{u}$, $yg = \tilde{u}'\hat{w}t\tilde{u}$ form defined in Definition 4.7. Write t_i for the parameter of the element x_i occurring in \tilde{u} and t_{i+24} for the parameter of the element x_i occurring in \tilde{u}' in the normal form of gx . Write t'_i for the parameter of the element x_i occurring in \tilde{u} and t'_{i+24} for the parameter of the element x_i occurring in \tilde{u}' in the normal form of yg . Go to Step 2.
2. For all i in $1, \dots, 24 + l(w)$ do $y_i = t'_i - t_i$. Go to Step 3.
3. Try to find valid choices for the parameters s_j, h_j such that the specializations of the y_i vanish for all $i \in 1, \dots, 24 + l(w)$. These choices may depend on the parametrization of x, y . If such a valid choice exists, append the corresponding choice(s) for the parameters of x, y as a list to l and remove the corresponding choice(s) of parameters for x, y from p . If p is empty after this, return l and finish. Else go to Step 4.
4. If $\mathcal{W} = \emptyset$, return l and finish. Else pick $w \in \mathcal{W}$ and let $g = u'\hat{w}tu$ be parametrized as usual ($u' = x_{j_1}(s_{24+1}), \dots, x_{j_{|U'|}}(s_{24+|U'|})$, $t = (h_1, h_2, h_3, h_4)$, $u = x_1(s_1) \dots x_{24}(s_{24})$ with $h_i \in \mathbb{F}_q^\times$ and $s_i \in \mathbb{F}_q$). Return to Step 1.

OUTPUT: The list l of tuples of allowable parameters for x and y such that x and y are conjugate in $F_4(q)$.

The following example illustrates one iteration of the above algorithm:

Example 7. Let $x = x_2(t_{53})$, $y = x_2(t_{54})x_3(t_{55})$ be two parametrized elements of U and $w = s_1 \in W$ an element of the Weyl group.

Then $g = u' \hat{w} h u = x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24})$ with parameters $t_1, \dots, t_{24}, t_{29} \in \mathbb{F}_q$ and $t_{25}, \dots, t_{28} \in \mathbb{F}_q^\times$. Let $t_{53}, t_{54} \in \mathbb{F}_q^\times$ and $t_{55} \in \mathbb{F}_q$. Then

$$\begin{aligned} gx &= x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24}) x_2(t_{53}) \\ &= x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_{53} + t_2) x_3(t_3) x_4(t_4) x_5(t_5) x_6(t_{53}t_4 + t_6) x_7(t_7) \cdot \\ &\quad x_8(t_8) x_9(t_{53}t_7 + t_9) x_{10}(t_{10}) x_{11}(t_{53}t_4t_5 + t_{11}) x_{12}(t_{12}) x_{13}(t_{53}t_{10} + t_{13}) \cdot \\ &\quad x_{14}(t_{53}t_{12} + t_{14}) x_{15}(t_{15}) x_{16}(t_{53}t_7^2 + t_{16}) x_{17}(t_{53}t_4t_5t_7 + t_{17}) \cdot \\ &\quad x_{18}(t_{53}t_4t_5^2 + t_{53}t_{15} + t_{18}) x_{19}(t_{53}t_4t_5t_{10} + t_{19}) x_{20}(t_{20}) x_{21}(t_{53}t_4t_5^2t_7 + t_{21}) \cdot \\ &\quad x_{22}(t_{53}t_{12}^2 + t_{22}) x_{23}(t_{53}t_4t_5^2t_7^2 + t_{53}t_7^2t_{15} + t_{23}) x_{24}(t_{53}^2t_4^2t_5^2t_{10} + t_{53}t_4t_5^2(t_{53}t_7^2 + t_{16}) + t_{53}t_{23} + t_{24}) \end{aligned}$$

and

$$\begin{aligned} yg &= x_2(t_{54}) x_3(t_{55}) x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24}) \\ &= x_1(t_{29}) x_2(t_{54}) x_3(t_{55}) x_5(t_{29}t_{55}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24}) \\ &= x_1(t_{29}) \hat{s}_1 x_2(t_{54}) x_5(t_{55}) x_3(t_{29}t_{55}) h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24}) \\ &= x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_2(t_{26}^2 t_{28}^{-1} t_{54}) x_5(t_{25} t_{27} t_{28}^{-1} t_{55}) x_3(t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_{29} t_{55}) x_1(t_1) x_2(t_2) \cdots x_{24}(t_{24}) \\ &= x_1(t_{29}) \hat{s}_1 h(t_{25}, t_{26}, t_{27}, t_{28}) x_1(t_1) x_2(t_{26}^2 t_{28}^{-1} t_{54} + t_2) x_3(t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_{29} t_{55} + t_3) \cdot \\ &\quad x_4(t_4) x_5(t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55} + t_5) x_6(t_6) x_7(t_7) \cdot \\ &\quad x_8((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55}) t_4 + t_8) x_9(t_9) x_{10}(t_{10}) x_{11}(t_{11}) x_{12}(t_{12}) x_{13}(t_{13}) x_{14}(t_{14}) \cdot \\ &\quad x_{15}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 + t_{15}) x_{16}(t_{16}) x_{17}(t_{17}) x_{18}(t_{18}) x_{19}(t_{19}) \cdot \\ &\quad x_{20}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6 + t_{20}) x_{21}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_9 + t_{21}) \cdot \\ &\quad x_{22}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_{13} + t_{22}) x_{23}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6 t_{10} + t_{23}) \cdot \\ &\quad x_{24}((t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_9^2 + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6 t_{13} + t_{24}) \end{aligned}$$

so for $gx = yg$ to hold, all of the following expressions have to vanish:

$$\begin{aligned} &t_{53} + t_{26}^2 t_{28}^{-1} t_{54}, \quad t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_{29} t_{55}, \quad t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55}, \quad t_{53} t_4, \\ &(t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55}) t_4, \quad t_{53} t_7, \quad t_{53} t_4 t_5, \quad t_{53} t_{10}, \quad t_{53} t_{12} (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4, \\ &t_{53} t_7^2 t_{53} t_4 t_5 t_7, \quad t_{53} t_4 t_5^2 + t_{53} t_{15}, \quad t_{53} t_4 t_5 t_{10}, \quad (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6, \\ &t_{53} t_4 t_5^2 t_7 + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_9, \quad t_{53} t_{12}^2 + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_{13}, \\ &t_{53} t_4 t_5^2 t_7^2 + t_{53} t_7^2 t_{15} + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6 t_{10}, \\ &t_{53}^2 t_4^2 t_5^2 t_{10} + t - 53 t_4 t_5^2 (t_{53} t_7^2 + t_{16}) + t_{53} t_{23} + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_9^2 \\ &\quad + (t_{25} t_{27} t_{28}^{-1} t_{55} + t_{25}^{-1} t_{27}^2 t_{28}^{-1} t_1 t_{29} t_{55})^2 t_4 t_6 t_{13}. \end{aligned}$$

Thus for $t_4 = t_7 = t_{10} = t_{12} = t_{15} = t_{23} = t_{55} = 0$ and $t_{28} t_{53} = t_{26}^2 t_{54}$, $gx = yg$. In particular, x and y are conjugate in $F_4(q)$ without any additional restrictions on the parameters t_{53}, t_{54}, t_{55} .

Theorem 10.1. *The fusions of the conjugacy classes of U into the conjugacy classes of $F_4(q)$ can be found in the Appendix A.9.*

11 Induction of Characters from U to G and Decomposition into Irreducible Characters

We have inserted the character values for representations of U into a generic character table, using the Maple-part of CHEVIE[CHEVIE] and Maple. CHEVIE provides functionalities to create generic

character tables, i.e. character tables with rows and columns indexed by parametrized sets of conjugacy classes respectively characters rather than single characters or classes. This allows one to deal with an entire series of finite reductive groups $G(q)$ at once without having to specialize the parameter q to a specific value.

In several cases, the number of conjugacy classes in a parametrized conjugacy class of the unipotent subgroup U depends on the number of solutions of certain polynomial equations. For example, for parametrized classes fusing into the conjugacy classes in G of the elements y_{17}, y_{18}, y_{19} , the number of conjugacy classes in some of the parametrized conjugacy classes depends on whether there exists, and if that be the case, how many solutions x there are to the equation $(m^2 + m)^3 + m^2 + m = x$ resp. $(m^2 + m + \eta)^3 + m^2 + m + \eta = x$ for some $m \in \mathbb{F}_q$ respectively $m^3 + m \neq x$ for all $m \in \mathbb{F}_q$. Similarly for the conjugacy classes in G of the elements y_{24}, y_{25}, y_{26} .

In some cases, the number of classes in a parametrized conjugacy class (i.e. the size of the parameter set) depends on whether $1 \in \mathbb{F}_q$ is in the image of the map $f : \mathbb{F}_q \rightarrow \mathbb{F}_q$, $x \mapsto x^2 + x$ or not. In these cases, we have used an “ α ” as a placeholder in the generic character table of G . “ α ” = 1 if $1 = m^2 + m$ for some $m \in \mathbb{F}_q$ and “ α ” = 0 otherwise. In some other cases of classes fusing into the conjugacy classes in G of the elements $y_{17}, y_{18}, y_{19}, y_{24}, y_{24}$ or y_{26} , it proved very difficult to determine $|V(I) \cap D_f|$ for I, f as described in Remark 10.

In all these cases, we have used placeholders in the generic character table. See Table 10 in Appendix A.10 for a list of the placeholders used. The sizes of the parameter sets of the parametrized conjugacy classes are included in Appendix A.9.

The values of the placeholders were determined later on using the orthogonality relations between characters (see Theorem 6.7). They are included in Table 10 in Appendix A.10.

Recall that for $\chi \in \text{Irr}(U)$, $\text{Ind}_U^G(\chi)(x) := \frac{1}{|U|} \sum_{g \in G} \hat{\chi}(x^g)$ (Definition 6.8). So for $x \in U$, $\text{Ind}_U^G(\chi)(x) = \frac{1}{|U|} \sum_{g \in G} \hat{\chi}(x^g) = \frac{1}{|U|} \sum_{g \in I} (\sum_{p \in P_g} \chi(g(p)) |g(p)^U|) |C_G(x)|$ if I is a set of (parametrized) representatives of the (parametrized) conjugacy classes of U fusing into the conjugacy class x^G of x in G , and P_g is the parameter set of g . Calculating $\sum_{p \in P_g} \chi(g(p)) |g(p)^U|$ can be quite difficult depending on the parameter set P_g . This is another reason why not all the parametrized irreducible characters of U arising from abelian cores (see Appendix A.6) where induced to G (see also Section 9).

Definition 11.1. [MS, (1.1)] An irreducible character χ of G is said to be **unipotent** if $(R_{\mathbf{T}}^G(1), \chi) \neq 0$, where $R_{\mathbf{T}}^G(1)$ is a generalized character defined by Deligne and Lusztig [DL, Definition 1.9, Section 1.20] (for a fixed prime $l \neq p = 2$) and \mathbf{T} is a fixed F -stable maximal torus of the algebraic group \mathbf{G} .

Definition 11.2. [Carter85, p.293] A character $\chi \in \text{Irr}(G)$ is **cuspidal** if for every proper standard parabolic subgroup $P_J \neq G$, the truncation map $T_{P_J/U_J}(\chi) = 0$, where

$$(T_{P_J/U_J}(\chi))(p) := \frac{1}{|U_J|} \sum_{u \in U_J} \chi(up), p \in P_J$$

and $W_J := \langle s_\alpha \mid \alpha \in J \rangle$, $\Phi_J := W_J(J)$, $U_J := \prod_{\alpha \in \Phi^+ \cap \Phi \setminus \Phi_J} U_\alpha$, and $P_J := \langle T, U_\alpha \mid \alpha \in \Phi^+ \cup \Phi_J \rangle$ for a subset $J \subset \Pi$ and the fixed torus T .

Theorem 11.3. [Lusztig, Theorems 4.23, 8.6] $G = F_4(p^n)$ has 37 unipotent characters that can be classified as follows (Here, B is a fixed Borel subgroup of G . For the notation of the characters, see also [Carter85, p.413], [MS, Theorem 1.3]):

- 25 characters appearing in $\text{Ind}_B^G(1)$ and denoted by $\phi_{i,j}$ for suitable i, j .
- 5 characters appearing in $\text{Ind}_P^G(\delta)$, where δ is the unique unipotent cuspidal character of L with L a Levi subgroup of a parabolic subgroup P of G of type B_2 . These are denoted by $B_{2,j}$ for suitable j .

- 7 cuspidal characters, denoted by $F_4[j]$ or $F_4^j[1]$ for suitable j .

The 37 irreducible unipotent characters of $F_4(q)$ are denoted

$$\begin{aligned} &\phi_{1,0}, \phi_{4,1}, \phi''_{2,4}, \phi'_{2,4}, \phi_{9,2}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \\ &\phi_{4,8}, \phi'_{6,6}, \phi''_{6,6}, \phi_{16,5}, \phi'_{8,9}, \phi''_{8,9}, \phi_{9,10}, \phi_{4,13}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, \\ &F_4[\theta], F_4[\theta^2], F_4[i], F_4[-i], F_4^I[1], F_4^{II}[1], F_4[-1], \\ &B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,r}. \end{aligned}$$

Their values on the unipotent conjugacy classes have been computed by [MS]. To find the multiplicities of the irreducible unipotent characters in the characters induced from U , use the orthogonality relation in Theorem 6.7.

Theorem 11.4. *The multiplicities of the irreducible unipotent characters of G in the induced characters of U are included in Appendix A.11. The ordering of the unipotent characters in Appendix A.11 is the same as in the generic character table of the unipotent characters of $F_4(q)$ on the unipotent classes implemented in **CHEVIE**.*

The values of the placeholders were determined by forming inner products of induced characters of U with linear combinations of the irreducible unipotent characters. The following example illustrates this:

Example 8. For the placeholders used in the conjugacy classes in G of the elements y_{24}, y_{25}, y_{26} and y_{17}, y_{18}, y_{19} , it is easy to find the size of the union of the parametrized classes of U corresponding to the placeholders in question in the union of the conjugacy classes in G of the elements y_{24}, y_{25}, y_{26} respectively y_{17}, y_{18}, y_{19} . By forming inner products of induced characters with linear combinations of irreducible unipotent characters, the sizes of the individual parametrized classes of U in the conjugacy classes in G of the different elements y_i can be determined.

Take the placeholders $\beta_{30}, \beta_{31}, \beta_{32}$. Then $\beta_{30} + \beta_{31} + \beta_{32} = q(\frac{q}{2} - 1)$ since $X \neq 0, Y \in \mathbb{F}_q$ is required for the polynomial equations corresponding to the placeholders (see Table 10 in Section A.10). Now the multiplicity of the unipotent character $B_{2,1}$ in the character T_7 induced from U is $(T_7, B_{2,1}) = \frac{-1}{4q^6}(q-1)^3(q^2 - 4\beta_{32} - 2q) \in \mathbb{C}$. Since this must be integral, $\beta_{30} + \beta_{31} + \beta_{32} = q(\frac{q}{2} - 1)$ and the $\beta_i \geq 0$, it follows that $q^2 - 4\beta_{32} - 2q = 0$, so $\beta_{32} = \frac{q}{4}(q - 2)$.

Certain placeholders do not occur linearly independently of one another in the values of the induced characters; in these cases only the value of this fixed linear combination of placeholders can be determined, but this is all that is needed.

Part IV

Decomposition Numbers of Unipotent Blocks of $F_4(q)$

The aim of this section is to consider the modular representation theory of $F_4(q)$ for $q = 2^n$ and in particular to obtain information on the l -decomposition numbers of $F_4(q)$ on unipotent blocks. For this, let $l \neq 2$ be a prime. If l does not divide the order of the group $F_4(q)$, the set of irreducible Brauer characters is simply the set of ordinary irreducible characters. Thus it is sufficient to consider primes l with $l \mid |F_4(q)|$. The representation theory for the case that $l \mid q$ is sufficiently different to justify the restriction $l \neq 2$. It turns out that the decomposition numbers can be described simultaneously for all primes l such that q has the same multiplicative order e modulo l . As $|F_4(q)| = q^{24}(q^{12} - 1)(q^8 - 1)(q^6 - 1)(q^2 - 1) = q^{24}\Phi_{12}\Phi_8\Phi_6^2\Phi_4^2\Phi_3^2\Phi_2^4\Phi_1^4$ for Φ_i the i -th cyclotomic polynomial evaluated at q , only the cases $e = 1, 2, 3, 4, 6, 8, 12$ need to be considered.

12 Modular Representation Theory

Let us introduce some basic notions of modular representation theory of finite groups. This section is largely based on [Navarro] and [Isaacs].

For the length of this section, let G be an arbitrary finite group, l a prime and (K, S, k) a split l -modular system. Here let R be the ring of algebraic integers over \mathbb{C} , $K = \mathbb{C}$, and $k = R/M$, for a maximal ideal M of R such that $lR \subset M$, is a field of characteristic l . Let $*$: $R \rightarrow k$ be the natural ring homomorphism from R to k . Note that M is not unique, but has to be chosen instead. Set $S = \{\frac{r}{s} \mid r \in R, s \in R \setminus M\}$, a complete valuation ring.

In the following we will only work on the level of characters and the argumentation will be independent of the choice of split l -modular system and in particular of M (indeed the decomposition matrix does not depend on the choice of M up to permutation of rows and columns). For more details on modular representation theory, see also [Navarro, Chapter 2].

Definition 12.1. [Navarro, Chapter 2] Call an element $g \in G$ **l -regular** if the order of g is not divisible by l . Denote the set of l -regular elements of G by G^0 .

Definition 12.2. [Navarro, Chapter 2] Let $R : G \rightarrow GL_n(k)$ be a k -representation of G and k the field of characteristic l from the split l -modular system (K, S, k) introduced above. Then one can associate to R a map $\phi : G^0 \rightarrow \mathbb{C}$, called the **Brauer character** of G afforded by the representation R . The map ϕ is uniquely determined by the equivalence class of the representation R (once M has been chosen). Call ϕ **irreducible** if R is irreducible and denote the set of irreducible Brauer characters of G by $\text{IBr}(G)$. The **degree** of ϕ is n if $R : G \rightarrow GL_n(k)$ is the representation affording ϕ .

An important notion in modular representation theory is the theory of **blocks**. For more details on this and the definition of a block, see for example [Navarro, Chapter 3].

Definition 12.3. [Navarro, (3.1)] Two ordinary characters $\chi_1, \chi_2 \in \text{Irr}(G)$ lie in the same l -block if

$$\left(|g^G| \frac{\chi_1(g)}{\chi_1(1)}\right)^* = \left(|g^G| \frac{\chi_2(g)}{\chi_2(1)}\right)^*$$

for all l -regular elements $g \in G$ (where $|g^G|$ denotes the length of the conjugacy class of g in G). By [Isaacs, (3.7)], for a character $\chi \in \text{Irr}(G)$ and an element $g \in G$, $|g^G| \frac{\chi(g)}{\chi(1)}$ is an algebraic integer. So in particular $(|g^G| \frac{\chi(g)}{\chi(1)})^*$ is defined.

Let us fix some notation:

1. For an integer $n \in \mathbb{N}$ and a prime l , write $n = mn_l$ with $m, n_l \in \mathbb{N}$, where m denotes the maximal factor of n not divisible by l and n_l is the highest power of l contained in n . Call an element m such that $l \nmid m$ an l' -**element**. Also, call a group G an l' -**group** if $l \nmid |G|$.
2. For a class function f of G , denote the restriction of f to l -regular elements of G by \check{f} .
3. Denote the set of irreducible (ordinary) characters in a given l -block B by $\text{Irr}(B)$ and set $k(B) := |\text{Irr}(B)|$.
4. Call the l -block of G containing the principal character 1_G the **principal block** of G and denote it by B_0 .

Theorem 12.4. [Navarro, (2.6),(2.9)] *Let $\chi \in \text{Irr}(G)$, then the restriction $\check{\chi}$ of χ to l -regular elements is a Brauer character and*

$$\check{\chi} = \sum_{\phi \in \text{IBr}(G)} d_{\chi\phi} \phi$$

for uniquely determined nonnegative integers $d_{\chi\phi}$, called (l) -**decomposition numbers**.

The matrix

$$D := (d_{\chi\phi})_{\chi \in \text{Irr}(G), \phi \in \text{IBr}(G)}$$

is called the **decomposition matrix** of G with respect to the prime l or l -**decomposition matrix**.

Remark 15. [Navarro, (3.3)]

- For an l -block B of G set

$$\text{IBr}(B) := \{\phi \in \text{IBr}(G) \mid d_{\chi\phi} \neq 0 \text{ for some } \chi \in \text{Irr}(B)\}$$

and set $l(B) := |\text{IBr}(B)|$. Then $l(B) \leq k(B)$.

- If the ordinary and Brauer characters are arranged according to the l -blocks, the l -decomposition matrix may be written as

$$D = \begin{pmatrix} D_{B_1} & 0 & \cdots & 0 \\ 0 & D_{B_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{B_t} \end{pmatrix},$$

where B_1, \dots, B_t are the l -blocks of G and D_{B_i} is the l -decomposition matrix of the l -block B_i .

Thanks to Remark 15, the decomposition numbers of each block can be considered separately.

Remark 16. [NT, Brauer Reciprocity, (6.8)] There is a relation between Brauer characters and projective characters which will allow us to gain information about the decomposition numbers without having to consider the Brauer characters as such. More precisely, a projective character of G is a character of a projective module of G , i.e. of a direct summand of the group algebra of G over S (for the fixed l -modular system (K, S, k)). If this module cannot be decomposed further into direct summands, it is called a **projective indecomposable module**, usually abbreviated as **PIM** and the associated character is called **projective indecomposable character**.

Now for each $\phi \in \text{IBr}(B)$ for a block B of G define $\Phi_\phi = \sum_{\chi \in \text{Irr}(B)} d_{\chi\phi} \chi$. Then Φ_ϕ is the character of a PIM and each projective indecomposable character arises in this way.

Definition 12.5. [Navarro, Chapter 3] An irreducible character $\chi \in \text{Irr}(G)$ is said to have **l -defect zero** if $\chi(1)_l = |G|_l$.

Remark 17. [Navarro, (3.18)] l -defect-zero characters are projective indecomposable and each of them forms an l -block all by itself. In particular, if $\text{Irr}(B) = \{\chi\}$ then $\text{IBr}(B) = \{\check{\chi}\}$ for such a block B .

The aim of this part will be to use Remarks 16 and 17 to find out something about the decomposition numbers of $F_4(q)$. In order to do so, we need to construct or obtain projective characters and we need to be able to discern whether they are indecomposable or not, and if they are not, find their summands. The following will be useful to produce projective characters:

Remark 18. [Alperin, III.8.5, II.7.4]

1. The induction of a projective character of a subgroup is again projective. In particular, all characters of l' -subgroups are projective and thus so are their inductions to the whole group.
2. Tensor products of ordinary characters with projective characters are again projective.

So in particular the characters of $F_4(q)$ induced from the unipotent subgroup U of $F_4(q)$ are projective for all primes $l \neq 2$.

As the decomposition matrix is rectangular, i.e. $|\text{Irr}(G)| \geq |\text{IBr}(G)|$ (see Remark 15) and has full rank, to obtain all decomposition numbers it is sufficient to find the decomposition numbers for a suitable subset of the ordinary characters. The restriction to l -regular elements of each of the other ordinary characters will then be a linear combination of the restrictions to l -regular elements of the ordinary characters whose decomposition numbers are known. In fact, for $l \neq 3$ it will be possible to choose a subset of the ordinary characters in such a way that the l -regular part of each of the remaining ordinary characters will be a \mathbb{Z} -linear combination of the l -regular parts of these and the coefficients will be easy to obtain if the values of these characters are known.

Let us introduce the notion of a basic set:

Definition 12.6. [Navarro, (7.3)] Let B be an l -block of G . Recall that $l(B) := |\text{IBr}(B)|$

1. A set $\{\phi_1, \dots, \phi_{l(B)}\}$ of (not necessarily irreducible) Brauer characters is called a **basic set** for B if it is a basis of the abelian group $\{\sum_{\chi \in \text{Irr}(B)} a_\chi \check{\chi} \mid a_\chi \in \mathbb{Z}\}$. In particular $\text{IBr}(B)$ is a basic set for B .
2. Let D be a basic set for B and $\chi \in \text{Irr}(B)$. Then $\check{\chi} = \sum_{\phi \in D} a_{\chi\phi} \phi$ for some uniquely defined integer coefficients $a_{\chi\phi}$ and this equation is called a **basic relation**.
3. Call a basic set D **ordinary** if $D = \{\check{\chi} \mid \chi \in M\}$ for some $M \subseteq \text{Irr}(B)$. In this case also call M an ordinary basic set.
4. We use the terms basic set and basic relation for unions of l -blocks as well.

Remark 19. [Hiß90, (2.2.2)] Let B be an l -block of G and $\text{Irr}(B) = \{\chi_1, \dots, \chi_{k(B)}\}$, $\text{IBr}(B) = \{\phi_1, \dots, \phi_{l(B)}\}$. If $D = \{\check{\chi}_1, \dots, \check{\chi}_{l(B)}\}$ is an ordinary basic set of B , then

$$d_{\chi_i \phi_j} = \sum_{m=1}^{l(B)} a_{im} d_{\chi_m \phi_j}$$

for all $i > l(B)$ and $1 \leq j \leq l(B)$, where

$$\check{\chi}_i = \sum_{m=1}^{l(B)} a_{im} \check{\chi}_m$$

is the basic relation of χ_i with respect to D . Thus the decomposition matrix of the block B is determined by the decomposition numbers of the elements in the basic set D and the basic relations of the elements $\chi \in \text{Irr}(B) \setminus \{\chi \in \text{Irr}(B) \mid \check{\chi} \in D\}$.

Remarks 16 and 19 can be used to test whether a projective character is indecomposable, more precisely:

Lemma 12.7. [Hiβ90, (2.2.7)] *Let Φ be a projective character of G and $D = \{\chi_1, \dots, \chi_{l(B)}\}$ an ordinary basic set of a given l -block B . Define $z_j := (\Phi, \chi_j)$ for all $1 \leq j \leq l(B)$. If for every $(0, \dots, 0) \neq (z'_1, \dots, z'_{l(B)}) \neq (z_1, \dots, z_{l(B)})$, $0 \leq z'_j \leq z_j$ for $1 \leq j \leq l(B)$, there exists a basic relation (with respect to D) $\check{\chi}_i = \sum_{m=1}^{l(B)} a_{im} \check{\chi}_m$ with $\sum_{j=1}^{l(B)} a_{ij} z'_j < 0$, then Φ is indecomposable.*

13 Unipotent Blocks of $F_4(q)$

Now the results of the previous section will be applied to the group $F_4(q)$.

The unipotent blocks of $F_4(q)$ are (by definition) precisely the blocks that contain at least one unipotent character.

Theorem 13.1. [GH91, (5.1)] *For every unipotent l -block B of $F_4(q)$, $q = 2^n$, and every prime $l \neq 2, 3$, $\{\check{\chi} \mid \chi \in \text{Irr}(B) \text{ unipotent}\}$ is an ordinary basic set of B .*

By [BMM], the decomposition of the irreducible unipotent characters into blocks depends solely on the order e of q modulo l if $l \neq 3$ and can be described generically for all $q = 2^n$ and $l \neq 3$ such that q has the same order e modulo l . So consider the cases $e = 1, 2, 3, 4, 6, 8, 12$ for $l \neq 3$. If $l = 3$, by [Enguehard] the decomposition of the irreducible unipotent characters into blocks again depends solely on the order e of q modulo 3 and can be described generically for all $q = 2^n$ such that q has the same order e modulo 3. So consider the cases $l \mid q + 1$ and $l \mid q - 1$ for $l = 3$. One can use Definition 12.3 and Remark 17 to partition the irreducible unipotent characters of $F_4(q)$ into unions of blocks. Since the values of the unipotent characters on $F_4(q) \setminus U$ are not known, a priori one can only partition the characters into unions of blocks using this criterion. However, the block decomposition is already known by [BMM] for $l \neq 2, 3$ and by [Enguehard] for $l = 3$. Or at least, [BMM] give the non-principal blocks with more than one character explicitly. So all other characters either lie in the principal block or have defect zero for $l \neq 2, 3$. Now [Enguehard] describes how the block decomposition of the unipotent characters of $F_4(q)$ changes for $l = 3$ as compared to $l \neq 2, 3$ for the different cases for e .

- $e = 1, l \neq 3$: The defect-zero characters are $F_4[\theta], F_4[\theta^2], F_4[i], F_4[-i], F_4^I[1], F_4^{II}[1], F_4[-1]$ and thus Remark 17 and [BMM, 5.24, 6.1, 6.2] yield 9 unipotent blocks: the principal block with 25 characters, one block with 5 characters and the 7 projective characters. The two blocks with more than one character are

$$\begin{aligned} &\phi_{1,0}, \phi_{4,1}, \phi''_{2,4}, \phi'_{2,4}, \phi_{9,2}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \\ &\phi_{4,8}, \phi'_{6,6}, \phi''_{6,6}, \phi_{16,5}, \phi'_{8,9}, \phi''_{8,9}, \phi_{9,10}, \phi_{4,13}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24} \end{aligned}$$

and

$$B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,r}$$

- $e = 1, l = 3$: The defect-zero characters are $F_4[i], F_4[-i], F_4^I[1], F_4[-1]$ and thus Remark 17 and [Enguehard, Theorem A] yield 6 unipotent blocks: the principal block with 28 characters, one block with 5 characters and the 4 projective characters. The two blocks with more than one character are

$$\begin{aligned} &\phi_{1,0}, \phi_{4,1}, \phi''_{2,4}, \phi'_{2,4}, \phi_{9,2}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \\ &\phi_{4,8}, \phi'_{6,6}, \phi''_{6,6}, \phi_{16,5}, \phi'_{8,9}, \phi''_{8,9}, \phi_{9,10}, \phi_{4,13}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, F_4[\theta], F_4[\theta^2], F_4^{II}[1] \end{aligned}$$

and

$$B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,r}$$

- $e = 2, l \neq 3$: The defect-zero characters are $\phi_{12,4}, \phi_{4,8}, \phi_{16,5}, F_4[\theta], F_4[\theta^2], F_4[i], F_4[-i]$ and thus Remark 17 and [BMM, 5.24, 6.1, 6.2] yield 9 unipotent blocks: the principal block with 25 characters, one block with 5 characters and the 7 projective characters. The two blocks with more than one character are

$$\begin{aligned} & \phi_{1,0}, \phi''_{2,4}, \phi'_{2,4}, \phi_{9,2}, \phi'_{8,3}, \phi''_{8,3}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \\ & \phi'_{6,6}, \phi''_{6,6}, \phi'_{8,9}, \phi''_{8,9}, \phi_{9,10}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, F_4^I[1], F_4^{II}[1], F_4[-1], B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''} \end{aligned}$$

and

$$\phi_{4,1}, \phi''_{4,7}, \phi'_{4,7}, B_{2,r}, \phi_{4,13}.$$

- $e = 2, l = 3$: The defect-zero characters are $\phi_{4,8}, \phi_{16,5}, F_4[i], F_4[-i]$ and thus Remark 17 and [Enguehard, Theorem A] yield 6 unipotent blocks: the principal block with 28 characters, one block with 5 characters and the 4 projective characters. The two blocks with more than one character are

$$\begin{aligned} & \phi_{1,0}, \phi''_{2,4}, \phi'_{2,4}, \phi_{9,2}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \\ & \phi'_{6,6}, \phi''_{6,6}, \phi'_{8,9}, \phi''_{8,9}, \phi_{9,10}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, F_4^I[1], F_4^{II}[1], F_4[-1], B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, F_4[\theta], F_4[\theta^2] \end{aligned}$$

and

$$\phi_{4,1}, \phi''_{4,7}, \phi'_{4,7}, B_{2,r}, \phi_{4,13}.$$

- $e = 3$: The 16 defect-zero characters are

$$\begin{aligned} & B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,r}, F_4[i], F_4[-i], F_4^I[1], F_4[-1], \\ & \phi_{9,2}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{6,6}, \phi'_{6,6}, \phi_{9,10} \end{aligned}$$

and thus Remark 17 and [BMM, 5.24, 6.1, 6.2] yield 17 unipotent blocks: the principal block with 21 characters and the 16 projective characters. The block with more than one character is

$$\begin{aligned} & \phi_{1,0}, \phi_{4,1}, \phi''_{2,4}, \phi'_{2,4}, \phi'_{8,3}, \phi''_{8,3}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \\ & \phi_{4,8}, \phi_{16,5}, \phi'_{8,9}, \phi''_{8,9}, \phi_{4,13}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, F_4[\theta], F_4[\theta^2], F_4^{II}[1] \end{aligned}$$

- $e = 4$: The 13 defect-zero characters are

$$\phi'_{8,3}, \phi''_{8,3}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{6,6}, \phi_{16,5}, \phi'_{8,9}, \phi''_{8,9}, F_4[\theta], F_4[\theta^2], F_4[-1]$$

and thus Remark 17 and [BMM, 5.24, 6.1, 6.2] yield 16 unipotent blocks: the principal block with 16, two blocks with 4 characters and the 13 projective characters. The three blocks with more than one character are

$$\begin{aligned} & \phi_{1,0}, \phi_{4,1}, \phi_{9,2}, \phi_{12,4}, \phi_{4,8}, \phi'_{6,6}, \phi_{9,10}, \phi_{4,13}, \phi_{1,24}, \\ & F_4[i], F_4[-i], F_4^I[1], F_4^{II}[1], B_{2,1}, B_{2,\epsilon}, B_{2,r}, \\ & \phi'_{2,4}, \phi'_{4,7}, \phi'_{2,16}, B_{2,\epsilon'}, \end{aligned}$$

and

$$\phi''_{2,4}, \phi''_{4,7}, \phi''_{2,16}, B_{2,\epsilon''}.$$

- $e = 6$: The 16 defect-zero characters are

$$\begin{aligned} & \phi_{4,1}, \phi_{9,2}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \phi_{4,8}, \phi'_{6,6}, \phi''_{6,6}, \phi_{16,5}, \phi_{9,10}, \phi_{4,13}, \\ & B_{2,r}, F_4[i], F_4[-i], F_4^{II}[1] \end{aligned}$$

and thus Remark 17 and [BMM, 5.24, 6.1, 6.2] yield 17 unipotent blocks: the principal block with 21 characters and the 16 projective characters. The block with more than one character is

$$\begin{aligned} & \phi_{1,0}, \phi''_{2,4}, \phi'_{2,4}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi'_{8,9}, \phi''_{8,9}, \phi'_{2,16}, \phi''_{2,16}, \phi_{1,24}, \\ & F_4[\theta], F_4[\theta^2], F_4^I[1], F_4[-1], B_{2,1}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,\epsilon} \end{aligned}$$

- $e = 8$: The 29 defect-zero characters are

$$\begin{aligned} & \phi_{4,1}, \phi''_{2,4}, \phi'_{2,4}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \phi_{4,8}, \phi'_{6,6}, \phi''_{6,6}, \\ & \phi'_{8,9}, \phi''_{8,9}, \phi_{4,13}, \phi'_{2,16}, \phi''_{2,16}, B_{2,1}, B_{2,\epsilon}, B_{2,\epsilon'}, B_{2,\epsilon''}, B_{2,r}, F_4[\theta], F_4[\theta^2], F_4^I[1], F_4^{II}[1] \end{aligned}$$

and [BMM, 5.24, 6.1, 6.2] yields the principal block with 8 characters. The principal block is

$$\phi_{1,0}, \phi_{9,2}, \phi_{16,5}, \phi_{9,10}, \phi_{1,24}, F_4[i], F_4[-i], F_4[-1].$$

- $e = 12$: The 25 defect-zero characters are

$$\begin{aligned} & \phi''_{2,4}, \phi'_{2,4}, \phi'_{8,3}, \phi''_{8,3}, \phi_{12,4}, \phi''_{9,6}, \phi'_{9,6}, \phi''_{1,12}, \phi'_{1,12}, \phi''_{4,7}, \phi'_{4,7}, \phi_{4,8}, \phi'_{6,6}, \\ & \phi'_{8,9}, \phi''_{8,9}, \phi'_{2,16}, \phi''_{2,16}, \phi_{9,2}, \phi_{16,5}, \phi_{9,10}, B_{2,\epsilon'}, B_{2,\epsilon''}, F_4^I[1], F_4^{II}[1], F_4[-1] \end{aligned}$$

and [BMM, 5.24, 6.1, 6.2] yields the principal block with 12 characters. The principal block is

$$\phi_{1,0}, \phi_{4,1}, \phi''_{6,6}, \phi_{4,13}, \phi_{1,24}, F_4[\theta], F_4[\theta^2], F_4[i], F_4[-i], B_{2,1}, B_{2,\epsilon}, B_{2,r}.$$

By Theorem 13.1 ordinary basic sets for all unipotent l -blocks, $l \neq 3$, are given by the unipotent characters lying in the corresponding block.

14 The Shape of the Decomposition Matrix

By the previous Section 12, the decomposition numbers of the (ordinary) irreducible unipotent characters in a block B are given by the multiplicities of the (ordinary) irreducible unipotent characters of G in the PIMs associated to the Brauer characters of the block B , which are constituents of the characters of G induced from the unipotent subgroup U of G . In particular, whenever the matrices in Appendix A.11 containing the multiplicities of the irreducible unipotent characters of G in the characters of G induced from U have a 1 on the diagonal, the same will hold for the column of an appropriately chosen PIM in the decomposition matrix of the ordinary unipotent characters into Brauer characters. Sadly, the characters we induced from the unipotent subgroup U of G were not sufficient to give the shape of the entire decomposition matrix. However, in the parts that have been obtained, the decomposition matrix is of triangular shape except possibly for one respectively two entries for the case $e = 2$.

15 Modular Harish-Chandra Theory

Keep the notation of Section 12. Modular Harish-Chandra theory is a generalization of ordinary Harish-Chandra theory that can be useful in finding decomposition numbers by sorting irreducible Brauer characters into modular Harish-Chandra series. These series can give information about the decomposition of projective characters into PIMs and thus via Brauer reciprocity also about the decomposition numbers. First of all fix some notation (see [Hiß93, section 3] or [MT, chapter 12.2]):

For a subset $J \subset \Pi$ and the fixed torus T , set $W_J := \langle s_\alpha \mid \alpha \in J \rangle$, $\Phi_J := W_J(J)$, $U_J := \prod_{\alpha \in \Phi^+ \cap \Phi \setminus \Phi_J} U_\alpha$, $L_J := \langle T, U_\alpha \mid \alpha \in \Phi_J \rangle$ and $P_J := \langle T, U_\alpha \mid \alpha \in \Phi^+ \cup \Phi_J \rangle$. Call P_J a **standard parabolic subgroup** of G and L_J a **standard Levi subgroup** or **standard Levi complement** of P_J . Then U_J is normal in

P_J and $P_J = L_J \rtimes U_J$. In particular, have $B = P$ for the fixed Borel subgroup B of G .

Recall that a (general) parabolic subgroup of G is simply a subgroup of G containing the fixed Borel subgroup B and all such P are conjugate to a P_J for some $J \subset \Pi$. Similarly, the conjugates of standard Levi complements are called **Levi subgroups** of G .

Now let $\phi \in \text{Irr}(L)$ be an irreducible character of L for a Levi subgroup L and a parabolic subgroup P containing L . The **Harish-Chandra induction** $R_{L \subseteq P}^G(\phi)$ is given by first inflating ϕ to a character of P and then inducing to G . It can be shown that this induction does not depend on the choice of P , therefore one also writes R_L^G for the Harish-Chandra induction, omitting the P (see e.g. [Carter85, 7.3.6]). For more details on ordinary Harish-Chandra theory, see also [Carter85, Chapter 9,10]. A more detailed description of modular Harish-Chandra theory can be found in [Hiß93].

Definition 15.1. Let ϕ be a Brauer character of G and Φ a projective character of G . Define

$$\langle \Phi, \phi \rangle := \frac{1}{|G|} \sum_{g \in G^0} \Phi(g)\phi(g^{-1}),$$

where G^0 is the set of l' -elements of G , i.e. of l -regular elements.

Definition 15.2. A Brauer character $\phi \in \text{IBr}(G)$ is called **cuspidal** if $\langle \text{Ind}_{U_J}^G(1), \phi \rangle = 0$ for all subsets $J \subsetneq \Pi$.

Remark 20. • $\text{Ind}_{U_J}^G(1)$ is projective by Remark 18 as U_J is a subgroup of the p -group U and $l \nmid p$

- If ϕ is a projective character of L , then $R_L^G \phi$ is again projective (see [Hiß93, Section 3]).

Theorem 15.3. [Hiß93, Section 5, in particular Theorem 5.5] Set

$$\mathfrak{S} := \{(L, \theta) \mid L \leq G \text{ Levi subgroup of } G, \theta \in \text{IBr}(L)\}$$

and define $(L', \theta') \leq (L, \theta)$ if $L' \leq L$ and $\langle R_{L'}^L \Theta_{\theta'}, \theta \rangle > 0$ for $(L', \theta'), (L, \theta) \in \mathfrak{S}$, where $\Theta_{\theta'}$ denotes the PIM associated to θ' . This induces a partial order on \mathfrak{S} and if $(L, \theta) \in \mathfrak{S}$ is minimal with respect to this order, then θ is a cuspidal Brauer character of L .

Moreover, if $\phi \in \text{IBr}(G)$, then there exists a unique minimal element $(L, \theta) \in \mathfrak{S}$ up to conjugation under W such that $(L, \theta) \leq (G, \phi)$.

This theorem allows one to define modular Harish-Chandra series analogously to the classical situation in characteristic 0.

Definition 15.4. [Hiß93, see Section 5]

- Let $J \subsetneq \Pi$ and $\theta \in \text{IBr}(L_J)$ be a cuspidal character. Then an irreducible Brauer character $\phi \in \text{IBr}(G)$ is said to belong to the **modular Harish-Chandra series** of the pair (L_J, θ) if there exists a minimal element $(L', \theta') \in \mathfrak{S}$ with $(L', \theta') \leq (G, \phi)$ and such that (L', θ') is conjugate to (L_J, θ) under the action of W .
- The trivial character of the Borel subgroup B of G is cuspidal and the corresponding modular Harish-Chandra series of G for the pair $(T, 1)$ is called the **principal series** of G .

The irreducible Brauer characters in a Harish-Chandra series can be described through the simple modules of an endomorphism ring. More precisely:

Theorem 15.5. [GHM, Theorem 2.4] Let $J \subsetneq \Pi$ and $\theta \in \text{IBr}(L_J)$ be cuspidal and let X be a simple kL_J -module affording the Brauer character θ (recall that k is a field of characteristic l). Then there is a bijection between the irreducible Brauer characters of G lying in the modular Harish-Chandra series of the pair (L_J, θ) and the simple modules of the endomorphism ring $\text{End}_{kG}(R_{L_J}^G(X))$.

Remark 21. • [GHM, Chapter 3] give a basis and multiplication rules for the basis elements of $\text{End}_{kG}(R_{L_J}^G(X))$.

- For the principal series, [Hiß90, 5.3] shows that the endomorphism ring is a specialization of the generic Iwahori-Hecke algebra of type F_4 . The decomposition numbers of the Brauer characters in the principal series into unipotent characters are precisely the decomposition numbers of this Hecke algebra, see [Hiß90, Theorem 5.1.2] and [Dipper, Corollary 4.10].
- The l -decomposition matrix of the Hecke algebra of type F_4 has been calculated in [GL]. It does not depend on the prime p , only on the prime l , or more precisely, on the order e of l modulo q and on whether $l = 2$, $l = 3$ or $l \neq 2, 3$. Thus for characters in the principal series of $F_4(q)$, the decomposition numbers are known.

Remark 22. • If a block B has cyclic defect group, one can define the **Brauer tree** of the block B (see [Navarro, Theorem 11.13 ff] for the definition of the Brauer tree and [Navarro, Theorem 4.3] for the definition of a defect group). It encodes the decomposition numbers of the block completely.

- For $e = 4$, the two unipotent blocks with unipotent characters

$$\phi'_{2,4}, \phi'_{4,7}, \phi'_{2,16}, B_{2,\epsilon'}$$

and

$$\phi''_{2,4}, \phi''_{4,7}, \phi''_{2,16}, B_{2,\epsilon''}$$

have cyclic defect group, for $e = 8$ and $e = 12$, the principal block has cyclic defect group. In all these cases, the Brauer trees are known (see [HL, Theorem 2.1]) and thus the decomposition numbers of these blocks are known as well.

The following are a few useful lemmata for decomposing projective characters into PIMs and finding upper bounds for decomposition numbers.

Lemma 15.6. [Hiß93, Lemma 5.6] *Let (L_J, θ) be a minimal pair of \mathfrak{S} and Θ_θ the PIM associated to θ . Moreover, let Θ be an indecomposable summand of $R_{L_J}^G(\Theta_\theta)$ with associated Brauer character ϕ . Then ϕ lies in the modular Harish-Chandra series of the pair (I, θ') for an I with $I^x \subseteq J$ for some $x \in W$ and $\theta' \in \text{IBr}(L_I)$.*

Lemma 15.7. [Hiß93, Lemma 6.1] *Let Θ be a projective character of G . Then there exist two characters χ, ψ of G with $\Theta = \chi + \psi$ and such that χ has only unipotent constituents and ψ has no unipotent constituents. Call χ the **unipotent quotient** of Θ and write $u(\Theta)$ for χ .*

Now let L be a maximally split Levi subgroup of G and Θ a projective character of kL . Then $u(R_L^G \Theta) = R_L^G(u(\Theta))$.

So in particular, to find decomposition numbers of unipotent blocks, it is sufficient to restrict oneself to the unipotent quotients of projective characters. As the decomposition numbers of unipotent blocks for the proper Levi subgroups of G are known ([FS82], [White95], [White00], [HN]), one can form the unipotent quotients of the PIMs of these Levi subgroups and Harish-Chandra induce these to G to gain the unipotent quotient of projective characters of G . The Harish-Chandra induction of unipotent characters of the Levi subgroups of G to G is implemented in **CHEVIE** ([S+, Chapter 84]), so Harish-Chandra inducing PIMs of a Levi subgroup can be done very easily.

Convention for the notation of the characters of the Levi subgroups: As the decomposition matrices for the unipotent blocks of the proper Levi subgroups are all unitriangular (see [FS82], [White95], [White00], [HN]), there is a canonical parametrization of the Brauer characters in the unipotent blocks of these

Levi subgroups in terms of the ordinary unipotent characters: If the characters are arranged such that the decomposition matrix has a lower unitriangular shape and the i -th ordinary unipotent character is denoted χ_i , then denote the i -th Brauer character by λ_{χ_i} (see also [GH97a, section 3]). Since the values of the irreducible unipotent characters of G are only known on the unipotent classes, one cannot use the orthogonality relations in Theorem 6.7 to decompose tensor products of irreducible unipotent characters into their irreducible constituents. Sadly, tensor products of irreducible unipotent characters with characters induced from U will typically have very many constituents and thus do not help to find upper bounds for the entries of the decomposition matrices for the unipotent blocks.

16 The Decomposition Matrix

Theorem 16.1. *In Appendix A.12, we have included our results for the l -decomposition matrices of irreducible unipotent characters of $F_4(q)$ into PIMs for $q = 2^n$ for some $n \in \mathbb{N}$ and $l \neq 2$ a prime. In most cases (for all columns not labelled “ps”), these are not the actual decomposition matrices, but only upper bounds for these (i.e. upper bounds entrywise), since several of the projective characters used are not necessarily PIMs. As the values of the non-unipotent characters included in the unipotent l -blocks are not known, we cannot use the basic relations (see Lemma 12.7) to test whether a projective character is in fact a PIM.*

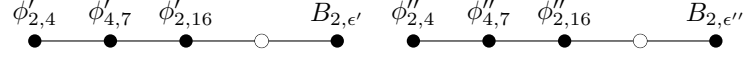
The non-empty entries of the matrices in Appendix A.12 however are of lower unitriangular shape for all the different cases of the order of q modulo l we studied.

The general procedure is to produce projective characters or at least unipotent quotients of projective characters either by inducing characters of U or by Harish-Chandra induction of projective characters of Levi subgroups of G . These can of course be restricted to the different blocks. Via Brauer reciprocity the multiplicities of the irreducible unipotent characters in these projective characters give information on the decomposition matrix of the unipotent block.

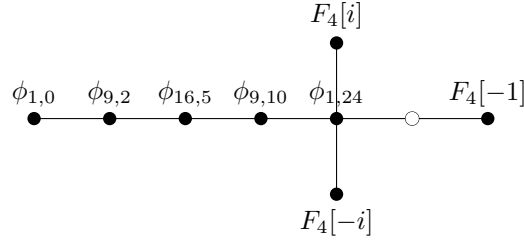
A note on the notation in the matrices: in the row labelled “HC”, whenever applicable the Levi subgroup from which a character was Harish-Chandra induced is given. In this case, the notation for the characters of the Levi subgroup is as in [HN, Section 3.3]. For characters lying in the principal series, this is denoted by a “ps” in the row. Zero-entries are denoted by a \cdot in the matrix. The projective characters lying in the principal series are simply denoted by S_i ; similarly, the missing entries of the matrices are denoted by S_i as well in the corresponding row, but of course without entries in the corresponding column of the matrix below.

- $e = 1$: For $l \neq 3$, this is the case of linear characteristic described in [Hiß90, Theorem 6.3.7]. Thus the decomposition numbers are known in this case. For $l = 3$, note that we have changed the ordering of the unipotent characters in Table 17 as compared to the ordering used in the tables in Appendix A.11. The position of the characters $\Phi_{12,4}$, $\Phi''_{9,6}$, $\Phi'_{9,6}$ and $\Phi_{4,13}$ have been changed to obtain a lower triangular shape of the matrix in the non-empty entries. Similarly, the ordering of the unipotent characters in Table 18 has been changed. The position of the character $B_{2,r}$ has been changed to obtain a lower triangular shape of the matrix in the non-empty entries.
- $e = 2$: If $(q+1)_l = 3$, then $\beta = \gamma = 1$, else if $(q+1)_l = 5$, then $\beta = \gamma = 2$. Finally, if $(q+1)_l \geq 5$, then $\beta = 3$ and $\gamma = 2$. Note that we have changed the ordering of the unipotent characters in the Tables 19, 20 and 22, 23 as compared to the ordering used in the tables in Appendix A.11. The position of the characters $\Phi_{12,4}$, $\Phi''_{9,6}$, $\Phi'_{9,6}$ and $B_{2,\epsilon}$ have been changed to obtain a nearly lower triangular shape of the matrices in the non-empty entries.

- $e = 3$: Note that we have changed the ordering of the unipotent characters in Table 25 as compared to the ordering used in the tables in Appendix A.11. The position of the character $\Phi_{4,13}$ has been changed to obtain a lower triangular shape of the matrix in the non-empty entries.
- $e = 4$: All characters not lying in the principal block have defect zero or lie in one of two blocks with cyclic defect group. The associated Brauer trees are as follows ([HL, Theorem 2.1]):

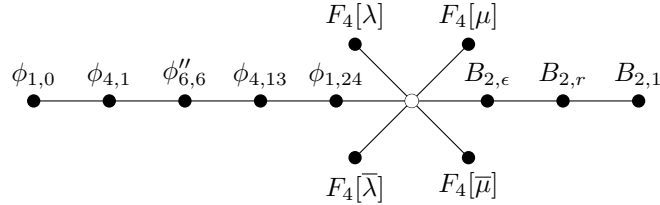


- $e = 6$: Note that we have changed the ordering of the unipotent characters in Table 27 as compared to the ordering used in the tables in Appendix A.11. The position of the character $B_{2,\epsilon}$ has been changed to obtain a lower triangular shape of the matrix in the non-empty entries.
- $e = 8$: All characters not lying in the principal block have defect zero, and the defect group of the principal block is cyclic. The associated Brauer tree is as follows ([HL, Theorem 2.1]):



Thus the decomposition numbers are fully known in this case.

- $e = 12$: All characters not lying in the principal block have defect zero, and the defect group of the principal block is cyclic. The associated Brauer tree is as follows ([HL, Theorem 2.1]):



(With $\{\lambda, \mu\} = \{i, \theta\}$, i.e. the planar embedding of the tree is not determined, and $\bar{i} = -i$, $\bar{\theta} = \theta^2$ denoting the complex conjugates of i, θ). Thus the decomposition numbers are fully known in this case.

Remark 23. Gerhard Hiss has suggested the following to find projective characters having some of the cuspidal unipotent characters as constituents:

Let P be the standard parabolic subgroup of $F_4(2^n)$ of type C_3 and let $V \leq U$ be the unipotent radical of P . Finally, let $V_0 := Z(V)$; then $V_0 \trianglelefteq U$ (as $p = 2$). I.e. $J = \{\alpha_1, \alpha_3, \alpha_4\}$, $P = P_J := \langle T, X_\alpha \mid \alpha \in \Phi_J \cup \Phi^+ \rangle$ where $\Phi_J := \mathbb{Z}\{\alpha_1, \alpha_3, \alpha_4\} \cap \Phi$, $V = U_J := \langle X_\alpha \mid \alpha \in \Phi^+ \setminus (\Phi_J \cap \Phi^+) \rangle = \langle X_{\alpha_i} \mid i \in \{2, 6, 9, 11, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24\} \rangle$ and $V_0 = \langle X_{\alpha_i} \mid i \in \{9, 11, 14, 17, 19, 21, 24\} \rangle$.

Many cuspidal unipotent characters χ of $F_4(2^n)$ fulfill $(\chi \downarrow_{V_0}, 1_{V_0}) > 0$; such χ are then constituents of $\theta^G := \text{Ind}_U^G \theta$ for suitable irreducible characters θ of U that have V_0 in the kernel.

Part V

A Appendix

A.1 The Unipotent Subgroups

Let $U_i := \langle x_j(u) \mid u \in \mathbb{F}_q, ht(\alpha_j) \geq i \rangle$. Then $U_{11} = \langle x_{24}(u) \mid u \in \mathbb{F}_q \rangle$,
 $U_{10} = \langle x_{23}(u), U_{11} \mid u \in \mathbb{F}_q \rangle$,
 $U_9 = \langle x_{22}(u), U_{10} \mid u \in \mathbb{F}_q \rangle$,
 $U_8 = \langle x_{21}(u), U_9 \mid u \in \mathbb{F}_q \rangle$,
 $U_7 = \langle x_{20}(u), x_{19}(u), U_8 \mid u \in \mathbb{F}_q \rangle$,
 $U_6 = \langle x_{18}(u), x_{17}(u), U_7 \mid u \in \mathbb{F}_q \rangle$,
 $U_5 = \langle x_{16}(u), x_{15}(u), x_{14}(u), U_6 \mid u \in \mathbb{F}_q \rangle$,
 $U_4 = \langle x_{13}(u), x_{12}(u), x_{11}(u), U_5 \mid u \in \mathbb{F}_q \rangle$,
 $U_3 = \langle x_{10}(u), x_9(u), x_8(u), U_4 \mid u \in \mathbb{F}_q \rangle$,
 $U_2 = \langle x_7(u), x_6(u), x_5(u), U_3 \mid u \in \mathbb{F}_q \rangle$,
 $U_1 = \langle x_1(u), x_2(u), x_3(u), x_4(u), U_2 \mid u \in \mathbb{F}_q \rangle$.

A.2 A Root System of Type F_4

In a fixed basis given by the simple roots, the positive roots have the following coordinate vectors:

$$\begin{aligned}
 \alpha_1 &= (1, 0, 0, 0) & \alpha_2 &= (0, 1, 0, 0) & \alpha_3 &= (0, 0, 1, 0) & \alpha_4 &= (0, 0, 0, 1) \\
 \alpha_5 &= (1, 0, 1, 0) & \alpha_6 &= (0, 1, 0, 1) & \alpha_7 &= (0, 0, 1, 1) & \alpha_8 &= (1, 0, 1, 1) \\
 \alpha_9 &= (0, 1, 1, 1) & \alpha_{10} &= (0, 0, 2, 1) & \alpha_{11} &= (1, 1, 1, 1) & \alpha_{12} &= (1, 0, 2, 1) \\
 \alpha_{13} &= (0, 1, 2, 1) & \alpha_{14} &= (1, 1, 2, 1) & \alpha_{15} &= (2, 0, 2, 1) & \alpha_{16} &= (0, 1, 2, 2) \\
 \alpha_{17} &= (1, 1, 2, 2) & \alpha_{18} &= (2, 1, 2, 1) & \alpha_{19} &= (1, 1, 3, 2) & \alpha_{20} &= (2, 1, 2, 2) \\
 \alpha_{21} &= (2, 1, 3, 2) & \alpha_{22} &= (2, 1, 4, 2) & \alpha_{23} &= (2, 1, 4, 3) & \alpha_{24} &= (2, 2, 4, 3)
 \end{aligned}$$

Here, the basis used is the same as in the **GAP**-package **Unipot**. In the standard basis of \mathbb{C}^4 , the simple roots are given as

$$\alpha_1 = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \quad \alpha_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \quad \alpha_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \alpha_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

The short roots have the numbers 1, 3, 5, 7, 8, 9, 11, 12, 14, 17, 19, 21 and consequently the long roots are those with numbers 2, 4, 6, 10, 13, 15, 16, 18, 20, 22, 23, 24.

The Cartan matrix with respect to the simple roots $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ has the form

$$\begin{pmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -2 & 2 \end{pmatrix}$$

and for

$$A = \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 2 & 0 & -1 \\ -1/2 & 0 & 1 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix},$$

the scalar product of two roots α, β in the basis given by the simple roots $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ is defined as $\langle \alpha, \beta \rangle := \alpha^T A \beta$.

A.3 Commutator Relations in U

Let $s, t \in \mathbb{F}_q$. The non-trivial commutator relations in U are given as follows:

Table 1: Commutator Relations in U

$[x_1(t), x_3(s)] = x_5(ts)$	$[x_2(t), x_{15}(s)] = x_{18}(ts)$	$[x_5(t), x_{17}(s)] = x_{21}(ts)$
$[x_2(t), x_4(s)] = x_6(ts)$	$[x_1(t), x_{16}(s)] = x_{17}(ts)x_{20}(t^2s)$	$[x_7(t), x_{18}(s)] = x_{21}(ts)x_{23}(t^2s)$
$[x_3(t), x_4(s)] = x_7(ts)x_{10}(t^2s)$	$[x_3(t), x_{17}(s)] = x_{19}(ts)$	$[x_6(t), x_{22}(s)] = x_{24}(ts)$
$[x_4(t), x_5(s)] = x_8(ts)x_{15}(ts^2)$	$[x_4(t), x_{18}(s)] = x_{20}(ts)$	$[x_8(t), x_9(s)] = x_{17}(ts)$
$[x_3(t), x_6(s)] = x_9(ts)x_{13}(t^2s)$	$[x_1(t), x_{19}(s)] = x_{21}(ts)$	$[x_8(t), x_{13}(s)] = x_{19}(ts)x_{23}(t^2s)$
$[x_1(t), x_7(s)] = x_8(ts)$	$[x_3(t), x_{20}(s)] = x_{21}(ts)x_{22}(t^2s)$	$[x_8(t), x_{14}(s)] = x_{21}(ts)$
$[x_2(t), x_7(s)] = x_9(ts)x_{16}(ts^2)$	$[x_4(t), x_{22}(s)] = x_{23}(ts)$	$[x_9(t), x_{12}(s)] = x_{19}(ts)$
$[x_2(t), x_8(s)] = x_{11}(ts)x_{20}(ts^2)$	$[x_2(t), x_{23}(s)] = x_{24}(ts)$	$[x_9(t), x_{15}(s)] = x_{21}(ts)x_{24}(t^2s)$
$[x_3(t), x_8(s)] = x_{12}(ts)$	$[x_5(t), x_6(s)] = x_{11}(ts)x_{18}(t^2s)$	$[x_{10}(t), x_{11}(s)] = x_{19}(ts)x_{24}(ts^2)$
$[x_1(t), x_9(s)] = x_{11}(ts)$	$[x_5(t), x_7(s)] = x_{12}(ts)$	$[x_{10}(t), x_{18}(s)] = x_{22}(ts)$
$[x_1(t), x_{10}(s)] = x_{12}(ts)x_{15}(t^2s)$	$[x_5(t), x_9(s)] = x_{14}(ts)$	$[x_{10}(t), x_{20}(s)] = x_{23}(ts)$
$[x_2(t), x_{10}(s)] = x_{13}(ts)$	$[x_6(t), x_{10}(s)] = x_{16}(ts)$	$[x_{11}(t), x_{12}(s)] = x_{21}(ts)$
$[x_3(t), x_{11}(s)] = x_{14}(ts)$	$[x_7(t), x_{11}(s)] = x_{17}(ts)$	$[x_{13}(t), x_{15}(s)] = x_{22}(ts)$
$[x_2(t), x_{12}(s)] = x_{14}(ts)x_{22}(ts^2)$	$[x_6(t), x_{12}(s)] = x_{17}(ts)x_{23}(ts^2)$	$[x_{13}(t), x_{20}(s)] = x_{24}(ts)$
$[x_1(t), x_{13}(s)] = x_{14}(ts)x_{18}(t^2s)$	$[x_7(t), x_{14}(s)] = x_{19}(ts)$	$[x_{15}(t), x_{16}(s)] = x_{23}(ts)$
$[x_4(t), x_{13}(s)] = x_{16}(ts)$	$[x_6(t), x_{15}(s)] = x_{20}(ts)$	$[x_{16}(t), x_{18}(s)] = x_{24}(ts)$
$[x_4(t), x_{14}(s)] = x_{17}(ts)x_{20}(ts^2)$	$[x_5(t), x_{16}(s)] = x_{19}(ts)x_{22}(t^2s)$	

A.4 The Action of the Torus T on U :

Let $h \hat{=} (\lambda_1, \dots, \lambda_4)$ be a generic element of the maximal torus T and consider $x_i(t)^h := h^{-1}x_i(t)h$.

