Analyzing Centrality Indices in Complex Networks: an Approach Using Fuzzy Aggregation Operators

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"It always seems impossible until it is done."

Nelson Mandela

Abstract

The identification of entities that play an important role in a system is one of the fundamental analyses being performed in network studies. This topic is mainly related to *centrality indices*, which quantify node centrality with respect to several properties in the represented network. The nodes identified in such an analysis are called central nodes. Although *centrality indices* are very useful for these analyses, there exist several challenges regarding which one fits best for a network. In addition, if the usage of only one index for determining central nodes leads to under- or overestimation of the importance of nodes and is insufficient for finding important nodes, then the question is how multiple indices can be used in conjunction in such an evaluation. Thus, in this thesis an approach is proposed that includes multiple indices of nodes, each indicating an aspect of importance, in the respective evaluation and where all the aspects of a node's centrality are analyzed in an explorative manner. To achieve this aim, the proposed idea uses fuzzy operators, including a parameter for generating different types of aggregations over multiple indices. In addition, several preprocessing methods for normalization of those values are proposed and discussed. We investigate whether the choice of different decisions regarding the aggregation of the values changes the ranking of the nodes or not. It is revealed that (1) there are nodes that remain stable among the top-ranking nodes, which makes them the most central nodes, and there are nodes that remain stable among the bottom-ranking nodes, which makes them the least central nodes; and (2) there are nodes that show high sensitivity to the choice of normalization methods and/or aggregations. We explain both cases and the reasons why the nodes' rankings are stable or sensitive to the corresponding choices in various networks, such as social networks, communication networks, and air transportation networks.

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List of Abbreviations and Notations

$V(G)$	Set of nodes in graph G
$\|V\|$	Number of nodes
$E(G)$	Set of edges in graph G
$\|E\|$	Number of edges
$N(v)$	Set of neighbours of node v
$E(G[N(v)])$	Set of edges in the induced subgraph $G[N(v)]$
$\eta(G)$	Density of graph
$\langle\overrightarrow{u,v}\rangle$	a tuple of nodes of source u and target v
$deg(v)$	*degree* of node v
$\langle deg\rangle$	Average *degree* of nodes
w_{uv}	Weight of an edge between two nodes
$\mathcal{W}(s,t)$	*walk* between pair of nodes s and t
$\mathcal{P}(s,t)$	*path* between the pair of nodes
$\|\mathcal{P}(s,t)\|$	Length of a *path*
$d(s,t)$	Distance between pair of nodes s and t
$d^w(s,t)$	Distance between the pair of nodes in a weighted network
$cc(v)$	The *Clustering Coefficient* of node v
$\mathcal{CC}(G)$	The average *Clustering Coefficient* of graph G
$\tau(G)$	The *transitivity* of network G
V°	Set of nodes with $deg(v) > 1$
$N_\triangle(v)$	Set of triangles of node v
$N_\triangle(G)$	Set of triangles of graph G
$N_\wedge(v)$	Set of triple of node v
$N_\wedge(G)$	Set of triple of graph G
$C_D(v)$	*degree* centrality index
$C_B(v)$	*betweenness* centrality index
$C_C(v)$	*closeness* centrality index
$s(v)$	*strength* of node v
$ecc(v)$	*eccentricity* of node v
$far(v)$	*farness* of node v
$\delta(s,t)$	Number of shortest paths between pair of nodes s and t
R	Set of relations in a multi-relational network
$\|L\|$	Number of layers in a multiplex network
V_i	Set of nodes in network layer l_i
V^*	Set of shared nodes between $\|L\|$ layers
$deg^{l_i}(v)$	*degree* of node v in layer l_i
$C_D^{l_i}(v)$	*degree* centrality of node v in layer l_i

$C_B^{l_i}(v)$	*betweenness* centrality of node v in layer l_i
$C_C^{l_i}(v)$	*closeness* centrality of node v in layer l_i
$d^{l_i}(s,t)$	Distance between pair of nodes s and t in layer l_i
$Rank_i(v)$	Rank of node v in layer l_i
$minRank(v)$	Minimal rank of node v
$maxRank(v)$	Maximal rank of node v
$D = (x_{mn})$	Decision matrix with m alternatives and n criteria
W	Weight vector
$orness(W)$	*orness* of weight vector W
$dispersion(W)$	*dispersion* of weight vector W
$Q(x)$	Linguistic quantifier
$\mathcal{A}$	Crisp set
$\underset{\sim}{\mathcal{A}}$	Fuzzy set of the crisp set A
$\mathcal{X}$	Universe of discourse (samples)
$\mu_{\underset{\sim}{\mathcal{A}}}(x)$	Membership value of x in fuzzy set
$\mathcal{S}$	Linguistic term set
s_i	A label in a term set
(s_i, α)	A 2-tuple including a label and the value of symbolic translation
MCDM	Multi-Criteria Decision Making
OWA	Ordered Weighted Averaging
MEOWA	Maximum Entropy Ordered Weighted Averaging
SNA	Social Network Analysis
TOPSIS	Technique for Order Performance by Similarity to Ideal Solution

To my parents

Chapter 1

Introduction

There exist many complex systems in the world that are comprised of a collection of individuals or components that are connected to each other. One way to study such systems is to represent them as a network [62]. A network encompasses a set of entities (so-called nodes) that are linked with respect to a specific type of connection (e.g., a social relation) that is represented by using an edge between the corresponding pair of nodes. This type of representation allows addressing many research questions that scientists face analyzing data from many different areas, such as sociology [12], medicine [4], biology [3], and economy [79]. In fact, network analysis is an interdisciplinary field, which provides a means for expressing concepts in networks using a set of formal definitions [90].

In recent decades, interesting network analytic methods and ideas have been proposed, ranging from graph theory [15] via statistics [49] to physics [1, 19], to qualitatively analyze the structure of a network and study the connections between a pair of nodes or those between larger groups. In addition to the analysis of the connections, there is great interest in identifying important nodes (also termed *influential* nodes) using network analytic methods such as centrality measures. These measures aim at obtaining indices that characterize important nodes and allow identifying the major structural center and/or quantifying the influence of individuals who control processes in a network [50, 51, 10]. One main source describing what centrality means and what centrality measures compute is a paper published by Freeman in 1979, where he states the three classical centrality measures: *degree*, which measures the direct influence of a node using the number of connections it has in a network; *betweenness*, which counts the number of shortest paths in a network that pass through a node; and *closeness*, which quantifies how close a node is to other nodes in terms of distance in a network [30]. Depending on the type of system and the interactions between nodes, representations other than simple representation are used, such as weighted networks [65, 61, 70] or temporal networks [38, 72, 20]. More complicated representations of systems include so-called multiplex networks, where the entities in the node set are linked using multiple types of relations [90] or interactions [47] in multiple layers of the network. An example of the latter would be the friendship and co-working relationships among members of an organization (who are represented as nodes) represented in two layers of a network. Accordingly, network analytic measures have been extended from simple networks to other types of networks in recent years [71, 81, 22, 23, 41, 7]. Researchers mostly use classical centrality measures or define their own measure depending on the property they are interested in capturing. In the context of such analyses, an indispensable part of presenting the results of almost all centrality measures is "centrality ranking". Simply put, centrality ranking allows nodes that are more important than others to be observed better; those that are at "the thick of things", as Freeman states [1]. There is no doubt regarding the usefulness of centrality measures and their applicability in network analysis— a huge number of studies ($130,000$) were found on Google Scholar in October 2017 for "*centrality ranking*". However, several challenges exist regarding their usage in various types of networks, which is the main topic studied in this thesis. One of the key questions is: What characterizes important nodes in a specific network? Since network analysis has to deal with a multitude of information for nodes, this thesis aims at analyzing node centrality when multiple indices contribute to this characterization. So the next question is: If those indices have conflicting views, how to analyze whether the nodes' ranking is sensitive or insensitive to different aggregations that can be made in the evaluation? Which nodes show stable ranking among the top-ranking nodes and which ones show stable ranking among the bottom-ranking nodes, making them the most central or the least central ones, respectively? And the last question is: Are centrality rankings sensitive to different type of normalizations that are performed prior to any aggregation, and if so, why is this the case? We will address these questions considering several aspects regarding the identification of important nodes in various complex networks. The proposed approach contains a sensitivity analysis designed with the help of fuzzy aggregation operators. Furthermore, several normalization methods are proposed and discussed that can be applied to multiple values of nodes in networks before using any aggregation. In the following, we will explain what motivated us to conduct this research and what its contribution is for the analysis of node centrality in complex networks.

1.1 Motivations

I. A search for network analytic methods immediately reveals that despite the simplicity of centrality measures, they are very helpful for analyzing the basics underlying many studies, such as controlling the transmission of disease in human contact networks [20], analyzing information diffusion in organizations [46], finding the best target in viral marketing campaigns [78], handling bottlenecks in traffic networks [83], or analyzing leaders' activities in preplanned networks [53]. Although each centrality measure is defined for a specific property of nodes in a network, in some

[1] Two definitions of centrality in the Oxford Dictionary of English are: (1) the quality or fact of being in the middle of somewhere or something; (2) the quality of being essential or of the greatest importance.

datasets, identifying the top entities requires using more properties in the evaluation of node centrality. For instance, in a study on the analysis of a chat-log data, it turned out that multiple properties needed to be used to completely quantify the activities of the participants in an online group chat, as there was some additional information about the activity of participants that could not be represented in any network representation. There are only two works [70, 25] that focus on including multiple node properties in the analysis of centrality in a network. However, both of them can not be applied to datasets under analysis without supplements. The study by Opsahl, and Agneessens, and Skvoretz, focuses on the combination of two indices values in weighted networks using a formula for quantifying the relative importance of nodes, and proposes a sensitivity analysis with respect to the trade-off between the results of two indices using a tuning parameter [70]. In their method, the limitation is on the number of involved measures, i.e., it includes only two, whereas in many datasets more than two properties can be captured for node centrality. This is our first motivation for proposing an approach where a similar sensitivity analysis using a parameter is conducted, but without any strict limitation on the number of measures. The second work considers the use of multiple classical centrality indices in the analysis of node centrality, as proposed by Du et al. in [25]. However, they assume a fixed weight vector attached to multiple centrality indices, which makes scaling between different aggregations over the values of centrality indices impossible without any supplement. We thus propose a generalized approach that can deal with the aforementioned limitations for analyzing node centrality and allows performing a sensitivity analysis on centrality rankings. This contribution will be described in the next section.

II. Since the majority of real-world complex systems have a more complicated structure than one type of relation among their entities, multilayer networks have been proposed in the field of network science to represent multiple types of relations. A vivid example of a multiplex network is a person who is connected to others based on (1) the relation of being co-workers and (2) seeking or getting advice from others. These two types of relations can be represented using two layers where each layer is a network itself. Since the influence of nodes in multiple layers might differ, identifying influential nodes becomes a more complicated evaluation. Multiple theoretical frameworks and mathematical models have been proposed in the literature to generalize the classical centrality measures to the new representation [41, 81, 23, 22]. Summarizing, most papers present attempts on generalizing the basic methods from simple to multiplex networks. The two main concerns are how to deal with conflicting rankings of centrality measures in one layer, and how to deal with conflicting rankings in multiple layers of interest. These motivated us to apply the approach that is able to reveal conflicting rankings of nodes in simple networks to multiplex network in order to identify (1) groups of nodes that have similar behavior in their rankings; (2) whether or not they stay within a layer in the best position with respect to multiple centrality indices; and/or (3) whether or not they have the same importance considering all layers of interest.

III. Regarding the ranking of nodes in multiplex networks and how to partition nodes with respect to their ranking behavior, it turned out that centrality rankings can fluctuate heavily in multiple layers. One of the reasons for this fluctuation is the centrality indices have totally different views about nodes' ranking in the layers that have different structures. This motivated us to focus on the normalization of one measure in order to make the results of nodes' ranking comparable across the layers. There are multiple ways to normalize the result of a centrality measure in multiplex networks. Surprisingly, we found that not many network analytic papers describe such a basic preprocessing method before performing any aggregation. This motivated us to conduct a sensitivity analysis in order to analyze the influence of the choices on both modeling decisions in identifying important nodes for a measure like *degree* in multiplex networks.

IV. Many real-world networks are deduced from incomplete data, but, not many network analytic methods consider that this incompleteness can be the origin of uncertainty in the results. Regarding node centrality, producing a precise ranking might overestimate the importance of a node. Instead of such a discrete result, we are interested in partitioning nodes into a set of centrality classes that simply distinguishes nodes into classes of centrality ranging from *very peripheral* to *very central*.

1.2 Contributions

I. As mentioned above, the previous method by Opsahl, Agneessens, and Skvoretz, can not be generalized easily to more than two centrality measures. Thus, in the first contribution, we propose an approach that has a similar scaling feature but including more than two properties in the analysis of nodes' importance in real chat-log data and in a communication network represented as weighted networks. We first explain those properties that can be used for indicating the importance of a node in deduced networks. We think of such an evaluation as a <u>M</u>ulti-<u>C</u>riteria <u>D</u>ecision <u>M</u>aking (MCDM) problem. An MCDM, is a problem where a decision maker aims at identifying the best solution from a set of alternatives that are assessed using multiple criteria. The decision maker might select the best alternative as being the one that satisfies at least one, some, most, or all of the criteria. Therefore, the result of the identification might differ depending on which decision is made. In this consideration, nodes play the role of alternatives and multiple properties play the role of multiple criteria. A fuzzy aggregation operator called <u>O</u>rdered <u>W</u>eighted <u>A</u>veraging (OWA) facilitates guiding different decisions in the selection of top-ranking nodes with respect to their overall *Score*.

The OWA operator first orders the values of the criteria associated with the nodes in descending order and multiplies them with a weight vector in the evaluation. We use a parameter β to generate a set of different weight vectors (different types of aggregations), including the classic ones, such as min, max, $average$. We will present several situations in two communication networks, in which the nodes are assigned both structural properties classically obtained from network analysis and properties obtained from data that cannot be readily represented in the network representation. We focus on those nodes that have a *robust* importance and stay always on top-ranking and those are sensitive to the choices of different aggregations. In a transportation network, we also exemplify a situation where multiple classical centrality indices have different opinions about a node's centrality. We use the same proposed approach to find which nodes stay at the top of the ranking, respectively at the bottom, and to identify most central or least central nodes. Some results of this contribution including figures have been published already in two papers [84, 86]. All the included parts from these papers were written by myself.

II. Proceeding to multiplex network representation, we extend the proposed idea and explore the conflicting rankings with a new visualization showing the influence of nodes within a layer with respect to multiple centrality centrality indices and allowing simultaneous comparison of all layers. We categorize nodes with respect to their ranking behaviors and discuss their behavior in several multiplex networks, such as a European air-transportation network (which contains three layers), a law firm network (with three layers), and a Twitter network (with four layers). We discuss that an air transportation network allows for a smaller number of different ranking behaviors than a social network in a medium size or a large dataset of Tweets. Some parts of the discussed results have been published in a paper [85].

III. Focusing on the behavior of node rankings in multiplex networks led us to focus on a topic in the field of network analysis that seemed to be simple, as many studies dismiss investigating it: sensitivity analysis of *degree centrality* in the identification of influential nodes in multiplex networks with respect to different modeling assumptions for normalizing the result of the corresponding measure. We show that even in such a seemingly simple case, very basic modeling using different assumptions result in very different centrality rankings. We state that an analysis in multiplex networks requires at least two preprocessing steps to compute a ranking of the nodes. The first step is normalization of all *centrality indices* to make them comparable over multiple layers. We propose a set of intuitive normalizations that leads to very different rankings. Second, we present different aggregations for the different centrality index values across all layers using an aggregation operator with the different values of a tuning parameter. We then visualize the results of the rankings using two measures that capture the sensitivity of a node's ranking considering the choices of normalization methods and aggregations. In the experimental results obtained from three multiplex network datasets, we observe that some nodes are very fragile to different modeling decisions and some are not sensitive to any one of them. Such variation in the sensitivity of nodes confirms that any normalization method as well as any aggregation needs to be taken in to account carefully in the analysis of node centrality in order to make the findings interpretable. The results of this work led us to a publication [89].

IV. We also discuss a problem regarding the usage of centrality ranking: it provides a delusive picture of node centrality and it can be sometimes very sensitive to the different choice of modeling decisions or be influenced by the incompleteness of data—this is a problem occurring in many real networks. In most studies, precise ranking is used to indicate the importance of nodes and to label them as either the most central or the least central node. We discuss this problem in a real network dataset and show the results using a 2-tuple fuzzy model, which contains a centrality label and the extent to which a node's normalized value is close to its label. We use a set of predefined labels (*Very Peripheral, Low, Medium, High, Very Central*) to determine the extent to which a node is central in a network, i.e., whether the node is among the group of nodes that have *Medium* importance or whether it is a *Very Central* node. The empirical results using visualizations emphasize the usefulness of this type of representation and the simplicity of the analysis of node centrality in a real network that satisfies the concern of imperfection. We discuss the pros and cons of this model at the end of analysis. Some parts of the obtained results are published in [88].

1.3 Thesis Outline

The structure of this thesis is shown in Figure 1.1 to help the reader find the connections between the different chapters. Following the contribution of this thesis as described above, the basics of complex networks, the fundamental concepts, and the definitions of centrality indices in simple, weighted networks and in multiplex networks are provided in the first section of Chapter 2. A theoretical background on fuzzy models and OWA operators employed in the main approach forming our contribution is given in the second section of Chapter 2. To facilitate reading, we dedicate Chapter 2 to the theoretical background including preliminaries and related work used for comparison with our approach. Figure 1.1 depicts which preliminaries from Chapter 2 are necessary to be read before reading one of the subsequent chapters.

Chapter 3 includes the findings using the proposed approach from communications networks and from a transportation network.

The extension of the idea presented in the first chapter is shown in Chapter 4 for the results of several multiplex networks. Multiple datasets, such as an air transportation network dataset, a law firm network dataset, and a twitter network dataset, are analyzed and various findings are discussed related to the ranking behaviors of nodes.

The results of the analysis regarding the impact of different modeling decisions in the evaluation of node centrality and the identification of important nodes from three different network datasets are explained in Chapter 5.

Chapter 6 is dedicated to node centrality being represented using labels in networks. The results of the partitioning of the nodes in to a set of classes of centrality in a real network dataset are presented in this chapter.

Finally, in Chapter 7, all the discussion from the applications of the proposed idea to various datasets are summarized to show how the contribution presented in this thesis allow investigating multiple aspects of node centrality

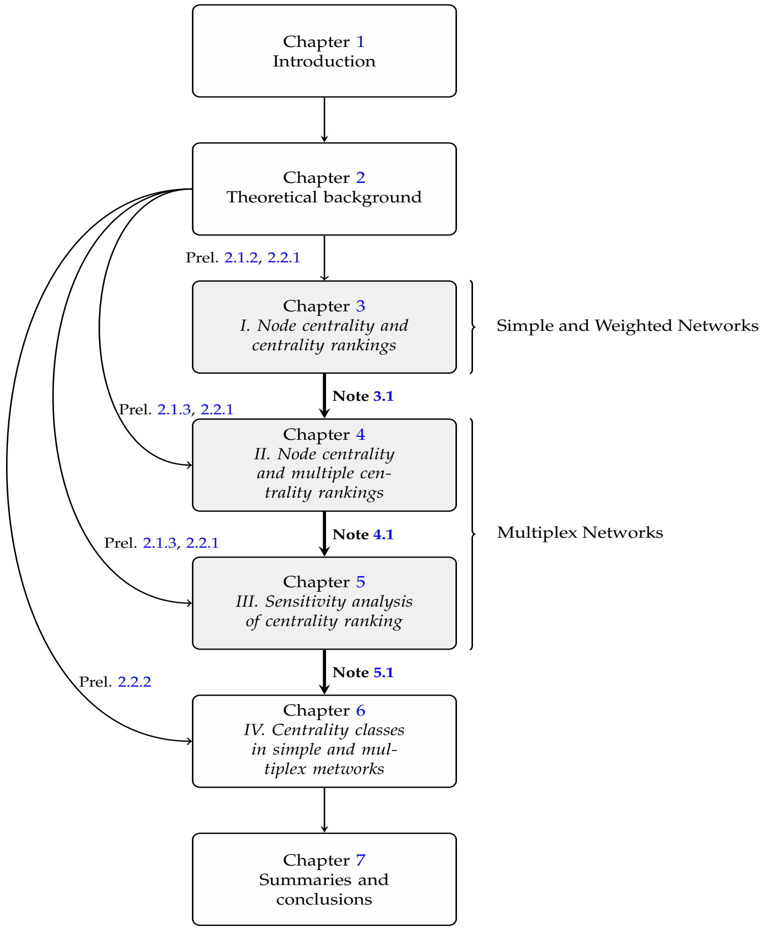


FIGURE 1.1: Structure of the thesis, including connections between background/preliminaries and main chapters. The darker squares indicate the main contribution presented in this thesis.

and conducting exploratory analyses of *centrality indices* in complex networks. We present several ideas regarding future studies of this topic as well.

Chapter 2

Theoretical background

Using a network representation, structural and behavioral analyses of the entities and the connections between them in different networked systems become more feasible. Since different disciplines such as economic, computer science, and mathematic, contribute to network analysis, different kinds of definitions or terms may exist for the same idea [62, pp. 1–3]. In addition to this, many interesting network methods and ideas inspired from different disciplines have been proposed in the literature by scientists; all are aimed at capturing the underlying patterns governing the connections between entities (or actors in social networks). Regarding the properties of nodes, many discoveries related to the concept of centrality in different types of network representations—from simple and weighted [65, 13, 70] to temporal [72, 20] and further on to multiplex networks [81, 22]—have been proposed. However, here we stick to the description of the classical *centrality indices* widely used in studies to date. We will first present the theoretical concepts and basic definitions of networks. Then we explain classical *centrality indices* in weighted and multiplex networks in detail. In order to deal with some of the challenges regarding node centrality we encountered, we propose the usage of fuzzy models and Ordered Weighted Averaging (OWA) aggregation operators that are very practical and useful in this matter. The approaches based on fuzzy logic have been used before to analyze social networks for different aims, such as community detecting [100] and fuzzy relations modeling [60]. In this thesis, we address the questions of how to use multiple properties for the exploratory analysis of node centrality and how to conduct a sensitivity analysis on the centrality ranking of nodes using aggregation operators.

We will thus provide the basics in fuzzy models and OWA operators, which are the preliminaries for the ideas proposed in Chapters 3, 4, 5, and 6. In addition, any related work used for comparison with our proposed approach will be noted and described in the current chapter.

2.1 Complex networks

Complex network analysis is described as a part of network science, which uses different theorems from graph theory that is also the basis of social network analysis [101, pp. 26-27]. Mathematically, networks are viewed as graphs in which a collection of nodes are connected using edges [82, pp. 18–19]. In fact, graph theory provides a wide range of vocabulary that can be used for the formalization of properties in networks [90, p. 93]. Therefore, in the following section, we describe some graph theoretic definitions and classic network analytic measures.

2.1.1 Basic definitions

A graph is defined as follows:

Definition 2.1.1. A simple undirected graph $G = (V, E)$ is a tuple of a set of nodes V, and a set of edges $E \subseteq V \times V$. If a pair of nodes $u, v \in V$ is connected, then there is an edge between them denoted by an unordered pair $\langle u, v\rangle$.

To denote a set of n nodes and m edges, we use $V(G) = \{v_1, v_2, \cdots, v_n\}$ and $E(G) = \{e_1, e_2, \cdots, e_m\}$, respectively. If there are multiple edges, let's say two, between a pair of nodes u and v, then the edges can be indexed as $e_1 = \langle u, v\rangle$ and $e_2 = \langle u, v\rangle$ [82, pp. 18–19]. If the graph is *weighted*, each edge has a real-valued number. The weights represent different measures depending on the modeled real network, e.g., the total energy flow between prey and predator in a food web [62, p. 112]. If the graph is *directed*, the set of edges consists of tuples of nodes $\langle\overrightarrow{u, v}\rangle$, where u is said to be the *source* and v is said to be the *target* of the edge. The neighbors of node v can be denoted as $N(v) = \{w \in V \mid \langle v, w\rangle \in E\}$. Accordingly, in a directed graph, $N_+(v) = \{w \in V \mid \langle v, w\rangle \in E\}$, where node w is the *target* and $N_-(v) = \{w \in V \mid \langle w, v\rangle \in E\}$, where node w is the *source*.[1].

Let G be a graph. The *degree* denoted by $deg(v)$ is the number of edges incident with v. Assume all the *degree* values of nodes in a graph have been obtained. Then $\langle deg\rangle$ denotes the average of the sums of all *degree* values. In a directed version of a graph, the *in-degree* $deg_-(v)$ is defined as the number of edges in which v is the target, while the *out-degree* $deg_+(v)$ is defined as the number of edges in which v is the source. One of the basic topics studied in graphs is the *degree sequence*, which is an ordered sequence of *degree*s, $\{deg(v_1), deg(v_2), \cdots, deg(v_n)\}$. By considering the definition of *in-degree* and *out-degree*, the *degree sequence* can be extended to directed graphs as well. There are several basic ways to display a graph, e.g, global edge list, local edge list and adjacency matrix.

Definition 2.1.2. Let A be the adjacency matrix of the undirected graph G, where an element a_{ij} indicates the existence of an edge between a pair of nodes; then in order to compute the *degree* of node i, $deg(i) = \sum_j a_{ij}$ can be used.

In a weighted graph, the cells of the corresponding adjacency matrix contain the weight of the edges. If the weight is zero, it can be interpreted that the edge between the corresponding pair of nodes does not exist [6]. Considering the

[1] Most of the definitions are based on those provided in two references [90, 82].

number of edges incident with a node in a graph, it is of interest to know how many edges exist in the total graph. This refers to the *density* of a graph, which is defined as:

Definition 2.1.3. Let $|E|$ be the number of edges presented in graph G. Then the *density* of the graph is defined based on the maximum possible number of edges in a graph $|V|(|V|-1)/2$ using the following division:

$$\eta(G) = \frac{2|E|}{|V|(|V|-1)}$$

Let G be a graph with $|E| = 20$ and $|V| = 16$ as depicted in Figure 2.2. Its *density* is then computed as: $\eta(G) = \frac{2\times 20}{16\times 15} = 0.167$.