Table 2: Torus action on U

$x_1(t)^h = x_1(\lambda_1^2\lambda_3^{-1}t)$	$x_{13}(t)^h = x_{13}(\lambda_1^{-2}\lambda_2\lambda_3^2\lambda_4^{-1}t)$
$x_2(t)^h = x_2(\lambda_2^2\lambda_4^{-1}t)$	$x_{14}(t)^h = x_{14}(\lambda_2\lambda_3\lambda_4^{-1}t)$
$x_3(t)^h = x_3(\lambda_1^{-1}\lambda_3^2\lambda_4^{-1}t)$	$x_{15}(t)^h = x_{15}(\lambda_1^2\lambda_2^{-1}t)$
$x_4(t)^h = x_4(\lambda_2^{-1}\lambda_3^{-2}\lambda_4^2t)$	$x_{16}(t)^h = x_{16}(\lambda_1^{-2}\lambda_4t)$
$x_5(t)^h = x_5(\lambda_1\lambda_3\lambda_4^{-1}t)$	$x_{17}(t)^h = x_{17}(\lambda_3^{-1}\lambda_4t)$
$x_6(t)^h = x_6(\lambda_2\lambda_3^{-2}\lambda_4t)$	$x_{18}(t)^h = x_{18}(\lambda_1^2\lambda_2\lambda_4^{-1}t)$
$x_7(t)^h = x_7(\lambda_1^{-1}\lambda_2^{-1}\lambda_4t)$	$x_{19}(t)^h = x_{19}(\lambda_1^{-1}\lambda_3t)$
$x_8(t)^h = x_8(\lambda_1\lambda_2^{-1}\lambda_3^{-1}\lambda_4t)$	$x_{20}(t)^h = x_{20}(\lambda_1^2\lambda_3^{-2}\lambda_4t)$
$x_9(t)^h = x_9(\lambda_1^{-1}\lambda_2t)$	$x_{21}(t)^h = x_{21}(\lambda_1t)$
$x_{10}(t)^h = x_{10}(\lambda_1^{-2}\lambda_2^{-1}\lambda_3^2t)$	$x_{22}(t)^h = x_{22}(\lambda_3^2\lambda_4^{-1}t)$
$x_{11}(t)^h = x_{11}(\lambda_1\lambda_2\lambda_3^{-1}t)$	$x_{23}(t)^h = x_{23}(\lambda_2^{-1}\lambda_4t)$
$x_{12}(t)^h = x_{12}(\lambda_2^{-1}\lambda_3t)$	$x_{24}(t)^h = x_{24}(\lambda_2t)$

A.5 Conjugacy Classes in U

The conjugacy classes in U are given as follows. They are sorted by size.

- 1-element classes:
 $\{x_{21}(u)x_{24}(v)\}_{u,v \in \mathbb{F}_q}$
- q -elements classes:
 $\{x_{21}(v)x_{23}(u)x_{24}(a) \mid a \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{19}(u)x_{21}(a)x_{24}(v) \mid a \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$
- q^2 -elements classes:
 $\{x_{21}(v)x_{22}(u)x_{23}(a)x_{24}(b) \mid a, b \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{19}(u)x_{21}(a)x_{23}(v)x_{24}(b) \mid a, b \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{24}(v) \mid a, b \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$
- q^3 -elements classes:
 $\{x_{19}(u)x_{21}(a)x_{22}(v)x_{23}(b)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{20}(u)x_{21}(a)x_{22}(a^2u^{-1} + v)x_{23}(b)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{23}(v)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{24}(au + v) \mid a, b, c \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$
- q^4 -elements classes:
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{23}(v)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{19}(u)x_{20}(v)x_{21}(a)x_{22}(b)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{17}(u)x_{19}(a)x_{20}(v)x_{21}(b)x_{22}(au^{-1}v + w)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{22}(v)x_{23}(au^{-1}v + w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{22}(v)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{18}(u)x_{19}(v)x_{20}(a)x_{21}(b)x_{22}(c)x_{23}(b^2u^{-1} + w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1} + v)x_{21}(acu^{-1} + w)x_{22}(b^2u^{-1} + t)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v=0, w,t \in \mathbb{F}_q}$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au + v)x_{23}(bu + t)x_{24}(ab + w) \mid a, b, c, d \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, v=0, w,t \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{24}(cu + t) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \in \mathbb{F}_q}$
- q^5 -elements classes:
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{24}(eu + t) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{23}(v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}((au^{-1})^2v + w)x_{23}(cu^{-1}v + t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \in \mathbb{F}_q}$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au + v)x_{23}(e)x_{24}(ab + w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, v \neq 0, w \in \mathbb{F}_q}$,
 $\{x_{14}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(au^{-1}v + w)x_{21}(c)x_{22}(d)x_{23}(b^2u^{-2}v + t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \in \mathbb{F}_q}$,
 $\{x_{15}(u)x_{18}(a)x_{19}(v)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(aeu^{-1} + w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1} + v)x_{21}(acu^{-1} + w)x_{22}(c)x_{23}(d)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, v \neq 0, w \in \mathbb{F}_q}$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1} + z)x_{18}(a^2u^{-1} + y)x_{19}(c)x_{20}(a^2bu^{-2} + x)x_{21}(acu^{-1} + w)x_{22}(d)x_{23}(c^2u^{-1} + v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v,w,x,y=0, z \in \mathbb{F}_q}$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{17}(c)x_{19}(d)x_{20}(au + v)x_{21}(e)x_{22}(ab^2u^{-1} + t)x_{23}(du + y)x_{24}(ad + z) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u,v=0, w=t, y, z \in \mathbb{F}_q}$
- q^6 -elements classes:
 $\{x_{17}(u)x_{18}(v)x_{19}(a)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{20}(v)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{14}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(au^{-1}v + w)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(v)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}((au^{-1})^2v + w)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{23}(v)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v + t)x_{21}(e)x_{22}(b^2u^{-2}v + w)x_{23}(eu^{-1}v + s)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t, s \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(efv^{-2} + w) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$

$$\begin{aligned}
& \{x_{11}(u)x_{14}(a)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(cu^{-1}v+w)x_{23}(cdu^{-1}+t)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \in \mathbb{F}_q}, \\
& \{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+t)x_{19}(d)x_{20}(a^2cu^{-1}+s)x_{21}(adu^{-1}+z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1}+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t, s, r=0, z, y \in \mathbb{F}_q}, \\
& \{x_{12}(u)x_{14}(a)x_{15}(v)x_{17}(b)x_{18}(au^{-1}v+t)x_{19}(c)x_{20}(bu^{-1}v+w)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(fau^{-1}+c^2u^{-2}v+s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, s \in \mathbb{F}_q}, \\
& \{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(acu^{-1}+w)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, x, y \neq 0, z \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+t)x_{23}(f)x_{24}(afu^{-1}+dvu^{-1}+z) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t, z \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(uf+x)x_{24}(bf+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=v=t=s=0, x, y, z \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(aeu^{-1}+z)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=s=y=0, z, x \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(ue+x)x_{23}(ae+y)x_{24}(be+s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t, s, y=z=0, x \in \mathbb{F}_q},
\end{aligned}$$

• $q^7/2$ -elements classes:

$$\begin{aligned}
& \{x_{14}(u)x_{16}(v)x_{17}(a)x_{18}(w)x_{19}(b)x_{20}((c^2+c)vw^2u^{-2}+wau^{-2}+t)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=vw^2u^{-2}\eta \text{ or } t=0 \in \mathbb{F}_q}, \\
& \{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(w)x_{21}(d)x_{22}((e^2+e)u^2v^2w^{-1}+a^2wu^{-2}+s)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=u^2v^2w^{-1}\eta \text{ or } s=0},
\end{aligned}$$

• q^7 -elements classes:

$$\begin{aligned}
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w=0=t, s, x=0=z, y \in \mathbb{F}_q}, \\
& \{x_{15}(u)x_{17}(v)x_{18}(a)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}, \\
& \{x_{16}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}, \\
& \{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+t)x_{19}(d)x_{20}(a^2cu^{-1}+s)x_{21}(adu^{-1}+z)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t, s, r, z \in \mathbb{F}_q}, \\
& \{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+t)x_{19}(d)x_{20}(a^2cu^{-1}+s)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(d^2u^{-1}+y) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t, s, r \neq 0, y \in \mathbb{F}_q}, \\
& \{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}, \\
& \{x_{14}(u)x_{16}(v)x_{17}(a)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times}, \\
& \{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{22}(v)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}, \\
& \{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v+t)x_{19}(e)x_{20}(a^2cu^{-2}+s)x_{21}(f)x_{22}(fu^{-1}v+x)x_{23}(d^2u^{-2}v+w)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t=0, s, x \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+t)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w, t \in \mathbb{F}_q, w \text{ and/or } t \neq 0}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=z=0, x', y'=0, z' \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(ceu^{-1}+x')x_{22}(b^2fu^{-2}+y')x_{23}(g)x_{24}(agu^{-1}+z') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=0=z, x', y'=0, z' \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(aeu^{-1}+z)x_{22}(a^2eu^{-2}+y)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w=t=0=s, y \neq 0, z \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w=t=0, s \neq 0, y=0, x \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=s=y=0, x \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(ue+x)x_{23}(ae+y)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t, y \neq 0, z=0, x \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(bgu^{-1}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t, s, y=0, z \neq 0 \in \mathbb{F}_q},
\end{aligned}$$

$$\begin{aligned}
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(bgu^{-1}+s) \mid \\
& a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, s, y=z=0 \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v+t)x_{21}(e)x_{22}(b^2u^{-2}v+w) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0 \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1}+s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(du^{-1}v+w)x_{23}(g)x_{24}(c^2vu^{-2}+z) \mid \\
& a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t=s=0, z \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(g)x_{24}(u^{-1}bg+y) \mid \\
& a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, v=t=s=0, y, z \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(bfu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, t, s, x=y=z=0 \in \mathbb{F}_q},
\end{aligned}$$

- $q^8/2$ -elements classes:

$$\begin{aligned}
& \{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}((f^2+f)u^{-2}+s)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \text{ or } s=u^{-2}\eta \in \mathbb{F}_q}, \\
& \{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{18}(w)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}((g^2+g)u^2w^{-1}v^{-1}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0 \text{ or } s=u^2w^2v^{-1}\eta \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w+t)x_{21}(e)x_{22}((f^2+f)u^2v^2w^{-1}+b^2wu^{-2}+s)x_{23}(g)x_{24}(h) \mid \\
& a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=0 \text{ or } s=u^2v^2w^{-1}\eta \in \mathbb{F}_q}, \\
& \{x_{12}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(au^{-1}v+t)x_{19}(c)x_{20}((d^2+d)v^2wu^{-2}+vbu^{-2}+s)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s=u^{-2}wv^2\eta, \text{ or } s=0 \in \mathbb{F}_q},
\end{aligned}$$

- q^8 -elements classes:

$$\begin{aligned}
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=z=0, y' \neq 0, z' \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=x=y=z=0, x', y' \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=0, x \neq 0, y=z=0, x', y' \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(ceu^{-1}+x')x_{22}(b^2fu^{-2}+y')x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=0=z, x', y' \neq 0 \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2}+y')x_{23}(h)x_{24}(ahu^{-1}+z') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=0, z \neq 0, y'=0, z' \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2}+y')x_{23}(h)x_{24}(ahu^{-1}+z') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o \neq 0, x=y=0=z, y' \neq 0, z' \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w=0, t \neq 0, x=0=z, y \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w=t=0, s, x=0, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(e)x_{19}(f)x_{20}(ab+w)x_{21}(g)x_{22}(h)x_{23}(af+y)x_{24}(bhu^{-1}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t, s, y \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, t=0, y \neq 0, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, y, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1}+z)x_{22}(a^2gu^{-2}+y)x_{23}(e^2u^{-1}+x)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s=y=0, z, x \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2}+y)x_{23}(g)x_{24}(h) \mid
\end{aligned}$$

$a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v, w=t=0, s \neq 0, y \neq 0 \in \mathbb{F}_q}$,
 $\{x_{14}(v)x_{15}(u)x_{16}(w)x_{17}(a)x_{18}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{15}(u)x_{16}(v)x_{17}(a)x_{18}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times}$,
 $\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w=0=t \in \mathbb{F}_q}$,
 $\{x_{12}(u)x_{14}(a)x_{16}(w)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$,
 $\{x_{12}(u)x_{14}(a)x_{16}(v)x_{17}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times}$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w + t)x_{21}(e)x_{22}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w=0, t \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w + t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v + t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{20}(v)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times}$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(gu^{-1}v + y)x_{23}(h)x_{24}(e^2u^{-2}v + hbu^{-1} + x) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, t=w=s=r=o=z, y, x \in \mathbb{F}_q}$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(u^{-1}bc^2 + z)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, w, v=0, t \neq 0, s=0, z \in \mathbb{F}_q}$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(cdu^{-1} + v)x_{20}((af + c^2)u^{-1} + y)x_{21}(g)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0, w, s, x=0, y \neq 0, z=0 \in \mathbb{F}_q}$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(cdu^{-1} + v)x_{20}((af + c^2)u^{-1} + y)x_{21}(g)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s, x=0, y, z=0 \in \mathbb{F}_q}$,

• $q^9/2$ -elements classes:

$\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}((g^2 + g)u^2t^2w^{-1} + x)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, z, x=0 \text{ or } x=u^2t^2w^{-1}\eta \in \mathbb{F}_q}$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(a^2fu^{-2} + y)x_{23}((h^2 + h)w^{-2}y^2u + x)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, w \neq 0, t=0, s, y \neq 0, x=0 \text{ or } x=w^{-2}y^2u \in \mathbb{F}_q}$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2 + i)u^2v^{-1}t^2 + z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s, z=0 \text{ or } z=u^2v^{-1}t^2\eta \in \mathbb{F}_q}$,
 $\{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2 + i)u^2t^2v^{-1} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0 \in \mathbb{F}_q, s=u^2t^2v^{-1}\eta, \text{ or } s=0 \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^{-2}v + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \text{ or } s=u^{-2}v\eta \in \mathbb{F}_q}$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^2t^2v^{-1} + ud + z)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0 \text{ or } z=u^2t^2v^{-1}\eta \in \mathbb{F}_q}$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}((g^2 + g)v^2wu^{-2} + dvu^{-1} + z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v, w \in \mathbb{F}_q^\times, t=0=s, z=0 \text{ or } z=v^2wu^{-2}\eta \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^2w^2v^{-1} + wd + cv^{-1}(wb + g) + r)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v, w \in \mathbb{F}_q^\times, t=s=0, r=0 \text{ or } r=u^2w^2v^{-1}\eta \in \mathbb{F}_q}$,
 $\{x_{11}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)v^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s=0 \text{ or } s=v^{-2}\eta \in \mathbb{F}_q}$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}((f^2 + f)u^2v^2w^{-1} + ua + s)x_{21}(g)x_{22}(b^2fu^{-2} + t)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=0 \text{ or } s=u^2v^2w^{-1}\eta \in \mathbb{F}_q}$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}((d^2 + d)uvw^{-2} + awv^{-2} + s)x_{19}(e)x_{20}(cv^{-1}w + b(d^2 + d)w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s=uvw^{-2}\eta \text{ or } s=0 \in \mathbb{F}_q}$,

• q^9 -elements classes:

$\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(cv^{-1}w + b(d^2 + d)w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u, v \in \mathbb{F}_q^\times, w=0, t \in \mathbb{F}_q}$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=0, x \neq 0, y, z=0, y' \in \mathbb{F}_q}$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(ch + x') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=r=o=0, y, z \neq 0, x', y' \in \mathbb{F}_q}$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$

$$\begin{aligned}
& r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=0=x=y=z=0, y' \neq 0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + \\
& r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, r=0=x=y=z=0, x', y' \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + \\
& r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o \neq 0, x=y=0, z \neq 0, y'=0 \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + \\
& r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=0, z, y' \neq 0 \in \mathbb{F}_q, o \text{ or } z \neq 0}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + \\
& o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1} + x')x_{22}(b^2hu^{-2} + y')x_{23}(i)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0=s, t \neq 0, r=0=x=y=0, x', y', z \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(uf + y)x_{21}(g)x_{22}(ayu^{-1} + w)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v, w \neq 0, x=0=z, y \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v, w=0, t \neq 0, x=0, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v, w \neq 0, t, x=0, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f + v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1} + w)x_{23}(b^2gu^{-2} + t)x_{24}(c^2xu^{-2} + \\
& s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v, w=0, t, s, x \neq 0 \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t=0, z \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(bhu^{-1} + s) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, s \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(bhu^{-1} + s) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t=0, s \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1} + z)x_{22}(a^2gu^{-2} + y)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, t \neq 0, s=0, y \neq 0, z \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(a^2fu^{-2} + y)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, w \neq 0, t=0, s, y=0 \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(e) \mid a, b, c, d, e, f \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=0=t \in \mathbb{F}_q}, \\
& \{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t, s \in \mathbb{F}_q, t \text{ or } s \neq 0}, \\
& \{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0 \in \mathbb{F}_q}, \\
& \{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, t=0, w \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=t=0 \in \mathbb{F}_q}, \\
& \{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t, s \in \mathbb{F}_q, t \text{ or } s \neq 0}, \\
& \{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(a^2bu^{-1} + t)x_{19}(e)x_{20}(a^2cu^{-1} + s)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w, t, s \in \mathbb{F}_q}, \\
& \{x_{13}(u)x_{14}(a)x_{15}(v)x_{16}(b)x_{17}(abu^{-1} + w)x_{18}(c)x_{19}(d)x_{20}(e) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}, \\
& \{x_{11}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}, \\
& \{x_{11}(w)x_{12}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1} + t) \mid a, b, c, d, e \in \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \in \mathbb{F}_q}, \\
& \{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + t) \mid a, b, c, d, e, f, g \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0 \in \mathbb{F}_q},
\end{aligned}$$

$$\begin{aligned}
& \{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + t) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0 \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b + v)x_{21}(g)x_{22}(u^{-1}bc^2 + z)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v=0, s \neq 0, z \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v \neq 0, t, s=0 \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(g)x_{20}((af + c^2)u^{-1} + y)x_{21}(h)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s, x=0, y, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag + c^2)u^{-1} + y)x_{21}(bgu^{-1} + t)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, t, s, x \neq 0, y=0 \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b + v)x_{21}(g)x_{22}(d^2s^{-2}v + z)x_{23}(h)x_{24}(j) \mid \\
& a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v \neq 0, s \neq 0, z \in \mathbb{F}_q},
\end{aligned}$$

• $q^{10}/2$ -elements classes:

$$\begin{aligned}
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=s, t=0, r=o=0=x, y \neq 0, z=0, z'=0 \text{ or } z'=u^2y \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2r^{-1}x^2 + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=0, r \neq 0, o=0, x \neq 0, y=z=0, z'=0 \text{ or } z'=u^2r^{-1}x^2 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0, z'=0 \text{ or } z'=u^2y \in \mathbb{F}_q, ry=x^2}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)u(y')^2x^{-2} + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r=0, o=0, x \neq 0, y=0, y' \neq 0, z'=0 \text{ or } z'=u(y')^2x^{-2} \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ux^{-2}(y')^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z'=0 \text{ or } z'=u(y')^2x^{-2} \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o=0, x=y=0, y' \neq 0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q, rx=y'}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \neq 0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q, rx=y', ur^2o=s^2x^2}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)v^{-1}u^2t^2 + s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t \neq 0, s=0 \text{ or } s=v^{-1}u^2t^2 \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(a^2gu^{-2} + y)x_{23}((i^2 + i)t^{-1}s^{-1}y^2u + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \neq 0, x=0 \text{ or } x=t^{-1}s^{-1}y^2u \in \mathbb{F}_q}, \\
& \{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i^2 + i)u^2v + uf + be + dhv^{-1} + z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, z=0 \text{ or } z=u^2v \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h^2v)x_{23}((i^2 + i)uv^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0, y=0, \text{ or } y=uv^2 \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^2w^2r^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, r \neq 0, o=z=0, y=0 \text{ or } y=u^{-2}v^2w^2r^{-1} \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2z^2v^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=r=o=z=0, y=0 \text{ or } y=v^2wu^{-2} \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2z^2v^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=0, s, r=o=0, z \neq 0, y=0 \text{ or } y=u^2z^2v^{-1} \in \mathbb{F}_q},
\end{aligned}$$

$$\begin{aligned}
& \{x_9(u)x_{11}(a)x_{14}(b)x_{15}(v)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(h)x_{24}((j^2 + j)u^2v + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, y=0 \text{ or } y=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2w^2v^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=z=0, y=0 \text{ or } y=u^2w^2v^{-1}\eta \in \mathbb{F}_q}, \\
& \{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)vz^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0, y=0 \text{ or } y=vz^2\eta \in \mathbb{F}_q}, \\
& \{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)v-1uw + bdwv^{-2} + (bi + dh)v^{-1} + ug + af + r) \mid a, b, c, d, e, f, g, h, j, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=z=0, r=0 \text{ or } r=v^{-1}uw\eta \in \mathbb{F}_q}, \\
& \{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1} + w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1} + t)x_{19}(f)x_{20}(a^2du^{-2} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)uv^2 + f^2u^{-1} + r) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=s, r=0 \text{ or } r=uv^2\eta \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + s)x_{16}(ub + w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)uv + y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, y=0 \text{ or } y=uv\eta \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^{-1}(u^2s + vv)^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s \neq vwu^{-2}, t=0=z, y=0 \text{ or } y=u^{-2}v^{-1}(u^2s + vv)^2\eta \in \mathbb{F}_q, w \text{ or } s=0}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)v^{-1}u^2z^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s, r=0, z \neq 0, y=0 \text{ or } y=v^{-1}u^2z^2\eta \in \mathbb{F}_q, sv \neq z^2}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)v^{-1}u^2z^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s=v^{-1}z^2, r=0, z \neq 0, y=0 \text{ or } y=v^{-1}u^2z^2\eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + w)x_{16}(v)x_{17}(e)x_{18}(bu + t)x_{19}(f)x_{20}(ab + e^2v^{-1} + s)x_{21}(g)x_{22}((h^2 + h)u^2v + y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=s, y=0 \text{ or } y=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^{-2}t^2v + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0, y, x=0 \text{ or } x=u^{-2}t^2v\eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^2v^{-1}w^2 + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=z=0, y, x=0 \text{ or } x=u^2v^{-1}w^2\eta \in \mathbb{F}_q}, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}((i^2 + i)us^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, z=0, y=0 \text{ or } y=us^2\eta \in \mathbb{F}_q, us^2=v^2t}, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}((i^2 + i)us^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, z=0, y=0 \text{ or } y=us^2\eta \in \mathbb{F}_q}, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fv^{-1}w + x)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=z=0, x, y=0 \text{ or } y=uv^{-2}w^2\eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1} + z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=r=0, z, y=0 \text{ or } y=u^2v^2w^{-1}\eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{10} -elements classes:

$$\begin{aligned}
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r \neq 0, o=0, x=y=z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=0, o=0, x, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r=0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r=0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q},
\end{aligned}$$

$$\begin{aligned}
& r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q, ry \neq x^2,} \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(bh+y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=0, r=o=0=y, z \neq 0, y' \in \mathbb{F}_q,} \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=0, r, o=0, x, y, z \in \mathbb{F}_q,} \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(ch+x') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r=0, o \neq 0, x \neq 0, y=z=0, x' \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r=0, o \neq 0, y=0, y'=0 \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r=0, o \neq 0, x \neq 0, y=0, y' \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r=0, o, x, y=0, y' \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r \neq 0, o, x=y=0, y' \neq 0 \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r \neq 0, o \neq 0, x=y=0, y'=0 \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, rx \neq y'} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, rx=y'} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, rx=y'} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, rx=y', ur^2 \neq s^2x^2,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o, x=0, y \neq 0, z \in \mathbb{F}_q,} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2}+y')x_{23}(j)x_{24}(f^2u^{-1}+z) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=0, o \neq 0, y=0, y', z \in \mathbb{F}_q,} \\
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab+z)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1}+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, s=t=0=z, y \in \mathbb{F}_q,} \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(a^2gu^{-2}+y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{t \neq 0, s \neq 0, y=0 \in \mathbb{F}_q,} \\
& \{x_{11}(v)x_{12}(t)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \in \mathbb{F}_q,} \\
& \{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{15}(v)x_{16}(t)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w, t \in \mathbb{F}_q \text{ or } t \neq 0,} \\
& \{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(t)x_{17}(d)x_{18}(w)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \in \mathbb{F}_q,} \\
& \{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1}+s)x_{19}(f)x_{20}(c^2dv^{-2}+t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}_{w, t, s \in \mathbb{F}_q, w, t, s \text{ not all } = 0,} \\
& \{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \} \\
& \mathbb{F}_q \}
\end{aligned}$$

$a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t, s \in \mathbb{F}_q, \text{ some of } w, t, s \neq 0$}
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(j)x_{24}(c^2xu^{-2}+s) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v, w \neq 0, s, x \neq 0 \in \mathbb{F}_q$}
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}((j^2+j)vs^2u^{-2}+y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, t=0=w=s, z, y, x \neq 0, y' \in \mathbb{F}_q$}
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(i)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, w, s, x \neq 0, y \neq 0 \in \mathbb{F}_q$}
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(aeu^{-1}+w)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t=z=0 \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s, r, o=svr^{-1}, z \in \mathbb{F}_q, s \text{ or } z \neq 0$}
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, t=0=w, s, r \neq 0, o=svr^{-1}, z \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, r=0, o=z=0 \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, r \neq 0, o=svr^{-1}, z=0 \in \mathbb{F}_q$}
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+tx_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=s=0=z, \in \mathbb{F}_q$}
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+tx_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t \neq 0, s=0=z, \in \mathbb{F}_q$}
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+tx_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t, s=0, z \neq 0, \in \mathbb{F}_q, w \text{ or } t \neq 0$}
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v+w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v+t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1}+s)x_{19}(f)x_{20}(a^2du^{-2}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, j, i \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=0, t, s, z \in \mathbb{F}_q$}
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v+w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v+t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1}+s)x_{19}(f)x_{20}(a^2du^{-2}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, j, i \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, t, s, z \in \mathbb{F}_q, s, t, z \text{ not all } = 0$}
 $\{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1}+w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1}+t)x_{19}(f)x_{20}(a^2du^{-2}+tk+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t, s \in \mathbb{F}_q, \text{ not all } = 0$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+s)x_{16}(ub+w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t=0, s \in \mathbb{F}_q, w \text{ or } s \neq 0$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0=z \in \mathbb{F}_q, u^2s \neq vw$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, s \neq vw u^{-2}, t=0, z \neq 0 \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, s=vw u^{-2}, t=0, z \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=0, s, r=z=0 \in \mathbb{F}_q$}
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=0, s, r \neq 0, z \in \mathbb{F}_q$}
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+w)x_{16}(v)x_{17}(e)x_{18}(bu+tx_{19}(f)x_{20}(ab+e^2v^{-1}+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=0, t, s \in \mathbb{F}_q, \text{ some of } t, s \neq 0$}
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad+y)x_{18}(bu+e^2v)x_{19}(f)x_{20}(e^2w+ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=z=0, y \in \mathbb{F}_q$}
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad+y)x_{18}(bu+e^2v)x_{19}(f)x_{20}(e^2w+ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t, z \neq 0, y \in \mathbb{F}_q, \text{ some of } w, t=0$}
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=s=z=0 \in \mathbb{F}_q$}
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t=0, s, z \in \mathbb{F}_q, w \text{ and/or } z \neq 0$}
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w, t \neq 0, s=v\sqrt{t/u}, z \in \mathbb{F}_q, w \text{ and/or } z \neq 0$}

• $q^{11}/2$ -elements classes:

- $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}s^2o + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=0, r \neq 0, o=0, x=0, y=z=0, x'=0 \text{ or } x'=u^{-2}s^2o \in \mathbb{F}_q}$
- $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2y^2o^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s=t=0, r \neq 0, o \neq 0, x=0, y \neq 0, z=0, x'=0 \text{ or } x'=u^2y^2o^{-1} \in \mathbb{F}_q}$
- $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}((j^2 + j)t^2wu^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0, o=x=y=z=0, y'=0 \text{ or } y'=t^2wu^{-2} \in \mathbb{F}_q}$
- $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}((j^2 + j)t^2wu^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r \neq 0, o=0, x \neq 0, y=or/u, z=0, x'=0 \text{ or } x'=u^2x^2t^{-1} \in \mathbb{F}_q}$
- $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2x^2t^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=or/u, z=0, x'=0 \text{ or } x'=u^2x^2t^{-1} \in \mathbb{F}_q, x^2=ry}$
- $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=0, y'=0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q, ur^2t=v^2x^2, v^2o=ur^2}$
- $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q, ur^2t=v^2x^2, y'=xr}$
- $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t=0, r \neq 0, o=x=y=0, y'=0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q}$
- $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o \neq 0, x=y=0, y'=0, z'=0 \text{ or } z'=ur^2 \in \mathbb{F}_q, v^2o=ur^2}$
- $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s, t=0, r, y=0 \text{ or } y=u^2v^2w^{-1} \in \mathbb{F}_q, \text{ some of } r, s \neq 0}$
- $\{x_6(u)x_8(v)x_9(a)x_{11}(bu + fv)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2d + w)x_{23}((j^2 + j)us^2 + y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, y=0 \text{ or } y=us^2 \in \mathbb{F}_q, us^2 \neq v^2t}$
- $\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=0, z \neq 0, y=0 \text{ or } y=uv^{-2}w^2 \in \mathbb{F}_q}$
- $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(vu^{-1}i + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=r=o=z=0, y, s=0 \text{ or } s=vw^2u^{-2} \in \mathbb{F}_q}$
- $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=tu^{-1}, s, t \neq 0, r=0, y=0 \text{ or } y=u^{-2}vt^2 \in \mathbb{F}_q}$
- $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s, t \neq 0, r=0, y=0 \text{ or } y=u^{-2}vt^2 \in \mathbb{F}_q}$
- $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}((j^2 + j)vt^2 + y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, s=r=0, z, y=0 \text{ or } y=vt^2 \in \mathbb{F}_q}$
- $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s=x, y, z \neq 0, r=0 \text{ or } r=u^2z^2 \in \mathbb{F}_q}$
- $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)uv^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=x, y, z, o=0 \text{ or } o=uv^{-2}w^2 \in \mathbb{F}_q}$
- $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x, y, z \neq 0, r=0 \text{ or } r=u^2z^2 \in \mathbb{F}_q, z=v\sqrt{s/u}}$
- $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} +$

$$\begin{aligned}
& \{y x_{18}(g) x_{19}(h) x_{20}((i^2 + i) u v^{-2} w^2 + o) x_{21}(j) x_{22}(v h + r) x_{23}(g b^2 u^{-2} + x) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s = 0, x, y, r, o = 0 \text{ or } o = u v^{-2} w^2 \eta \in \mathbb{F}_q, v t = w} \\
& \{x_5(v) x_7(u) x_8(a) x_9(b) x_{10}(w) x_{11}(c) x_{12}(d) x_{13}(b u^{-1} w + s) x_{14}(e) x_{15}((f^2 + f) v^2 u^2 w^{-1} + y) x_{16}(c v^{-1} w + t) x_{17}(g) x_{18}(a^2 b u^{-3} + r) x_{19}(h) x_{20}(c f v^{-1} + a c^2 u^{-1} v^{-2} w^2 + o) x_{21}(i) x_{22}(j) x_{23}(k) x_{24}(u^{-1} b k + z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t = s = 0 = r = o, z, y = 0 \text{ or } y = v^2 u^2 w^{-1} \eta \in \mathbb{F}_q} \\
& \{x_2(u) x_6(a) x_9(b) x_{11}(c) x_{12}(v) x_{13}(d) x_{14}(e) x_{16}((b^2 + a d) u^{-1} + t) x_{17}(f) x_{18}(g) x_{19}(h) x_{20}((c^2 + a g) u^{-1} + s) x_{21}(i) x_{22}((j^2 + j) u w^2 + x) x_{23}(v f + w) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t = 0 = s, x = 0 \text{ or } x = u v^2 \eta \in \mathbb{F}_q} \\
& \{x_1(u) x_5(a) x_6(v) x_8(b) x_9(a u^{-1} v + t) x_{11}(c) x_{12}(d) x_{13}(a^2 u^{-2} v + w) x_{14}(e) x_{15}(d u + s) x_{16}(u^{-1} v d + x) x_{17}(f) x_{18}(g) x_{19}(a f u^{-1} + z) x_{20}(h) x_{21}(i) x_{22}(d e + a^2 h u^{-2} + y) x_{23}((j^2 + j) v s^2 u^{-2} + y') x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t = 0 = w, s \neq 0, z, y, x, y' = 0 \text{ or } y' = v s^2 u^{-2} \eta \in \mathbb{F}_q, v s \neq u^2 x} \\
& \{x_1(u) x_5(a) x_6(v) x_8(b) x_9(a u^{-1} v + t) x_{11}(c) x_{12}(d) x_{13}(a^2 u^{-2} v + w) x_{14}((e^2 + e) u t^2 v^{-1} + z') x_{15}(d u + s) x_{16}(u^{-1} v d + x) x_{17}(f) x_{18}(g) x_{19}(a f u^{-1} + z) x_{20}(h) x_{21}(i) x_{22}(d e + a^2 h u^{-2} + y) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w = 0, s, z, y, x, z' = 0 \text{ or } z' = u t^2 v^{-1} \eta \in \mathbb{F}_q} \\
& \{x_1(u) x_5(a) x_8(b) x_{11}(c) x_{12}(d) x_{14}(e) x_{15}(u d + x) x_{16}(v) x_{17}(f) x_{18}(e u + z) x_{19}(g) x_{20}((h^2 + h) u^2 v + y) x_{21}(i) x_{22}(d e + w) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, z = x = 0, y = 0 \text{ or } y = u^2 v \eta \in \mathbb{F}_q}
\end{aligned}$$

• q^{11} -elements classes:

$$\begin{aligned}
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(c u + s) x_{14}(e) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(f) x_{18}(a d + w) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = 0 = w, s, t = 0, r, o = 0, y, z \neq 0 \in \mathbb{F}_q, \text{ some of } r, y \neq 0} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(c u + s) x_{14}(e) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(f) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = 0, s = t = 0 = r, o \neq 0, x = 0, y = z = 0 \in \mathbb{F}_q} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(c u + s) x_{14}(e) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(f) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = 0, s \neq 0, t = 0 = r, o \neq 0, x = 0, y \neq 0, z = 0 \in \mathbb{F}_q} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(c u + s) x_{14}(e) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(f) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = 0, s, t = 0, r, o \neq 0, x, y, z = 0 \in \mathbb{F}_q, r \text{ or } x \neq 0} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(e) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(a d + w) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, t \neq 0, r = 0, o, x, y, z = 0 \in \mathbb{F}_q, \text{ some of } o, y \neq 0} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(e) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(a d + w) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, t \neq 0, r \neq 0, o, x = 0, y, z = 0 \in \mathbb{F}_q} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(e) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(a d + w) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y = o r / u, z = 0 \in \mathbb{F}_q, x^2 \neq r y} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(a c u^{-1} + z) x_{12}(d) x_{13}(e) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(b e u^{-1} + c d u^{-1} + x) x_{18}(a d + w) x_{19}(g) x_{20}(a b d u^{-1} + y) x_{21}(h) x_{22}(i) x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, t \neq 0, r \neq 0, o, x \neq 0, y \neq o r / u, z = 0 \in \mathbb{F}_q} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(d) x_{12}(e) x_{13}(c u + s) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(g) x_{18}(a e + w) x_{19}(h) x_{20}(a b e u^{-1} + y) x_{21}(i) x_{22}(j) x_{23}(k) x_{24}(c i + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = 0 = s, t, r = 0 = o = y, x' \in \mathbb{F}_q} \\
& \{x_3(u) x_5(a) x_7(b) x_8(a b u^{-1} + v) x_9(c) x_{10}(b u + t) x_{11}(d) x_{12}(e) x_{13}(c u + s) x_{14}(f) x_{15}(a^2 b u^{-1} + o) x_{16}(c b + c u^{-1} t + r) x_{17}(g) x_{18}(a e + w) x_{19}(h) x_{20}(a b e u^{-1} + y) x_{21}(i) x_{22}(j) x_{23}(k) x_{24}(c i + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = s = 0, t, r = 0, o \neq 0, y = 0, x' \in \mathbb{F}_q} \\
& \{x_4(u) x_6(a) x_7(b) x_8(c) x_9(a b u^{-1} + v) x_{10}(b^2 u^{-1} + t) x_{11}(a c u^{-1} + s) x_{12}(d) x_{13}(a b^2 u^{-1} + w) x_{14}(a b c u^{-2} + r) x_{15}(c^2 u^{-1} + o) x_{16}(e) x_{17}(f) x_{18}(a c^2 u^{-2} + y) x_{19}(g) x_{20}(h) x_{21}(i) x_{22}(b^2 h u^{-2} + y') x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, s, t \neq 0, r, o, y = 0, y' \in \mathbb{F}_q, \text{ some of } s, r \neq 0} \\
& \{x_4(u) x_6(a) x_7(b) x_8(c) x_9(a b u^{-1} + v) x_{10}(b^2 u^{-1} + t) x_{11}(d) x_{12}(b c u^{-1} + x) x_{13}(a b^2 u^{-1} + w) x_{14}(a b c u^{-2} + r) x_{15}(c^2 u^{-1} + o) x_{16}(e) x_{17}(f) x_{18}(a c^2 u^{-2} + y) x_{19}(g) x_{20}(h) x_{21}(i) x_{22}(b^2 h u^{-2} + y') x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = 0, s, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y = 0, y' \neq 0 \in \mathbb{F}_q, u r^2 t = v^2 x^2, v^2 o = u r^2} \\
& \{x_4(u) x_6(a) x_7(b) x_8(c) x_9(a b u^{-1} + v) x_{10}(b^2 u^{-1} + t) x_{11}(d) x_{12}(b c u^{-1} + x) x_{13}(a b^2 u^{-1} + w) x_{14}(a b c u^{-2} + r) x_{15}(c^2 u^{-1} + o) x_{16}(e) x_{17}(f) x_{18}(a c^2 u^{-2} + y) x_{19}(g) x_{20}(h) x_{21}(i) x_{22}(b^2 h u^{-2} + y') x_{23}(j) x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = 0, s, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y = 0, y' \neq 0 \in \mathbb{F}_q}
\end{aligned}$$

$$\begin{aligned}
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, ur^2t=v^2x^2, y' \neq xr'} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=0, y'=0 \in \mathbb{F}_q, ur^2t=v^2x^2, v^2o \neq ur^2} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \in \mathbb{F}_q, ur^2t \neq v^2x^2} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t=0, r \neq 0, o \neq 0, x=y=0, y'=0 \in \mathbb{F}_q, v^2o \neq ur^2} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t=0, r \neq 0, o, x=y=0, y' \neq 0 \in \mathbb{F}_q} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r=0, o, x, y=0, y' \in \mathbb{F}_q} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o, x=y=0, y' \in \mathbb{F}_q} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=0, r \neq 0, o, x \neq 0, y=0, y' \in \mathbb{F}_q} \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \\
& \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r, o, x, y \neq 0 \in \mathbb{F}_q, \text{ some } s, x, r \neq 0} \\
& \{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + w)x_{16}(v)x_{17}(e)x_{18}(bu + kw)x_{19}(f)x_{20}(ku^{-1}wa + ab + e^2v^{-1} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad)x_{18}(bu + e^2v + kt)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad)x_{18}(bu + e^2v + kt)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab + z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(bu + fv)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(d)x_{16}(d)x_{17}(e)x_{18}(b^2u)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2d + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{18}(v)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(d)x_{15}(v)x_{16}(ub + w)x_{17}(e)x_{18}(bu^{-1}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(a^2u^{-2}g + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(a^2eu^{-2} + w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\
& \{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}
\end{aligned}$$

$a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, t, s \neq 0, \in \mathbb{F}_q}$,
 $\{x_9(u)x_{11}(a)x_{12}(t)x_{13}(w)x_{14}(b)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, t \in \mathbb{F}_q}$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{15}(w)x_{16}(t)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, t \in \mathbb{F}_q}$,
 $\{x_8(u)x_9(t)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, t, s \in \mathbb{F}_q}$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(((ucs^{-1} + ab)^2 + ucs^{-1} + ab)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, t=0, s \neq 0, y \in \mathbb{F}_q}$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(aev^{-1} + z) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, t, s=0, z \in \mathbb{F}_q}$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, s, t=0, r \in \mathbb{F}_q}$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, s, t \neq 0, r \neq 0, \in \mathbb{F}_q}$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w \neq 0, s, t \neq 0, r=0, \in \mathbb{F}_q, uv \neq t}$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, t, s, r=0, z \in \mathbb{F}_q}$ some of $s, z \neq 0$ or $t=0$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=0=s, x, y, z=0 \in \mathbb{F}_q}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x, y, z=0 \in \mathbb{F}_q}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x, y, z=0 \in \mathbb{F}_q}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x, y, z \neq 0, \in \mathbb{F}_q, z \neq v\sqrt{s/u}}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq 0, x, y, z \in \mathbb{F}_q}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0, x, y, z \in \mathbb{F}_q, vt \neq w}$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq 0, x, y, r \in \mathbb{F}_q, vt = w}$,
 $\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + 0)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + 0)x_{19}(g)x_{20}(h)x_{21}(hav^{-1} + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=s=0, z \in \mathbb{F}_q}$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfu^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(u^{-1}bk + z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=t=s=0, z \in \mathbb{F}_q}$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2 + ag)u^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(vf +aju^{-1} + w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, t=0, s \neq 0 \in \mathbb{F}_q}$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, t=0=w, s, z, y, x \in \mathbb{F}_q, vs = u^2x}$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(dgu^{-1} + w)x_{23}(diu^{-1} + t)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w=0, t, x=0 \in \mathbb{F}_q}$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w, z \neq 0, x=0 \in \mathbb{F}_q}$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(cdu^{-1} + s)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, s, x=z=0 \in \mathbb{F}_q}$

- $q^{12}/4$ -elements classes:

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r \neq 0, z, y, x=0}$ or $x=u^2v^{-1}w^2\eta, x'=0}$ or $x'=u^{-2}vs^2\eta \in \mathbb{F}_q, vr=w^2$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}((k^2 + k)vs^2u^{-2} + y')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s \neq 0, z, y, z'=0}$ or $z'=ut^2v^{-1}\eta, y'=0}$ or $y'=vs^2u^{-2}\eta \in \mathbb{F}_q, vw=t^2$

- $q^{12}/2$ -elements classes:

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)u^{-2}v^2t^2s^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, y=0}$ or $y=u^{-2}v^2t^2s^{-1}\eta \in \mathbb{F}_q$,

$$\begin{aligned}
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}((h^2+h)v^2t^2u^{-2}s^{-1}+z)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z=0 \text{ or } z=v^2t^2u^{-2}s^{-1} \eta \in \mathbb{F}_q}, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}((f^2+f)(v^2t+us^2)^{-1}uw^2t+ \\
& y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s \neq v\sqrt{t/u}, y=0 \text{ or } y=u(v^2t+us^2)^{-2}w^2t\eta \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+(l^2+l)w^{-1}vt+z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, z \in \mathbb{F}_q, z=0 \text{ or } z=w^{-1}vt\eta \in \mathbb{F}_q}, \\
& \{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}((h^2+h)wv^2u^{-2}+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+ \\
& o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, r, o, z, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q, z, r, o \text{ not all } =0}, \\
& \{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w+s)x_{14}(e)x_{15}((f^2+f)v^2u^2w^{-1}+y)x_{16}(cu^{-1}w+ \\
& t)x_{17}(g)x_{18}(a^2bu^{-3}+r)x_{19}(h)x_{20}(cfv^{-1}+ac^2u^{-1}v^{-2}w^2+o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s, r, o, y=0 \text{ or } y=v^2u^2w^{-1}\eta \in \mathbb{F}_q, t, s, r, o \text{ not all } =0}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+ \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-2}vs^2+x')x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t=0, s \neq 0, r=0, z, y, x'=0 \text{ or } x'=u^{-2}vs^2\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+ \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-2}vs^2+x')x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=0, z, y, x'=0 \text{ or } x'=u^{-2}vs^2\eta \in \mathbb{F}_q, s=ut}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+ \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}((i^2+i)u^2v^{-1}w^2+x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s=ut, r=0, z, y, x=0 \text{ or } x=u^2v^{-1}w^2\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+ \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}((i^2+i)u^2v^{-1}w^2+x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=ut, r \neq 0, z, y, x=0 \text{ or } x=u^2v^{-1}w^2\eta \in \mathbb{F}_q, vr=w^2}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+ \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-2}vs^2+x')x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq ut, r=v^{-1}w^2, z, y, x'=0 \text{ or } x'=u^{-2}vs^2\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}((e^2+e)ut^2v^{-1}+z')x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1}+ \\
& z)x_{20}(i)x_{21}(j)x_{22}(de+a^2gu^{-2}+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z, y, z'=0 \text{ or } z'=ut^2v^{-1}\eta \in \mathbb{F}_q, vw=t^2}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+ \\
& s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}((k^2+k)vy^2+y')x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r, z, y \neq 0, x, z', y'=0 \text{ or } y'=vy^2\eta \in \mathbb{F}_q}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+ \\
& g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s=t \neq 0, r=0, z, y, x, z', y'=0 \text{ or } y'=vu^{-2}t^2\eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+ \\
& z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}((l^2+l)v^{-2}u^{-1}(v^2z'+ux)^2+x') \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z, y, x, z', x'=0 \text{ or } x'=v^{-2}u^{-1}(v^2z'+ux)^2\eta \in \mathbb{F}_q, v^2z' \neq ux}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+ \\
& r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}(l) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, z, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q, v^2z'=ux}, \\
& \{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(acu^{-1}+s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2+j)uv^2+x)x_{21}(k)x_{22}(f^2u^{-1}+t)x_{23}(iv+ \\
& w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s=0, t, x=0 \text{ or } x=uv^2\eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1}+w)x_{14}(e)x_{15}(au+t)x_{16}(f)x_{17}(g)x_{18}((h^2+h)u^2v+y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui+ \\
& x)x_{23}(g^2v^{-1}+z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, z, x, y=0 \text{ or } y=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1}+t)x_{16}(f)x_{17}(g)x_{18}((h^2+h)u^2v+x)x_{19}(i)x_{20}(ug+ \\
& z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1}+(u+ug)tu^{-2}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, x=0 \text{ or } x=u^2v\eta, y, z \in \mathbb{F}_q}
\end{aligned}$$

• q^{12} -elements classes:

$$\begin{aligned}
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+ \\
& r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s, t=0, r, o \neq 0, y, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ad+ \\
& w)x_{19}(h)x_{20}(abdu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, r, o, y, z \neq 0 \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+ \\
& w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, s, t, r, o=0, y \in \mathbb{F}_q, \text{ some of } s, r, y \neq 0}
\end{aligned}$$

$$\begin{aligned}
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s, t, r, o = 0, y \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s, t, r, o \neq 0, y \in \mathbb{F}_q, w, s, r, y \text{ not all } = 0}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v = w = 0, s, t \neq 0, r, o, y \neq 0 \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = 0, s, t, r, o, x, y \neq 0 \in \mathbb{F}_q}, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w \neq 0, s, t, o, x, y \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + t)x_{23}(l)x_{24}(ht + v^{-1}c + y) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = t = x = 0, y \in \mathbb{F}_q}, \\
& \{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, t \neq 0, s = 0, x, y \in \mathbb{F}_q}, \\
& \{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + gb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq 0, x, y, r \in \mathbb{F}_q, vt \neq w}, \\
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s = 0, t \neq 0 \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s = 0, t \neq 0 \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(v)x_{16}(d)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}, \\
& \{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, s \in \mathbb{F}_q, \text{ some of } w, t, s \neq 0}, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s = v\sqrt{t/u} \in \mathbb{F}_q}, \\
& \{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, s, r \neq 0 \in \mathbb{F}_q}, \\
& \{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})uv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, r, o, s \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \in \mathbb{F}_q}, \\
& \{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(lk) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s, r, o \neq sr v^{-1} \in \mathbb{F}_q}, \\
& \{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfu^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, s \in \mathbb{F}_q, w, t, s \text{ not all } = 0}, \\
& \{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s, r \in \mathbb{F}_q}, \\
& \{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s, r \in \mathbb{F}_q}, \\
& \{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u + acv^{-1} + \sqrt{ev^{-1}}w + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s, t, r \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(dgu^{-1} + w)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, x = 0 \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, t, s = r = 0, z, y \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, t \neq 0, s \neq 0, r = 0, z, y \in \mathbb{F}_q, s \neq ut}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, t, s, r \neq 0, z, y \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s = ut, r \neq 0, z, y \in \mathbb{F}_q, vr \neq w^2}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t = 0, s \neq ut, r \neq v^{-1}w^2, z, y \in \mathbb{F}_q},
\end{aligned}$$

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euw^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq ut, r, z, y \in \mathbb{F}_q}$
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2gu^{-2}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, y \in \mathbb{F}_q}$
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2gu^{-2}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, y \in \mathbb{F}_q, vw \neq t^2}$
 $\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1}+w)x_{16}(g)x_{17}(afu^{-1}+t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1}+aiu^{-1}+z)x_{21}(cfu^{-1}+biu^{-1}+x)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=z, x \in \mathbb{F}_q}$
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r, z, y=0, x, z' \in \mathbb{F}_q}$
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{16}(v)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(de+w)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, x \neq 0 \in \mathbb{F}_q}$
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z, y, x, z' \in \mathbb{F}_q, v^2z'=ux'}$
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w+t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2}+s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s \neq 0 \in \mathbb{F}_q}$
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2+ad)u^{-1}+s)x_{17}(hvw^{-1}+t)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0, t \in \mathbb{F}_q}$
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, x=0, z \neq 0 \in \mathbb{F}_q}$
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(vf+aku^{-1}+w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0 \in \mathbb{F}_q}$
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ab+ud+x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(g^2uv^{-2}w+ue^2v^{-2}w+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, x=0, y \in \mathbb{F}_q}$

- $q^{13}/4$ -elements classes:

$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(biu^{-1}+z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1}+y)x_{23}(l)x_{24}((m^2+m)v^{-2}ux^2+x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z, y, x \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2\eta, x'=0 \text{ or } x'=v^{-2}ux^2\eta \in \mathbb{F}_q}$

- $q^{13}/2$ -elements classes:

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2gu^{-2}+y)x_{23}((l^2+l)vs^2u^{-2}+y')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s \neq 0, y, y'=0 \text{ or } y'=vs^2u^{-2}\eta \in \mathbb{F}_q, vw \neq t^2}$
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2gu^{-2}+y)x_{23}((l^2+l)vs^2u^{-2}+y')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s \neq 0, y, y'=0 \text{ or } y'=vs^2u^{-2}\eta \in \mathbb{F}_q}$
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(biu^{-1}+z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z, y, x=0, y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q}$
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q, v^2z' \neq ux'}$
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q}$
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1}+y)x_{23}(m)x_{24}((n^2+n)ux^2t(uw^2+v^2t)^{-1}+x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r=0, y, x \neq 0, x'=0 \text{ or } x'=ux^2t(uw^2+v^2t)^{-1}\eta \in \mathbb{F}_q, v^2t \neq uw^2}$
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2+ad)u^{-1}+s)x_{17}((f^2+f)v^{-1}ws+t)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t=0 \text{ or } t=v^{-1}ws\eta \in \mathbb{F}_q}$
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq v^{-1}u^2w, r=0, z, y, x, y'=0 \text{ or } y'=vu^{-2}(t+v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}((h^2+h)vu^{-2}(t+$

$$\begin{aligned}
& v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq v^{-1}u^2w, r=0, z, x, z', y'=0 \text{ or } y'=vu^{-2}(t+v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(ghv^{-1} + i^2u + \\
& z)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, z, y, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f) + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + \\
& r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(ul+y)x_{23}(i^2v^{-1} + bl+z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=s=t=r=0, y, z, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + \\
& y)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(vj + u^{-2}a^2k + x)x_{23}(m)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r, z, x, y=0 \text{ or } y=uv^2\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(fju^{-1} + \\
& x)x_{23}(lv + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, z, x, y=0 \text{ or } y=uv^2\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ab + ud + x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}((k^2 + \\
& k)u^{-1}x^2v^2w^{-1} + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, x \neq 0, y=0 \text{ or } y=u^{-1}x^2v^2w^{-1}\eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{13} -elements classes:

$$\begin{aligned}
& \{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, x \neq 0 \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(i^2v^{-1} + jtu^{-2} + \\
& y)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, t, y \in \mathbb{F}_q, \text{ some of } w, t \neq 0 \text{ or } wt=0}, \\
& \{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + \\
& t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, t, x \in \mathbb{F}_q, \text{ some of } w, t, x \neq 0}, \\
& \{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + \\
& x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + y)x_{23}(l)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, t=0, z, y, x \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + \\
& z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y, x, z' \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} + \\
& z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, t, z \in \mathbb{F}_q, \text{ some of } w, t, z \neq 0}, \\
& \{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(iv + w)x_{24}(m) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, s \neq 0 \in \mathbb{F}_q}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + \\
& s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r, z, y, x \in \mathbb{F}_q}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + \\
& s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=v^{-1}u^2w, r=0, z, x, z' \in \mathbb{F}_q}, \\
& \{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + \\
& ahv^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, t=0, x, y, z \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}(b^2u^{-1} + w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}, \\
& \{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + \\
& w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(dbmv^{-2} + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, s=t=0=z, y, x \in \mathbb{F}_q}, \\
& \{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + \\
& z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, z \in \mathbb{F}_q}, \\
& \{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + \\
& z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, z \in \mathbb{F}_q}, \\
& \{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + \\
& w)x_{20}(k)x_{21}(ciu^{-1} + z)x_{22}(b^2u^{-2}k + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w, s=t=0, z, y=0, x \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(v^{-2}d^2 + t)x_{23}(l)x_{24}(m) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, s=0, t \in \mathbb{F}_q},
\end{aligned}$$

• $q^{14}/2$ -elements classes:

$$\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, y, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q},$$

$$\begin{aligned}
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}((n^2 + n)v^{-2}u^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, y, x \neq 0, x' = 0 \text{ or } x' = v^{-2}u^2 \eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = r = 0, y, x, y' = 0 \text{ or } y' = v^{-1}uw^2 \eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t = 0, r \neq 0, z, x, y' = 0 \text{ or } y' = v^{-1}uw^2 \eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t = uw^2/v^2, r \neq 0, z, x, y' = 0 \text{ or } y' = v^{-1}uw^2 \eta \in \mathbb{F}_q}, \\
& \{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}((m^2 + m)wt^2u^{-2} + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, z = 0 \text{ or } z = wt^2u^{-2} \eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}((m^2 + m)w^2v^{-1}t^2u^{-2} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, y = 0 \text{ or } y = w^2v^{-1}t^2u^{-2} \eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}((l^2 + l)w^2sv^{-2} + t)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t = 0 \text{ or } t = w^2sv^{-2} \eta \in \mathbb{F}_q}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(dmv^{-1} + f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = s = t = 0, r \neq 0, z, y = 0 \text{ or } y = uv^2 \eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(v^{-1}bn + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = t = r = z = 0, y, x, s = 0 \text{ or } s = u^2v \eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(lv + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, z, y = 0 \text{ or } y = uv^2 \eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(i^2/v + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = s = t = 0, r \neq 0, z, x = 0 \text{ or } x = u^2v \eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(j^2/v + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, t \neq 0, r = z = 0, x, y, s = 0 \text{ or } s = u^2v \eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{14} -elements classes:

$$\begin{aligned}
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, y, x=0 \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, y, x \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r=0, y, x=0 \in \mathbb{F}_q, v^2t \neq uu^2}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r=0, y, x \in \mathbb{F}_q, v^2t \neq uu^2}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t \neq 0, r \neq 0, z, x \in \mathbb{F}_q, v^2t \neq uu^2}, \\
& \{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s \neq 0, t = 0 = z, y \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(i^2v^{-1} + bl + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = s = 0, t \neq 0, r = 0, z \in \mathbb{F}_q}, \\
& \{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(l)x_{22}(b^2u^{-2}k + x)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s \neq 0, t = 0, y = 0, x \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu +
\end{aligned}$$

$$\begin{aligned}
& \{x_1x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(cnv^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=0, z, y=0 \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(cju^{-1} + lw^{-1}v + y)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0=z, y \in \mathbb{F}_q}, \\
& \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, r, z, y, x \in \mathbb{F}_q}, \\
& \{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(c^2u^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biv^{-1} + x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2} + z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, z, y, x, r \in \mathbb{F}_q}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(i^2/u + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r=0, z \in \mathbb{F}_q},
\end{aligned}$$

• $q^{15}/2$ -elements classes:

$$\begin{aligned}
& \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(gv^2w^{-1} + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)(sx + v^2z)^2/(v^2s) + y)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s \neq 0, t=0, r, z, x, y=0 \text{ or } y=(sx+v^2z)^2/(v^2s)\eta \in \mathbb{F}_q, sx \neq v^2z}, \\
& \{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu + y)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2 + o)v^2y^2u^{-2}w^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=0, z=0 \text{ or } z=v^2y^2u^{-2}w^{-1}\eta, y \neq 0 \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t=0, r \neq 0, x, y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t=uw^2/v^2, r \neq 0, x, y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2) + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=r=0, z, y'=0 \text{ or } y'=vu^{-2}(t+v^{-1}u^2w^2)\eta \in \mathbb{F}_q}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}((k^2 + k)v^{-1}u^2w^2t^{-1}s + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r=0, z, x=0 \text{ or } x=v^{-1}u^2w^2t^{-1}s\eta \in \mathbb{F}_q, vt \neq u^2w}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}((k^2 + k)v^{-1}u^2w^2t^{-1}s + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r=0, z, x=0 \text{ or } x=v^{-1}u^2w^2t^{-1}s\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}((l^2 + l)v^{-1}wz + y)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0, z \neq 0, y=0 \text{ or } y=v^{-1}wz\eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(v^{-1}bo + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=r=z=0, x, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)u^{-2}t^2r + z)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r \neq 0, z=0 \text{ or } z=t^2u^{-2}r\eta \in \mathbb{F}_q}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2 + o)v^{-2}tr^2 + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r \neq 0, z=0, \text{ or } z=v^{-2}tr^2\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + s)x_{16}((h^2 + h)uw^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,w,v \in \mathbb{F}_q^\times, t=0, s, z, y=0 \text{ or } y=uv^2\eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{15} -elements classes:

$$\begin{aligned}
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s, t, r \neq 0, x \in \mathbb{F}_q}, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s, t, r \neq 0, x \in \mathbb{F}_q},
\end{aligned}$$

$$\begin{aligned}
& r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2 t \neq uw^2, \\
& \{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + \\
& w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(dbnv^{-2} + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, z = 0, x \in \mathbb{F}_q, \\
& \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(gv^2w^{-1} + \\
& x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t = 0, r, z, x \in \mathbb{F}_q, sx = v^2z, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + \\
& s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(d^2v^{-1} + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, vt \neq u^2w, \\
& \{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l) \\
& x_{21}(m)x_{22}(b^2u^{-2}l + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, s, t = 0, y \neq 0, x \in \mathbb{F}_q, \\
& \{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(cju^{-1} + \\
& z)x_{22}(b^2u^{-2}m + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, s = 0, t \neq 0, z, x \in \mathbb{F}_q, \\
& \{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + \\
& s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s, z \in \mathbb{F}_q,
\end{aligned}$$

• $q^{16}/4$ -elements classes:

$$\begin{aligned}
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + \\
& h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w, s, t = 0, r \neq 0, x = 0 \text{ or } x = vr^2\eta, y' = 0 \text{ or } y' = vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q, ws = 0, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + \\
& h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w = 0, y' = 0 \text{ or } y' = vu^{-2}(t + v^{-1}u^2w)^2\eta, s, t \neq 0, r \neq 0, x = 0 \text{ or } x = vr^2\eta \in \mathbb{F}_q, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(u^2w^2/v((hr)^2 + \\
& h(ws + vr^2))^2/(wvs)^2 + ((hr)^2 + h(ws + vr^2))/(wvs)) + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vw^{-1}r^2, s \neq 0, t = 0, r \neq 0, y' \in \mathbb{F}_q, q/4 \text{ different choices for } y' \\
& \{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + \\
& t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}((p^2 + p)u^{-2}w^2t + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, r = z = 0, s = 0 \text{ or } s = u^2v\eta \in \mathbb{F}_q, x = 0 \text{ or } x = u^{-2}w^2t\eta \in \mathbb{F}_q, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + \\
& s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = 0, r = 0 \text{ or } r = vw^{-2}s^2\eta \in \mathbb{F}_q, x = 0 \text{ or } x = u^2v\eta \in \mathbb{F}_q, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + jvw^{-2}st + x)x_{14}(g)x_{15}(a^2bu^{-1} + \\
& s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r = 0 \text{ or } r = vw^{-2}s^2\eta \in \mathbb{F}_q, x = 0 \text{ or } x = u^2w^2s^{-1} \text{ for some } z \in \mathbb{F}_q, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + \\
& y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = 0, r = 0 \text{ or } r = v^2w^2s^{-1}\eta \in \mathbb{F}_q, y = 0 \text{ or } y = uv^2\eta \in \mathbb{F}_q, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + (jv^{-1}w^{-1}s)^2ut + \\
& y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, y, r = 0 \text{ or } r = v^2w^2s^{-1}\eta \in \mathbb{F}_q, t = (z^2 + z)uw^{-2}s^2 \text{ for some } z \in \mathbb{F}_q, y = 0 \text{ or } y = uv^2\eta \in \mathbb{F}_q, \\
& \{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + \\
& y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x, y, z = 0 \in \mathbb{F}_q, x = 0 \text{ or } x = w^2v\eta \in \mathbb{F}_q, y = 0 \text{ or } y = w^2u^2t^{-1}\eta \in \mathbb{F}_q, \\
& \{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}((j^2 + \\
& j)u^2t^2v^{-1} + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, z = 0 \text{ or } z = u^2w\eta, y = 0 \text{ or } y = u^2t^2v^{-1}\eta \in \mathbb{F}_q,
\end{aligned}$$