In the definition of the *density* of a directed graph, the maximum possible number of edges is equal to $|V|(|V|-1)$. In order to describe the concept of connectivity between nodes and to measure the distance between two nodes, some definitions need to be provided. A graph G is called *connected* if all pairs of distinct nodes are reachable, i.e., nodes v and u are reachable if there is a path between them. To clarify this, consider a network that represents the connections among a number of students of a college. If all students are linked and reachable on, e.g., Facebook, then any information transmission is feasible among them. If G is not connected, then it is called a disconnected graph. In order to understand the indirect influence of nodes on each other, several distance measures are defined for graphs, such as *walk*, *trail* and *path* described in the following:

Definition 2.1.4. A *walk* $\mathcal{W}(s,t)$ is an alternating sequence of nodes and edges $[v_0 = s, e_1, v_1, e_2, ..., v_{k-1}, e_k, v_k = t]$ with $e_i = \langle v_{i-1}, v_i \rangle$, $0 < i \leq k$. Duplication of nodes and edges is allowed.

The *length of a walk* is the number of edges occurring in it. A *trail* is a walk in which the duplication of edges is not allowed. The *length of a trail* is defined similarly [82, pp. 37–38].

Definition 2.1.5. A *path* $\mathcal{P}(s,t)$ between a pair of nodes $\langle s,t \rangle$ is a sequence of nodes $[v_0 = s, v_1, ..., v_k = t]$ such that no node and no edge is contained more than once and that for all subsequent node pairs $\langle v_{i-1}, v_i \rangle$, $0 < i \leq k$, there exists an edge $e_i \in E$.

The *length of a path* $|\mathcal{P}(s,t)|$ is defined as the number of edges in the path. The *shortest path* between s and t is the path with minimal length in the set of all paths between s and t.

The distance $d(s,t)$ between the pair s and t is defined as the length of the shortest path between s and t. If no such path exists, $d(s,t)$ is ∞ by definition. In a directed graph, an edge in the definitions above is defined as $e_i = \langle \overrightarrow{v_i, v_{i+1}} \rangle$ [82, p. 61]. Back to the concept of connectivity: A directed graph is called *strongly connected* if a directed *path* exists between every pair of nodes in the graph. This graph is called *weakly connected* if its undirected version is connected.

If a graph is disconnected, then the nodes may be divided into two or more than two subsets. After such a division, it contains more than one *subgraph*. A *subgraph* G_s of graph G is a graph whose set of nodes is a subset of node set of G, $V_s \subseteq V$ and whose set of edges is a subset of edge set of G, $E_s \subseteq E$. A *component* is known as a maximal connected *subgraph* in which all pairs of nodes are reachable; there exists a path between all pairs in the component and there is no path between any chosen node in the component and any node out of the component [90, p. 109]. The definitions of *weakly connected component* and *strongly connected component* are similarly defined for a directed graph.

Clustering coefficient and *transitivity* are two fundamental ideas in networks, which focus on two aspects. The first one tries to provide an insight in to the connections between the neighbors of a node v, whether or not they are neighbors and linked to each other—as Watts and Strogatz describe in [91], this provides information about the local *clustering coefficient* and the local connectedness of a node. When all its neighbors are connected, the best result will be achieved. This means the neighbors of v can form a *complete graph*, i.e., a graph in which all pairs of nodes are connected via an edge [82, p. 144]. The outcome of such an analysis in networks can indicate that the node is able to spread information or transmit an epidemic to its neighbors [26]. On the other hand, network *transitivity* considers a global view on the stated property for the whole network [82]. Considering the definition of *density* in Eq. 2.1.3, the *clustering coefficient* of a node $v \in V(G)$ with a *degree* greater than one in a simple undirected graph G is defined as follows:

Definition 2.1.6. Let $N(v)$ be the set of neighbors of node v and $E(G[N(v)])$ be the set of edges in the subgraph that the neighbors of v form, then, we have,

$$cc(v) = \begin{cases} \frac{2|E(G[N(v)])|}{|N(v)|(|N(v)|-1)} & if \quad deg(v) > 1 \\ undefined & otherwise \end{cases}$$

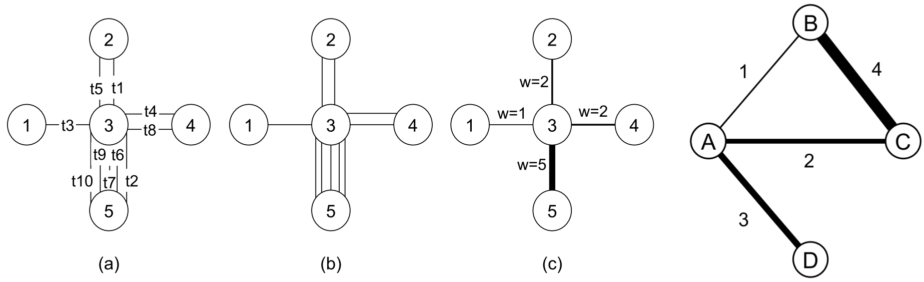
Now consider the whole network G, in which all the *Clustering coefficient* of nodes are obtained. Then the average *clustering coefficient* of G can be simply obtained by computing the average as $\mathcal{CC}(G) = \frac{1}{|V^\circ|} \sum_{v \in V^\circ} cc(v)$, where V° is the set of all nodes with a *degree* greater than one.

In 2003, Newman showed that by using the number of triangles for a node $v \in V(G)$ as $|N_\triangle(v)|$ and the number of its triples as denoted by $|N_\wedge(v)|$ in a graph, it is possible to measure the local *clustering coefficient* as $\frac{|N_\triangle(v)|}{|N_\wedge(v)|}$, [64]. The definition of network *transitivity* then is described as follows:

Definition 2.1.7. *Transitivity* is defined as $\tau(G) = \frac{3|N_\triangle(G)|}{|N_\wedge(G)|}$, where $|N_\triangle(G)|$ is the total number of distinct triangles, and $|N_\wedge(G)|$ in the denominator is the total number of triples.

2.1.2 Centrality indices in simple and weighted Networks

Going further into detail regarding the structural and positional properties of nodes in networks, several measures are defined formally for the centrality concept in simple networks, where the connections are represented using undirected edges, and in weighted networks, where the connections have some form of weights. To understand which nodes are at the center among all nodes in a network, different centrality measures for quantifying a node's centrality



(A) (B)

FIGURE 2.1: (A) Transformation of different types of graphs from temporal ones via multi-graphs including multiple edges to weighted graphs. (B) A weighted graph comprised of four nodes, where two nodes A and C have a similar *strength* value but different *degree* values.

are defined that show to which degree a node is important considering local or global structural properties. The most commonly used measure among them is *degree centrality*, which is described as follows:

Definition 2.1.8. The *degree centrality* of a node v is equal to its *degree*, and is denoted by an index $C_D(v) = deg(v)$. If the graph is directed, the *in-* and *degree* can be used as *degree centrality* as well.

Let n denote the number of nodes in the network. This measure is standardized using division by $n - 1$. This standardization is performed to make the measure independent of the network order (i.e., the number of nodes) [90, pp. 178–179]. The interpretation of this measure is: A node with a large *degree* is at the top of direct influence or communication, or has a high accessibility to first-hand information [101, p. 269], and thus is called the most central node. The remaining nodes in the network—those that have low *degree* values—are called least central. Freeman illustrates his point of view by using a star graph with five nodes (consider an unweighted form of Figure 2.1(c)). Node 3 has the maximum *degree*, i.e., $deg(n_3) = 4$, in the graph with the size of $n = 5$ and its standardized centrality value equals 1. According to Freeman, this index shows the "potential communication activity" of the node in a network [30].

A wide range of complex systems, such as communication, biological, and collaboration data, have been represented as weighted networks to explore patterns with respect to the strength of the connections between the nodes [56, 99, 80, 68, 71]. In order to analyze weighted networks, all the structural properties of nodes in simple networks have been extended to the new framework of weighted networks [61, 63].

Consider a weighted graph as shown in Figure 2.1(c). The definition of *degree centrality* is generalized in terms of *strength* $s(v)$ as shown by Barrat et al. [5] as follows:

Definition 2.1.9. Let W be the weighted adjacency matrix of a graph. Then if node v has a connection to node u, the element ω_{vw} is greater than 0. Then the *strength* of node v is:

$$s(v) = C_D^w(v) = \sum_u w_{vu}$$

In the aforementioned generalization, one point is missing: the number of edges that is the basis of the original definition for *degree centrality*; see the graph shown in Figure 2.1 (B), where the two nodes of "A" and "C" have the same *strength* but a different number of connections. Therefore, a new description for *degree centrality* in weighted networks is proposed by Opsahl, Agneessens, and Skvoretz which focuses on the relative importance of weights on the connections to the number of communication partners using the following equation:

$$C_D^{w\alpha}(v) = deg(v) \cdot \left(\frac{s(v)}{deg(v)}\right)^{\alpha} \tag{2.1}$$

where α is a tuning parameter and setting it to 0 and 1 convert the measure to *degree* and *strength*, respectively. A value of α between 0 and 1 indicates having a high *degree* is favored and a value above 1 indicates having a low *degree* is favored [70].

Other classic *centrality indices* are *eccentricity*, defined as the inverse of the maximal distance of node v to any node u in the graph as $ecc(v) = \frac{1}{max_{u \in V} d(v,u)}$, *farness* $far(v)$, defined as the sum of the distances of v to all other nodes, and *closeness* $C_C(v)$, defined as the inverse of the *farness* of v.

Definition 2.1.10. Assume $d(v, u)$ is the distance between nodes v and u. The *closeness centrality* is defined as the inverse of the sum of distances to all other nodes in a network.

$$C_C(v) = \frac{1}{\sum_u d(v,u)}$$

To make the result comparable over the networks, it can be standardized by multiplying $C_C(v)$ with $n - 1$. When a node is adjacent to all the other nodes, the centrality index reaches its maximum value which is $\frac{1}{n-1}$. This measure tells about the potential of a node in passing a piece of information to all other nodes. Note that this measure is defined for connected networks [90, pp. 178–190]. In a direct version of a network, for measuring the distance between nodes v and u, the in-closeness centrality considers paths to node v and the *out-closeness* measures paths from node v.

If a node lies on many shortest paths in a network, it is assumed that it has some control over the interaction between those nodes in the network and it is regarded as the most central node because it plays an important role as a mediator in the network.

Definition 2.1.11. Let $\delta(s, t)$ denote the number of shortest paths between s and t, and $\delta_v(s, t)$ denote the number of shortest paths between $s \neq v$ and $t \neq v$ containing v. Then *betweenness centrality* is equated as follows:

$$C_B(v) = \sum_{s,t \in V} \frac{\delta_v(s,t)}{\delta(s,t)} \tag{2.2}$$

If a node is located between many nodes via their geodesics, then it obtains a large *betweenness centrality*. In order to compare the *betweenness* values across

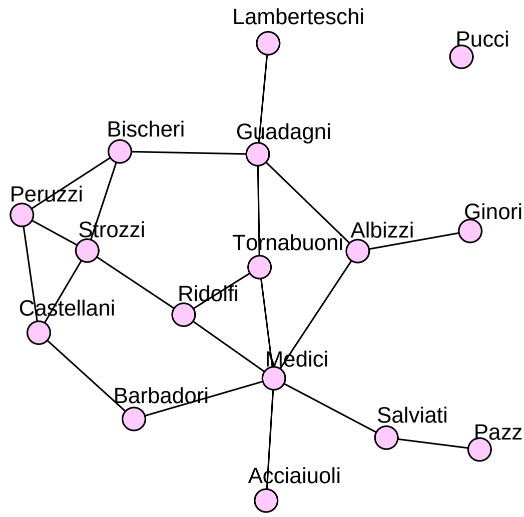


FIGURE 2.2: The small graph of Florentine families of Padgett. The figure is redrawn from the graph discussed in [90].

networks, this measure is standardized by dividing it by $\frac{(n-1)(n-2)}{2}$. In contrast to *closeness*, it can be computed even if the network is disconnected [90, pp. 178–190]. The computation of the aforementioned measures using efficient algorithms is described in detail in [42].

To exemplify the operation of the three aforementioned *centrality indices*, we obtain them for 15 families in the graph of Padgett's Florentine families, which is a simple network as shown in Figure 2.2 with respect to marital relationships [90]. There is an undirected edge between two nodes if a member of family A married a member of family B. The results of the centrality indices values of *degree*, *betweenness*, and *closeness* are sorted from the highest to the lowest values for each corresponding index and are listed in Table 2.1. It can be observed that the measures sometimes agree on a node's importance (e.g., Medici family and Pazzi) and sometimes have different opinions about it, e.g., Ridolfi and Albizzi [2].

The measures above are generalized to weighted networks in two studies [13, 63] by applying Dijkstra's algorithm to the inverted edge weights in order to find the shortest paths between a pair of nodes. In this generalization, the weights are considered as costs, meaning less weight is preferable. However, again, as stated by Opsahl, Agneessens, and Skvoretz, the problem mentioned earlier still remains: The number of edges is missing in the formulation. Opsahl, Agneessens, and Skvoretz argue that a piece of information or disease might be transformed quicker through strong connections than through weak connections [70]. In their generalization, the weights are normalized with the

[2] Both families have the same importance with respect to the normalized *degree centrality*; however, using *betweenness*, Albizzi has a higher value than Ridolfi. Conversely, Ridolfi gets a better rank than Albizzi, with respect to *closeness centrality*.

TABLE 2.1: The centrality of 15 members of the Florentine families of Padgett is obtained using the three classical centrality indices—the isolated node ("Pucci") is removed from the computation. This exemplification is based on the example provided in [90].

Names (sorted by standardized $C_D(v)$)	Names (sorted by standardized $C_B(v)$)	Names (sorted by standardized $C_C(v)$)
Medici(0.429)	Medici(0.522)	Medici(0.56)
Guadagni(0.286)	Guadagni(0.255)	**Ridolfi(0.5)**
Strozzi(0.286)	**Albizzi(0.212)**	**Albizzi(0.483)**
Albizzi(0.214)	Salviati(0.143)	Tornabuoni(0.483)
Bischeri(0.214)	**Ridolfi(0.114)**	Guadagni(0.467)
Castellani(0.214)	Bischeri(0.104)	Barbadori(0.438)
Peruzzi(0.214)	Strozzi(0.103)	Strozzi(0.438)
Ridolfi(0.214)	Barbadori(0.093)	Bischeri(0.4)
Tornabuoni(0.214)	Tornabuoni(0.092)	Castellani(0.389)
Barbadori(0.143)	Castellani(0.055)	Salviati(0.389)
Salviati(0.143)	Peruzzi(0.022)	Acciaiuoli(0.368)
Acciaiuoli(0.071)	Acciaiuoli(0)	Peruzzi(0.368)
Ginori(0.071)	Ginori(0)	Ginori(0.333)
Lamberteschi(0.071)	Lamberteschi(0)	Lamberteschi(0.326)
Pazzi(0.071)	Pazzi(0)	Pazzi(0.286)

average distance, and both the weights and the number of intermediary nodes are considered in path length before using the Dijkstra's algorithm.

Opsahl, Agneessens, and Skvoretz, defined the binary distance measure as the minimum number of edges directly or indirectly connecting nodes v and u as follows:

$$d(v,u) = min(a_{vh} + \cdots + a_{hu})$$

where h are intermediary nodes on paths between node v and u and A is the adjacency matrix. The generalized distance between these two nodes in a weighted network then is defined as follows:

$$d^{w\alpha}(v,u) = min\left(\frac{1}{(w_{vh})^{\alpha}} + ... + \frac{1}{(w_{hu})^{\alpha}}\right)$$

where α is the same tuning parameter as used in Eq.2.1. When $\alpha = 0$, the measure acts as same as the binary distance measure. When $\alpha = 1$, it produces the same result as Dijkstra's algorithm as implemented in [13, 63]. For $\alpha < 1$, the shortest path constituted of weak connections is preferred and for $\alpha > 1$, a path with more intermediaries is favored. Following this generalization, Opsahl, Agneessens, and Skvoretz extended the *closeness centrality* and *betweenness centrality* as well (see [70] for more details).

2.1.3 Centrality indices in Multiplex Networks

As the complexity of systems increases, a network representation needs to be equipped with some means that make it possible to reveal the inherently complex nature of the corresponding systems. For example, when the dynamics of interaction are of interest with respect to a period of time, a temporal network representation is used most frequently [38]. There is other type of network such as multi-relational [90, p. 81] or multiplex networks, that has been

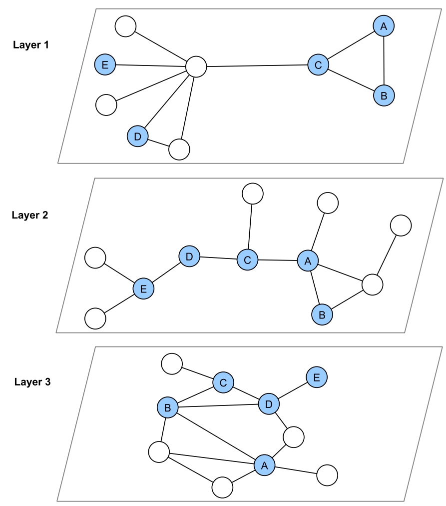


FIGURE 2.3: A three-layer multiplex network where the five blue nodes are shared.

recently in the center of attention in many studies [47]. In a multi-relational network, multiple relations exist between a set of actors. Assume $|\mathcal{R}|$ relations between $|V|$ nodes. A multi-relational network can be represented using a three-dimensional matrix of the size $|V| \times |V| \times |\mathcal{R}|$, where each entry indicates the existence of a relation r between a pair of nodes $v_i \xrightarrow{r} v_j$ as described in the sociology literature [90]. Sometimes, for the sake of simplicity, the relations are aggregated in order to produce weights between a pair of nodes [57]. But in some studies, it has been shown that this type of aggregation might dismiss useful information in the actual layers of a network [58]. A multiplex network representation is of interest for the representation of different complex systems such as biological organs, transportation systems, and social networks. Different types of interactions between the entities in the corresponding systems are demonstrated in multiple layers. For instance, in an air transportation network, cities can be represented as nodes and flights operated by different airlines can form the edges in multiple different layers of the network. For the sake of clarity, assume Figure 2.3 to be a small multiplex network comprised of three layers sharing some nodes (blue nodes). In this type of representation, the individual network layers are used to show the difference between the position of nodes with respect to different types of relations or interactions. The definition of a multiplex network based on this representation is given below.

Definition 2.1.12. A multiplex network is a network comprised of $|L|$ layers $L = \{l_1, l_2, \cdots, l_m\}$, where each layer l_i itself is a network comprised of $|V_i|$ nodes and $|E_i|$ edges. Each edge set E_i represents a different type of relation or interaction, and in almost all multiplex networks, a set of nodes (sometimes all nodes) are contained in multiple layers, which are denoted by V^*.

In other studies [9, 23], a multiplex network is defined as an interconnected network that allows interconnections between nodes in different layers. They

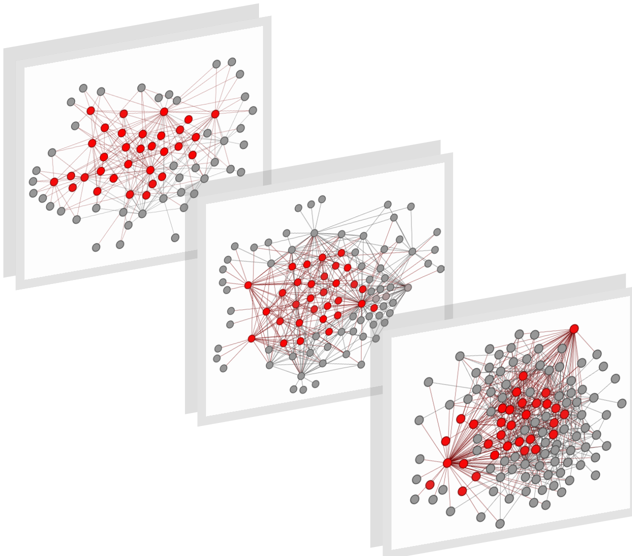


FIGURE 2.4: A multiplex network comprised of three layers with a set of shared nodes, which are colored red. The numbers of nodes and edges vary between the layers.

define a multiplex (interconnected) network using a pair $(L, \mathcal{C})$, where L similarly denotes a family of layers and $\mathcal{C}$ denotes a set of interconnections between nodes between different layers, i.e., the set of edges connecting nodes in different layers. They explain that most researchers use a vector-type node *degree*, by which obtaining the overlapping of the degrees using an aggregation in a multiplex (interconnected) network is feasible [9]. Based on this definition, the authors in a study [23] propose a model to identify the most central node in a multiplex (interconnected) network, whose role is to bridge the different types of relations. Accordingly, they consider interconnections between the layers and define the *shortest path* $\mathcal{P}^*_{[s\sigma \to t\gamma]}$, as the path with the minimum cost from node s in layer σ to node t in layer γ [23]. They define that *betweenness centrality* of node v is proportional to the number of times that node v belongs to the set $\mathcal{P}^*$ for any possible pair (s, t) irrespective of the layers [23]. Based on this consideration, a number of nodes are identified less central in an interconnected network; these nodes were more central in the aggregated network [23].

There are different ways to describe centrality measures in a multiplex framework and researchers continue to explore new ways regarding this matter in different attempts and for several purposes [41, 7, 22]. If we assume multiple (non-interconnected) layers and check the position of nodes with respect to the different relations as exemplified in Figure 2.3, then the centrality measures can be extended from a simple network to a multiplex (unweighted) network as follows:

Definition 2.1.13. The *degree* $deg^{l_i}(v)$ of node v is defined as the number of edges connected to v that are contained in layer l_i. Then, the *degree centrality* $C_D^{l_i}(v)$ of the node is equal to its *degree*.

To compute the *degree* of node v in layer l_i, an undirected and unweighted adjacency matrix of layer l_i denoted by A^{l_i} can be used [41]. In the matrix, the corresponding cell element $a_{vu}^{l_i}$ equals 1 if node v is connected to node u and 0 otherwise. Then the *degree* of node v equals $deg^{l_i}(v) = \sum_{u \in V_i} a_{vu}^{i}$.

Definition 2.1.14. Let $d^{l_i}(v,u)$ denote the distance between nodes v and u in layer l_i, which is defined if and only if $v, u \in V_i$. Similarly, the *closeness centrality* is described by an index of $C_C^{l_i}(v)$ for node v in layer l_i, as the inverse of the sum of all distances of v to all other nodes in V_i.

Definition 2.1.15. The *betweenness centrality* of node v in layer l_i is defined as follows:

$$C_B^{l_i}(v) = \sum_{s,t \in V_i} \frac{\delta_v^{l_i}(s,t)}{\delta^{l_i}(s,t)}$$

where $\delta_v^{l_i}(s,t)$ denotes the number of shortest paths between any pair s and t that contains v in layer l_i and $\delta^{l_i}(s,t)$ denotes the number of all shortest paths between s and t in layer l_i.

In a multiplex network, the layers might have different numbers of nodes and edges, and some nodes might be inactive or might not exist in one or two layers as depicted in Figure 2.4. Therefore, it is worth noting that a preprocessing step is required to make the results of any centrality measure obtained for a specific node comparable across multiple layers. Therefore, four different normalization methods are proposed for *degree centrality*, which will be explained in Chapter 5.

Although many researches focus on the analysis of node centrality in simple, weighted, and multiplex networks in network analytic studies, not many of them consider that the analysis of node centrality might relate to more than one or two aspects of importance in networks. The search for a useful approach that allows including multiple indices in the evaluation of node centrality and is capable of providing different aggregations over the used indices brought to our attention fuzzy models and OWA aggregation operators as a very promising approach of exploration for our aims. In the following section, we will thus provide basic definitions in the area of fuzzy logic and then continue with Subsection 2.2.1 required for the approach in Chapters 3, 4, and 5 and Subsection 2.2.2 required for the analysis in Chapter 6.

2.2 Fuzzy models and aggregation operators

Fuzzy logic was introduced by Zadeh in his seminal work in 1965 in order to deal with problems for which imprecise information exists [98]. The idea he proposed had a deep impact on how we think about analytical models with

TABLE 2.2: A decision matrix D with the size $m \times n$.

	Multiple criteria			
Alternatives	a_1	a_2	$\cdots$	a_n
x_1	d_{11}	d_{12}	$\cdots$	d_{1n}
x_2	d_{21}	d_{22}	$\cdots$	d_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
x_m	d_{m1}	d_{m2}	$\cdots$	d_{mn}

respect to uncertainty originating from various problems such as incompleteness of data. Although most of the information we get from complex systems is fuzzy, our decision making processes are binary [77]. We say an element either belongs to a crisp set $\mathcal{A}$ or not. In contrast, as introduced by Zadeh, a fuzzy set $\underset{\sim}{\mathcal{A}}$ contains elements that have degrees of membership, which are measured using a function [98, 77, 48].

Definition 2.2.1. Let $\mathcal{X}$ be a set of objects. A fuzzy set $\underset{\sim}{\mathcal{A}}$ in the so-called universe of discourse $\mathcal{X}$, is denoted by a membership function that maps an element x to a real number in the interval $[0, 1]$. Therefore, the element x is characterized by $\mu_{\underset{\sim}{\mathcal{A}}}(x)$ in the set $\underset{\sim}{\mathcal{A}}$: the closer the corresponding value to unity, the higher the degree of membership for x.

2.2.1 Aggregation operators

Aggregation operators are a means for aggregating a set of values into a single value. These operators are generally classified as either *conjunctive* or *disjunctive*, depending on whether they combine the values by a logical AND or an OR operator, respectively, and between these categories there is the category of averaging. A number of studies show that these operators are very practical for resolving Multi-Criteria Decision Making (MCDM) problems [94, 28, 8, 97].