• $q^{16}/2$ -elements classes:

$$\begin{aligned}
& \{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t + \\
& x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x = 0 \text{ or } x = u^2t\eta \in \mathbb{F}_q, \\
& \{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)u^2z + \\
& w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w = 0 \text{ or } w = u^2z\eta, s, t = 0, z \neq 0 \in \mathbb{F}_q,
\end{aligned}$$

$$\begin{aligned}
& \{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}((h^2 + h)us^2 + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t=0, z, r, y=0 \text{ or } y=us^2\eta \in \mathbb{F}_q\}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(u)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)uv^2w^{-1}s + z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v, w \in \mathbb{F}_q^\times, s \neq 0, z=0 \text{ or } z=uv^2w^{-1}s\eta \in \mathbb{F}_q\}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0, r \neq 0, y' = vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q, ws = vr^2\}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r \neq 0, x=0 \text{ or } x = v^{-1}u^2w^2t^{-1}s\eta \in \mathbb{F}_q, vt \neq u^2w\}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w, s, t = v^{-1}u^2w, r \neq 0, x=0 \text{ or } x = vr^2\eta \in \mathbb{F}_q, ws = 0\}, \\
& \{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = v^{-1}u^2w, r \neq 0, x=0 \text{ or } x = v^{-1}u^2w^2t^{-1}s\eta \in \mathbb{F}_q\}, \\
& \{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w, t, r, z, s = 0 \text{ or } s = u^2v\eta \in \mathbb{F}_q, r \text{ or } z \neq 0\}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w = 0, s \neq 0, t = 0, x = 0 \text{ or } x = u^2v\eta \in \mathbb{F}_q\}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s = t = 0, x = 0 \text{ or } x = u^2v\eta \in \mathbb{F}_q\}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + js^{-1}t + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w = 0, s \neq 0, t \neq 0, x = 0 \text{ or } x = u^2v\eta \in \mathbb{F}_q\}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + \sqrt{jv}w^{-1}t + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t \neq 0, x = 0 \text{ or } x = u^2v\eta \in \mathbb{F}_q\}, \\
& \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r = 0 \text{ or } r = vw^{-2}s^2\eta \in \mathbb{F}_q, t = (z^2 + z + \eta)u^2w^2s^{-1} \text{ for some } z \in \mathbb{F}_q\}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s = t = 0, y = 0 \text{ or } y = uv^2\eta \in \mathbb{F}_q\}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w = 0, s \neq 0, t = 0, y = 0 \text{ or } y = uv^2\eta \in \mathbb{F}_q\}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w = 0, s \neq 0, t \neq 0, y = 0 \text{ or } y = uv^2\eta \in \mathbb{F}_q\}, \\
& \{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r = 0 \text{ or } r = v^2w^2s^{-1}\eta \in \mathbb{F}_q, t = (z^2 + z + \eta)uw^{-2}s^2 \text{ for some } z \in \mathbb{F}_q\}, \\
& \{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2v + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v, w \in \mathbb{F}_q^\times, t = 0, y = 0 \text{ or } y = w^2v\eta \in \mathbb{F}_q\}, \\
& \{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\} \\
& \{u, v, w \in \mathbb{F}_q^\times, t \neq 0, y, z \in \mathbb{F}_q, y = 0 \text{ or } y = w^2u^2t^{-1}\eta \in \mathbb{F}_q, z = vu^{-2}t^2\eta \in \mathbb{F}_q\},
\end{aligned}$$

$$\begin{aligned}
& \{x_5(u)x_6(v)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times}, \\
& \{x_1(u)x_5(a)x_8(b)x_9(v)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2+g)u^2w+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, y=0 \text{ or } y=u^2w\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2+g)u^2w+z)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, z=0 \text{ or } z=u^2w\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_8(v)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)uw^2 + t)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0 \text{ or } t=uv^2\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)ut^2 + z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, z=0 \text{ or } z=ut^2\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(h)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, y=0 \text{ or } y=u^2w\eta \in \mathbb{F}_q}, \\
& \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(h)x_{16}((h^2+h)uv^2+t)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0 \text{ or } t=uv^2\eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{16} -elements classes:

$$\begin{aligned}
& \{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1}+s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2+j)u^2s+(j^2+j)^2u^2v^{-2}w^2t+y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t,s,y \in \mathbb{F}_q, t \text{ or } s \neq 0}, \\
& \{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1}+y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2}+z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, y \in \mathbb{F}_q, w^2t=us^2}, \\
& \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w+s)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1}+r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s, t \neq 0, r, z \in \mathbb{F}_q}, \\
& \{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1}+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2u^{-2}m+x)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, x \in \mathbb{F}_q},
\end{aligned}$$

• $q^{17}/4$ -elements classes:

$$\begin{aligned}
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g+t)x_{16}((i^2+i)vw^2+s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2+o)u^{-2}vt^2+x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s=0 \text{ or } s=vw^2\eta, x=0 \text{ or } x=u^{-2}vt^2\eta \in \mathbb{F}_q}, \\
& \{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(v^{-1}wd+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+(h^2+h)uw^2+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2+o)vy^2+x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, y \neq 0, s=0 \text{ or } s=uv^2\eta, x=0 \text{ or } x=vy^2\eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}((g^2+g)u^2w+s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi+x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}((q^2+q)v^2x^2w^{-1}+y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0 \text{ or } s=u^2w\eta, t=0, x \neq 0, y=0 \text{ or } y=v^2x^2w^{-1}\eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2+h)u^2w+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2+m)u^2v^{-1}t^2+x)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s=0 \text{ or } s=u^2w\eta, x=0 \text{ or } x=u^2v^{-1}t^2\eta \in \mathbb{F}_q},
\end{aligned}$$

• $q^{17}/2$ -elements classes:

$$\begin{aligned}
& \{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2+m)u^2z+w)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0 \text{ or } w=u^2z\eta, t \neq 0, z \neq 0, \eta \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g+t)x_{16}((i^2+i)vw^2+s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s=0 \text{ or } s=vw^2\eta \in \mathbb{F}_q}, \\
& \{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(v^{-1}wd+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+(h^2+h)uw^2+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, y, s=0 \text{ or } s=uw^2\eta \in \mathbb{F}_q}, \\
& \{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}((g^2+g)u^2w+s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi+x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0 \text{ or } s=u^2w\eta, t=0=x \in \mathbb{F}_q}, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2+h)u^2w+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s=0 \text{ or } s=u^2w\eta \in \mathbb{F}_q},
\end{aligned}$$

• q^{17} -elements classes:

$$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(h)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, r \in \mathbb{F}_q, w^2 t \neq us^2},$$

$$\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x \in \mathbb{F}_q},$$

- $q^{18}/4$ -elements classes:

$$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}((r^2 + r)w^2x^2v^{-3} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \text{ or } t=u^2v\eta, s=0, x \neq 0, y=0 \text{ or } y=w^2x^2v^{-1}\eta \in \mathbb{F}_q},$$

$$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}((p^2 + p)wy^2 + x)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, y \neq 0, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q, x=0 \text{ or } x=y^2w\eta \in \mathbb{F}_q},$$

- $q^{18}/2$ -elements classes:

$$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}((i^2 + i)w^2t + y)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0, t \neq 0, y=0 \text{ or } y=w^2t\eta \in \mathbb{F}_q},$$

$$\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + (i^2 + i)uw^2 + s)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=0 \text{ or } s=uw^2\eta \in \mathbb{F}_q},$$

$$\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{10}(t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)vt + x)x_{14}(h)x_{15}(i)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w, t \in \mathbb{F}_q^\times, x=0 \text{ or } x=vt\eta \in \mathbb{F}_q},$$

$$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \text{ or } t=u^2v\eta, s=x=0 \in \mathbb{F}_q},$$

$$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \text{ or } t=u^2v\eta, s \neq 0, x \in \mathbb{F}_q},$$

$$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, y, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q},$$

$$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, y=0, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q},$$

- $q^{19}/4$ -elements classes:

$$\{x_1(w)x_2(y)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + c^2v^{-1} + z)x_{11}(g)x_{12}(h)x_{13}(i)x_{14}(j)x_{15}((k^2 + k)w^2u^2v + x)x_{16}(l)x_{17}(m)x_{18}(n)x_{19}(o)x_{20}(p)x_{21}(q)x_{22}(r)x_{23}(s)x_{24}(t) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t \in \mathbb{F}_q\}_{u, v, w, y \in \mathbb{F}_q^\times, z=0 \text{ or } z=u^2v\eta, x=0 \text{ or } x=w^2u^2v\eta \in \mathbb{F}_q},$$

so $4(q-1)^4q^2/4$ classes.

A.6 Parametrization of Irreducible Characters of U via the Reduction-Method

Write F_Σ for the set of irreducible characters of U corresponding to the antichain Σ and for $\Sigma = \{\alpha_{i_1}, \dots, \alpha_{i_j}\}$ also write $F_\Sigma = F_{i_1, \dots, i_j}$. Moreover, $F_{lin} := \bigcup_{\Sigma \in \Pi} F_\Sigma$. For each $\chi_{\underline{b}}^{\underline{a}} = \overline{\mu_{\underline{b}}^{\underline{a}}}$ corresponding to the linear character $\mu_{\underline{b}}^{\underline{a}}$ of an abelian core $C = (S, Z, A, L, K)$ or $\chi_{\underline{c}}$ corresponding to the character $\{\nu_{\underline{c}} \mid \underline{c} \in J_C\}$ for some indexing set J_C and $C = (S, Z, A, L, K)$ a nonabelian core, let I_A reps. I_L denote the set of root numbers of the roots in A resp. L .

Table 3: Parametrization of Irreducible Characters of U , Part I

F	χ	I	#	$\chi(1)$	Inductions in G
F_{lin}	$\chi_{b_1, b_2, b_3, b_4}$		q^4	1	T_1 for $b_1 = b_2 = b_3 = b_4 = 0$, T_2 for $b_1 \neq 0 = b_2 = b_3 = b_4$, T_3 for $b_2 \neq 0 = b_1 = b_3 = b_4$, T_4 for $b_1 = b_2 = b_4 = 0 \neq b_3$, T_5 for $b_1 = b_2 = b_3 = 0 \neq b_4$ $T_{41} = T_2 T_3$, $T_{42} = T_2 T_4$, $T_{43} = T_2 T_5$, $T_{44} = T_3 T_5$, $T_{45} = T_3 T_4$, $T_{51} = T_4 T_5$, $T_{46} = T_2 T_3 T_5$, $T_{47} = T_2 T_3 T_4$, $T_{48} = T_2 T_4 T_5$, $T_{49} = T_3 T_4 T_5$, $T_{50} = T_2 T_3 T_4 T_5$
F_5	χ^{a_5}	$I_A = \{1\}, I_L = \{3\}$	$(q-1)$	q	T_9
F_6	χ^{a_6}	$I_A = \{2\}, I_L = \{4\}$	$q-1$	q	T_8
F_7	χ^{a_7}	$I_A = \{3\}, I_L = \{4\}$	$q-1$	q	T_7
F_8	$\chi_{b_3}^{a_8}$	$I_A = \{1, 5\}, I_L = \{4, 7\}$	$q(q-1)$	q^2	T_{34} for $b_3 \neq 0$, T_{38} for $b_3 = 0$
F_9	$\chi_{b_4}^{a_9}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q(q-1)$	q^2	T_{36} for $b_4 \neq 0$, T_{40} for $b_4 = 0$
F_{10}	$\chi^{a_{10}}$ χ	$I_A = \{\}, I_L = \{\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_1 in section A.7	$(q-1)$	q	T_{12}
F_{11}	$\chi^{a_7, a_{11}}$ $\chi_{b_3, b_4}^{a_{11}}$	$I_A = \{1, 2, 3, 5\}, I_L = \{4, 6, 8, 9\}$ $I_A = \{1, 2, 5\}, I_L = \{6, 8, 9\}$	$(q-1)^2$ $q^2(q-1)$	q^4 q^3	T_{30} T_{81} for $b_3 = b_4 = 0$, T_{82} for $b_3 \neq 0 = b_4$ T_{83} for $b_3 = 0 \neq b_4$, T_{84} for $b_3, b_4 \neq 0$
F_{12}	$\chi_{b_4}^{a_{12}}$	$I_A = \{1, 3, 5\}, I_L = \{7, 8, 10\}$	$q(q-1)$	q^3	T_{85} for $b_4 = 0$, T_{86} for $b_4 \neq 0$
F_{13}	$\chi_{b_6, b_9}^{a_7, a_{13}}$ χ	$I_A = \{2, 3\}, I_L = \{4, 10\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_2 in section A.7	$q^2(q-1)^2$	q^2	
F_{14}	$\chi_{b_4, b_6, b_7, b_8, b_{10}}^{a_{14}}$	$I_A = \{1, 2, 3, 5\}, I_L = \{9, 11, 12, 13\}$	$q^5(q-1)$	q^4	
F_{15}	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_3 in section A.7			
F_{16}	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_4 in section A.7			
F_{17}	$\chi_{b_3, b_5}^{a_{13}, a_{17}}$ $\chi_{b_2, b_5, b_5, b_{10}}^{a_{17}}$	$I_A = \{1, 2, 4, 6, 7, 8\}, I_L = \{9, 10, 11, 12, 14, 16\}$ $I_A = \{1, 4, 6, 7, 8\}, I_L = \{9, 11, 12, 14, 16\}$	$q^2(q-1)^2$ $q^4(q-1)$	q^6 q^5	
F_{18}	$\chi_{b_4, b_6, b_9, b_{11}, b_{13}, b_{14}}^{a_{12}, a_{18}}$ $\chi_{b_6, b_9, b_{13}}^{a_8, a_{14}, a_{18}}$ $\chi_{b_{11}}^{a_8, a_9, a_{18}}$ $\chi_{b_3, b_6, b_{11}}^{a_8, a_{18}}$ χ	$I_A = \{1, 2, 3, 5\}, I_L = \{7, 8, 10, 15\}$ $I_A = \{1, 2, 3, 5\}, I_L = \{4, 7, 11, 15\}$ $I_A = \{1, 2, 3, 5\}, I_L = \{4, 6, 7, 15\}$ $I_A = \{1, 2, 5\}, I_L = \{4, 7, 15\}$ $I_A = \{\}, I_L = \{\}$, see $\mathfrak{C}_i, 5 \leq i \leq 9$ in section A.7	$q^6(q-1)^2$ $q^3(q-1)^3$ $q(q-1)^3$ $q^3(q-1)^2$	q^4 q^4 q^4 q^3	
F_{19}	$\chi_{b_1}^{a_6, a_{19}}$ $\chi_{b_1, b_2, b_4}^{a_{19}}$	$I_A = \{2, 3, 5, 7, 8, 9, 11\}, I_L = \{4, 10, 12, 13, 14, 16, 17\}$ $I_A = \{3, 5, 7, 8, 9, 11\}, I_L = \{10, 12, 13, 14, 16, 17\}$	$q(q-1)^2$ $q^3(q-1)$	q^7 q^6	

Table 4: Parametrization of Irreducible Characters of U , Part II

F	χ	I	#	$\chi(1)$	Inductions in G
F_{20}	$\chi_{b_7, b_8, b_{10}, b_{16}, b_{17}}^{a_{14}, a_{20}}$	$I_A = \{1, 2, 4, 6, 9, 11\}, I_L = \{3, 5, 12, 13, 15, 18\}$	$q^5(q-1)^2$	q^6	
	$\chi_{b_{16}, b_{17}}^{a_{11}, a_{12}, a_{13}, a_{20}}$	$I_A = \{1, 4, 6, 7, 8, 10\}, I_L = \{2, 3, 5, 9, 15, 18\}$	$q^2(q-1)^4$	q^6	
	$\chi_{b_2, b_9, b_{11}, b_{16}, b_{17}}^{a_{12}, a_{20}}$	$I_A = \{1, 4, 6, 7, 8\}, I_L = \{3, 5, 10, 15, 18\}$	$q^5(q-1)^2$	q^5	
	$\chi_{b_{16}}^{a_5, a_{13}, a_{17}, a_{20}}$	$I_A = \{1, 2, 4, 6, 7, 8\}, I_L = \{3, 9, 10, 11, 15, 18\}$	$q(q-1)^4$	q^6	
	$\chi_{b_2}^{a_5, a_{10}, a_{17}, a_{20}}$	$I_A = \{1, 4, 6, 7, 8\}, I_L = \{3, 9, 11, 15, 18\}$	$q(q-1)^4$	q^5	
	$\chi_{b_7, b_8, b_9, b_{11}, b_{16}}^{a_5, a_{13}, a_{20}}$	$I_A = \{1, 2, 4, 6\}, I_L = \{3, 10, 15, 18\}$	$q^5(q-1)^3$	q^4	
	$\chi_{b_2}^{a_5, a_{17}, a_{20}}$	$I_A = \{1, 4, 6, 7, 8\}, I_L = \{3, 9, 11, 15, 18\}$	$q(q-1)^3$	q^5	
	$\chi_{b_8, b_{11}}^{a_5, a_9, a_{10}, a_{20}}$	$I_A = \{1, 2, 4, 6\}, I_L = \{3, 7, 15, 18\}$	$q^2(q-1)^4$	q^4	
	$\chi_{b_8, b_{11}}^{a_5, a_9, a_{20}}$	$I_A = \{1, 2, 4, 6\}, I_L = \{3, 7, 15, 18\}$	$q^2(q-1)^3$	q^4	
	χ	$I_A = \{\}, I_L = \{\},$ see $\mathfrak{C}_i, 10 \leq i \leq 19$ in section A.7			
	F_{21}	$\chi_{b_2}^{a_{16}, a_{21}}$	$I_A = \{1, 3, 4, 5, 6, 7, 8, 9, 11\}, I_L = \{10, 12, 13, 14, 15, 17, 18, 19, 20\}$	$q(q-1)^2$	q^9
$\chi_{b_4, b_6}^{a_{13}, a_{21}}$		$I_A = \{1, 2, 3, 5, 7, 8, 9, 11\}, I_L = \{10, 12, 14, 15, 17, 18, 19, 20\}$	$q^2(q-1)^2$	q^8	
$\chi_{b_{10}}^{a_6, a_{21}}$		$I_A = \{1, 2, 3, 5, 7, 8, 9, 11\}, I_L = \{4, 12, 14, 15, 17, 18, 19, 20\}$	$q(q-1)^2$	q^8	
$\chi_{b_2, b_4, b_{10}}^{a_{21}}$		$I_A = \{1, 3, 5, 7, 8, 9, 11\}, I_L = \{12, 14, 15, 17, 18, 19, 20\}$	$q^3(q-1)$	q^7	
F_{22}	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{20} in section A.7			
F_{23}	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{21} in section A.7			
F_{24}	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{22} in section A.7			
$F_{1,6}$	χ^{a_1, a_6}	$I_A = \{2\}, I_L = \{4\}$	$(q-1)^2$	q	T_{57}
$F_{1,7}$	χ^{a_1, a_7}	$I_A = \{3\}, I_L = \{4\}$	$(q-1)^2$	q	T_{54}
$F_{1,9}$	$\chi_{b_4}^{a_1, a_9}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q(q-1)^2$	q^2	T_{37} for $b_4 \neq 0,$ T_{52} for $b_4 = 0$
$F_{1,10}$	$\chi^{a_1, a_{10}}$	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{23} in section A.7	$(q-1)^2$	q	T_{63}
	$\chi_{\mathfrak{C}_{3,4}}^{a_1, a_7, a_{10}}$	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{23} in section A.7	$4(q-1)^3$	$q/2$	
$F_{1,13}$	$\chi_{b_6, b_9}^{a_1, a_7, a_{13}}$	$I_A = \{2, 3\}, I_L = \{4, 10\}$	$q^2(q-1)^3$	q^2	
	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{24} in section A.7			
$F_{1,16}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{25} in section A.7			
$F_{2,5}$	χ^{a_2, a_5}	$I_A = \{1\}, I_L = \{3\}$	$(q-1)^2$	q	T_{60}
$F_{2,7}$	χ^{a_2, a_7}	$I_A = \{3\}, I_L = \{4\}$	$(q-1)^2$	q	T_{55}
$F_{2,8}$	$\chi_{b_3}^{a_2, a_8}$	$I_A = \{1, 5\}, I_L = \{4, 7\}$	$q(q-1)^2$	q^2	T_{35} for $b_3 \neq 0,$ T_{39} for $b_3 = 0$
$F_{2,10}$	$\chi^{a_2, a_{10}}$	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{26} in section A.7	$(q-1)^2$	q	T_{64}
	$\chi_{\mathfrak{C}_{3,4}}^{a_2, a_7, a_{10}}$	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C} in section A.7	$4(q-1)^3$	$q/2$	
$F_{2,12}$	$\chi_{b_4}^{a_2, a_{12}}$	$I_A = \{1, 3, 5\}, I_L = \{7, 8, 10\}$	$q(q-1)^2$	q^3	T_{87} for $b_4 = 0,$ T_{88} for $b_4 \neq 0$
$F_{2,15}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{27} in section A.7			
$F_{3,6}$	χ^{a_3, a_6}	$I_A = \{2\}, I_L = \{4\}$	$(q-1)^2$	q	T_{58}
$F_{4,5}$	χ^{a_4, a_5}	$I_A = \{1\}, I_L = \{3\}$	$(q-1)^2$	q	T_{61}
$F_{5,6}$	χ^{a_5, a_6}	$I_A = \{1, 2\}, I_L = \{3, 4\}$	$(q-1)^2$	q^2	T_{32}
$F_{5,7}$	$\chi_{b_1}^{a_5, a_7}$	$I_A = \{3\}, I_L = \{4\}$	$q(q-1)^2$	q	T_{11} for $b_1 = 0,$ T_{69} for $b_1 \neq 0$
$F_{5,9}$	$\chi_{b_1, b_4}^{a_5, a_9}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q^2(q-1)^2$	q^2	T_{20} for $b_1 = b_4 = 0,$ T_{75} for $b_1 \neq 0 = b_4,$ T_{76} for $b_1 \neq b_4,$ T_{77} for $b_1, b_4 \neq 0$
$F_{5,10}$	$\chi_{b_4, b_7}^{a_5, a_{10}}$	$I_A = \{1\}, I_L = \{3\}$	$q^2(q-1)^2$	q	
$F_{5,13}$	$\chi_{b_4, b_6, b_7, b_9}^{a_5, a_{13}}$	$I_A = \{2, 3\}, I_L = \{1, 10\}$	$q^4(q-1)^2$	q^2	
$F_{5,16}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathfrak{C}_{28} in section A.7			
$F_{6,7}$	$\chi_{b_2}^{a_6, a_7}$	$I_A = \{4\}, I_L = \{3\}$	$q(q-1)^2$	q	T_{10} for $b_2 = 0, \neq 0,$ T_{66} for $b_2 \neq 0,$

Table 5: Parametrization of Irreducible Characters of U , Part III

F	χ	I	#	$\chi(1)$	Inductions in G
$F_{6,8}$	$\chi_{b_2, b_3}^{a_6, a_8}$	$I_A = \{1, 4\}, I_L = \{5, 7\}$	$q^2(q-1)^2$	q^2	T_{19} for $b_2 = b_3 = 0$, T_{72} for $b_2 \neq 0 = b_3$ T_{73} for $b_2 = 0 \neq b_3$, T_{74} for $b_2, b_3 \neq 0$
$F_{6,10}$	$\chi_{b_3, b_7}^{a_6, a_{10}}$	$I_A = \{4\}, I_L = \{2\}$	$q^2(q-1)^2$	q	T_{152} for $b_3 = b_7 = 0$, T_{153} for $b_3 \neq 0 = b_7$, T_{156} for $b_3 = 0 \neq b_7$, T_{157} for $b_3, b_7 \neq 0$ and, $\chi_{b_3}(x_3(s_0)) = -1$ for s_0 s.t. $\chi_{b_3, b_7}^{a_6, a_{10}}(x_3(s_0)) \neq 0$ T_{160} for $b_3, b_7 \neq 0$ and, $\chi_{b_3}(x_3(s_0)) = 1$ for s_0 s.t. $\chi_{b_3, b_7}^{a_6, a_{10}}(x_3(s_0)) \neq 0$
$F_{6,12}$	$\chi^{a_6, a_{12}}$	$I_A = \{1, 2, 3, 5\}, I_L = \{4, 7, 8, 10\}$	$(q-1)^2$	q^4	T_{31}
$F_{6,15}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{29} in section A.7			
$F_{8,9}$	$\chi_{b_1}^{a_8, a_9}$	$I_A = \{3, 5, 7\}, I_L = \{2, 4, 6\}$	$q(q-1)^2$	q^3	T_{27} for $b_1 = 0$, T_{28} for $b_1 \neq 0$
$F_{8,10}$	$\chi_{b_3}^{a_8, a_{10}}$	$I_A = \{1, 4\}, I_L = \{5, 7\}$	$q(q-1)^2$	q^2	T_{121} for $b_3 = 0$, T_{122} for $b_3 \neq 0$
$F_{8,13}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{30} in section A.7			
$F_{8,16}$	$\chi_{b_2, b_3, b_9}^{a_8, a_{16}}$ χ	$I_A = \{4, 6, 7\}, I_L = \{1, 10, 13\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{31} in section A.7	$q^3(q-1)^2$	q^3	
$F_{9,10}$	$\chi_{b_4}^{a_9, a_{10}}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q(q-1)^2$	q^2	
$F_{9,12}$	$\chi^{a_6, a_9, a_{12}}$ $\chi_{b_2, b_4}^{a_9, a_{12}}$	$I_A = \{1, 2, 3, 5\}, I_L = \{4, 7, 8, 10\}$ $I_A = \{1, 3, 7\}, I_L = \{5, 8, 10\}$	$(q-1)^3$ $q^2(q-1)^2$	q^4 q^3	T_{134}
$F_{9,15}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{32} in section A.7			
$F_{10,11}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{33} in section A.7			
$F_{11,12}$	$\chi^{a_6, a_{11}, a_{12}}$ $\chi_{b_2, b_4}^{a_{11}, a_{12}}$ χ	$I_A = \{1, 2, 3, 5\}, I_L = \{4, 7, 8, 10\}$ $I_A = \{1, 5, 8\}, I_L = \{3, 7, 10\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{34} in section A.7	$(q-1)^3$ $q^2(q-1)^2$	q^4 q^3	T_{139}
$F_{11,13}$	$\chi^{a_7, a_{11}, a_{13}}$ $\chi_{b_3, b_4}^{a_{11}, a_{13}}$ χ	$I_A = \{1, 2, 3, 6\}, I_L = \{4, 5, 9, 10\}$ $I_A = \{1, 2, 6\}, I_L = \{5, 9, 10\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{35} in section A.7	$(q-1)^3$ $q^2(q-1)^2$	q^4 q^3	T_{129} T_{130} for $b_3 = b_4 = 0$, T_{131} for $b_3 \neq 0 = b_4$, T_{132} for $b_3 = 0 \neq b_4$, T_{133} for $b_3, b_4 \neq 0$
$F_{11,15}$	$\chi_{b_{12}}^{a_7, a_{11}, a_{15}}$ $\chi_{b_3, b_4, b_{12}}^{a_{11}, a_{15}}$ χ	$I_A = \{1, 3, 5, 8\}, I_L = \{2, 4, 6, 9\}$ $I_A = \{1, 5, 8\}, I_L = \{2, 6, 9\}$ $I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{36} in section A.7	$q(q-1)^3$ $q^3(q-1)^2$	q^4 q^3	
$F_{11,16}$	$\chi_{b_3, b_5, b_7}^{a_{11}, a_{16}}$	$I_A = \{1, 2, 4, 6\}, I_L = \{8, 9, 10, 13\}$	$q^3(q-1)^2$	q^4	
$F_{12,13}$	$\chi_{b_9}^{a_6, a_{12}, a_{13}}$ χ	$I_A = \{2, 3, 7, 10\}, I_L = \{1, 4, 5, 8\}$ $I_A = \{\}, I_L = \{\}$, see $\mathfrak{C}_i, i = 37, 38$ in section A.7	$q(q-1)^3$	q^4	
$F_{12,16}$	$\chi_{b_1, b_2, b_9}^{a_{12}, a_{16}}$	$I_A = \{3, 4, 7, 10\}, I_L = \{5, 6, 8, 13\}$	$q^3(q-1)^2$	q^4	
$F_{13,15}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{39} in section A.7			
$F_{14,15}$	$\chi_{b_4, b_6, b_7, b_8, b_{10}}^{a_{14}, a_{15}}$	$I_A = \{1, 2, 3, 5\}, I_L = \{9, 11, 12, 13\}$	$q^5(q-1)^2$	q^4	
$F_{14,16}$	$\chi^{a_8, a_{14}, a_{16}}$ $\chi_{b_1, b_7}^{a_{14}, a_{16}}$	$I_A = \{1, 2, 3, 4, 5, 6\}, I_L = \{7, 9, 10, 11, 12, 13\}$ $I_A = \{1, 2, 3, 5, 6\}, I_L = \{4, 9, 10, 11, 12\}$	$(q-1)^3$ $q^2(q-1)^2$	q^6 q^5	
$F_{15,16}$	χ	$I_A = \{\}, I_L = \{\}$, see \mathfrak{C}_{40} in section A.7			
$F_{15,17}$	$\chi_{b_3, b_5}^{a_{13}, a_{15}, a_{17}}$ $\chi_{b_2, b_3, b_5, b_{10}}^{a_{15}, a_{17}}$	$I_A = \{1, 2, 4, 6, 7, 8\}, I_L = \{9, 10, 11, 12, 14, 16\}$ $I_A = \{1, 4, 6, 7, 8\}, I_L = \{9, 11, 12, 14, 16\}$	$q^2(q-1)^3$ $q^4(q-1)^2$	q^6 q^5	

Table 6: Parametrization of Irreducible Characters of U , Part IV

F	χ	I	#	$\chi(1)$	Inductions in G
$F_{15,19}$	$\chi_{b_1}^{a_6, a_{15}, a_{19}}$ $\chi_{b_1, b_2, b_4}^{a_{15}, a_{19}}$	$I_A = \{2, 3, 5, 7, 8, 9, 10\}, I_L = \{4, 11, 12, 13, 14, 16, 17\}$ $I_A = \{3, 5, 7, 8, 9, 10\}, I_L = \{11, 12, 13, 14, 16, 17\}$	$q(q-1)^3$ $q^3(q-1)^2$	q^7 q^5	
$F_{16,18}$	$\chi_{b_1, 4}^{a_{11}, a_{12}, a_{16}, a_{18}}$ $\chi^{a_8, a_{14}, a_{16}, a_{18}}$ $\chi_{b_3, b_5, b_9, b_{11}}^{a_8, a_{16}, a_{18}}$ $\chi_{b_1, b_9, b_{14}}^{a_{12}, a_{16}, a_{18}}$ χ	$I_A = \{1, 2, 3, 4, 5, 6\}, I_L = \{7, 8, 9, 10, 13, 15\}$ $I_A = \{1, 2, 3, 4, 5, 6\}, I_L = \{7, 9, 10, 11, 13, 15\}$ $I_A = \{1, 2, 4, 6\}, I_L = \{7, 10, 13, 15\}$ $I_A = \{2, 3, 5, 10, 13\}, I_L = \{4, 6, 7, 8, 15\}$ $I_A = \{\}, I_L = \{\},$ see $\mathcal{C}_i, i = 41, 42$ in section A.7	$q(q-1)^4$ $(q-1)^4$ $q^4(q-1)^3$ $q^3(q-1)^3$	q^6 q^6 q^4 q^5	
$F_{17,18}$	$\chi_{b_3, b_5, b_{10}, b_{13}}^{a_{17}, a_{18}}$	$I_A = \{1, 2, 4, 6, 7, 8\}, I_L = \{9, 11, 12, 14, 15, 16\}$	$q^4(q-1)^2$	q^6	
$F_{18,19}$	$\chi_{b_1, b_4, b_6}^{a_{18}, a_{19}}$	$I_A = \{2, 3, 5, 7, 9, 11, 13\}, I_L = \{8, 10, 12, 14, 15, 16, 17\}$	$q^3(q-1)^2$	q^7	
$F_{19,20}$	$\chi_{b_1, b_2}^{a_{19}, a_{20}}$	$I_A = \{3, 4, 6, 7, 8, 9, 11, 16\}, I_L = \{5, 10, 12, 13, 14, 15, 17, 18\}$	$q^2(q-1)^2$	q^8	
$F_{1,2,7}$	χ^{a_1, a_2, a_7}	$I_A = \{3\}, I_L = \{4\}$	$(q-1)^3$	q	T_{56}
$F_{1,2,10}$	$\chi^{a_1, a_2, a_{10}}$ $\chi_{\mathcal{C}_{3,4}}^{a_1, a_2, a_7, a_{10}}$	$I_A = \{\}, I_L = \{\},$ see \mathcal{C} in section A.7 $I_A = \{\}, I_L = \{\},$ see \mathcal{C}_{43} in section A.7	$(q-1)^3$ $4(q-1)^4$	q $q/2$	T_{65}
$F_{1,3,6}$	χ^{a_1, a_3, a_6}	$I_A = \{2\}, I_L = \{4\}$	$(q-1)^3$	q	T_{59}
$F_{1,6,7}$	$\chi_{b_2}^{a_1, a_6, a_7}$	$I_A = \{4\}, I_L = \{3\}$	$q(q-1)^3$	q	T_{67} for $b_2 = 0$, T_{68} for $b_2 \neq 0$
$F_{1,6,10}$	$\chi_{b_3, b_7}^{a_1, a_6, a_{10}}$	$I_A = \{4\}, I_L = \{2\}$	$q^2(q-1)^3$	q	T_{154} for $b_3 = b_7 = 0$, T_{155} for $b_3 \neq 0 = b_7$, T_{158} for $b_3 = 0 \neq b_7$, T_{159} for $b_3, b_7 \neq 0$ and $\chi_{b_3}(x_3(s_0)) = -1$ for s_0 s.t. $\chi_{b_3, b_7}^{a_1, a_6, a_{10}}(x_3(s_0)) \neq 0$ T_{161} for $b_3, b_7 \neq 0$ and $\chi_{b_3}(x_3(s_0)) = 1$ for s_0 s.t. $\chi_{b_3, b_7}^{a_1, a_6, a_{10}}(x_3(s_0)) \neq 0$
$F_{1,9,10}$	$\chi_{b_4}^{a_1, a_9, a_{10}}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q(q-1)^3$	q^2	
$F_{2,4,5}$	χ^{a_2, a_4, a_5}	$I_A = \{1\}, I_L = \{3\}$	$(q-1)^3$	q	T_{62}
$F_{2,5,7}$	$\chi_{b_1}^{a_2, a_5, a_7}$	$I_A = \{3\}, I_L = \{4\}$	$q(q-1)^3$	q	T_{70} for $b_1 = 0$, T_{71} for $b_1 \neq 0$
$F_{2,5,10}$	$\chi_{b_4, b_7}^{a_2, a_5, a_{10}}$	$I_A = \{3\}, I_L = \{1\}$	$q^2(q-1)^3$	q	
$F_{2,8,10}$	$\chi_{b_3}^{a_2, a_8, a_{10}}$	$I_A = \{1, 4\}, I_L = \{5, 7\}$	$q(q-1)^3$	q^2	T_{123} for $b_3 = 0$, T_{124} for $b_3 \neq 0$
$F_{5,6,7}$	χ^{a_5, a_6, a_7}	$I_A = \{1, 4\}, I_L = \{2, 3\}$	$(q-1)^3$	q^2	T_{17}
$F_{5,6,10}$	$\chi_{b_7}^{a_5, a_6, a_{10}}$	$I_A = \{1, 4\}, I_L = \{2, 3\}$	$q(q-1)^3$	q^2	T_{53} for $b_7 = 0$, T_{18} for $b_7 \neq 0$
$F_{5,9,10}$	$\chi_{b_1, b_4}^{a_5, a_9, a_{10}}$	$I_A = \{2, 3\}, I_L = \{6, 7\}$	$q^2(q-1)^3$	q^2	
$F_{6,8,10}$	$\chi_{b_2, b_3}^{a_6, a_8, a_{10}}$	$I_A = \{1, 4\}, I_L = \{5, 7\}$	$q^2(q-1)^3$	q^2	
$F_{8,9,10}$	$\chi_{b_1}^{a_8, a_9, a_{10}}$	$I_A = \{3, 5, 7\}, I_L = \{2, 4, 6\}$	$q(q-1)^3$	q^3	T_{23} for $b_1 = 0$, T_{29} for $b_1 \neq 0$
$F_{11,12,13}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathcal{C}_{44} in section A.7			
$F_{11,12,16}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathcal{C}_{45} in section A.7			
$F_{11,13,15}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathcal{C}_{46} in section A.7			
$F_{11,15,16}$	χ	$I_A = \{\}, I_L = \{\},$ see \mathcal{C}_{47} in section A.7			
$F_{14,15,16}$	$\chi^{a_8, a_{14}, a_{15}, a_{16}}$ $\chi_{b_1, b_7}^{a_{14}, a_{15}, a_{16}}$	$I_A = \{1, 2, 3, 4, 5, 10\}, I_L = \{6, 7, 9, 11, 12, 13\}$ $I_A = \{2, 3, 5, 10, 13\}, I_L = \{4, 6, 9, 11, 12\}$	$(q-1)^4$ $q^2(q-1)^3$	q^6 q^5	

A.7 The Nonabelian Cores

Table 7: The Nonabelian Cores, Part I

\mathfrak{C}_i	Σ	S	Z	A	L	$ S $	$ Z $	$ comrel $
\mathfrak{C}_1	{10}	{3, 4, 7, 10}	{10}	{}	{}	4	1	1
\mathfrak{C}_2	{13}	{3, 4, 6, 9, 13}	{13}	{2}	{10}	5	1	1
\mathfrak{C}_3	{15}	{1, 3, 4, 5, 7, 8, 10, 12, 15}	{15}	{}	{}	9	1	7
\mathfrak{C}_4	{16}	{2, 3, 7, 9, 16}	{16}	{4, 6}	{10, 13}	5	1	1
\mathfrak{C}_5	{18}	{1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 18}	{8, 10, 18}	{2}	{15}	11	3	10
\mathfrak{C}_6	{18}	{1, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 18}	{10, 18}	{2}	{15}	12	2	8
\mathfrak{C}_7	{18}	{1, 5, 6, 7, 9, 11, 13, 14, 18}	{7, 18}	{2, 3}	{4, 15}	7	2	4
\mathfrak{C}_8	{18}	{3, 6, 8, 9, 11, 13, 18}	{8, 13, 18}	{1, 2, 5}	{4, 7, 15}	7	3	1
\mathfrak{C}_9	{18}	{1, 3, 4, 5, 6, 9, 11, 13, 14, 18}	{18}	{2}	{15}	10	1	7
\mathfrak{C}_{10}	{20}	{1, 9, 12, 13, 16, 17, 20}	{12, 13, 20}	{4, 6, 7, 8, 10}	{2, 3, 5, 15, 18}	7	3	1
\mathfrak{C}_{11}	{20}	{2, 5, 7, 8, 9, 10, 11, 16, 17, 20}	{5, 10, 16, 17, 20}	{1, 4, 6}	{3, 15, 18}	10	5	4
\mathfrak{C}_{12}	{20}	{1, 3, 7, 8, 9, 11, 13, 16, 17, 20}	{13, 20}	{2, 4, 6}	{10, 15, 18}	10	2	5
\mathfrak{C}_{13}	{20}	{2, 5, 7, 8, 9, 11, 16, 17, 20}	{5, 16, 17, 20}	{1, 4, 6}	{3, 15, 18}	9	4	4
\mathfrak{C}_{14}	{20}	{1, 2, 3, 7, 8, 9, 10, 11, 16, 17, 20}	{10, 20}	{4, 6}	{15, 18}	11	2	7
\mathfrak{C}_{15}	{20}	{2, 5, 7, 8, 9, 10, 11, 16, 20}	{5, 10, 16, 20}	{1, 4, 6}	{3, 15, 18}	9	4	2
\mathfrak{C}_{16}	{20}	{1, 2, 3, 7, 8, 9, 11, 16, 17, 20}	{20}	{4, 6}	{15, 18}	10	1	7
\mathfrak{C}_{17}	{20}	{2, 5, 7, 8, 9, 11, 16, 20}	{5, 16, 20}	{1, 4, 6}	{3, 15, 18}	8	3	2
\mathfrak{C}_{18}	{20}	{2, 5, 7, 8, 10, 11, 20}	{5, 10, 20}	{1, 4, 6}	{3, 15, 18}	7	3	1
\mathfrak{C}_{19}	{20}	{2, 5, 7, 8, 11, 20}	{5, 20}	{1, 4, 6}	{3, 15, 18}	6	2	1
\mathfrak{C}_{20}	{22}	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22}	{22}	{}	{}	22	1	43
\mathfrak{C}_{21}	{23}	{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23}	{23}	{4}	{22}	21	1	37
\mathfrak{C}_{22}	{24}	{1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24}	{24}	{2, 6}	{22, 23}	20	1	32
\mathfrak{C}_{23}	{1, 10}	{1, 3, 4, 7, 10}	{1, 10}	{}	{}	5	2	1
\mathfrak{C}_{24}	{1, 13}	{1, 3, 4, 6, 9, 13}	{1, 13}	{2}	{10}	9	2	1

Table 8: The Nonabelian Cores, Part II

\mathcal{C}_i	Σ	S	Z	A	L	$ S $	$ Z $	$ comrel $
\mathcal{C}_{25}	{1, 16}	{1, 2, 3, 7, 9, 16}	{1, 16}	{4, 6}	{10, 13}	6	2	1
\mathcal{C}_{26}	{2, 10}	{2, 3, 4, 7, 10}	{2, 10}	{}	{}	5	2	1
\mathcal{C}_{27}	{2, 15}	{1, 2, 3, 4, 5, 7, 8, 10, 12, 15}	{2, 15}	{}	{}	10	2	7
\mathcal{C}_{28}	{5, 16}	{2, 5, 7, 9, 16}	{5, 16}	{1, 4, 6}	{3, 10, 13}	5	2	1
\mathcal{C}_{29}	{6, 15}	{1, 2, 3, 5, 6, 7, 8, 10, 12, 15}	{6, 15}	{4}	{2}	10	2	5
\mathcal{C}_{30}	{8, 13}	{3, 6, 8, 9, 16}	{8, 13}	{1, 2, 5}	{4, 7, 10}	5	2	1
\mathcal{C}_{31}	{8, 16}	{2, 5, 7, 8, 9, 16}	{5, 8, 16}	{1, 4, 6}	{3, 10, 13}	6	3	1
\mathcal{C}_{32}	{9, 15}	{1, 4, 5, 8, 9, 10, 12, 15}	{9, 15}	{3, 7}	{2, 6}	8	2	2
\mathcal{C}_{33}	{10, 11}	{3, 4, 7, 10, 11}	{10, 11}	{1, 2, 5}	{6, 8, 9}	5	2	1
\mathcal{C}_{34}	{11, 12}	{2, 3, 4, 5, 6, 7, 8, 9, 11, 12}	{9, 11, 12}	{1}	{10}	10	3	9
\mathcal{C}_{35}	{11, 13}	{3, 6, 8, 9, 11, 13}	{8, 11, 13}	{1, 2, 5}	{4, 7, 10}	6	3	1
\mathcal{C}_{36}	{11, 15}	{3, 4, 7, 10, 11, 12, 15}	{10, 11, 15}	{1, 5, 8}	{2, 6, 9}	7	3	1
\mathcal{C}_{37}	{12, 13}	{1, 2, 4, 5, 7, 9, 10, 12, 13}	{9, 12, 13}	{3}	{8}	9	3	4
\mathcal{C}_{38}	{12, 13}	{1, 2, 4, 10, 12, 13}	{12, 13}	{3, 5}	{7, 8}	6	2	2
\mathcal{C}_{39}	{13, 15}	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15}	{13, 15}	{}	{}	13	2	11
\mathcal{C}_{40}	{15, 16}	{1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 15, 16}	{15, 16}	{4}	{13}	12	2	8
\mathcal{C}_{41}	{16, 18}	{1, 3, 4, 5, 7, 9, 11, 13, 14, 16, 18}	{7, 16, 18}	{2, 6}	{10, 15}	11	3	7
\mathcal{C}_{42}	{16, 18}	{1, 3, 5, 9, 11, 14, 16, 18}	{16, 18}	{2, 6, 13}	{4, 10, 15}	8	2	4
\mathcal{C}_{43}	{1, 2, 10}	{1, 2, 3, 4, 7, 10}	{1, 2, 10}	{}	{}	6	3	1
\mathcal{C}_{44}	{11, 12, 13}	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}	{11, 12, 13}	{}	{}	13	3	14
\mathcal{C}_{45}	{11, 12, 16}	{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 16}	{11, 12, 16}	{4}	{13}	12	3	11
\mathcal{C}_{46}	{11, 13, 15}	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15}	{11, 13, 15}	{}	{}	14	3	14
\mathcal{C}_{47}	{11, 15, 16}	{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16}	{11, 15, 16}	{4}	{13}	13	3	11

A.8 Representatives of the Unipotent Conjugacy Classes in G

The following is a table listing representatives for each unipotent conjugacy class in $F_4(q)$ and giving the order of the centralizer of this representative in $F_4(q)$. These results are taken from ([Shinoda]).

The elements $\eta, \zeta \in \mathbb{F}_q$ are chosen as follows: Let $\eta \in k - f(k)$ for $f : t \mapsto t^2 + t$ ($k = \mathbb{F}_q$; since $\ker(f) = \{0, 1\}$ and thus f not surjective, such an η exists), so $X^2 + X + \eta$ is an irreducible polynomial in the polynomial ring $k[X]$ over \mathbb{F}_q . Furthermore, let $\zeta \in k - f'(k)$ for $f' : t \mapsto t^3 + t$ (like f , f' is not surjective, so such a ζ exists). Thus $X^3 + X + \zeta$ is an irreducible polynomial in $k[X]$ as well (if it were reducible, it would have to have a zero, since it is of degree 3. But ζ was chosen in $k - f'(k)$). To facilitate the notation later on, the conjugacy classes are enumerated as in [Shinoda] with y_i 's.

Table 9: Representatives of the Unipotent Conjugacy Classes in G

Representative	$C_G(x)$
$y_0 = 1$	$q^{24}(q^2 - 1)(q^6 - 1)(q^8 - 1)(q^{12} - 1)$
$y_1 = x_{21}(1)$	$q^{24}(q^2 - 1)(q^4 - 1)(q^6 - 1)$
$y_2 = x_{24}(1)$	$q^{24}(q^2 - 1)(q^4 - 1)(q^6 - 1)$
$y_3 = x_{21}(1)x_{24}(1)$	$q^{24}(q^2 - 1)(q^4 - 1)$
$y_4 = x_{16}(1)x_{21}(1)$	$q^{20}(q^2 - 1)^2$
$y_5 = x_9(1)x_{16}(1)x_{18}(1)$	$2q^{17}(q^2 - 1)(q^3 - 1)$
$y_6 = x_9(1)x_{16}(1)x_{18}(1)x_{23}(\eta)$	$2q^{17}(q^2 - 1)(q^3 + 1)$
$y_7 = x_9(1)x_{12}(1)x_{16}(1)$	$2q^{17}(q^2 - 1)(q^3 - 1)$
$y_8 = x_9(1)x_{12}(1)x_{16}(1)x_{22}(\eta)$	$2q^{17}(q^2 - 1)(q^3 + 1)$
$y_9 = x_9(1)x_{15}(1)$	$2q^{14}(q^2 - 1)(q^4 - 1)$
$y_{10} = x_9(1)x_{15}(1)x_{24}(\eta)$	$2q^{14}(q^2 - 1)(q^4 - 1)$
$y_{11} = x_9(1)x_{10}(1)x_{20}(1)$	$q^{16}(q^2 - 1)$
$y_{12} = x_9(1)x_{12}(1)x_{20}(1)$	$q^{16}(q^2 - 1)$
$y_{13} = x_9(1)x_{15}(1)x_{16}(1)$	$q^{14}(q^2 - 1)$
$y_{14} = x_9(1)x_{10}(1)x_{15}(1)$	$q^{14}(q^2 - 1)$
$y_{15} = x_9(1)x_{12}(1)x_{16}(1)x_{18}(1)$	q^{16}
$y_{16} = x_9(1)x_{12}(1)x_{15}(1)x_{16}(1)$	q^{14}
$y_{17} = x_5(1)x_9(1)x_{15}(1)x_{16}(1)$	$6q^{12}$
$y_{18} = x_5(1)x_9(1)x_{15}(1)x_{16}(1)x_{20}(\eta)$	$2q^{12}$
$y_{19} = x_8(1)x_9(1)x_{10}(1)x_{15}(1)x_{18}(\zeta)$	$3q^{12}$
$y_{20} = x_2(1)x_7(1)x_{15}(1)$	$2q^8(q^2 - 1)$
$y_{21} = x_2(1)x_7(1)x_{15}(1)x_{16}(\eta)x_{24}(\eta)$	$2q^8(q^2 - 1)$
$y_{22} = x_3(1)x_6(1)x_8(1)$	$2q^8(q^2 - 1)$
$y_{23} = x_3(1)x_6(1)x_8(1)x_{13}(\eta)x_{19}(\eta)x_{23}(\eta)$	$2q^8(q^2 - 1)$
$y_{24} = x_1(1)x_5(1)x_6(1)x_{10}(1)$	$8q^8$
$y_{25} = x_1(1)x_5(1)x_6(1)x_{10}(1)x_{15}(\eta)$	$4q^8$
$y_{26} = x_1(1)x_5(1)x_6(1)x_{10}(1)x_{15}(\eta)x_{18}(\eta)$	$8q^8$
$y_{27} = x_5(1)x_6(1)x_7(1)x_{10}(1)x_{13}(\eta)$	$4q^8$
$y_{28} = x_5(1)x_6(1)x_7(1)x_{10}(1)x_{13}(\eta)x_{15}(\eta)$	$4q^8$
$y_{29} = x_2(1)x_3(1)x_4(1)x_{15}(1)$	$2q^6$
$y_{30} = x_2(1)x_3(1)x_4(1)x_{10}(\eta)x_{15}(1)x_{22}(\eta)x_{23}(\eta)$	$2q^6$
$y_{31} = x_1(1)x_2(1)x_3(1)x_4(1)$	$4q^4$
$y_{32} = x_1(1)x_2(1)x_3(1)x_4(1)x_{10}(\eta)$	$4q^4$
$y_{33} = x_1(1)x_2(1)x_3(1)x_4(1)x_{15}(\eta)$	$4q^4$
$y_{34} = x_1(1)x_2(1)x_3(1)x_4(1)x_{10}(\eta)x_{15}(\eta)$	$4q^4$

A.9 Fusions of the Conjugacy Classes of U in $F_4(q)$

The unipotent conjugacy classes are named as their representatives y_i given in Appendix A.8. I.e. y_i is the conjugacy class in $F_4(q)$ of the element y_i .

- y₁** $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t, s=x=0=z=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=0=z=x'=y'=z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(ue+x)x_{23}(ae+y)x_{24}(be+s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t, s=y=z=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(uf+x)x_{24}(bf+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=v=t=s=0=x=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+b^2u^{-2}v+t)x_{23}(du+y)x_{24}(ad+z) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{24}(eu+t) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{24}(cu+t) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au+v)x_{23}(bu+t)x_{24}(ab+w) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{24}(au+v) \mid a, b, c \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b) \mid a, b \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)$,
 $\{x_{19}(u)x_{21}(a) \mid a \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)$,
 $\{x_{21}(u)\}_{u \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)$,
- y₂** $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(bfu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=s=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(ceu^{-1}+x')x_{22}(b^2fu^{-2}+y')x_{23}(g)x_{24}(agu^{-1}+z') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=z=x'=y'=z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(aeu^{-1}+z)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0=s=y=z=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+s)x_{19}(d)x_{20}(a^2cu^{-1}+t)x_{21}(adu^{-1}+z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1}+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=s=r=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(acu^{-1}+w)x_{22}(d)x_{23}(c^2u^{-1}+v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{15}(u)x_{18}(a)x_{19}(v)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(aeu^{-1}+w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1}+v)x_{21}(abu^{-1}+w)x_{22}(b^2u^{-1}+t)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{18}(u)x_{19}(v)x_{20}(a)x_{21}(b)x_{22}(c)x_{23}(b^2u^{-1}+w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{20}(u)x_{21}(a)x_{22}(a^2u^{-1}+v)x_{23}(b)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)$,
 $\{x_{22}(u)x_{23}(a)x_{24}(b) \mid a, b \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times}$,
 $\{x_{23}(u)x_{24}(a) \mid a \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)$,
 $\{x_{24}(u)\}_{u \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)$,
- y₃** $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t=s, x=0=z, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0=s, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(b^2gu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t=s, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(bgu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=t=s=0, x \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=x=0, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$(q-1)^2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(bfu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t=s=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=z=x'=y'=0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(bh+y')x_{24}(ch+x') \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, r=o=x=y=z=0=x'=y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=x=y=z=0=x'=y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(ceu^{-1}+x')x_{22}(b^2fu^{-2}+y')x_{23}(g)x_{24}(agu^{-1}+z') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=z=0=x'=y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2}+y')x_{23}(h)x_{24}(ahu^{-1}+z') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o \neq 0, x=y=0=z, x'=y'=0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1}+x')x_{22}(b^2hu^{-2}+y')x_{23}(i)x_{24}(f^2u^{-1}+z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=o=x=y=z=0, x'=y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(ue+x)x_{23}(ae+y)x_{24}(be+s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t, s=y=z=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(bgu^{-1}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=t=s=y, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(e)x_{19}(f)x_{20}(ab+w)x_{21}(g)x_{22}(h)x_{23}(af+y)x_{24}(bhu^{-1}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t=s=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1}+z)x_{22}(a^2gu^{-2}+y)x_{23}(e^2u^{-1}+x)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s=y=0, z=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0, s \neq 0, y=0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(adu^{-1}+v)x_{20}(e)x_{21}(aeu^{-1}+z)x_{22}(a^2eu^{-2}+y)x_{23}(d^2u^1+x)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0=s=y=z, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(uf+x)x_{24}(bf+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=v=t=s=0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(gu^{-1}v+y)x_{23}(h)x_{24}(e^2u^{-2}v+hbv^{-1}+x) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=s=r=o=z=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(g)x_{24}(u^{-1}bg+y) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, v=t=s=0=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+b^2u^{-2}v+t)x_{23}(du+y)x_{24}(ad+z) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1}+s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(du^{-1}v+w)x_{23}(g)x_{24}(c^2vu^{-2}+z) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+b^2u^{-2}v+t)x_{23}(f)x_{24}(afu^{-1}+dvv^{-1}+z) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{24}(eu+t) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v+t)x_{21}(e)x_{22}(b^2u^{-2}v+w)x_{23}(eu^{-1}v+s)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=w=t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v+t)x_{19}(e)x_{20}(a^2cu^{-2}+s)x_{21}(f)x_{22}(fu^{-1}v+z)x_{23}(d^2u^{-2}v+w)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, z=w=t=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+s)x_{19}(d)x_{20}(a^2cu^{-1}+t)x_{21}(adu^{-1}+\sqrt{wg}+z)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t=s=r=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+s)x_{19}(d)x_{20}(a^2cu^{-1}+t)x_{21}(adu^{-1}+z) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t=s=r=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=s=r=0=z, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{24}(cu + t) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}((au^{-1})^2v + w)x_{23}(cu^{-1}v + t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(cu^{-1}v + w)x_{23}(cdu^{-1} + t)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{17}(b)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}(bu^{-1}v + w)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(fau^{-1} + c^2u^{-2}v + s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au + v)x_{23}(bu + t)x_{24}(ab + w) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au + v)x_{23}(e)x_{24}(ab + w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1} + z)x_{18}(a^2u^{-1} + y)x_{19}(c)x_{20}(a^2bu^{-2} + x)x_{21}(acu^{-1} + \sqrt{ye} + w)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, y \neq 0, w=x=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1} + z)x_{18}(a^2u^{-1} + y)x_{19}(c)x_{20}(a^2bu^{-2} + x)x_{21}(acu^{-1} + \sqrt{yv} + w)x_{22}(d)x_{23}(c^2u^{-1} + v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, y=w=x=0, v \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{24}(au + v) \mid a, b, c \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{22}(v)x_{23}(au^{-1}v + w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(au^{-1}v + w)x_{21}(c)x_{22}(d)x_{23}(b^2u^{-2}v + t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{15}(u)x_{18}(a)x_{19}(v)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(aeu^{-1} + c^2u^{-1} + w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1} + v)x_{21}(abu^{-1} + \sqrt{vc} + w)x_{22}(c)x_{23}(d)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1} + v)x_{21}(abu^{-1} + \sqrt{vt} + w)x_{22}(b^2u^{-1} + t)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{24}(v) \mid a, b \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$, $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{23}(v)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{17}(u)x_{19}(a)x_{20}(v)x_{21}(b)x_{22}(au^{-1}v + w)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{18}(u)x_{19}(v)x_{20}(a)x_{21}(b)x_{22}(c)x_{23}(b^2u^{-1} + w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{19}(u)x_{21}(a)x_{24}(v) \mid a \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{19}(u)x_{21}(a)x_{23}(v)x_{24}(b) \mid a, b \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{19}(u)x_{21}(a)x_{22}(v)x_{23}(b)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{20}(u)x_{21}(a)x_{22}(a^2u^{-1} + v)x_{23}(b)x_{24}(c) \mid a, b, c \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{21}(u)x_{24}(v)\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{21}(v)x_{23}(u)x_{24}(a) \mid a \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{21}(v)x_{22}(u)x_{23}(a)x_{24}(b) \mid a, b \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,

y₄ $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=0=z=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agv^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=z=y=x=z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w=s=z=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(uf + y)x_{21}(g)x_{22}(ayu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(c^2xu^{-2} + s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0=t, s \neq 0, x=0=z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(uf + y)x_{21}(g)x_{22}(ayu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, x=0=z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(uf + y)x_{21}(g)x_{22}(ayu^{-1} + w)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, x=0=z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0, s \neq 0, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(b^2gu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t, s, x \neq 0 \in \mathbb{F}_q, \text{ or } s \neq 0}$ with parameter set of size $(q-1)^3(q+1)$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(j)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z=y=x=z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(i)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s=0, x \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(bgu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, t \neq 0, s=0, x \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(bgu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, t=s=0, x \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(h)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s=x=0, y, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(bfu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, s=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s=x=0, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s=x=0, y, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s=x=0, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r \neq 0, o=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=s=t=0=r, o \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r=0, o \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(cg+x') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=0, z, x' \neq 0, y'=0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(gu+z')x_{23}(bg+y')x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=0, z, y' \neq 0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdu^{-1}+x)x_{18}(ad+w)x_{19}(f)x_{20}(abdu^{-1}+y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$