Definition 2.2.2. Let $D_{m\times n}$ be a decision matrix with n criteria that are used to assess m alternatives, where each d_{ij} denotes the degree to which an alternative x_i satisfies a criterion a_j, as shown in Table 2.2. MCDM is then a problem for which the best solution among the set of alternatives needs to satisfy at least one, some, few, most, or all criteria provided in an evaluation.

Zadeh explains that the aforementioned linguistic terms allow us to express what we desire from the corresponding aggregation [97]. This type of problem solving has vast applications in many different disciplines. In complex systems, it can be considered for modeling complicated situations where the best entities have to be identified with respect to important roles [3].

[3]More specifically, it is proposed for the exploratory analysis of node centrality in complex and multiplex networks as will be elaborated in Chapters 3, 4, and 5.

Ordered Weighted Averaging (OWA)

In 1988, Yager proposed an aggregation operator that is between the two extreme cases of the min and max operators [94]. According to him, the aggregation of multiple criteria is mostly done through the guidance of a quantifier that characterizes a linguistic term, e.g., most. In fact, there are several decision making problems for which it does not matter which criteria are actually satisfied, as long as enough of them are fulfilled. Yager assumes that the degree to which a criterion j is satisfied can be expressed by a real positive number in a unit interval $[0, 1]$, and that 1 means full satisfaction and 0 means no satisfaction for the respective criterion. In the extreme case of "all", an alternative x should satisfy all of the criteria. This aggregation is described by an "anding" of the values [94]. In the case of "at least one", the alternative x must satisfy at least one criterion with the best value to be selected as the best solution. This aggregation is accordingly described by "oring" of the values [94]. [4]

Definition 2.2.3. Let A be the set of satisfaction values for all criteria, let B be a descending version of it, and $W = [w_1, w_2, ..., w_n]$ (where $n = |A|$) be a weight vector such that $w_i \in [0, 1]$ and $\sum_i w_i = 1$.

Then the operator OWA is defined as a mapping function $I^n \to I$ (where $I = [0, 1]$) as follows:

$$OWA(a_1, a_2, \cdots, a_n) = \sum_{i=1}^{n} w_i b_i, \tag{2.3}$$

where b_i is the i^{th} largest element in A. The function uses the inner product of a weight vector W and B[5].

Yager explains that the OWA operators need to satisfy four properties of being monotonic, symmetric, idempotent and being bounded by max and min [93, 28, 95]. The two extreme cases of OWA (max, min) can be obtained using two weight vectors in the following way: If we set $W = [1, 0, 0, ...0]$, the operator returns the maximal satisfaction value among n ordered criteria and $W = [0, 0, ..., 1]$ results in the minimal satisfaction value among them.

Example 1. Assume the weight vector $W = [0.4, 0.3, 0.2, 0.1]$ is associated with $n = 4$ criteria. For an alternative x with the satisfaction values $A = [0.6, 1, 0.1, 0.3]$, whose ordered version is $B = [1, 0.6, 0.3, 0.1]$, the operator results in:

$$OWA(0.6, 1, 0.1, 0.3) = 0.4 \times 1 + 0.3 \times 0.6 + 0.2 \times 0.3 + 0.1 \times 0.1 = 0.65.$$

It is obvious that the weights are not associated with a specific criterion but with an ordered position. Therefore, there is no fixed weight for any particular criterion. The only fixed weighting vector that is included in OWA operators is the regular average, as it can be obtained by $W = [1/n, 1/n, ..., 1/n]$ [94].

Example 2. Let $W = [0.25, 0.25, 0.25, 0.25]$ be a weight vector. Then all the values of the $n = 4$ criteria have the same importance and the operator gives

[4] In multi-criteria decision making, no compensation is allowed if "anding" is used, i.e., a high satisfaction in one criterion does not compensate a low satisfaction in other criteria [94].

[5] Because of the ordering process, OWA is a nonlinear aggregation.

the result as follows:

$$OWA(0.6, 1, 0.1, 0.3) = 0.25 \times 1 + 0.25 \times 0.6 + 0.25 \times 0.3 + 0.25 \times 0.1 = 0.5$$

Regarding the semantics of the extreme cases of OWA operators, the weight vector $W = [1, 0, 0, \cdots 0]$ indicates that there is complete satisfaction if "at least one criterion is satisfied" and, analogously, the weight vector $W = [0, 0, \cdots, 1]$ indicates that there is no satisfaction unless "all criteria are satisfied" [94].

Example 3. Assume two candidates x_1 and x_2 are assessed using $n = 4$ criteria in a selection process. The two candidates have the values $[0.4, 0.8, 0.1, 0.7]$ and $[0.5, 0.4, 0.8, 0.3]$, respectively. If a decision maker prefers to choose the one who has at least one criterion with the best value and if the weight vector equals $[1, 0, 0, 0]$, x_1 and x_2 get the same score, as their highest value is equal to 0.8. Now consider the selection of the one who has a higher score on average. Both candidates would get the same score of 0.5. However, if a decision maker wants to reward the one whose least value is maximal, the results vary between the candidates:

$$OWA_{x_1}(0.4, 0.8, 0.1, 0.7) = 0 \times 0.8 + 0 \times 0.7 + 0 \times 0.4 + 1 \times 0.1 = 0.1$$

$$OWA_{x_2}(0.5, 0.4, 0.8, 0.3) = 0 \times 0.8 + 0 \times 0.5 + 0 \times 0.4 + 1 \times 0.3 = 0.3$$

In order to scale between different aggregations, Yager states that the degree to which an aggregation operator is close to either of the aforementioned extreme cases can be computed by a measure called *orness*.

Definition 2.2.4. The *orness* of weight vector W can be measured by:

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} ((n-i)w_i)$$

where n is the number of criteria to aggregate.

Therefore, the *orness* of a weight vector like $[\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}]$ is 0.5, the *orness* of the $[1, 0, 0, \cdots, 0]$ vector is 1, and the *orness* of the $[0, 0, 0, \cdots, 1]$ vector is 0 (*andness*=1-*orness* as defined in [28]). Although this measure allows one to recognize the type of the chosen decision strategy, there might be some weight vectors that have the same *orness* but that are different in the sense of information usage. Assume two weight vectors $[0, 1, 0]$ and $[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ for $n = 3$ criteria. Both of them have an *orness* of 0.5. However, the second vector uses more information. Yager therefore introduces a measure of entropy to obtain the dispersion degree of weight vectors.

Definition 2.2.5. Let W be a weight vector including weights for n criteria in an OWA operator. Its *dispersion* is defined as follows:

$$dispersion(W) = -\sum_{i=1}^{n} w_i \ln w_i$$

Considering the operators "all" and "at least one", it reveals that they have minimum *dispersion* and the regular average has maximum *dispersion* according to [94].

In real-world problems, we use linguistic quantifiers to express our expectations in making decisions, such as few, most, many. In 1983, Zadeh elaborated that these quantifiers can be categorized into two classes with respect to their expression: either the number of criteria or the proportion of criteria [97]. For the sake of formality, Zadeh introduced a representation using fuzzy sets for any kind of quantifier. Any relative quantifier such as most or many can be represented as a fuzzy subset Q, where for any proportion of criteria, $x \in [0,1]$, $Q(x)$ gives the degree to which x expresses the corresponding concept. Yager explains that a Regular Increasing Monotone (RIM), e.g., all, many, at least α, results in an aggregation saying *the more criteria satisfied the better the solution* [95]. Briefly, a fuzzy subset Q of a real line is RIM quantifier when $Q(0) = 0; Q(1) = 1; Q(x) \geq Q(y)$ if $x \geq y$. Furthermore, the weights are obtained as follows:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right) \text{ for } i = 1, \cdots n \tag{2.4}$$

If we consider an RIM quantifier such as $Q(r) = r^{\beta}$ with $\beta \geq 0$, then the *orness* of this quantifier can be obtained as follows:

$$orness = \int_0^1 r^{\beta} dr = \frac{1}{\beta + 1}$$

which is the area under the quantifier Q. If $\beta > 1$, then $orness < 0.5$ and if $\beta < 1$, then $orness > 0.5$ [28].

By changing the value of β, we obtain a wide range of weight vectors and, consequently, various aggregation types. When $\beta = 0$, the first term is 1 (based on $0^0 = 0$) and all other values are 0 ("or" aggregation). If $\beta = 1$, all values are $1/n$ (averaging). If $\lim_{\beta \to \infty}$, the last term is 1 and all others are 0 ("and" aggregation). Tables 2.3 summarize some weight vectors, each of which can be obtained with a value of β.

For a long time, obtaining the weights of OWA operators was a challenging problem [29]. For instance, in the weights generated by the quantifier $Q(r) = r^{\beta}$, the dispersion of OWA weights around $\beta = 1$ does not indicate symmetric behavior. When $\beta \geq 1$, the weights are close to the maximum; however, when $\beta < 1$, the weights are less close to it [54].

Maximum Entropy Ordered Weighted Averaging (MEOWA)

Regarding the OWA weights with having maximum entropy between them, Yager analyzed an approach proposed by O'Hagan [67]. In 1995, he proposed a method to provide the OWA weights with Maximum Entropy directly as follows:

He stated that the weight vector for n criteria can be obtained based on a parameter β [28]:

$$w_i = \frac{e^{\beta \frac{n-i}{n-1}}}{\sum_{j=1}^{n} e^{\beta \frac{n-j}{n-1}}}, i = (1, n) \tag{2.5}$$

In order to make relevant any chosen β and the degree of *orness* of the corresponding operator, Yager used a non-linear equation [28], which is as follows:

TABLE 2.3: A diverse range of weights obtained using different values of β is listed in the upper table. * denotes that in these cases the weight vector depends on the number of criteria and the chosen value of β. The lower table shows the associated weight vectors for some numbers of criteria ($n = 3$ and $n = 4$). The numbers in the tables are rounded.

β	Associated Weights in OWA	$orness$
$\beta \to 0$	$w = [1, 0, ..., 0]$	1.0
$\beta \to 1$	$w = [1/n, 1/n, ...1/n]$	0.5
$\beta \to \infty$	$w = [0, 0, ..., 1]$	0

β	$n = 3$	$n = 4$	$orness$
$\beta \to 0$	$w = [1, 0, 0]$	$w = [1, 0, 0, 0]$	1
$\beta \to 0.5$	$w = [0.58, 0.24, 0.18]$	$w = [0.50, 0.21, 0.16, 0.13]$	0.67
$\beta \to 1$	$w = [0.33, 0.33, 0.33]$	$w = [0.25, 0.25, 0.25, 0.25]$	0.5
$\beta \to 2$	$w = [0.11, 0.33, 0.56]$	$w = [0.06, 0.19, 0.31, 0.44]$	0.33
$\beta \to 5$	$w = [0, 0.13, 0.87]$	$w = [0, 0.03, 0.21, 0.76]$	0.17
$\beta \to \infty$	$w = [0, 0, 1]$	$w = [0, 0, 0, 1]$	0

TABLE 2.4: MEOWA weights, *orness* values, and the entropy of weights for different β-values on the aggregation of $n = 3$ criteria.

β	w_1	w_2	w_3	$orness(W)$	$dispersion(W)$
-20	0	0	1	0	0
-10	0	0.01	0.99	0.01	0.04
-5	0.01	0.07	0.92	0.04	0.30
-2	0.09	0.24	0.67	0.21	0.83
0	0.33	0.33	0.33	0.5	1
2	0.67	0.24	0.09	0.79	0.83
5	0.92	0.07	0.01	0.96	0.30
10	0.99	0.01	0	0.99	0.04
20	1	0	0	1	0

$$orness = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) \frac{e^{\beta \frac{n-i}{n-1}}}{\sum_{j=1}^{n} e^{\beta \frac{n-j}{n-1}}}$$

The produced weights are always between $[0, 1]$ and the sum of the resulting weights is equal to 1, the same as the conditions of OWA weights. For $\beta \to 20$, the resulting MEOWA weight vector is $[1, 0, \ldots 0]$ and the *orness* of the operator equals 1. When $\beta \to -20$, the weight vector is $[0, \ldots 0, 1]$ and the *orness* of the operator equals 0. When $\beta = 0$, then (for all n), the weight vector is simply given by $[\frac{1}{n}, \frac{1}{n}, \cdots \frac{1}{n}]$ and its *orness* value is 0.5. The MEOWA weight vectors for $n = 3$ criteria and their *orness* and *dispersion* values are listed in Table 2.4.

To clarify the operation of OWA even better, we extend the example stated earlier to an $MCDM$ problem including $m = 6$ alternatives that are assessed with $n = 3$ criteria, where each has a value between $[0, 1]$, i.e., 0 means no

TABLE 2.5: A 6×3 decision matrix containing the satisfaction values of three criteria for six students. For each student, the maximum value is highlighted in bold.

Students	Multiple Criteria		
x_1	0.7	**0.9**	0.8
x_2	**0.8**	0.3	0.4
x_3	0.2	**0.8**	0.3
x_4	**0.4**	0.2	0.3
x_5	0.1	**0.2**	0.1
x_6	0.1	**0.1**	0.1

satisfaction and 1 means full satisfaction.

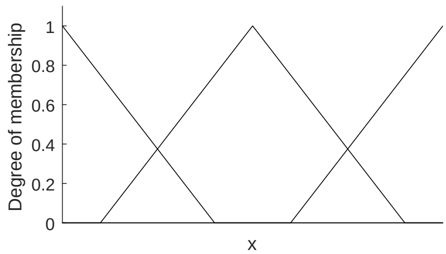
Example 4. Assume the values of a decision matrix on choosing the most promising student among six students with respect to their multiple activities in Table 2.5. Considering an OWA operator with the weight vectors $W = [1, 0, 0]$ and $W = [0, 0, 1]$, respectively, different outcomes will be obtained for the students. Using the weight vector $W = [1, 0, 0]$, the result will be the best activity level that each student has: $0.9, 0.8, 0.8, 0.4, 0.2, 0.1$ x_1 to x_6, respectively. x_1 has the best result and x_6 has the worst one. Now, consider an aggregation that results in a score for students with respect to their least activity level; this means the one whose least activity level is maximal can be selected as the most promising student. The resulting scores for students x_1 to x_6 are $0.7, 0.3, 0.2, 0.2, 0.1, 0.1$, respectively. Again, the best observed value is for the student x_1 and similarly, x_6 has the worst value. Obviously, both decision strategies can be used in the evaluation process. However, in some cases the selection with respect to the second aggregation will be an strict decision. Assume a student has the values $[0.8, 0.7, 0.1]$. In the first aggregation he/she will be among the top 4 students. But, using the second aggregation he/she will be among the bottom 3 students. Instead, if the aggregation uses a weight vector that gives more importance to highest and second highest activity values, e.g., $[0.6, 0.3, 0.1]$, then he/she will be among top 3 students.

2.2.2 A 2-tuple fuzzy representation model

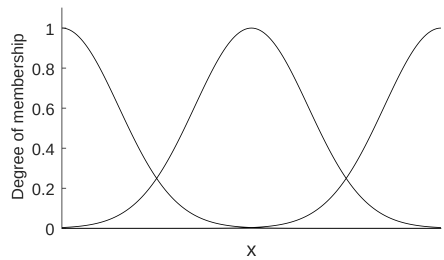
Let x be a value in $[0, 1]$. Herrera and Martinez propose a model to transform x into a linguistic 2-tuple in [36, 37]. They assume a set of labels $S = \{s_0, s_1, \cdots, s_g\}$, where each label is represented using a membership function, e.g., triangular, or Gaussian as shown in Figure 2.5 (A). In the transformation process, the value is converted into a fuzzy set in S. Then the fuzzy set is transformed into a linguistic 2-tuple, i.e., it requires an aggregation operation over multiple obtained membership values, which is defined by Herrera and Martinez as follow:

Definition 2.2.6. Let $\mathcal{S} = \{(s_0, \mu_{s_0}), (s_1, \mu_{s_1}), \cdots, (s_g, \mu_{s_g})\}$ be a fuzzy set, then a symbolic aggregation operation is used as follows:

$$\theta = \frac{\sum_{j=0}^{g} j \cdot \mu_{s_j}}{\sum_{j=0}^{g} \mu_{s_j}} \tag{2.6}$$



(A) Triangular



(B) Gaussian

FIGURE 2.5: Two basic membership functions for variable x.

The result of the aggregation is a linear projection onto the sequence of linguistic term set denoted by θ, which is$\in [0, g]$.

Definition 2.2.7. Let θ be the result of symbolic aggregation. As Herrera explain the equivalent information of θ in the linguistic term set S can be expressed using the 2-tuple model using a function $\Delta : [0, g] \to \mathcal{S} \times [-0.5, 0.5)$:

$$\Delta(\theta) = (s_i, \alpha), with \begin{cases} s_i, & i = round(\theta) \\ \alpha = \theta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{2.7}$$

where *round* is the regular operation of rounding [36]. This results in a 2-tuple comprised of a label to which value x mostly belongs to and a parameter $\alpha \in [-0.5, 0.5)$ that indicates the value of the symbolic translation, i.e., the corresponding value expresses the *difference of information* between θ and the closest index in $\{0, \cdots, g\}$.

The inverse function of Δ is: $\Delta^{-1} : \mathcal{S} \times [-0.5, 0.5) \to [0, g]$ defined as follows:

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \theta, \tag{2.8}$$

Chapter 3

Analyzing node centrality in complex networks

Centrality measures are well known and mostly used to focus on different structural properties of nodes and to quantify the corresponding importance of nodes in a network [10, 50]. They are functions on the nodes of a graph that take the structure of the graph and assign a real-value to all nodes; higher index values of nodes emphasize that the corresponding entities are more central and lower index values of nodes identify them as less central in the networked system. Depending on the network process under investigation, the most central nodes are described as most mobile nodes [46], key mediators [83], or brokers [14]; there are many names, but only a few semantic descriptions of *centrality*. Again, depending on the type of network representation, such as directed [92], weighted [65, 70], temporal [72, 20, 2], or multi-layer representations [81, 22], a number of *centrality indices* have been developed or have been extended from one type of network representation to another one. Such a wide range of studies and developments does, on the one hand, indicate the importance of "node centrality" and of "identifying the most influential nodes"; on the other hand, it is surprising that such a large range of measures has been proposed to do the same thing, namely, "rank the nodes" according to their influence.

The most frequently cited work related to the concept of centrality and different methods for measuring node centrality is a paper published in 1979 by Freeman [30]. He discusses many ways to define the concept of centrality in human groups and illustrates multiple aspects of centrality using a star graph with five nodes. He exemplifies that if node centrality is conceived in terms of *degree*, in a human group, the purpose of using it is to analyze the activities of persons with respect to a specific process of communication. Then the nodes can be assessed based on their position and their potential to control the information or knowledge flow in the network. In this scenario, as Freeman says [30] the concept of centrality and its measurement refer to the question of whether or not a node has an important role in a network, and if so, to which degree.

Going beyond simple networks, there have been many attempts to generalize classical centrality measures into more complicated frameworks such as weighted networks as already sketched in Section 2.1.2.

In this type of networks, weights can be indicators of the intensity of emotions, functions regarding the duration of links in social networks [33], or quantifiers for a specific capacity of links between a pair of nodes in non-social networks [70].

Opsahl, Agneessens, and Skvoretz argue that in all the extensions of centrality measures to weighted networks by several researchers (e.g., in [5, 65]), the key feature of the original measures proposed by Freeman is missing: the number of communication partners. They mention a situation concerning node centrality in weighted networks that focuses on the relative importance of weights, which are attached to links, to the number of communication partners of a node.

They found a trade-off between two properties, where each can be captured using a centrality index: The first one is quantified by *strength* and the second one is computed as *degree*. Using a tuning parameter, they show a scaling between different decisions that can be made using two properties in the analysis of node centrality [70]. This is an interesting idea with respect to the *degree centrality* measure in weighted networks, which has been highly cited in network analytic papers; with about 1290 citations until 2017. However, a crucial question is raised here: What if more than two indices are able to characterize important nodes? We realized that their measure cannot be generalized easily to include more than two *centrality indices*. We show that using an approach based on a fuzzy aggregation operator enables us to include more than two properties in the analysis of node centrality [1]. We will discuss how the proposed approach using a tuning parameter controls the trade-offs between multiple properties and how it provides insights into a node's importance with respect to the worst and the best rankings yielded by multiple properties and whether the importance of the nodes is *robust* to the choices of different decisions, which are guided by the parameter. Thus, in this chapter, we will explain this idea in the analysis of a chat-log network and a communication network. We will show how the stated case can be turned into an MCDM problem and how this problem can be solved.

Besides the problem mentioned above, which encouraged us to propose the idea of including multiple properties in the analysis of node centrality, there is another problem with centrality measures in many real networks. It is not always clear which measure of centrality fits best for a specific network. The main reason for Freeman to introduce several centrality measures was that according to him, each classical centrality measure has a certain limitation as regards the analysis of node importance in a simple network. Briefly explained, *degree centrality* considers the involvement of a node in a network [30]; however, it only focuses on the local structural property of the respective node. *Closeness centrality* considers the reachability of nodes from any other node in the network and focuses on the global structure of a network, but it also has its own limitation, as it results in an infinity value for some nodes in disconnected networks. *Betweenness centrality* focuses on the flow control of a node in the network and measures the degree to which the corresponding node lies on the shortest path

[1] In many real-world problems, to analyze the importance of an entity, we need to consider multiple properties, each of which is characteristic of a specific aspect of importance.

between two other nodes. It thus provides information about a node with respect to the global structure of the network and can also be applied to disconnected networks. However, a large number of nodes might not lie on the shortest paths between any pair of nodes; therefore, it results in the value of 0 for these [70]. Borgatti also argues that each centrality measure is defined to capture a specific flow process in a network. He discusses different topologies of flow processes and categorizes them according to two dimensions of trajectory (e.g., geodesics, paths) and transmission (e.g., duplication, transfer) [10]. Based on his categorization, there are some measures that can be used for a specific flow process, e.g., the measures of both *betweenness* and *closeness* can be used to analyze the importance of nodes if a process passes along geodesics [10]. Once again, for a long time, no generalized method existed that included all of the centrality measures in a scalable evaluation and also allowed checking whether multiple measures agree on a node's importance or have totally different views.

The usage of multiple centrality measures in such a generalized method has only been pointed out in one paper, by Du et al. [25]. They use a handful of classical *centrality indices* to identify most central nodes using the TOPSIS technique [40]. They consider the evaluation of influential nodes as a decision making problem and compute the difference of each node's centrality values to a hypothetical, ideal solution determined by the respective decision maker. However, the TOPSIS approach requires a fixed weight vector that assigns a value between 0 and 1 to each of the *centrality indices* whose sum of weights is 1. This opens a huge range of possible weight vectors that cannot possibly be explored fully. Instead, in our proposed approach based on a fuzzy aggregation operator, which contains a weighting equation, it is possible to meaningfully produce a range of weight vectors as lying between the aggregation that either satisfying one aspect maximally is favored, or doing well on average, or satisfying all aspects. The nodes at the top-ranking and at the bottom-ranking with a stable behavior are revealed in multiple network datasets as described below.

3.1 Description of node centrality in different data sets

3.1.1 Communication networks

There are many rich data in the world that can be represented as networks and can then be analyzed using network analytic methods. Although this type of representation allows for better exploration of the interactions among entities in the corresponding data, the entities and/or the interactions in the data might have additional properties that cannot be easily mapped to the represented network.

Chat-log data is one of those types of rich data that is used for various kinds of research. Recent studies argue that many mental disorders can be partially improved in online group psychotherapy sessions [59, 32]. Thus, constructing a social network deduced from digitally logged interactions allows for much more extensive exploration of the activity of persons in a group chat and their involvement with respect to the logged communication. In a psychotherapy group chat, the findings will provide more insights about the productivity of a session and will be helpful for therapists. In a study, we preprocessed a large amount of semi-structured chat-log data[2] and deduced a weighted social network from multiple group chat sessions. A network representation was chosen where each participant represented by a node and interaction in the form of a message sent between a pair of participants is represented using an edge. Since in a chat session, multiple interactions exist between the same participants, these multiple interactions can be represented by multi-edges or by weighted edges, i.e., the number of statements sent from person A to person B can be summed up to result in a weight. In total, a network was constructed from multiple sessions including 52 nodes and $29,590$ multiple edges [84].

What characterizes important nodes? In fact, in a human group the importance of the participants can be determined in different ways. Here the question is which properties can show the activity level of one participant. Obviously, in the represented weighted network, *degree centrality* and *strength* can be two of these properties (see the definition of these two measures in Chapter 2.1.2.). The chat-log data encompasses features such as the number of words or the exact timing of when the statement was submitted. Therefore, it is obvious that one way to operationalize the importance of a participant in a chat is to count the number of words a person submits. Another way is to sum up the reaction time of the other participants to his or her statements. We operationalize this idea by measuring the *total reaction time*, i.e., the total time until the next participant starts sending a statement after A has submitted a statement (disregarding the 10 first and last sentences of the sessions, which only refer to hello and farewell statements). A participant might have a large number of communication partners, but might have sent only a small number of statements, or spent a very short time in the chat session. This may vary from participant to participant. Thus, any meaningful approach should be able to draw conclusions from these multiple trade-offs and possibly conflicting rankings.

Freeman's EIES communication data set is the second dataset that contains multiple features for nodes. It was originally compiled and analyzed by Freeman [31]. This dataset contains three networks of researchers working in the Social Network Analysis area. The first and the second network contain the personal relationships between a group of researchers at the start and at the end of the study. Opsahl, Agneessen, and Skvoretz used the third network of this dataset to analyze the researchers' importance in the represented (weighted) network [70, 69]. The network illustrates the interactions among 32 researchers in terms of sending messages to each other—we obtained the network from `http://toreopsahl.com/datasets/#FreemansEIES`. The network consists of 460 multiple edges, which can also be represented by a simple, weighted network where the weights on the edges denote the number of messages sent between a pair of nodes.