$\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=r=o=0=x=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(cg + x') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=x=y=z=0, x' \neq 0, y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=x=y=z=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, r=o=x=y=z=0, x' \neq 0, y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=0=w, t \neq 0, r=o=x=y=z=0, x', y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(ahu^{-1} + z') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=x=y=z=0, z \neq 0, y' = 0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(ahu^{-1} + z') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=s=t=0=r, o \neq 0, x=y=z=y'=0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=s=t=0=r, o \neq 0, x=y=0, z \neq 0, y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1} + x')x_{22}(b^2hu^{-2} + y')x_{23}(i)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=0=s, t \neq 0, r=o=y=0, z \neq 0, x' = y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1} + x')x_{22}(b^2hu^{-2} + y')x_{23}(i)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=0=s, t \neq 0, r=o=y=0, x' \neq 0, y' = 0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=0=s, t \neq 0, r=0, o \neq 0, y=0=y', z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(beu^{-1} + z)x_{20}(f)x_{21}(ceu^{-1} + x')x_{22}(b^2fu^{-2} + y')x_{23}(g)x_{24}(agu^{-1} + z') \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0=r, o=x=y=0=z, x' \neq 0, y' = 0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o, x \neq 0, y=y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=r=0, o, x=y=0=y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\} \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r=o=x=y=y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(ue + x)x_{23}(ae + y)x_{24}(be + s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(ue + x)x_{23}(ae + y)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(ae + y)x_{24}(bgu^{-1} + s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(ae + y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(ae + y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(ae + y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(af + y)x_{24}(bhu^{-1} + s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t=s, y \in \mathbb{F}_q, s \text{ or } y \neq 0}$ with parameter set of size $(q-1)^3(q+1)$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t=0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(bhu^{-1} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t=0, s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(a^2gu^{-2} + y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(a^2fu^{-2} + y)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, t=0, s, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1}+z)x_{22}(a^2gu^{-2}+y)x_{23}(e^2u^{-1}+x)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s=y=0, z \neq 0, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1}+z)x_{22}(a^2gu^{-2}+y)x_{23}(e^2u^{-1}+x)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s=y=z=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2} + y)x_{23}(d^2u^1 + x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=0, s \neq 0, y=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2} + y)x_{23}(d^2u^1 + x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=0, s \neq 0, y=0, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2} + y)x_{23}(d^2u^1 + x)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=s=y=0, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(aeu^{-1} + z)x_{22}(a^2eu^{-2} + y)x_{23}(d^2u^1 + x)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=t=s=y=0, z \neq 0, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(gu^{-1}v + y)x_{23}(h)x_{24}(e^2u^{-2}v + hbu^{-1} + x) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=s=r=o=z, y, x \in \mathbb{F}_q, y \text{ or } x \neq 0}$ with parameter set of size $(q-1)^3(q+1)$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, r=o=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=s=0, r \neq 0, o=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(u^{-1}bc^2 + z)x_{23}(uf + x)x_{24}(bf + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=v=t=s=0, x, y, z \in \mathbb{F}_q, y \text{ or } z \neq 0}$ with parameter set of size $(q-1)^2q(q+1)$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v \neq 0, t=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(g)x_{24}(u^{-1}bg+y) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, v=t=s=0, y, z \in \mathbb{F}_q, y \text{ or } z \neq 0}$ with parameter set of size $(q-1)^3(q+1)$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(du^{-1}v + w)x_{23}(g)x_{24}(c^2vu^{-2} + z) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t=s=0, z \in \mathbb{F}_q, w \text{ or } z \neq 0}$ with parameter set of size $(q-1)^3(q+1)$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q}$

$\mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{17}(c)x_{19}(d)x_{20}(au + v)x_{21}(e)x_{22}(ab^2u^{-1} + b^2u^{-2}v + t)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ with parameter set of size $(q-1)^2q$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{17}(c)x_{19}(d)x_{20}(au + v)x_{21}(e)x_{22}(ab^2u^{-1} + b^2u^{-2}v + t)x_{23}(f)x_{24}(afu^{-1} + dvu^{-1} + z) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{17}(c)x_{19}(d)x_{20}(au + v)x_{21}(e)x_{22}(ab^2u^{-1} + b^2u^{-2}v + t)x_{23}(du + y)x_{24}(ad + z) \mid a, b, c, d, e \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{23}(v)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{21}(e)x_{22}(v)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{17}(c)x_{19}(d)x_{20}(v)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v + t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v + t)x_{21}(e)x_{22}(b^2u^{-2}v + w)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{14}(b)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-2}v + t)x_{21}(e)x_{22}(b^2u^{-2}v + w)x_{23}(eu^{-1}v + s)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(fu^{-1}v + z)x_{23}(d^2u^{-2}v + w)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,
 $\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(fu^{-1}v + z)x_{23}(d^2u^{-2}v + w)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q^2$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + \sqrt{wg} + z)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q^4$,
 $\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}((au^{-1})^2v + w)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}((au^{-1})^2v + w)x_{23}(cu^{-1}v + t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(cu^{-1}v + w)x_{23}(cdv^{-1} + t)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3(q+1)$,
 $\{x_{11}(u)x_{14}(a)x_{15}(v)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{23}(v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{11}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(v)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(cv^{-1}w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, e, f, g, h, i \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{17}(b)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}(bu^{-1}v + w)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(fau^{-1} + c^2u^{-2}v + s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q(q+1)$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{17}(b)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}(bu^{-1}v + w)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(fau^{-1} + c^2u^{-2}v + s) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,

- $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{16}(v)x_{17}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(v)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{12}(u)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(au+v)x_{23}(bu+t)x_{24}(ab+w) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w \neq 0, t \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^2$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(acu^{-1}+w)x_{22}(d)x_{23}(c^2u^{-1}+v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w, x=0, y, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^3$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(acu^{-1}+\sqrt{ye}+w)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, y \neq 0, w \neq 0, x=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(acu^{-1}+\sqrt{yv}+w)x_{22}(d)x_{23}(c^2u^{-1}+v)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, y=0, w \neq 0, x=0, v, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{23}(v)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{21}(c)x_{22}(v)x_{23}(au^{-1}v+w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{14}(u)x_{17}(a)x_{19}(b)x_{20}(v)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(au^{-1}v+w)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{14}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(au^{-1}v+w)x_{21}(c)x_{22}(d)x_{23}(b^2u^{-2}v+t)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{14}(u)x_{16}(v)x_{17}(a)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{14}(v)x_{15}(u)x_{17}(a)x_{18}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{15}(u)x_{17}(v)x_{18}(a)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{15}(u)x_{18}(a)x_{19}(v)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(aeu^{-1}+w) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1}+v)x_{21}(abu^{-1}+\sqrt{vc}+w)x_{22}(c)x_{23}(d)x_{24}(e) \mid a, b, c, d, e \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{16}(u)x_{17}(a)x_{19}(b)x_{20}(a^2u^{-1}+v)x_{21}(abu^{-1}+\sqrt{vt}+w)x_{22}(b^2u^{-1}+t)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_{17}(u)x_{19}(a)x_{21}(b)x_{22}(v)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{17}(u)x_{18}(v)x_{19}(a)x_{20}(b)x_{21}(c)x_{22}(d)x_{23}(e)x_{24}(f) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{17}(u)x_{19}(a)x_{20}(v)x_{21}(b)x_{22}(au^{-1}v+w)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{18}(u)x_{19}(v)x_{20}(a)x_{21}(b)x_{22}(c)x_{23}(b^2u^{-1}+w)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_{19}(u)x_{20}(v)x_{21}(a)x_{22}(b)x_{23}(c)x_{24}(d) \mid a, b, c, d \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
- y5** $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(euw^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-2}vs^2+x')x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, r=0=z=y, x'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}((l^2+l)v^{-2}u^{-1}(v^2z'+ux)^2+x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=z=y, x \neq 0, z'=0, x'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}((j^2+j)vs^2u^{-2}+y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, z=y=x=0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=s, t \neq 0, r=0=z=y=x=z', y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(bcu^{-1}+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1}+w)x_{20}(k)x_{21}(ciu^{-1}+z)x_{22}(b^2u^{-2}k+x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}((h^2+h)uw^{-2}w^2+o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=x=y=z=o=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1}+w)x_{16}(g)x_{17}(afu^{-1}+t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1}+aiu^{-1}+z)x_{21}(cfu^{-1}+biu^{-1}+x)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=z=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(bfu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, t, s \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+s)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=x=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, s \neq 0, x=0, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(bgu^{-1}+t)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, t, s \neq 0, x \neq 0, y=0 \in \mathbb{F}_q, t^2x^{-1}s^{-2}=(m^2+m) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag+c^2)u^{-1}+y)x_{21}(i)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s \neq 0, x \neq 0, y \neq 0 \in \mathbb{F}_q, w=(m^2+m)x^{-1}y^{-1}s^2u \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(g)x_{20}((af+c^2)u^{-1}+y)x_{21}(h)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s \neq 0, x=0, y, z \neq 0 \in \mathbb{F}_q, w=(m^2+m)z^{-2}s^2u \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+x)x_{17}((ae+bc)u^{-1}+z)x_{18}(f)x_{19}(cdu^{-1}+v)x_{20}((af+c^2)u^{-1}+y)x_{21}(g)x_{22}(e^2u^{-1}+w)x_{23}(ae^2u^{-2}+s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s \neq 0, x=0, y \neq 0, z=0 \in \mathbb{F}_q, v^2yu^{-1}s^{-2}=(m^2+m) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q(q/2-1)$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdx_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2+k)u^{-2}s^2o+x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=0, r=0, o \neq 0, x=y=z=0, x' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdx_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}((j^2+j)t^2wu^{-2}+y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0, o \neq 0, x=y=z=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(beu^{-1}+cdx_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}((j^2+j)t^2wu^{-2}+y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0, o \neq 0, x=y=z=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(ceu^{-1}+x')x_{22}(b^2fu^{-2}+y')x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(beu^{-1}+z)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2}+y')x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=0, r=0, o \neq 0, x \neq 0, z, y' \neq 0 \in \mathbb{F}_q, (oz)^2/(u(y')^2)=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3(q/2+1)$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}((j^2+j)u(y')^2x^{-2}+z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}((j^2+j)ux^{-2}(y')^2+z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1}+x')x_{22}(b^2hu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=0, o=y=0, x', y' \neq 0 \in \mathbb{F}_q, (ut(x')^2)/(uy')^2=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}((i^2+i)y'\sqrt{u/t}+x')x_{22}(b^2hu^{-2}+y')x_{23}(i)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=0, o \neq 0, y=0, y' \neq 0, x' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fs^{-1}y+z)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=0, o \neq 0, x=0, y \neq 0 \in \mathbb{F}_q, s^2o/uy^2=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(iw^{-1}v + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t, o=x=y=z=0 \in \mathbb{F}_q, v^2t/(uw^2)=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q((q-1)/2+1)$,

$\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r=0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^{-2}t^2v + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(vu^{-1}i + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=r=o=z=y=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fv^{-1}w + x)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=z=x=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(hav^{-1} + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=s=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(aeu^{-1} + w)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(aeu^{-1} + z)x_{22}(a^2eu^{-2} + y)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v,w=t=0=s, y \neq 0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^2$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2} + y)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v,w=t=0, s \neq 0, y \neq 0 \in \mathbb{F}_q, v^2su^{-1}y^{-2}=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(a^2fu^{-2} + y)x_{23}((h^2 + h)w^{-2}y^2u + x)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, t=0, s, y \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(a^2gu^{-2} + y)x_{23}((i^2 + i)t^{-1}s^{-1}y^2u + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1} + z)x_{22}(a^2gu^{-2} + y)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, t \neq 0, s=0, y \neq 0, z \in \mathbb{F}_q, z^2tu^{-1}y^{-2}=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^2w^2r^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, r \neq 0, o=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}((h^2 + h)v^2wu^{-2} + y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=r=o=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^{-1}(u^2s + vw)^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)w^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}((g^2 + g)v^2wu^{-2} + dvu^{-1} + z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}((g^2 + g)u^2sv^{-2} + o)x_{23}(d^2u^{-2}v + w)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w,t,s \neq 0, o=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^{-2}v + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w,t \neq 0, s=0, r \neq 0, z,y \in \mathbb{F}_q, r^2w/(ut^2)=m+m^2 \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^3/2$,

$\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(a^2bu^{-1} + s)x_{19}(e)x_{20}(a^2cu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w,t=0, s \neq 0 \in \mathbb{F}_q, v^2w/(us^2)=m+m^2 \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w,t,s \neq 0, r=0, z,y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^4$,

$\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} + z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w,t \neq 0, s=r=0, z,y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^3$,

$\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}((f^2 + f)u^{-2} + s)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdv^{-1} + t)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)v^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}((d^2 + d)uvw^{-2} + awv^{-2} + s)x_{19}(e)x_{20}(cv^{-1}w + b(d^2 + d)w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(bu)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}((d^2 + d)v^2wu^{-2} + vbu^{-2} + s)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0=s}$ with parameter set of size $(q-1)^3$,
 $\{x_{13}(u)x_{14}(a)x_{15}(v)x_{16}(b)x_{17}(abu^{-1} + w)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1} + z)x_{18}(a^2u^{-1} + y)x_{19}(c)x_{20}(a^2bu^{-2} + x)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, x \neq 0, y, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1} + z)x_{18}(a^2u^{-1} + y)x_{19}(c)x_{20}(a^2bu^{-2} + x)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, x \neq 0, y, z \neq 0 \in \mathbb{F}_q, z^2y/(u^2x^2)=m+m^2 \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,
 $\{x_{14}(u)x_{16}(v)x_{17}(a)x_{18}(w)x_{19}(b)x_{20}((c^2 + c)vwu^{-2} + wau^{-2} + t)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{15}(u)x_{16}(v)x_{17}(a)x_{18}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,
 $\{x_{16}(u)x_{17}(a)x_{18}(v)x_{19}(b)x_{20}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^2$,

y6 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, r=0=z=y, x'=u^{-2}vs^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=r=0=z=y, x \neq 0, z'=0, x'=v^{-2}u^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, z=y=x=0, y'=vs^2u^{-2}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0=s, t \neq v^{-1}u^2w, r=0=z=y=x=z', y'=vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)uv^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, x=y=z=0, o=uv^{-2}w^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag + c^2)u^{-1} + y)x_{21}(i)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, t, s \neq 0, x \neq 0, y=0 \in \mathbb{F}_q, t^2xu^{-1}s^{-2}=(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((ag + c^2)u^{-1} + y)x_{21}(i)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s \neq 0, x \neq 0, y \neq 0 \in \mathbb{F}_q, w=(m^2+m+\eta)x^{-1}y^{-1}s^2u \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(g)x_{20}((af + c^2)u^{-1} + y)x_{21}(h)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, s \neq 0, x=0, y, z \neq 0 \in \mathbb{F}_q, w=(m^2+m+\eta)z^{-2}s^2u \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(g)x_{20}((af + c^2)u^{-1} + y)x_{21}(h)x_{22}(e^2u^{-1} + w)x_{23}(ae^2u^{-2} + s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, s \neq 0, x=0, y \neq 0, z=0 \in \mathbb{F}_q, v^2yu^{-1}s^{-2}=(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}s^2o + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=0=r, o \neq 0, x=y=z=0, x'=u^{-2}s^2o\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}((j^2 + j)t^2wu^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0=x=y=z=0, y'=t^2wu^{-2}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}((j^2 + j)t^2wu^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0=x=y=z=0, y'=t^2wu^{-2}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r=0, o \neq 0, x=y=z=0, y'=t^2wu^{-2}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2fu^{-2} + y')x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=s=t=0, r=o, x=y=0, z, y' \neq 0 \in \mathbb{F}_q, (oz)^2/(u(y')^2)^2=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)u(y')^2x^{-2} + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z'=u(y')^2x^{-2}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ux^{-2}(y')^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=s=t=r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, z'=u(y')^2x^{-2}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(cfu^{-1} + x')x_{22}(b^2hu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=o=y=0, x', y' \neq 0 \in \mathbb{F}_q, (ut(x')^2)/(uy')^2=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}((i^2 + i)y'\sqrt{u/t} + x')x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r=0, o \neq 0, x \neq 0, y=0, y' \neq 0, x'=y'\sqrt{u/t}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fs^{-1}y + z)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r=z, o=x=0, y \neq 0 \in \mathbb{F}_q, s^2o/uy^2=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(iw^{-1}v + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v, w \neq 0, s=0, t, o=x=y=z=0 \in \mathbb{F}_q, v^2t/(uw^2)=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r=0, y=u^{-2}vt^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^{-2}t^2v + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0, y=x=u^{-2}t^2v\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(w)x_7(u)x_8(ax_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(vu^{-1}i + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t=r=o=z=0, y, s=uv^{-2}w^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_8(v)x_9(ax_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fv^{-1}w + x)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t=s=z=0, x, y=uv^{-2}w^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(ax_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(adu^{-1} + v)x_{20}(e)x_{21}(f)x_{22}(a^2eu^{-2} + y)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v, w=t=0, s \neq 0, y \neq 0 \in \mathbb{F}_q, v^2su^{-1}y^{-2}=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_6(u)x_9(ax_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(a^2fu^{-2} + y)x_{23}((h^2 + h)w^{-2}y^2u + x)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, w \neq 0, t=0, s, y \neq 0, x=w^{-2}y^2u\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_6(u)x_9(ax_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(a^2gu^{-2} + y)x_{23}((i^2 + i)t^{-1}s^{-1}y^2u + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \neq 0, x=t^{-1}s^{-1}y^2u\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_9(ax_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(agu^{-1} + z)x_{22}(a^2gu^{-2} + y)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, t \neq 0, s=0, y \neq 0, z \in \mathbb{F}_q, z^2tu^{-1}y^{-2}=m^2+m+\eta$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_7(u)x_8(ax_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^2w^2r^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, r \neq 0, o=z=0, y=u^{-2}v^2w^2r^{-1}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(ax_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}((h^2 + h)v^2wu^{-2} + y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=r=o=z=0, y=v^2wu^{-2}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(ax_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^{-1}(u^2s + vw)^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=0, z, y=u^{-2}v^{-1}(u^2s + vw)^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)vz^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0, y=vz^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}((g^2 + g)v^2wu^{-2} + dvu^{-1} + z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, s, z=v^2wu^{-2}\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,

$\{x_9(u)x_{11}(a)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v+t)x_{19}(e)x_{20}(a^2cu^{-2}+s)x_{21}(f)x_{22}((g^2+g)u^2sv^{-2}+o)x_{23}(d^2u^{-2}v+w)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, s \neq 0, o = u^s v^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3 q^2$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2+h)u^{-2}v+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s = u^{-2} v \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(acu^{-1}+r)x_{18}(a^2bu^{-1}+s)x_{19}(d)x_{20}(a^2cu^{-1}+t)x_{21}(adu^{-1}+z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1}+y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, s=0, r \neq 0, z, y \in \mathbb{F}_q, r^2 w / (ut^2) = m + m^2 + \eta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q^3 / 2$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1}+v)x_{15}(a^2u^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(a^2bu^{-1}+s)x_{19}(e)x_{20}(a^2cu^{-1}+t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w, t=0, s \neq 0 \in \mathbb{F}_q, v^2 w / (us) = m + m^2 + \eta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q / 2$,
 $\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}((f^2+f)u^{-2}+s)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s = u^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1}+t)x_{21}(f)x_{22}(g)x_{23}((h^2+h)v^{-2}+s)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s = v^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}((d^2+d)uvw^{-2}+awv^{-2}+s)x_{19}(e)x_{20}(cv^{-1}w+b(d^2+d)w+t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \in \mathbb{F}_q, s = uvv^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(bu)x_{18}(au^{-1}v+t)x_{19}(c)x_{20}((d^2+d)v^2wu^{-2}+vbu^{-2}+s)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s = v^2 w u^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(abu^{-1}+z)x_{18}(a^2u^{-1}+y)x_{19}(c)x_{20}(a^2bu^{-2}+x)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, x \neq 0, y, z \neq 0 \in \mathbb{F}_q, z^2 y / (ux^2) = m + m^2 + \eta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q / 2$,
 $\{x_{14}(u)x_{16}(v)x_{17}(a)x_{18}(w)x_{19}(b)x_{20}((c^2+c)vuw^{-2}+wau^{-2}+t)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t = vw^2 u^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

y7 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(evv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}((i^2+i)u^2v^{-1}w^2+x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t = s = r = z = y = x = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(du+s)x_{14}(g)x_{15}(h)x_{16}(bd+z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1}+w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1}+y)x_{24}(dbmv^{-2}+x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = s = t = 0 = z = y = x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t = uw^2 / v^2, r = z = y = x = z' = 0, y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}((e^2+e)ut^2v^{-1}+z')x_{15}(du+x)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0 = s = z = y = x, z' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h+x)x_{15}(f)x_{16}(cv+w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2}+t)x_{23}(l)x_{24}(ht+v^{-1}c+y) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = t = x = 0 = y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(dgu^{-1}+w)x_{23}(diu^{-1}+t)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = t = x = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0 = t, s, x=0 = z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2 q^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, x=0 = z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3 q$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(uf+y)x_{21}(g)x_{22}(ayu^{-1}+w)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, x=0 = z, y \in \mathbb{F}_q, y = (o^2+o)u^2v^2w^{-1}}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q / 2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w = t = 0, s, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3 q$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, x=0, z \neq 0 \in \mathbb{F}_q, t = (o^2+o)u^2v^2z^{-1}}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 (q/2 - 1)$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(ue+z)x_{19}(au^{-1}f+v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1}+w)x_{23}(b^2zu^{-2}+t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t, x=0, z \neq 0 \in \mathbb{F}_q, t = (o^2+o)u^2v^2z^{-1}}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4 q / 2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(b^2gu^{-2}+t)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, s, x \neq 0 \in \mathbb{F}_q, s = (o^2+o)x^{-1}u^2v^2}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q^2 / 2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud+x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f+v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1}+w)x_{23}(j)x_{24}(c^2xu^{-2}+s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s, x \neq 0 \in \mathbb{F}_q, s = (o^2+o)x^{-1}u^2v^2}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4 q / 2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x=y^2u^{-2}v, z'=0, y'=vy^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_7(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(ajv^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x=y=0, z \neq 0, r=uz^2 \in \mathbb{F}_q, z=v\sqrt{s/u}}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(cg + x') \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=o=0, x \neq 0, y=z=0, x', y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q^2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=0, x \neq 0, y=z=0, y' \in \mathbb{F}_q, sy'=u^2x^2(m^2+m)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^2x^2s^{-1} + y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r \neq 0, o=0, x \neq 0, y=z=0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2r^{-1}x^2 + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=0, r \neq 0, o=0, x \neq 0, y=z=0, z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r=0, o=0, x \neq 0, y=z=0, x' \in \mathbb{F}_q, tx'=u^2x^2(m^2+m)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2x^2t^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=z=0, x'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=r=o=0=y, z \neq 0, x', y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(bh + y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=0=y, z \neq 0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=r=o=x=y=0, z \neq 0 \in \mathbb{F}_q, sw=u^2z^2(m^2+m)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(ci + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=s=0, t, r=0, o \neq 0, y=0, x'=0 \in \mathbb{F}_q, to=u^2v^2(m^2+m)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(ci + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=s, t, r=o=0=y=x' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)s^2t + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r \neq 0, o=y=y'=0, z'=0 \in \mathbb{F}_q, ts^2=ur^2}$ with parameter set of size $(q-1)^3$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=y'=0, z'=0 \in \mathbb{F}_q, ur^2t=v^2x^2, v^2o=ur^2}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=0, r \neq 0, o \neq 0, x=y=0, y'=0, z'=0 \in \mathbb{F}_q, v^2o=ur^2}$ with parameter set of size $(q-1)^3$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfv^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(u^{-1}bk + z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(u^{-1}bk + z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=r=o=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab + z)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1} + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=t=0=z=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au+t)x_{16}(au^{-1}w+s)x_{17}(e)x_{18}((f^2+f)u^2v^2w^{-1}+y)x_{19}(g)x_{20}(ab+af^2w^{-1}u^{-1}+r)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1}+z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=r=0, z, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}((g^2+g)u^2t^2w^{-1}+x)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, z, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(e)x_{19}(f)x_{20}(ab+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)v^{-1}u^2t^2+s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(e)x_{19}(f)x_{20}(ab+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(bhu^{-1}+s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, s \in \mathbb{F}_q, s=(m^2+m)v^{-1}u^2t^2}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(bgu^{-1}+s) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w=0, t \neq 0, s, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w=0, t \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+v)x_{17}(bcu^{-1}+t)x_{18}(bu+z)x_{19}(e)x_{20}(ab+w)x_{21}(f)x_{22}(g)x_{23}(ae+y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w=0, t \neq 0, y, z \neq 0 \in \mathbb{F}_q, y=(m^2+m)u^2t^2z^{-1}}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cu^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}((j^2+j)v^2t^2+y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t^2u^{-2}, t \neq 0, s=r=z=0, y=vt^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}((i^2+i)us^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, z=y=0 \in \mathbb{F}_q, us^2=v^2t}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)u^2z^2v^{-1}+y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=0, s, r=0, z \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}((i^2+i)v^{-1}u^2z^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=0, z \neq 0, y=0}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w=v=0, t \neq 0, s=0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(bcu^{-1}+t)x_{16}(ub+w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b+v)x_{21}(f)x_{22}(u^{-1}bc^2+z)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w \neq 0, v=0, t \neq 0, s=0, z=m^2+m}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(d)x_{16}(ub+w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b+v)x_{21}(g)x_{22}(u^{-1}bc^2+z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v=0, s \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+s)x_{12}(c)x_{14}(d)x_{16}(ub+w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b+v)x_{21}(g)x_{22}(d^2s^{-2}v+z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, s \neq 0, z=0, w=(m^2+m)u^2s^2v^{-1}}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2}+t)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}((f^2+f)u^2v^2w^{-1}+ua+s)x_{21}(g)x_{22}(b^2fu^{-2}+t)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1}+s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2+i)u^2v^{-1}t^2+z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2+h)u^2t^2v^{-1}+ud+z)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+b^2u^{-2}v+t)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t=(m^2+m)u^2w^2v^{-1}}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+w)x_{17}(c)x_{19}(d)x_{20}(au+v)x_{21}(e)x_{22}(ab^2u^{-1}+b^2u^{-2}v+t)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v+t)x_{19}(e)x_{20}(a^2cu^{-2}+s)x_{21}(f)x_{22}(g)x_{23}((h^2+h)u^2w^2v^{-1}+wd+cv^{-1}(wb+g)+r)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w+t)x_{21}(e)x_{22}((f^2+f)u^2v^2w^{-1}+b^2wu^{-2}+s)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w+t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v+w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v+t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1}+s)x_{19}(f)x_{20}(a^2du^{-2}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)v-1uw+bdwv^{-2}+(bi+dh)v^{-1}+ug+af+r) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=z=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{19}(c)x_{21}(d)x_{22}(e)x_{23}(f)x_{24}(efv^{-2} + w) \mid a, b, c, d, e, f \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,

$\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(w)x_{21}(d)x_{22}((e^2 + e)u^2v^2w^{-1} + a^2wu^{-2} + s)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0}$ with parameter set of size $(q-1)^3$,

$\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{18}(w)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}((g_1^2 + g_1)u^2w^2v^{-1} + ((g_2dw)^2 + g_2dw)u^2v^2w^{-1} + vbu^{-4} + a^2d^2u^{-4}w^{-1} + dfw^{-1} + s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0}$ with parameter set of size $(q-1)^3$,

$\{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2 + i)u^2t^2v^{-1} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

y8 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(egw^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=r=0=z=y, x=u^2v^{-1}w^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=z=y=x=z'=0, y'=v^{-1}uw^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=0=s=z=y=x, z'=ut^2v^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(uf + y)x_{21}(g)x_{22}(ayu^{-1} + w)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, x=0=z, y \in \mathbb{F}_q, y=(o^2+o+\eta)u^2v^2w^{-1} \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, x=0, z \neq 0 \in \mathbb{F}_q, t=(o^2+o+\eta)u^2v^2z^{-1} \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(ue + z)x_{19}(au^{-1}f + v)x_{20}(g)x_{21}(h)x_{22}(agu^{-1} + w)x_{23}(b^2zu^{-2} + t)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t, x=0, z \neq 0 \in \mathbb{F}_q, t=(o^2+o+\eta)u^2v^2z^{-1} \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f + v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1} + w)x_{23}(b^2gu^{-2} + t)x_{24}(c^2xu^{-2} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, s, x \neq 0 \in \mathbb{F}_q, s=(o^2+o+\eta)x^{-1}u^2v^2 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2/2$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(au^{-1}f + v)x_{20}(h)x_{21}(i)x_{22}(ahu^{-1} + w)x_{23}(j)x_{24}(c^2xu^{-2} + s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s, x \neq 0 \in \mathbb{F}_q, s=(o^2+o+\eta)x^{-1}u^2v^2 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x=y^2u^{-2}v, z'=0, y'=(o^2+o+1+\eta)vy^2 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x=y=0, z \neq 0, r=(n^2+n+1+\eta)uz^2 \text{ for some } n \in \mathbb{F}_q, z=v\sqrt{s/u}}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(bg + y')x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=o=0, x \neq 0, y=z=0, y' \in \mathbb{F}_q, sy'=u^2x^2(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^2x^2s^{-1} + y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y=z=0, y'=u^2x^2s^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2r^{-1}x^2 + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s=t=0, r \neq 0, o=0, x \neq 0, y=z=0, z'=u^2r^{-1}x^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(ch + x') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r=0=o, x \neq 0, y=z=0, x' \in \mathbb{F}_q, tx'=u^2x^2(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2x^2t^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=z=0, x'=u^2x^2t^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=0=o=x=y=0, z \neq 0 \in \mathbb{F}_q, sw=u^2z^2(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(ci + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=0=o=x=y=0, z \neq 0 \in \mathbb{F}_q, sw=u^2z^2(m^2+m+\eta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,

$\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w = s = 0, t, r = 0, o \neq 0, y = x' = 0 \in \mathbb{F}_q, to = u^2 v^2 (m^2 + m + \eta)$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)s^2t + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v = w = 0, s, t \neq 0, r \neq 0, o = y = y' = 0, z' = s^2 t \eta \in \mathbb{F}_q, ts^2 = ur^2$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w = 0, s, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y = y' = 0, z' = ur^2 \eta \in \mathbb{F}_q, ur^2 t = v^2 x^2, v^2 o = ur^2$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w = 0, t = 0, r \neq 0, o \neq 0, x = y = 0, y' = 0, z' = ur^2 \eta \in \mathbb{F}_q, v^2 o = ur^2$ with parameter set of size $(q-1)^3$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^2w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(u^{-1}bk + z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t = s = 0 = r = o = z, y = v^2 u^2 w^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1} + z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, s = t = r = 0 = z, y = u^2 v^2 w^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}((g^2 + g)u^2t^2w^{-1} + x)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v = 0, w \neq 0, t \neq 0, z, x = u^2 t^2 w^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)v^{-1}u^2t^2 + s) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t \neq 0, s = v^{-1}u^2t^2 \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(e)x_{19}(f)x_{20}(ab + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(bhu^{-1} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w = 0, t \neq 0, s \in \mathbb{F}_q, s = (m^2 + m + \eta)v^{-1}u^2t^2$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + v)x_{17}(bcu^{-1} + t)x_{18}(bu + z)x_{19}(e)x_{20}(ab + w)x_{21}(f)x_{22}(g)x_{23}(ae + y)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v = 0 = w, t \neq 0, y, z \neq 0 \in \mathbb{F}_q, y = (m^2 + m + \eta)u^2t^2z^{-1}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}((j^2 + j)vt^2 + y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w = t^2 u^{-2} v, t \neq 0, s = r = 0 = z, y = (m^2 + m + 1 + \eta)vt^2$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}((i^2 + i)us^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w = 0, t \neq 0, s \neq 0, z = 0, y = us^2 \eta \in \mathbb{F}_q, us^2 = v^2 t$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2z^2v^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t = w = 0, s, r = o = 0, z \neq 0, y = u^2 z^2 v^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)v^{-1}u^2z^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w = t = s = r = 0, z \neq 0, y = v^{-1}u^2z^2 \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(u^{-1}bc^2 + z)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, w \neq 0, v = 0, t \neq 0, s = 0, z = (m^2 + m + \eta)$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b + v)x_{21}(g)x_{22}(d^2s^{-2}v + z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, v = (m^2 + m + \eta)u^2s^2w^{-1}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}((f^2 + f)u^2v^2w^{-1} + ua + s)x_{21}(g)x_{22}(b^2fu^{-2} + t)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t = 0, s = u^2 v^2 w^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2 + i)u^2v^{-1}t^2 + z) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t \neq 0, s, z = u^2 v^{-1} t^2 \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^2t^2v^{-1} + ud + z)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w = 0, t \neq 0, z = u^2 t^2 v^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{17}(c)x_{19}(d)x_{20}(au + v)x_{21}(e)x_{22}(ab^2u^{-1} + b^2u^{-2}v + t)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, t = (m^2 + m + \eta)u^2 w^2 v^{-1}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q/2$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(g)x_{23}((h^2 + h)u^2w^2v^{-1} + wd + cv^{-1}(wb + g) + r)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t = s = 0, r = u^2 w^2 v^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w + t)x_{21}(e)x_{22}((f^2 + f)u^2v^2w^{-1} + b^2wu^{-2} + s)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t = 0, s = u^2 v^2 w^{-1} \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)v^{-1}uw + bdwv^{-2} + (bi + dh)v^{-1} + ug + af + r) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t = s = z = 0, r = v^{-1}uw \eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,

$\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{19}(c)x_{20}(w)x_{21}(d)x_{22}((e^2 + e)u^2v^2w^{-1} + a^2wu^{-2} + s)x_{23}(f)x_{24}(g) \mid a, b, c, d, e, f, g \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=u^2v^2w^{-1}, \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{12}(v)x_{14}(a)x_{17}(b)x_{18}(w)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}((g_1^2 + g_1)u^2w^2v^{-1} + ((g_2dw)^2 + g_2dw)u^2v^2w^{-1} + vbu^{-4} + a^2d^2u^{-4}w^{-1} + dfw^{-1} + s)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=u^2w^2v^{-1}, \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}((i^2 + i)u^2t^2v^{-1} + s) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0 \in \mathbb{F}_q, s=u^2t^2v^{-1}, \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

y₉ $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(c^2v^{-2} + bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r=w^2v^{-1}, z, y=v^{-1}u(wz+w^2s), x=x'=0 \in \mathbb{F}_q, v^2t^2x+u^2w^2x'=u^2vz^2+u^2vwtz}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=z=y=0, x'=0 \in \mathbb{F}_q, s=ut}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=ut, r \neq 0, z=y=0, x=u^2v^{-1}w^2 \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=z=y, x=0, z' \neq 0, x'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=z=y=x=0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w=s=z=y, x \neq 0, y'=m^2+m \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q/2$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z=y=0, z'=0 \in \mathbb{F}_q, vw=t^2}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + x)x_{19}(i)x_{20}(ug + z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1} + (u + ug)tu^{-2} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(ghv^{-1} + i^2u + z)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=z=y, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}((h^2 + h)u^2v + y)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=z=x=0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x=0, z'=0, y'=vy^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0=z=x=z', y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=x=y=0, z \neq 0, r=uz^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}((i^2 + i)uv^{-2}w^2 + o)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=x=0, y, r=v^2y^2u^{-1}t^{-2}, o=0 \in \mathbb{F}_q, vt=w}$ with parameter set of size $(q-1)^3q$,
 $\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(fju^{-1} + x)x_{23}(lv + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=z=s, y=uv^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2 + j)uv^2 + x)x_{21}(k)x_{22}(f^2u^{-1} + t)x_{23}(iv + w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=0=t, x=uv^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2 + ag)u^{-1} + s)x_{21}(i)x_{22}((j^2 + j)uv^2 + x)x_{23}(vf + w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, x=uv^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$

$r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s=rw/y, t=0, r, o=0, x=\sqrt{yr}, y \neq 0, z=0, z'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=w=s=t=0, r, o=0, x=\sqrt{yr}, y \neq 0, z=0, z'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2y^2o^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=s=t=0, r, o \neq 0, x=0, y \neq 0, z=0, x'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2x^2t^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=or/t, z=0, x'=0 \in \mathbb{F}_q, x^2=ry\}$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f) + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(ul + y)x_{23}(i^2v^{-1} + bl + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=s=t=r=y=x, s=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(v^{-1}bn + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=t=r=z=0=y=x, s=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o=x=y=y'=z'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=0 \in \mathbb{F}_q, rx=y'\}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o=s^2(y')^2/(ur^4), x \neq 0, y=0, y' \neq 0, z'=0 \in \mathbb{F}_q, rx=y'\}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=0 \in \mathbb{F}_q, ur^2i=v^2x^2, y'=xr\}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u \in \mathbb{F}_q^\times, v \neq 0, w=t=0, r \neq 0, o=x=y=0, y'=0, z'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(vj + u^{-2}a^2k + x)x_{23}(m)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z=x, y=uv^2 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui + x)x_{23}(g^2v^{-1} + z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=t=0, z=x=y=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(f^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw, r=0, y=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^2v^{-1}w^2 + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w \neq 0, t=z=0=y, x=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + w)x_{16}(v)x_{17}(e)x_{18}(bu + t)x_{19}(f)x_{20}(ab + e^2v^{-1} + s)x_{21}(g)x_{22}((h^2 + h)u^2v + y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=t=s=y=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}((j^2 + j)vt^2 + y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=r=z=0, y=vt^2 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}((i^2 + i)us^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, z=y=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h^2v)x_{23}((i^2 + i)uv^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, w=t=s=y=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^2o + x')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, r \neq 0, o=srv^{-1}, z=\sqrt{so}, x'=0 \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)v^{-1}u^2z^2 + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, s=v^{-1}z^2, r=0, z \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^{-2}v^{-1}(u^2s + vw)^2 + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + s)x_{16}(ub + w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)uv + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=s=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i^2 + i)u^2v + uf + be + dhv^{-1} + z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2w^2v^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_9(u)x_{11}(a)x_{14}(b)x_{15}(v)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(h)x_{24}((j^2 + j)u^2v + y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, y=0}$ with parameter set of size $(q-1)^2$,

$\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)u^2t + o) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=\sqrt{vt}, t \neq 0, s=z=0, o=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1} + w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1} + t)x_{19}(f)x_{20}(a^2du^{-2} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2 + j)uv^2 + f^2u^{-1} + r) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=s=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

Y10 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evu^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r=w^2v^{-1}, z, y=v^{-1}u(wz + w^2s), x=u^2v^{-1}w^2\eta, x'=u^{-2}vs^2\eta \in \mathbb{F}_q, v^2t^2x + u^2w^2x' = u^2vz^2 + u^2vwtz}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evu^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=z=y=0, x'=u^{-2}vs^2\eta \in \mathbb{F}_q, s=ut}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evu^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=ut, r \neq 0, z=y=0, x=u^2v^{-1}w^2(\eta+1) \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=r=0=z=y, x=0, z' \neq 0, x'=v^{-2}u^{-1}(v^2z')^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=z=y=x=0, y'=v^{-1}uw^2(\eta) \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0=w=s=z=y, x \neq 0, y'=vs^2u^{-2}(m^2 + m + \eta)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q/2$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z=y=0, z'=ut^2v^{-1}\eta \in \mathbb{F}_q, v=w=t^2}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + x)x_{19}(i)x_{20}(ug + z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1} + (u + ug)tu^{-2} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, x=u^2v\eta, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(ghv^{-1} + i^2u + z)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0=z=y, x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}((h^2 + h)u^2v + y)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=z=x=0, y=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x=0, z'=0, y'=(1+\eta)vy^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0=z=x, z'=y'=vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} +$

$y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q$
 $\left. \begin{array}{l} \{x_2(u)x_5(v)x_6(ax_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}((i^2 + i)uv^{-2}w^2 + o)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\ \{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(fju^{-1} + x)x_{23}(lv + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\} \\ \{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2 + j)uw^2 + x)x_{21}(k)x_{22}(f^2u^{-1} + t)x_{23}(iv + w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\} \\ \{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2 + ag)u^{-1} + s)x_{21}(i)x_{22}((j^2 + j)uw^2 + x)x_{23}(vf + w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\} \\ \{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(ul + y)x_{23}(i^2v^{-1} + bl + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\} \end{array} \right\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=y=z, x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(v^{-1}bn + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=r=z=0=y=x, s=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w=s, t=r=0=o=x, y \neq 0, z=0, z'=u^2y\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s=rw/y, t=0, r, o=0, x=\sqrt{y}r, y \neq 0, z=0, z'=u^2y\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2y^2o^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s=t=0=r, o \neq 0, x \neq 0, y \neq 0, z=0, x'=u^2y^2o^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}((h^2 + h)u^2y + z')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0, z'=u^2y\eta \in \mathbb{F}_q, ry=x^2}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^2x^2t^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=or/t, z=0, x'=u^2x^2t^{-1}\eta \in \mathbb{F}_q, x^2=ry}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r \neq 0, o=0, x=y=0, y'=0, z'=ur^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s=t=0, r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=ur^2\eta \in \mathbb{F}_q, rx=y'}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}((j^2 + j)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o=s^2(y')^2/(ur^4), x \neq 0, y=0, y' \neq 0, z'=ur^2\eta \in \mathbb{F}_q, rx=y'}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t=v^2(y')^2/(ur^4), r \neq 0, o=0, x \neq 0, y=0, y' \neq 0, z'=ur^2\eta \in \mathbb{F}_q, y'=xr}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}((k^2 + k)ur^2 + z') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=t=0, r \neq 0, o=x=y=0, y'=0, z'=ur^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(vj + u^{-2}a^2k + x)x_{23}(m)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z=x, y=(1+\eta)uv^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui + x)x_{23}(g^2v^{-1} + z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=z=x, y=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw, r=0, y=u^{-2}vt^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(av)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad+y)x_{18}(bu+e^2v)x_{19}(f)x_{20}(ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}((i^2+i)u^2v^{-1}w^2+x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=z=0=y, x=u^2v^{-1}w^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(ax_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+w)x_{16}(v)x_{17}(e)x_{18}(bu+t)x_{19}(f)x_{20}(ab+e^2v^{-1}+s)x_{21}(g)x_{22}((h^2+h)u^2v+y)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=s, y=u^2v \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_6(v)x_7(u)x_8(ax_9(b)x_{11}(c)x_{12}(cv^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}((j^2+j)vt^2+y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=r=0=z, y=(1+\eta)vt^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_6(u)x_8(v)x_9(ax_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}((i^2+i)us^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, z=0, y=us^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_6(u)x_9(ax_{11}(b)x_{12}(v)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h^2v)x_{23}((i^2+i)uv^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0, y=uv^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_7(u)x_8(ax_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}((i^2+i)v^{-1}u^2z^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s=v^{-1}z^2, r=0, z \neq 0, y=v^{-1}u^2z^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_7(u)x_8(ax_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)u^{-2}v^{-1}(u^2s+vv)^2+y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq vuv^{-2}, t=0=z, y=u^{-2}v^{-1}(u^2s+vv)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_7(u)x_8(ax_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+s)x_{16}(ub+w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}((i^2+i)uv+y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, y=uv \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_7(u)x_8(ax_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}((i^2+i)u^2o+x')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, r \neq 0, o=svv^{-1}, z=\sqrt{so}, x'=u^2o \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)u^2w^2v^{-1}+y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=z=0, y=u^2w^2v^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1}+s)x_{19}(f)x_{20}(c^2dv^{-2}+t)x_{21}(g)x_{22}(h)x_{23}(i^2+u^2v+uf+be+dhv^{-1}+z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, z=u^2v \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_9(u)x_{11}(a)x_{14}(b)x_{15}(v)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(h)x_{24}((j^2+j)u^2v+y) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, y=u^2v \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

$\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v+w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v+t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1}+s)x_{19}(f)x_{20}(a^2du^{-2}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)u^2t+o) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=\sqrt{vt}, t \neq 0, s=z=0, o=u^2t \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1}+w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1}+t)x_{19}(f)x_{20}(a^2du^{-2}+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}((j^2+j)uv^2+f^2u^{-1}+r) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=s, r=uv^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^2$,

y11 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(ax_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf+r)x_{14}(g)x_{15}(c^2u^{-1}+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1}+x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2}+z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=z=y=x=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_2(v)x_5(ax_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(evv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=0=z, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_2(v)x_5(ax_6(b)x_8(c)x_9(cu^{-1}v+w)x_{11}(d)x_{12}(evv^{-1}+t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-2}vs^2+x')x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, r=0=z, y \neq 0, x'=u^{-2}vs^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(ax_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=z, y \neq 0, x'=z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(ax_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(bhu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}((l^2+l)v^{-2}u^{-1}(v^2z'+ux)^2+x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0=z, y \neq 0, x \neq 0, z'=0, x'=0 \text{ or } x'=v^{-2}u^{-1}(v^2z'+ux)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_5(ax_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}((j^2+j)vs^2u^{-2}+y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, z=0, y \neq 0, x=0, y'=0 \text{ or } y'=vs^2u^{-2} \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_5(ax_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+$

$z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w=s=z, y \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0=z=y=x, z' \neq 0, y'=0 \text{ or } y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r=z=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=z=y=0=x, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z=y/x, y=\sqrt{t}s, x, y'=0 \in \mathbb{F}_q, vtx/s=u^2x^2/s^2}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(ciu^{-1} + z)x_{22}(b^2u^{-2}k + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=0, z, y=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(ciu^{-1} + z)x_{22}(b^2u^{-2}k + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0, z \neq 0, y=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)uv^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, x \neq 0, y=z=0, o=0 \text{ or } o=uv^{-2}w^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ahx^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0=x=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} + z)x_{21}(cfu^{-1} + biu^{-1} + x)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t=z, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(cdu^{-1} + s)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s \neq 0, x=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}((b^2 + ad)u^{-1} + x)x_{17}((ae + bc)u^{-1} + z)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, x=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=0, r \neq 0, o=0=x=y=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r \neq 0, o \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r \neq 0, o \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r \neq 0, o \neq 0, x=y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=sx/o, w \neq 0, s \neq 0, t, o \neq 0, x \neq \sqrt{t}o, y=0 \in \mathbb{F}_q, uw^2o^3=(to+x^2)^2s^2}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=t=r=o=x, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$$\begin{aligned}
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r=0, o \neq 0, x=0, y \neq 0 \in \mathbb{F}_q, y=vo/\sqrt{tu}} \text{ with parameter set of size } (q-1)^4, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r=0, o=x=y=0, y' \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s, t=0=r, o, x \neq 0, y \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=r=0, o, x=y=0, y' \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fs^{-1}y + z)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=0=r, o, x=0, y \neq 0, z \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(is^{-1}y + z)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0=r, z \neq 0, o, x=0, y \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4q, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(iw^{-1}v + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w \neq 0, s=0, t=0=r, o, x=y=0, z \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q^2, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(iw^{-1}v + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w \neq 0, s \neq 0, t=0=r, o=x=y=0, z \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q^2, \\
& \{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(iw^{-1}v + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s=0, t=0=r, o, x \neq 0, y=0, z \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q^2, \\
& \{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=0, t, r \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t, z \neq 0, y=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(vu^{-1}i + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=r=o=z=0, y \neq 0, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q} \text{ with parameter set of size } 2(q-1)^4, \\
& \{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, r=t^2/(wv), o=vr, s=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=r=0, z \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=r=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(fv^{-1}w + x)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=z=0, x \neq 0, y=0 \text{ or } y=uv^{-2}w^2\eta \in \mathbb{F}_q} \text{ with parameter set of size } 2(q-1)^4, \\
& \{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=s=z \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(hav^{-1} + z)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3,
\end{aligned}$$

$a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(aeu^{-1} + w)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v +$
 $o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s \neq 0, r=o=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v +$
 $o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t=0, s \neq 0, r \neq 0, o=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v +$
 $s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f +$
 $r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=t=0=s, r \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1} + s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in$
 $\mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in$
 $\mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} +$
 $z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} +$
 $z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(acu^{-1} + r)x_{18}(a^2bu^{-1} + s)x_{19}(d)x_{20}(a^2cu^{-1} + t)x_{21}(adu^{-1} +$
 $z)x_{22}(e)x_{23}(f)x_{24}(d^2u^{-1} + y) \mid a, b, c, d, e, f \in \mathbb{F}_q\}$ $_{u \in \mathbb{F}_q^\times, v=0, w, t, s \neq 0, r \neq 0, z, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^4$,
 $\{x_{10}(u)x_{12}(a)x_{13}(b)x_{14}(abu^{-1} + v)x_{15}(a^2u^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(a^2bu^{-1} + s)x_{19}(e)x_{20}(a^2cu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid$
 $a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ $_{u \in \mathbb{F}_q^\times, v \neq 0, w, t \neq 0, s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,
 $\{x_{11}(v)x_{12}(t)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in$
 $\mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(cv^{-1}w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in$
 $\mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{12}(v)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}((d^2+d)uvw^{-2} + awv^{-2} + s)x_{19}(e)x_{20}(cv^{-1}w + b(d^2+d)w + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid$
 $a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \in \mathbb{F}_q, s=uvw^{-2}\eta \text{ or } s=0 \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_{12}(u)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(au^{-1}v + t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4$,
 $\{x_{12}(u)x_{14}(a)x_{16}(w)x_{17}(b)x_{18}(v)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^3$,
 $\{x_{13}(u)x_{14}(a)x_{15}(v)x_{16}(b)x_{17}(abu^{-1} + w)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3$,
 $\{x_{14}(v)x_{15}(u)x_{16}(w)x_{17}(a)x_{18}(b)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times}$ with parameter set of size $(q-1)^3$,

Y12 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w +$
 $z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in$
 $\mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times, s=t=0=r=z=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} +$
 $z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=t=s=r=0, z \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} +$
 $z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in$
 $\mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=r=0, z \neq 0, y=0, x=0 \text{ or } x=u^2v^{-1}w^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(dbmv^{-2} + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=z, y=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(dbmv^{-2} + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=z, y \neq 0, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z \neq 0, y=x=z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, z \neq 0, y=x=z'=0, y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0=y, x=v^2u^{-1}z', z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0=y, x=0, z'=uws/v, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0=y, x=vws, z'=0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w=s, z \neq 0, y=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0=s, z \neq 0, y=x=0, z'=0 \text{ or } z'=ut^2v^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0=z=y=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + t)x_{23}(l)x_{24}(ht + v^{-1}c + y) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=x=0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(dgu^{-1} + w)x_{23}(diu^{-1} + t)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(dgu^{-1} + w)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r, z \neq 0, y=0=x=z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r, z \neq 0, y \neq 0, x=y^2u^{-2}v, z'=0, y'=0 \text{ or } y'=vy^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s=x, y \neq 0, z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x=0, y \neq 0, z \neq 0, r=0 \text{ or } r=uz^2\eta \in \mathbb{F}_q, z=v\sqrt{s/u}}$ with parameter set of size $2(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(ci + x') \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=s, t, o, r=0=y, x' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae +$

$w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, s=t, r \neq 0, o=0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s=0=t=r=0, o, 0=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=v^2s^3/(u^2r^2), s \neq 0, t, r \neq 0, o=(vs)^2t/(u^2r^2)+(vs)v/r, y=(vs)^2/(u^2r) \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(bh+y')x_{24}(ch+x') \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=s=t=r=0=o=y, z \neq 0, x', y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(bh+y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s \neq 0, t=r=0=o=y, z \neq 0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s=t=r=0=o, x \neq 0, y=0, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0=w, s, t=0, r \neq 0, o=y=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w=t^2/r, t \neq 0, r \neq 0, o=ty/r, y=t^2z^2/(u^2r), z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(cu+s)x_{14}(e)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w=s=t=0=r, o \neq 0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(acu^{-1}+z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(f)x_{18}(ad+w)x_{19}(g)x_{20}(abdu^{-1}+y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r=0, o \neq 0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2}+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2}+y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r=0, o, x \neq 0, y=0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(acu^{-1}+s)x_{12}(d)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2}+y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t \neq 0, r, o \neq 0, y=0, y'=0 \in \mathbb{F}_q, ts^2=ur^2}$ with parameter set of size $(q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+v)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(bcu^{-1}+x)x_{13}(ab^2u^{-1}+w)x_{14}(abcu^{-2}+r)x_{15}(c^2u^{-1}+o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2}+y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2}+y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r, o, x, y=y'=0 \in \mathbb{F}_q, ur^2=v^2o, x^2 \neq to}$ with parameter set of size $(q-1)^3q^2$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(bu+w)x_{17}(g)x_{18}(bfu^{-1}+t)x_{19}(h)x_{20}(cfv^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(u^{-1}bk+z) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(bu+w)x_{17}(g)x_{18}(bfu^{-1}+t)x_{19}(h)x_{20}(cfv^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(bfu^{-1}+t)x_{17}(g)x_{18}(bfu^{-1}+t)x_{19}(h)x_{20}(cfv^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(bfu^{-1}+t)x_{17}(g)x_{18}(bfu^{-1}+t)x_{19}(h)x_{20}(cfv^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=0=r=0, z \neq 0, y=0 \text{ or } y=v^2u^2w^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(bfu^{-1}+t)x_{17}(g)x_{18}(bfu^{-1}+t)x_{19}(h)x_{20}(cfv^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=uv/\sqrt{sr}, s \neq 0, r, o=tr/s, y=0 \in \mathbb{F}_q, u^2st=wt^2}$ with parameter set of size $2(q-1)^4$,

$\{x_5(w)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(f)v^{-1}w^2+s)x_{16}(f)x_{17}(g)x_{18}(cw+a^2u^{-2}v+t)x_{19}(h)x_{20}(cfv^{-1}w+r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(f)v^{-1}u+acv^{-1}+\sqrt{ev^{-1}w}+s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1}+t)x_{19}(h)x_{20}(efv^{-1}+r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab + z)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1} + y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=t=0=z, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab + z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(aiu^{-1} + z)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=r=0, z \neq 0, y=0 \text{ or } y=u^2v^2w^{-1} \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=z=0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t^2u^{-2}v, t, s \neq 0, r=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t, s=v\sqrt{t/u}, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=w=s=0, r \neq 0, o=svr^{-1}, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(a^2eu^{-2} + w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=\sqrt{rv^{-1}ut}, s, r \neq 0, o=svr^{-1} \in \mathbb{F}_q, sw=vt^2}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(a^2u^{-2}g + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=r=0, \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(d)x_{15}(v)x_{16}(ub + w)x_{17}(e)x_{18}(bu^{-1}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{19}(f)x_{20}(a^2u^{-1}b + v)x_{21}(g)x_{22}(u^{-1}bc^2 + z)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v, s \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + s)x_{12}(c)x_{14}(bcu^{-1} + t)x_{16}(ub + w)x_{17}(d)x_{19}(e)x_{20}(a^2u^{-1}b + v)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, w, v \neq 0, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(aev^{-1} + z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(w)x_{17}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(b^2fu^{-2} + t)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(t)x_{17}(d)x_{18}(w)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1} + w)x_{16}(v)x_{17}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v + t)x_{19}(e)x_{20}(a^2cu^{-2} + s)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_9(u)x_{11}(a)x_{12}(v)x_{14}(b)x_{16}(w)x_{17}(c)x_{19}(d)x_{20}(a^2u^{-1}w + t)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,
 $\{x_{11}(w)x_{12}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1} + t)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

y13 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(c^2u^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2} + z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$

$\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, s=t=0, z, y \neq 0, x=r=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0, r \neq 0, z=0, y \neq 0, x=0$ or $x=u^2v^{-1}w^2\eta \in \mathbb{F}_q, vr=w^2$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq 0, r=v^{-1}w^2, z, y, x' \in \mathbb{F}_q, uz^2=wsz$ with parameter set of size $2 * (q-1)^4q$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=0, z=\sqrt{vty/u}, y \neq 0, x'=0$ or $x'=u^{-2}vs^2\eta \in \mathbb{F}_q, s=ut$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq ut, r=0, z=\sqrt{vty/u}, y \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r \neq 0, z \neq 0, y=u(zt+z^2)/(vt), x=0$ or $x=u^2v^{-1}w^2\eta, x'=0$ or $x'=u^{-2}vs^2\eta \in \mathbb{F}_q, vr=w^2, z \neq wt, v^2t^2x+u^2w^2x'=0$ with parameter set of size $2(q-1)^4(q-2)$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq ut, r=w^2/v, z, y \in \mathbb{F}_q, uvty=u^2z^2+uwz(ut+s)+uw^2st$ with parameter set of size $(q-1)^5q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z, y \neq 0, x=0, y'=0$ or $y'=v^{-1}uw^2\eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}((m^2 + m)v^{-2}ux^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z, y, x \neq 0, y'=0$ or $y'=v^{-1}uw^2\eta, x'=0 \in \mathbb{F}_q, xz/w=v^2z^2/(uw^2)$ with parameter set of size $4(q-1)^4q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z=\sqrt{yz'}, y, x \neq 0, z' \neq 0 \in \mathbb{F}_q, v^2z'=ux$ with parameter set of size $(q-1)^3q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z=\sqrt{yz'}, y \neq 0, x, z' \neq 0, x'=0$ or $x'=v^{-2}u^{-1}(v^2z' + ux)^2\eta \in \mathbb{F}_q, v^2z' \neq ux$ with parameter set of size $2(q-1)^3(q-2)$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z=\sqrt{yz'}, y \neq 0, x, z' \neq 0, x'=0$ or $x'=v^{-2}u^{-1}(v^2z' + ux)^2\eta \in \mathbb{F}_q, v^2z' \neq ux$ with parameter set of size $2(q-1)^5$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z, y \neq 0, z'=0$ or $z'=ut^2v^{-1}\eta \in \mathbb{F}_q, vw=t^2$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}((k^2 + k)vs^2u^{-2} + y')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s \neq 0, z, y, z'=0$ or $z'=ut^2v^{-1}\eta, y'=0 \in \mathbb{F}_q, vw=t^2, uz^2=tsz$ with parameter set of size $4(q-1)^4q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, w=s \neq 0, z=0, y, x \neq 0, y' \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=w=s \neq 0, z=0, y \neq 0, x \neq 0, y' \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, w=s \neq 0, z=0, y, x \neq 0, y'=0$ or $y'=vs^2u^{-2}\eta \in \mathbb{F}_q, vs \neq u^2x$ with parameter set of size $2(q-1)^3(q-2)q$,
 $\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + x)x_{19}(i)x_{20}(ug + z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1} + (u + ug)tu^{-2} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=t=0, x=0$ or $x=u^2v\eta, y=0, z \neq 0 \in \mathbb{F}_q$

with parameter set of size $2(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(i^2v^{-1} + jtu^{-2} + y)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(ghv^{-1} + i^2u + z)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, z \neq 0, y=0, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, y=0, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, z \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(de + w)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s=0, t \neq v^{-1}u^2w, r=0, z \neq 0, y \neq 0, x=0, z' \neq 0, y' = 0 \in \mathbb{F}_q, tz/y = u^2z^2/(vy^2)}$ with parameter set of size $2(q-1)^4q$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z=yx/s, y, x \neq 0, y' = 0 \in \mathbb{F}_q, s \neq y^2/t, u^2x^2/(vs^2)=tx/s}$ with parameter set of size $2(q-1)^5$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z \neq yx/s, y, x \neq 0, y' = 0 \in \mathbb{F}_q, s \neq y^2/t, u^2x^2/(vs^2)=tx/s}$ with parameter set of size $(q-1)^6$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x=0, z' \neq 0, y' = 0 \text{ or } y' = vy^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r=z, y \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z=\sqrt{wz'}, x=0, z' \neq 0, y' = 0 \text{ or } y' = vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + s)x_{20}(k)x_{21}(ciu^{-1} + z)x_{22}(b^2u^{-2}k + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r=0, z=\sqrt{wz'}, x=0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(ciu^{-1} + z)x_{22}(b^2u^{-2}k + x)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s=t=0, z, y=0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(l)x_{22}(b^2u^{-2}k + x)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=y=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(cju^{-1} + z)x_{22}(b^2u^{-2}m + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=0, t \neq 0, z=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)uw^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, x, y, z \neq 0, o=0 \in \mathbb{F}_q, wy/z = v^2y^2/(uz^2)}$ with parameter set of size $2(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0=s, x \neq 0, y=0, z \neq 0, r=0 \text{ or } r=uz^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}((i^2 + i)uv^{-2}w^2 + o)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0, x \neq 0, y=\sqrt{v^{-1}utx}, r, o=0 \text{ or } o=uv^{-2}w^2\eta \in \mathbb{F}_q, vt=w}$ with parameter set of size $2(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0, x, y=\sqrt{v^{-1}utx} \in \mathbb{F}_q, vt \neq w}$ with parameter set of size $(q-1)^3(q-2)q^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0, x, y=\sqrt{v^{-1}utx} \in \mathbb{F}_q, vt \neq w}$ with parameter set of size $(q-1)^3(q-2)q^2$,

$y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0, x, y = \sqrt{v^{-1}ux} \in \mathbb{F}_q$}
 with parameter set of size $(q-1)^3q$,
 $\{x_2(u)x_5(v)x_6(ax_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} +$
 $ahu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v, w \in \mathbb{F}_q^\times, t=0, x \neq 0, y=0=z \in \mathbb{F}_q$} with parameter set of size
 $(q-1)^4$,
 $\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(cju^{-1} +$
 $lw^{-1}v + y)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ _{$u, v, w \in \mathbb{F}_q^\times, s=0=z=y \in \mathbb{F}_q$} with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(fju^{-1} +$
 $x)x_{23}(lv + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=0, z \neq 0, x=0, y=0$ or $y=uv^2\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(v^{-2}d^2 + t)x_{23}(l)x_{24}(m) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v, w \in \mathbb{F}_q^\times, s=0=t \in \mathbb{F}_q$} with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2 + j)uv^2 + x)x_{21}(k)x_{22}(f^2u^{-1} + t)x_{23}(iv +$
 $w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, s=0=t, x=0$ or $x=uv^2\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} +$
 $z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=z \in \mathbb{F}_q$} with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} +$
 $z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=0, t, z \neq 0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^3q$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2 + ad)u^{-1} + s)x_{17}(hvv^{-1} + t)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ _{$u, v, w \in \mathbb{F}_q^\times, s=0=t \in \mathbb{F}_q$} with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2 + ag)u^{-1} + s)x_{21}(i)x_{22}((j^2 + j)uv^2 +$
 $x)x_{23}(vf + w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, x=0$ or $x=uv^2\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{13}(d)x_{14}(e)x_{15}(v)x_{16}(b^2u^{-1} + w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} +$
 $s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(ul + y)x_{23}(i^2v^{-1} + bl + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in$
 $\mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=s=t=r=0, y \neq 0, z=0, x=0$ or $x=u^2v\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad +$
 $r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(i^2v^{-1} + bl + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r=0=z \in \mathbb{F}_q$} with
 parameter set of size $(q-1)^3$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad +$
 $t)x_{16}(i)x_{17}(j)x_{18}(abd v^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(v^{-1}bn + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in$
 $\mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w=t=r=z=0, y \neq 0, x=0, s=0$ or $s=u^2v\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad +$
 $t)x_{16}(i)x_{17}(j)x_{18}(abd v^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(v^{-1}bo + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in$
 $\mathbb{F}_q\}$ _{$u, v \in \mathbb{F}_q^\times, w \neq 0, t=r=z=0, x, s=0$ or $s=u^2v\eta \in \mathbb{F}_q$} with parameter set of size $2(q-1)^3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$
 $r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, w, s \neq 0, t=r=o=0, x, y \neq 0, z=0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^3q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb +$
 $cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=0=r, o \neq 0, x=0, y \neq 0, z=0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$
 $r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, r=0, o, x=0, y \neq 0, z=0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^3q^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$
 $r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, w, s \neq 0, t=0, r \neq 0, o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q, x^2=yr$} with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$
 $r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, w \neq 0, s \neq wr/y, t=0, r \neq 0, o=0, x = \sqrt{ry}, y \neq 0, z=0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^5$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t +$
 $r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$
 $\mathbb{F}_q\}$ _{$u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r \neq 0, o, x = \sqrt{ry}, y \neq 0, z=0 \in \mathbb{F}_q$} with parameter set of size $(q-1)^5q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb +$
 $cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$