What characterizes important nodes? Although having a high number of collaborators and strong relationships would give insights into the activity and importance of a researcher, we assume that in order to characterize whether

[2]Some statements in this data did not include the intended username of a receiver and were termed "misaddressed statements". We proposed six methods for reliably predicting a receiver of such statements by using a set of prediction rules that follow human communication behaviors. The dataset is not public. See paper [84] for more details.

a researcher is an active researcher or not, additional information may be required in some cases. This dataset contains other information for each researcher, such as the number of citations and the disciplinary affiliation in 1978. Therefore, in addition to the *out-degree* and the *out-strength* (the number of sent messages), which are measures in the represented weighted network, we use the *number of citations* as additional information that describes a researcher's activity in a field in a year. Again, a researcher might have a large number of communications, but weak relationships, or a low number of citations. In another case, the situation may be the reverse: a low number of collaborators, but strong relationships and a large number of citations. Such trade-offs make finding the top researchers a more complicated problem to be solved. Therefore, any practical approach should be able to deal with conflicting rankings resulting from such an evaluation.

3.1.2 Air transportation network

As mentioned above, it is not always clear which centrality measure fits best for a network. In addition, no generalized method has existed to date that includes all of them to show the conflicting views of classical *centrality indices*. In a network of **airline transportation data**, airports are represented as nodes; two nodes are connected using an edge if a flight exists between them that is operated by a specific airline. This network was selected from a large dataset comprised of 37 networks, each containing data about the flights of a European airline [17]. We use the data from the low-cost airline, AirBerlin, provided at `http://complex.unizar.es/~atnmultiplex/`. It encompasses an undirected network of 75 airports and 239 edges.

What characterizes important nodes? The three classical *centrality indices* can all together show the importance of an airport in this network. We assume that *degree* is correlated with the number of people willing to go to the other cities via a low-cost comfortable airline. The average distance to other airports can be captured by *closeness*. *Betweenness* can indicate the importance of an airport in losing a network process if the airport is shut down. We thus measure three *centrality indices*: *degree, betweenness* and *closeness* for all nodes in this dataset. Any practical approach should be able to find the top nodes with respect to multiple indices of centrality. Since each centrality measure has a different view on a node's ranking, finding the top-ranking nodes with respect to different types of aggregations and drawing appropriate conclusions from all the conflicting rankings should be done using the proposed approach.

3.2 The evaluation of node centrality in networks

We aim at analyzing node centrality with respect to multiple aspects of importance, where the importance itself is characterized differently depending on the type of dataset under analysis. Different properties are measured for nodes depending on the network data. We thus use the term "multiple criteria" for multiple aspects of importance in the evaluation process.

Having multiple criteria obtained to identify the best node(s) among all initiates the elements of an MCDM problem (see Definition 2.2.2). In this configuration, the nodes play the role of *alternatives*, which are assessed using multiple criteria. As explained in Chapter 2.2.1, OWA operators are one of the best methods for dealing with MCDM problems as they provide a global evaluation of alternatives with respect to their best and worst results. The steps of the proposed method are listed in the following box.

Step 1: Measure multiple properties and normalize the values of each one between $[0,1]$: Assume $A = \{\vec{a}_1, \vec{a}_2, \cdots, \vec{a}_m\}$ is a set of numerical vectors where each vector a_j corresponds to one measure. For each vector a_j, normalization is computed by the max and min-values of all $a_j[i]$, and $a'_j[i] = \frac{a_j[i]-\min}{\max-\min}$. Set multiple criteria for all nodes in the network.

Step 2: Compute the aggregated *scores* for all nodes using the $OWA(a'_1, a'_2, \cdots, a'_m)$ operator 2.2.1 with respect to a set of different values of the β-parameter in Eq. 2.5.

Step 3: Rank all nodes with respect to the aggregated *scores* for all values of β-parameter.

Step 4: Find the minimal and maximal rank for all nodes over the set of β-values. For any node i these are denoted by $MinRank(i)$ and $MaxRank(i)$.

In this evaluation, 0 always means no satisfaction and 1 indicates full satisfaction for each criterion. Using the different aggregations guided by the β-parameter (in $MEOWA$ weighting function), node i gets different *scores*. The *scores* will be between $[0,1]$. The aggregations scale between two cases: taking either at least one criterion with the best value of satisfaction, or all of them. This consequently leads to different rankings for node i with respect to its best and worst value.

Here, we are especially interested in whether the ranking changes strongly depending on the choice of different types of aggregations, or whether their importance is *robust*. Therefore, we apply the OWA operator accompanied by a range of different β-values, which scales between two cases: The $-\infty$-symbol represents a large value for β that is used to obtain the weight vector $[0, \cdots, 0, 1]$. The ∞ shows a large value that is used to result in the weight vector $[1, 0, \cdots, 0]$. When $\beta = 0$, a regular average is performed. Note that taking only a range of values before and after $\beta = 0$ would be sufficient to show the changes in the rankings. However, in this chapter, the values are more detailed on the x-axis.

Once all the nodes have been evaluated with all the possible aggregations, they are ranked according to their resulting *score*s in non-increasing order for all β-values. A node whose ranking remains stable among top-ranking nodes with respect to the different values of β is speculated to have *robust* importance, whereas one whose ranking changes only fulfills some aspects of importance but not all in the same way.

The results are presented using a table of the criteria' values of all nodes in the corresponding datasets, a plot of the pairwise correlations of the multiple criteria, a visualization of the rankings of the nodes over the different values of the β-parameter, and a visualization of $MinRank(i)$ versus $MaxRank(i)$ obtained over all β-values for all nodes. If the values of $MinRank(i)$ and $MaxRank(i)$ for any node i are too far apart, the node does not have a stable ranking, i.e., it might have a high satisfaction value in one or two criteria but not in all.

3.3 Results

Chat-log network

In the deduced network, the questions to be addressed are: *How active are the participants in a group-chat session?* and *Who is (are) the most* active participant(s) among all? It can be conjectured that in a group chat session with a fixed topic, the responsibility of the moderator (e.g., the psychotherapist) demands a higher level of activity compared to that of the other members of the group [96]. For example, the therapist might need to spend more time communicating with more other members than a normal member. Sometimes, however, the therapist should talk less than other members and moderate the chat in order to make the session productive. Due to the informal and basically unstructured nature of a chat, some participants will attempt to dominate it by sending more statements, and others will isolate themselves. This might lead the therapist to have many interactions with the same persons to either activate or moderate them; this results in low *degree* and high *strength* for the therapist. However, in most therapy sessions, a reasonable measure should identify the therapist(s) as the most active or most central person(s).

Following the steps of the proposed approach, the four criteria are set for node i as follows:

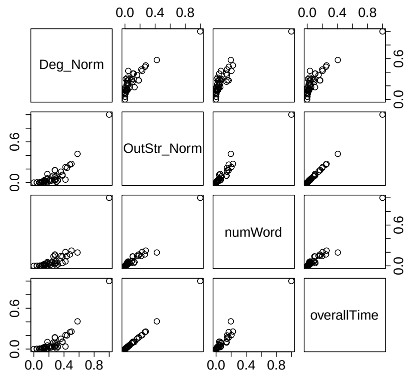
$a_1[i]$: the number of communication partners (*in- plus out-degree*).

$a_2[i]$: the total number of statements sent by the participant i (*out-strength*).

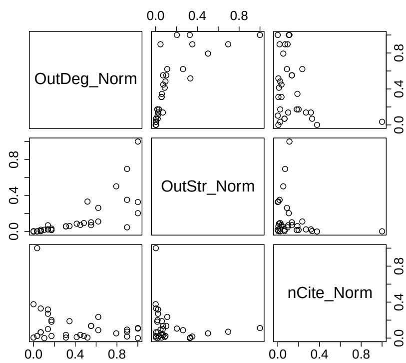
$a_3[i]$: the total number of words submitted by the participant i.

$a_4[i]$: the overall reaction time.

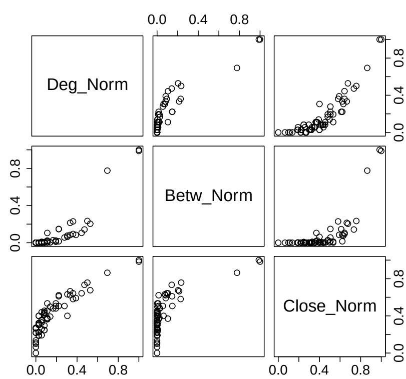
Then all the values are normalized and the results for different values of the β-parameter are computed. Then the nodes are ranked for the corresponding values of β.



(A) Chat-log network



(B) Freeman's EIES network



(C) Air transportation network

FIGURE 3.1: The figures show (A) the scatter plot of pairwise correlation of four criteria for chat-log data. (B) The scatter plot of three criteria for Freeman's EIES dataset. (C) Pairwise correlation of three criteria for an air transportation network.

TABLE 3.1: The nodes are listed from the highest rank 52 to the lowest rank 1 with respect to the result of $a'_1[i]$, which is the normalized *degree centrality* of node i. The other criteria are, respectively: *out-strength*, number of words, and overall reaction time.

Rank	Participant (i)	$a'_1[i]$	$a'_2[i]$	$a'_3[i]$	$a'_4[i]$
52	Therapist	1	1	1	1
51	P6	0.58	0.4225	0.1957	0.4074
50	P1	0.5	0.2764	0.2237	0.2568
49	P36	0.48	0.2651	0.1678	0.2491
48	P37	0.44	0.2167	0.1446	0.1995
47	P11	0.42	0.2289	0.2036	0.2045
46	P22	0.42	0.0436	0.0636	0.0366
45	P3	0.38	0.1162	0.0423	0.1087
44	P30	0.38	0.1037	0.1365	0.1051
43	P39	0.36	0.1612	0.145	0.1506
42	P27	0.32	0.0333	0.0198	0.0317
41	P24	0.3	0.0608	0.0482	0.0588
40	P28	0.3	0.0103	0.0081	0.0108
39	P41	0.3	0.0924	0.0731	0.0831
38	P5	0.28	0.1826	0.1524	0.1735
37	P10	0.28	0.0314	0.0442	0.0311
36	P18	0.28	0.1823	0.167	0.1679
35	P23	0.28	0.0427	0.0272	0.0429
34	P25	0.28	0.0912	0.0659	0.0898
33	P4	0.26	0.1024	0.1388	0.0962
32	P16	0.24	0.0334	0.0562	0.0279
31	P14	0.22	0.0432	0.0365	0.0392
30	P20	0.22	0.0242	0.014	0.0197
29	P15	0.2	0.034	0.0304	0.0346
28	P7	0.18	0.1269	0.056	0.1041
27	P33	0.18	0.003	0.003	0.0028
26	P51	0.18	0.063	0.0404	0.0527
25	P17	0.16	0.0004	0.0004	0.0003
24	P42	0.16	0.0265	0.0201	0.0256
23	P8	0.14	0.0221	0.0238	0.0202
22	P31	0.14	0.0032	0.0041	0.0023
21	P35	0.14	0.0024	0.0033	0.0029
20	P43	0.14	0.0043	0.0056	0.0044
19	P46	0.14	0.0314	0.0236	0.0283
18	P50	0.14	0.0363	0.0268	0.0277
17	P52	0.14	0.0261	0.011	0.0241
16	P21	0.12	0.0086	0.0096	0.0067
15	P45	0.12	0.0202	0.0066	0.0187
14	P47	0.12	0.0139	0.0141	0.0123
13	P48	0.12	0.0152	0.0103	0.0128
12	P12	0.1	0.0011	0.0008	0.0011
11	P13	0.1	0.0037	0.0032	0.0026
10	P19	0.1	0.0006	0.0004	0.0001
9	P49	0.1	0.0049	0.0044	0.0043
8	P9	0.08	0.0013	0.0007	0.0005
7	P32	0.08	0.0024	0.0016	0.0023
6	P34	0.08	0.0014	0.0016	0.0015
5	P40	0.08	0.0059	0.0029	0.0076
4	P44	0.08	0	0	0
3	P26	0.04	0.0023	0.0011	0.0027
2	P29	0.04	0.0024	0.002	0.001
1	P38	0	0.0009	0.0004	0.0005

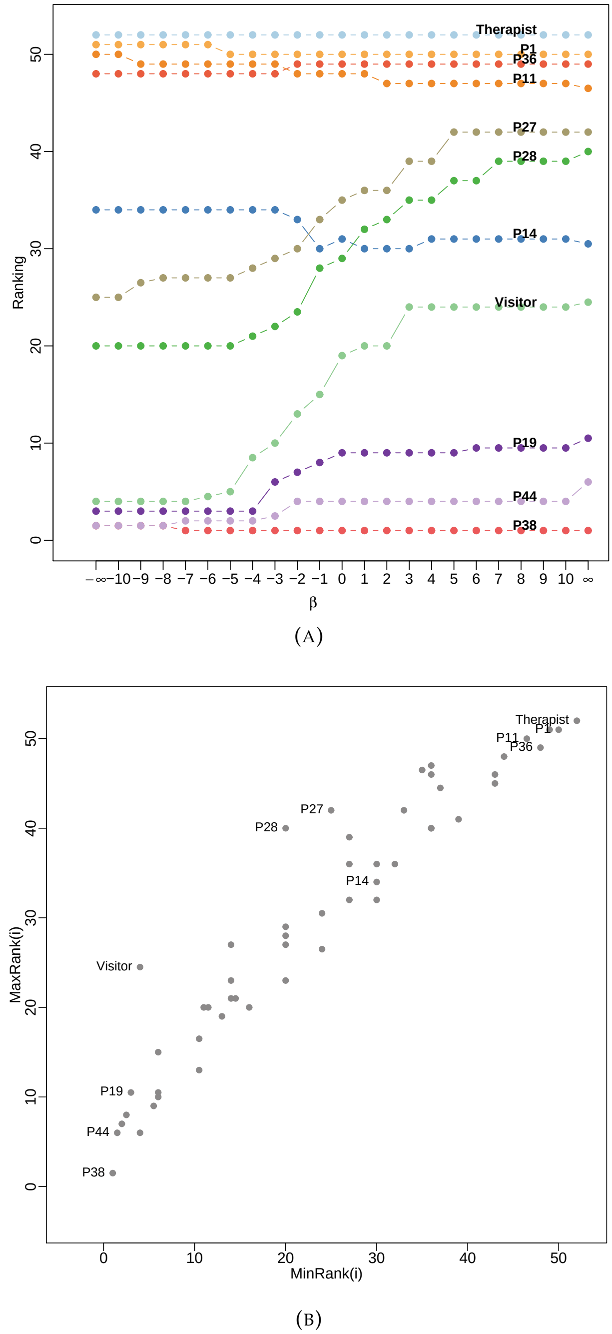


FIGURE 3.2: Chat-log data. (A) The detailed rankings of eleven nodes are depicted over using the different aggregations guided by the β parameter in MEOWA operator (see Definition 2.2.1). (B) Minimal ranking versus Maximal ranking of the nodes are obtained over all β values (from $-\infty$, via 0, to ∞). The details of the four criteria used in the evaluation are listed in Table 3.1.

A pairwise correlation analysis is a good first step to understand whether there are trivial relationships between the values of the criteria associated with the nodes or not. However, in a pairwise comparison, it is not easily possible to track the same node's behavior in different correlations. Thus, a human observer of the different correlation plots still has a hard time to decide which nodes do best with respect to multiple criteria in general. The pairwise correlations between four associated criteria are demonstrated in Fig. 3.1 (A). It is obvious that the correlations are not negligible; however, we apply the proposed approach to the data to see how it performs the evaluation based on the associated criteria and how nodes compete for the top position.

The visualization of Fig. 3.2 (A) starts with the ranks of the nodes obtained in the extreme case of $\beta \rightarrow -\infty$ and as it proceeds to the right side, we see some variations on the ranks of the *score* of the nodes until $\beta \rightarrow \infty$ is reached.

At first glance, it can be observed that the therapist—because of his/her responsibility—gets a high *score* and consequently gets the highest rank for all the values of β; regardless of whether at least one of the criteria is favored to reach the highest importance value or all of them or any mixture of them, the therapist is the most active participant. After him, four participants (P1, P6, P36, P11) are the most active ones.

We choose three interesting cases to emphasize the exploratory feature of our proposed approach. These cases are the nodes labeled $P14$, $P17$, and $P28$, which differ in their ranking on the extreme scales of β. The first case is $P14$: This person communicated with only 13 participants out of the 52 participants of the chat-log. If all nodes were ranked by the normalized *degree* (a'_1) alone, (s)he would be ranked right in the middle (rank 31 out of 52) as listed in Table 3.1. His level of activity involving these 13 persons, measured by the number of words used in the statements, is medium, as is his level of activity with respect to the number of messages and the overall reaction time. In the extreme case of $\beta = -\infty$, when the weight vector equals $[0, 0, 1]$, he has a medium ranking until $\beta = \infty$ is reached. Therefore, he is a *medium active* person.

Another case is $P28$ who has a higher rank than $P14$ if only the highest satisfaction level of the nodes is regarded (in Table 3.1)—this is given by the normalized *degree*. In the extreme case $\beta \rightarrow \infty$, his rank, however, is much lower than $P14$. As can be seen in Fig. 3.2 (A), going to $\beta \rightarrow \infty$, when the aggregation rewards nodes that have at least one criterion with the best value, $P28$ is ranked higher than $P14$. By referring back to the full chat-log data, we found that he did indeed communicate with more distinct members than $P14$, but using only single-word statements. Thus, by using the aggregation $\beta = 0$, his ranking is still less than that of $P14$ as he does not have on average a better *score* to get a better ranking. The participant $P27$ has a similar behavioral ranking to $P28$, but he has a better *score* and a better ranking than those of $P14$ and $P28$ on average.

Another interesting case is $P17$, who has a medium number of distinct communication partners, namely 8. However, all his other criteria satisfaction values are extremely close to 0, such that the lower the value of β is, the lower his ranking becomes. Looking at the chat session, it became clear that $P17$ was not actually a patient, but a psychologist who visited the chat-log and was only a bystander; therefore, (s)he greeted many persons at the beginning but was just an observer for the remaining time.

TABLE 3.2: The values of three criteria (normalized *out-degree*, *out-strength*, and number of citations) associated with 32 researchers in Freeman's EIES dataset are listed here. The nodes are ranked from the highest rank 32 to the lowest rank 1 with respect to the results of the normalized *out-degree* (a'_1).

Rank	Researcher (i)	$a'_1[i]$	$a'_2[i]$	$a'_3[i]$
32	Lin Freeman	1	1	0.1118
31	Nick Mullins	1	0.2036	0.1059
30	Sue Freeman	1	0.3275	0
29	Doug White	0.8966	0.3519	0.0176
28	Phipps Arabie	0.8966	0.0465	0.0941
27	Barry Wellman	0.8966	0.6955	0.0706
26	Russ Bernard	0.7931	0.5021	0.0529
25	Ron Burt	0.6207	0.1113	0.2353
24	Pat Doreian	0.6207	0.2624	0.0882
23	Richard Alba	0.5517	0.1021	0.1353
22	Jack Hunter	0.5517	0.0711	0.1353
21	Lee Sailer	0.5172	0.3329	0.0059
20	Steve Seidman	0.4828	0.0939	0.0235
19	Carol Barner-Barry	0.4483	0.073	0.0353
18	Al Wolfe	0.4138	0.0869	0.0118
17	Paul Holland	0.3448	0.0601	0.1882
16	John Boyd	0.3103	0.0569	0.0353
15	Davor Jedlicka	0.3103	0.0569	0.0059
14	Charles Kadushin	0.1724	0.018	0.2
13	Nan Lin	0.1724	0.0158	0.1824
12	Don Ploch	0.1724	0.0319	0.0235
11	Claude Fischer	0.1379	0.0234	0.3176
10	Mark Granovetter	0.1379	0.0231	0.2706
9	Maureen Hallinan	0.1379	0.0692	0.1
8	Nick Poushinsky	0.1034	0.019	0
7	Sam Leinhardt	0.069	0.0044	0.0647
6	Joel Levine	0.069	0.0196	0.0647
5	John Sonquist	0.069	0.0051	0.3294
4	Ev Rogers	0.0345	0.0019	1
3	Brian Foster	0.0345	0.0022	0.0235
2	Gary Coombs	0	0.0038	0.0059
1	Ed Laumann	0	0	0.3765

Obviously, making decisions over a set of multiple criteria with conflicting opinions about the importance of a node is not a simple task to do. However, to draw a conclusion out of all possible aggregation, the best top-ranking nodes and the worst bottom-ranking nodes can be selected when nodes have *robust* and high importance, and have low importance with a stable ranking, respectively. As illustrated in Figure 3.2 (B), it is clear from the results that the therapist was the most active person and always remained at the top. His values of $MinRank$ and $MaxRank$ are equal. The visitor is the one who frequently stays in the bottom ranking and is not even identified as a medium-active person, i.e., the nodes in the center of the visualization. The cases of $P28$ and $P27$ are observed only very few times among top 10 nodes. And the case $P14$ has a medium stable ranking as shown on the diagonal. The cases $P38$, $P44$, and are always among bottom 10 nodes.

Freeman's EIES network

The questions to be addressed in the represented weighted network are: *How do we evaluate the researchers with respect to their activity in a field?* and *Who is (are) the most active researcher(s)?*

As the first step of the proposed approach, three properties are measured as follows:

$a_1[i]$: the number of communication partners (*out-degree*).

$a_2[i]$: the total number of messages sent by researcher i (*out-strength*).

$a_3[i]$: the total number of citations of researcher i.

Following along the steps, normalization is then performed using the maximum and minimum values of each measured property. Looking at the pairwise correlation between the three aforementioned criteria, there is no strong correlation between any pair (s.Figure 3.1 (B)), and even for such a small network, tracking the ranking of nodes for a human observer would not be easy. Then the overall aggregation *score* is computed with respect to different aggregations guided by β for all nodes. Looking at the visualizations in Fig. 3.3 (A) and (B), we can see that there are much more fluctuations than in the first data set, even though it has less nodes [3].

For all values of β and the produced aggregations, the most active researcher with *robust* importance is Lin Freeman, who achieves maximal satisfaction values for two criteria. There is no one found having all three criteria with a full satisfaction to get 1 in his/her *score*. And, Lin Freeman is so far not the one with the most citations as listed in Table 3.2 (a value of 0.112). However, when $\beta = -\infty$, all the nodes are ranked with respect to their least value, he is the one whose least value is maximal. Thus, he stays at the top. Sue Freeman and Nick Mullins also have the highest satisfaction values (for the normalized *out-degree* centrality) and Rogers has a maximal satisfaction value of 1 for number of citations (he has 170 citations and Sue Freeman has 0 citation). Thus they share the first rank with Lin Freeman in $\beta = \infty$.

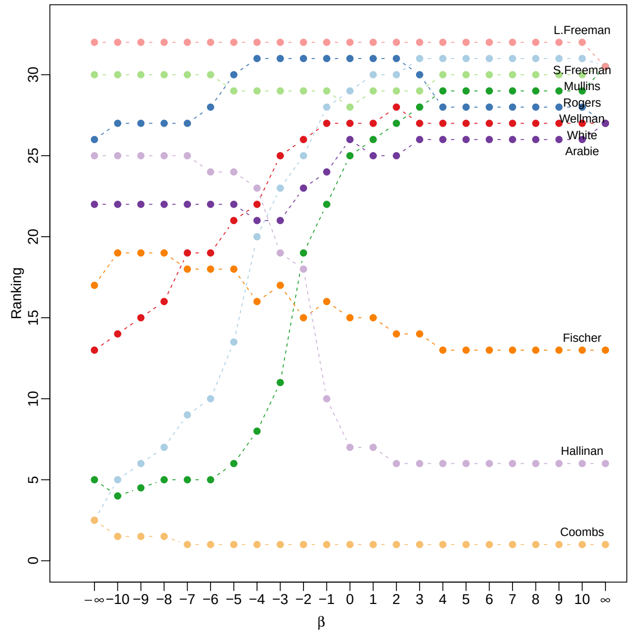
Claude Fischer is the one with a medium and rather stable ranking in the middle of Figure 3.3 (A), and all of his values result in a medium rank for him among all the researchers. A considerable change can also be seen in the ranks of Rogers (dark green curve) and Hallinan (purple curve) if different values of β are used. If they were ranked only by their least satisfaction value, Rogers would get a much lower rank than Hallinan. However, they switch their positions when the value of β is increased in Fig. 3.3 (A). Regarding the selection of the top researchers, it turns out that they are not robustly among the top-ranking nodes, as shown in Fig. 3.3 (B).

We assume that those nodes with the most stable behavior among top-ranking nodes are the most central nodes. In this dataset, the examples are Freeman, Mullins, and Wellman. In contrast, the least central nodes are in the category that contains nodes with weak importance and have *stable* behavior, e.g., Coombs, i.e., he is always (on average, min, and max aggregations) among the bottom 2 rankings. After him, Poushinsky and Foster are those who stay often among the bottom-ranking nodes.

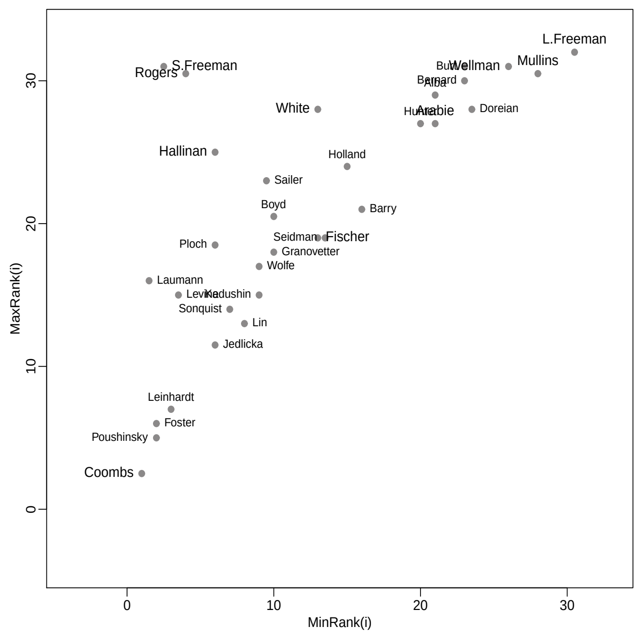
Air transportation network

An airline transportation network data including information about flights between airports in Europe is analyzed. Considering a process in the network

[3]The results obtained here are included in a journal paper, which is in preparation [87].