$\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0, r \neq 0, o=x=y=0, y' \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au+t)x_{16}(f)x_{17}(g)x_{18}((h^2+h)u^2v+y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui+x)x_{23}(g^2v^{-1}+z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=t=0=z, x \neq 0, y=0$ or $y=u^2v\eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^3$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au+t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u+cv^{-1}a+w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1}+s)x_{18}(cev^{-1}+u^2fv^{-1}+t)x_{19}(g)x_{20}(efv^{-1}+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq u, r=0, \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u+cv^{-1}a+w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1}+s)x_{18}(cev^{-1}+u^2fv^{-1}+t)x_{19}(g)x_{20}(efv^{-1}+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=\sqrt{u^{-1}vwr}, t, r \neq 0, \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad)x_{18}(bu+e^2v+kt)x_{19}(f)x_{20}(e^2w+ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, z=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad+y)x_{18}(bu+e^2v)x_{19}(f)x_{20}(e^2w+ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, z \neq 0, y=\sqrt{zw} \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+w)x_{16}(v)x_{17}(e)x_{18}(bu+t)x_{19}(f)x_{20}(ab+e^2v^{-1}+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+w)x_{16}(v)x_{17}(e)x_{18}(bu+kw)x_{19}(f)x_{20}(ku^{-1}wa+ab+e^2v^{-1}+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, s=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2}+uaw^{-1}v^{-1})wv^2u^{-2}+s)x_{14}(d)x_{15}(a^2u^{-2}v+r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v+o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, o, z=0 \in \mathbb{F}_q, u^2o^2/r^2=uvw/r$ with parameter set of size $2(q-1)^4$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=r=0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_8(v)x_9(bv^{-1}u+t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1}+s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1}+r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=t\sqrt{r/u}, r=v^2t^4u^{-3}w^2 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}((ucs^{-1}+ab)^2+ucs^{-1}+ab)uw^{-2}w^2+y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t=0, s \neq 0, y=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq 0, z=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1}+cdv^{-1}+w)x_{15}(c^2v^{-1}+t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2}+c^2dv^{-2}+s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1}+cdv^{-1}+w)x_{15}(c^2v^{-1}+t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2}+c^2dv^{-2}+s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w, t=0, s \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t=z=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{15}(v)x_{16}(d)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, w \neq 0, s \neq 0, r \neq 0, o=svr^{-1}, z=\sqrt{so} \in \mathbb{F}_q$ with parameter set of size $(q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=0, s \neq 0, r \neq 0, o=vs^{-1}, o \neq 0, z=\sqrt{so} \in \mathbb{F}_q$ with parameter set of size $(q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, t=s=0, r, o \neq 0, z=\sqrt{so} \in \mathbb{F}_q$ with parameter set of size $(q-1)^5$,

size $(q-1)^3q$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq 0, z = \sqrt{vs} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, r \neq 0, z = \sqrt{vs} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + s)x_{16}(ub + w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_9(t)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v, w \in \mathbb{F}_q^\times, t=s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t = \sqrt{ws}, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{15}(v)x_{16}(t)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_9(u)x_{11}(a)x_{12}(t)x_{13}(w)x_{14}(b)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v, w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w = \sqrt{vt}, t \neq 0, s, z \in \mathbb{F}_q, s, z \text{ not all } = 0}$ with parameter set of size $(q-1)^4(q+1)$,

$\{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1} + w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1} + t)x_{19}(f)x_{20}(a^2du^{-2} + kt + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t, s \in \mathbb{F}_q} (s \text{ and/or } t \neq 0)$ with parameter set of size $(q-1)^3(q+1)$,

y14 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v, w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=y=0, x=uv\sqrt{r/w} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t, s=ut, r \neq 0, z = \sqrt{(v^2 + ruvt)/(u^2r)}, y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=0, z \neq 0, y=0, x'=0 \text{ or } x'=u^{-2}vs^2\eta \in \mathbb{F}_q, s=ut}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r \neq 0, z=y=uw(z+wt)/v, x=0, x'=u^{-2}vs^2\eta \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $(q-1)^4q$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r \neq 0, z=y=uw(z+wt)/v, x=u^2v^{-1}w^2\eta, x'=0 \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $(q-1)^4q$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r \neq 0, z \neq 0, y=uw(wt+z)/(vt), x=0 \text{ or } x=u^2v^{-1}w^2\eta, x'=0 \text{ or } x'=u^{-2}vs^2\eta \in \mathbb{F}_q, vr=w^2, z \neq wt, v^2t^2x + u^2w^2x' = 0}$ with parameter set of size $2(q-1)^4(q-2)$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, s=ut, r \neq 0, z \neq 0, y=uwz/v, x=0 \text{ or } x=u^2v^{-1}w^2\eta \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t, s=ut, r \neq 0, z=wt + \sqrt{(v^2 + (w^2 + vr)uty)/(u^2r)}, y \in \mathbb{F}_q, vr \neq w^2}$ with parameter set of size $(q-1)^3(q-2)q^2$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evw^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t, s=ut, r \neq 0, z=wt + \sqrt{(v^2 + (w^2 + vr)uty)/(u^2r)}, y \in \mathbb{F}_q, vr \neq w^2}$ with parameter set of size $(q-1)^3(q-2)q^2$,

$\mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=ut, r=0, z, y, x=0 \in \mathbb{F}_q, y/s=v y^2/(w^2 s^2)}$ with parameter set of size $2(q-1)^4 q$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(du+s)x_{14}(g)x_{15}(h)x_{16}(bd+z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1}+w)x_{21}(l)x_{22}(m)x_{23}(bm v^{-1}+y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=z, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1}+w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(dbn v^{-2}+x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bju^{-1}+w)x_{21}(l)x_{22}(m)x_{23}(bm v^{-1}+y)x_{24}(dbm v^{-2}+x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0=z, y, x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3 q^2$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2 u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2 u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s, t=uw^2/v^2, r=0, y=w^2 z', x=vw s, z' \neq 0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2 u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2 u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s, t=uw^2/v^2, r=0, y=x=0, z' \neq uws/v, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2 u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2 u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(ihu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1}+y)x_{23}(k)x_{24}((l^2+l)v^{-2}u^{-1}(v^2 z'+ux)^2+x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=s=t=r=0, z \neq 0, y=x=0, z' \neq 0, x'=0 \text{ or } x'=v^{-2}u^{-1}(v^2 z'+ux)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2 u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2 u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(biu^{-1}+z)x_{20}(j)x_{21}(k)x_{22}(bju^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z \neq 0, y=wz, x=0, y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2 u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2 u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1}+z')x_{17}(h)x_{18}(agu^{-1}+fv+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=y=x=0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v+w)x_{14}((e^2+e)ut^2 v^{-1}+z')x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2 hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=0, s=0, z, 0=y, x \neq 0, z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v+w)x_{14}((e^2+e)ut^2 v^{-1}+z')x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2 hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=0, s=z, y=t^2 u^2 v^{-2}, x \neq 0, z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2 gu^{-2}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t, w \neq 0, s=0=y \in \mathbb{F}_q, vw \neq t^2}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2 hu^{-2}+y)x_{23}((j^2+j)vs^2 u^{-2}+y')x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t=0=w=s, z \neq 0, y=0, x \neq 0, y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 q$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v+w)x_{14}((e^2+e)ut^2 v^{-1}+z')x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1}+z)x_{20}(i)x_{21}(j)x_{22}(de+a^2 gu^{-2}+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z \neq 0, y=tuzv^{-1}, z'=0 \text{ or } z'=ut^2 v^{-1} \eta \in \mathbb{F}_q, vw=t^2}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w+t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w+z)x_{17}(h)x_{18}(fu+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times, t=0, z \neq 0, y=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(c w v^{-1}+s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu+y)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(c n v^{-1}+z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times, t=s=z=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h+x)x_{15}(f)x_{16}(cv+w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2}+t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w=0, t, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3 q$,
 $\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h+x)x_{15}(f)x_{16}(cv+w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2}+t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ab+ud+x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(g^2 u v^{-2} w+u e^2 v^{-2} w+y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $_{u,v,w \in \mathbb{F}_q^\times, x=0=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + x)x_{19}(i)x_{20}(ug + z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1} + (u + ug)tu^{-2} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, x=0 \text{ or } x=u^2v\eta, y \neq 0, z=0 \in \mathbb{F}_q}$
 with parameter set of size $2(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(i^2v^{-1} + jtu^{-2} + y)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(ghv^{-1} + i^2u + z)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0=z, y \neq 0, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}((h^2 + h)u^2v + y)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, z=x=0, y=0 \text{ or } y=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=r=z=y=0, x \neq 0, z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r, z \neq 0, y=0, x \neq 0, z'=uz\sqrt{x/v} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x \neq 0, z'=0, y'=0 \text{ or } y'=vy^2\eta \in \mathbb{F}_q, x \neq y^2u^{-2}v}$ with parameter set of size $2(q-1)^3(q-2)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r=z, y \neq 0, x \neq y^2u^{-2}v, z'=z\sqrt{(u^2x+vy^2)/v}, y'=0 \text{ or } y'=vy^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^5$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0=r, z \neq 0, x=0, z'=0, y'=0 \text{ or } y'=vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0=r, z=\sqrt{(v(z')^2 + u^2wxz' + u^2x^2y')/x}, u, x \neq 0, z', y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0=r, z=\sqrt{(v(z')^2 + u^2wxz' + u^2x^2y')/x}, u, x \neq 0, z', y'=vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0=r, z=\sqrt{(v^2s+uz^2)/u}, y, z \in \mathbb{F}_q, z \neq v\sqrt{s/u}}$ with parameter set of size $(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2 + j)uz^2 + r)x_{23}(aju^{-1} + fb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0=s, x=y, z \neq 0, r=0 \text{ or } r=uz^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=vt, t \neq 0, s \neq 0, x, y, r \in \mathbb{F}_q, s(ut^2r + v^2y^2 + uvtx)=ux^2}$ with parameter set of size $(q-1)^4q^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}((i^2 + i)uv^{-2}w^2 + o)x_{21}(j)x_{22}(vh + r)x_{23}(gb^2u^{-2} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0=x, y, r \neq v^2yu^{-1}t^{-2}, o=0 \text{ or } o=uv^{-2}w^2\eta \in \mathbb{F}_q, vt=w}$ with parameter set of size $2(q-1)^4q$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(fju^{-1} + x)x_{23}(lv + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0=z, x \neq 0, y=0 \text{ or } y=uv^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2 + h)uv^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(lv + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, z=0, y=0 \text{ or } y=uv^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2 + j)uv^2 + x)x_{21}(k)x_{22}(f^2u^{-1} + t)x_{23}(iv + w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, x=0 \text{ or } x=uv^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^3$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(iv + w)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2 + ad)u^{-1} + t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2 + ag)u^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(vf +aju^{-1} + w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=s=t=0=r, o \neq 0, x, y \neq 0, z=0 \in \mathbb{F}_q, ry \neq x^2$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o=yt/r, x, y \neq 0, z=0 \in \mathbb{F}_q, x^2 \neq ry$ with parameter set of size $(q-1)^5$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(vj + u^{-2}a^2k + x)x_{23}(m)x_{24}(f^2u^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=0=r, z \neq 0, x=0, y=0$ or $y=uv^2 \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^3$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(i^2/u + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s, t=0, r \neq 0, o, x, y=0, y' \in \mathbb{F}_q, rx=y', ur^2 \neq s^2x^2$ with parameter set of size $(q-1)^3q^2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r \neq \sqrt{ts^2/u}, o=y=y'=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r \neq 0, x=\sqrt{t}, y=y'=0 \in \mathbb{F}_q, ur^2 \neq v^2o$ with parameter set of size $(q-1)^4q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o=x=y=y'=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t=0, r \neq 0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, u(y')^2=(v^2o+ur^2)*x^2$ with parameter set of size $(q-1)^5$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, ur^2t=v^2x^2, v^2o \neq ur^2, ur(y')=x(v^2o+ur^2)$ with parameter set of size $(q-1)^4(q-2)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=w=0, s, t \neq 0, r, o \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q, u(y')^2=(ts^2+ur^2)*(to)$ with parameter set of size $(q-1)^4q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t \neq 0, r \neq 0, x, y=0, y' \neq 0 \in \mathbb{F}_q, ur^2t \neq v^2x^2, u(y')^2=(v^2o+ur^2)*(to+x^2)$ with parameter set of size $(q-1)^5(q-2)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r=0, o, x, y=0, y' \neq 0 \in \mathbb{F}_q, u(y')^2=(v^2o)*(to+x^2)$ with parameter set of size $(q-1)^4q$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui + x)x_{23}(g^2v^{-1} + z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=t=0, z \neq 0, x=0, y=0$ or $y=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^3$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=0=z \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfu^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfu^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq u\sqrt{w}/v, s \neq 0, r, o=tr/s, y=0 \in \mathbb{F}_q, wr/s=vu\sqrt{r}/s$ with parameter set of size $2(q-1)^5$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=0=r, o, y=0 \in \mathbb{F}_q, wo/t=uv\sqrt{o}/t$ with parameter set of size $2(q-1)^4$,
 $\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=wu\sqrt{t}/v, r=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(f)v^{-1}u + acv^{-1} + \sqrt{ev^{-1}w} + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab+z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s \neq 0, t=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au+t)x_{16}(au^{-1}w+s)x_{17}(e)x_{18}((f^2+f)u^2v^2w^{-1}+y)x_{19}(g)x_{20}(ab+af^2w^{-1}u^{-1}+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t=0, r, y=0 \in \mathbb{F}_q, wr/s=uv\sqrt{r/s}}$ with parameter set of size $2(q-1)^4$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u+cv^{-1}a+w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1}+s)x_{18}(cev^{-1}+u^2fv^{-1}+t)x_{19}(g)x_{20}(efv^{-1}+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2+k)u^{-2}vt^2+y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=tu^{-1}, s \neq 0, t \neq 0, r=0, y=0 \text{ or } y=u^{-2}vt^2 \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d+ev)x_{15}(au^2+t)x_{16}(av+w)x_{17}(aev+ew+ad+y)x_{18}(bu+e^2v)x_{19}(f)x_{20}(e^2w+ae^2v+ab+z)x_{21}(g)x_{22}(h)x_{23}((i^2+i)u^2v^{-1}w^2+x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=z=0, y \neq 0, x=0 \text{ or } x=u^2v^{-1}w^2 \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au+w)x_{16}(v)x_{17}(e)x_{18}(bu+t)x_{19}(f)x_{20}(ab+e^2v^{-1}+s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}((j^2+j)v^2+y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=r=0=z, y=0 \text{ or } y=vt^2 \in \mathbb{F}_q, u^2v \neq t^2v}$ with parameter set of size $2(q-1)^3(q-2)$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=r=0=z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u+t)x_{13}(d)x_{14}(bcv^{-1}+s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1}+w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1}+c^2ev^{-2}+z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq t^2u^{-2}v, t, s \neq 0, r=0, z=us\sqrt{w/v} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+fv)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2d+w)x_{23}((j^2+j)us^2+y)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, y=0 \text{ or } y=us^2 \in \mathbb{F}_q, us^2 \neq v^2t}$ with parameter set of size $2(q-1)^3(q-2)$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+fv)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2d+w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(bu+ev)x_{12}(au^{-1}v+s)x_{13}(a^2u^{-1}+t)x_{14}(ab+z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c+w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=sz, t=0, s \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1}+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=\sqrt{st}, t \neq 0, s \in \mathbb{F}_q}$ with parameter set of size $(q-1)3q$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(a^2eu^{-2}+w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, r \neq 0, o=srv^{-1}, z \neq \sqrt{so} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(d)x_{15}(a^2u^{-2}v+r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v+o)x_{19}(g)x_{20}(a^2eu^{-2}+w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w, s, r=vw^2u^{-2}t^{-2}, o=srv^{-1} \in \mathbb{F}_q, sw \neq vt^2}$ with parameter set of size $(q-1)^3q + (q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v+s)x_{19}(g)x_{20}(a^2u^{-2}g+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0, r=ut\sqrt{s/v} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}((i^2+i)v^{-1}u^2z^2+y)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, r=0, z \neq 0, y=0 \text{ or } y=v^{-1}u^2z^2 \in \mathbb{F}_q, sv \neq z^2}$ with parameter set of size $2(q-1)^3(q-2)$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, r=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(d)x_{15}(v)x_{16}(ub+w)x_{17}(e)x_{18}(bu^{-1}v+s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(d)x_{16}(ub+w)x_{17}(e)x_{18}(v)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+s)x_{16}(ub+w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w+t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2}+s)x_{19}(g)x_{20}(aev^{-1}+$

$$\begin{aligned}
& z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=0, z=uv\sqrt{t/w} \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid \\
& a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s=vt^2u^{-2}w^{-2} \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3q, \\
& \{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid \\
& a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=t=0, s \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{14}(c)x_{15}(w)x_{16}(t)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, t=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av+w)x_{14}(ab+t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid \\
& a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0=z \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{15}(v)x_{16}(t)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3, \\
& \{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} + \\
& z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq \sqrt{vt}, t \neq 0, s=z=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1} + w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1} + t)x_{19}(f)x_{20}(a^2du^{-2} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid \\
& a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0=t=s \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^3,
\end{aligned}$$

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$$\begin{aligned}
& \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + \\
& z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, s=t=0=r=z=y, x \neq 0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(c^2u^{-1} + \\
& y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biv^{-1} + x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2} + z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \\
& \mathbb{F}_q \}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z, y \neq 0, x, r=0 \in \mathbb{F}_q, w^2x^2=vyz, w^2z=vy^2} \text{ with parameter set of size } (q-1)^4, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=s=r=0, z, y \neq 0, x=0 \text{ or } x=u^2v^{-1}w^2 \in \mathbb{F}_q} \text{ with parameter set of size } 2(q-1)^4q, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + \\
& z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=0=t, s=r=0, z \neq 0, y \neq 0 \in \mathbb{F}_q} \text{ with parameter set of} \\
& \text{size } (q-1)^4, \\
& \{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(evv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + \\
& r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=0=t, s \neq 0, r=0, z \neq 0, y, x'=0 \text{ or } x'=u^{-2}vs^2 \in \mathbb{F}_q} \text{ with parameter set of size } 2(q-1)^4q, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + \\
& z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z \neq 0, y \neq 0, x=z'=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + \\
& z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z \neq 0, y, x \neq 0, z'=0, x'=0 \text{ or } x'=v^{-2}u^{-1}(v^2z' + ux)^2 \in \mathbb{F}_q} \text{ with parameter set of size} \\
& 2(q-1)^4q, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + \\
& r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, z, y \neq 0, x=z'=0, y'=0 \text{ or } y'=v^{-1}uw^2 \in \mathbb{F}_q} \text{ with parameter set of size} \\
& 2(q-1)^4q, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + \\
& z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \\
& \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0=y, x \neq 0, z'=0 \in \mathbb{F}_q} \text{ with parameter set of size } (q-1)^4, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + \\
& y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid \\
& a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0, y=vs^2(y')^2/(uw^2), x=vs+vsy'/(uw), z'=0, y'=v^{-1}uw^2 \in \mathbb{F}_q} \text{ with pa-} \\
& \text{rameter set of size } (q-1)^4, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + \\
& r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, \\
& i, j, k, l, m \in \mathbb{F}_q \}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0, y=w^2z'+uw^2x/(v^2)+ws(vvs+x)^2/(v^3s), x=v^2sy'/(uw), z'=uws/v, y'=v^{-1}uw^2 \in \mathbb{F}_q} \text{ with} \\
& \text{parameter set of size } (q-1)^4, \\
& \{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + \\
& r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j,
\end{aligned}$$

$$k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = uw^2/v^2, r=0, y=w^2z'+ws(vws+x)^2/(v^3s)+(z')^2(vuws+v^2z'+ux)/(u^2s^2), x=vws+v^2sy'/(uw)+v^2(z')^2/(u^2ws), z' \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2\eta \in \mathbb{F}_q, vz' \neq uws}$$

with parameter set of size $2(q-1)^4(q-2)$,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, t=0=w=s, z \neq 0, y \neq 0, x=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}((j^2+j)vs^2u^{-2}+y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, t=0=w=s, z \neq 0, y \neq 0, x=0 \in \mathbb{F}_q}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}((e^2+e)ut^2v^{-1}+z')x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=0=s=0, z, y \neq 0, x=0, z'=0 \text{ or } z'=ut^2v^{-1}\eta \in \mathbb{F}_q}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w+t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w+z)x_{17}(h)x_{18}(fu+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, t=0, z=v/u\sqrt{xw}, y=zx/w, x \neq 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0=s, t \neq 0, r=0, z \neq 0, y=0=x, z', y'=0 \text{ or } y'=v^{-2}(t+v^{-1}u^2w)^2\eta \in \mathbb{F}_q}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z=yx/s+v/u^2\sqrt{(y^2+st)^3}, y \neq \sqrt{st}, x, y'=0 \text{ or } y'=v/u^2(t+u^2w/v)^2\eta \in \mathbb{F}_q, y^2+st=\sqrt{t^2s^2((u^2x/(vts))^2+u^2x/(vts))+u^2y'/v}}$$
 with parameter set of size $(q-1)^4(q+q-2)$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r, z \neq 0, y=x=0, z' \neq 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}((k^2+k)vy^2+y')x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=s=t=0=r, z, y \neq 0, x=y^2u^{-2}v, z' \neq 0, y'=0 \text{ or } y'=vy^2\eta \in \mathbb{F}_q}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r=z, y \neq 0, x=y^2u^{-2}v \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}((h^2+h)uw^{-2}w^2+o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, x, y \neq 0, z=0, o=0 \text{ or } o=uv^{-2}w^2\eta \in \mathbb{F}_q}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=t=0=s, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2+j)uz^2+r)x_{23}(aju^{-1}+fb^2u^{-2}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q \}_{u,v \in \mathbb{F}_q^\times, w=t=0, s \neq 0, x \neq 0, y, z \neq 0, r=0 \text{ or } r=uz^2\eta \in \mathbb{F}_q, z=v\sqrt{s/u}}$$
 with parameter set of size $2(q-1)^4q$,

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1}+x)x_{16}(g)x_{17}(afu^{-1}+y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1}+ahu^{-1}+z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \}_{u,v,w \in \mathbb{F}_q^\times, t=0, x=v\sqrt{z/u}, y=\sqrt{uxz/w}/v, z \neq 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0=s, t, r \neq 0, o \neq 0, y \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4q$,

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s \neq 0, t, r \neq 0, o=0, y=(vs)^2/(u^2r) \in \mathbb{F}_q, rw=ys, s+rt \neq 0}$$
 with parameter set of size $(q-1)^5$,

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s=0, t \neq 0, r=0, o, y=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4q$,

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1}+v)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(cu+s)x_{14}(f)x_{15}(a^2bu^{-1}+o)x_{16}(cb+cu^{-1}t+r)x_{17}(g)x_{18}(ae+w)x_{19}(h)x_{20}(abeu^{-1}+y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \}_{u \in \mathbb{F}_q^\times, v \neq 0, w=sy/r, s, t, r \neq 0, o \neq 0, y=(vs)^2/(u^2r) \in \mathbb{F}_q, or+yt \neq sy/r}$$
 with parameter set of size $(q-1)^6$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=0, r=0, o \neq 0, y=0, z \neq 0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq t, z^2/r, t \neq 0, r \neq 0, o=t^3z^2/(u^2r^2), y=or/t, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=0, r \neq 0, o=0, x, y=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q^2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, s, t=0, r \neq 0, o=0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=r=0, o \neq 0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(f)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, s, t=0, r \neq 0, o \neq 0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, t \neq 0, r \neq 0, o \neq 0, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r, o, x \neq 0, y=z=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q^2$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2gu^{-2} + y')x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=r=0, o, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t \neq 0, r=\sqrt{ts^2}/u, o, y=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r \neq 0, o \neq 0, x, y=0, y' \neq 0 \in \mathbb{F}_q, v^2o=ur^2}$ with parameter set of size $(q-1)^4q^2$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2hu^{-2} + y')x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r=0, o, x \neq 0, y=0, y' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(d)x_{17}(e)x_{18}(ac^2u^{-2} + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0=r, o, x \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v=w=0, s \neq 0, t=0=r, o, x \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q, (v)^2=utr^2, (ur^2+v^2o)^2=uv^2ty^2}$ with parameter set of size $(q-1)^5 + (q-1)^4$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s=0, t=0, o=0, x \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s \neq \sqrt{v^2y}/w, t, o \neq yt/w, x, y \neq 0 \in \mathbb{F}_q, u^2w^2y(t^2y+x^2w)=((x+ts)^2+ty+w)(ty+w), v^2(ty+w)^2=u^2w^2((v^2y+s^2w)(t^2y+x^2w)+(ty+w)wy)}$ with parameter set of size $(q-1)^5q$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s=r=0, o \neq 0, y=0 \text{ or } y=v^2u^2w^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=uv/\sqrt{sr}, s \neq 0, r, o=tr/s, y=0 \text{ or } y=v^2u^2w^{-1}\eta \in \mathbb{F}_q, v^2(u^2st+wt^2)+u^2s^2y \neq 0}$ with parameter set of size $(q-1)^4(q+q-2)$,

$\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s \neq 0, r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u + acv^{-1} + \sqrt{ev^{-1}}w + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(ce v^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0, t \neq 0, r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, y=0 \text{ or } y=u^2v^2w^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t, r \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}((k^2 + k)u^{-2}vt^2 + y) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, y=0 \text{ or } y=u^{-2}vt^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t, z \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}((i^2 + i)u^{-2}t^2v + x)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, z=0, y \neq 0, x=0 \text{ or } x=u^{-2}t^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0=r=o, z \neq 0, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, r \neq 0, o, z \neq 0, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q, u^4z^2=w^2v^3r, wu^2sr^2=wu^2vro + (u^2o + wvr)^2}$ with parameter set of size $2(q-1)^4$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, o, s \in \mathbb{F}_q, t \neq \sqrt{vr}wu^{-1}, (t^2 + wo)^2u^6t^2 = (u^2t^2 + w^2vr)^3, uv t(u^2t^2 + w^2vr)(3/2) + v(u^2t^2 + w^2vr)^2 + wu^4t^2rs}$ with parameter set of size $(q-1)^4(q-2)$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t^2u^{-2}v, t \neq 0, s=0, r \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t^2u^{-2}v, t \neq 0, s=r=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t^2u^{-2}v, t, s \neq 0, r=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s=r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{15}(w)x_{16}(c)x_{17}(d)x_{18}((e^2 + e)uv^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=0, z \neq 0, y=0 \text{ or } y=uv^{-2}w^2\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1})x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=v\sqrt{t/u}, z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0=s, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s \neq 0, r=o=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s, r=0, o=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, w \neq 0, s=0, r \neq 0, o=srv^{-1}, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(bcu^{-1}+z)x_{15}(a^2u^{-2}v+r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v+o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, s \neq 0, r \neq 0, o=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(bu^{-1}v+s)x_{14}(d)x_{15}(a^2u^{-2}v+r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v+o)x_{19}(g)x_{20}(a^2eu^{-2}+w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, w \neq vt^2s^{-1}, s \neq 0, r=v^3t^2u^{-2}s^{-2}, o=srv^{-1} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1}+au^{-1}v+z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v+s)x_{19}(f)x_{20}(a^2u^{-2}f+r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, s=0, r \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v+s)x_{19}(g)x_{20}(a^2u^{-2}g+r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0, r \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(bcu^{-1}+z)x_{15}(v)x_{16}(ub+w)x_{17}(d)x_{18}(bu^{-1}v+s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1}+t)x_{12}(c)x_{14}(d)x_{15}(v)x_{16}(ub+w)x_{17}(e)x_{18}(bu^{-1}v+s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w+tx_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2}+s)x_{19}(g)x_{20}(aev^{-1}+z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{16}(t)x_{17}(d)x_{18}(w)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_{10}(v)x_{11}(a)u x_{12}(b)x_{13}(av+w)x_{14}(ab+tx_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_{10}(v)x_{11}(a)u x_{12}(b)x_{13}(av+w)x_{14}(ab+tx_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1}+s)x_{19}(f)x_{20}(cdv^{-1}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{15}(v)x_{16}(w)x_{17}(c)x_{18}(avu^{-1}+s)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_8(u)x_{11}(a)x_{12}(b)x_{14}(abu^{-1}+t)x_{16}(w)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{16}(t)x_{17}(c)x_{18}(v)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_9(u)x_{11}(a)x_{12}(w)x_{13}(v)x_{14}(b)x_{16}(c)x_{17}(d)x_{18}(a^2u^{-2}v+tx_{19}(e)x_{20}(a^2cu^{-2}+s)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v+wx_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v+tx_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1}+s)x_{19}(f)x_{20}(a^2du^{-2}+z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, s, z \in \mathbb{F}_q, s, z \text{ not all } = 0}$ with parameter set of size $(q-1)^4(q+1)$,

$\{x_{11}(u)x_{12}(w)x_{14}(a)x_{16}(v)x_{17}(b)x_{18}(t)x_{19}(c)x_{20}(d)x_{21}(e)x_{22}(f)x_{23}(g)x_{24}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_{11}(w)x_{12}(v)x_{13}(u)x_{14}(a)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(bdu^{-1}+tx_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_{11}(u)x_{12}(t)x_{14}(a)x_{15}(v)x_{16}(w)x_{17}(b)x_{18}(c)x_{19}(d)x_{20}(e)x_{21}(f)x_{22}(g)x_{23}(h)x_{24}(i) \mid a, b, c, d, e, f, g, h, i \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_{11}(v)x_{12}(t)x_{13}(u)x_{14}(a)x_{15}(w)x_{16}(b)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

Y16 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w+s)x_7(b)x_8(c)x_9(d)x_{10}(bu+tx_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g+zx_{16}(c^2v^{-2}w+z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1}+r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1}+y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=y=0, x \neq vu\sqrt{r/w} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+tx_{11}(e)x_{12}(f)x_{13}(w^{-1}vf+rx_{14}(g)x_{15}(c^2u^{-1}+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1}+x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2}+z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, z \neq 0, y \neq w\sqrt{z/v}, x=w\sqrt{vuyz/w}, r=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+wx_{11}(d)x_{12}(euv^{-1}+tx_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}((i^2+i)u^2v^{-1}w^2+x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=0, r \neq 0, z \neq 0, y \neq zwv^{-1}, x=0 \text{ or } x=u^2v^{-1}w^2 \eta \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $2(q-1)^5$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+wx_{11}(d)x_{12}(euv^{-1}+tx_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1}+s)x_{16}(bev^{-1}+r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1}+z)x_{20}((i^2+i)u^2v^{-1}w^2+x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1}+y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=s=0, r \neq 0, z \neq 0, y \neq zwv^{-1}, x=0 \text{ or } x=u^2v^{-1}w^2 \eta \in \mathbb{F}_q, vr=w^2}$ with parameter set of size $2(q-1)^5$,

$r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q^2 + q(q-2))$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^5$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^6q$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q^2$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4 * (q^2 + q(q-2))$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4 * (q^2 + q(q-2))$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}((i^2 + i)u^2v^{-1}w^2 + x)x_{21}(j)x_{22}((k^2 + k)u^{-2}vs^2 + x')x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $4(q-1)^6$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(dbnv^{-2} + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^5$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}((m^2 + m)v^{-2}ux^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4(q^2 + q(q-2))$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4(q-2)$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4(q-2)$,

$\{z'x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}((l^2 + l)v^{-2}u^{-1}(v^2z' + ux)^2 + x') \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=s=t=r=0, z \neq \sqrt{yz'}, y \neq 0, x, z' \neq 0, x' = 0 \text{ or } x' = v^{-2}u^{-1}(v^2z' + ux)^2 \eta \in \mathbb{F}_q, v^2z' \neq ux}$ with parameter set of size $2(q-1)^6$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0=y, x \neq 0, z' \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uu^2/v^2, r=0, y, x=0, z' \neq 0, y'=v^{-1}uw^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uu^2/v^2, r=0, y \neq w^2z', y \neq 0, x=0, z' \neq 0, y'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4(q-2)$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uu^2/v^2, r=0, y, x, z', y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, vusy=y'(v^2sz' + usx + vus^2) + uwxz', v^3(z')^2 \neq us(v^2y' + uwx + vuw^2s)}$ with parameter set of size $(q-1)^5q$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bjv^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uu^2/v^2, r=0, y=wz'/(vs), x \neq 0, z' \neq \sqrt{(u^2wsx + vu^2w^2s^2)/v^3}, y'=0 \in \mathbb{F}_q, x \neq vus}$ with parameter set of size $(q-1)^5(q-2)$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s=0, z \neq 0, y \neq tuz/v, z'=0 \text{ or } z'=ut^2v^{-1} \eta \in \mathbb{F}_q, vw=t^2}$ with parameter set of size $2(q-1)^5$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de + a^2gu^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t, w \neq 0, s=0, y \neq 0 \in \mathbb{F}_q, vw \neq t^2}$ with parameter set of size $(q-1)^5$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0, s \neq 0, z, y, x=0, z'=0 \text{ or } z'=ut^2v^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4q^2$

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(agu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(de + a^2gu^{-2} + y)x_{23}((k^2 + k)vs^2u^{-2} + y')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s \neq 0, z, y, z'=0 \text{ or } z'=ut^2v^{-1} \eta, y'=0 \text{ or } y'=vs^2u^{-2} \eta \in \mathbb{F}_q, vw=t^2, y' \neq v(uz^2 + tsz)/(ut^2)}$ with parameter set of size $2(q-1)^4(q^2 + q(q-2))$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, z \neq 0, y, x \neq 0 \text{ or } y'=vs^2u^{-2} \eta \in \mathbb{F}_q, vs=us^2x}$ with parameter set of size $2(q-1)^4(q-2)q$

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w, s \neq 0, z \neq 0, y, x \neq 0 \in \mathbb{F}_q, vs=u^2x}$ with parameter set of size $(q-1)^4q$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}((j^2 + j)vs^2u^{-2} + y')x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t=0=w=s, z \neq 0, y \neq 0, x \neq 0, y' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5q$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0=s, z, y, x \neq 0, z'=ut^2v^{-1} \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q^2$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v + w)x_{14}((e^2 + e)ut^2v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0=s, z, y \neq t^2u^2x/v^2, y \neq 0, x \neq 0, z'=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4(q-2)q$,

$\{x_1(u)x_5(a)x_6(v)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, z \neq v/u\sqrt{xw}, y=zx/w, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu + y)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(cnv^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=0, z \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} +$

$t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ab + ud + x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(g^2uv^{-2}w + ue^2v^{-2}w + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, x=0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + x)x_{19}(i)x_{20}(ug + z)x_{21}(j)x_{22}(k)x_{23}(i^2v^{-1} + (u + ug)tu^{-2} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=t=0, x=0 \text{ or } x=u^2v\eta, y \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(i^2v^{-1} + jtu^{-2} + y)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t=0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2v + x)x_{16}(h)x_{17}(i)x_{18}(ahv^{-1} + e^2u + w)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(fi + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, y \neq 0, x=0 \text{ or } x=u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(i^2v^{-1} + jtu^{-2} + y)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(eu + z)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(de + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, z \neq 0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_8(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ud + x)x_{16}(v)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(de + w)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s=t \neq 0, r=0, z, y \neq 0, x=0, z', y'=0 \text{ or } y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \in \mathbb{F}_q, y' \neq tz/y + u^2z^2/(vy^2)}$ with parameter set of size $(q-1)^4(q^2 + q(q-2))$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s=t \neq 0, r=0, z, y=u\sqrt{x/v}, x \neq 0, z', y'=0 \text{ or } y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4q^2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z=\sqrt{s/ty'}, y, x=sy'/t, y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \in \mathbb{F}_q, st \neq y^2 + u^2x/v}$ with parameter set of size $(q-1)^5$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z=x\sqrt{t/s}, y=\sqrt{ts}, x \neq sy'/t, y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \text{ or } y'=0 \in \mathbb{F}_q, u^2x^2 \neq vs(tx + sy')}$ with parameter set of size $(q-1)^4(q-2) + (q-1)^5$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z, y \neq \sqrt{ts}, x \neq sy'/t, y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \text{ or } y'=0 \in \mathbb{F}_q, u^2x^2 \neq vs(tx + sy')}$ with parameter set of size $(q-1)^5(q-2) + (q-1)^4(q-2)^2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0, r=z \neq 0, y \neq u\sqrt{x/v}, x, z' \neq z\sqrt{(u^2x + vy^2)/v}, y'=0 \text{ or } y'=vy^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^6 + (q-1)^5$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}((k^2 + k)vy^2 + y')x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=s=t=0, r=z, y \neq u\sqrt{x/v}, x \neq 0, y' \neq 0 \text{ or } y'=vy^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4(q-2) + (q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0, r=z, y \neq u\sqrt{x/v}, x \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z \neq \sqrt{wz'}, x=0, z' \neq 0, y'=0 \text{ or } y'=vu^{-2}(t + v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^5$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=t=r=0, z \neq \sqrt{(v(z')^2 + u^2wxz' + u^2x^2y')/x}, x \neq 0, z', y'=0 \text{ or } y'=u^2v^{-1}w^2 \eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^5q$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r=0, z \neq \sqrt{wz'}, x=0, z' \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5q$,
 $\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} +$

$w)x_{20}(k)x_{21}(l)x_{22}(b^2u^{-2}k+x)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1}+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(cju^{-1}+z)x_{22}(b^2u^{-2}m+x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}((h^2+h)uv^{-2}w^2+o)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q^2+q(q-2))$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}((j^2+j)uz^2+r)x_{23}(aju^{-1}+fb^2u^{-2}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^5$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+fb^2u^{-2}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2+w)x_{16}(aeu^{-1}+s)x_{17}(afu^{-1}+y)x_{18}(g)x_{19}(h)x_{20}((i^2+i)uv^{-2}w^2+o)x_{21}(j)x_{22}(vh+r)x_{23}(gb^2u^{-2}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^5q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v+z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2+w)x_{16}(adu^{-1}+s)x_{17}(aeu^{-1}+y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2+w)x_{16}(aeu^{-1}+s)x_{17}(afu^{-1}+y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1}+x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2+w)x_{16}(aeu^{-1}+s)x_{17}(afu^{-1}+y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(vh+r)x_{23}(gb^2u^{-2}+x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1}+x)x_{16}(g)x_{17}(afu^{-1}+y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1}+ahu^{-1}+z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1}+z)x_{21}(cju^{-1}+lw^{-1}v+y)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2+h)uv^2+y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1}+w)x_{21}(l)x_{22}(fju^{-1}+x)x_{23}(lv+z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{16}((h^2+h)uv^2+y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1}+w)x_{21}(l)x_{22}(m)x_{23}(lv+z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1}+s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(v^{-2}d^2+t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1}+s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}((j^2+j)uw^2+x)x_{21}(k)x_{22}(f^2u^{-1}+t)x_{23}(iv+w)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{16}(aeu^{-1}+s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(iv+w)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1}+w)x_{16}(g)x_{17}(afu^{-1}+t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1}+aiu^{-1}+z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2+ad)u^{-1}+s)x_{17}(hvw^{-1}+t)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}((c^2+ag)u^{-1}+s)x_{21}(i)x_{22}(j)x_{23}(vf+aju^{-1}+w)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{16}((b^2+ad)u^{-1}+t)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(vf+aku^{-1}+w)x_{24}(l) \mid$

$a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(ul + y)x_{23}(i^2v^{-1} + bl + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=r=0, y \neq 0, z \neq 0, x=0$ or $x=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(i^2/v + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=t=0, r \neq 0, z \neq 0, x=0$ or $x=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(i^2v^{-1} + bl + z)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=s=0, t \neq 0, r=0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(v^{-1}bn + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=t=r=z=0, y \neq 0, x \neq 0, s=0$ or $s=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(mu + y)x_{23}(n)x_{24}(j^2/v + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, t \neq 0, r=z=0, y \neq 0, s=0$ or $s=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(v^{-1}bo + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w \neq 0, t=r=z=0, x \neq 0, s=0$ or $s=u^2v \eta \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=0, s \neq 0, t, r, o \neq 0, y=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=s=0, t, r \neq 0, o, y \neq 0 \in \mathbb{F}_q, o^2r^2 + u^2v^2ry + t^2y^2 \neq 0$ with parameter set of size $(q-1)^5q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s \neq 0, t, r, o, y \in \mathbb{F}_q, s^2o^2 + u^2v^2sw + t^2w^2 \neq 0, u^2r^2w \neq v^2s^3, sy=rv$ with parameter set of size $(q-1)^6q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s=0, t \neq 0, r=0, o, y \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w, s, t=0, r \neq 0, o=0, y \neq 0, z \neq 0 \in \mathbb{F}_q, r^2w^2 + s^2y^2 + u^2z^2ry \neq 0$ with parameter set of size $(q-1)^5q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, r, o \neq 0, y, z \neq 0 \in \mathbb{F}_q, ty=or, tw^2 \neq u^2z^2o, t^3z^2 \neq u^2or^2$ with parameter set of size $(q-1)^6$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w \neq 0, t \neq 0, r, o=y=0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v \neq 0, w=s=0, t \neq 0, r=0, o, y \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w \neq 0, s \neq 0, t=r=o=0, x, y \neq 0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(ad + w)x_{19}(g)x_{20}(abdu^{-1} + y)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w=s \neq 0, t=r=o=0, y \neq 0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, s \neq 0, t=r=0, o \neq 0, y \neq 0, z \neq 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^5$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(cu + s)x_{14}(e)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(beu^{-1} + cdu^{-1} + x)x_{18}(ad + w)x_{19}(f)x_{20}(abdu^{-1} + y)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ $u \in \mathbb{F}_q^\times, v=0, w \neq 0, s \neq 0, t=r=o=0, x \neq 0, y \neq 0, z=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^5$,

$\mathbb{F}_q\}$ with parameter set of size $(q-1)^6q$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(e^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $\mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}((h^2 + h)u^2v + y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(ui + x)x_{23}(g^2v^{-1} + z)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(b^2v^{-1} + w)x_{14}(e)x_{15}(au + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(g^2v^{-1} + z)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(f)x_{16}(bu + w)x_{17}(g)x_{18}(bfu^{-1} + t)x_{19}(h)x_{20}(cfv^{-1} + s)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4q$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q+q-2)$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q+q-2)$,
 $\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u + acv^{-1} + \sqrt{ev^{-1}}w + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(ab + z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}((f^2 + f)u^2v^2w^{-1} + y)x_{19}(g)x_{20}(ab + af^2w^{-1}u^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q+q-2)$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_5(u)x_8(a)x_{10}(v)x_{11}(fv^{-1}u + cv^{-1}a + w)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(adu^{-1} + s)x_{18}(cev^{-1} + u^2fv^{-1} + t)x_{19}(g)x_{20}(efv^{-1} + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad + y)x_{18}(bu + e^2v)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_5(u)x_8(au)x_{11}(b)x_{12}(c)x_{13}(v)x_{14}(d + ev)x_{15}(au^2 + t)x_{16}(av + w)x_{17}(aev + ew + ad)x_{18}(bu + e^2v + kt)x_{19}(f)x_{20}(e^2w + ae^2v + ab + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + w)x_{16}(v)x_{17}(e)x_{18}(bu + t)x_{19}(f)x_{20}(ab + e^2v^{-1} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + w)x_{16}(v)x_{17}(e)x_{18}(bu + kw)x_{19}(f)x_{20}(ku^{-1}wa + ab + e^2v^{-1} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,
 $\mathbb{F}_q\}$ with parameter set of size $(q-1)^5$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, r \neq u^2t^2/(vw^2), o \neq t^2/w, s \in \mathbb{F}_q, u^2v^3(t^2+wo)^2+v^2(t^2+wo)(w^2vr+u^2t^2)+ws(w^2vr+u^2t^2), vt(t^2+wo)/u+(w^2vr+u^2t^2)(3/2) \neq 0}$
with parameter set of size $(q-1)^4(q-2)^2$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, z = u^{-1}\sqrt{vr^{-1}(wsr^2+u^2o^2+wwr)}, s = wv^2u^{-2}\eta \text{ or } s = 0 \in \mathbb{F}_q, u^4z^2 \neq w^2v^3r}$ with parameter set of size $(q-1)^4(2q-4)$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq u\sqrt{w/v}, s = 0, r \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq u\sqrt{w/v}, s = r = 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(dhv^{-1} + c^2ev^{-2} + z)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq t^2u^{-2}v, t, s \neq 0, r = 0, z \neq us\sqrt{w/v} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5q$,

$\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t, s = t\sqrt{r/u}, r \neq 0 \in \mathbb{F}_q, v^2t^4 \neq u^3w^2r}$ with parameter set of size $(q-1)^5$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(((ucs^{-1} + ab)^2 + ucs^{-1} + ab)uw^{-2}w^2 + y)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t = 0, s \neq 0, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s = v\sqrt{t/u} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(bu + fv)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(b^2u)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(b^2d + w)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq v\sqrt{t/u} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_6(u)x_8(v)x_9(a)x_{11}(bu + ev)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(ab + z)x_{16}(c)x_{17}(d)x_{18}(b^2u)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(b^2c + w)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq sz, t = 0, s \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s = 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t = 0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_6(u)x_9(a)x_{11}(b)x_{12}(v)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + w)x_{16}(c)x_{17}(d)x_{18}(b^2u^{-1} + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq \sqrt{ts}, t \neq 0, s \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + (t^2+l)w^{-1}vt + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t = 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t = 0, s \neq 0, r \neq osv^{-1}, o \neq 0, z \neq \sqrt{so} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^6$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t = s = 0, r, o \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(a^2eu^{-2} + w)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t = 0, w \neq 0, s \neq 0, r \neq 0, o = srv^{-1}, z \neq \sqrt{so} \in \mathbb{F}_q}$
with parameter set of size $(q-1)^6$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(a^2eu^{-2} + w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w \neq \sqrt{rv^{-1}ut}, s \neq \sqrt{v^3r^{-1}t^2u^{-2}}, r \neq 0, o = srv^{-1} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5 + (q-1)^4(q-2)^2 + (q-1)^4(q-2)$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(a^2eu^{-2} + w)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w \neq 0, s, r = 0, o = srv^{-1} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t = 0, s \neq 0, z \neq \sqrt{vs} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(bcu^{-1} + au^{-1}v + z)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(a^2u^{-2}v + s)x_{19}(f)x_{20}(a^2u^{-2}f + r)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w = t = 0, s \neq 0, r \neq 0, z \neq \sqrt{vs} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(a^2u^{-2}g + r)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w = 0, t \neq 0, s \neq 0, r \neq \sqrt{s/vut} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + z)x_{15}(v)x_{16}(ub + w)x_{17}(d)x_{18}(bu^{-1}v + s)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(d)x_{15}(v)x_{16}(ub + w)x_{17}(e)x_{18}(bu^{-1}v + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(d)x_{16}(ub + w)x_{17}(e)x_{18}(v)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{14}(bcu^{-1} + s)x_{16}(ub + w)x_{17}(d)x_{18}(v)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(aev^{-1} +$
 $z)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=0, z \neq uv\sqrt{t/w} \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_8(u)x_9(w)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(c^2dv^{-2} + t)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq uv\sqrt{s/v}, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_8(u)x_9(v)x_{11}(a)x_{12}(b)x_{14}(c)x_{15}(w)x_{16}(t)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_8(u)x_9(t)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(cdv^{-1} + z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_8(u)x_{10}(v)x_{11}(au)x_{12}(b)x_{13}(av + w)x_{14}(ab + t)x_{15}(c)x_{16}(d)x_{17}(e)x_{18}(acu^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid$
 $a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq \sqrt{ws}, s \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_9(u)x_{11}(a)x_{12}(w)x_{14}(b)x_{15}(v)x_{16}(t)x_{17}(c)x_{18}(d)x_{19}(e)x_{20}(f)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in$
 $\mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_9(u)x_{11}(a)x_{12}(t)x_{13}(w)x_{14}(b)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_9(u)x_{10}(v)x_{11}(a)x_{12}(au^{-1}v + w)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-2}v + t)x_{16}(d)x_{17}(e)x_{18}(a^2bv^{-1} + s)x_{19}(f)x_{20}(a^2du^{-2} +$
 $z)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq \sqrt{vt}, t \neq 0, s, z \in \mathbb{F}_q, s, z \text{ not all } = 0}$ with parameter set of size
 $(q-1)^5(q+1)$,
 $\{x_{10}(u)x_{11}(v)x_{12}(a)x_{13}(b)x_{14}(c)x_{15}(a^2u^{-1} + w)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-1} + t)x_{19}(f)x_{20}(a^2du^{-2} + s)x_{21}(g)x_{22}(h)x_{23}(i)x_{24}(j) \mid$
 $a, b, c, d, e, f, g, h, i, j \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s \in \mathbb{F}_q}$ (*s and/or t $\neq 0$*) with parameter set of size $(q-1)^4(q+1)$,

Y17 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w +$
 $z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0=r=z, y \neq 0, x \in \mathbb{F}_q, 1=p+p^2 \text{ for some } p \in \mathbb{F}_q, p^3v^4u^2w^2y=1 \text{ for some } p \in \mathbb{F}_q}$ with parameter set of size $\alpha(q-1)^3q\beta_{10}$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w +$
 $z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=0, y \neq 0, x \in \mathbb{F}_q, \text{ let } o \in \mathbb{F}_q^\times \text{ s.t. } v^2u^{-2}wr^{-3}y^2=(o^2v^2u^2wr)^{-1}+(o^2v^2u^2wr)^{-3}, o^3v^4u^2w^2y=p+p^2 \text{ for some } p \in \mathbb{F}_q}$ with
parameter set of size $(q-1)^4q(\alpha\beta_4 + (1-\alpha)\beta_5)$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(h)x_{15}(c^2u^{-1} +$
 $y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bju^{-1} + x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2} + z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z=vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, (n')^3w^2v^4u^2(vuyz+w^2x^2)=1, 1=p^2+p \text{ for some } p, n' \in \mathbb{F}_q}$ with parameter set of size
 $(q-1)^3q\alpha\beta_{10}$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(h)x_{15}(c^2u^{-1} +$
 $y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bju^{-1} + x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2} + z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in$
 $\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z \neq vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, \text{ let } n \in \mathbb{F}_q \text{ s.t. } vu^2(vy^2 + w^2z)(n + n^3) = w^4(vuyz + w^2x^2)^2, w^4v^3u^2(vuyz +$
 $w^2x^2)^4(vy^2 + w^2z)^3 = n^3(p^2 + p) \text{ for some } p \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q(\alpha\beta_4 + (1-\alpha)\beta_5)$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} +$
 $r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in$
 $\mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq ut, r, z, y \in \mathbb{F}_q, vr \neq w^2, uvty(vr+w^2)=vr(uz+ws)^2+(o^2+o)(vr+w^2)^2*(ut+s)^2 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size
 $(q-1)^5q^3/2$
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} +$
 $r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in$
 $\mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t=0, s \neq 0, r \neq v^{-1}w^2, z, y \in \mathbb{F}_q, vr(uz+ws)^2=(o^2+o)(vr+w^2)^2*s^2 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q^2+(q-1)^5q^2/2$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} +$

$w)x_{21}(l)x_{22}(m)x_{23}(bm v^{-1}+y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0=z, y=u^2 w^2 v^{-2} s^{-1}(p^2+p)}$ for some $p \in \mathbb{F}_q$
with parameter set of size $(q-1)^4 q/2$,

$\{x_1(v)x_3(x)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(dbnv^{-2}+x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, z=0, x=u^2 w^2 v^{-2} t^{-1}(p^2+p)}$ for some $p \in \mathbb{F}_q$
with parameter set of size $(q-1)^4 q/2$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}((n^2 + n)u x^2 t(uw^2 + v^2 t)^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w, s=0, t \neq 0, r=0, y, x \neq 0, x'=0 \in \mathbb{F}_q, v^2 t \neq u w^2}$ with parameter set of size $(q-1)^5 q$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=uw^2/v^2, r=0, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, uvvsy'+vs(vy'z'+uy)+usxy'+uwxz' \neq 0,}$
 $let n \in \mathbb{F}_q^\times \text{ s.t. } uvvsy'+vs(vy'z'+uy)+usxy'+uwxz'=1/(n^3 us^2)+(w(uvws+ux)/(vs)+vy'+v^2(z')^2/(us^2))/n,$
 $n^2 us^2(w(uvws+ux)/(vs)+vy'+v^2(z')^2/(us^2))=(o^2+o) \text{ for some } o \in \mathbb{F}_q$

with parameter set of size $2(q-1)^4 q(\alpha((q-1)\beta_4 + \beta_{10}) + (1-\alpha)(q-1)\beta_5)$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, z, y, x \neq 0, z' \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, v^2 z' = ux,}$ with parameter set
 $v^2 y + vy'x = (o^2 + o + 1)uw^2 x$ for some $o \in \mathbb{F}_q$

of size $(q-1)^4 q^2$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, y, x \neq 0, z' \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, v^2 z' \neq ux,}$ with parameter set of
 $v^2 uw^2 z'x + v^4 z'y + v^3 y'z'x = (o^2 + o)u^2 w^2 x^2$ for some $o \in \mathbb{F}_q$

size $(q-1)^4 (q-2)q$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z'=0 \in \mathbb{F}_q, 1=o^2+o \text{ for some } o \in \mathbb{F}_q, o^3 v^4 u^2 s^2 y=1 \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $\alpha(q-1)^3 q \beta_{10}$,

$\{x_1(v)x_4(x)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2 u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2 u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z' \neq 0 \in \mathbb{F}_q, let n \in \mathbb{F}_q^\times \text{ s.t. } u^2 s^2 y=v^2(z')^3((n^{-1}v^{-2}s(z')^{-1})^3+n^{-1}v^{-2}s(z')^{-1}), n^3 v^4 u^2 s^2 y=o^2+o \text{ for some } o \in \mathbb{F}_q}$

with parameter set of size $(q-1)^4 q(\alpha\beta_4 + (1-\alpha) * \beta_5)$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v + w)x_{14}(e)x_{15}(du + s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de + a^2 gu^{-2} + y)x_{23}((l^2 + l)vs^2 u^{-2} + y')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t, w \neq 0, s \neq 0, y, v \neq 0 \in \mathbb{F}_q, v w \neq t^2}$ with parameter
set of size $(q-1)^5 q$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{11}(c)x_{12}(d)x_{13}(a^2 u^{-2}v + w)x_{14}((e^2 + e)ut^2 v^{-1} + z')x_{15}(du + s)x_{16}(u^{-1}vd + x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1} + z)x_{20}(h)x_{21}(i)x_{22}(de + a^2 hu^{-2} + y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, t \neq 0, w=0, s \neq 0, z, y, x \neq 0, z'=0 \text{ or } z'=ut^2 v^{-1} \eta \in \mathbb{F}_q, u^2 xy + uz' sx = t^2 s^2 (o^2 + o) \text{ for some } o \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5 q^2$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t=0, z=v/u\sqrt{wx}, y \neq zx/w, x \in \mathbb{F}_q,}$
 $1=p+p^2 \text{ for some } p \in \mathbb{F}_q,$
 $p^3 u^4 v^2 w(wy+zx)=1 \text{ for some } p \in \mathbb{F}_q$

with parameter set of size $\alpha(q-1)^3 q \beta_{10}$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2} + y)x_{23}(l)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t=0, z \neq v/u\sqrt{wx}, y \neq zx/w, x \in \mathbb{F}_q,}$
 $let o \in \mathbb{F}_q^\times \text{ s.t. } u^2 v^4 w^2 (v^2 wx + (zu)^2)^{-3} (wy + zx)^2$
 $= (o^2 u^2 (v^2 wx + (zu)^2))^{-1} + (o^2 u^2 (v^2 wx + (zu)^2))^{-3},$
 $o^3 u^4 v^2 w (wy + zx) = p + p^2 \text{ for some } p \in \mathbb{F}_q$

with parameter set of size $(q-1)^4 q(\alpha\beta_4 + (1-\alpha)\beta_5)$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu + y)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2 + o)v^2 y^2 u^{-2} w^{-1} + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, t=s=z=0, y \neq 0 \in \mathbb{F}_q}$

with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h + x)x_{15}(f)x_{16}(cv + w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2 ju^{-2} + t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t, x \neq 0 \in \mathbb{F}_q, t=v^2 x^2 w^{-1}(o^2+o) \text{ for some } o \in \mathbb{F}_q}$ with parameter set of
size $(q-1)^4 q/2$,

$\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(ab + ud + x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}((k^2 + k)u^{-1}x^2 v^{-1} w^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$
 $_{u,v,w \in \mathbb{F}_q^\times, x \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2 fv^{-1} + t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}((m^2 + m)w^2 v^{-1} t^2 u^{-2} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$
 $_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $\alpha(q-1)^3q\beta_{10}$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(\alpha\beta_4 + (1-\alpha)\beta_5)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1} + z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4q(\alpha(\beta_4(q-1) + \beta_{10}) + (1-\alpha)\beta_5(q-1))$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q^2/2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1} + x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1} + z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(cju^{-1} + z)x_{22}(b^2u^{-2}m + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + fb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + fb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q/2 + (q-1)^4$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + fb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ahu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q\alpha\beta_{10}$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ahu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(\alpha\beta_4 + (1-\alpha)\beta_5)$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}((l^2 + l)v^{-1}wz + y)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}((l^2 + l)w^2sv^{-2} + x)x_{23}(m)x_{24}(n)x_{25}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(\alpha\beta_4 + (1-\alpha)\beta_5)$,

$t)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2 + ad)u^{-1} + s)x_{17}((f^2 + f)v^{-1}ws + t)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)u^{-2}t^2r + z)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}((p^2 + p)u^{-2}w^2t + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(q/2 - 1)$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2(2 - q)/3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2/2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2(q/2 - 1)$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(q/2 - 1)$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2(q/2 - 1)$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3(q-2)q(\alpha * (\beta_4(q-1) + \beta_{10}) + (1-\alpha)\beta_5(q-1)) + q(q-1)^4(2-q)/3$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q(\alpha(\beta_4(q-1) + \beta_{10}) + (1-\alpha)\beta_5(q-1))$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2 + o)v^{-2}tr^2 + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(bcu^{-1} + x)x_{14}(ab^2u^{-1} + w)x_{15}(abcu^{-2} + r)x_{16}(c^2u^{-1} + o)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(q/2 - 1)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(bcu^{-1} + x)x_{14}(ab^2u^{-1} + w)x_{15}(abcu^{-2} + r)x_{16}(c^2u^{-1} + o)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q(q/2 - 1)$,

$\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t \neq 0, x \neq 0, y, s \in \mathbb{F}_q$ let $n \in \mathbb{F}_q^\times$ s.t. $(vy)^2uw + u^2wx(wy + sx) + (uw^2x + (wy + sx)v^2)t^2 \neq 0$, with parameter set of size
 $((vy)^2uw + u^2wx(wy + sx) + (uw^2x + (wy + sx)v^2)t^2)v^2wx = ((uwx + v^2y)^2 + (wy + sx)uv^2x + wv^2t^2x)n + n^3$,
 $((uwx + v^2y)^2 + (wy + sx)uv^2x + wv^2t^2x)/n^2 = m^2 + m + 1$ for some $m \in \mathbb{F}_q$
 $(q - 1)^5(\alpha(\beta_4(q - 1) + \beta_{10}) + (1 - \alpha)\beta_5(q - 1))$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}$ $u, v, w \in \mathbb{F}_q^\times, t=0, r \neq 0, o, z, s=0$ or $s=wv^2u^{-2}\eta \in \mathbb{F}_q, wvr^2s \neq wv^2ro + u^2vo^2 + u^2z^2r$ let $n \in \mathbb{F}_q^\times$ s.t. with parameter set of size $2(q - 1)^4(\alpha(\beta_4(q - 1) + \beta_{10}) + (1 - \alpha)(q - 1)\beta_5)$,
 $n^2w^2(w^2v^2r^2 + u^2(wvror + u^2o^2 + wr^2s)) + n^3w^4u^2r(wv^2ro + u^2vo^2 + u^2z^2r + wvr^2s) = 1$,
 $n^3w^4u^2r(wv^2ro + u^2vo^2 + u^2z^2r + wvr^2s) = m^2 + m$ for some $m \in \mathbb{F}_q$
 $1) + \beta_{10}) + (1 - \alpha)(q - 1)\beta_5)$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with
 $u, v \in \mathbb{F}_q^\times, w=0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q, 1 = m + m^2$ for some $m \in \mathbb{F}_q$ with
 $m^3u^4s^2v^2r = 1$ for some $m \in \mathbb{F}_q$
parameter set of size $(q - 1)^3q\alpha\beta_{10}$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with pa-
 $u, v \in \mathbb{F}_q^\times, w \neq 0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q$ let $n \in \mathbb{F}_q^\times$ s.t. with pa-
 $u^2s^{-2}vw^{-3}r^2 = (n^2u^2s^2vw)^{-1} + (n^2u^2s^2vw)^{-3}$
 $n^3u^4s^2v^2r = m + m^2$ for some $m \in \mathbb{F}_q$
parameter set of size $(q - 1)^4q(\alpha\beta_4 + (1 - \alpha)\beta_5)$,
 $\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abu^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parame-
 $u, v, w \in \mathbb{F}_q^\times, t, s \neq t\sqrt{r}/u, r \in \mathbb{F}_q$ let $n \in \mathbb{F}_q^\times$ s.t. with parame-
 $n^3u^3v^4w^2(t^2r + us^2) + n^2u^3v^2w^2r = 1$
 $n^3u^3v^4w^2(t^2r + us^2) = m + m^2$ for some $m \in \mathbb{F}_q$
ter set of size $(q - 1)^3q(\alpha(\beta_4(q - 1) + \beta_{10}) + (1 - \alpha)\beta_5(q - 1))$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}((f^2 + f)(v^2t + us^2)^{-1}uw^2t + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size
 $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s \neq \sqrt{t}/u, y = 0$ with parameter set of size
 $(q - 1)^5$,
 $\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size
 $u, v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq 0 \in \mathbb{F}_q, w^2 = (m^2 + m)us^2t^{-1}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q - 1)^4q/2$
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1} + t)x_{14}(abu^{-1} + (l^2 + l)w^{-1}vt + z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q - 1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q - 1)^4q/2$,
 $u, v \in \mathbb{F}_q^\times, t \neq 0, r = 0, o \neq srv^{-1}, s = t^2v^2u^{-2}o^{-1}(m^2 + m)$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q - 1)^4q/2$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q - 1)^4(q/2 - 1)$,
 $u, v \in \mathbb{F}_q^\times, t \neq 0, s = 0, o \neq srv^{-1}, r = u^2o^2t^{-2}v^{-1}(m^2 + m)$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q - 1)^4(\alpha\beta_4 + (1 - \alpha)\beta_5)$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with pa-
 $u, v \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, r \neq 0, o \neq srv^{-1} \in \mathbb{F}_q$, let $n \in \mathbb{F}_q^\times$ solve with pa-
 $1 \neq nvt^3u^2ts(sr + vo) + n^2(v^2t^2 + u^2so) = 1$,
 $n^3u^2ts(sr + vo) = m^2 + m$ for some $m \in \mathbb{F}_q$
parameter set of size $(q - 1)^4(\alpha\beta_4(q - 1) + \beta_{10}) + (1 - \alpha)\beta_5(q - 1) - (q/2 - 1)$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size
 $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s = 0 \in \mathbb{F}_q, 1 = m + m^2$ for some $m \in \mathbb{F}_q$ with parameter set of size
 $m^3u^4t^2v^2w = 1$ for some $m \in \mathbb{F}_q$
 $(q - 1)^3\alpha\beta_{10}$,
 $\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of
 $u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq 0 \in \mathbb{F}_q$ let $n \in \mathbb{F}_q^\times$ s.t. $u^2t^{-2}vs^{-3}w^2 = (n^2u^2t^2vs)^{-1} + (n^2u^2t^2vs)^{-3}$, with parameter set of
 $n^3u^4t^2v^2w = m + m^2$ for some $m \in \mathbb{F}_q$
size $(q - 1)^4(\alpha\beta_4 + (1 - \alpha)\beta_5)$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size
 $u, v, w \in \mathbb{F}_q^\times, t = 0, s \neq 0 \in \mathbb{F}_q, 1 = m + m^2$ for some $m \in \mathbb{F}_q, m^3vw = us$ for some m with parameter set of size
 $(q - 1)^4\alpha\beta_{10}$,
 $\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of
 $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0 \in \mathbb{F}_q$, let $n \in \mathbb{F}_q^\times$ s.t. $u^2v^{-2}wt^{-3}s^2 = (n^2u^2v^2wt)^{-1} + (n^2u^2v^2wt)^{-3}$, with parameter set of
 $n^3u^4v^2w^2s = m + m^2$ for some $m \in \mathbb{F}_q$
size $(q - 1)^4(\alpha\beta_4 + (1 - \alpha)\beta_5)$,
 $\{x_8(u)x_9(t)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q - 1)^4q/2$,
 $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s = t^2w^2u^{-2}v^{-1}(m^2 + m)$ for some $m \in \mathbb{F}_q$

$y_{18} \{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bj u^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=0, y \neq 0, x \in \mathbb{F}_q, 1=p+p^2+\eta}$ for some $p \in \mathbb{F}_q, p^3v^4u^2w^2y=1$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,

$\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bj u^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=0, y \neq 0, x \in \mathbb{F}_q, let o \in \mathbb{F}_q^\times s.t. v^2u^{-2}wr^{-3}y^2=(o^2v^2u^2wr)^{-1}+(o^2v^2u^2wr)^{-3}, o^3v^4u^2w^2y=p+p^2+\eta}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bj u^{-1} + x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2} + z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z=vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, (n')^3w^2v^4u^2(vuyz+w^2x^2)=1, 1=p^2+p+\eta}$ for some $p, n' \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bj u^{-1} + x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2} + z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z \neq vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, let n \in \mathbb{F}_q s.t. vu^2(vy^2 + w^2z)(n + n^3) = w^4(vuyz + w^2x^2)^2, w^4v^3u^2(vuyz + w^2x^2)^4(vy^2 + w^2z)^3 = n^3(p^2 + p + \eta)}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq ut, r, z, y \in \mathbb{F}_q, vr \neq w^2, uvty(vr+w^2)=vr(uz+ws)^2+(o^2+o+\eta)(vr+w^2)^2*(ut+s)^2}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^3/2$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + w)x_{11}(d)x_{12}(euv^{-1} + t)x_{13}(e)x_{14}(f)x_{15}(eu^2v^{-1} + s)x_{16}(bev^{-1} + r)x_{17}(g)x_{18}(h)x_{19}(cfu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(eguv^{-1} + y)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t=0, s \neq ut, r \neq v^{-1}w^2, z, y \in \mathbb{F}_q, uvty(vr+w^2)=vr(uz+ws)^2+(o^2+o+\eta)(vr+w^2)^2*(ut+s)^2}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bj u^{-1} + w)x_{21}(l)x_{22}(m)x_{23}(bmv^{-1}+y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=0, z, y=u^2w^2/v^2/s(p^2+p+\eta)}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(dbnv^{-2}+x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, z=0, x=u^2w^2/v^2/t(p^2+p+\eta)}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}((n^2 + n)ux^2t(uw^2 + v^2t)^{-1} + x') \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, s, s=0, t \neq 0, r=0, y, x \neq 0, x'=ux^2t(uw^2+v^2t)^{-1} \eta \in \mathbb{F}_q, v^2t \neq uw^2}$ with parameter set of size $(q-1)^5q$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(bhu^{-1} + z)x_{20}(i)x_{21}(j)x_{22}(biu^{-1} + y)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, y, x \neq 0, z' \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, v^2z' \neq ux, v^2y+vy'x=(o^2+o+1+\eta)uw^2x}$ for some $o \in \mathbb{F}_q$

of size $(q-1)^4q^2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bj u^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r=0, y, x \neq 0, z' \neq 0, y'=0 \text{ or } y'=v^{-1}uw^2 \eta \in \mathbb{F}_q, uvvsy'+vs(vy'z'+uy)+usxy'+uwxz' \neq 0, let n \in \mathbb{F}_q^\times s.t. uvvsy'+vs(vy'z'+uy)+usxy'+uwxz'=1/(n^3us^2)+(w(uvvs+ux)/(vs)+vy'+v^2(z')^2/(us^2))/n, n^2us^2(w(uvvs+ux)/(vs)+vy'+v^2(z')^2/(us^2))=(o^2+o+\eta)}$ for some $o \in \mathbb{F}_q$

with parameter set of size $2(q-1)^4q((1-\alpha)((q-1)\beta_4 + \beta_{10}) + \alpha(q-1)\beta_5)$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bj u^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z'=0 \in \mathbb{F}_q, 1=o^2+o+\eta}$ for some $o \in \mathbb{F}_q, o^3v^4u^2s^2y=1$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bj u^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$

$\mathbb{F}_q\}$ $u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z' \neq 0 \in \mathbb{F}_q$, let $n \in \mathbb{F}_q^\times$ s.t. $u^2 s^2 y = v^2 (z')^3 ((n^{-1} v^{-2} s (z')^{-1})^3 + n^{-1} v^{-2} s (z')^{-1})$, $n^3 v^4 u^2 s^2 y = o^2 + o + \eta$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4 q((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}(e)x_{15}(du+s)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(de+a^2gu^{-2}+y)x_{23}((l^2+l)v s^2u^{-2}+y')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t, w \neq 0, s \neq 0, y, y' = v s^2 u^{-2} \eta \in \mathbb{F}_q, v w \neq t^2}$ with parameter set of size $(q-1)^5 q$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+t)x_{11}(c)x_{12}(d)x_{13}(a^2u^{-2}v+w)x_{14}((e^2+e)ut^2v^{-1}+z')x_{15}(du+s)x_{16}(u^{-1}vd+x)x_{17}(f)x_{18}(g)x_{19}(afu^{-1}+z)x_{20}(h)x_{21}(i)x_{22}(de+a^2hu^{-2}+y)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, w=0, z, y, x \neq 0, z'=0 \text{ or } z'=ut^2v^{-1}\eta \in \mathbb{F}_q, u^2xy+uz'sx=t^2s^2(o^2+o+\eta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5 q^2$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w+t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w+z)x_{17}(h)x_{18}(fu+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2}+y)x_{23}(l)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, z=v/u\sqrt{xw}, y \neq zx/w, x \in \mathbb{F}_q, 1=p+p^2+\eta}$ for some $p \in \mathbb{F}_q$,
 $p^3 u^4 v^2 w(wy+zx)=1$ for some $p \in \mathbb{F}_q$

with parameter set of size $(q-1)^3 q(1-\alpha)\beta_{10}$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w+t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w+z)x_{17}(h)x_{18}(fu+x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2}+y)x_{23}(l)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, z \neq v/u\sqrt{xw}, y \neq zx/w, x \in \mathbb{F}_q, \text{let } o \in \mathbb{F}_q^\times \text{ s.t. } u^2v^4w^2(v^2wx+(zu)^2)^{-3}(wy+zx)^2 = (o^2u^2(v^2wx+(zu)^2))^{-1} + (o^2u^2(v^2wx+(zu)^2))^{-3}, o^3u^4v^2w(wy+zx)=p+p^2+\eta}$ for some $p \in \mathbb{F}_q$

with parameter set of size $(q-1)^4 q((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuw^{-1}+s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(fu+y)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2+o)v^2y^2u^{-2}w^{-1}+z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=s=0, z=v^2y^2u^{-2}w^{-1}\eta, y \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{14}(u^{-1}h+x)x_{15}(f)x_{16}(cu+w)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2}+t)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, x \neq 0 \in \mathbb{F}_q, t=v^2x^2w^{-1}(o^2+o+\eta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4 q/2$,

$\{x_1(u)x_5(a)x_8(b)x_9(v)x_{11}(c)x_{12}(d)x_{14}(e)x_{15}(abu+ud+x)x_{16}(w)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}((k^2+k)u^{-1}x^2v^2w^{-1}+y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, x \neq 0, y=u^{-1}x^2v^2w^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_8(b)x_9(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(a^2fv^{-1}+t)x_{16}(f)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}((m^2+m)w^2v^{-1}t^2u^{-2}+y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, y=w^2v^{-1}t^2u^{-2}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r, z \neq 0, y, x=y^2u^{-2}v \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$, $m^3v^2u^2zs=1$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3 q(1-\alpha)\beta_{10}$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}(g)x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0=r, z \neq 0, y, x \neq y^2v^{-2} \in \mathbb{F}_q}$ let $n \in \mathbb{F}_q^\times$ s.t. $vu^4s^2z^2(vy^2+u^2x)^{-3}=(n^2v(vy^2+u^2x))^{-1}+(n^2v(vy^2+u^2x))^{-3}$, with parameter set of size $(q-1)^4 q((1-\alpha)\beta_4 + \alpha\beta_5)$,
 $n^3v^2u^2zs=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0=s, t \neq 0, r=0, z, y \neq u\sqrt{x/v}, x \neq 0, z', y'=0 \text{ or } y'=vu^{-2}(t+v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5 q^2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(dv^{-1}u+y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1}+s)x_{16}((g^2+g)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(fbu^{-1}+z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1}+x)x_{21}(j)x_{22}(k)x_{23}(bku^{-1}+z')x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq v^{-1}u^2w, r=0, z \neq 0, y, x, z', y'=0 \text{ or } y'=vu^{-2}(t+v^{-1}u^2w)^2 \eta \in \mathbb{F}_q}$ let $n \in \mathbb{F}_q^\times$ s.t. $(v(ts+y^2)+n/t)^2+vs^2(vt^2((v^{-1}u^2t^{-1}s^{-1}x)^2+v^{-1}u^2t^{-1}s^{-1}x)+u^2y') \neq 0$,
 $n^3+n(t^2(v^2(ts+y^2)^2+u^4x^2+vu^2tsx+vu^2s^2y'))=vu^2t^3s(u^2tx^2+v(ts+y^2)(sy'+tx)+u^2sz^2)$,
 $((u^2tx+vt(ts+y^2))^2+vu^2t^2s(sy'+tx))/n^2=m^2+m+1+\eta}$ for some $m \in \mathbb{F}_q$

parameter set of size $2(q-1)^4 q((1-\alpha)(\beta_4(q-1)+\beta_{10})+\alpha\beta_5(q-1))$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(gbu^{-1}+z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1}+x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1}+z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq v^{-1}u^2w, t \neq 0, r=0, z, x \neq 0, z', u^4w^2(tx+z')^2+vu^2t^2z^2x=v^2t^4x^2(o^2+o+\eta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q-2)q^2/2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(gbu^{-1}+z)x_{18}(i)x_{19}(j)x_{20}(d^2v^{-1}+x)x_{21}(k)x_{22}(l)x_{23}(blu^{-1}+z')x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=v^{-1}u^2w, r=0, z, x \neq 0, z' \in \mathbb{F}_q, w(z')^2+tz^2x=wt^2x^2(o^2+o+1+\eta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4 q^2/2$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + w)x_{20}(k)x_{21}(l)x_{22}(b^2u^{-2}k + x)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter $\left. \begin{array}{l} u, v \in \mathbb{F}_q^\times, w, s \neq 0, t, y = 0, x \neq 0 \in \mathbb{F}_q, \\ w^2 = (p^2 + p + \eta)u^2s^{-1} \text{ for some } p \in \mathbb{F}_q \end{array} \right\}$

set of size $(q-1)^4q/2$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(cju^{-1} + z)x_{22}(b^2u^{-2}m + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + gb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(au^{-1}v^2 + w)x_{16}(aeu^{-1} + s)x_{17}(afu^{-1} + y)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(aku^{-1} + gb^2u^{-2} + x)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{11}(c)x_{12}(bu^{-1}v + z)x_{13}(d)x_{14}(e)x_{15}(au^{-1}v^2 + w)x_{16}(adu^{-1} + s)x_{17}(aeu^{-1} + y)x_{18}(f)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(aju^{-1} + x)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ahx^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ahx^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}((l^2 + l)v^{-1}wz + y)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(w)x_{16}(aeu^{-1} + s)x_{17}(g)x_{18}(h)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}((l^2 + l)w^2sv^{-2} + t)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_9(b)x_{10}(v)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2v^{-1} + w)x_{16}(g)x_{17}(afu^{-1} + t)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + aiu^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(u)x_6(a)x_9(b)x_{11}(c)x_{12}(v)x_{13}(d)x_{14}(e)x_{15}(w)x_{16}((b^2 + ad)u^{-1} + s)x_{17}((f^2 + f)v^{-1}ws + t)x_{18}(g)x_{19}(h)x_{20}(i)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(ad + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)u^{-2}t^2r + z)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}((p^2 + p)u^{-2}w^2t + x) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^4$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2((1-\alpha)((q-1)\beta_4 + \beta_{10}) + \alpha\beta_5(q-1))$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q^2/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^3/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^3/2$,

$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q(q-2)((1-\alpha)(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1))$,
 $\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q((1-\alpha)(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1))$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(ac^2u^{-2} + r)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}((o^2 + o)v^{-2}tr^2 + z) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(d)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3(q + (q-1)^2)((1-\alpha)(\beta_4 * (q-1) + \beta_{10}) + \alpha\beta_5(q-1))$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q^2/2 + q(q-2)/2 + q^2/2 + (q-1)(q-2)q/2)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q^2/2 + q(q-2)/2 + q^2/2 + (q-1)(q-2)q/2)$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^3/2$,
 $\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q$,
 $\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $2(q-1)^5q((1-\alpha)\beta_4 + \alpha\beta_5)$,
 $\{x_5(w)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w +$

$r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s, r \neq 0 \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q, m^3w^4u^2v^2r=1$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,
 $\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s, r \neq 0 \in \mathbb{F}_q, \text{let } n \in \mathbb{F}_q^\times \text{ s.t. } w^2u^{-2}vt^{-3}r^2=(n^2w^2u^2vt)^{-1}+(n^2w^2u^2vt)^{-3}, n^3w^4u^2v^2r=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,
 $\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u+acv^{-1}+\sqrt{ev^{-1}w+s})x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1}+t)x_{19}(h)x_{20}(efv^{-1}+r) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0, t, r \neq 0 \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q, m^3u^4w^2v^2r=1$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,
 $\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u+acv^{-1}+\sqrt{ev^{-1}w+s})x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1}+t)x_{19}(h)x_{20}(efv^{-1}+r) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s \neq 0, t, r \neq 0 \in \mathbb{F}_q, \text{let } n \in \mathbb{F}_q^\times \text{ s.t. } u^2w^4/v^2/s^6r^2=(n^2u^2v^2s^2)^{-1}+(n^2u^2v^2s^2)^{-3}, n^3u^4w^2v^2r=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{14}(d)x_{15}(au + t)x_{16}(s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)v^2t^2u^{-2}s^{-1} + z)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z=v^2t^2u^{-2}s^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_5(u)x_8(a)x_9(v)x_{11}(b)x_{12}(c)x_{13}(w)x_{14}(d)x_{15}(au + t)x_{16}(au^{-1}w + s)x_{17}(e)x_{18}(f)x_{19}(g)x_{20}((h^2 + h)u^{-2}v^2t^2s^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, y=u^{-2}v^2t^2s^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, r=0, o \neq 0, s \in \mathbb{F}_q, wo \neq t^2, u^2so=(m^2+m+\eta)v^2(wo+t^2)}$ with parameter set of size $(q-1)^4(q-2)q/2$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, r \neq 0, z, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q, zu\sqrt{wo}/(v(wo+t^2))=(m^2+m+\eta)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, o, s \in \mathbb{F}_q \text{ let } n \in \mathbb{F}_q^\times \text{ s.t. } (vy)^2uw+u^2wx(wy+sx)+(uw^2x+(wy+sx)v^2)t^2 \neq 0, (vy)^2uw+u^2wx(wy+sx)+(uw^2x+(wy+sx)v^2)t^2v^2wx=w((uwx+v^2y)^2+(wy+sx)uv^2x+uv^2t^2x)n+n^3, ((uwx+v^2y)^2+(wy+sx)uv^2x+uv^2t^2x)/n^2=m^2+m+\eta+1}$ for some $m \in \mathbb{F}_q$
 $(q-1)^5((1-\alpha)(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1))$,
 $\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, r \neq 0, o, z, s=0 \text{ or } s=vw^2u^{-2}\eta \in \mathbb{F}_q, wvr^2s \neq uv^2r^2o+u^2v^2o^2+u^2z^2r \text{ let } n \in \mathbb{F}_q^\times \text{ s.t. } n^2w^2(w^2v^2r^2+u^2(wvr^2o+u^2o^2+wr^2s))+n^3w^4u^2r(wv^2r^2o+u^2v^2o^2+u^2z^2r+wvr^2s)=1, n^3w^4u^2r(wv^2r^2o+u^2v^2o^2+u^2z^2r+wvr^2s)=m^2+m+\eta}$ for some $m \in \mathbb{F}_q$
 $\alpha(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1)$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q, m^3u^4s^2v^2r=1$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^3q(1-\alpha)\beta_{10}$,
 $\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q \text{ let } n \in \mathbb{F}_q^\times \text{ s.t. } u^2s^{-2}vw^{-3}r^2=(n^2u^2s^2vw)^{-1}+(n^2u^2s^2vw)^{-3}, n^3u^4s^2v^2r=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$
parameter set of size $(q-1)^4q((1-\alpha)\beta_4 + \alpha\beta_5)$,
 $\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h) \mid a, b, c, d, e, f, g, h \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t, s \neq t\sqrt{r/u}, r \in \mathbb{F}_q \text{ let } n \in \mathbb{F}_q^\times \text{ s.t. } n^3u^3v^4w^2(t^2r+us^2)+n^2u^3v^2w^2r=1, n^3u^3v^4w^2(t^2r+us^2)=m+m^2+\eta}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^3q((1-\alpha)(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1))$,
 $\{x_6(u)x_8(v)x_9(a)x_{11}(b)x_{12}(au^{-1}v + s)x_{13}(a^2u^{-1} + t)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}((f^2 + f)(v^2t + us^2)^{-1}uw^2t + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s \neq v\sqrt{t/u}, y=u(v^2t+us^2)^{-2}w^2t\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5$,
 $\{x_6(u)x_9(a)x_{10}(v)x_{11}(b)x_{12}(c)x_{13}(d)x_{14}(abu^{-1} + cdv^{-1} + w)x_{15}(c^2v^{-1} + t)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-2} + c^2dv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w, t \neq 0, s \neq 0 \in \mathbb{F}_q, w^2=(m^2+m+\eta)us^2t^{-1}}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^4q/2$,
 $\{x_6(u)x_9(a)x_{11}(b)x_{12}(w)x_{13}(a^2u^{-1}+t)x_{14}(abu^{-1}+(l^2+l)w^{-1}vt+z)x_{15}(v)x_{16}(c)x_{17}(d)x_{18}(e)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, z=w^{-1}vt\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(lk) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, r=0, o \neq sr/v, s=t^2v^2/u^2/o(m^2+m+\eta)}$ for some $m \in \mathbb{F}_q$
with parameter set of size $(q-1)^4q/2$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(lk) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, s=0, o \neq sr/v, r=u^2o^2/t^2/v(m^2+m+\eta)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(lk) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, r \neq 0, o \neq sr v^{-1} \in \mathbb{F}_q, \text{ let } n \in \mathbb{F}_q^\times \text{ solve } 1 \neq nvt, n^3 u^2 ts(sr+vo) + n^2(v^2 t^2 + u^2 so) = 1, n^3 u^2 ts(sr+vo) = m^2 + m + \eta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4((1-\alpha)(\beta_4(q-1) + \beta_{10}) + \alpha\beta_5(q-1) - (q/2))$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s=0 \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q, m^3 u^4 t^2 v^2 w = 1$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3(1-\alpha)\beta_{10}$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq 0 \in \mathbb{F}_q, \text{ let } n \in \mathbb{F}_q^\times \text{ s.t. } u^2 t^{-2} v s^{-3} w^2 = (n^2 u^2 t^2 v s)^{-1} + (n^2 u^2 t^2 v s)^{-3}, n^3 u^4 t^2 v^2 w = m + m^2 + \eta}$ for some $m \in \mathbb{F}_q$ size $(q-1)^4((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q, 1=m+m^2+\eta}$ for some $m \in \mathbb{F}_q, m^3 v w = u s$ for some m with parameter set of size $(q-1)^4(1-\alpha)\beta_{10}$,

$\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0 \in \mathbb{F}_q, \text{ let } l \in \mathbb{F}_q^\times \text{ s.t. } u^2/v^2 w/t^3 s^2 = (l^2 u^2 v^2 w t)^{-1} + (l^2 u^2 v^2 w t)^{-3} l^3 u^4 v^2 w^2 s = m + m^2 + \eta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4((1-\alpha)\beta_4 + \alpha\beta_5)$,

$\{x_8(u)x_9(t)x_{11}(a)x_{12}(b)x_{13}(v)x_{14}(c)x_{15}(w)x_{16}(d)x_{17}(e)x_{18}(c^2v^{-1} + s)x_{19}(f)x_{20}(g)x_{21}(h)x_{22}(i)x_{23}(j)x_{24}(k) \mid a, b, c, d, e, f, g, h, i, j, k \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s=t^2 w^2 u^{-2} v^{-1} (m^2+m+\eta)}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

Y19 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, r \neq 0, z=0, y \neq 0, x \in \mathbb{F}_q, v^2 u^{-2} w r^{-3} y^2 = m^3 + m + \zeta}$ for some $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=r=z=0, y \neq 0, x \in \mathbb{F}_q, v^4 u^2 w^2 y m^3 \neq 1}$ for any $m \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bju^{-1} + x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2} + z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, z=vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, (n')^3 w^2 v^4 u^2 (vuyz+w^2 x^2) \neq 1}$ for any $n' \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(c^2u^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2} + z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=t=0, z \neq vy^2/w^2, y, x \neq \sqrt{vuyz}/w, r=0 \in \mathbb{F}_q, v^{-1} u^{-2} w^4 (vuyz+w^2 x^2)^2 (vy^2+w^2 z)^{-1} = (p^3+p+\zeta)}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z' \neq 0 \in \mathbb{F}_q, u^2 s^2 y = v^2 (z')^3 (o^3 + o + \zeta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=r=0, y \neq 0, x, z'=0 \in \mathbb{F}_q, 1 \neq v^3 u^2 s^2 y}$ for any $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = uw^2/v^2, r=0, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2 \in \mathbb{F}_q, uvv'sy' + vs(vy'z' + uy) + usxy' + uwxz' \neq 0, w(uvvs + ux)/s = v^2 y' + v^3 (z')^2 / (us^2), 1 \neq n^3 us^2 (uvv'sy' + vs(vy'z' + uy) + usxy' + uwxz')}^2 = w(uvvs + ux)/s + v^2 y' + v^3 (z')^2 / (us^2)}$ for any $n \in \mathbb{F}_q^\times$ with parameter set of size $2(q-1)^4q(q-1-\beta_{10})$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(fuv^{-1} + z')x_{17}(h)x_{18}(agu^{-1} + fv + x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(bju^{-1} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = uw^2/v^2, r=0, y, x, z', y'=0 \text{ or } y'=v^{-1}uw^2 \in \mathbb{F}_q, uvv'sy' + vs(vy'z' + uy) + usxy' + uwxz' \neq 0, w(uvvs + ux)/s \neq v^2 y' + v^3 (z')^2 / (us^2), (uvv'sy' + vs(vy'z' + uy) + usxy' + uwxz')^2 = w(uvvs + ux)/s + v^2 y' + v^3 (z')^2 / (us^2)}$ for any $n \in \mathbb{F}_q^\times$ with parameter set of size $2(q-1)^5q(q-1-\beta_4-\beta_5)$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu +$

$$x)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v, w \in \mathbb{F}_q^\times, t=0, z \neq v/u\sqrt{xw}, y \neq zx/w, x \in \mathbb{F}_q, \\ u^2v^4w^2(v^2wx+(zu)^2)^{-3}(wy+zx)^2=p^3+p+\zeta \\ \text{for some } p \in \mathbb{F}_q \end{array} \right\}$$

with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(gu^{-2}w + z)x_{17}(h)x_{18}(fu + y)x_{19}(i)x_{20}(j)x_{21}(k)x_{22}(a^2ju^{-2} + y)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v, w \in \mathbb{F}_q^\times, t=0, z=v/u\sqrt{xw}, y \neq zx/w, x \in \mathbb{F}_q, \\ u^4v^2w(wy+zx)p^3 \neq 1 \text{ for any } p \in \mathbb{F}_q \end{array} \right\}$$

with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}(g)x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0, r, z \neq 0, y, x \neq y^2vu^{-2} \in \mathbb{F}_q, vu^4s^2z^2(vy^2+u^2x)^{-3}=m+m^3+\zeta \text{ for some } m \in \mathbb{F}_q \\ (q-1)^4q(q-1-\beta_4-\beta_5), \end{array} \right\}$$

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t=0, r, z \neq 0, y, x=y^2vu^{-2} \in \mathbb{F}_q, v^2u^2szm^3 \neq 1 \text{ for any } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^3q(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z \neq 0, y, x, y'=0 \text{ or } y'=vu^{-2}t^2 \in \mathbb{F}_q, v^2(ts+y^2)^2+vs^2(vt^2((v^{-1}u^2t^{-1}s^{-1}x)^2+v^{-1}u^2t^{-1}s^{-1}x)+u^2y') \neq 0, \\ \text{with parameter set of size } (q-1)^5q(q-1-\beta_4-\beta_5), \\ \text{parameter set of size } (q-1)^5q(q-1-\beta_4-\beta_5), \end{array} \right\}$$

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(dv^{-1}u + y)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + s)x_{16}((g^2 + g)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(fbu^{-1} + z)x_{18}(h)x_{19}(i)x_{20}(d^2v^{-1} + x)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r=0, z \neq 0, y, x, y'=0 \text{ or } y'=vu^{-2}t^2 \in \mathbb{F}_q, v^2(ts+y^2)^2=vs^2(vt^2((v^{-1}u^2t^{-1}s^{-1}x)^2+v^{-1}u^2t^{-1}s^{-1}x)+u^2y'), \\ \text{with parameter set of size } (q-1)^4q(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ah^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v, w \in \mathbb{F}_q^\times, t=0, x \neq v\sqrt{z/u}, y \neq \sqrt{uxzw}/v, z \in \mathbb{F}_q, u^{-1}v^{-2}w^4(uwxz+v^2y^2)^2(ux^2+v^2z)^{-3}=o^3+o+\zeta \text{ for some } o \in \mathbb{F}_q \\ (q-1)^4q(q-1-\beta_4-\beta_5), \end{array} \right\}$$

$$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(d^2w^{-1} + x)x_{16}(g)x_{17}(afu^{-1} + y)x_{18}(h)x_{19}(i)x_{20}(c^2u^{-1} + ah^{-1} + z)x_{21}(j)x_{22}(k)x_{23}(l)x_{24}(m) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q \left. \begin{array}{l} u, v, w \in \mathbb{F}_q^\times, t=0, x=v\sqrt{z/u}, y \neq \sqrt{uxzw}/v, z \in \mathbb{F}_q, u^4v^2w^2(uwxz+v^2y^2)o^3 \neq 1 \text{ for any } o \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^3q(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s, t, r \neq 0, o, y \neq v^2s^2/(u^2r) \in \mathbb{F}_q, sy \neq rw, u^4v^2r^2s(wr+sy)^2/(v^2s^2+u^2ry)^3=m+m^3+\zeta \text{ for some } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^5q^2(q-1-\beta_4-\beta_5) + q^2(q-1)^4(1+q)/3, \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(cu + s)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ae + w)x_{19}(h)x_{20}(abeu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v \neq 0, w \neq 0, s, t, r \neq 0, o, y=v^2s^2/(u^2r) \in \mathbb{F}_q, sy \neq rw, rm^3 \neq (u^2v(sy+rw))^2 \text{ for any } m \in \mathbb{F}_q^\times \\ \text{with parameter set of size } (q-1)^4q^2(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, r \neq 0, o \neq 0, y=t^2z^2/(u^2r), z \neq 0 \in \mathbb{F}_q, ty \neq or, 1 \neq m^3u^2z^2r(ty+or) \text{ for any } m \in \mathbb{F}_q \\ (q-1)(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(g)x_{18}(ad + w)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v=0, w, t \neq 0, r \neq 0, o \neq 0, y, z \neq 0 \in \mathbb{F}_q, t^2z^2 \neq u^2ry, ty \neq or, (u^2rz(ty+or))^2/(u^2ry+t^2z^2)^3=(m+m^3+\zeta) \text{ for some } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^4q(q-2)(q-1-\beta_4-\beta_5) + q(q-1)^4(1+q)/3, \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v=0, s, t=0, r \neq 0, o \neq 0, y \neq 0, z \neq 0 \in \mathbb{F}_q, ur(zo)^2/(uy)^3=m+m^3+\zeta \text{ for some } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^4q(q-1-\beta_4-\beta_5), \end{array} \right\}$$

$$\{x_3(u)x_5(a)x_7(b)x_8(abu^{-1} + v)x_9(c)x_{10}(bu + t)x_{11}(acu^{-1} + z)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(a^2bu^{-1} + o)x_{16}(cb + cu^{-1}t + r)x_{17}(f)x_{18}(g)x_{19}(h)x_{20}(abdu^{-1} + y)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v=0, s, t=0, r \neq 0, o \neq 0, y=0, z \neq 0 \in \mathbb{F}_q, u^2zor^2m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^3q(q-1-\beta_{10}), \end{array} \right\}$$

$$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q \left. \begin{array}{l} u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r, o, x \neq \sqrt{to}, y \neq 0 \in \mathbb{F}_q, uty^2v^2 \neq (v^2o+ur^2)^2, (uy^2v^2)^2(v^2x^2+tur^2)^2=(uty^2v^2+(v^2o+ur^2)^2)^3(m^3+m+\zeta) \text{ for some } m \in \mathbb{F}_q \\ \text{with parameter set of size } (q-1)^4(q+(q-1)^2)(q-1-\beta_4-\beta_5), \end{array} \right\}$$

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(abcu^{-2} + r)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v \neq 0, w=0, t, r, o, x \neq \sqrt{t}, y \neq 0 \in \mathbb{F}_q, v^2x^2 + tur^2 \neq 0, uty^2v^2 = (v^2o + ur^2)^2, (u^2yv)^2(v^2x^2 + tur^2)m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3(q+(q-1)^2)(q-1-\beta_{10})$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with

$u \in \mathbb{F}_q^\times, v, w \neq 0, s, t, o, x \neq \sqrt{t}, y \in \mathbb{F}_q, v^2y \neq s^2w, (u^2(to+x^2)((ty+wo)(ts^2+v^2o+uwy)+(to+x^2)(v^2y+s^2w)))/(u^2((tw+oy)^2+(to+x^2)((ts^2+v^2o)/u+wy)))^3 = m^3 + m + \zeta$ for some $m \in \mathbb{F}_q$ parameter set of size $(q-1)^4q(q+(q-1)^2)(q-1-\beta_4-\beta_5) + (q+1)(q-1)^4(q^2-1)/3$,

$\{x_4(u)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + v)x_{10}(b^2u^{-1} + t)x_{11}(acu^{-1} + s)x_{12}(bcu^{-1} + x)x_{13}(ab^2u^{-1} + w)x_{14}(d)x_{15}(c^2u^{-1} + o)x_{16}(e)x_{17}(f)x_{18}(ac^2u^{-2} + y)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u \in \mathbb{F}_q^\times, v, w \neq 0, s, t, o, x \neq \sqrt{t}, y \in \mathbb{F}_q, v^2y \neq s^2w, (tw+oy)^2 = (to+x^2)((ts^2+v^2o)/u+wy), m^3u^2(to+x^2)((ty+wo)(ts^2+v^2o+uwy)+(to+x^2)(v^2y+s^2w)) \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q(q+(q-1)^2)(q-1-\beta_{10})$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq u/v\sqrt{sr}, s \neq 0, r, o, y=0 \text{ or } y=v^2u^2w^{-1} \in \mathbb{F}_q, v^2u^4s^2(so+tr)^2/(u^2sr+v^2t^2)^3 = (m^3+m+\zeta) \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^5q(q-1-\beta_4-\beta_5)$,

$\{x_5(v)x_7(u)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(bu^{-1}w + s)x_{14}(e)x_{15}((f^2 + f)v^2u^2w^{-1} + y)x_{16}(cv^{-1}w + t)x_{17}(g)x_{18}(a^2bu^{-3} + r)x_{19}(h)x_{20}(cfv^{-1} + ac^2u^{-1}v^{-2}w^2 + o)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t = u/v\sqrt{sr}, s \neq 0, r, o \neq tr/s, y=0 \text{ or } y=v^2u^2w^{-1} \in \mathbb{F}_q, v^4u^2s(so+tr)m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4q(q-1-\beta_{10})$,

$\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s, r \neq 0 \in \mathbb{F}_q, w^2u^{-2}vt^{-3}r^2 = m^3 + m + \zeta \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_5(w)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(d)x_{13}(v)x_{14}(e)x_{15}(fv^{-1}w^2 + s)x_{16}(f)x_{17}(g)x_{18}(cw + a^2u^{-2}v + t)x_{19}(h)x_{20}(cfv^{-1}w + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s, r \neq 0 \in \mathbb{F}_q, w^4u^2v^2rm^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u + acv^{-1} + \sqrt{ev^{-1}}w + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t, r \neq 0 \in \mathbb{F}_q, uw^2v^{-1}s^{-3}r = m^3 + m + \zeta \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_5(u)x_8(a)x_9(w)x_{10}(v)x_{11}(fv^{-1}u + acv^{-1} + \sqrt{ev^{-1}}w + s)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(e)x_{16}(f)x_{17}(g)x_{18}(cev^{-1} + t)x_{19}(h)x_{20}(efv^{-1} + r)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0, t, r \neq 0 \in \mathbb{F}_q, u^4w^2v^2rm^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, o, s \in \mathbb{F}_q, wu^2sr^2 + w^2v^2r^2 + u^4vt^2r + wu^2vro + u^6o^2 \neq 0, u^2(w^2(u^2t^2 + w^2vr)(wu^2sr^2 + w^2v^2r^2 + u^4vt^2r + wu^2vro + u^6o^2) + u^2(u^2t^2 + w^2vr)^3 + w^2u^6(uto)^2 + u^8t^6)^2 / (w^6(wu^2sr^2 + w^2v^2r^2 + u^4vt^2r + wu^2vro + u^6o^2)^3) = m^3 + m + \zeta \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^6(q-1-\beta_4-\beta_5)$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((u^2a^2w^{-2}v^{-2} + uaw^{-1}v^{-1})wv^2u^{-2} + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, r \neq 0, o, s \in \mathbb{F}_q, wu^2sr^2 + w^2v^2r^2 + u^4vt^2r + wu^2vro + u^6o^2 = 0, u^2o^2wv + w^2vr(vo+sr) + (wv^2r + (vo+sr)u^2)t^2 \neq 0, u^2vr(u^2o^2wv + w^2vr(vo+sr) + (wv^2r + (vo+sr)u^2)t^2)m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5(q-1-\beta_{10})$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, r \neq 0, o, z, s=0 \text{ or } s=wv^2u^{-2} \in \mathbb{F}_q, wvr^2s \neq wv^2ro + u^2vo^2 + u^2z^2r, (wvr^2s + wv^2ro + u^2vo^2 + u^2z^2r)u^2 \neq (w^2v^3r + u^4z^2)r, w^2u^4v^3r^2(wvr^2s + wv^2ro + u^2vo^2 + u^2z^2r)^2 / ((wvr^2s + wv^2ro + u^2vo^2 + u^2z^2r)u^2 + (w^2v^3r + u^4z^2)r)^{-3} = m^3 + m + \zeta \text{ for some } m \in \mathbb{F}_q}$ parameter set of size $2(q-1)^5(q-1-\beta_4-\beta_5)$,

$\{x_6(w)x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}((h^2 + h)wv^2u^{-2} + s)x_{14}(bcu^{-1} + z)x_{15}(a^2u^{-2}v + r)x_{16}(d)x_{17}(e)x_{18}(a^2bu^{-3}v + o)x_{19}(f)x_{20}(g)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, r \neq 0, o, z, s=0 \text{ or } s=wv^2u^{-2} \in \mathbb{F}_q, wvr^2s \neq wv^2ro + u^2vo^2 + u^2z^2r, u^4z^2 \neq w^2v^3r, 1 = u^2o / (wvr) + (u^2o / (wvr))^2 + u^2s / (wv^2)},$ with parameter set of size $2(q-1)^4(q-1-\beta_{10})$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$

$\mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q, u^2/s^2v/w^3r^2 = m^3 + m + \zeta \text{ for some } m \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,

$\{x_6(v)x_7(u)x_8(a)x_9(b)x_{11}(c)x_{12}(cv^{-1}u + t)x_{13}(d)x_{14}(bcv^{-1} + s)x_{15}(r)x_{16}(e)x_{17}(f)x_{18}(c^2v^{-1} + w)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, t, s \neq 0, r \neq 0 \in \mathbb{F}_q, u^4s^2v^2rm^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4(q-1-\beta_{10})$,

$\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s \neq t\sqrt{r/u}, r \neq 0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q(q-1-\beta_4-\beta_5)$,
 $v^2(t^2r+us^2)^2/(u^3w^2r^3)=m^3+m+\zeta$ for some $m \in \mathbb{F}_q$

$\{x_6(u)x_8(v)x_9(bv^{-1}u + t)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(abv^{-1} + s)x_{15}(d)x_{16}(e)x_{17}(f)x_{18}(dfw^{-1} + r)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t, s \neq 0, r=0 \in \mathbb{F}_q, v^4u^4w^2s^2m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3q(q-1-\beta_{10})$,

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s, r \neq 0, o \neq sr v^{-1} \in \mathbb{F}_q}$
with parameter set of size $(q-1)^5(q-1-\beta_4-\beta_5)$,
 $u^2t^4r^2(sr+vo)^2(u^2o^2+vt^2r)^{-3}=m^3+m+\zeta$ for some $m \in \mathbb{F}_q$

$\{x_7(u)x_8(a)x_9(b)x_{10}(v)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(bu^{-1}v + s)x_{14}(d)x_{15}(a^2u^{-2}v + r)x_{16}(e)x_{17}(f)x_{18}(a^2bu^{-3}v + o)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, r \neq 0, o \neq sr/v \in \mathbb{F}_q, v^2t^2=u^2s_0}$
with parameter set of size $(q-1)^4(q-1-\beta_{10})$,
 $u^2ts(sr+vo)m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q$

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq 0 \in \mathbb{F}_q, u^2t^{-2}vs^{-3}w^2=m^3+m+\zeta \text{ for some } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4(q-1-\beta_4-\beta_5)$,

$\{x_7(u)x_8(a)x_9(b)x_{11}(abu^{-1} + t)x_{12}(c)x_{13}(v)x_{14}(d)x_{15}(w)x_{16}(e)x_{17}(f)x_{18}(a^2u^{-2}v + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, t \neq 0, s \neq 0 \in \mathbb{F}_q, tw/(uv)m^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3(q-1-\beta_{10})$,

$\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0 \in \mathbb{F}_q, u^2v^{-2}wt^{-3}s^2=m^3+m+\zeta \text{ for some } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4(q-1-\beta_4-\beta_5)$,

$\{x_8(u)x_9(v)x_{10}(w)x_{11}(a)x_{12}(b)x_{13}(c)x_{14}(d)x_{15}(a^2v^{-2}w + t)x_{16}(e)x_{17}(f)x_{18}(a^2cv^{-2} + s)x_{19}(g)x_{20}(h)x_{21}(i)x_{22}(j)x_{23}(k)x_{24}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s \neq 0 \in \mathbb{F}_q, u^4v^2w^2sm^3 \neq 1 \text{ for any } m \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3(q-1-\beta_{10})$,

y20 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0, t \neq 0, r, z=0 \in \mathbb{F}_q, u^2r=(y^2+y)v^2wt^2 \text{ for some } y \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(c^2u^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(biu^{-1} + x)x_{20}(k)x_{21}(l)x_{22}(b^2ku^{-2} + z)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, z, y, x, r \neq 0 \in \mathbb{F}_q, v(ry+z)=w^2r^2(p^2+p) \text{ for some } p \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q^3/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}((h^2 + h)us^2 + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t=0, z, r=0, y=0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2 + h)u^2w + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s=0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=vr/w, t=0, r \neq 0, z, x, y'=0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q^2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=uvwr/(t(v^2+u^2)), t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, tx=(p^2+p)ur^2(uw^2+v^2t) \text{ for some } p \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q(q-2)/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s=t \neq 0, r \neq 0, x \in \mathbb{F}_q, x=v^2ur^2(p^2+p) \text{ for some } p \in \mathbb{F}_q}$
with parameter set of size $(q-1)^4q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=0, s, t=0, r \neq 0, x \in \mathbb{F}_q, x=(p^2+p)v^2r \text{ for some } p \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3q^2/2$,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, z=0 \in \mathbb{F}_q}$
with parameter set of size $(q-1)^3$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)^2u^2v^{-2}w^2t +$

$y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4$,

$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2+f)u^2v+s)x_{11}(g)x_{12}((f^2+f)uva+u^{-1}as+uw^{-1}g+y)x_{13}(h)x_{14}(i)x_{15}(ad+tx_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(v^{-1}wd+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+(h^2+h)uw^2+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}((h^2+h)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(gbu^{-1}+z)x_{18}(i)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(gbu^{-1}+z)x_{18}(i)x_{19}(j)x_{20}((k^2+k)v^{-1}u^2w^2t^{-1}s+x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^5q$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1}+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2u^{-2}m+x)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(bcu^{-1}+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(b^2u^{-2}l+x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3q^2/2$,

$\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v+t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)ut^2+z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(w)x_{16}((h^2+h)uv^2+t)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

$\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)uv^2w^{-1}s+z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4$,

$\{x_2(u)x_6(a)x_8(v)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)uv^2+w^2+t)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1}+w)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}((f^2+f)u^2v+x)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2+f)u^2v+cu+s)x_{11}(aeu^{-1}+z)x_{12}(g)x_{13}(ue+w)x_{14}(h)x_{15}(ad+tx_{16}(i)x_{17}(j)x_{18}(abdv^{-1}+r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3q^2$,

$\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+w)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+s)x_{14}(f)x_{15}((g^2+g)uv^2+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3q$,

$\{x_5(u)x_6(v)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2+j)u^2v+y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

Y21 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w+s)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1}+r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf+r)x_{14}(h)x_{15}(c^2u^{-1}+y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(bjw^{-1}+x)x_{20}(l)x_{21}(m)x_{22}(b^2lu^{-2}+z)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q^3/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf+r)x_{14}(g)x_{15}((h^2+h)us^2+y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2}+z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2+h)u^2w+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(biu^{-1}+z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q \}$ with parameter set of size $(q-1)^4q^2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+s)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+r)$

$r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)q/2$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q^2/2$,
 $\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuvv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uv^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(v^{-1}wd + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + (h^2 + h)uw^2 + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2 + y')x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(gbu^{-1} + z)x_{18}(i)x_{19}(j)x_{20}((k^2 + k)v^{-1}u^2w^2t^{-1}s + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q$,
 $\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2u^{-2}m + x)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(v)x_4(u)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(bcu^{-1} + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(b^2u^{-2}l + x)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q^2/2$,
 $\{x_2(u)x_5(v)x_6(a)x_8(au^{-1}v + t)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)ut^2 + z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(w)x_{16}((h^2 + h)uv^2 + t)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_2(u)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(b^2wv^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)uv^2w^{-1}s + z)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(u)x_6(a)x_8(v)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)uv^2 + t)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q^2$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,
 $\{x_5(u)x_6(v)x_8(a)x_9(b)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2v + y)x_{19}(k)x_{20}(l) \mid a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,

Y22 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^3/2$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}((i^2 + v)u^2w^{-1}u^2s + x)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g + t)x_{16}((i^2 + v)vw^2 + s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)u^2z + w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,
 $\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2 + m)u^2z + w)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)u^2w + s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q/(q/2-1)$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5q$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4q^2$,
 $\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(w)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2w + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4(q-2)$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + s)x_{16}((h^2 + h)w^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, w, v \in \mathbb{F}_q^\times, t=0=s, z, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, w, v \in \mathbb{F}_q^\times, t \neq 0, s=0, z \in \mathbb{F}_q, t^2z=uw^2v^4(p^2+p)}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, r=0, s=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q^2$,

$\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s=0, t, y=uv^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

Y23 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(v^2w^{-1}g + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}(bmu^{-1} + y)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=t=0, r, z \neq 0, y, x \in \mathbb{F}_q, r \neq (p^2+p)v^2w}$ for any $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^3/2$,

$\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}((i^2 + i)v^2w^{-1}u^2s + x)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t=0, r, z=svv^{-2}, x=v^2w^{-1}u^2s\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, y=0 \in \mathbb{F}_q, w^2t=us^2, w^2z=vu^2s^4(r^2+r+\eta)}$ for some $r \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g + t)x_{16}((i^2 + i)vw^2 + s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s=vw^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(du + s)x_{14}(g)x_{15}(h)x_{16}(bd + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)u^2z + w)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=u^2z\eta, s, t=0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3q$,

$\{x_1(v)x_3(u)x_5(a)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(bd + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2 + m)u^2z + w)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w=u^2z\eta, t \neq 0, z \neq 0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=u^2v(n), s=x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)u^2w + s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=u^2w(\eta), t=0=x \in \mathbb{F}_q}$ with parameter set of size $(q-1)^3$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, s \neq 0, t \neq 0, r=0, y, x \in \mathbb{F}_q, v^2t \neq uw^2, utx^2=s^2(uw^2+v^2t)^2(o^2+o+\eta)}$ for some $o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(bku^{-1} + y)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t=r=0, y, x, y'=v^{-1}uw^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q^2$,

$\{x_1(u)x_5(a)x_6(w)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(au^{-1}w + t)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x=u^2t\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0, s \neq 0, y=u^2s\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_7(v)x_8(b)x_9(c)x_{11}(d)x_{12}(e)x_{13}(w)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2w + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$

$a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_1(u)x_5(a)x_8(b)x_9(v)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2+g)u^2w+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}((h^2+h)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)v^{-1}u^2w^2t^{-1}s+x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q-2)$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)vr^2+x)x_{21}(m)x_{22}(m)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+r)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)vr^2+x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2+s)x_{16}((h^2+h)wv^2+y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1}+z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2+t(b^2+w^2a^2u^{-2})v^{-2}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1}+z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1}+w)x_9(c)x_{10}(bu+t)x_{11}(d)x_{12}(e)x_{13}((f^2+f)u^2v+\sqrt{v}w^{-1}t+x)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2+f)u^2v+cu+s)x_{11}(aeu^{-1}+z)x_{12}(g)x_{13}(ue+w)x_{14}(h)x_{15}(ad+t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1}+r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q^2$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+w)x_{10}(b^2u^{-1}+t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+s)x_{14}(f)x_{15}((g^2+g)uv^2+y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3q$,
 $\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2+j)w^2v+y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^3$,

y24 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w+s)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(gv^2w^{-1}+x)x_{16}(c^2v^{-2}w+z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1}+r)x_{21}(l)x_{22}(m)x_{23}((n^2+n)(sx+v^2z)^2/(v^2s)+y')x_{24}(o) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5(q^2/2)$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w+s)x_7(b)x_8(c)x_9(d)x_{10}(bu+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w+z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1}+r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(\beta_{33}+q/2-1)$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf+r)x_{14}(g)x_{15}((h^2+h)us^2+y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2}+z)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q/2$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w+s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1}+y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2}+z)x_{23}(o)x_{24}(p) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q/2-1)q/4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g+t)x_{16}((i^2+i)vv^2+s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2+o)u^{-2}vt^2+x)x_{23}(p)x_{24}(q) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v+t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2+h)u^2w+s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2+m)u^2v^{-1}t^2+x)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid$
 $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set

of size $(q-1)^4$,

$$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2+f)u^2v+tx_{11}(g)x_{12}(h)x_{13}(ue+s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl+ce+x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}((r^2+r)w^2x^2v^{-3}+y) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, s=0, x \neq 0, y=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu+tx_{11}(e)x_{12}(f)x_{13}((g^2+g)u^2w+sx_{14}(h)x_{15}(i)x_{16}(v^{-2}wi+x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}((q^2+q)v^2x^2w^{-1}+y) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0, t=0, x \neq 0, y=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, z, urx=(p^2+p)(vs)t, s^2t=(z^2+z)ur^2, y^2+y=(p^2+p)(1+z)^3v^3u^2r^3}$$
 for some $p, z, y \in \mathbb{F}_q$ with parameter set of size $(q-1)^3\beta_43$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t=0, r \neq 0, x, y'=0 \in \mathbb{F}_q, rx=(p^2+p)(vr+ws)^2}$$
 for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q/2$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t=uw^2/v^2, r \neq 0, x, y'=0 \in \mathbb{F}_q, x=v^2r(p^2+p)}$$
 for some $p \in \mathbb{F}$ with parameter set of size $(q-1)^4q/2$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, x=v^2r(p^2+p), (p^2+p)v^2t=(q^2+q)uw^2}$$
 for some $p, q \in \mathbb{F}$ with parameter set of size $(q-1)^4((q/2-1)(q/2-2)+q-2)$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=vr/w, t=uw^2/v^2, r \neq 0, z, x, y'=0 \in \mathbb{F}_q, let a \in \mathbb{F}_q}$$
 s.t. $1 = a^2 + ax = v^2r(p^2+p)$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/2\alpha$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}((e^2+e)v^{-1}uw^2+v^{-1}g+y')x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t=uw^2/v^2, r \neq 0, x, y'=0 \in \mathbb{F}_q, let a \in \mathbb{F}_q}$$
 s.t. $ws=(a^2+a)vr, x=v^2r(p^2+p)$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2(\alpha(q/2-2)+(1-\alpha)(q/2-1))$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=vr/w, t \neq 0, r \neq 0, z, x \in \mathbb{F}_q, v^2t \neq uw^2, let a \in \mathbb{F}_q}$$
 s.t. $a \neq 1, v^2t=(a^2+a)uw^2, tx=a^3uw^2r(q^2+q), a^3(a+1)^8(uw)^5(vr)^3+a^2+1+a^3(q^2+q)=(a+1)^3(p^2+p)}$ for some $p, q \in \mathbb{F}_q$ with parameter set of size $2(q-1)^3q/2\beta_36$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v+w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1}+sx_{10}(b^2u^{-1}+tx_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2}+rx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1}+fv+x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, let a \in \mathbb{F}_q}$$
 s.t. $avr \neq ws, (a+1)vr \neq ws, s^2t=(a^2+a)ur^2, tx=aruw^2((a+1)vr/(ws+1))^2(q^2+q), v^2s^6r^2(a+1)^2+vu^5r^2(a+1)^2((a+1)vr+ws)^3(avr+ws)^6+u^2s^4((a+1)vr+ws)^3(avr+ws)^2(q^2+q)=u^2s^4r(a+1)((a+1)vr+ws)(avr+ws)^4(p^2+p)}$ for some $p, q \in \mathbb{F}_q$ with parameter set of size β_39 ,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v+tx_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2+g)u^2w+z)x_{16}(h)x_{17}(i)x_{18}((j^2+j)u^2t^2v^{-1}+y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, z=0, y=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1}+sx_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2+j)u^2s+(j^2+j)^2u^2v^{-2}w^2t+y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \in \mathbb{F}_q, let x \in \mathbb{F}_q}$$
 s.t. $v^2s=tw^2(x^2+x), y=x^2u^2tv^{-1}w^2(z^2+z)}$ for some $z \in \mathbb{F}_q, x^2u^{-2}tv^{-2}w^2s^{-2}y=(z^2+z)}$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(\beta_30)$,

$$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2+f)u^2v+sx_{11}(g)x_{12}((f^2+f)uwa+u^{-1}as+uw^{-1}g+y)x_{13}(h)x_{14}(i)x_{15}(ad+t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}((p^2+p)wy^2+x)x_{23}(q)x_{24}(r) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, y \neq 0, s=x=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1}+tx_{11}(e)x_{12}(v^{-1}wd+y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1}+(h^2+h)uw^2+sx_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2+o)vy^2+x)x_{23}(p)x_{24}(q) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, y \neq 0, x=0, s=0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v+w)x_7(b)x_8(abu^{-1}+rx_{9}(c)x_{10}(bu+tx_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1}+s)x_{16}((h^2+h)vu^{-2}(t+v^{-1}u^2w)^2+y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2+l)vr^2+x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s=u^2r^2t^{-1}(z^2+z)}$$
 for some $z \in \mathbb{F}_q, t \neq 0, r \neq 0, x=0 \in \mathbb{F}_q, y'=0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = t = 0, r \neq 0, x = v^2u^{-2}w^{-2}r^2y', y' = 0 \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(u^2w^2/v(((hr)^2 + h(ws + vr^2))^2/(vvs)^2 + ((hr)^2 + h(ws + vr^2))/(vvs)) + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vu^{-2}w^{-2}r^2(u^2w + vt), s \neq 0, t = 0, r \neq 0, y' \in \mathbb{F}_q, vy' = (y^2 + y + \eta)u^2w^2, (1 + ws/(vr^2))^2(z^2 + z) = y^2 + y + \eta}$ for some z, y

parameter set of size $(q - 1)^4(q - 2)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(d^2v^{-1} + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, 0 = s, t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x = vr^2(o^2 + o)(1 + vt/(u^2w))^2}$ for some $o = ((o')^2 + o')/(vt/(u^2w) + v^2t^2/(u^2w^2)), o' \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4(q - 2)q/4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vu^{-2}w^{-2}r^2(u^2w + vt), s \neq 0, t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x \in \mathbb{F}_q, ts = u^2r^2(z^2 + z)}$ for some $z \in \mathbb{F}_q$, with parameter set of size $(q - 1)^4(q - 2)(q/2 - 2)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t = v^{-1}u^2w, r \neq 0, x \in \mathbb{F}_q, ws = vr^2(z^2 + z)}$ for some $z \in \mathbb{F}_q, x = vr^2(z^2 + z)^2 + (z^2 + z)ws}$ for some $z \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4(q/2 - 1)$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + s)x_{16}((h^2 + h)w^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t = 0, s \neq 0, z, y = 0 \in \mathbb{F}_q, w^2z = us^2(p^2 + p)}$ for some $p \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4q/2$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, z \in \mathbb{F}_q, ts = w^2v^2(p^2 + p)}$ for some $p \in \mathbb{F}_q, w^2z = us^2((w^2v^2t^{-1}s^{-1}(p^2 + p))^2 + (w^2v^2t^{-1}s^{-1}(p^2 + p)))}$ for some $p \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4((q/2 - 1)q/2)$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)((f^2 + f)u^2v + jvw^{-2}st + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t, r = x = 0, t = (z^2 + z)u^2w^2s^{-1}}$ for some $z \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4q/2$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w, t, r \neq 0, z \neq 0, s = 0 \in \mathbb{F}_q, w = (\alpha^2 + \alpha)u^2z^2r^{-1}}$ for some $\alpha \in \mathbb{F}_q, t = (\alpha^2 + \alpha)vz^{-2}r^2}$ for some $\alpha \in \mathbb{F}_q}$, with parameter set of size $(q - 1)^4q^2/4$,

$\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + (jv^{-1}w^{-1}s)^2)ut + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t, r = y = 0 \in \mathbb{F}_q, t = (z^2 + z)u^{-2}s^2}$ for some $z \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4q/2$,

$\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x = y = z = 0 \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4$,

y25 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s = 0, t \neq 0, r, z \neq 0 \in \mathbb{F}_q, let x \in \mathbb{F}_q^\times, s.t. u^2z = wt^2(x^2 + x) r = x^2v^2wu^{-1}t^2(y^2 + y + \eta)}$ for some $y \in \mathbb{F}_q, x^2v^{-2}wu^{-2}t^2z^{-2}r = (y^2 + y)}$ for some $y \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4\beta_{34}$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}((h^2 + h)us^2 + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t = 0, z, r \neq 0, y = us^2\eta \in \mathbb{F}_q, v(ry + z) = w^2r^2(x^2 + x)}$ for some $x \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^5q/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}((h^2 + h)us^2 + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t = 0, z, r \neq 0, y = 0 \in \mathbb{F}_q, v(ry + z) = w^2r^2(x^2 + x + \eta)}$ for some $x \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^5q/2$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, y \neq 0 \in \mathbb{F}_q, w^2t = us^2, y = us^2(r^2 + r + \eta)}$ for some $r \in \mathbb{F}_q, let n \in \mathbb{F}_q, s.t. z = w^{-2}vy^2(n^2 + n), yn = us^2(r^2 + r + \eta)}$ for some $r \in \mathbb{F}_q}$ with parameter set of size $(q - 1)^4q^2/4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2 + h)u^2w + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2 +$