(A)



(B)

FIGURE 3.3: Freeman's EIES network. (A) The detailed rankings of the nodes are depicted over using the different aggregations guided by the β parameter in MEOWA operator. (B) Minimal ranking versus Maximal ranking of the nodes are obtained over all β values.The details of the three criteria used in the evaluation are listed in Table 3.2. The results of MEOWA are ranked by tied ranking.

that can be captured in multiple ways, the approach should be able to reveal which nodes are identified as important airports considering one measure but unimportant with respect to the other two measures. Questions to be answered include: *Which airports are on top with respect to all the three aspects of centrality?*

Following the steps of the proposed approach, multiple criteria for node i are set as follows:

$a_1[i]$: *degree centrality*.

$a_2[i]$: *betweenness centrality*.

$a_3[i]$: *closeness centrality*.

All the results of the three criteria are listed in Table 3.3, following the normalization step taken in accordance with box 3.2. In this table, the nodes can only be ranked by one of the criteria—we chose the normalized *degree centrality*.

Another representation, the pairwise correlation among the normalized three classical *centrality indices*, is shown in Fig. 3.1 (C). Looking at the correlations and the values make it clear that the normalized *closeness* index values for the majority of the nodes are greater than the values of the other two indices. Therefore, the corresponding criterion will have mostly a dominant role in the aggregation results, i.e., 79% of nodes have a higher normalized *closeness* value than those of the other two indices (i.e., $a'_2 < a'_1 < a'_3$). We explain this problem in Chapter 5 in detail.

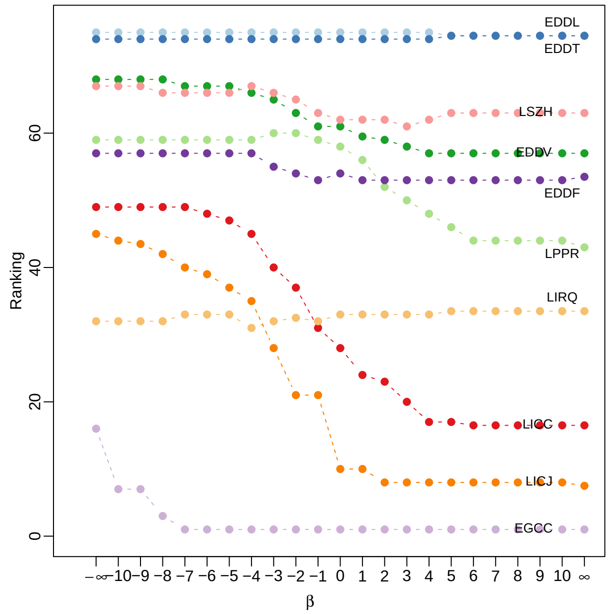
Performing the remaining steps of the procedure results in the rankings of 75 airports depicted in Fig. 3.4 (A) with respect to the different aggregation schemes over the three criteria employed. The visualization shows that the top-ranking airports are always EDDT (Berlin) and EDDL (Düsseldorf); the two lines at the top in Fig. 3.4 (A). In the extreme case of $\beta = -\infty$, Berlin achieves a slightly better ranking than Düsseldorf. However, they share the first rank where another kind of aggregation with $\beta = \infty$ is used. Note that not all of the nodes' ranking curves are depicted in the visualization 3.4 (A). But, Figure 3.4 (B) depicts all the nodes with respect to their values of minimal and maximal ranking. The airports EDDT (Berlin) and EDDL (Düsseldorf) stay at the top right in the plot and have *robust* importance. The airport EGCC (Manchester) has stable weak importance as it mostly stays among the bottom 20 rankings. There are many nodes whose rankings are more stable as visualized on the diagonal rather than those staying far from it.

EDDV (Hanover) and LSZH (Zurich) airports are two interesting cases that remain among the top 15 rankings, like Berlin. Looking at the values of their *centrality indices* in Table 3.3, they both have the same number of connections towards other airports, so they have the same importance for a network process. Hanover airport has higher influence than Zurich airport with respect to being in between pairs of other airports and has less influence with respect to being close to other airports. Using our approach, they both stay almost close to each other among the top-ranking nodes. If we ask how fast a process can reach the other airports from these two, and to what extent these airports are important in terms of losing a process if they shut down, they will get a pretty equal *score*. LIRQ (Florence) has a stable ranking in the middle of the plot. Two airports, LICJ (Falcone–Borsellino), LICC (Catania), stay often among the

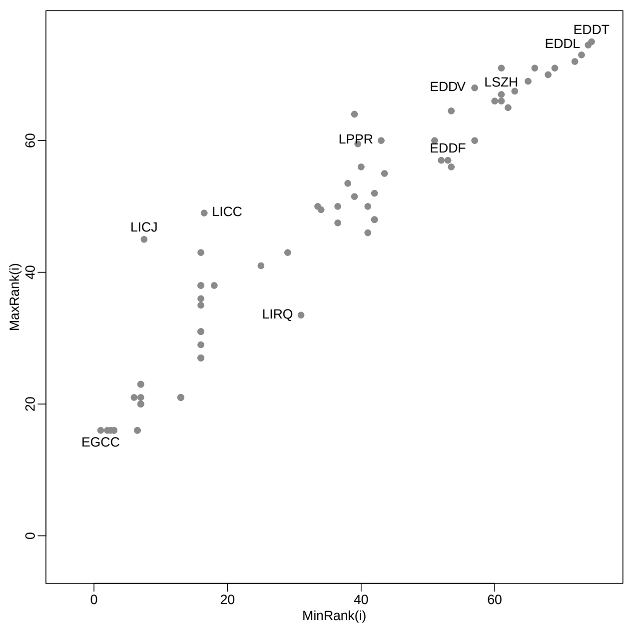
TABLE 3.3: The 75 airports in the air transportation network are ranked with respect to the results of normalized *degree*. The other criteria are normalized *betweenness*, and *closeness centrality*.

Rank	Code	$a'_1[i]$	$a'_2[i]$	$a'_3[i]$
75	EDDT (Berlin)	1	0.988	1
74	EDDL (Düsseldorf)	1	1	0.9838
73	LEPA	0.6944	0.7763	0.8637
72	LOWW	0.5278	0.2043	0.6758
71	EDDM	0.5	0.2339	0.7582
70	EDDS	0.4722	0.1422	0.7337
69	EDDH	0.4444	0.106	0.6429
68	LGTS	0.3889	0.0838	0.5912
67	EDDN	0.3611	0.2292	0.5813
66	LGKO	0.3611	0.094	0.6429
65	EDDK	0.3333	0.2139	0.6647
64	LPFR	0.3333	0.0829	0.6218
63	LEAL	0.3056	0.0763	0.6323
62	LPPR (Porto)	0.3056	0.0634	0.4
61	LOWS	0.2778	0.056	0.5065
60	EDDV (Hanover)	0.2222	0.1475	0.5065
59	LSZH (Zurich)	0.2222	0.1459	0.6115
58	LEMG	0.2222	0.0147	0.6218
57	EDDG	0.1944	0.0226	0.4803
56	LICA	0.1944	0.0113	0.5336
55	EDXW	0.1944	0.0131	0.5065
54	EDDF	0.1667	0.0272	0.4803
53	LIRF	0.1389	0.0027	0.4976
52	LIRN	0.1389	0.0053	0.3925
51	LEBL	0.1111	0.0018	0.5336
50	LGRP	0.1111	0.0147	0.3633
49	EDLP (Paderborn)	0.1111	0.1048	0.3704
48	EDDC	0.1111	0.0233	0.3777
47	EDSB	0.1111	0.0026	0.4552
46	LIPZ	0.0833	0.0018	0.4552
45	LFSB	0.0833	0.0027	0.431
44	EFHK	0.0833	0.0011	0.4389
43	LICC (Catania)	0.0833	0.0081	0.2469
42	LIEO	0.0833	0.0004	0.2469
41	LEBB	0.0833	0.0062	0.4153
40	EDDR	0.0833	0.0034	0.4153
39	LIPX	0.0833	0.0011	0.431
38	LGMK	0.0833	0.0024	0.2958

Rank	Code	$a'_1[i]$	$a'_2[i]$	$a'_3[i]$
37	EGSS	0.0556	0.01	0.3286
36	LIMC	0.0556	0.0009	0.4389
35	LGSA	0.0556	0	0.3562
34	LGSR	0.0556	0	0.3562
33	LGMT	0.0556	0	0.3491
32	LEVC	0.0556	0.0013	0.2587
31	LIRQ (Florence)	0.0556	0.0003	0.3286
30	LICJ (Falcone–B.)	0.0556	0.0032	0.1912
29	EDDP	0.0278	0	0.186
28	EKCH	0.0278	0.0009	0.4
27	LEIB	0.0278	0	0.4
26	LOWG	0.0278	0	0.3219
25	LOWL	0.0278	0	0.3219
24	LFPO	0.0278	0	0.3219
23	ENKR	0.0278	0	0.3022
22	LOWK	0.0278	0	0.3022
21	LEZL	0.0278	0	0.2239
20	ESGG	0.0278	0	0.3087
19	EGKK	0.0278	0	0.1912
18	LEST	0.0278	0	0.2239
17	LEJR	0.0278	0	0.2239
16	LEAS	0.0278	0	0.2239
15	LEAM	0.0278	0	0.2239
14	LPPD	0.0278	0	0.3022
13	LFPG	0	0	0.2648
12	EHAM	0	0	0.2184
11	LTAI	0	0	0.2648
10	EGBB	0	0	0.0652
9	ESSA	0	0	0.2708
8	ENGM	0	0	0.2708
7	LLBG	0	0	0.2708
6	LEMD	0	0	0.2184
5	LGIR	0	0	0.1128
4	EGCC (Manchester)	0	0	0
3	LFMN	0	0	0.2648
2	LDRI	0	0	0.136
1	LYTV	0	0	0.0993



(A)



(B)

FIGURE 3.4: Air transportation network. (A) The detailed ranking of the nodes is depicted over the different aggregations guided by the β parameter in MEOWA operator. (B) Minimal ranking versus Maximal ranking of nodes are obtained over all β values. The values of the three criteria are listed in Table 3.3.

bottom-ranking nodes but with different frequency. Looking at their values in Table 3.3 reveals that all three values of LICC are higher than those of LICJ and thus its line of ranking is always above that of LICJ. However, in comparison with LIRQ (Florence) airport, when the aggregation rewards airports that have at least one best value, they both are at the bottom.

3.4 Discussion and Conclusion

In many real networked systems, analyzing the importance of entities and the activity of actors requires the use of multiple aspects, each of which is able to characterize importance. Network analysis often also has to deal with a multitude of information known about a node: We focused on some situations in two communication networks where a node was assigned both structural properties classically known from network analysis and properties deduced from data that are not directly presented in the network (number of words, total reaction time in a chat-log communication data; number of citation in a collaboration network data). While multiple criteria all focus on one possible operationalization of activity, they create conflicting rankings.

The exploratory analysis using the proposed approach based on the OWA operator allows understanding how *stable* the ranks are, and thus provides more insights than, e.g., pairwise correlation of the values or just a single ranking based on one node centrality. We inspected some cases whose ranking changed extremely by applying multiple criteria to the analysis of their activity *score*. Then we discussed the nodes in both datasets that have *robust* importance and those which stay in the top ranking with respect to all types of aggregations, each of which could be used for the aggregation of multiple criteria associated with the nodes. In the chat-log dataset, we observed fewer fluctuations in the ranking of the nodes since the correlations between the criteria were stronger than the correlation between the criteria in the Freeman's EIES network. But, we still found some interesting cases that could not be identified easily using a single ranking with respect to only one or two measures.

In an airline transportation network data, we focused on situations where multiple *centrality indices* are used together to analyze node centrality. The main concern was how to select top nodes that have conflicting rankings produced because of the trade-offs between multiple measures.

We observed that some nodes have *robust* importance with respect to different aggregations, while the rankings of the rest fluctuate. In this dataset, we observe that the values of a centrality index have a dominant role in the aggregation, and this might not allow other criteria to have a high impact in the results of the aggregation due to the ordering process. We discuss this in Chapter3 in detail. Based on our experience, the nodes that change their ranks the least and the most, are very interesting to inspect. It is concluded that those nodes that have high and weak importance with high stability in their rankings, respectively, are the most central and the least central nodes.

The observations indicated that there are some nodes that, based on a single ranking, might be totally ignored or be identified as the most important node. We show that the different decisions made over the used multiple criteria sometimes change the rankings of nodes strongly, except those have the best values in all of them, or in other words; those whose values give them a comparable ranking.

Comparing the main idea of our approach with the basics of other methods in general, it turns out that the proposed approach provides a means for dealing with such complicated cases that the others would not be able to handle without supplements; e.g., the method proposed by Opsahl, Agneessen, and Skvoretz needs improvement regarding the number of measures that can be contained in the evaluation (s. Chapter 2.1, Equation 2.1), and the method proposed by Du et al. needs improvement with regard to the production of a range of different weight vectors, as explained at the beginning of this chapter.

The proposed approach help to deal with other types of conflicting information about node centrality in networks:

Note 3.1 *Consider multiple centrality indices obtained for a node in a multiplex network. If the node has different positions in different layers, then the approach can show how its rankings change among all nodes with respect to some or all measures over some or all layers.*

In the following chapter, we will elaborate on this note and show what led us to generalize the proposed method for analyzing multiple rankings of nodes in several multiplex networks.

Chapter 4

Analysis of multiple centrality rankings in multiplex networks

Regarding the concept of node centrality in complex networks, the success of classical *centrality indices* proposed by Freeman in 1979 has been documented well in recent decades in a wide range of applications such as the analysis of influence spreading in social networks [45], anomalous centrality in transportation networks [34], and gene importance in cancer networks [99].

In addition, according to Borgatti (2005), each centrality index is tied to a specific network flow [10], and various network processes can take place in networks, such as the spread of a rumor [24] and the transmission of an infection [44]. According to his categorization, if a process transmits through geodesics, e.g., package delivery, classical betweenness and closeness measures can be employed to predict the importance of nodes. However, for a long time there existed no significant general characterization method aggregating all these known, commonly used *centrality indices* and exploring node centrality with respect to multiple aspects of importance.

In the previous chapter, we asked what happens if one wants to explore the importance of nodes regarding multiple *centrality indices*. We include multiple indices into the analysis of node centrality and reveal the ranking behaviors of the nodes over different aggregations. We discussed a vivid example where the activity level of a researcher in a collaboration with others could be measured by quantifying the *strength* of his/her relationships with other researchers and by measuring the number of people with whom he/she had at least one collaboration in a field.

Along the same line, imagine now that the relations among the corresponding researchers with respect to friendship in social networks were also available for this analysis. This could be represented in a new network in which the importance of the researchers is analyzed similarly using two measured properties. If both of these networks were available, they could be considered in a coherent framework comprised of two network layers. Recently, it has been shown that articulating the complicated nature of complex systems found in the world requires a more complicated framework that goes beyond single-layer network representation, such as multi-layer or multiplex networks [47]. In multiplex networks, the same set of nodes is linked using multiple types of relations or interactions [90, 47], as exemplified above. Based on different analyses of such networks, various ideas have been suggested in the literature regarding structural measures [7], centrality rankings [81], and the extraction of information from multiplex networks [41]. Similarly, the fundamental concepts of centrality have been extended to the new framework in several attempts to explore nodes' centrality with respect to their different relations or interactions. In this regard, it is crucial to have an approach that can deal with a node's centrality index in multiple layers or, in the worst case, is even able to deal with multiple *centrality indices* of a node in multiple layers.

A simple approach is to aggregate the results of the *centrality indices* over the layers, e.g., by averaging over all indices in all layers. However, several studies show that the aggregation of the classical centrality values over the layers yields misleading results [81, 23]. For example, De Domenico et al. use a tensorial model to capture multiple rankings of nodes within multiple layers of an interconnected network [23], i.e., they define a multiplex network as an interconnected network in which the interconnections between layers are allowed. However, if we consider a multiplex network with separate layers (non-interconnected), we are able to visualize the ranking positions of nodes in individual layers and compare them. Going back to the example provided above, we can analyze the importance of a researcher with respect to his/her position in two layers, where the extent of the importance itself is quantified in multiple ways, i.e., by looking at the number of communication partners the researcher has and/or by looking at the strength of their relationships in two layers—this results in two rankings in two layers for a researcher. Once we have obtained all the results for all researchers over multiple layers, the questions for this example will be which one is (are) the most central node(s) and which one is (are) the least central node(s). This might seem a simple problem to solve. However, in this chapter we will show that having multiple *centrality indices* (more than two), such as three classical centrality measures, in a multiplex network can turn this into a more complicated problem. We will show that these centrality measures might have totally different views on a node's importance and thus produce more conflicting rankings. It is even worse when such results differ among the layers as well. In such a case, arriving at a final conclusion is more challenging, as it requires an exploratory approach that is able to deal with this situation and to conveniently outline the nodes' ranking behavior.

Questions regarding multiple rankings

Considering multiple *centrality indices* for nodes in a network, it will be very interesting to explore the importance of a node with respect to *at least one* index, *all* of them, or any combination of them within a layer and over multiple layers. Then we will be able to observe whether or not a particular node is especially important for at least one of the network measures, or we can aim at investigating whether or not there is a node that is never the top-ranking one over all layers, but has been identified as a node with moderate influence regarding all classical centrality measures [85]. Analyzing the ranking of nodes with respect to the differences among their rankings within a layer and the differences among all layers will yield insights into the behavioral patterns of ranking and will allow partitioning nodes into groups that each interpret a specific manner. Summarizing our interests in this chapter, the following questions will be addressed:

1. Do rankings based on a set of *centrality indices* rather correlate or conflict?
2. If they conflict, how can multiple measures of centrality be used in the identification of top nodes within a layer?
3. How can multiple aspects of centrality be explored for each node within all layers of interest?
4. Do nodes exhibit similar ranking behavior with respect to multiple aspects of centrality in multiple layers?
5. Do their rankings sensitive to the choice of aggregations within a layer and among all layers?

To address these questions, we use a similar approach using OWA aggregation operator and two measures that allow partitioning nodes with respect to their overall ranking behavior in each layer with respect to multiple indices of centrality. We will apply this approach to several multiplex network datasets with more than two network layers and demonstrate the partitions of nodes based on their behavioral patterns within a layer and their ranking over all layers of interest.

4.1 Analyzing multiple rankings of nodes in multiple layers of a network

So far in this thesis, we have explained some situations regarding node centrality in which a node could be identified as central using multiple centrality measures a single-layer network. Generalizing this analysis into a multiplex framework, it turns out that this problem will be even more complicated because the involvement of nodes in multiple types of interaction most likely will not result in the same quantity if the positions of nodes in the structure of one network layer differ from their positions in the remaining layer(s).

To exemplify this, imagine that person A is a very trustworthy coworker but rather isolated in terms of socializing and establishing friendships. Conversely, imagine that person B is a naturally solitary worker but open to advice by others, with strong friendships ties outside work. This trade-off is more complicated when different aspects of the importance result in different, possibly conflicting, rankings for nodes. Thus, we assume the analysis of the different normalized *centrality indices* (*degree*, *betweenness*, and *closeness*) of a node within one layer as an MCDM problem. The nodes are considered as the alternatives in this decision-making process, where the best solution (the best node) can be selected based on the satisfaction of either *at least one* criterion, or *all* of them, or anything in between.

In a multiplex network with $|L|$ layers, where $|V*|$ nodes are shared, we compute the overall *score* of node v within layer l_i considering multiple indices of centrality using a set of aggregations (denoted by Γ) guided by some β-parameter. Then, as a result, we have $|\Gamma|$ rankings for node v.

We visualize the ranking positions of node v by using a colored curve. Considering the same scenario in multiple layers, we end up with $|L|$ colored curves of rankings for node v. By inspecting all curves of the same color, we can compare the *within-layer importance* for all nodes regarding the chosen measures.

The concise aforementioned steps of the proposed method are detailed in the following.

Step 1: For all layers in a multiplex network with $|L|$ layers:

- (a) Set the criteria by measuring multiple *centrality indices* for all $|V^*|$ shared nodes and normalizing the values of each between $[0, 1]$ by the $\max$ and $\min$-values of all.
- (b) Compute the overall *score*s using OWA operator with Eq. 2.5 for all $|V^*|$ nodes with respect to a set of β-values in Γ.
- (c) Rank all $|V^*|$ nodes with respect to the aggregated *score*s for the corresponding values of β-parameter.

Step 2: Measure Δagg and $\Delta layers$ for all $|V^*|$ nodes using Algorithm 1.

Step 3: Normalize the computed values of Δagg and $\Delta layers$ between $[0, 1]$.

Step 4: Partition all $|V^*|$ nodes with respect to the two normalized values of Δagg and $\Delta layers$.

To proceed with the remaining steps, we need to understand the behavioral patterns of nodes' ranking in a broader view. To do this, we propose two measures for partitioning nodes with respect to their various ranking manners.

Partitioning nodes with respect to the two measures of Δagg and $\Delta layers$

First, we obtain the minimal rank of node v within layer l_i over all β-values; and we denote it by $minRank(v, l_i)$ and obtain $maxRank(v, l_i)$ accordingly. Then we obtain a measure that computes the difference of ranking $\Delta agg(v) := \max\{maxRank(v, l_i) - minRank(v, l_i) | 1 \leq i \leq |L|\}$; a large value of Δagg means the *centrality indices* were more conflicting, i.e., the nodes have a large value in one or two centrality indices but not in all. In general, Δagg indicates the maximum difference in ranking positions fixing a layer[1].

The maximal differences among all layers for node v for any β-value is computed using $maxRank(v, \beta)$, which is the maximal rank of v based on any layer, and $minRank(v, \beta)$ is defined as the minimal rank for any β-value. The overall maximum difference of node v is then defined as $\Delta layers(v) := \max\{maxRank(v, \beta) - minRank(v, \beta) | \beta \in \Gamma\}$, where Γ is a set of β-values. A large value of $\Delta layers$ indicates that the node v is more central in one or two layers and not central in the rest[2]. The algorithm for computing Δagg and $\Delta layers$ taking $|\Gamma|$ rankings of $|V^*|$ nodes in $|L|$ layers as input is described as follows: Having both

Algorithm 1: Measuring sensitivity of ranking within a layer over different aggregations and in all layers.

```
for all nodes in V* do
  for all layers in L do
    Find maxRank among all β-values
    Find minRank among all β-values
    Compute maxRank-minRank
  end for
  return Max over all maxRank-minRank {Δagg}
end for
for all nodes in V* do
  for all β-values in Γ do
    Find maxRank among all layers
    Find minRank among all layers
    Compute maxRank-minRank
  end for
  return Max over all maxRank-minRank {Δlayers}
end for
```

measures of Δagg and $\Delta layers$ standardized between 0 and 1, we are able to partition the nodes based on the behavioral patterns they exhibit in their ranking. One behavioral pattern is conceived in this way: The nodes have very different importance among all types of interactions (all layers L) and show different importance with respect to different aggregation schemes of multiple centrality indices; this can happen if the maximum and minimum values in the corresponding criteria by which the nodes are assessed are too far apart ($\boldsymbol{L+A+}$). A second behavioral pattern is that the nodes have very similar ranking within a layer with respect to multiple criteria but there exists a large difference between multiple layers of interest ($\boldsymbol{L+A0}$). The third behavioral pattern is one in which the nodes show the opposite behavior than in the second pattern ($\boldsymbol{L0A+}$). The fourth pattern contains nodes that have very similar importance (less fluctuation of ranking) within all layers and indicate very similar

[1] This can be extended to the minimum difference as $\Delta agg(v) := \min\{maxRank(v, l_i) - minRank(v, l_i) | 1 \leq i \leq |L|\}$ and the average in a similar way. We will discuss this in future work in Chapter 7

[2] This can be extended to the minimum difference using $\Delta layers(v) := \min\{maxRank(v, \beta) - minRank(v, \beta) | \beta \in \Gamma\}$ and the average accordingly.

stable importance considering the different aggregations with respect to multiple centrality aspects (***L*0*A*0**). Thus, in the experiments, we identify the nodes in the corresponding four partitions as ***L*+*A*+**, ***L*+*A*0**, ***L*0*A*+**, and ***L*0*A*0**, respectively.

4.2 Data sets represented as multiplex networks

1. A large dataset comprising the transportation links between airports based on European airlines was developed in 2013 by Cardillo et al. [17]. It contains the flight records between cities of 37 European airlines. The connections between airports concerning a specific airline compose an undirected and unweighted network layer. Therefore, the data of 37 network layers are available, where the airports represent the nodes and a pair of nodes are connected if there is at least one connecting flight between them operated by the corresponding airline. In this large data, one interesting category was used for inspection, namely the one comprised of the low-cost airlines AirBerlin, Easyjet, and Ryanair. For the experiments in this chapter, we built a multiplex network including three layers, where each layer represents the flights between airports by the aforementioned low-cost airlines. In total, 20 airports are shared between the three airlines. We obtained the network from `http://complex.unizar.es/~atnmultiplex/`. The details of this dataset are listed in Table 4.1 (A). We measured several properties in the three layers; the values are demonstrated in Figure 4.1. All the results obtained regarding the layers of Ryanair and Easyjet are almost comparable in terms of *density*, but the maximum *degree* values vary between the three layers.

2. A large dataset comprising a large number of tweets posted on Twitter over the course of one week was collected by De Domenico et al. as a means of analyzing the dynamics of information spreading (a scientific rumor about the Higgs boson particle) in a social network [24]. All collected posts were tweeted on Twitter between 1 and 7 July, 2012. The authors built a network consisting of $456,631$ nodes and $14,855,875$ directed edges. The nodes constitute the authors of the tweets and there is a directed edge between a pair of nodes if there is follower/followee relation between them. In addition to this network, they deduced three directed, unweighted networks based on three different types of interactions: a user replying to another one; retweeting a post; or a user mentioning other users in his/her tweet concerning the same topic. We obtained the network from `https://snap.stanford.edu/data/higgs-twitter.html`. We built a multiplex network comprised of three layers representing the three aforementioned relationships [85]. Since some users might or might not be engaged in an activity (e.g., user A might retweet a tweet but never reply to others), we restrict our analysis to those users who participated in a minimum number of activities, and were therefore active in a layer, i.e., user A replied at least once to other users. Considering this assumption, 127 same nodes exist in the largest, strongly *connected component* of each of the three networks. Considering this, the details of the dataset are listed in Table 4.1 (C). All the basic properties are measured

separately for the four types of relations and demonstrated in Figure 4.1. Among them, the layer that represents the relation of follower/followee includes some nodes with a maximum *degree* of more than 40,000. Among the four layers, the layer that represents the communications among users who got involved into the action of replying to posts about the Higgs boson is the densest network.