$m)u^2v^{-1}t^2 + x)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s = u^2w\eta, x = u^2v^{-1}t^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2 + h)u^2w + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2 + m)u^2v^{-1}t^2 + x)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s = 0, x = u^2v^{-1}t^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, z, urx = (p^2 + p(vs)^2t, s^2t = (z^2 + z)ur^2, y^2 + y + \eta = (p^2 + p)(1 + z)^3v^3u^2r^3}$ for some $p, z, y \in \mathbb{F}_q$ with parameter set of size $(q-1)^3\beta_{44}$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t=0, r \neq 0, x, y' = v^{-1}uw^2\eta \in \mathbb{F}_q, rx = (p^2 + p + \eta)(vr + ws)^2}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t=0, r \neq 0, x, y' = 0 \in \mathbb{F}_q, rx = (p^2 + p + \eta)(vr + ws)^2}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t = uw^2/v^2, r \neq 0, x, y' = v^{-1}uw^2\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, x = v^2r(p^2 + p + \eta), (p^2 + p + \eta)v^2t = (q^2 + q)uw^2}$ for some $p, q \in \mathbb{F}$ with parameter set of size $(q-1)^4(q/2 - 1)q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s=0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, x = v^2r(p^2 + p), (p^2 + p)v^2t = (q^2 + q + \eta)uw^2}$ for some $p, q \in \mathbb{F}$ with parameter set of size $(q-1)^4(q/2 - 1)q/2$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t = uw^2/v^2, r \neq 0, z, x, y' = v^{-1}uw^2\eta \in \mathbb{F}_q, \text{let } a \in \mathbb{F}_q \text{ s.t. } 1 = a^2 + ax = v^2r(p^2 + p + \eta) + v^2ra\eta}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2\alpha$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t = uw^2/v^2, r \neq 0, x, y' = v^{-1}uw^2\eta \in \mathbb{F}_q, \text{let } a \in \mathbb{F}_q^\times \text{ s.t. } ws = (a^2 + a)vr, xv = v^2r(p^2 + p + \eta) + v^2ra^2\eta}$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q(\alpha(q/2 - 2) + (1 - \alpha)(q/2 - 1))$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t \neq 0, r \neq 0, z, x \in \mathbb{F}_q, v^2t \neq uw^2, \text{let } a \in \mathbb{F}_q^\times \text{ s.t. } a \neq 1, v^2t = (a^2 + a)uw^2, tx = a^3uw^2r(q^2 + q + \eta), a^3(a+1)^8(uw)^5(vr)^3 + a^2 + 1 + a^3(q^2 + q + \eta) = (a+1)^3(p^2 + p)}$ for some $p, q \in \mathbb{F}_q$

$(q-1)^3\beta_{37}$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, \text{let } a \in \mathbb{F}_q^\times \text{ s.t. } avr \neq ws, (a+1)vr \neq ws, s^2t = (a^2 + a)ur^2, tx = aruw^2((a+1)vr/(ws) + 1)^2(q^2 + q + \eta), vus^6r^2(a+1)^2 + vu^5r^2(a+1)^2((a+1)vr + ws)^3(avr + ws)^6 + u^2s^4((a+1)vr + ws)^3(avr + ws)^2(q^2 + q + \eta) = u^2s^4r(a+1)((a+1)vr + ws)(avr + ws)^4(p^2 + p)}$ for some $p, q \in \mathbb{F}_q$

with parameter set of size β_{40} ,

$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t^2v^{-1} + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, z=0 \text{ or } z = u^2w\eta, y = u^2t^2v^{-1}\eta \in \mathbb{F}_q}$ with parameter set of size $2(q-1)^4$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \in \mathbb{F}_q, \text{let } x \in \mathbb{F}_q^\times \text{ s.t. } tv^2s = tw^2(x^2 + y) = x^2u^2tv^{-1}w^2(z^2 + z + \eta)}$ for some $z \in \mathbb{F}_q, x^2u^{-2}tv^{-2}w^2s^{-2}y = (z^2 + z)}$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4\beta_{31}$,

$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}((p^2 + p)wy^2 + x)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t=0, y \neq 0, x=0, s = u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad +$

$t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}((p^2 + p)wy^2 + x)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(v^{-1}wd + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + (h^2 + h)uw^2 + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2 + o)vy^2 + x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(v^{-1}wd + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + (h^2 + h)uw^2 + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2 + o)vy^2 + x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q-2)q/2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/2$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/4$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/4$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + (jv^{-1}w^{-1}s)^2ut + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

y26 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(gv^2w^{-1} + x)x_{16}(c^2v^{-2}w + z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bjv^{-1} + r)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)(sx + v^2z)^2/(v^2s) + y')x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^5q^2/2$,

with parameter set of size $(q-1)^4\beta_{35}$,

$$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}((h^2 + h)us^2 + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, r \neq 0, y = us^2 \eta \in \mathbb{F}_q, v(ry+z) = w^2r^2(x^2+x+\eta) \text{ for some } x \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^5q/2$,

$$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, y \neq 0 \in \mathbb{F}_q, w^2t = us^2, y = us^2(r^2+r) \text{ for some } r \in \mathbb{F}_q, \text{ let } n \in \mathbb{F}_q \text{ s.t. } z = w^{-2}vy^2(n^2+n), yn = us^2(r^2+r+\eta) \text{ for some } r \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4(q/2-1)q/4$,

$$\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g + t)x_{16}((i^2 + i)vw^2 + s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2 + o)u^{-2}vt^2 + x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s = vw^2\eta, x = 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(u)x_2(v)x_5(a)x_6(b)x_8(c)x_9(cu^{-1}v + t)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}((h^2 + h)u^2w + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}((m^2 + m)u^2v^{-1}t^2 + x)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s = u^2w\eta, x = 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}((r^2 + r)w^2x^2v^{-3} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t = u^2v\eta, s = 0, x \neq 0, y = w^2x^2v^{-1}\eta \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)u^2w + s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}((q^2 + q)v^2x^2w^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s = u^2w\eta, t = 0, x \neq 0, y = v^2x^2w^{-1}\eta \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w = 0, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, zurx = (p^2+p+\eta)(vs)^2t, s^2t = (z^2+z)ur^2, y^2+y+\eta = (p^2+p+\eta)(1+z)^3v^3u^2r^3 \text{ for some } p, z, y \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^3\beta_{45}$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, t = 0, r \neq 0, x, y' = v^{-1}uw^2\eta \in \mathbb{F}_q, rx = (p^2+p)(vr+ws)^2 \text{ for some } p \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^5q/2$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t = uw^2/v^2, r \neq 0, x, y' = 0 \in \mathbb{F}_q, x = v^2r(p^2+p+\eta) \text{ for some } p \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4q/2$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, x = v^2r(p^2+p+\eta) \text{ for some } p \in \mathbb{F}_q, v^2t = (q^2+q+\eta)uw^2 \text{ for some } p, q \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4(q/2-1)q/2$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t = uw^2/v^2, r \neq 0, z, x, y' = 0 \in \mathbb{F}_q, \text{ let } a \in \mathbb{F}_q \text{ s.t. } 1 = a^2 + ax = v^2r(p^2+p+\eta) \text{ for some } p \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4q^2/2\alpha$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t = uw^2/v^2, r \neq 0, x, y' = 0 \in \mathbb{F}_q, \text{ let } a \in \mathbb{F}_q \text{ s.t. } ws = (a^2+a)vr, x = v^2r(p^2+p+\eta) \text{ for some } p \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4q/2(\alpha(q/2-2) + (1-\alpha)(q/2-1))$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, \text{ let } a \in \mathbb{F}_q \text{ s.t. } a \neq 1, v^2t = (a^2+a)uw^2, tx = a^3uw^2r(q^2+q+\eta), a^3(a+1)^8(uw)^5(vr)^3 + a^2 + 1 + a^3(q^2+q+\eta) = (a+1)^3(p^2+p+\eta) \text{ for some } p, q \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^3q\beta_{38}$,

$$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vr/w, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, v^2t \neq uw^2, \text{ let } a \in \mathbb{F}_q \text{ s.t. } avr \neq ws, (a+1)vr \neq ws, s^2t = (a^2+a)ur^2, tx = aruw^2((a+1)vr/(ws)+1)^2(q^2+q+\eta), vws^6r^2(a+1)^2 + vu^5r^2(a+1)^2((a+1)vr+ws)^3(avr+ws)^6 + u^2s^4((a+1)vr+ws)^3(avr+ws)^2(q^2+q+\eta) = u^2s^4r(a+1)((a+1)vr+ws)(avr+ws)^4(p^2+p+\eta) \text{ for some } p, q \in \mathbb{F}_q}$$
 with parameter set of size β_{41} ,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t^2v^{-1} + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, z = u^2w\eta, y = 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t^2v^{-1} + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, z = u^2w\eta, y = 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

with parameter set of size β_{41} ,

$$\{x_1(u)x_5(a)x_6(v)x_8(b)x_9(au^{-1}v + t)x_{10}(w)x_{11}(c)x_{12}(d)x_{13}(e)x_{14}(f)x_{15}((g^2 + g)u^2w + z)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2t^2v^{-1} + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, z = u^2w\eta, y = 0 \in \mathbb{F}_q}$$
 with parameter set of size $(q-1)^4$,

$\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v, w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, y \in \mathbb{F}_q$, let $x \in \mathbb{F}_q^\times, s, t, v^2s = tw^2(x^2 + x) = yx^2u^2tv^{-1}w^2(z^2 + z + \eta)$ for some $z \in \mathbb{F}_q, x^2u^{-2}tv^{-2}w^2s^{-2}y = (z^2 + z + \eta)$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4\beta_{32}$,

$\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)wv + u^{-1}as + uv^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}((p^2 + p)wy^2 + x)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}$
 $u, v, w \in \mathbb{F}_q^\times, t=0, y \neq 0, s = u^2v\eta, x = y^2w\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(b)x_7(c)x_8(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(v^{-1}wd + y)x_{13}(f)x_{14}(g)x_{15}(c^2u^{-1} + (h^2 + h)uw^2 + s)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2 + o)vy^2 + x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}$
 $u, v, w \in \mathbb{F}_q^\times, t=0, y \neq 0, x = vy^2\eta, s = \eta uw^2 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w=0, s = u^2r^2t^{-1}(z^2 + z)$ for some $z \in \mathbb{F}_q, t \neq v^{-1}u^2w, r \neq 0, x = vr^2\eta, y' = 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s = t=0, r \neq 0, x = v^2u^{-2}w^{-2}r^2y', y' = vu^{-2}(t + v^{-1}u^2w)^2\eta \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(u^2w^2/v((hr)^2 + h(ws + vr^2))^2/(wvs)^2 + ((hr)^2 + h(ws + vr^2))/(wvs) + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vu^{-2}w^{-2}r^2(u^2w + vt), s \neq 0, t=0, r \neq 0, y' \in \mathbb{F}_q, vy' = (y^2 + y + \eta)u^2w^2$, with parameter set of size $2(q-1)^4(q/2 - 1)$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(d^2v^{-1} + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s = t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x = vr^2(o^2 + o)(1 + vt/(u^2w))^2$ for some $o = ((o')^2 + o' + \eta)/(vt/(u^2w) + v^2t^2/(u^2w)^2), o' \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q-2)q/4$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq vu^{-2}w^{-2}r^2(u^2w + vt), s \neq 0, t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x \in \mathbb{F}_q, ts = u^2r^2(z^2 + z)$ for some $z \in \mathbb{F}_q$, with parameter set of size $(q-1)^4(q-2)(q/2 - 2)$,

$\{x_2(v)x_3(u)x_4(w)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s = 0, t = v^{-1}u^2w, r \neq 0, x \in \mathbb{F}_q, ws = vr^2(z^2 + z)$ for some $z \in \mathbb{F}_q, x = vr^2(z^2 + z + \eta)^2 + (z^2 + z + \eta)ws$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q/2 - 1)$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + s)x_{16}((h^2 + h)uw^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$
 $u, w, v \in \mathbb{F}_q^\times, t=0, s \neq 0, z, y = uv^2\eta \in \mathbb{F}_q, w^2z = us^2(p^2 + p)$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}$
 $u, w, v \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, z \in \mathbb{F}_q, ts = w^2v^2(p^2 + p)$ for some $p \in \mathbb{F}_q, w^2z = us^2((w^2v^2t^{-1}s^{-1}(p^2 + p + \eta))^2 + (w^2v^2t^{-1}s^{-1}(p^2 + p + \eta)))$ for some $p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4((q/2 - 1)q/4)$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + jvw^{-2}st + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t, r = 0, x = u^2v\eta \in \mathbb{F}_q, t = (z^2 + z)u^2w^2s^{-1}$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w, t, r \neq 0, z \neq 0, s = u^2v\eta \in \mathbb{F}_q, w = (\alpha^2 + \alpha)u^2z^2r^{-1}$ for some $\alpha \in \mathbb{F}_q, t = (\alpha^2 + \alpha)vz^{-2}r^2$ for some $\alpha \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/2$,

$\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}((g^2 + g)uv^2 + (jv^{-1}w^{-1}s)^2)ut + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t, y = uv^2\eta, r = 0 \in \mathbb{F}_q, t = (z^2 + z)uw^{-2}s^2$ for some $z \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q/2$,

$\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)w^2v + x)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$
 $u, v, w \in \mathbb{F}_q^\times, t \neq 0, x = w^2v\eta, y = z = 0 \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

y27 $\{x_1(v)x_2(w)x_3(u)x_4(a)x_5(bu^{-1}w + s)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(gv^2w^{-1} + x)x_{16}(c^2v^{-2}w +$

$z)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(bju^{-1} + r)x_{21}(l)x_{22}(m)x_{23}((n^2 + n)(sx + v^2z)^2/(v^2s) + y')x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$
with parameter set of size $(q-1)^5q^2$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/4$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(q/2 - 1)q/2$,
 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(u^2v^{-1}g + t)x_{16}((i^2 + i)vw^2 + s)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}((o^2 + o)u^{-2}vt^2 + x)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $2(q-1)^4$,
 $\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}((r^2 + r)w^2x^2v^{-3} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}((r^2 + r)w^2x^2v^{-3} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)u^2w + s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}((q^2 + q)v^2x^2w^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)u^2w + s)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}((q^2 + q)v^2x^2w^{-1} + y) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/4$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/2(\frac{q}{2}\alpha + (1-\alpha)(\frac{q}{2}-1))$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2/2(\frac{q}{2}\alpha + (1-\alpha)(\frac{q}{2}-1))$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q^2(1-\alpha)$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q(\frac{q}{2}\alpha + (1-\alpha)(\frac{q}{2}-1))$,
 $\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4(\frac{q}{2} + \beta_{42})$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4q$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q$ with parameter set of size $(q-1)^4$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(u^2w^2/v(((hr)^2 + h(ws + vr^2))^2/(wvs)^2 + ((hr)^2 + h(ws + vr^2))/(wvs)) + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vv^{-1}r^2, s \neq 0, t=0, r \neq 0, y' \in \mathbb{F}_q, vy' = u^2w^2(y^2 + y + x(z^2 + z + \eta)), \text{ with parameter } (1 - x + ws/(vr^2))^2(z^2 + z + \eta) = y^2 + y \text{ for some } z, y \in \mathbb{F}_q, x \in \{0, 1\}}$

set of size $4(q-1)^4(\frac{q}{2}-1)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(d^2v^{-1} + x)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, 0 = s, t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x = vr^2(o^2 + o + \eta) \text{ for some } o \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4(q-2)\frac{q}{2}$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq vu^{-2}w^{-2}r^2(u^2w + vt), s \neq 0, t \neq v^{-1}u^2w, t \neq 0, r \neq 0, x=0 \text{ or } x = v^{-1}u^2w^2t^{-1}s \eta \in \mathbb{F}_q, ts = u^2r^2(z^2 + z) \text{ for some } z \in \mathbb{F}_q, \text{ with parameter } x = (z^2 + z + \eta)v^{-1}u^{-4}w^{-2}r^{-2}(vr^2(u^2w + vt) + u^2w^2s)^2 \text{ for some } z \text{ in } \mathbb{F}_q$

set of size $(q-1)^4(q-2)(\frac{q}{2}-2)$,

$\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t = v^{-1}u^2w, r \neq 0, x=0 \text{ or } x = v^{-1}u^2w^2t^{-1}s \eta \in \mathbb{F}_q, ws = vr^2(z^2 + z) \text{ for some } z \in \mathbb{F}_q, \text{ with parameter set of size } (q-1)^4(\frac{q}{2}-1), \text{ with parameter set of size } x = (z^2 + z + \eta)v^{-1}u^{-4}w^{-2}r^{-2}(vr^2(u^2w + vt) + u^2w^2s)^2 \text{ for some } z \text{ in } \mathbb{F}_q$

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + s)x_{16}((h^2 + h)uw^2 + y)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, t=0, s \neq 0, z, y=0 \text{ or } y = uv^2 \eta \in \mathbb{F}_q, w^2z = us^2(p^2 + p + \eta) \text{ for some } p \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q$,

$\{x_2(u)x_5(w)x_6(a)x_7(v)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,w \in \mathbb{F}_q^\times, t \neq 0, s \neq 0, z \in \mathbb{F}_q, w^2z = us^2(p^2 + p + \eta) \text{ for some } p \in \mathbb{F}_q, ts = w^2v^2(p^2 + p) \text{ for some } p \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4((q/2 - 1)q/2)$,

$\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}((f^2 + f)u^2v + jvw^{-2}st + x)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t, r = vv^{-2}s^2 \eta, x=0 \text{ or } x = u^2v \eta \in \mathbb{F}_q, t = (z^2 + z)u^2w^2s^{-1} \text{ for some } z \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q$,

$\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w, t, r \neq 0, z \neq 0, s=0 \text{ or } s = u^2v \eta \in \mathbb{F}_q, w = (a^2 + \alpha)u^2z^2r^{-1} \text{ for some } \alpha \in \mathbb{F}_q, t = (a^2 + \alpha + \eta)vz^{-2}r^2 \text{ for some } \alpha \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q^2/2$,

$\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s \neq 0, t \neq 0, r=0, t = (z^2 + z + \eta)uw^{-2}s^2 \text{ for some } z \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q/2$,

$\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, t \neq 0, y=0, z = vu^{-2}t^2 \eta \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4$,

Y28 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s=0, t \neq 0, r, z \in \mathbb{F}_q, u^2z = wt^2(y^2 + y + \eta) \text{ for some } y \in \mathbb{F}_q, wt^2r = v^2u^2z^2(y^2 + y + \eta) \text{ for some } y \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q^2/4$,

$\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + y)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(b^2mu^{-2} + z)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u,v,w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, z, y \neq 0 \in \mathbb{F}_q, w^2t = us^2, w^2z = vy^2(r^2 + r + \eta) \text{ for some } r \in \mathbb{F}_q, y = vu^2(r^2 + r + \eta) \text{ for some } r \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q^2/4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w=0, s \neq 0, t \neq 0, r \neq 0, x \in \mathbb{F}_q, urx = (p^2 + p + \eta)v^2s^2t \text{ for some } p \in \mathbb{F}_q, s^2t = ur^2(p^2 + p + \eta) \text{ for some } p \in \mathbb{F}_q \text{ with parameter set of size } (q-1)^4q^2/4$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q\}_{u,v \in \mathbb{F}_q^\times, w \neq 0, s = vr/w, t \neq 0, r \neq 0, z, x \in \mathbb{F}_q, v^2t \neq uw^2, urx(uw^2 + v^2t) = (p^2 + p + \eta)(sv^2t)^2 \text{ for some } p \in \mathbb{F}_q, \text{ with parameter set of size } t(sv^2t)^2 = (p^2 + p + \eta)ur^2(uw^2 + v^2t)^2 \text{ for some } p \in \mathbb{F}_q$

$(q-1)^4\frac{q^2}{2}(\frac{q}{2}\alpha + (1-\alpha)(\frac{q}{2}-1))$,

$\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} +$

$r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 \frac{q}{2} (\alpha((\frac{q}{2}-1)(q-1)-1) + (1-\alpha)(\frac{q}{2}-1)(q-1))$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(biu^{-1} + z)x_{20}(j)x_{21}(k)x_{22}(l)x_{23}(m)x_{24}(n) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q^2 (1-\alpha)$,
 $\{x_1(v)x_4(u)x_5(bu^{-1}v + w)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + s)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}((e^2 + e)v^{-1}uw^2 + v^{-1}g + y')x_{13}(ab^2u^{-2} + r)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}(agu^{-1} + fv + x)x_{19}(j)x_{20}(k)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q(\frac{q}{2}\alpha + (1-\alpha)(\frac{q}{2}-1))$,
 $\{x_1(u)x_5(a)x_6(t)x_7(v)x_8(b)x_9(c)x_{10}(w)x_{11}(d)x_{12}(e)x_{13}(cuv^{-1} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)u^2s + (j^2 + j)^2u^2v^{-2}w^2t + y)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 (q/2 - \beta_{42})$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}((h^2 + h)vu^{-2}(t + v^{-1}u^2w)^2 + y')x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)vr^2 + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 (q-2)q/2$,
 $\{x_2(v)x_3(u)x_5(a)x_6(bu^{-1}v + w)x_7(b)x_8(abu^{-1} + r)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}((l^2 + l)v^{-1}u^2w^2t^{-1}s + x)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q/2$,
 $\{x_2(u)x_5(w)x_6(a)x_7(b)x_8(b)x_9(c)x_{10}(t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(au^{-1}w^2 + t(b^2 + w^2a^2u^{-2})v^{-2} + s)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(aju^{-1} + z)x_{21}(l)x_{22}(m)x_{23}(n)x_{24}(o) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q^2/4$,
 $\{x_3(u)x_5(a)x_6(v)x_7(b)x_8(abu^{-1} + w)x_9(c)x_{10}(bu + t)x_{11}(d)x_{12}(e)x_{13}(f)x_{14}(g)x_{15}(a^2bu^{-1} + s)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)vw^{-2}s^2 + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q/2$,
 $\{x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + cu + s)x_{11}(aeu^{-1} + z)x_{12}(g)x_{13}(ue + w)x_{14}(h)x_{15}(ad + t)x_{16}(i)x_{17}(j)x_{18}(abdv^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q^2/4$,
 $\{x_4(u)x_5(v)x_6(a)x_7(b)x_8(c)x_9(abu^{-1} + w)x_{10}(b^2u^{-1} + t)x_{11}(d)x_{12}(e)x_{13}(ab^2u^{-2} + s)x_{14}(f)x_{15}(g)x_{16}(h)x_{17}(i)x_{18}((j^2 + j)v^2w^2s^{-1} + r)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4 q/2$,
 $\{x_5(w)x_6(v)x_7(u)x_8(a)x_9(b)x_{10}(t)x_{11}(c)x_{12}(d)x_{13}((e^2 + e)vu^{-2}t^2 + z)x_{14}(f)x_{15}((g^2 + g)w^2u^2t^{-1} + y)x_{16}(h)x_{17}(i)x_{18}(j)x_{19}(k)x_{20}(l)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q^{\times}$, with parameter set of size $(q-1)^4$,
 $\{u, v, w \in \mathbb{F}_q^{\times}, t \neq 0, y, z \in \mathbb{F}_q, y = w^2u^2t^{-1}\eta, z = vu^{-2}t^2\eta \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^4$,

y29 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{10}(t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)vt + x)x_{14}(h)x_{15}(i)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w, t \in \mathbb{F}_q^{\times}, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}$ with parameter set of size $(q-1)^5 q^2/2$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}((i^2 + i)w^2t + y)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^{\times}, s=0, t \neq 0, y=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(h)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^{\times}, s \neq 0, t \neq 0, r \in \mathbb{F}_q, w^2t \neq us^2, w^2us^2r^2 = v^2t(us^2 + w^2t)^2(z^2 + z) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4 (q-2)q/2$,

$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \text{ or } t=u^2v\eta, s \neq 0, x \in \mathbb{F}_q, x=ws(z^2+z+1) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x \in \mathbb{F}_q, x=vt(z^2+z+1) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, y, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q, u^{-2}v^{-1}t^{-1}y^2 + u^{-2}v^{-1}s = (z^2+z) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + (i^2 + i)uw^2 + s)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=uw^2 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

y30 $\{x_1(u)x_2(v)x_5(a)x_6(b)x_7(w)x_8(c)x_9(d)x_{10}(t)x_{11}(e)x_{12}(f)x_{13}((g^2 + g)vt + x)x_{14}(h)x_{15}(i)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w, t \in \mathbb{F}_q^\times, x=vt\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(v)x_2(w)x_3(u)x_5(a)x_6(bu^{-1}w + s)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(c^2v^{-2}w + z)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(bku^{-1} + r)x_{21}(m)x_{22}(n)x_{23}(o)x_{24}(p) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, r, z \in \mathbb{F}_q, z=st(y^2+y+1+\eta) \text{ for some } y \in \mathbb{F}_q}$ with parameter set of size $(q-1)^5q^2/2$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}((i^2 + i)w^2t + y)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s=0, t \neq 0, y=w^2t\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,
 $\{x_1(w)x_2(v)x_4(u)x_5(bu^{-1}w + s)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(w^{-1}vf + r)x_{14}(g)x_{15}(h)x_{16}(i)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, s \neq 0, t \neq 0, r \in \mathbb{F}_q, w^2t \neq us^2, w^2us^2r^2 = v^2t(us^2+w^2)^2(z^2+z+\eta) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4(q-2)q/2$,

$\{x_1(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + t)x_{11}(g)x_{12}(h)x_{13}(ue + s)x_{14}(i)x_{15}(j)x_{16}(w^{-2}vl + ce + x)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t=0 \text{ or } t=u^2v\eta, s \neq 0, x \in \mathbb{F}_q, x=ws(z^2+z+1+\eta) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_1(v)x_3(u)x_5(a)x_6(w)x_7(b)x_8(c)x_9(d)x_{10}(bu + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(i)x_{16}(v^{-2}wi + x)x_{17}(j)x_{18}(k)x_{19}(l)x_{20}(m)x_{21}(n)x_{22}(o)x_{23}(p)x_{24}(q) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, x \in \mathbb{F}_q, x=vt(z^2+z+1+\eta) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,
 $\{x_2(w)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + s)x_{11}(g)x_{12}((f^2 + f)uva + u^{-1}as + uw^{-1}g + y)x_{13}(h)x_{14}(i)x_{15}(ad + t)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, y, s=0 \text{ or } s=u^2v\eta \in \mathbb{F}_q, u^{-2}v^{-1}t^{-1}y^2 + u^{-2}v^{-1}s = (z^2+z+\eta) \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q$,
 $\{x_2(v)x_4(u)x_5(w)x_6(a)x_7(b)x_8(c)x_9(d)x_{10}(b^2u^{-1} + t)x_{11}(e)x_{12}(f)x_{13}(g)x_{14}(h)x_{15}(c^2u^{-1} + (i^2 + i)uw^2 + s)x_{16}(j)x_{17}(k)x_{18}(l)x_{19}(m)x_{20}(n)x_{21}(o)x_{22}(p)x_{23}(q)x_{24}(r) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r \in \mathbb{F}_q\}_{u, v, w \in \mathbb{F}_q^\times, t \neq 0, s=(z^2+z+1+\eta)uw^2 \text{ for some } z \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4q/2$,

y31 $\{x_1(w)x_2(y)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + c^2v^{-1} + z)x_{11}(g)x_{12}(h)x_{13}(i)x_{14}(j)x_{15}((k^2 + k)w^2u^2v + x)x_{16}(l)x_{17}(m)x_{18}(n)x_{19}(o)x_{20}(p)x_{21}(q)x_{22}(r)x_{23}(s)x_{24}(t) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t \in \mathbb{F}_q\}_{u, v, w, y \in \mathbb{F}_q^\times, z=0, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

y32 $\{x_1(w)x_2(y)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + c^2v^{-1} + z)x_{11}(g)x_{12}(h)x_{13}(i)x_{14}(j)x_{15}((k^2 + k)w^2u^2v + x)x_{16}(l)x_{17}(m)x_{18}(n)x_{19}(o)x_{20}(p)x_{21}(q)x_{22}(r)x_{23}(s)x_{24}(t) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t \in \mathbb{F}_q\}_{u, v, w, y \in \mathbb{F}_q^\times, z=u^2v\eta, x=0 \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

y33 $\{x_1(w)x_2(y)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + c^2v^{-1} + z)x_{11}(g)x_{12}(h)x_{13}(i)x_{14}(j)x_{15}((k^2 + k)w^2u^2v + x)x_{16}(l)x_{17}(m)x_{18}(n)x_{19}(o)x_{20}(p)x_{21}(q)x_{22}(r)x_{23}(s)x_{24}(t) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t \in \mathbb{F}_q\}_{u, v, w, y \in \mathbb{F}_q^\times, z=0, x=w^2u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$,

y34 $\{x_1(w)x_2(y)x_3(u)x_4(v)x_5(a)x_6(b)x_7(c)x_8(d)x_9(e)x_{10}((f^2 + f)u^2v + c^2v^{-1} + z)x_{11}(g)x_{12}(h)x_{13}(i)x_{14}(j)x_{15}((k^2 + k)w^2u^2v + x)x_{16}(l)x_{17}(m)x_{18}(n)x_{19}(o)x_{20}(p)x_{21}(q)x_{22}(r)x_{23}(s)x_{24}(t) \mid a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t \in \mathbb{F}_q\}_{u, v, w, y \in \mathbb{F}_q^\times, z=u^2v\eta, x=w^2u^2v\eta \in \mathbb{F}_q}$ with parameter set of size $(q-1)^4$

A.10 List of Placeholders used for the Sizes of Parametrized Conjugacy Classes of U fusing into Conjugacy Classes of G

There are two types of placeholders. For a rational function $f \in \mathbb{F}_q[l, m, n](X, Y)$, define the associated placeholder of the first type by $\alpha(f)$. A placeholder of the first type (I) takes the value $\alpha(f) = 1$ if there exists a solution $t \in \mathbb{F}_q^2$ for the equation $f(t) = 0$ for a fixed tuple l, m, n and $\alpha(f) = 0$ otherwise. The second type (II) of placeholder is defined as $\beta(f_i)_{i \in J} := |\{t \in \mathbb{F}_q^2 \mid f_i(t) = 0 \forall i \in J \text{ for some } l, m, n \in \mathbb{F}_q\}|$ for rational functions $f_i \in \mathbb{F}_q[l, m, n](X, Y, Z, T, W, C, U)$ and J an index set, and l, m, n allowed to vary. However, if the rational function(s) do in fact already lie in e.g. $\mathbb{F}_q[l, m, n](X)$, then $\beta(f_i) := |\{t \in \mathbb{F}_q \mid f_i(t) = 0 \forall i \in J \text{ for some } l, m \in \mathbb{F}_q\}|$. In several cases, it will moreover be required that the solutions should be nonzero in some of the variables.

Table 10: Placeholders

rational function	placeholder	type	value
$X^2 + X + 1$	α	(I)	
$X + (m^2 + m)^{-3} + (m^2 + m)^{-1}, X \neq 0$	β_4	(II)	
$X + (m^2 + m + \eta)^{-3} + (m^2 + m + \eta)^{-1}, X \neq 0$	β_5	(II)	$\frac{q-2+(q-4\eta)\alpha}{6(1-2\alpha)}$
$1 + m^3 X$	β_{10}	(II)	$\frac{(q-1)^2}{1+2\alpha} - (q-1)\beta_4 - \frac{(q-2)}{1+2\alpha}(\frac{q-2}{2} + \alpha)$
$X + m^2 + m, m^2 Y + X^2(t^2 + l), Y + m^2(n^2 + n), X \neq 0$	β_{30}	(II)	$\frac{q}{8}(q-2)$
$X + m^2 + m, m^2 Y + X^2(t^2 + l + \eta), Y + m^2(n^2 + n), X \neq 0$	β_{31}	(II)	$\frac{q}{8}(q-2)$
$X + m^2 + m, m^2 Y + X^2(t^2 + l + \eta), Y + m^2(n^2 + n + \eta), X \neq 0$	β_{32}	(II)	$\frac{q}{8}(q-2)$
$X + m^2 + m, m^4(n^2 + n) + (m^2 + m)(t^2 + l), Y + m^2(n^2 + n), X, Y \neq 0$	β_{33}	(II)	$\frac{q}{8}(q-2) - (\frac{q}{2} - 1)$
$X + m^2 + m, m^4(n^2 + n) + (m^2 + m)(t^2 + l + \eta), Y + m^2(n^2 + n), X, Y \neq 0$	β_{34}	(II)	$\frac{q}{8}(q-2)$
$X + m^2 + m, m^4(n^2 + n + \eta) + (m^2 + m)(t^2 + l + \eta), Y + m^2(n^2 + n + \eta), X, Y \neq 0$	β_{35}	(II)	$\frac{q}{8}(q-2)$
$X + m^2 + m, Y + m^3(n^2 + n), m^3(1 + m^8)Z^3 + m^3(n^2 + n) + (1 + m)^3(t^2 + l), X, Z \neq 0$	β_{36}	(II)	$(q-1)(q-2)q - \beta_{37} - \beta_{38} - (q-1)q(\alpha\frac{q}{2} + (1-\alpha)(\frac{q}{2} - 1))$
$X + m^2 + m, Y + m^3(n^2 + n + \eta), m^3(1 + m^8)Z^3 + m^3(n^2 + n + \eta) + (1 + m)^3(t^2 + l), X, Z \neq 0$	β_{37}	(II)	
$X + m^2 + m, Y + m^3(n^2 + n + \eta), m^3(1 + m^8)Z^3 + m^3(n^2 + n + \eta) + (1 + m)^3(t^2 + l + \eta), X, Z \neq 0$	β_{38}	(II)	
$X + m^2 + m, Y + m((m+1)T + 1)^2(n^2 + n), (m+1)^2 J^3 W / (UV) + W^3 Z^3 / (U^3 V)(m+1)^2((m+1)T + 1)^3(mT + 1)^6 + WZ((m+1)T + 1)^3(mT + 1)^2(n^2 + n) + Z^2(m+1)(m+1)T + 1)(mT + 1)^4(t^2 + l), X, Z, T, W, V, U \neq 0$	β_{39}	(II)	$(q-1)^4(q-2)q(\frac{q}{2} - 2) - q(q-1)^4(1-\alpha)(\frac{q}{2} - 1) - q(q-1)^4(\alpha(\frac{q}{2} - 2)) - (\beta_{40} + \beta_{41})$
$X + m^2 + m, Y + m((m+1)T + 1)^2(n^2 + n + \eta), (m+1)^2 J^3 W / (UV) + W^3 Z^3 / (U^3 V)(m+1)^2((m+1)T + 1)^3(mT + 1)^6 + WZ((m+1)T + 1)^3(mT + 1)^2(n^2 + n + \eta) + Z^2(m+1)(m+1)T + 1)(mT + 1)^4(t^2 + l), X, Z, T, W, V, U \neq 0$	β_{40}	(II)	$3\frac{q}{4}((q-1)^4 - (q-1)^5) - \beta_{37}(q-1)^3 + \beta_{44}(q-1)^4$
$X + m^2 + m, Y + m((m+1)T + 1)^2(n^2 + n + \eta), (m+1)^2 J^3 W / (UV) + W^3 Z^3 / (U^3 V)(m+1)^2((m+1)T + 1)^3(mT + 1)^6 + WZ((m+1)T + 1)^3(mT + 1)^2(n^2 + n + \eta) + Z^2(m+1)(m+1)T + 1)(mT + 1)^4(t^2 + l + \eta), X, Z, T, W, V, U \neq 0$	β_{41}	(II)	$3\frac{q}{8}((q-1)^4 - (q-1)^5) - \beta_{38}(q-1)^3 + \beta_{45}(q-1)^4$
$X = n^2 + n, nY = m^2 + m, t^2 + l + (m^2 + m)(1 + n)^3 Z, X, Z \neq 0$	β_{43}	(II)	$\frac{q}{8}(q-1)(q-2)$
$X = n^2 + n, nY = m^2 + m, t^2 + l + \eta + (m^2 + m)(1 + n)^3 Z, X, Z \neq 0$	β_{44}	(II)	$\frac{q}{8}(q-1)(q-2)$
$X = n^2 + n, nY = m^2 + m + \eta, t^2 + l + \eta + (m^2 + m + \eta)(1 + n)^3 Z, X, Z \neq 0$	β_{45}	(II)	$\frac{q}{8}(q-1)(q-2)$

Table 13: The Characters Induced from U , Part III

$\text{Irr}(G)$	T_{51}	T_{46}	T_{47}	T_{55}	T_{58}	T_{61}	T_{63}	T_{66}	T_{67}	T_{69}	T_{153}	T_{160}	T_{32}	T_{39}	T_{72}	T_{37}	T_{75}	T_{23}	T_{28}	T_{87}	T_{132}	T_{31}	T_{128}	T_{134}	T_{139}	T_{157}	
$\Phi_{1,0}$
$\Phi_{4,1}$
$\Phi_{2,4}''$
$\Phi_{2,4}$
$B_{2,1}$
$\Phi_{9,2}$
$\Phi_{8,3}'$
$\Phi_{8,3}''$
$\Phi_{12,4}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\Phi_{9,6}$	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	2	1	2	1	1	1	1	1	2	1	1
$\Phi_{9,6}'$	1	1	1	1	1	1	1	1	1	1	.	2	1	1	1
$\Phi_{1,12}''$
$\Phi_{1,12}$	1	1
$\Phi_{4,7}''$	2	1
$\Phi_{4,7}'$	1
$\Phi_{4,8}$	1	.	1	1	1	1	1	1	1	.	.	1	1	1	1	1	1
$\Phi_{6,6}$.	1	1	1	1
$\Phi_{6,6}'$	1	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\Phi_{16,5}$	2	1	1	2	2	2	2	2	2	1	1	1	2	2	2	1	1	1	1	2	1	1	1	1	1	1	1
$B_{2,r}$
$B_{2,e'}$
$B_{2,e''}$
$F_4[\theta]$
$F_4[\theta^2]$
$F_4[\psi]$
$F_4[-\psi]$
$F_4[1]$
$F_4'[1]$
$F_4''[1]$
$F_4[-1]$
$\Phi_{8,9}'$	2	2	1	$q+1$	$q+1$	2	2	$q+1$	1	1	$q+2$	$q+2$	$q+1$	$q+1$	$q+1$	1	1	1	1	$q+1$	2 q	2 q	q	q	q	q	
$\Phi_{8,9}''$	2	1	2	2	$q+1$	$q+1$	2	$q+1$	2	$q+2$	1	1	$q+1$	$q+1$	$q+1$	$q+1$	$q+2$	$q+1$	2 $q+1$	2 q	1	2 q	2 q	3 q	3 q	1	
$\Phi_{9,10}$	3	2	2	$q+2$	$q+2$	$q+2$	$q+2$	$q+2$	2	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	2 q	2 q	2 q	3 q	2 q	$(q+2)q$	$q(q+1)$	$q(q+1)$	$q(q+1)$	$q+1$	
$\Phi_{4,13}$	2	1	1	$q+1$	$q+1$	$q+1$	$q+1$	$q+1$	1	$q+1$	$q+1$	$q+1$	2 q	2 q	2 q	q	2 q	$\frac{q}{2}(q+1)$	$(q+1)q(q+1)q$	q^2	$\frac{q}{2}(q+1)^2$	$\frac{q}{2}(q+1)^2$	$\frac{q}{2}(q+1)^2$	$\frac{q}{2}(q+1)^2$	$\frac{q}{2}(q+1)^2$	q	
$\Phi_{2,16}'$	1	1	q	q	1	1	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q^2	q^2	q^2	q^2	q^2	q	
$\Phi_{2,16}''$	1	.	1	1	1	q	q	1	1	$q+1$.	q	q	q	q	q	2 q	$\frac{q}{2}(q+1)$	$(q+1)q$	q^2	$\frac{q}{2}(q^2+1)$	$\frac{q}{2}(q^2+1)$	$\frac{q}{2}(q^2+1)$	$\frac{q}{2}(q^2+1)$	$\frac{q}{2}(q+1)^2$	1	
$B_{2,e}$
$\Phi_{1,24}$	1	1	1	q	q	q	q	q	q	q	q	q	q^2	q^2	q^2	q^2	q^2	q^3	q^3	q^3	q^4	q^4	q^4	q^4	q^4	q	

Table 14: The Characters Induced from U , Part IV

$\text{Irr}(G)$	T_{49}	T_{62}	T_{65}	T_{154}	T_{158}	T_{53}	T_{123}	T_{29}	T_{88}	T_{48}	T_{56}	T_{59}	T_{68}	T_{70}	T_{17}	T_{35}	T_{73}	T_{74}	T_{76}	T_{77}	T_{84}	T_{133}	T_{18}	T_{124}	T_{71}	T_{155}	T_{161}	T_{159}	T_{50}		
$\Phi_{1,0}$	
$\Phi_{4,1}$	
$\Phi''_{2,4}$	
$\Phi_{2,4}$	
$B_{2,1}$	
$\Phi_{9,2}$	
$\Phi'_{8,3}$	
$\Phi''_{8,3}$	
$\Phi_{12,4}$	
$\Phi''_{9,6}$	
$\Phi'_{9,6}$	
$\Phi''_{1,12}$	
$\Phi_{1,12}$	
$\Phi''_{4,7}$	
$\Phi'_{4,7}$	
$\Phi_{4,8}$	
$\Phi_{6,6}$	
$\Phi''_{6,6}$	
$\Phi_{16,5}$	
$B_{2,r}$	
$B_{2,\epsilon'}$	
$B_{2,\epsilon''}$	
$F_4[\theta]$	
$F_4[\theta^2]$	
$F_4[\delta]$	
$F_4[-\delta]$	
$F_4^I[1]$	
$F_4^{II}[1]$	
$F_4^I[-1]$	
$\Phi'_{8,9}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
$\Phi''_{8,9}$	
$\Phi_{9,10}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
$\Phi_{4,13}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
$\Phi'_{2,16}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
$\Phi''_{2,16}$
$B_{2,\epsilon}$
$\Phi_{1,24}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

A.12 Upper Bounds for the Decomposition Matrices of the Irreducible Unipotent Characters of G into Irreducible Brauer Characters

Table 15: $e = 1, l \neq 3$ (i.e. $l \mid q - 1$), the principal block B_0

Irr(G)	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}
HC	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps
$\Phi_{1,0}$	1
$\Phi_{4,1}$.	1
$\Phi_{2,4}$.	.	1
$\Phi'_{2,4}$.	.	.	1
$\Phi_{9,2}$	1
$\Phi'_{8,3}$	1
$\Phi''_{8,3}$	1
$\Phi_{12,4}$	1
$\Phi''_{9,6}$	1
$\Phi_{9,6}$	1
$\Phi''_{1,12}$	1
$\Phi'_{1,12}$	1
$\Phi''_{4,7}$	1
$\Phi'_{4,7}$	1
$\Phi_{4,8}$	1
$\Phi'_{6,6}$	1
$\Phi''_{6,6}$	1
$\Phi_{16,5}$	1
$\Phi'_{8,9}$	1
$\Phi''_{8,9}$	1
$\Phi_{9,10}$	1
$\Phi_{4,13}$	1	.	.	.
$\Phi'_{2,16}$	1	.	.
$\Phi''_{2,16}$	1	.
$\Phi_{1,24}$	1

Table 16: $e = 1, l \neq 3$ (i.e. $l \mid q - 1$), B_1 , the non-principal block with more than one character

Irr(G)	S_{26}	S_{27}	S_{28}	S_{29}	S_{30}
HC	B_2	B_2	B_2	B_2	B_2
$B_{2,1}$	1
$B_{2,r}$.	1	.	.	.
$B_{2,\epsilon'}$.	.	1	.	.
$B_{2,\epsilon''}$.	.	.	1	.
$B_{2,\epsilon}$	1

Table 17: $e = 1, l = 3$ (i.e. $3 \mid q - 1$), the principal block B_0

Irr(G)	$S_1 S_2 S_3 S_4 S_5 S_6 S_7 T_{69} _{E_0} T_{153} _{E_0} T_{47} _{E_0} T_{46} _{E_0} S_{12} S_{13} S_{14} S_{15} S_{16} S_{17} S_{18} S_{19} S_{20} S_{21} T_{49} _{E_0} T_{48} _{E_0} S_{24} T_{161} _{E_0} T_{71} _{E_0} T_{159} _{E_0} T_{50}$														
	HC	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps	ps
$\Phi_{1,0}$	1
$\Phi_{4,1}$
$\Phi'_{2,4}$	1
$\Phi'_{2,4}$	1
$\Phi_{9,2}$
$\Phi'_{8,3}$
$\Phi'_{8,3}$	1
$\Phi'_{1,12}$
$\Phi'_{1,12}$
$\Phi'_{4,7}$
$\Phi'_{4,7}$
$\Phi'_{9,6}$
$\Phi'_{9,6}$
$\Phi_{4,8}$	1
$\Phi'_{6,6}$
$\Phi'_{6,6}$
$\Phi'_{6,6}$
$\Phi_{12,4}$
$\Phi_{16,5}$
$F_4[\emptyset]$
$F_4[\emptyset^2]$
$F_4'[\emptyset^2]$
$\Phi'_{8,9}$
$\Phi'_{8,9}$
$\Phi_{9,10}$
$\Phi_{2,16}$
$\Phi_{4,13}$
$\Phi'_{2,16}$
$\Phi_{1,24}$

Table 18: $e = 1, l = 3$ (i.e. $3 \mid q - 1$), B_1 , the non-principal block with more than one character

Irr(G)	$R_L^G(u(\lambda_{[1,-,3]}))$	$R_L^G(u(\lambda_{[-,1,3]}))$	$R_L^G(u(\lambda_{[-,1,3]})')$	$S_{32} T_{159} _{B_1}$
HC	B_3	B_3	C_3	
$B_{2,1}$	1	.	.	.
$B_{2,\epsilon'}$.	1	.	.
$B_{2,\epsilon''}$	1	.	1	.
$B_{2,r}$	1	1	1	.
$B_{2,\epsilon}$.	1	1	1

Table 19: $e = 2, l \neq 3$ (i.e. $l \mid q + 1$), the principal block B_0 , Part I

HC	Irr(G)	ps ps ps		B ₂		ps																		
		S ₁	S ₂	S ₃	R _L ^G (u(λ _{₋, -3}))	B ₀	S ₅	T ₄₄	B ₀	T ₄₂	B ₀	T ₆₉	B ₀	T ₁₅₃	B ₀	T ₄₇	B ₀	T ₅₁	B ₀	T ₁₅₇	B ₀	T ₄₆	B ₀	
Φ _{1,0}	1
Φ _{2,4} ''	.	1
Φ _{2,4} '	.	.	1
B _{2,1}	.	.	.	1
Φ _{9,2}	.	1	1
Φ _{8,3} '	1	.	1
Φ _{8,3} ''	1	1
Φ _{1,12} '	1
Φ _{1,12} ''	1
Φ _{6,6} '	1	.	.	.	γ
Φ _{6,6} ''	1
Φ _{9,6} '	.	1	1	.	γ
Φ _{9,6} ''	.	1	1	.	γ
B _{2,ε'}	γ
B _{2,ε''}	1
F ₄ [1]
F ₄ [1]
F ₄ [1]
F ₄ [-1]
Φ _{6,9}	1	1	.	.	2γ
Φ _{8,9}	1	.	1	.	2γ
Φ _{9,10}	.	1	1	.	3γ
B _{2,ε}	1
Φ _{2,16} '	.	1	.	.	γ
Φ _{2,16} ''	.	.	1	.	γ
Φ _{1,24}	1	.	.	.	γ

Table 20: $e = 2, l \neq 3$ (i.e. $l \mid q + 1$), the principal block B_0 , Part II

Irr(G)	$R_L^G(u(\lambda_{[-,1,3]})) _{B_0}$		$R_L^G(u(\lambda_{[-,1,3]}))' _{B_0}$		S_{16}		S_{17}		S_{18}		$T_{48} _{B_0}$		$T_{18} _{B_0}$		T_{159}		$T_{161} _{E_0}$		$T_{71} _{E_0}$		T_{50}		
	HC	B_3	C_3	B_3	C_3																		
$\Phi_{1,0}$																							
$\Phi''_{2,4}$																							
$\Phi'_{2,4}$																							
$B_{2,1}$																							
$\Phi_{9,2}$																							
$\Phi_{8,3}$																							
$\Phi''_{8,3}$																							
$\Phi'_{1,12}$																							
$\Phi'_{1,12}$																							
$\Phi'_{6,6}$																							
$\Phi''_{6,6}$																							
$\Phi_{9,6}$																							
$\Phi'_{9,6}$																							
$\Phi_{9,6}$																							
$B_{2,\epsilon'}$		1																					
$B_{2,\epsilon''}$					1																		
$F_4[1]$																							
$F_4''[1]$																							
$F_4[-1]$																							
$\Phi_{6,9}$										1													
$\Phi_{8,9}$											1												
$\Phi_{9,10}$												1											
$B_{2,\epsilon}$		1																					
$\Phi_{2,16}$																							
$\Phi_{2,16}$																							
$\Phi_{1,24}$																							

Table 21: $e = 2, l \neq 3$ (i.e. $l \mid q + 1$), B_1 , the non-principal block with more than one character

Irr(G)	S_{26}	$T_{47} _{B_1}$	$T_{46} _{B_1}$	$R_L^G(u(\lambda_{[-,1,3]})) _{B_1}$	$T_{71} _{B_1}$
HC	ps			B_3	
$\Phi_{4,1}$	1
$\Phi''_{4,7}$	1	1	.	.	.
$\Phi'_{4,7}$	1	.	1	.	.
$B_{2,r}$.	.	.	1	.
$\phi_{4,13}$	1	1	1	γ	1

Table 22: $e = 2, l = 3$ (i.e. $3 \mid q + 1$), the principal block B_0 , Part I

HC	B_2											B_3						
	$S_1 S_2 S_3$	R_L^G	$(u(\lambda_{[-, -, -3]}) _{B_0})$	S_5	$T_{44} _{B_0}$	$T_{42} _{B_0}$	$T_{69} _{B_0}$	$T_{153} _{B_0}$	$T_{47} _{B_0}$	$T_{51} _{B_0}$	$T_{46} _{B_0}$	$T_{157} _{B_0}$	R_L^G	$(u(\lambda_{[13, -, -1]}) _{B_0})$				
	ps	ps	ps															
$\Phi_{1,0}$	1				
$\Phi_{2,4}^{\prime}$.	1				
$\Phi_{2,4}^{\prime\prime}$.	.	1				
$B_{2,1}$.	.	.	1				
$\Phi_{9,2}$.	1	1				
$\Phi_{8,3}$	1	.	1	1				
$\Phi_{8,3}^{\prime\prime}$	1	1	.	.	1				
$\Phi_{1,12}^{\prime\prime}$	1	.	.	.	1	1				
$\Phi_{1,12}^{\prime}$	1	.	.	.	1	1	.	.	1	.	.	.	1	.				
$\Phi_{6,6}$	1	.	.	1	1	1	.	.	1				
$\Phi_{6,6}^{\prime\prime}$	1	.	.	1	1	1	.	.	1				
$\Phi_{12,4}$	2	.	2	2	2	1	1	1	1	1	1	.	.	.				
$\Phi_{9,6}^{\prime\prime}$.	1	1	1	.	3	2	.	1	1	1	.	1	.				
$\Phi_{9,6}$.	1	1	1	3	.	.	.	1	1	1	.	1	.				
$B_{2,\epsilon'}$				
$B_{2,\epsilon''}$				
$F_4[\theta]$				
$F_4[\theta^2]$				
$F_4^I[1]$				
$F_4^{II}[1]$				
$F_4[-1]$				
$\Phi_{8,9}^{\prime}$	1	1	.	4	1	1	1	1	1	2	2	2	1	1				
$\Phi_{8,9}^{\prime\prime}$	1	.	1	1	4	1	1	1	1	2	2	1	1	.				
$\Phi_{9,10}$.	1	1	3	3	3	1	1	1	2	3	2	1	1				
$B_{2,\epsilon}$				
$\Phi_{2,16}^{\prime}$.	1	.	2	.	.	1	1	1	1	1	1	1	1				
$\Phi_{2,16}^{\prime\prime}$.	.	1	.	2	1	1	1	1	1	1	1	1	1				
$\Phi_{1,24}$	1	.	.	1	1	1	1	1	1	1	1	1	1	1				

Table 23: $e = 2, l = 3$ (i.e. $3 \mid q + 1$), the principal block B_0 , Part II

Irr(G)	$R_L^G(u(\lambda_{[-,1,3]})) _{B_0}$	$R_L^G(u(\lambda_{[-,1,3]}))' _{B_0}$	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	$T_{49} _{B_0}$	$T_{48} _{B_0}$	$T_{18} _{B_0}$	T_{159}	$T_{161} _{B_0}$	$T_{71} _{B_0}$	T_{50}
HC	B_3	C_3												
$\Phi_{1,0}$
$\Phi''_{2,4}$
$\Phi'_{2,4}$
$B_{2,1}$
$\Phi_{9,2}$
$\Phi'_{8,3}$
$\Phi''_{8,3}$
$\Phi''_{1,12}$
$\Phi'_{1,12}$
$\Phi'_{6,6}$
$\Phi''_{6,6}$
$\Phi_{12,4}$
$\Phi''_{9,6}$
$\Phi_{9,6}$
$B_{2,\epsilon'}$	1
$B_{2,\epsilon''}$.	1
$F_4[\theta]$
$F_4[\theta^2]$
$F_4^I[1]$
$F_4^{II}[1]$
$F_4[-1]$
$\Phi'_{8,9}$	γ	1
$\Phi''_{8,9}$.	γ	1
$\Phi_{9,10}$	γ	γ	1	1	1
$B_{2,\epsilon}$	1	1	$\frac{q}{2}$	1	.	.	.
$\Phi'_{2,16}$	γ	1	.	$\frac{q}{2}$.	1	.	.
$\Phi''_{2,16}$.	γ	1	$\frac{q}{2}$	1	.	1	.
$\Phi_{1,24}$	γ	γ	1	1	q^2	q	q	q	1

Table 24: $e = 2, l = 3$ (i.e. $3 \mid q + 1$), B_1 , the non-principal block with more than one character

Irr(G)	S_{29}	$T_{47} _{B_1}$	$T_{46} _{B_1}$	$R_L^G(u(\lambda_{[-,1,3]})) _{B_1}$	$T_{71} _{B_1}$
HC	ps	B_3			
$\Phi_{4,1}$	1
$\Phi''_{4,7}$	1	1	.	.	.
$\Phi'_{4,7}$	1	.	1	.	.
$B_{2,r}$.	.	.	1	.
$\phi_{4,13}$	1	1	1	γ	1

Table 25: $e = 3$ (i.e. $l \mid q^2 + q + 1$), the principal block B_0 (all characters not in the principal block have defect zero)

Irr(G)	$S_1 \ S_2 \ S_3 \ S_4 \ S_5 \ S_6 \ R_L^C(u(\lambda_{[1^3, -1]}))' \Big _{B_0}$			$R_L^C(u(\lambda_{[1^3, -1, -1]}))' \Big _{B_0}$			$T_{47} \Big _{B_0}$			$T_{46} \Big _{B_0}$			$S_{11} \ S_{12} \ S_{13} \ S_{14} \ S_{15}$			$T_{49} \Big _{B_0}$			$T_{48} \Big _{B_0}$			$T_{155} \Big _{B_0}$			$T_{71} \Big _{B_0}$			$T_{159} \Big _{B_0}$			T_{50}					
	HC	ps	ps	ps	ps	ps	C ₃	B ₃	C ₃	B ₃	C ₃	B ₃	ps	ps	ps	T ₄₉	T ₄₈	T ₁₅₅	T ₇₁	T ₁₅₉	T ₅₀															
$\Phi_{1,0}$	1			
$\Phi_{4,1}$.	1		
$\Phi_{2,4}''$	1	.	1		
$\Phi_{2,4}'$	1	.	.	1	
$\Phi_{8,3}'$.	1	.	.	1	
$\Phi_{8,3}''$.	1	.	.	.	1	
$\Phi_{1,12}'$.	.	1	1	
$\Phi_{1,12}''$.	.	.	1	1	
$\Phi_{4,7}'$	1	1	
$\Phi_{4,7}''$	1	1	
$\Phi_{4,8}$	1	.	1	1	1	
$\Phi_{16,5}$.	1	.	.	1	1	1	
$F_4[\theta]$
$F_4[\theta^2]$
$F_4^I[\Gamma]$
$\Phi_{8,9}'$	1	1	2	.	1
$\Phi_{8,9}''$	1	2	1	.	1
$\Phi_{2,16}'$	1
$\Phi_{4,13}$	1	.	.	1
$\Phi_{2,16}''$.	.	1	1	.	.	1
$\Phi_{1,24}$	1	1	.	1

Table 26: $e = 4$ (i.e. $l \mid q^2 + 1$), the principal block B_0 . The two other blocks with more than one character have cyclic defect group.

Irr(G)	$S_1 R_L^G(u(\lambda_{[1,-3]})) _{B_0}$		$S_3 S_4 S_5 R_L^G(u(\lambda_{[-,12,1]})) _{B_0}$		$S_7 R_L^G(u(\lambda_{[-,1,3]})) _{B_0}$		$S_9 S_{10} S_{11} S_{12} R_L^G(u(\lambda_{[-,1^3,1]})) _{B_0}$		$T_{71} _{B_0}$		$T_{159} _{B_0}$		T_{50}
	ps	ps	B_3	ps	ps	ps	B_3	B_3	B_3	q	q	q	1
$\Phi_{1,0}$	1
$\Phi_{4,1}$.	1
$B_{2,1}$.	.	1
$\Phi_{9,2}$	1	.	.	1
$\Phi_{12,4}$.	1	.	1	1
$\Phi_{4,8}$.	.	.	1
$\Phi'_{6,6}$	1	.	.	1	.	1
$B_{2,r}$.	.	1	.	.	.	1
$F_4[i]$
$F_4[-i]$
$F'_4[1]$
$F''_4[1]$
$\Phi_{9,10}$.	.	.	1	1	1	.	.	.	1	.	.	.
$\Phi_{4,13}$	1	.	.	.
$B_{2,\epsilon}$	1	.	.	.	1	.
$\Phi_{1,24}$	1	.	.	.	q	.	q	1

Table 27: $e = 6$ (i.e. $l \mid q^2 - q + 1$), the principal block B_0 . All unipotent characters not lying in the principal block have defect zero.

HC	B_2			B_3			C_3													
	S_1	S_2	S_3	$R_L^C(u(\lambda_{[-,-,3]})) _{B_0}$	S_5	S_6	S_7	S_8	S_9	$R_L^C(u(\lambda_{[-,-,3]})) _{B_0}$	S_{12}	S_{13}	S_{14}	S_{15}	$T_{48} _{B_0}$	$T_{159} _{B_0}$	$T_{155} _{B_0}$	$T_{71} _{B_0}$	T_{50}	
$\Phi_{1,0}$	1
$\Phi''_{2,4}$.	1
$\Phi'_{2,4}$.	.	1
$B_{2,1}$.	.	.	1
$\Phi'_{8,3}$	1	.	1	.	1
$\Phi''_{8,3}$	1	1
$\Phi_{12,4}$	1	.	.	.	1	1
$\Phi''_{9,6}$.	1	.	.	1	1	.	1
$\Phi'_{9,6}$.	.	1	.	1	.	.	1
$B_{2,\epsilon'}$.	.	.	1	1
$B_{2,\epsilon''}$.	.	.	1
$F_4[\emptyset]$
$F_4[\emptyset^2]$
$F_4^I[1]$
$F_4^I[-1]$
$\Phi'_{8,9}$	1	1	.	1	1
$\Phi''_{8,9}$	1	1	1
$B_{2,\epsilon}$.	.	.	1	1
$\Phi'_{2,16}$	1
$\Phi''_{2,16}$	1
$\Phi_{1,24}$	1	1	1	1	1	1	1

A.13 Our GAP-Programs

A.13.1 The Unipotent Subgroup U of $F_4(q)$:

```

LoadPackage(" unipot ");
U_F4:=UnipotChevSubGr("F",4,GF(2));
R_F4:=RootSystem(U_F4);
SimpleSystem(R_F4);
PositiveRootsFC(R_F4);

```

To define the unipotent subgroup for arbitrary q , we define indeterminates t_i over \mathbb{F}_q and then use generic unipotent elements $x_{\alpha_i}(t_i)$ to define the unipotent subgroups U and $U_i = \langle x_i(t) \mid ht(\alpha_i) \geq i \rangle$:

```

#write a function that takes n and returns the Ui and y[i] for the
  finite field \mathbb{F}_{2^n}
LoadPackage(" unipot ");
uni_subgr:=function(n) local U,y,U_F4,R_F4; U:=[]; y:=[];
U_F4:=UnipotChevSubGr("F",4,GF(2^n));
R_F4:=RootSystem(U_F4);
for i in [1..24] do y[i]:=[];for j in [0..2^n-2] do y[i][j+1]:=
  UnipotChevElemByRN(U_F4,i,Z(2^n)^j);od;od;
U[1]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17],y[16],y[15],y[14],y[13],y[12],y[11],y[10],y[9],y[8],y[7],y[6],y
  [5], y[4],y[3],y[2],y[1]));
U[2]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17],y[16],y[15],y[14],y[13],y[12],y[11],y[10],y[9],y[8],y[7],y[6],y
  [5]));
U[3]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17],y[16],y[15],y[14],y[13],y[12],y[11],y[10],y[9],y[8]));
U[4]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17],y[16],y[15],y[14],y[13],y[12],y[11]));
U[5]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17],y[16],y[15],y[14]));
U[6]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19],y[18],y
  [17]));
U[7]:=Group(Concatenation(y[24],y[23],y[22],y[21],y[20],y[19]));
U[8]:=Group(Concatenation(y[24],y[23],y[22],y[21]));
U[9]:=Group(Concatenation(y[24],y[23],y[22]));
U[10]:=Group(Concatenation(y[24],y[23]));
U[11]:=Group(y[24]);
return [U,y[24],y[23],y[22],y[21],y[20],y[19],y[18],y[17],y[16],y[15],y
  [14],y[13],y[12],y[11],y[10],y[9],y[8],y[7],y[6],y[5], y[4],y[3],y
  [2],y[1]];end;

```

A.13.2 A Function to Calculate Conjugacy Classes in U :

```

#Define indeterminates:
t1:=Indeterminate(GF(2));

```

```

t2:=Indeterminate(GF(2),[t1]);
t3:=Indeterminate(GF(2),[t1,t2]);
t4:=Indeterminate(GF(2),[t1,t2,t3]);
t5:=Indeterminate(GF(2),[t1,t2,t3,t4]);
t6:=Indeterminate(GF(2),[t1,t2,t3,t4,t5]);
t7:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6]);
t8:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7]);
t9:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8]);
t10:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9]);
t11:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10]);
t12:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11]);
t13:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12]);
t14:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13]);
t15:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
]);
t16:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15]);
t17:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16]);
t18:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17]);
t19:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18]);
t20:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18,t19]);
t21:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18,t19,t20]);
t22:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18,t19,t20,t21]);
t23:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18,t19,t20,t21,t22]);
t24:=Indeterminate(GF(2),[t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14
,t15,t16,t17,t18,t19,t20,t21,t22,t23]);

```

```

#Define the  $x_{\alpha}$  in  $U$  (let  $x_{[i]}=x_{\{\alpha_i\}}$  be the element of  $U$ 
associated to the  $i$ -th root)

```

```

x:=[];
x[1]:=UnipotChevElemByRN(U_F4,1,t1);
x[2]:=UnipotChevElemByRN(U_F4,2,t2);
x[3]:=UnipotChevElemByRN(U_F4,3,t3);
x[4]:=UnipotChevElemByRN(U_F4,4,t4);
x[5]:=UnipotChevElemByRN(U_F4,5,t5);
x[6]:=UnipotChevElemByRN(U_F4,6,t6);
x[7]:=UnipotChevElemByRN(U_F4,7,t7);
x[8]:=UnipotChevElemByRN(U_F4,8,t8);
x[9]:=UnipotChevElemByRN(U_F4,9,t9);

```

```

x[10]:=UnipotChevElemByRN(U_F4,10,t10);
x[11]:=UnipotChevElemByRN(U_F4,11,t11);
x[12]:=UnipotChevElemByRN(U_F4,12,t12);
x[13]:=UnipotChevElemByRN(U_F4,13,t13);
x[14]:=UnipotChevElemByRN(U_F4,14,t14);
x[15]:=UnipotChevElemByRN(U_F4,15,t15);
x[16]:=UnipotChevElemByRN(U_F4,16,t16);
x[17]:=UnipotChevElemByRN(U_F4,17,t17);
x[18]:=UnipotChevElemByRN(U_F4,18,t18);
x[19]:=UnipotChevElemByRN(U_F4,19,t19);
x[20]:=UnipotChevElemByRN(U_F4,20,t20);
x[21]:=UnipotChevElemByRN(U_F4,21,t21);
x[22]:=UnipotChevElemByRN(U_F4,22,t22);
x[23]:=UnipotChevElemByRN(U_F4,23,t23);
x[24]:=UnipotChevElemByRN(U_F4,24,t24);

```

```

#####
#x is the list of all x[i]=UnipotChevElemByRN(U_F4,i,ti); <\leq i\leq
    24, y is the element which is to be conjugated by the x[i], and k is
    a natural number telling GAP to conjugate y only by the x[i] with i\
    geq k (i.e. in which subgroup U_i of U one wishes to stay)
#####

```

```

#####
conjclass:=function(x,y,k)
local l, l1, m, m1, j;
l:=0; l1:=0; m:=[]; m1:=[];
for i in [k..24] do m[i]:=CanonicalForm(y^x[i]); od;m:=DuplicateFreeList
(m);
j:=1;l:=Length(m);
while (l=l1)=false do l:=Length(m);
while j in [1..l] do for i in [k..24] do m1[(j-1)*(25-k)+i]:=
CanonicalForm(m[j]^x[i]); od;j:=j+1;od;
m1:=DuplicateFreeList(m1); l1:=Length(m1);m:=m1; od;
return m1; end;
#####

```

A.13.3 Finding Representations of Factorgroups U/V for a Fixed $q = 2^n$ and $V \trianglelefteq U$:

The field \mathbb{F}_{2^n} viewed as additive group is isomorphic to $\mathbb{F}_2[T]/(T^n + 1)$ and all elements of $\mathbb{F}_2[T]/(T^n + 1)$ can be expressed as powers of T . Since the multiplication in U is given by $x_i(t)x_i(s) = x_i(t + s)$ and otherwise just concatenation and some relations, this interpretation of \mathbb{F}_q can be used to define the groups U/V in **GAP**. Elements of U are defined as $x_i(t) = x_i(T^l)$ for some l . To define U/V , at first a free group on $n * \{j \mid x(\alpha_j) \notin V\}$ elements is created. Then a subgroup generated by all the relations needed in U/V is factored out.

```

# k1,k2 as exponents (k1=1 \hat{=} T^0 etc.), x=List[i_1,i_2,...,i_k],
    then get factor group of U spanned by the x_{i_j}(t), factoring out
    all other x_l; let q=2^n.
uni_factor_grp:=function(n,x) local v,w,y,g,m,l,Rel,k1,k2; y:=[];

```

```

v:=FreeGroup(n*Size(x)); g:=GeneratorsOfGroup(v);
Rel:=[[3,1],[1,3,5,0]], [[4,2],[2,4,6,0]], [[4,3],[3,4,7,10]],
[[5,4],[4,5,8,15]],[[6,3],[3,6,9,13]], [[7,1],[1,7,8,0]],
[[7,2],[2,7,9,16]], [[8,2],[2,8,11,20]],[[8,3],[3,8,12,0]],
[[9,1],[1,9,11,0]], [[10,1],[1,10,12,15]],[[10,2],[2,10,13,0]],
[[11,3],[3,11,14,0]], [[12,2],[2,12,14,22]],[[13,1],[1,13,14,18]],
[[13,4],[4,13,16,0]],[[14,4],[4,14,17,24]],[[15,2],[2,15,18,0]],
[[16,1],[1,16,17,20]],[[17,3],[3,17,19,0]],[[18,4],[4,18,20,0]],
[[19,1],[1,19,21,0]],[[20,3],[3,20,21,22]],[[22,4],[4,22,23,0]],
[[23,2],[2,23,24,0]], [[6,5],[5,6,11,18]], [[7,5],[5,7,12,0]],
[[9,5],[5,9,14,0]],[[10,6],[6,10,16,0]], [[11,7],[7,11,17,0]],
[[12,6],[6,12,17,23]],[[14,7],[7,14,19,0]],[[15,6],[6,15,20,0]],
[[16,5],[5,16,19,22]],[[17,5],[5,17,21,0]],[[18,7],[7,18,21,23]],
[[22,6],[6,22,24,0]], [[9,8],[8,9,17,0]], [[11,10],[10,11,19,24]],
[[12,9],[9,12,19,0]],[[13,8],[8,13,19,23]],[[14,8],[8,14,21,0]],
[[15,9],[9,15,21,24]],[[18,10],[10,18,22,0]],[[20,10],[10,20,23,0]],
[[12,11],[11,12,21,0]],[[15,13],[13,15,22,0]],[[20,13],[13,20,24,0]],
[[16,15],[15,16,23,0]],[[18,16],[16,18,24,0]];
for i in [1..n*Size(x)] do y[i]:=g[i]^2; od; # x_i^2=1
l:=n*Size(x)+1;#l counts, how long y is (y[l] is the next empty slot)
#now define the relations, using "Rel". x shall be ordered in such a way
that all  $(2^n)-1$   $x_i(k), k \in \mathbb{F}_q$  are behind one another in a
row, i.e.  $x_1(1), x_1(\dots), \dots, x_2(1), \dots$ . The ordering of the entries
in x is obviously relevant for defining the relations.
for i in [1..Size(x)] do for j in [i..Size(x)] do m:=
PositionFirstComponent(Rel,[x[j],x[i]]);
if m in [1..Size(Rel)] then #if  $x_i, x_j$  do not commute in U
if Rel[m][2][3] in x then #if  $x_i, x_j$  do not commute in  $\langle x_{\{a,b\}}(t) \mid a,b \in x \rangle$  either
if not Rel[m][2][4]=0 then
if Rel[m][2][4] in x then
if i in [1,3,5,7,8,9,11,12,14,17,19,21] then
for k1 in [0..n-1] do for k2 in [0..n-1] do
if k1+k2<n and 2*k1+k2<n then y[l]:=g[(i-1)*n+k1
+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+
k2+1]*g[(Position(x,Rel[m][2][3])-1)*n+(k1+k2)
mod n +1]*g[(Position(x,Rel[m][2][4])-1)*n+(2*
k1+k2) mod n +1];l:=l+1;
elif k1+k2<n and not 2*k1+k2 <n and not 2*k1+k2=2*
n-1 then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*
g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x,
Rel[m][2][3])-1)*n+(k1+k2) mod n +1]*g[(
Position(x,Rel[m][2][4])-1)*n+(2*k1+k2) mod n
+1]*g[(Position(x,Rel[m][2][4])-1)*n+QuoInt(2*
k1+k2,n)+(2*k1+k2)mod n +1];l:=l+1;

```



```

elif k1+k2<n and not 2*k1+k2 <n and 2*k1+k2=2*n-1
    then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+n]*g[(Position(x, Rel[m][2][4]) -1)*n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+2];l:=l+1;
elif not 2*k1+k2=2*n-1 then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][3]) -1)*n+QuoInt(k1+k2, n)+(k1+k2) mod n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+(2*k1+k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+QuoInt(2*k1+k2, n)+(2*k1+k2)mod n +1];l:=l+1;
else y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][3]) -1)*n+QuoInt(k1+k2, n)+(k1+k2) mod n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+n]*g[(Position(x, Rel[m][2][4]) -1)*n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+2];l:=l+1; fi; od; od;
else for k1 in [0..n-1] do for k2 in [0..n-1] do
    if k1+k2<n and k1+2*k2<n then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+(k1+2*k2) mod n +1];l:=l+1;
    elif k1+k2<n and not k1+2*k2 <n and not k1+2*k2=2*n-1 then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+(k1+2*k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+QuoInt(k1+2*k2, n)+(k1+2*k2)mod n +1];l:=l+1;
    elif k1+k2<n and not k1+2*k2 <n and k1+2*k2=2*n-1 then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x, Rel[m][2][4]) -1)*n+n]*g[(Position(x, Rel[m][2][4]) -1)*n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+2];l:=l+1;
    elif not k1+2*k2=2*n-1 then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+(k1+k2)

```

```

    mod n +1]*g[(Position(x, Rel[m][2][3]) -1)*n+
    QuotInt(k1+k2, n)+(k1+k2) mod n +1]*g[(Position(x,
    Rel[m][2][4]) -1)*n+(k1+2*k2) mod n +1]*g[(
    Position(x, Rel[m][2][4]) -1)*n+QuotInt(k1+2*k2, n)
    +(k1+2*k2)mod n +1];l:=l+1;
  else y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)
  *n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m
  ][2][3]) -1)*n+(k1+k2) mod n +1]*g[(Position(x,
  Rel[m][2][3]) -1)*n+QuotInt(k1+k2, n)+(k1+k2) mod
  n+1]*g[(Position(x, Rel[m][2][4]) -1)*n+n]*g[(
  Position(x, Rel[m][2][4]) -1)*n+1]*g[(Position(x,
  Rel[m][2][4]) -1)*n+2];l:=l+1; fi; od; od; fi;
else for k1 in [0..n-1] do for k2 in [0..n-1] do
  if k1+k2<n then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g
  [(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m
  ][2][3]) -1)*n+ (k1+k2) mod n +1];l:=l+1;
  else y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1
  +1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+
  (k1+k2) mod n +1]*g[(Position(x, Rel[m][2][3]) -1)*n+
  QuotInt(k1+k2, n)+(k1+k2) mod n +1];l:=l+1; fi; od; od; fi;
else
for k1 in [0..n-1] do for k2 in [0..n-1] do
  if k1+k2<n then y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i
  -1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3])
  -1)*n+ (k1+k2) mod n +1];l:=l+1;
  else y[l]:=g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g
  [(j-1)*n+k2+1]*g[(Position(x, Rel[m][2][3]) -1)*n+ (k1+k2)
  mod n +1]*g[(Position(x, Rel[m][2][3]) -1)*n+QuotInt(k1+k2, n
  )+(k1+k2) mod n +1];l:=l+1; fi; od; od; fi;
  else for k1 in [0..n-1] do for k2 in [0..n-1] do y[l]:=g[(i-1)*n+k1
  +1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1];l:=l+1; od; od
  ; fi;
  else for k1 in [0..n-1] do for k2 in [0..n-1] do y[l]:=g[(i-1)*n+k1
  +1]*g[(j-1)*n+k2+1]*g[(i-1)*n+k1+1]*g[(j-1)*n+k2+1];l:=l+1; od; od;
  fi; od; od;
w:=NormalClosure(v, Subgroup(v, y)); Display (y);
return v/w; end;
#returns <x[1], ..., x[Size(x)]>/<x_k|k\notin x >.

# then set v:=uni_factor_grp(n, x); and calculate CharacterDegrees(v); or
tbl:=CharacterTable(v); or SizesConjugacyClasses(tbl); or Irr(v); or
LinearCharacters(tbl);

```

A.13.4 The Weyl Group of F_4 :

```
#####
```

```

#The Weyl group as a permutation group, permuting roots.
# s[i] denotes the permutation associated to the i-th root. The Weyl
  group is then the subgroup of the permutation group on 48 elements
  generated by s[1], s[2], s[3], and s[4]. Moreover, the Weyl group
  elements are factorized as products of the generators.
#####
LoadPackage(" unipot ");
LoadPackage(" Floats ");
p:=[];n:=[];e:=[[[]],[[]],[[]],[[]]];a:=[];s:=[];
U_F4:=UnipotChevSubGr("F",4,GF(2));
R_F4:=RootSystem(U_F4);
SimpleSystem(R_F4);
p:=PositiveRootsFC(R_F4);
n:=NegativeRootsFC(R_F4);
#e[i] is the reflection associated to the i-th root, written as a matrix
  (so  $e[k](v)=v-\langle v,p[k]\rangle/\langle p[k],p[k]\rangle p[k]$ ) for  $v\in\mathbb{C}^4$  in a
  basis given by the simple roots
for k in [1..24] do
e[k]:=[[[]],[[]],[[]],[[]]];
e[k][1]:=[1,0,0,0]-Int(2*(-0.5*p[k][3]+p[k][1]))/(p[k][1]*(p[k][1]-0.5*p[k]
  [3])+p[k][2]*(2*p[k][2]-p[k][4])+p[k][3]*(p[k][3]-p[k][4]-0.5*p[k]
  [1])+p[k][4]*(2*p[k][4]-p[k][2]-p[k][3]))*p[k];
e[k][2]:=[0,1,0,0]-Int(2*(2*p[k][2]-p[k][4]))/(p[k][1]*(p[k][1]-0.5*p[k]
  [3])+p[k][2]*(2*p[k][2]-p[k][4])+p[k][3]*(p[k][3]-p[k][4]-0.5*p[k]
  [1])+p[k][4]*(2*p[k][4]-p[k][2]-p[k][3]))*p[k];
e[k][3]:=[0,0,1,0]-Int(2*(-0.5*p[k][1]-p[k][4]+p[k][3]))/(p[k][1]*(p[k]
  [1]-0.5*p[k][3])+p[k][2]*(2*p[k][2]-p[k][4])+p[k][3]*(p[k][3]-p[k]
  [4]-0.5*p[k][1])+p[k][4]*(2*p[k][4]-p[k][2]-p[k][3]))*p[k];
e[k][4]:=[0,0,0,1]-Int(2*(2*p[k][4]-p[k][2]-p[k][3]))/(p[k][1]*(p[k]
  [1]-0.5*p[k][3])+p[k][2]*(2*p[k][2]-p[k][4])+p[k][3]*(p[k][3]-p[k]
  [4]-0.5*p[k][1])+p[k][4]*(2*p[k][4]-p[k][2]-p[k][3]))*p[k];#ergibt
  das Transponierte, d.h. e[k][1] ist der erste Spaltenvektor
e[k]:=TransposedMat(e[k]);#TransposedMatMutable gives again a mutable
  matrix
od;
#Now define the permutation of the roots associated to each reflection (
  GAP can compute much faster with permutations than with matrices)
for k in [1..24] do
for i in [1..24] do for j in [1..24] do
if e[k]*p[i]=p[j] then a[i]:=j; a[i+24]:=j+24; fi;
if e[k]*p[i]=n[j] then a[i]:=j+24; a[i+24]:=j; fi; od; od;
s[k]:=PermList(a); od;
g:=Group(s[1],s[2],s[3],s[4]);
#denote the Weyl group elements associated to the simple roots by s1,...,
  s4

```

```

hom:=EpimorphismFromFreeGroup(g:names=["s1","s2","s3","s4"]);
a:=[];
for i in [1..24] do a[i]:=Factorization(g,s[i]);od;
#####

#####

#Action of a Weyl group element s on a n-tupel x of roots (given as
  numbers, so "i" for the ith root). orb_on_tupels gives the orbit
  of a tupel of roots under the Weyl group action intersected with U+.
#####
on_tupels:=function(x,s) local h,n; h:=[];n:=Length(x); for i in [1..n]
  do h[i]:=x[i]^s;od;return h; end;
orb_on_tupels:=function(x) local a,l,k,h,n,m; k:=1;h:=[];a:=[]; n:=Length
(x);m:=0;
a:=Orbit(g,x,on_tupels); l:=Length(a);
for i in [1..l] do for j in [1..n] do if a[i][j]>24 then m:=m+1;
  continue;else if j=n then if m=0 then h[k]:=a[i];k:=k+1;fi;fi;fi;od;m
:=0;od; return h;end;
#Membership test: returns 1 if x in orbit of y, 0 otherwise (x element, y
  an element-> x in orbit of y?)
in_test:=function(x,y) local z,k; k:=0;z:=[]; z:=Orbit(SymmetricGroup(
  Length(x)),x,Permuted); for i in [1..Length(z)] do if \in(z[i],
  orb_on_tupels(y))=true then k:=k+1;fi;od; return k;end;
#####

```

A.13.5 Bruhat Normal Form:

```

#Algo I 6.15 (Koehler): Bruhat decomposition of parametrised elements
# A,u are paramterized elements of the form  $u' \hat{\{w\}} hu$ ,  $u' \in U_w$ ,  $w \in W$ ,  $t \in T$ ,  $u \in U$  (so  $A[1] \in U_w$ ,  $A[2] \in W$ ,  $A[3] \in T$ ,  $A[4] \in U \rightarrow A[3]=h([\lambda_1, \lambda_2, \lambda_3, \lambda_4])$ ),  $P$  is a list of
  parameters, encoding whether the indeterminate  $t[i]$  may be zero or
  not. I.e.  $P[i]=1$  means  $t[i] \in k^*$ , and  $P[i]=2$  means  $t[i] \in k$ ;  $g$ 
  is the Weyl group. Then Bruhat_Decompara(A,P,u,g) returns the
  Bruhat decomposition of  $Au$ . If this requires a case-by-case study,
  this is done and the function returns a list of Bruhat
  decompositions for the different cases.
Bruhat_Decompara:=function(A,P,u,g)
local i, erg, list ,X,Px,x; list:=[[StructuralCopy(A),StructuralCopy(u)
  ],StructuralCopy(P)]; erg:=[]; i:=1;
#if a case-by-case study is required, the Bruhat decomposition is
  computed seperatly for each of the cases list[i] and list[i+1]
while not i>Length(list) do
X:=list[i][1][1]; x:=list[i][1][2];Px:=list[i][2];
X:=Bruhat_Decompara(X,x,i,list ,g);#g is the Weyl group
Add(erg,[X,list[i][2]]); i:=i+1;od;return erg;end;

```

```

#Algo I 5.8 (Koehler): Bruhat form of  $g_1 g_2$  (for  $g_1, g_2$  in Bruhat
form)
#  $g_i = u_i' \hat{w}_i h_{i,u_i}$ , and  $g_i$  is saved as a tuple  $(u', w, h, u)$ . Here
 $u, u'$  in  $U$ , so they can be written uniquely as  $x_1(u_1) \dots x_{24}(u_{24})$ . For the sake of efficiency save only the non-zero entries as a
tuple  $[[i_1, \dots, i_n], [u_{i_1}, \dots, u_{i_n}]]$ .  $w \in W$  is saved
as a permutation of the roots and  $h \in T$  as a tuple  $(\lambda_1, \dots, \lambda_4)$ . list[i][2] is a list of parameters, encoding for each
indeterminate  $t[i]$  which values it may take. If different cases have
to be considered, list can be elongated accordingly
Bruhat_Decomp:=function(g1,g2,n,list,g)
local liste,erg,Rel,l,w_2; liste:=[]; erg:=g1;l:=[0,list[n][2]]; w_2:=g2
[2];
#Relations between the elements of the unipotent subgroup
Rel:=[[[3,1],[1,3,5,0]], [[4,2],[2,4,6,0]], [[4,3],[3,4,7,10]],
[[5,4],[4,5,8,15]],[[6,3],[3,6,9,13]], [[7,1],[1,7,8,0]],
[[7,2],[2,7,9,16]], [[8,2],[2,8,11,20]],[[8,3],[3,8,12,0]],
[[9,1],[1,9,11,0]], [[10,1],[1,10,12,15]],[[10,2],[2,10,13,0]],
[[11,3],[3,11,14,0]], [[12,2],[2,12,14,22]],[[13,1],[1,13,14,18]],
[[13,4],[4,13,16,0]],[[14,4],[4,14,17,24]],[[15,2],[2,15,18,0]],
[[16,1],[1,16,17,20]],[[17,3],[3,17,19,0]],[[18,4],[4,18,20,0]],
[[19,1],[1,19,21,0]],[[20,3],[3,20,21,22]],[[22,4],[4,22,23,0]],
[[23,2],[2,23,24,0]], [[6,5],[5,6,11,18]], [[7,5],[5,7,12,0]],
[[9,5],[5,9,14,0]],[[10,6],[6,10,16,0]], [[11,7],[7,11,17,0]],
[[12,6],[6,12,17,23]],[[14,7],[7,14,19,0]],[[15,6],[6,15,20,0]],
[[16,5],[5,16,19,22]],[[17,5],[5,17,21,0]],[[18,7],[7,18,21,23]],
[[22,6],[6,22,24,0]], [[9,8],[8,9,17,0]], [[11,10],[10,11,19,24]],
[[12,9],[9,12,19,0]],[[13,8],[8,13,19,23]],[[14,8],[8,14,21,0]],
[[15,9],[9,15,21,24]],[[18,10],[10,18,22,0]],[[20,10],[10,20,23,0]],
[[12,11],[11,12,21,0]],[[15,13],[13,15,22,0]],[[20,13],[13,20,24,0]],
[[16,15],[15,16,23,0]],[[18,16],[16,18,24,0]]];
Append(g1[4][1],g2[1][1]);Append(g1[4][2],g2[1][2]);
erg[4]:=Coll_from_left([g1[4][1],g1[4][2]],Rel);#  $u_1 u_2'$  has to brought
into the right shape via "Collecting from the left".(Sims, S. 401ff)
w_2:=w_fac(w_2,g);# $w_2$  written as a word in the  $s[i]$ ,  $i$  in  $[1..4]$ .
for i in [1..Size(w_2)] do erg:=Cases_i(erg,w_2[i],Rel,1);# $i:=i-1$ ;
while not l[1]=0 do Display([" $x_k(t)u'$ "=,l[1]]);
liste:=InputFromUser("Which paramters need to be changed (as
list)?");
Add(list,StructuralCopy(list[n])); #for each new case the
parameters need to be adapted.
for j in [1..Size(liste)] do
list[n][2][liste[j]]:=InputFromUser(["new ",liste[j]," th
parameter="]);

```

```

list [Size(list)][2][liste[j]]:=InputFromUser(["other new",liste
[j],"th parameter="]);od;#Wenn ein Parameter 0 wird, sollte
das vielleicht auch in list[?][1] eingetragen werden:
Display(["erg=",erg]); liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed in erg[4]?(as list)");
for j in [1..Size(liste)] do
    erg[4][2][liste[j]]:=InputFromUser(["erg[4]",liste[j]]);od;
    liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed in erg[1]?");
for j in [1..Size(liste)] do
    erg[1][2][liste[j]]:=InputFromUser(["erg[1]",liste[j]]);od;
    liste:=[];
Display(["u=",g2]); liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed in u[4]?(as list)");
for j in [1..Size(liste)] do
    g2[4][2][liste[j]]:=InputFromUser(["u[4]",liste[j]]);od;
    liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed in u[1]?");
for j in [1..Size(liste)] do
    g2[1][2][liste[j]]:=InputFromUser(["u[1]",liste[j]]);od;
    liste:=[];
Display(["And now the new case",list[Size(list)]]);
liste:=InputFromUser("In which positions does something have to
be changed here? (list(Size(list))[1][1][1])");#X[1]
for j in [1..Size(liste)] do
    list[Size(list)][1][1][1][2][liste[j]]:=InputFromUser(["
list(Size(list))[1][1][1]",liste[j]]);od;liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed here? (list(Size(list))[1][1][4])");#X[4]
for j in [1..Size(liste)] do
    list[Size(list)][1][1][4][2][liste[j]]:=InputFromUser(["
list(Size(list))[1][4]",liste[j]]);od;liste:=[];
liste:=InputFromUser("In which positions does something have to
be changed here? (list(Size(list))[1][2][1])");#x[1]
for j in [1..Size(liste)] do
    list[Size(list)][1][2][1][2][liste[j]]:=InputFromUser(["
list(Size(list))[1][2][1]",liste[j]]);od;
liste:=InputFromUser("In which positions does something have to
be changed here? (list(Size(list))[1][2][4])");#x[4]
for j in [1..Size(liste)] do

```

```

        list[Size(list)][1][2][4][2][liste[j]]:=InputFromUser(["
            list(Size(list))[1][4]",liste[j]);od;
        #list[Size(list)][1] has the same format as erg, i.e. lists
            of lists..
    erg[4]:=Coll_from_left(erg[4],Rel); erg[1]:=Coll_from_left(erg[1],
        Rel);l:=[0,list[n][2]];
    Display(erg);
    erg:=Cases_i(erg,w_2[i],Rel,1);
    od;
od;#case-by-case study for w_2
for i in [1..4] do erg[3][i]:=erg[3][i]*g2[3][i];od;
erg[4]:=Torus_act(erg[4],g2[3]);
Append(erg[4][1],g2[4][1]);Append(erg[4][2],g2[4][2]);
erg[4]:=Coll_from_left([erg[4][1],erg[4][2]],Rel);
erg[1]:=Coll_from_left([erg[1][1],erg[1][2]],Rel);
return erg; end;

#Collection from the left: (Sims, S.401,403..)
#brings an element u in U in normal form. i.e.  $u=x_1(u_1)\dots x_{24}(u_{24})$  and removes any  $x_i(0)$ .
Coll_from_left:=function(u,Rel)
local l,n,i,j; l:=Size(u[1]); i:=2;j:=0;
while i in [2..Size(u[1])] do
    if u[1][i]<u[1][i-1] then n:=PositionFirstComponent(Rel,[u[1][i-1],u
        [1][i]]);
        if n in [1..Size(Rel)] then Add(u[1],u[1][i-1],i+1); Add(u[2],u
            [2][i-1],i+1);
            Remove(u[1],i-1);Remove(u[2],i-1);
            Add(u[1],Rel[n][2][3],i+1);Add(u[2],u[2][i-1]*u[2][i],i
                +1);
            if not Rel[n][2][4]=0 then
                if u[1][i-1] in [1,3,5,7,8,9,11,12,14,17,19,21] then
                    Add(u[1],Rel[n][2][4],i+2);Add(u[2],u[2][i-1]^2*u[2][i
                        ],i+2);
                else Add(u[1],Rel[n][2][4],i+2);Add(u[2],u[2][i-1]*u[2][i
                    ]^2,i+2);fi;
                if i+2<Size(u[1]) then
                    if u[1][i+2]=u[1][i+3] then u[2][i+2]:=u[2][i+2]+u[2][i
                        +3];
                        Remove(u[2],i+3);Remove(u[1],i+3);
                    if u[2][i+2]=0 then Remove(u[2],i+2);Remove(u[1],i
                        +2);fi;fi;fi;fi;
                if i+1<Size(u[1]) then
                    if u[1][i+1]=u[1][i+2] then u[2][i+1]:=u[2][i+1]+u[2][i+2];
                        Remove(u[2],i+2);Remove(u[1],i+2);

```

```

        if u[2][i+1]=0 then Remove (u[2], i+1); Remove(u[1], i+1); fi;
        fi; fi;
    else Add(u[1], u[1][i-1], i+1); Add(u[2], u[2][i-1], i+1);
        Remove (u[1], i-1); Remove(u[2], i-1); fi;
#jetzt noch Identitäten (also z.B. x_1(0)) rausschmeißen
    if i < Size(u[1]) then
        if u[1][i]=u[1][i+1] then u[2][i]:=u[2][i]+u[2][i+1];
            Remove(u[2], i+1); Remove(u[1], i+1); fi; fi;
        if u[2][i]=0 then Remove (u[2], i); Remove(u[1], i);
        elif u[1][i]=u[1][i-1] then u[2][i-1]:=u[2][i-1]+u[2][i];
            Remove(u[2], i); Remove(u[1], i); fi;
        if u[2][i-1]=0 then Remove (u[2], i-1); Remove(u[1], i-1);
        elif i > 2 then
            if u[1][i-1]=u[1][i-2] then u[2][i-2]:=u[2][i-2]+u[2][i-1];
                Remove(u[2], i-1); Remove(u[1], i-1);
            if u[2][i-2]=0 then Remove (u[2], i-2); Remove(u[1], i-2); fi;
            fi; fi;
        i:=2;
    else if u[2][i]=0 then Remove (u[2], i); Remove(u[1], i); j:=j+1;
        elif u[1][i]=u[1][i-1] then u[2][i-1]:=u[2][i-1]+u[2][i];
            Remove(u[2], i); Remove(u[1], i); j:=j+1; fi;
        if u[2][i-1]=0 then Remove (u[2], i-1); Remove(u[1], i-1); j:=j+1; fi;
        ;
        i:=i+1-j; j:=0; fi; od;
l:=Size(u[1]);
for i in [0..l-1] do if u[2][l-i]=0 then Remove (u[2], l-i); Remove(u[1], l-i); fi; od;
return u; end;

#Action of Torus on U. m[i] describes action of h on x_i, i.e. x_i(u)^h=
x_i(m[i]*u)
Torus_act:=function(u, h)
local m; m:=[h[1]^2*h[3]^(-1), h[2]^2*h[4]^(-1), h[1]^(-1)*h[3]^2*h[4]^(-1), h[2]^(-1)*h[3]^(-2)*h[4]^2, h[1]*h[3]*h[4]^(-1), h[2]*h[3]^(-2)*h[4], h[1]^(-1)*h[2]^(-1)*h[4], h[1]*h[2]^(-1)*h[3]^(-1)*h[4], h[1]^(-1)*h[2], h[1]^(-2)*h[2]^(-1)*h[3]^2, h[1]*h[2]*h[3]^(-1), h[2]^(-1)*h[3], h[1]^(-2)*h[2]*h[3]^2*h[4]^(-1), h[2]*h[3]*h[4]^(-1), h[1]^2*h[2]^(-1), h[1]^(-2)*h[4], h[3]^(-1)*h[4], h[1]^2*h[2]*h[4]^(-1), h[1]^(-1)*h[3], h[1]^2*h[3]^(-2)*h[4], h[1], h[3]^2*h[4]^(-1), h[2]^(-1)*h[4], h[2]];
for i in [1..Size(u[1])] do
    for j in [1..24] do
        if u[1][i]=j then u[2][i]:=u[2][i]*m[j]; fi; od; od;
return u; end;

```



```

#factorization of w into a product of reflections associated to the
  simple roots.
w_fac:=function(w,g)
  local v,u,hom;
hom:=EpimorphismFromFreeGroup(g:names:=["s1","s2","s3","s4"]);
v:=[];u:=Factorization(g,w);#In Listenform bringen
for i in [1..Length(u)] do
  for j in [1..4] do
    if Subword(u,i,i)=PreImagesRepresentative(hom,s[j])^(-1) then v[i
      ]:=j;fi;od;od;
#v:=Coll_from_left_W(v);
return v;end;

#brings element w in W into form w=s_{i_1}...s_{i_r}
Coll_from_left_W:=function(w)
  local l,i,Rel2; l:=Size(w);i:=2;
#Relations for W
Rel2:=[[[3,1],[1,5]],[[4,2],[2,6]],[[4,3],[3,10]],[[5,4],[4,8]],[
  [[6,3],[3,13]],[[7,1],[1,8]],[[7,2],[2,9]],[[8,2],[2,11]],[
  [[8,3],[3,12]],[[9,1],[1,11]],[[10,1],[1,15]],[[10,2],[2,13]],[
  [[11,3],[3,14]],[[12,2],[2,14]],[[13,1],[1,18]],[[13,4],[4,16]],[
  [[14,4],[4,17]],[[15,2],[2,18]],[[16,1],[1,20]],[[17,3],[3,19]],[
  [[18,4],[4,20]],[[19,1],[1,21]],[[20,3],[3,22]],[[22,4],[4,23]],[
  [[23,2],[2,24]],[[6,5],[5,18]],[[7,5],[5,12]],[[9,5],[5,14]],[
  [[10,6],[6,16]],[[11,7],[7,17]],[[12,6],[6,17]],[[14,7],[7,19]],[
  [[15,6],[6,20]],[[16,5],[5,22]],[[17,5],[5,21]],[[18,7],[7,23]],[
  [[22,6],[6,24]],[[9,8],[8,17]],[[11,10],[10,19]],[[12,9],[9,19]],[
  [[13,8],[8,23]],[[14,8],[8,21]],[[15,9],[9,24]],[[18,10],[10,22]],[
  [[20,10],[10,23]],[[12,11],[11,21]],[[15,13],[13,22]],[
  [[20,13],[13,24]],[[16,15],[15,23]],[[18,16],[16,24]],[
  [[5,1],[1,3]],[[6,2],[2,4]],[[10,3],[3,4]],[[8,4],[4,5]],[
  [[13,3],[3,6]],[[8,1],[1,7]],[[9,2],[2,7]],[[11,2],[2,8]],[
  [[12,3],[3,8]],[[11,1],[1,9]],[[15,1],[1,10]],[[13,2],[2,10]],[
  [[14,3],[3,11]],[[14,2],[2,12]],[[18,1],[1,13]],[[16,4],[4,13]],[
  [[17,4],[4,14]],[[18,2],[2,15]],[[20,1],[1,16]],[[19,3],[3,17]],[
  [[20,4],[4,18]],[[21,1],[1,19]],[[22,3],[3,20]],[[23,4],[4,22]],[
  [[24,2],[2,23]],[[18,5],[5,6]],[[12,5],[5,7]],[[14,5],[5,9]],[
  [[16,6],[6,10]],[[17,7],[7,11]],[[17,6],[6,12]],[[19,7],[7,14]],[
  [[20,6],[6,15]],[[22,5],[5,16]],[[21,5],[5,17]],[[23,7],[7,18]],[
  [[24,6],[6,22]],[[17,8],[8,9]],[[19,10],[10,11]],[[19,9],[9,12]],[
  [[23,8],[8,13]],[[21,8],[8,14]],[[24,9],[9,15]],[[22,10],[10,18]],[
  [[23,10],[10,20]],[[21,11],[11,12]],[[22,13],[13,15]],[
  [[24,13],[13,20]],[[23,15],[15,16]],[[24,16],[16,18]]];
while i in [2..l] do
  if w[i]<w[i-1]then n:=PositionFirstComponent(Rel2,[w[i-1],w[i]]);

```

```

    if n in [1..Size(Rel2)] then Remove (w,i-1); Add (w,Rel2[n][2][2], i
    );
    else Add(w,w[i-1],i+1); Remove (w,i-1); fi;
    if i<1 then if w[i]=w[i+1] then Remove (w,i+1); Remove (w,i); fi; fi
    ;
    if i>2 then if w[i-1]=w[i-2] then Remove (w,i-1);Remove(w,i-2); fi;
    fi;
    i:=2;
    else i:=i+1;fi;od;
return w; end;

#Case-by-case study for w.
Cases_i:=function(erg,k,Rel,n)
local u,v,l, coroot_coef; u:=StructuralCopy(erg[4]);l:=0;
#Coefficients of coroots expressed as combination of simple coroots.
coroot_coef:=[[1,0,0,0],[0,1,0,0], [0,0,1,0],[0,0,0,1],
[1,0,1,0],[0,1,0,1],[0,0,1,2],[1,0,1,2],
[0,2,1,2],[0,0,1,1],[1,2,1,2],[1,0,2,2],
[0,1,1,1],[1,2,2,2],[1,0,1,1],[0,1,1,2],
[1,2,2,4],[1,1,1,1],[1,2,3,4],[1,1,1,2],
[2,2,3,4],[1,1,2,2],[1,1,2,3],[1,2,2,3]];
if \in(k,u[1]) then
    if k^(erg[2]^(-1)) in [1..24] then v:=xv(u,k,Rel); erg[4]:=v[2];#u=x_k
    (t)u''
    v[1]:=Torus_act([[v[1][1]],[v[1][2]]],[erg[3][1]^(-1),erg[3][2]^(-1),
    erg[3][3]^(-1),erg[3][4]^(-1)]);
    v[1][1]:=on_tupels(v[1][1],erg[2]^(-1));
    Append(erg[1][1],v[1][1]); Append(erg[1][2],v[1][2]);
    erg[1]:=Coll_from_left(erg[1],Rel);#u'->u'(x_k(t))^(wh)^(-1)
    erg[2]:=erg[2]*s[k]; #w->w*s[k]
    erg[3]:=W_act_T(erg[3],s[k]); #h->h.s[k]
    v:=v1v2(erg,k,Rel);#u'(x_k(t))^(wh)^(-1)=u_1'*u_2', u_1' in U_{ws[k]
    }, u_2' not
    v[2]:=Torus_act([on_tupels(v[2][1],erg[2]),v[2][2]],erg[3]);
    erg[4][1]:=on_tupels(erg[4][1],s[k]);
    Append(v[2][1],erg[4][1]);
    Append(v[2][2],erg[4][2]);
    erg[4]:=Coll_from_left(v[2],Rel); #u''->(u_2')^(ws[k]*h.s[k])(u'')^s[
    k]
    erg[1]:=v[1];#u_1'#case ii)
    n[1]:=0;
    else v:=xv(u,k,Rel); Display ("Can this be zero?",v[1][2]); Display
    ("These are the parameters:",n[2]);n[1]:=InputFromUser("ok? If
    yes, please type in 0, else type in 1");#u=x_k(t)u'' #n[2] are the
    parameters
    if n[1]=0 then erg[4]:=v[2];#i.e. everything ok

```

```

v[1]:=Torus_act([[v[1][1]+24],[v[1][2]^(-1)]],[erg[3][1]^(-1),erg
  [3][2]^(-1),erg[3][3]^(-1),erg[3][4]^(-1)]);
v[1][1]:=on_tupels(v[1][1],erg[2]^(-1));
Append(erg[1][1],v[1][1]); Append(erg[1][2],v[1][2]);
erg[1]:=Coll_from_left(erg[1],Rel);#u'->u'(x_k(t^(-1)))^((wh)
  ^(-1))
  for i in [1..4] do erg[3][i]:=erg[3][i]*((-v[1][2][1])^(-
    coroot_coef[k][i]));od;
v[1]:=Torus_act([v[1][1],[v[1][2][1]^(-1)]],[(-v[1][2][1])^(-
  coroot_coef[k][1]),(-v[1][2][1])^(-coroot_coef[k][2]),(-v
  [1][2][1])^(-coroot_coef[k][3]),(-v[1][2][1])^(-coroot_coef[k
  ][4])]);
erg[4][1]:=on_tupels(erg[4][1],s[k]);
Append(v[1][1],erg[4][1]);
Append(v[1][2],erg[4][2]);
erg[4]:=v[1];
erg[4]:=Coll_from_left(erg[4],Rel);
v:=v1v2(erg,k,Rel);#u'(x_k(t))^((wh)^(-1))=u_1'*u_2', u_1' in U_{
  ws[k]}, u_2' not
v[2]:=Torus_act([on_tupels(v[2][1],erg[2]),v[2][2]],erg[3]);
Append(v[2][1],erg[4][1]);
Append(v[2][2],erg[4][2]);
erg[4]:=Coll_from_left(v[2],Rel); #u''->(u_2')^(ws[k]*h.s[k])(u''
  ^s[k]
erg[1]:=v[1];#u_1'
n[1]:=0;
else n[1]:=v;#return case-by-case study
fi;fi;#case iii)
else if k^(erg[2]^(-1)) in [1..24] then erg[2]:=erg[2]*s[k];
erg[3]:=W_act_T(erg[3],s[k]);
else erg[2]:=erg[2]*s[k];
  for i in [1..4] do erg[3][i]:=(-1)^(coroot_coef[k][i])*W_act_T
    (erg[3],s[k])[i];od;fi;
v:=v1v2(erg,k,Rel);
v[2]:=Torus_act([on_tupels(v[2][1],erg[2]),v[2][2]],erg[3]);
Append(v[2][1],on_tupels(erg[4][1],s[k]));
Append(v[2][2],erg[4][2]);
erg[4]:=Coll_from_left(v[2],Rel);
erg[1]:=v[1];n[1]:=0;fi;
#case i), should be fine
return erg;end;

#Weyl group action (conjugation) on torus, returns h^w=w^(-1)hw
# h=(\lambda_1,..,\lambda_4). For all possible coefficients (a,b,c,d) of
  roots, (\lambda_1^a*\lambda_2^b*\lambda_3^c*\lambda_4^d)\overset
  {!}{=}{\mu_1^{(c-a)}*\mu_2^b*\mu_3^c*\mu_4^d} ->Generalise this for w

```

```

in W: Write \alpha_j.w in terms of coefficients of simple roots (j
=1..4)-> get List a_j. Then \mu_j=\lambda_1^a_j[1]*..\lambda_4^a_j
[4].
#PositiveRootsFC(R_F4); gives coefficients of i-th root in terms of
simple roots.
W_act_T:=function(h,w)
local m,h1; m:=[[ ],[ ],[ ],[ ]];h1:=[];
for i in [1..4] do
if i^w in [1..24] then m[i]:=PositiveRootsFC(R_F4)[i^w];
else m[i]:=-PositiveRootsFC(R_F4)[i^w-24];fi;
h1[i]:=h[1]^(m[i][1])*h[2]^(m[i][2])*h[3]^(m[i][3])*h[4]^(m[i][4]);od
;
return h1;end;

#writing erg[1] as v_1*v_2, v_1\in U_{erg[2]}, v_2^{erg[2]}\in U, erg
[1]:=v_1,erg[4]:=v_2^{erg[2]*erg[3]}*[on_tupels(erg[4][1],s[k]),erg
[4][2]]
# erg[2] is old erg[2]*s[k], erg[1] in U_{old erg[2]};
v1v2:=function(erg,k,Rel)
local v1,v2,u,l1,l2,i,j,l; u:=StructuralCopy(erg[1]); v1:=[];v2
:=[[ ],[ ]];i:=1;j:=1;
while i in [1..Size(u[1])] do
if u[1][i]^erg[2] in [1..24] then j:=i;
while j in [i..Size(u[1])-1]do#altes u[1][j], dass in v2 soll,
kommt an j+1. Stelle
l1:= PositionFirstComponent(Rel,[u[1][j+1],u[1][j]] );
l2:= PositionFirstComponent(Rel,[u[1][j],u[1][j+1]] );
if l1 in [1..Size(Rel)] then Add (u[1],u[1][j],j+2); Add (u
[2],u[2][j],j+2);
Remove(u[1],j);Remove(u[2],j);
Add (u[1],Rel[l1][2][3],j+2); Add(u[2],u[2][j]*u[2][j
+1],j+2);
if not Rel[l1][2][4]=0 then
if u[1][j] in [1,3,5,7,8,9,11,12,14,17,19,21]then
Add(u[1],Rel[l1][2][4],j+3);Add(u[2],u[2][j]^2*u
[2][j+1],j+3);
else Add(u[1],Rel[l1][2][4],j+3);Add(u[2],u[2][j]*u
[2][j+1]^2,j+3);fi;fi;
elif l2 in [1..Size(Rel)] then Add (u[1],u[1][j],j+2); Add (
u[2],u[2][j],j+2);
Remove(u[1],j);Remove(u[2],j);
Add (u[1],Rel[l2][2][3],j+2); Add(u[2],u[2][j]*u[2][j
+1],j+2);
if not Rel[l2][2][4]=0 then
if u[1][j] in [1,3,5,7,8,9,11,12,14,17,19,21]then

```

```

        Add(u[1], Rel[12][2][4], j+3); Add(u[2], u[2][j]^2*u
            [2][j+1], j+3);
        else Add(u[1], Rel[12][2][4], j+3); Add(u[2], u[2][j]*u
            [2][j+1]^2, j+3); fi; fi;
        else Add(u[1], u[1][j], j+2); Add(u[2], u[2][j], j+2);
            Remove(u[1], j); Remove(u[2], j); fi; j:=j+1; od;
Add(v2[1], u[1][Size(u[1])], 1); 1:=u[2][Size(u[1])];
Add(v2[2], 1); v2[2]:=Reversed(v2[2]); #Add(v2[2], u[2][Size(u[1])
], 1);
Remove(u[2], Size(u[1])); Remove(u[1], Size(u[1]));
    else i:=i+1; fi; od; v1:=u;
v2:=Coll_from_left(v2, Rel); v1:=Coll_from_left(v1, Rel);
return [v1, v2]; end;

```

#write $u=x_k(t)*v$ s.t. no x_k turns up anymore in v .

```

xv:=function(u, k, Rel)
local l, n, x, v; l:=Position(u[1], k); n:=0; #v:=[]; for i in [1..Size(u)] do
    v[i]:=u[i]; od;
while l in [2..Size(u[1])] do n:=PositionFirstComponent(Rel, [u[1][1], u
[1][l-1]]);
    if n in [1..Size(Rel)] then Add(u[1], u[1][l-1], l+1); Remove(u[1],
l-1);
        Add(u[2], u[2][l-1], l+1); Remove(u[2], l-1);
        Add(u[1], Rel[n][2][3], l+1); Add(u[2], u[2][1]*u[2][l-1], l+1);
        if not Rel[n][2][4]=0 then
            if u[1][1] in [1, 3, 5, 7, 8, 9, 11, 12, 14, 17, 19, 21] then Add
                (u[1], Rel[n][2][4], l+2);
                Add(u[2], u[2][1]^2*u[2][l-1], l+2);
            else Add(u[1], Rel[n][2][4], l+2); Add(u[2], u[2][1]*u
                [2][l-1]^2, l+2); fi; fi;
        else Add(u[1], u[1][l-1], l+1); Remove(u[1], l-1);
            Add(u[2], u[2][l-1], l+1); Remove(u[2], l-1); fi;
        l:=l-1; od;
x:=[u[1][1], u[2][1]]; Remove(u[1], 1); Remove(u[2], 1);
u:=Coll_from_left(u, Rel);
return [x, u]; end;

```

#defines indeterminate $no.i$ over a field k

```

ite_Variable:=function(k, i)
local ti; ti:=Indeterminate(k, i); return ti; end;

```

A.13.6 Conjugacy Tests:

```

#defines indeterminate  $t[i]$  over field  $k$ , and the indeterminate number
of  $t[i]$  really is  $i$ , i.e.  $t[i]=x_i$ 
ite_Variable:=function(k, i)
local ti; ti:=Indeterminate(k, i); return ti; end;

```

```

# for an element w of the Weyl group g, this returns the set U_w (as a
  list of i_j\in [1..24])
U_w:=function(w,g)
  local u_w,l; u_w:=[];l:=1;
  for i in [1..24] do if i^w in [25..48] then u_w[l]:=i; l:=l+1; fi;od;
  return u_w;end;

#analyses the generators and returns a list s containing only the n
  indeterminates turning up in the generators, gen are the generators,
  t is the list of indeterminates (i.e. t[i] really is the
  indeterminate no. i.)
WhichIndets:=function(gen,t)
  local n,s1,i,g,k,h,o,u; u:=[];s1:=[]; i:=Size(gen);
  for j in [1..i] do g:=ExtRepNumeratorRatFun(gen[j]); #make sure that
    polynomial is represented in the desired way (i.e g=[mon, coeff, mon,
    coeff,...] list of monomials with corresponding coefficients. Each
    monomial is then given as [indetnr, exp, indetnr, exp,...] where
    indetnr is the number of an indeterminate and exp the corresponding
    exponent).
    k:=Size(g);
    for a in [1..k/2] do h:=g[2*a-1];#now analyse each monomial h of g
      for a1 in [1..(Size(h))/2] do o:=Position(u,h[2*a1-1]);
        if not o in [1..Size(u)] then Add(u,h[2*a1-1]); fi;
      od;od;od;
  #u only knows the number of the indeterminate. In s1, one might want to
    replace each number by the corresponding indeterminate, such that one
    has a list of indeterminates one can give to the vanishing_set
    function.
  for j in [1..Size(t)] do for a in [1..Size(u)] do if Indeterminate(GF(2)
    ,u[a])=t[j] then s1[a]:=t[j]; fi;od;od; # remark: by construction t[i
    ] really is the indeterminate no. i.
  n:=Size(s1);
  return [n,s1];end;

# Let GAP calculate generators of ideal for each w in U1, then display
  these and let user type in appropriate values. gen are the generators
  , as given by ideal_BuuB2,
vanishing_set_by_user:=function(gen,l,t,u_w)
  local o,o2,m,i,n,t2,c,o3,t3,j,c1,c2,g1,vgen;o:=[];m:=0;n:=0;c:=1;o2:=[];
  o3:=[];t3:=[];j:=0;c1:=0;g1:=[];vgen:=0;
  Sort(gen);
  for k in [1..Size(gen)] do gen[k]:=Value(gen[k],t3,o3);
    if gen[k]=Z(2)^0 then return 0; fi;
    if IsRationalFunction(gen[k])=true then
      g1:=ExtRepNumeratorRatFun(gen[k]);
      if Size(g1)=2 or Size(g1)=1 then

```

```

    if Value(gen[k],[25,26,27,28,53],[Z(2)^0,Z(2)^0,Z(2)^0,Z(2)^0,Z
      (2)^0])=Z(2)^0 then return 0; fi; fi;
  #ExtRepNumeratorRatFun(gen[k]) gives the external representation of
    the numerator of the rational function gen[k], i.e. g is a list
    g=[mon, coeff, mon, coeff,...] where "mon" is a monomial and the
    following "coeff" its coefficient. So ask whether gen[k] is a
    monomial. If this is the case and Value(gen[k
      ],[25,26,27,28,53],[1,1,1,1,1])=1, then can indeed return 0.
  #teach GAP to see that from e.g. gen[k]=t[25]*t[4] follows t[4]=0:
  vgen:=Value(gen[k],[25,26,27,28,53],[Z(2)^0,Z(2)^0,Z(2)^0,Z(2)^0,Z(2)
    ^0]);
  if IsRationalFunction(vgen)=true then
  if IsUnivariateRationalFunction(vgen)=true then
    if DegreeOfLaurentPolynomial(vgen)=1 # check for univar. linear
      poly.
      then i:=IndeterminateNumberOfUnivariateRationalFunction(vgen)
        ;
      if Value(gen[k],[i],[0*Z(2)])=0*Z(2) or Value(gen[k],[i],[0*Z
        (2)])=Zero(PolynomialRing(GF(2))) then Display(["t[i]=0
          for i=",i]);#Zero(PolynomialRing(GF(2),"x")) instead of 0*
          Z(2)!
          if i in [1..24] or i in [29..52] or i in [54..65] then
            #if i in [54,55] then return 0; fi; #if know that e.g. t
              [54],t[55] are \neq0, can tell GAP so...
            gen[k]:=0*Z(2);
            if IsBound(o[i]) then
              if not o[i]=0*Z(2) then return 0;
              fi;#if two different univariate generators fix same
                indeterminate to
            else o[i]:=0*Z(2); j:=j+1; fi;
            else if i in [25..28] then return 0; fi; fi; fi;
          #reorder fixed indeterminates in o2 s.t. they are positioned as in
            t
            while c in [1..1] do
              if t[c]=Indeterminate(GF(2),i) then if IsBound(o[i]) then
                o2[c]:=o[i]; o3[j]:=o[i]; t3[j]:=t[c]; fi; fi;
                c:=c+1; od; c:=1; fi; fi; fi; fi; od; j:=0;
          for k in [1..Size(gen)] do if IsRationalFunction(gen[k])=true #not gen[k
            ]=0*Z(2)
            then gen[k]:=Value(gen[k],t3,o3); fi; od;# in o3, there are no "
              holes"
          while m=0 do
            Display([gen[1],gen[2],gen[3],gen[4],gen[5],gen[6],gen[7],gen[8],gen
              [9],gen[10]]);#user can switch m to 1 when is finished, or to 2 if |
              C_w|=0.

```

```

Display("Type in required values for indeterminates such that
generators vanish.\n To do this, please type in first i, then
value of t[i].");
i:=InputFromUser("i=");o[i]:=InputFromUser("t[i]=");#(read in values
)
for k in [1..Size(o)] do if IsBound(o[k]) then if IsRationalFunction
(o[k]) then#not o[k]=0*Z(2)
if DegreeIndeterminate(NumeratorOfRationalFunction(o[k]),
Indeterminate(GF(2),i))>0 or DegreeIndeterminate(
DenominatorOfRationalFunction(o[k]),Indeterminate(GF(2),i))>0
then o[k]:=Value(o[k],[i],[o[i]]);fi;fi;fi;od;# check
whether Indeterminate(GF(2),i) turned up in o before, e.g.
first find t[4]=t[30], and later find out t[30]=1...
for k in [1..Size(o)] do
if IsBound(o[k])then j:=j+1; #reorder fixed indeterminates in
o2 s.t. they are positioned as in t
while c in [1..1] do
if t[c]=Indeterminate(GF(2),k) then o2[c]:=o[k]; o3[
j]:=o[k];t3[j]:=t[c];fi;
c:=c+1;od;fi;c:=1;od;c:=1; j:=0;
Display([t3,o3]);
n:=InputFromUser("Do you want to modify some of the other values? If
so, type in 1, else 0.\n");
while n=1 do i:=InputFromUser("i="); o[i]:=InputFromUser("t[i]="); n
:=InputFromUser("Do you want to modify some of the other values?
If so, type in 1, else 0.\n");
for k in [1..Size(o)] do if IsBound(o[k]) then if
IsRationalFunction(o[k]) then#not o[k]=0*Z(2) # check
whether Indeterminate(GF(2),i) turned up in o before
if DegreeIndeterminate(NumeratorOfRationalFunction(o[k]),
Indeterminate(GF(2),i))>0 or DegreeIndeterminate(
DenominatorOfRationalFunction(o[k]),Indeterminate(GF(2),i
))>0 then o[k]:=Value(o[k],[i],[o[i]]);fi;fi;fi;od;od;#
GAP cannot apply DegreeIndeterminate to rational functs
that are no polys
for k in [1..Size(o)] do
if IsBound(o[k]) then j:=j+1; #reorder fixed indeterminates in
o2 s.t. they are positioned as in t
while c in [1..1] do
if t[c]=Indeterminate(GF(2),k) then o2[c]:=o[k];o3[j
]:=o[k];t3[j]:=t[c];fi;
c:=c+1;od;fi;c:=1;od;c:=1;j:=0;
Display([t3,o3]);
for k in [1..Size(gen)] do if IsRationalFunction(gen[k])=true #not
gen[k]=0*Z(2)

```



```

        then gen[k]:=Value(gen[k],t3,o3);fi;od;# in o3,
            there are no "holes"
    Display(gen);
    m:=InputFromUser("If there is no solution , please type in 2. If all
        indeterminates have been set to appropriate values such that gen
        vanishes , type in 1. If you want to continue finding values for
        indeterminates , type in 0.\n");od;
return;end;

# returns the ideal <Bu-uB> over the polynomial ring in the variables t
over the field F_2, and a list of generators of the ideal
ideal_BuuB2:=function(Bu,uB,t)
local R,gen,kgv,I,l1,l2,l,s1,s2,gen2,ord; R:=PolynomialRing(GF(2),t);gen
:=[];kgv:=1; l1:=Size(Bu[1][1][4][2]);l2:=Size(uB[1][1][4][2]); s1:=
StructuralCopy(uB[1][1][4][1]); s2:=StructuralCopy(uB[1][1][4][1]);
gen2:=[];
for i in [1..l1] do l:=Position(uB[1][1][4][1],Bu[1][1][4][1][i]);
if l in [1..l2] then
if not Bu[1][1][4][2][i]=uB[1][1][4][2][1] then
gen[i]:=Bu[1][1][4][2][i]-uB[1][1][4][2][1];
gen[i]:=gen[i]*DenominatorOfRationalFunction(gen[i]);fi;
else gen[i]:=Bu[1][1][4][2][i];
gen[i]:=gen[i]*DenominatorOfRationalFunction(gen[i]);fi;od;
IntersectSet(s2,Bu[1][1][4][1]);
SubtractSet(s1,s2);#gives the x_i in uB, that do not turn up in Bu.
for i in [1..Size(s1)] do gen[l1+i]:=uB[1][1][4][2][Position(uB
[1][1][4][1],s1[i])];
gen[l1+i]:=gen[l1+i]*DenominatorOfRationalFunction(gen[l1+i]); od;
l1:=Size(Bu[1][1][1][2]);l2:=Size(uB[1][1][1][2]); s1:=StructuralCopy(uB
[1][1][1][1]); s2:=StructuralCopy(uB[1][1][1][1]);
for i in [1..l1] do l:=Position(uB[1][1][1][1],Bu[1][1][1][1][i]);
if l in [1..l2] then
if not Bu[1][1][1][2][i]=uB[1][1][1][2][1] then
gen2[i]:=Bu[1][1][1][2][i]-uB[1][1][1][2][1];
gen2[i]:=gen2[i]*DenominatorOfRationalFunction(gen2[i]);fi;
else gen2[i]:=Bu[1][1][1][2][i];
gen2[i]:=gen2[i]*DenominatorOfRationalFunction(gen2[i]);fi;od;
IntersectSet(s2,Bu[1][1][1][1]);
SubtractSet(s1,s2);#gives the x_i in uB, that do not turn up in Bu.
for i in [1..Size(s1)] do gen2[l1+i]:=uB[1][1][1][2][Position(uB
[1][1][1][1],s1[i])];
gen2[l1+i]:=gen2[l1+i]*DenominatorOfRationalFunction(gen2[l1+i]); od
;
Append(gen,gen2);
gen:=Compacted(gen);
I:=Ideal(R,gen*kgv);

```

```

return([I, gen*kgv]);end;

# u =list[i_1,..,i_l], g the Weyl group, a a counting index (telling GAP
to start with the a^{th} element of the Weyl group (which is viewed
as a list))
conj_test:=function(u1,u2,g,a)
local ,u_w,BwB,t,x,Bu,uB,I,U1,j,j1,l,v,gen,y,y2,d,d1,st,w,c1;d:=0; u_w
:=[];t:=[];x:=[];y:=[];y2:=[];c1:=0;d1:=0; #u_w=U_w, t list of
indeterminates, x list of indeterminates needed for the elements in
U_w, v vanishing set of ideal I generated by Bu1-u2B
for k in [1..52] do t[k]:=ite_Variable(GF(2),k);od;
U1:=[]; l:=1; j:=1;j1:=1;
d:=a-1;
for w in g do U1[1]:=w; l:=l+1;od;
for i in [a.. Size(U1)] do w:=U1[i];
u_w:=U_w(w,g);x:=[];
for k in [1.. Size(u_w)] do x[k]:=t[28+k];od;
BwB:=[[u_w,x],w,[t[25],t[26],t[27],t[28]],
[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,
21,22,23,24], [t[1],t[2],t[3],t[4],t[5],t[6],t[7],t[8],t
[9],t[10],t[11],t[12],t[13],t[14],t[15],t[16],t[17],t
[18],t[19],t[20],t[21],t[22],t[23],t[24]]]];
#c1:=InputFromUser("Should arbitrary parameters be used or
just 1? If the latter, type in 1, else 0.");
#if c1=1 then
#y:=[];for k in [1.. Size(u1)] do y[k]:=1;od;
#y2:=[];for k in [1.. Size(u2)] do y2[k]:=1;od;#get the list
of parameters for u
#else y:=[];
#for k in [1.. Size(u)] do t[k+52]:=ite_Variable(GF(2),k+52)
;# y[k]:=t[k+52];
#od;# fi ;

for k in [1..48] do t[52+k]:=ite_Variable(GF(2),52+k);od;
#for k in [1.. Size(u2)] do y2[k]:=t[k+52];od;
y2:=[_,_,_,...];#type in parameters of u2 #t[53] is to be
nonzero!
y:=[_,_,_,...];# type in parameters of u1,
Bu:=Bruhat_Decomp_Para(BwB,[],[[[]],[[]]),(),[1,1,1,1],[u1,y
[]],g);
uB:=Bruhat_Decomp_Para([[[]],[[]]),(),[1,1,1,1],[u2,y2]],[],
BwB,g);
y:=ideal_BuuB2(Bu,uB,t);
I:=y[1]; gen:=y[2];
y:=WhichIndets(gen,t);
Display(["now comes conj_set for w=",Factorization(g,w)]);

```

```
vanishing_set_by_user(gen,y[1],y[2],Size(u_w));# uses t
[53]\neq0!!! and sets t[54],..=0 if modified accordingly
d:=d+1; Display(d);
od;
return;end;
```

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