3. A dataset has been provided by Lazega in 2001, containing several types of interactions among a number of attorneys in a law firm [55]. He studied the ways in which 71 attorneys communicate in a law firm on a law case. His investigation is based on several relational aspects such as seeking advice from others, co-working, and having friendships outside the firm. Using these three different relationships, a multiplex network comprised of three layers can be deduced. In this construction, the nodes indicate the attorneys and there exists a directed, unweighted edge between a pair of nodes if there is communication between them with respect to the three mentioned social relations, i.e., in the first layer, a node is connected to someone an attorney might approach for advice on a task. Note that the person who is the advisor is not necessarily a co-worker or vice versa. The data is obtained from `https://www.stats.ox.ac.uk/~snijders/siena/Lazega_lawyers_data.htm`. The details of this dataset are listed in Table 4.1 (B). We depict the differences between the basic properties measured in the three layers in Figure 4.1. The results indicate that the characteristics of those layers that contain the interactions of seeking/getting advice and those of co-working, respectively, do not vary a lot. However, it seems that not many of the attorneys engage in a friendship relation outside the firm, as can be interpreted from the lower values obtained for the layer of friendship.

4.3 Experimental Results

4.3.1 European airlines network

We interpret the classical centrality measures on this data as follows: A direct property that can be captured by *degree* indicates the importance of an airport with respect to the number of connections that the city has to the other cities. We assume that the respective property can be correlated with the number of passengers who tend to go to their final destination via a particular airline. The second centrality measure *betweenness* can be interpreted as a measure of the importance of an airport in terms of losing the process if it shuts down [85]. An indicator of importance based on the average distance to an airport directly corresponds to *closeness*. We think of this property as the ease by which a process reaches the airport operated by a specific airline.

The aforementioned three *centrality indices* were computed in each layer and the steps of the proposed method as listed in the box, were applied to the network layers.

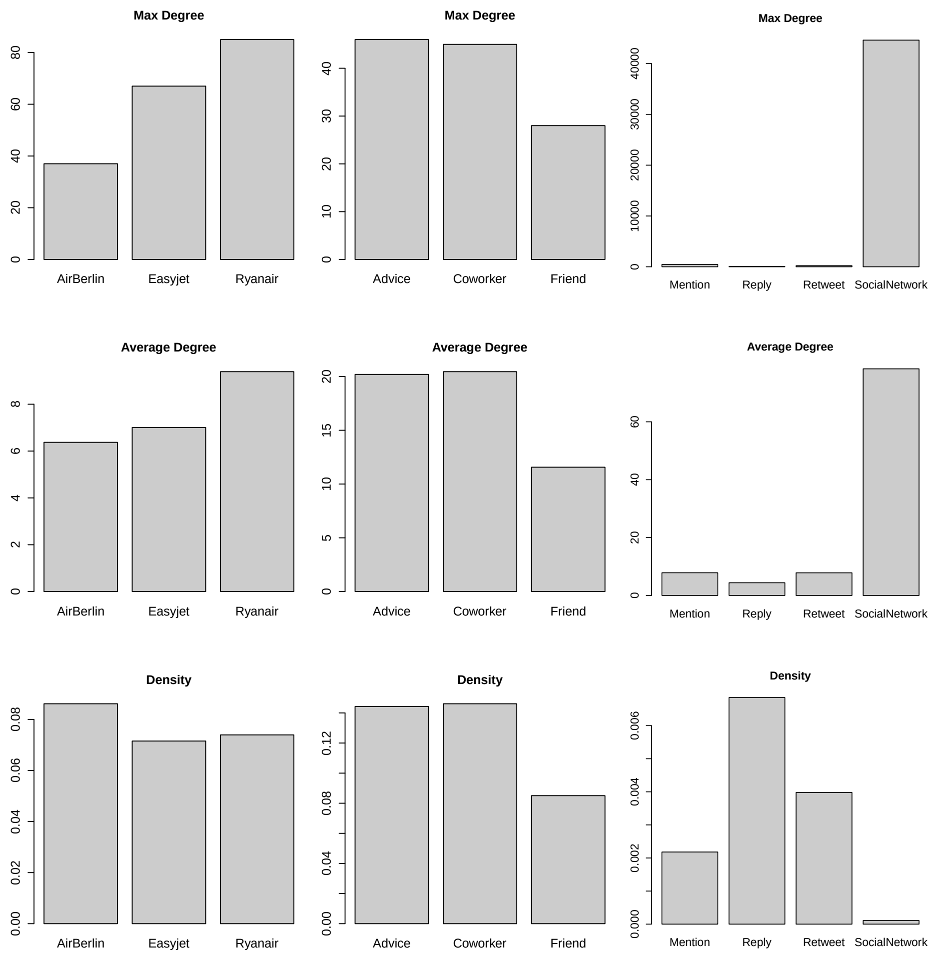


FIGURE 4.1: The basic properties of multiple layers are measured in the three multiplex networks: European airlines network, Twitter network, and a law firm network, respectively. All the definitions for the measured properties are provided in Sections 2.1.1 and 2.1.6. Note that for the max and average *degree* values, and the out-degree plus the in-degree of the nodes are computed in directed networks.

TABLE 4.1: Properties of all the layers of the three multiplex network data; V^* is defined as the set of nodes shared by all layers of the respective dataset.

(A) European airlines network. $|V^*| = 20$.

Properties	Air-Berlin	Easyjet	Ryanair
$\|V_i\|$	75	99	128
$\|E_i\|$	239	347	601

(B) Twitter network. $|V^*| = 127$.

Properties	Mention	Reply	Retweet	Social Network
$\|V_i\|$	1801	322	984	360210
$\|E_i\|$	7069	708	3850	14102605

(C) Law firm network. $|V^*| = |V_i| = 71$

Properties	Advice	Coworker	Friend
$\|V_i\|$	71	71	71
$\|E_i\|$	717	726	399

For ease of reading, we follow the research questions as listed in the introduction. The first question is whether the nodes' rankings regarding the chosen *centrality indices* conflict or correlate. To address this issue, a pairwise scatter plot of *centrality indices* is used as shown in Figure 4.2, which provides a general insight. While there is a generally positive correlation, there always exists conflicting views on the same node. This turns the identification of the top (best) nodes with respect to multiple importance values into an MCDM problem, for which the usage of the proposed approach allows the exploration of conflicting rankings in a convenient manner. After obtaining the *score*s over the different values of β, the ranking for each of the shared airports within each of the three low-cost airlines AirBerlin, Ryanair, and Easyjet is demonstrated in Figure 4.3. Note that in the visualization, only some values of the β-parameter are shown in the x-axis in order to reveal the changes better.

Focusing on each curve separately, it is possible to compare the rankings within a layer. In the layer of AirBerlin, the airports of Palma de Mallorca and Kos Island achieve the highest and the second highest ranking among the airports, independent of β, for all types of aggregation scaling from ($\beta = -20$) to ($\beta = 20$). Considering the airports of Faro and Alicante, it turns out that Faro airport with the centrality values $[0.48, 0.107, \mathbf{0.72}]$ and Alicante with the centrality values $[0.44, 0.098, \mathbf{0.732}]$ have almost stable ranking positions.

Imagine a single ranking of airports with respect to either *betweenness* or *closeness*. The outcome would be definitely different if one were to ask which airport is more important in case it was shut down, or which one is easier to reach for a process that is indivisible. However, if one were to ask which airport is of importance in general considering all cases, the answer should not be too different. We will come back to this point later on. Looking at all three

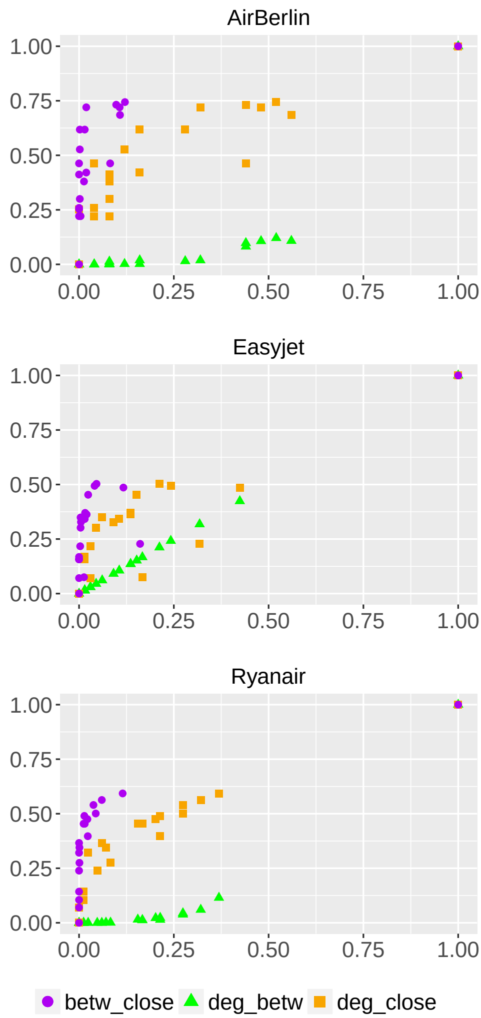


FIGURE 4.2: European airlines network. The correlations between the three normalized *centrality indices* are depicted for each layer of the multiplex network, respectively. The figure is reprinted from [85].

layers of interest (three airlines), both Faro and Alicante show almost similar behavior; their positions in the three layers are similar. In the layer of Easyjet, independent of the β-value, Gatwick airport is the one that always occupies the best rank; similar to London in the layer of Ryanair. As can be seen in Figure 4.2 (B), the pairwise correlation between the normalized *degree* and the *betweenness* centrality is strong; however, with respect to the other two pairs of measures, there are some nodes that produce conflicting rankings, e.g., the Barcelona airport. Consider the layer of Ryanair: London gets the highest importance considering all chosen centrality measures as conjectured, while the airports of Alicante and Madrid are always second and third.

Which nodes show similar ranking behavior? Going back to the point

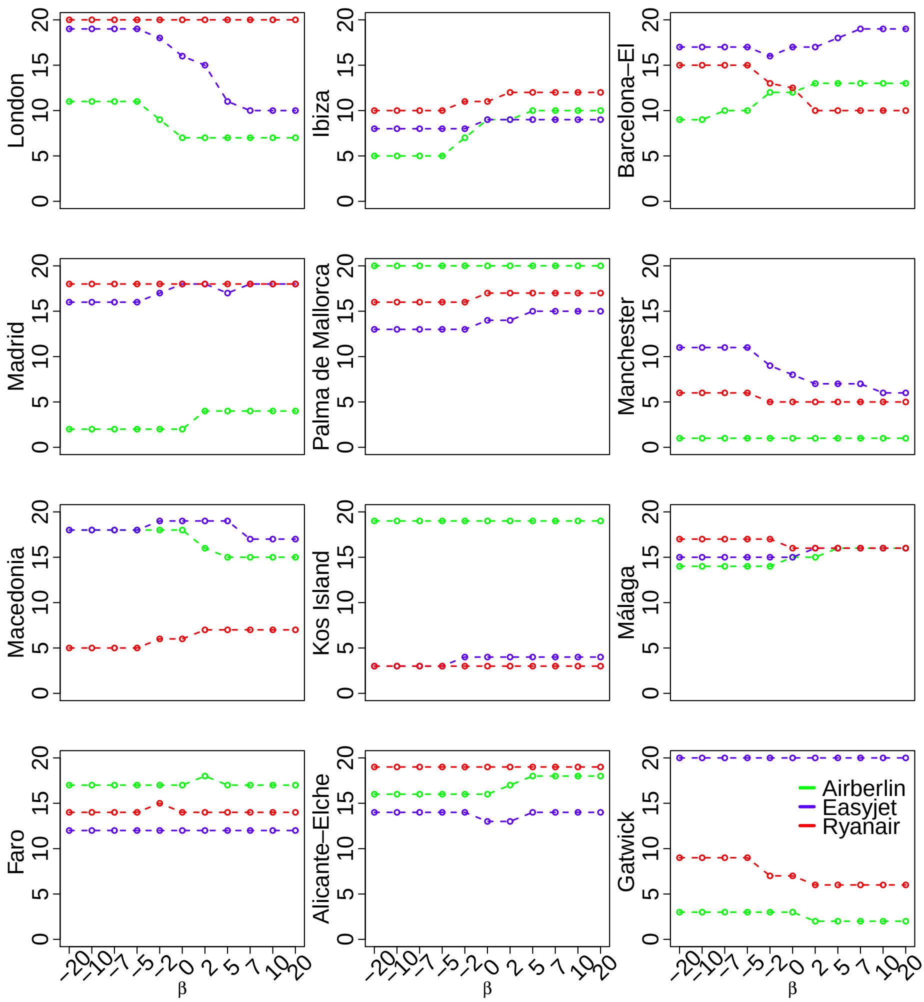


FIGURE 4.3: European airlines network. Rankings of some airports out of 20 shared airports between the three layers of airlines using different values of β.

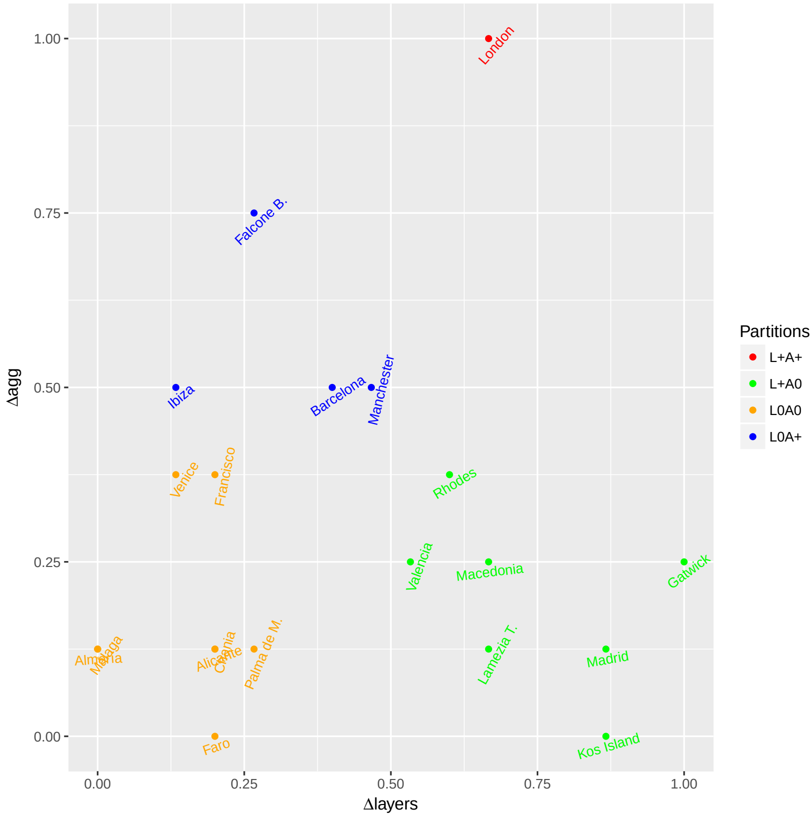


FIGURE 4.4: European airlines network. Partitioning of the 20 shared nodes using the two measures of Δagg and $\Delta layers$.

mentioned above, we use the two measures Δagg and $\Delta layers$ to explore the general behavior of nodes with respect to multiple measures within multiple layers of interest to check which ones show less (or more) sensitivity in their rankings? Following the steps of the proposed approach, the partitioning result is demonstrated for all nodes in Figure 4.4. As can be seen, the three airports of Alicante, Faro, Malága are both in the same partition (***L*0*A*0**) as they have almost similar behavior if one considers multiple indices of centrality. They show similar ranking behavior considering all three layers—they are both among the most important airports almost persistently. This means their ranking experiences less sensitivity within a layer considering multiple measures, and their importance within all layers stays almost within the same range. In contrast, an airport like London shows a pretty sensitive ranking within all layers and within multiple indices of centrality; its high value of Δagg indicates that, in general, this airport is identified as very important with respect to one centrality, but not important for the rest. Also, it shows a medium value of sensitivity within all the layers, $\Delta layers$, which means it is very important in one layer but not very important in the rest. This behavior is not as extreme as that revealed for Gatwick, which has a very high value of $\Delta layers$ meaning that this airport

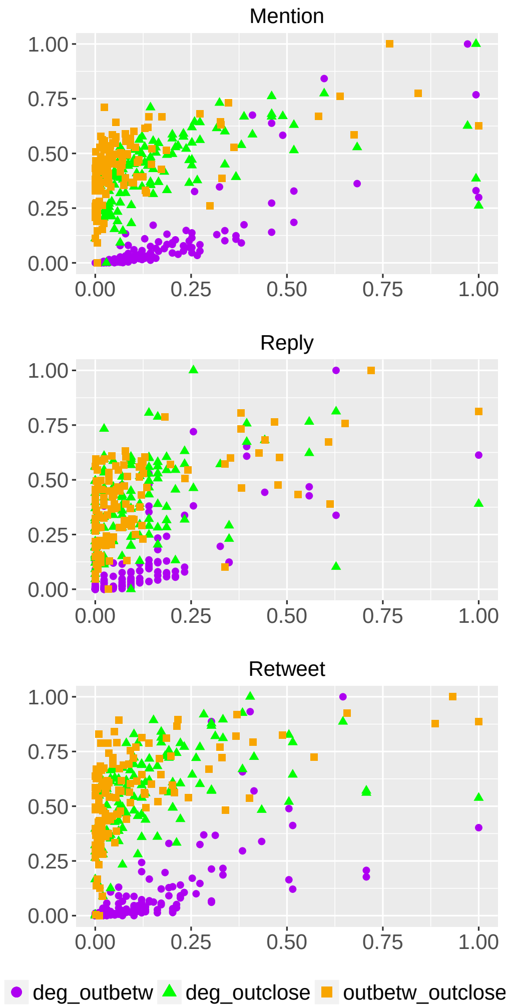


FIGURE 4.5: Twitter network. The correlations between the three normalized *centrality indices* are depicted for each layer of the multiplex network. The figure is reprinted from [85].

is at the center of attention of one or two specific airlines but not for all. However, it almost shows stable ranking within each layer as its Δagg value is small, similar to Madrid and Kos Island airports; thus they are all in the partition of $\boldsymbol{L+A0}$.

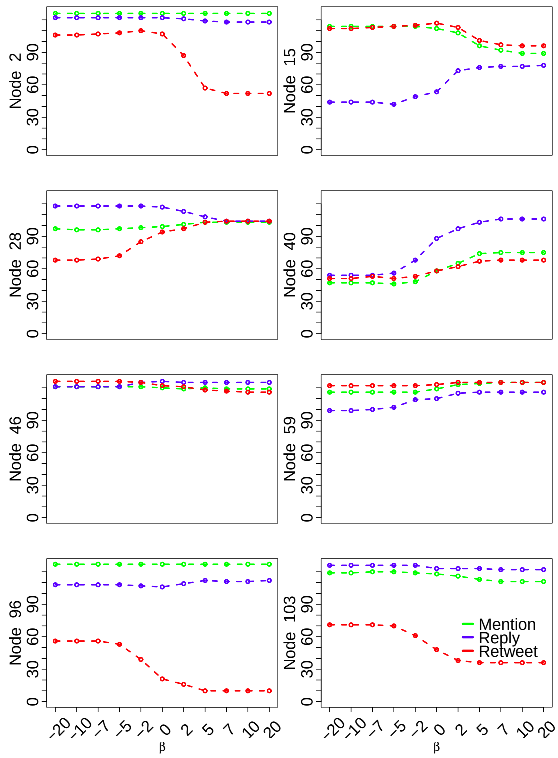
4.3.2 Twitter network

The proposed approach is also applied to a data set containing information about the tweets exchanged between a large number of users in a social network. For the sake of clarity, we interpret the centrality measures for this data as follows: *degree* indicates direct influence of a user in communicating with others. As explained in Section 4.2, this dataset was developed and also analyzed by De Domenico et al. in [24]. They stated that many "information hubs"— which are basically high-degree nodes— had more exchanges of information with low-degree nodes, which they named "information consumers" [24]. Considering the other aspects of centrality, *betweenness* and *closeness* and according to Borgatti (2005), it is pretty clear that this network would most likely not support any process that takes the shortest paths [10].

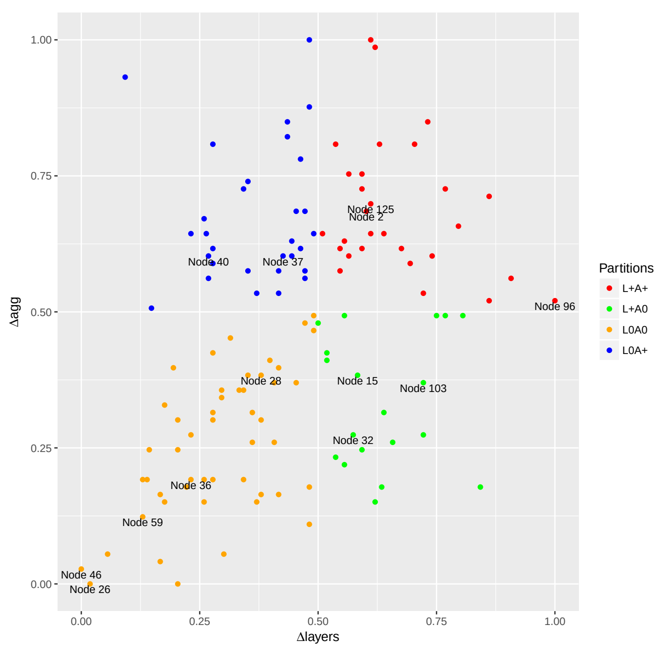
The reason for this is that, as the tweets were about a scientific rumor, it is reasonable to assume that the users wanted to communicate with others with the same frequency. However, we use this large multiplex dataset to show how the proposed approach can deal with multiple conflicting rankings of nodes in multiple layers using a pure demonstration. Therefore, we also measure for the 127 nodes the normalized indices of *out-betweenness* and *out-closeness*, since the directed version of the relations is available for three layers: mentioning users in tweets about the Higgs boson, replying to others, and retweeting the post. It is revealed that these *centrality indices* do not correlate very strongly as depicted in Figure 4.5. And, applying the proposed approach to analyze the multiple conflicting rankings is reasonable as there won't be any centrality index that leads the result of some aggregations, e.g., at least one criterion with the best value. Similar to the last two datasets, focusing on one curve shows information regarding the ranking of a user in one type of interaction in Figure 4.6 (A). A user such as node 2 is almost among the top 10 nodes with respect to the interaction type of Mentioning. Users 59 and 96 are among the top 10 as well with respect to this type of interaction. It would be expected that the number of friends that these users have would be pretty close to each other. To inspect this, we additionally obtain the *degree* of the 127 nodes in the Social network layer, which represents the relation of followee/followers on Twitter. The *degree* here indicates the number of friends a user has in total. What we observed, however, was different than expected. It turns out that there is no correlation between the number of direct friends and their centrality considering various aspects of communication on Twitter [85]. Going back to the aforementioned nodes, the number of followers/followees varies between them. Node 96 had the smallest number of friends (322 users), while node 59 was linked to $33,664$ users on twitter.

Similarly, the person named node 15 was connected to $11,880$ users, which is about 40 times higher than the number of friends of node 28. However, they show almost similar ranking with respect to the Mentioning activity. Conversely, there were some users who were similar in terms of the number of friends/followers, but nonetheless showed distinct ranking behavior; e.g., nodes 40 and 46. Both of these users had about 500 friends/followers on Twitter.

Which nodes show similar ranking behavior? Consider all the curves in the visualizations. It is interesting to do a broad investigation on which nodes show similar ranking behavior with respect to all multiple aspects of centrality over multiple types of interactions. Following along the steps of the proposed method, the partitioning results of the nodes are obtained, which are depicted in Figure 4.6 (B). In general, we observe nodes that have almost the same ranking behavior considering different aspects of importance over all layers in the partition $\boldsymbol{L0A0}$, such as nodes 46, 26, and 59— these show consistent ranking behavior with respect to their activity level. Also, the nodes in the partition $\boldsymbol{L+A0}$ show different behavior within all the layers of activities, such as node



(A)



(B)

FIGURE 4.6: Twitter network. (A) Rankings obtained using different values of the β-parameter for some shared nodes between the three layers of the Higgs Boson dataset. (B) Partitioning of the 127 shared nodes using the two measures ΔAgg and $\Delta Layers$.

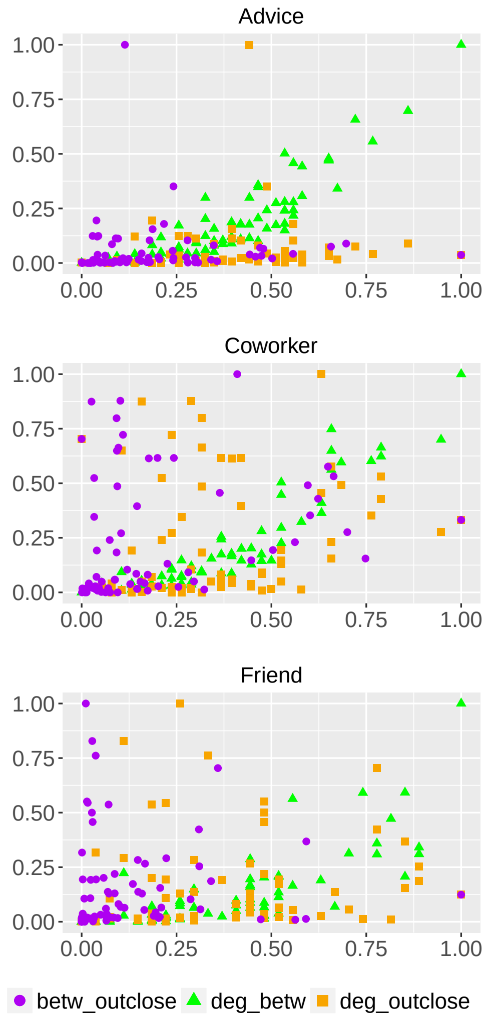


FIGURE 4.7: Law firm network. The correlations between the three normalized *centrality indices* are depicted for each layer of the multiplex network. The figure is reprinted from [85].

15; i.e., the ranking difference Δagg in this partition is less than the ranking difference within all layers $\Delta layers$.

4.3.3 Law firm network

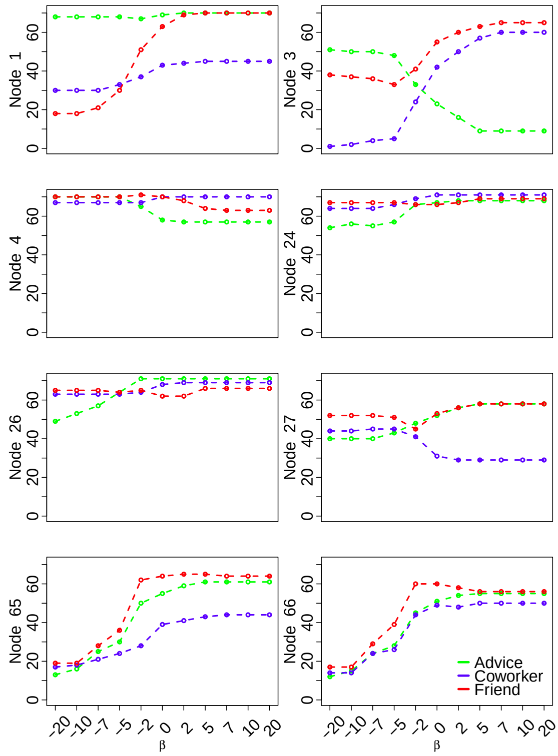
Nodes in the deduced network indicate the attorneys of a law firm and links show their communications with respect to three different relations. One interpretation for the *degree* centrality in this network can be the direct impact that someone has on others in the firm. Analogously, the *in-degree* quantifies the direct influence that node A receives from others and the *out-degree* indicates the extent that node A influences others. We interpret the *betweenness* centrality as quantity reflecting the importance of a person with respect to communication flows in a human group, and the last property of interest is considered to be the average minimal number of steps needed to give a note to another person. Since we have directed relations in this data, this property can be quantified by *out-closeness*. As explained in Section 4.2, these measures are obtained for three different relations: seeking/getting advice, co-working, and having friendships outside the firm. Figure 4.7 demonstrates the different views that pairwise *centrality indices* have on who is most influential with respect to the properties they measure.

As shown in Figure 4.8 (A), by focusing on one curve, the ranking of a node within one layer is revealed; e.g., node 1 is among the top three nodes in the Advice layer. This person gets the three normalized indices values $[0.442, 0.114, \mathbf{1}]$. One surprising point regarding his/her activity is that the number of people seeking advice from this attorney is not maximal; i.e., this property is measured by *degree*. However, since he/she achieves a maximal value in *out-closeness*— when the aggregations requires at least one property with a high value— he/she stays in the top ranking. This means that this is an important attorney in the firm with respect to giving advice to others as the person is close to the other members with a minimal number of intermediaries. Correspondingly, the other top attorneys with respect to seeking/giving advice are: node 26 in the highest place and node 24 among the top 4.Considering the Coworker layer, the two lawyers 24 and 4 are at the top when $\beta = 20$ with respect to their normalized centrality index values of $[\mathbf{1}, \mathbf{1}, 0.332]$ and $[0.632, 0.41, \mathbf{1}]$. These two nodes are also among the top 5 nodes in terms of having friendships outside the firm. **Which nodes show similar ranking behavior?** We compute the two measures Δagg and $\Delta layers$ for all nodes and normalize them as explained in the steps. Then we partition the nodes into four partitions; the results are depicted in Figure 4.8 (B). As can be seen, nodes 24 and 4 are among the nodes in partition $\boldsymbol{L0A0}$, which indicates that they are less sensitive to different types of aggregations and their ranking behavior is similar within all types of relations. Imagine a selection among attorneys in a law firm. The best person is the one who is identified most of the time as top-ranking with respect to all types of relations and all aspects of importance. Node 3, on the other hand, is among the nodes that show extremely varying behavior with in a layer (considering different aggregations over multiple chosen criteria) and within all layers of relations, as both their normalized Δagg and $\Delta layers$ values are high. There is one interesting node in the partition $\boldsymbol{L0A+}$, 66, which shows similar ranking behavior considering all relations but has very different behavior if different types of aggregations are used, meaning he/she has a high important in terms of one or two aspects of centrality but not in all.

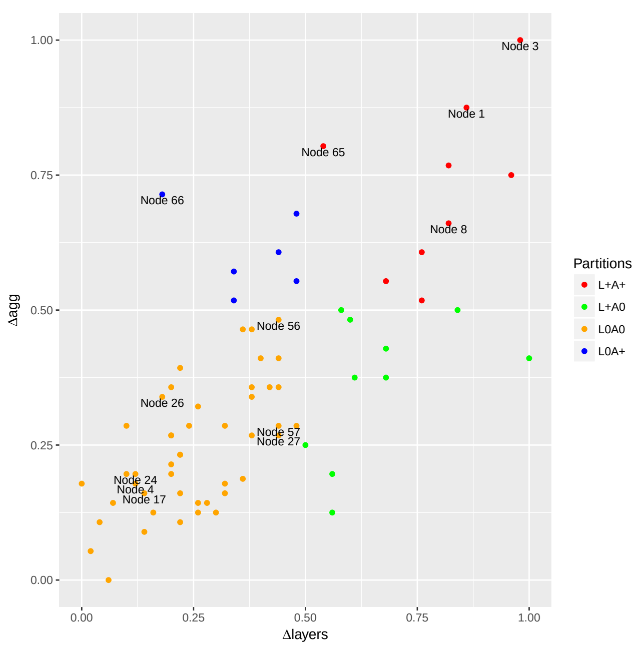
4.4 Summary

Our main goal in this chapter was to provide a means for exploring multiple rankings of nodes in multiple layers of a network.

We use a similar approach to the previous chapter, including the OWA operator and a partitioning for application to three different multiplex network datasets, each of which consists of more than two layers. We explained in the previous chapter that having a set of nodes evaluated by multiple measures



(A)



(B)

FIGURE 4.8: Law firm network. (A) Rankings obtained using different values of the β-parameter for some selected nodes out of 71 nodes in all three layers of relations. (B) Partitioning of the 71 shared nodes using the two measures Δagg and $\Delta layers$.

turns the exploration of node centrality into an MCDM problem. The reason is that the *centrality indices* mostly result in conflicting rankings. This is even worse when multiple types of interactions exist in the network; nodes have different roles in the layers, and in each layer their various aspects of communication can be quantified in multiple ways. We started this type of analysis by addressing a number of research questions in order to explore how the nodes' rankings change over the layer and over the measured multiple *centrality indices*. By means of two new ways of visualization and by comparing the resulting ranking curves of one node, we are able to explore the overall importance of a node. Providing a means for a very detailed exploration of the ranking of nodes is the strength of the proposed approach. In a transportation network, we observed airports that are very important for one or two specific EU airlines (e.g., London via Ryanair) but not important for the rest. We showed that if one takes a single ranking on a specific aspect of importance, the outcome will be different than what we get when considering multiple aspects of importance, as in the cases of Faro and Alicante; these two show more stable rankings over all aspects considering all layers of interest. In a law firm dataset, we identified top attorneys who are well-known for seeking/giving advice on a legal case. The *centrality indices* produced less conflicting rankings in this dataset in comparison to the first one. In a tweet-based network, we observed that the number of direct followers/followees is not necessarily correlated with other aspects of importance, such as mentioning or replying to the others' tweets on Twitter. We focused on some interesting users who, although top-ranked regarding various aspects of centrality, were in contact with only a small number of users.

To get a broad view of the ranking-related behavioral patterns of the nodes, four leading types of ranking behavior were conceived. We partitioned the nodes with respect to the changes they exhibit in their ranking within one layer and with respect to the changes they show within all the layers of interest. In one of the partitions, we observed the group of nodes that have more stable ranking, i.e., the nodes are either always most central in all layers or consistently least centrals in all layers. Some nodes are at the center of attention of one or two specific layers but not for all in one of the partitions. However, these nodes show stable importance within each layer of interest, e.g., Gatwick and Kos Island airports in the European airline network, or Node 103 in Tweets network, which was in the partition $\boldsymbol{L+A0}$. In the remaining partitions, e.g., $\boldsymbol{L+A+}$, the nodes' positions within a layer and/or within all layers give rise to more conflicting centrality rankings, which makes nodes more fragile to the choices.

Using the proposed approach in general, it is very interesting to observe whether or not the importance of a node is stable within a layer or over all layers. The approach can also be applied to any other case in a multiplex network, with multiple ways available to assess the activity of individuals or groups, such as communities. However, the observed changes regarding centrality rankings raise a question of great concern here that needs to be addressed:

Note 4.1 *If the centrality rankings are very sensitive to the choices regarding aggregation, what about other modeling decisions, e.g., normalization methods? Does the choice of normalization methods for the result of a centrality measure before performing any aggregation influence the rankings?*

In Chapter 3, we pointed out that if one of the *centrality indices* has always a higher value than the values of other *centrality indices*, the corresponding index plays a dominant role in the aggregation's result and does not allow other indices to get a comparable chance to contribute to some types of aggregations. This is more important in multiplex networks. Thus, it will be the main topic of the next chapter. We will propose a set of different normalization methods along with the same set of different aggregations used in this chapter for the three multiplex networks in order to elaborate the aforementioned concerns.

Chapter 5

Sensitivity analysis of centrality rankings in multiplex networks

A wide range of studies exist on node centrality in the field of network science, which shows the importance of this topic. However, not many of the works document all their modeling assumptions and discuss whether or not the choice of different decisions influences the results of the produced centrality ranking. Considering only one centrality measure in different networks, Freeman discusses that a comparison of the centrality index values of different nodes requires careful normalization. Assume two networks with the orders (i.e., number of nodes) 150 and 500, and consider two nodes with a *degree* of 100 occurred in the first network and a *degree* of 200 occurred in the second network, respectively [89]. In this scenario, a classic normalization method can be to use the division of the actual *degree* by the corresponding order of the network— which results in higher importance for the first node than for the second node in the example above—or to use the division by the maximal observed value among the *degree* values, or to use the subtraction of the minimal observed *degree* divided by the difference between the maximal and minimal values in the network. In order to perform a meaningful comparison over networks that are most likely to have different sizes and orders, normalization of the measured values is crucial.

Imagine the same scenario in a multiplex network. It has been well documented in the literature that the understanding of complex systems demands more comprehensive models and frameworks, such as multiplex networks [47]. As explained in the previous chapters, some researchers consider the existence of edges between nodes from different layers in interconnected networks [23, 24, 22].

Different strategies have been proposed for the evaluation of node centrality in multiplex networks. For instance, in a study, the authors propose the usage of a vector containing the centrality index values, where each entry belongs to the node's value in one specific layer [7]. The problem here is that it is not easy to perform a single ranking of all nodes based on such a vector. Thus, other researchers have suggested either computing a sum or a regular average of all centrality values [9]. In all strategies comparing or aggregating *centrality indices*, many preprocessing steps need to be described in a reproducible manner. However, as mentioned earlier, no concrete normalization method or aggregations regarding node centrality in multiplex networks have been discussed in the literature so far [1]. As we have shown in the previous chapter, the trade-offs between the different views of multiple centrality indices on a node's importance in multiple layers of a network result in more complicated ranking behavior, as a node's position might vary between the layers. This gives rise to concern regarding the sensitivity of centrality rankings if the layers have very different structures.

Thus in this chapter, we consider different definitions of normalizing centrality index values prior to applying a set of aggregations, and investigate whether or not the choice of different modeling decisions influences a node's centrality ranking. We focus on a very simple measure and show that even sticking to the most frequently used centrality index, the *degree*, but applying various normalization and aggregations reveals the extent of the rankings' sensitivity to the corresponding choices. We conduct this sensitivity analysis on the same multiplex networks we used in the previous chapters: A multiplex network representing a subset of the European air transportation network, a multiplex network of interactions between people who engaged in tweets about the Higgs-Boson particle, and a dataset describing three types of relations between employees of a law firm.

5.1 Comparing node degrees between network layers with different structures

In a multiplex network in which the position of the nodes varies between the layers, we will most likely get different views of importance for a single node using the same measure over the layers (as defined in Section 2.1.3). To discuss this, consider the simplest node centrality measure, *degree*, in the three following datasets, where each is represented as a multiplex network. Note that the datasets are the same datasets used in Chapter 3. However, we provide the details in the current chapter as well in order to make referring easier.

In the first dataset, which is a European airlines network comprised of 37 layers of European airlines (obtained from [17]), the number of airports shared between the 37 airlines is extremely small. Therefore, we use two subsets of airlines to construct two multiplex networks; in these networks, nodes always represent airports and there is an undirected edge between a pair of nodes if there exists a flight operated by the corresponding airlines. First, we consider a multiplex network comprised of AirBerlin, Easyjet, Lufthansa, and Ryanair. This constructed network has nine airports shared by the four aforementioned

[1] The possible reason might be, the choices of such preprocessing steps seem to be inconsequential as we discussed in our paper [89].

airlines. Second, we exclude the Lufthansa layer from the first subset, which results in a subset of three low-cost airlines. This consideration results in twenty airports shared by the three airlines. In Figure 5.1, the red nodes in the three airline layers depict the airports shared between them. It is obvious that the positions of the red nodes and their degree vary between layers, which results in different centrality rankings for these red nodes. Some airports have more connections to other cities, and thus have a high *degree* within, for example, the AirBerlin layer, but are not shared by other airlines in the market, while some high-*degree* nodes in the remaining layers are shared by all three low-cost airlines. For the subset including four layers, the structural properties are listed in Table 5.1a. Taking a closer look at the number of nodes in a layer in this data, it is obvious that the results are not as extreme as those we obtained for the Twitter network data. However, some characteristics still vary between the layers; e.g., while the layer of Air-Berlin has only 75 nodes, the layer of Ryanair contains 128 nodes. If we want to compare the *degree* of the same airport over the airlines, it is reasonable to do so by observing its position among all shared airports or among all airports that are available in all three layers. This can be done by performing normalization prior to any aggregation over the results of the same airport if the aim is to get the overall degree of importance.

The second multiplex network consists of Twitter network data representing the interactions among users who were active in tweets regarding the Higgs-Boson particle [24]. As mentioned, the four layers of this network represent, respectively: *mentioning* the users in tweets, *replying* to the tweets of other users, *re-tweeting* the tweets, and the social network of followers/followees [24]. These four layers share $|V^*| = 127$ nodes. Since the layers contain directed edges between the nodes, we measure the total *degree* (*out-degree* plus *in-degree*) of these nodes in all four layers per se showing the importance or the activity level of the corresponding users. The details of the properties of each layer are shown in Table 5.1(B). In this data, the orders are even more different; one network layer, the Reply layer, contains 322 nodes and the last layer, which is the social network of follower/followee relations, has the order of $360,210$ nodes. Recalling the same example that we used at the beginning of this chapter, consider that user A has replied to 45 users and retweeted 50 other users' posts. His first activity level might be higher than the second one if we compare their normalized values with the maximum activity level that is obtained among all active users (V^*) with respect to the corresponding activities, i.e., 45 and 101, respectively. Thus, before any comparison or aggregation of the result of the same user, meaningful normalization is required.

In the third dataset, the Law firm data, the communications among 71 attorneys in a law firm compiled by Lazega in 2001 [55] are recorded. Similar to the other datasets, the properties are listed in Table 5.1c. Figure 5.2 (A) to (C) demonstrates the *out-degree* and *in-degree* by node color and size, respectively, in the three layers of this dataset. As can be observed, all 71 nodes are available in all three layers, but the size of the layer that represents friendship is different than the values obtained in the remaining layers. It can be conjectured that the rankings in this network will have less fluctuation in comparison to the other two datasets. However, since the total degree (*out-degree* plus *in-degree*) of the nodes in the layers is different, and considering the different orders and sizes of the network layers, at least a few nodes would show some extent of sensitivity

TABLE 5.1: Three multiplex network data sets. In the listed properties, V^* denotes the set of nodes shared between all layers of a multiplex network. The tables are reprinted from [89].

(A) European airlines network. The number of shared nodes in the subset of Lufthansa, AirBerlin, Ryanair and Easyjet is $|V^*| = 9$ and in the subset of AirBerlin, Ryanair, and Easyjet equals 20.

Properties	Air-Berlin	Easyjet	Lufthansa	Ryanair
$\|V_i\|$	75	99	106	128
$\|E_i\|$	239	347	244	601
$\max_{v \in V_i}\{deg(v)\}$	37	67	78	85
$\max_{v \in V^*}\{deg(v)\}$	26	17	5	28
$\min_{v \in V_i}\{deg(v)\}$	1	1	1	1
$\min_{v \in V^*}\{deg(v)\}$	1	2	1	5

(B) Twitter network. The number of shared nodes, based on the corresponding largest strongly*connected component* among the layers, equals $|V^*| = 127$.

Properties	Mention	Reply	Retweet	SocialNetwork
$\|V_i\|$	1801	322	984	360210
$\|E_i\|$	7069	708	3850	14102605
$\max_{v \in V_i}\{deg(v)\}$	466	45	212	44611
$\max_{v \in V^*}\{deg(v)\}$	141	45	101	33664
$\min_{v \in V_i}\{deg(v)\}$	2	2	2	2
$\min_{v \in V^*}\{deg(v)\}$	2	2	2	27

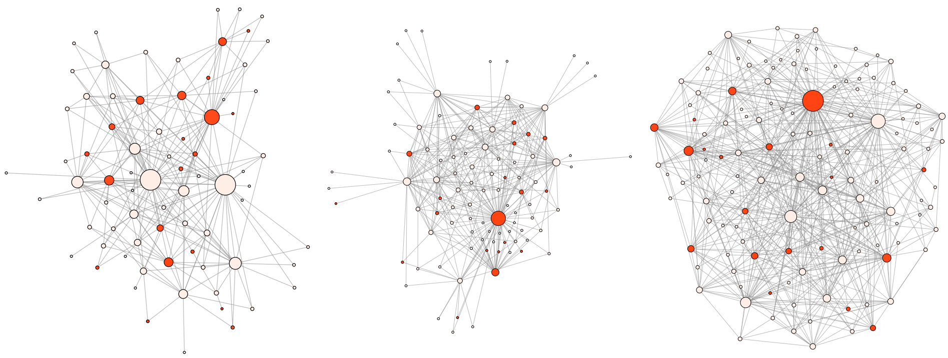
(C) Law firm network. The three layers of this network share the same 71 nodes, i.e., $V_i = V^*$ in all layers.

Properties	Advice	Coworker	Friend
$\|V_i\|$	71	71	71
$\|E_i\|$	717	726	399
$\max_{v \in V_i}\{deg(v)\} = \max_{v \in V^*}\{deg(v)\}$	46	45	28
$\min_{v \in V_i}\{deg(v)\} = \min_{v \in V^*}\{deg(v)\}$	3	7	1

to the choice of different normalization methods and, consequently, different aggregations, e.g., node 15. This means that the corresponding attorney has different numbers of communication partners with respect to seeking/getting advice, co-working with others, and being in a friendship relation outside the firm. Thus it is important to compare his/her degrees with those of the other attorneys.

5.2 Sensitivity analysis on centrality rankings

A set of normalization methods are proposed to be used prior to any aggregation in order to make the results of the nodes over the layers comparable. Afterwards, visualization is used to characterize a node's sensitivity to either the choice of a normalization or aggregation, to both, or to none of them in several multiplex networks. This type of sensitivity analysis allows making a decision on whether or not the choice of a specific normalization method can be defended. Most of the results in this chapter are published in a paper [89].

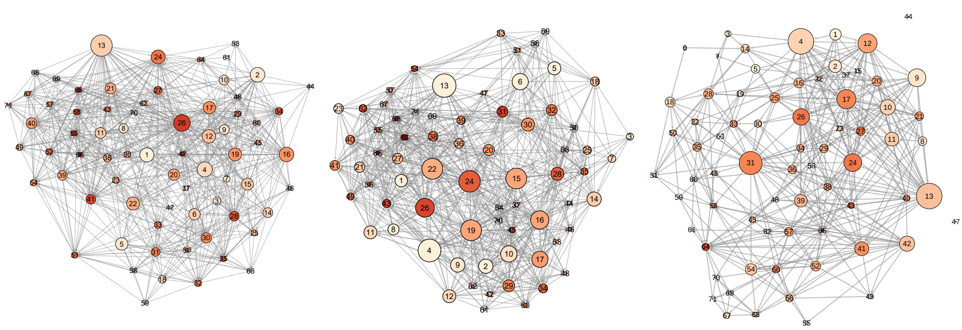


(A) Air-Berlin

(B) Easy-jet

(C) Ryanair

FIGURE 5.1: European airlines network. The nodes' sizes show their actual *degree*; the nodes shared between the three layers of low-cost airlines are colored red. The *degree*s vary between layers for any shared node.



(A) Ad-vice

(B) Co-working

(C) Friend-ship

FIGURE 5.2: Law firm network. The nodes' sizes show the actual *out-degree* and their color indicates the *in-degree*; e.g., in (A), node 13 has a high *out-degree*, but a rather low *in-degree* represented by a pale red.

5.2.1 Different normalization methods for degree centrality

Normalization can be performed differently based on the combination of various assumptions, such as: the minimum *degree* that a node obtains among all nodes within a layer, the minimum *degree* that a node achieves among all nodes within all the layers or, accordingly, among only those nodes that are active with respect to all types of interaction or relations. Other possible definitions include the maximum *degree* that a node gets among all nodes within a layer, or the maximum *degree* that a node has among all nodes over all the layers; accordingly, among only active nodes. The methods we propose are described as follows[2]:

NormMethod 1 considers $deg_i(v)$ for all $v \in V^*$ in layer l_i and normalizes it with the minimum and maximum values observed in the set of common nodes. Using this method, we then achieve a vector of normalized indices of $[0, 1]$ for layer l_i.

$$\mathcal{C}_1(v,i) = \frac{deg_i(v) - min\{deg_i(v)|v \in V^*\}}{max\{deg_i(v)|v \in V^*\} - min\{deg_i(v)|v \in V^*\}}$$

NormMethod 2 is a commonly applied normalization method in a lot of studies. Like the last method, this method uses minimal and maximal values. However, it takes these values from the set of all nodes (V_i) in the layer l_i. Therefore, after ranking the nodes available in V^*, a node with a normalized value of 0 or 1 might or might not be found among them. This means if the nodes having minimal and maximal *degree* values in l_i are also included in V^*, then a normalized value of 0 or 1 can be obtained.

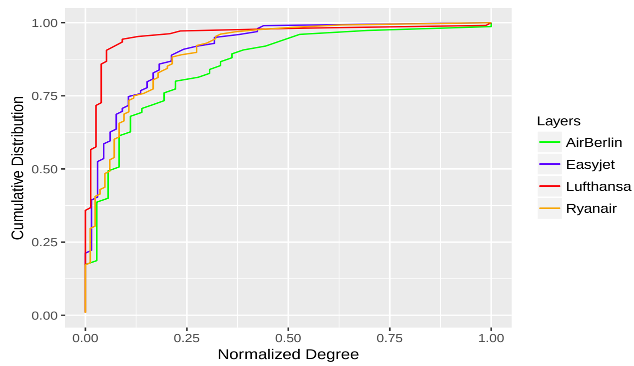
$$\mathcal{C}_2(v,i) = \frac{deg_i(v) - min\{deg_i(v)|v \in V_i\}}{max\{deg_i(v)|v \in V_i\} - min\{deg_i(v)|v \in V_i\}}$$

NormMethod 3 uses the results obtained by *NormMethod 2* and multiplies them with the fraction of the maximum degree in layer l_i and the maximum degree among all nodes in all $|L|$ layers. This results in a vector of indices of nodes ($v \in V_i$) between $[0, \frac{\max\{deg_i(v)|v \in V_i\}}{\max\{deg_i(v)|v \in \bigcup V_j, 1 \leq i \leq |L|\}}]$.

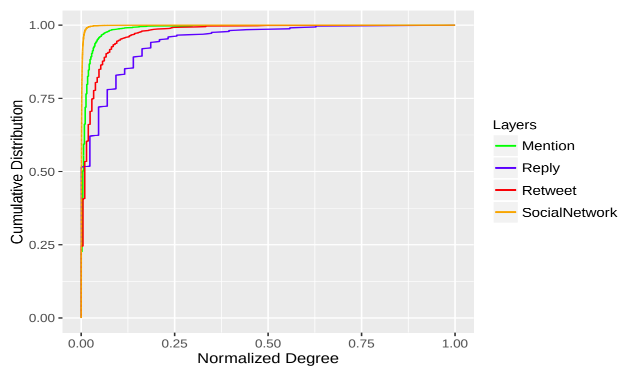
$$\mathcal{C}_3(v,i) = \mathcal{C}_2(v,i) \cdot \left(\frac{max\{deg_i(v)|v \in V_i\}}{max\{deg_i(v)|v \in \bigcup V_j, i \in [1, \ldots, |L|]\}} \right)$$

NormMethod 4 is proposed as a means of considering each node degree's ranking position within each layer. *NormMethod 4* ranks the nodes with respect to their degree ($deg_i(v)$) in each layer non-increasingly. Then it denotes the ranking of node v by $Rank_i(v)$ and normalizes it with the order of the layer (the number of nodes existing in the corresponding layer).

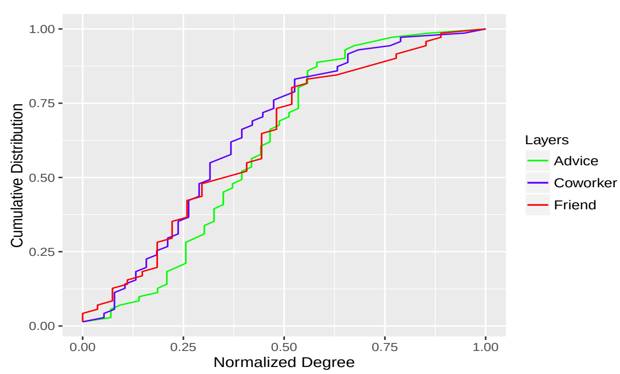
$$\mathcal{C}_4(v,i) = \frac{Rank_i(v)}{|V_i|}$$



(A) European airlines network.



(B) Twitter network.



(C) Law firm network.

FIGURE 5.3: The normalization of *NormMethod 2*—described in definition 5.2.1— is applied to the three networks and the cumulative distributions of the normalized degree are depicted. The figures are reprinted from [89].

The cumulative distribution of *degree* normalized using NormMethod 2

The cumulative distributions of normalized *degree* centrality for *NormMethod 2* are shown for all three multiplex network datasets in Figure 5.3 (A) to (C). Freeman discusses in his seminal work that some networks are more *centralized* than other networks. In such networks, a node that has a large *degree* would dominate the other nodes' *degree*. Freeman states that in a *centralized* network, the majority of nodes show a very small normalized *degree*, while some nodes have a large normalized *degree*. In Figure 5.3 (A) to (C), in the cumulative distribution represented for the normalized degree values, we visualize the percentage of nodes with at least normalized degree x against x. As can be seen, an increase followed by a long tail until reaching 1 is observed in a strongly *centralized* network layer. In contrast, for a less *centralized* network layer, the cumulative distribution of the values stays closer to the diagonal. Looking at the Figure 5.3 (A) reveals that the cumulative distributions of the normalized degree values are quite similar for the Easyjet and Ryanair layers. Conversely, the other two layers, AirBerlin and Lufthansa, show different behaviors. In the Lufthansa layer, it is obvious that more than 90% of the normalized degrees of nodes are smaller than about 70% of the normalized degrees in the Air Berlin layer.

If one wants to find the node that is the most important node in at least one layer of interest, this aggregation would not be able to identify most central nodes in the Lufthansa layer as even a node with a medium normalized *degree* in the AirBerlin layer would dominate a larger normalized *degree* centrality in the other layers. Similarly, the social network layer in the Twitter network data is a strongly centralized network layer, which gives it less chance to contribute in the result of the aggregation. Thus, *NormMethod 4* is proposed to deal with such cases. In contrast to the other two datasets, the distribution of the values for the law firm network for all three layer is close to the diagonal, which indicates these layers are less centralized. Assume a total ordering in the results of three layers (e.g., $l_1 < l_2 < l_3$) on the set of all values. In this dataset, less than 9% of nodes have the exemplified order on their values.

5.2.2 Different aggregations over normalized degrees

After performing all the normalization methods for the measure of *degree* centrality in multiplex networks, we can find out which nodes are top-ranking with respect to multiple layers of relations using a set of aggregations.

Assume $C_x(v, l_\gamma)$ as the normalized degree value of node v in layer l_γ that is computed using *NormMethod* x. Then, having all four normalized values for a set of shared nodes initiates the elements of an MCDM problem in the following way:

[2] These methods and their results have been published in a paper [89].

`Alternatives`: a set of $|V^*|$ shared nodes.

`Multiple criteria`: the normalized *degree* values obtained using a specific normalization method x in $|L| = m$ layers.

`Decision matrix`:

v_1	$C_x(v_1, l_1)$	$C_x(v_1, l_2)$	$\cdots$	$C_x(v_1, l_m)$
v_2	$C_x(v_2, l_1)$	$C_x(v_2, l_2)$	$\cdots$	$C_x(v_2, l_m)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v_{\lvert V^*\rvert}$	$C_x(v_{\lvert V^*\rvert}, l_1)$	$C_x(v_{\lvert V^*\rvert}, l_2)$	$\cdots$	$C_x(v_{\lvert V^*\rvert}, l_m)$

`Problem`: which node(s) is (are) the *most central* one(s) with respect to different aggregations used over multiple criteria?

Note that in the previous chapter, we evaluated node centrality by considering three classical centrality indices as multiple criteria, where each had a value in $[0, 1]$, for example, using *NormMethod 1*. In contrast, in the current chapter, we have one aspect of importance that is captured by *degree* centrality in multiple layers, but is normalized with respect to different assumptions using four normalization methods. Therefore, a similar decision matrix is considered for these four normalization methods. In such an evaluation, different aggregations can be created using a parameter in the MEOWA operator, as described in Chapter 2.2.1. In this setting, we consider the same range of the β-parameter (a set of values in $[-20, 20]$) as used in the previous chapter, where we aimed at understanding the ranking behaviors of nodes in multiple layers and partitioned them based on their changes. However, here we rather focus on the details regarding the sensitivity of the rankings.

For each node, we visualize its ranking by considering different normalization methods over a set of β-values using colored curves.

5.2.3 Partitioning nodes with respect to their sensitivity

Similar to the previous chapter, we propose computing two measures as a means of analyzing the sensitivity of the nodes' ranking to the choices of different normalization and aggregation using Algorithm 2.

For all nodes in the results of all normalization methods, we find the max and min values over all β-values and compute the maximum difference between the respective max and min. To describe this, let $minRank(v, C_i)$ denote the minimal rank of node v based in a normalization method C_i over all β-values and define $maxRank(v, C_i)$ accordingly. Then, $\Delta agg(v) := \max\{maxRank(v, C_i) - minRank(v, C_i) | 1 \leq i \leq 4\}$ shows the overall sensitivity of a node on the chosen aggregation.

Likewise, for all nodes over all β-values, find the max and min values among all normalization results and compute the maximum difference between the obtained max and min. For the sake of formality, assume $maxRank(v, \beta)$ denotes the maximal rank of v based on any normalization method and let

Algorithm 2: Measuring sensitivity of ranking to the choice of different normalizations and aggregations.

```
for all nodes in V* do
  for all normalizations C_i do
    Find maxRank among all β-values
    Find minRank among all β-values
    Compute maxRank-minRank
  end for
  return Max over all max-min {Δagg}
end for
for all nodes in V* do
  for all β-values in Γ do
    Find maxRank among all normalizations
    Find minRank among all normalizations
    Compute maxRank-minRank
  end for
  return Max over all maxRank-minRank {Δnorms}
end for
```

$minRank(v, \beta)$ be defined accordingly. The overall sensitivity of a node in the chosen normalization method is then defined as $\Delta norm(v) := \max\{maxRank(v, \beta) - minRank(v, \beta) | \beta \in \Gamma\}$, where Γ is a set of different β-values. If we plot nodes with respect to their Δagg and $\Delta norm$ values and take half of the maximum observed value for each measure to draw a vertical and a horizontal line, a node will be placed in one of the four categories $\boldsymbol{A0N+}$, $\boldsymbol{A+N0}$, $\boldsymbol{A+N+}$, $\boldsymbol{A0N0}$. In the first category, we observe nodes that are sensitive only to the choice of the normalization method; in the second category, nodes that are sensitive only to the choice of the aggregation will be observed; the third category includes nodes that are sensitive to both models; and the last one contains nodes that are not sensitive to either one. In the second visualization, we use the actual values of Δagg and $\Delta norm$ and inspect those nodes that are more sensitive in their rankings. All the contributions in this chapter have been published in a paper [89].

5.3 Results in three multiplex network datasets

For all the experimental results obtained for the three different multiplex network datasets, we visualize the aggregation of the normalized degree values for a set of common nodes using a set of different values of β. We demonstrate the scatter of nodes into four groups with respect to their sensitivity to the choices of normalization and aggregation.

European airlines network (excluding Lufthansa layer)

Assume that the Lufthansa layer is removed from the constructed multiplex network[3]. Then the outcome is a multiplex network comprised of three layers —those of AirBerlin, Easyjet, and Ryanair— in which twenty airports are shared. The ranking positions of the nodes are depicted in Figure. 5.4.

If we look at the rankings of airports as shown in Figure 5.4, an airport like Chania obtains very similar ranking positions using different aggregation and normalization methods. The two airports Gatwick and Kos Island show strongly more conflicting rankings if we use the four different normalization methods over a set of different β values. Consider the rankings in a simple, average aggregation, which can be obtained when $\beta = 0$. Even in this case, the normalization methods do not agree on the positions of these two airports. Fig. 5.5 demonstrates a scatter plot of the aforementioned two sensitivity values for each node in the European airlines data set.

We have four different groups of nodes. At the bottom left, we observe nodes that have the least sensitivity to both choices of normalization and aggregation. At the top right, on the other hand, nodes that are very sensitive to both can be identified. In the figure, a correlation between the obtained two sensitivity measures can be observed for the majority of the airports. However, some of them, like Barcelona and Venice, are rather more sensitive to the aggregation than to the normalization method. In contrast, there is no airport whose sensitivity to the normalization method is rather considerable; no airports are found in the $\boldsymbol{A0N+}$ group.

As can be observed in Fig. 5.5, Chania airport is located in the $\boldsymbol{A0N0}$ group, which indicates its robust ranking; it does not have any sensitivity to the choice of normalization method since all four curves are on top of each other. Nodes like Gatwick and Kos Island have the highest sensitivity to both modeling decisions and are therefore located in the top most right part of the group $\boldsymbol{A+N+}$. They both have sensitivity values of $\Delta agg = 10$ and $\Delta norm = 9$. Venice airport is much more sensitive to the choice of different types of aggregations than to the normalization method and is thus located in the $\boldsymbol{A+N0}$ group as it gets the values $\Delta agg(Venice) = 10$ and $\Delta norm(Venice) = 3$, respectively.

The sensitivity analysis of this small dataset provides few but interesting cases to inspect. Therefore, we aim at conducting the same analysis for larger datasets in order to explore the influence of the choices regarding the normalization and aggregation used.

Twitter network

In this data, we observe 127 Twitter users who were active with respect to four different types of interactions, which are represented by four layers of the deduced multiplex network. Similar to the previous dataset, we obtain the ranking position of the nodes using the four normalization methods. Looking at the cumulative distribution of the degree values normalized by *NormMethod 2* in Figure 5.3 (B), it turns out that the portion of nodes in the *social network* layer that have a small index value is much higher than the corresponding portion of nodes in the *retweet* and *reply* layers.

[3] An example including four layers in this dataset is elaborated in Appendix A.2.

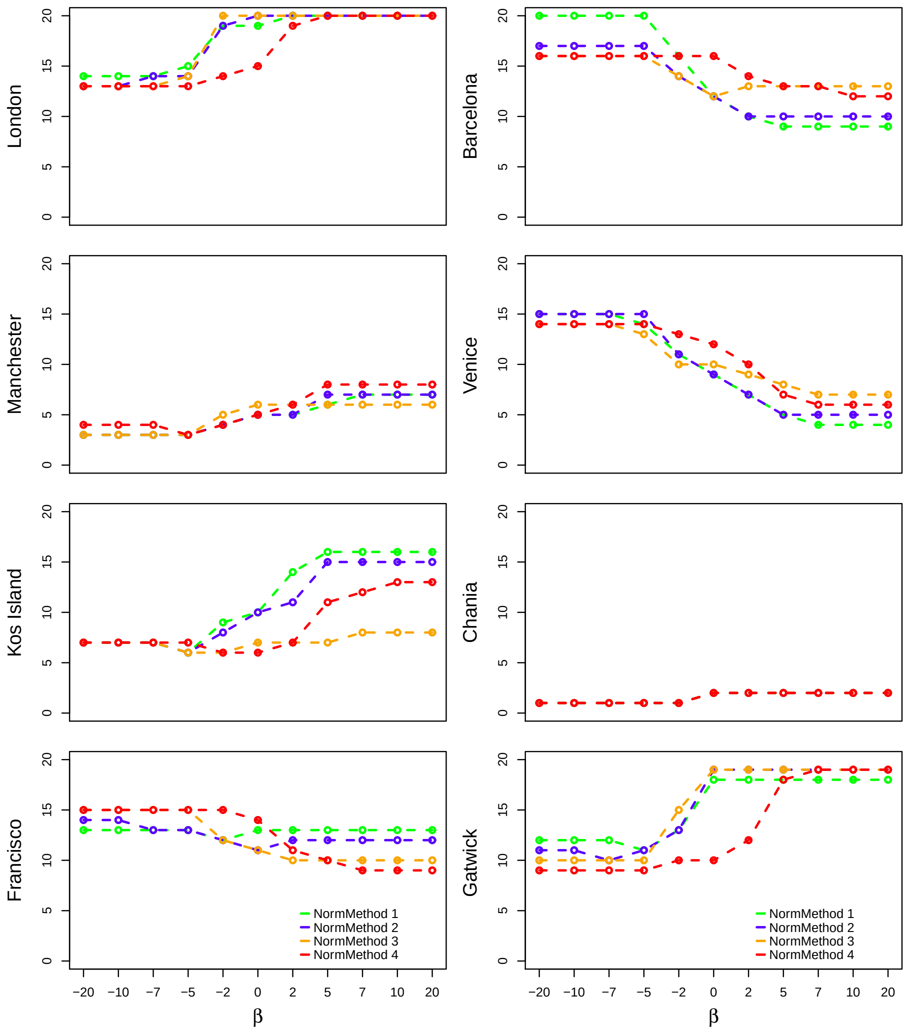


FIGURE 5.4: European airlines network. The normalized degree of the nodes in the three layers representing AirBerlin, Easyjet, and Ryanair are aggregated using different aggregations obtained using the MEOWA operator scaled by the β parameter. The four colored curves show the ranking positions of the indicated node based on its normalized degrees, depending on β. The figures are reprinted from [89].

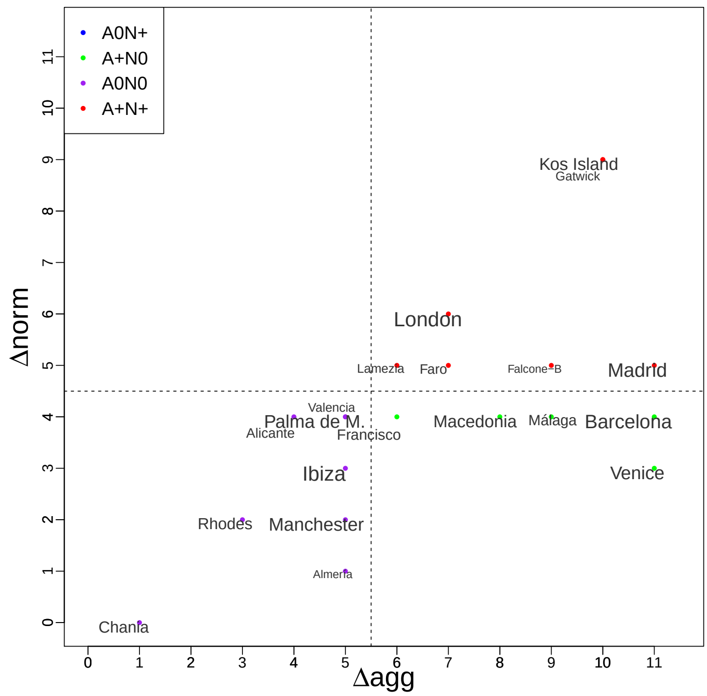


FIGURE 5.5: European airlines data set. Among the three layers of AirBerlin, Easyjet, and Ryanair, twenty airports are shared. The sensitivity of this set with respect to the choice of aggregation and normalization method is quantified by Δagg and $\Delta norm$, respectively. The four sections of the plot contain the group of nodes sensitive to only one choice ($\boldsymbol{A0N+}$ or $\boldsymbol{A+N0}$), those nodes that are sensitive to none ($\boldsymbol{A0N0}$), and those that are sensitive to both models ($\boldsymbol{A+N+}$), respectively. The figure is reprinted from [89].

This indicates that it might happen that the *social network* layer will have less of a chance to participate in an aggregation if nodes that have at least one high *degree* value among the four normalized *degree* values are selected. Figure 5.6 depicts the scatter plot of the obtained values of Δagg and $\Delta norm$ for all 127 shared nodes. It can be observed that in contrast to the previous dataset, although a correlation between two measures exists, a few number of nodes appear in the $\boldsymbol{A0N+}$ and $\boldsymbol{A+N0}$ groups. These groups contain those nodes that are rather sensitive to one model. Consider four nodes out of 127 shared nodes, selected to illuminate the characteristic of the four categories of sensitivity in Figure 5.6. Their rankings, based on the different aggregations computed over the results of the four layers, are shown by the four colored curves in Figure 5.7. Looking at the rankings of Node 59 in the bottom most left part of Figure 5.6 in the group of nodes with almost stable ranking ($\boldsymbol{A0N0}$), we see that it has actual degrees of $[141, 29, 101, 33664]$ in the four layers *mentioning*,

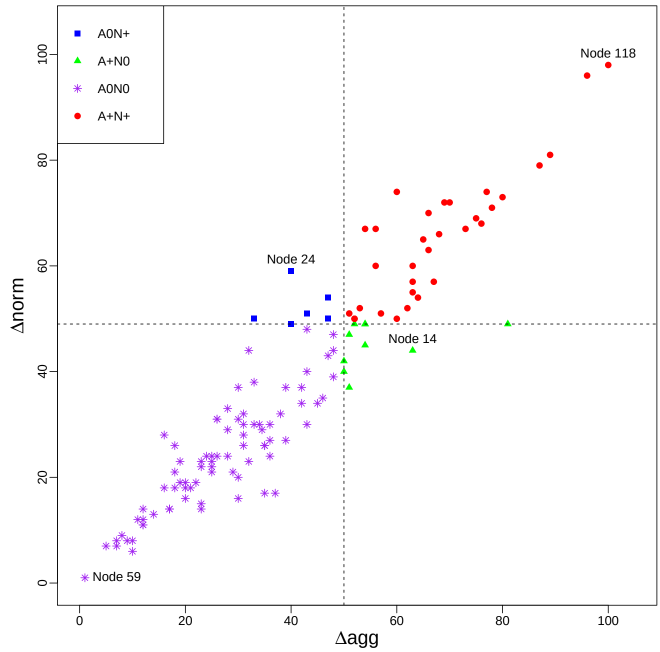


FIGURE 5.6: Twitter network. Sensitivity of nodes to different aggregations (Δagg) and different normalization methods ($\Delta norm$). The four sections of the plot encompass the group of nodes sensitive to only one choice ($\boldsymbol{A0N+}$ or $\boldsymbol{A+N0}$), those nodes that are sensitive to none ($\boldsymbol{A0N0}$), and those that are sensitive to both models ($\boldsymbol{A+N+}$), respectively. The figures are reprinted from [89].

replying, *retweeting*, and *social network*, respectively. Figure 5.7 depicts its ranking curves on top of each other as it has an almost stable position, using all four normalization methods and different aggregations. Recalling the maximum values listed in Table 5.1, it can be recognized that node 59 has maximum total *degree* between $|V^*| = 127$ shared nodes in the three layers *mentioning*, *replying* and the *social network*. These high importance values are sufficient to ensure that the corresponding user gets stable high ranking among the top five users over any aggregation that is desired in the evaluation. This means his/her ranking would not be sensitive to the choices. Instead, in the opposite side in $\boldsymbol{A+N+}$, a very conspicuous case is observed, which is Node 118. Its sensitivity is extremely high to both normalization and aggregation to turn it from being among the *least central* nodes to being among *most central* nodes. The corresponding user has very different numbers of communication partners in terms of *mentioning*, *replying*, *retweeting*, and having *followers/followees* relation on Twitter; the actual degree values are $[6, 2, 2, 1396]$. Obviously, he mentioned (and/or was mentioned by) six users in the tweets related to the Higgs-Boson particle. His *degree* related to the replying activity is as low as his *degree* with respect to the retweeting activity, even though he has a large number of friends on Twitter. The ranking curves of this user for the four normalization methods are depicted in Figure 5.7. As a first impression, its ranking based on *NormMethod 3* varies from 3 to 103, which results in a range of different rankings between staying in the bottom 3 and staying in the top 24 nodes— depending on which aggregation is used. *NormMethod 4* results in similar behavior for this user, but the differences in ranking between these *NormMethod 3* and the remaining methods are the highest for any aggregation, particularly when $\beta = 0$, i.e., when a regular average is computed.

Among the group of nodes that are rather more sensitive to one model, an interesting case is observed in $\boldsymbol{A0N+}$ group. Node 24 is rather more sensitive to the choice of normalization method than to the choice of aggregation. The degrees of its sensitivity are $\Delta norm = 60$ and $\Delta norm = 40$. The values of its actual *degree* vary between four communication partners with respect to the replying activity and 188 communication partners with respect to the relation of followers/followees. This user contacted 35, resp. 22 other users by mentioning, resp. retweeting them. The ranking curves in Figure 5.7 reveal that *NormMethod 3* produces rankings with a downward trend as this node's actual degrees are normalized by the maximum degree found in the four layers (e.g., 44611 in the social network as shown Table 5.1 (B)). In contrast, *NormMethod 1* delivers a ranking curve with an upward trend, i.e., its normalized degree values for mentioning and retweeting activities become higher if this normalization used. *NormMethod 2* and *NormMethod 4* both produce quite similar ranking curves.

The $\boldsymbol{A+N0}$ group is not empty, either. One of the interesting cases is Node 14, which has the sensitivity degrees $\Delta agg = 65$ and $\Delta norm = 46$. This reveals that this node is rather more sensitive to different aggregations than to different normalization methods. Its ranking curves in the bottom right sub-plot in Figure 5.7 detail the discussed sensitivity.

Law firm network

In this medium-sized network comprised of three layers, we observe less sensitivity in the rankings of the nodes. This is an interesting example that explains in which cases the rankings would be rather more stable if any choice of aggregation is defended by a decision maker. Looking at the cumulative distributions of the normalized degree values as demonstrated in Figure 5.3(C), it is clear that the three network layers of seeking *advice*, *co-working*, and having *friendships* are less centralized than what we observed in the layers of the other multiplex networks as mentioned earlier. This means that there are not only a few nodes that have almost high *degree* values in the network layers and that the percentage of nodes that have a specific normalized degree value is almost comparable in all layers.

As shown in Figure 5.8, a large portion of the nodes shows less sensitivity in their ranking with respect to the choice of normalization methods and aggregations. For example, the number of nodes that are separated by the vertical line and are sensitive to the aggregation used is pretty small; 6 out of 71 attorneys. However, some cases are still worth inspecting more closely: $25, 64, 24$, and 19 from the four groups, respectively. Their ranking curves are depicted for all

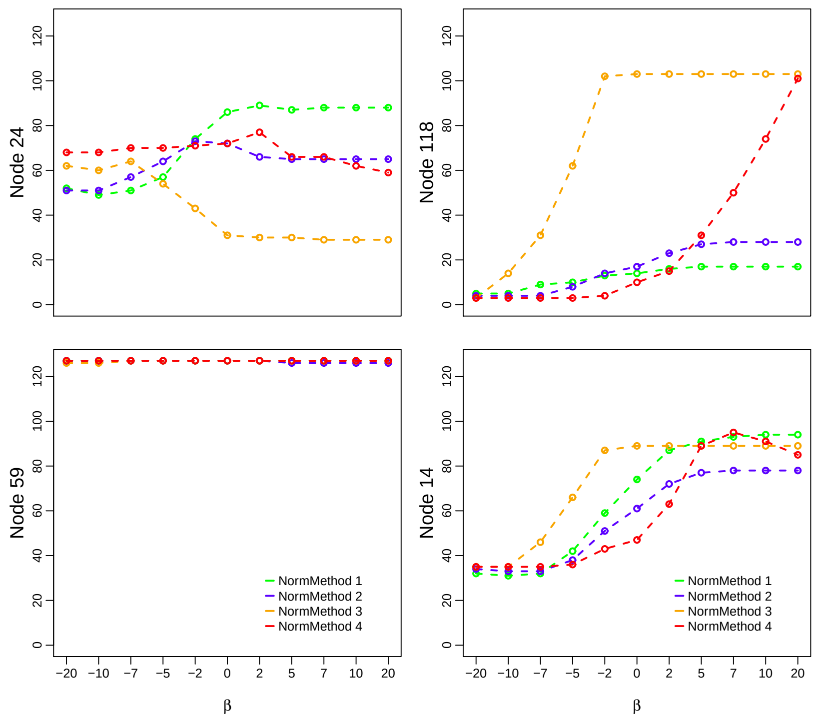


FIGURE 5.7: Twitter network. The rankings of the four nodes that are positioned in the four sections of the Δagg and $\Delta norm$ scatter plot are depicted here. Their ranking positions are obtained using different aggregations (guided by β) for the four layers *mentioning*, *replying*, *retweeting*, and *social network*). The curves show the results obtained using the four normalization methods. The figures are reprinted from [89].

three normalization methods in Figure 5.9. Note that we applied three normalization methods to the *degree* values because all 71 nodes are shared between all layers; i.e., $|V_i| = |V^*| = 71$, and the results of *NormMethod 1* and *NormMethod 2* are the same.

Node 24 in the $\boldsymbol{A0N0}$ group has a very robust ranking for all normalization methods; as depicted, it gets almost the same ranking position. In the $\boldsymbol{A+N+}$ group, the most sensitive node has Δagg of 56 and $\Delta norm = 13$, as illustrated in Figure 5.8; i.e., this is Node 15, as conjectured at the beginning of this section. The second-highest value of sensitivity to different aggregations belongs to Node 19—it has $\Delta agg = 47$. The main observation is that if various aggregations, such as maximum, minimum, or average, are used over the result of the normalized degree values, Node 19 obtains more conflicting rankings, but shows less sensitivity to the choices for normalization methods. Therefore, it is located in $\boldsymbol{A+N0}$. The second-highest value of sensitivity to different normalization methods is obtained for Node 25, which has $\Delta norm = 21$; this node is hence placed in the $\boldsymbol{A0N+}$ group. Following its ranking curves in Figure 5.9, it turns out that *NormMethod 3* results in a different ranking compared to the

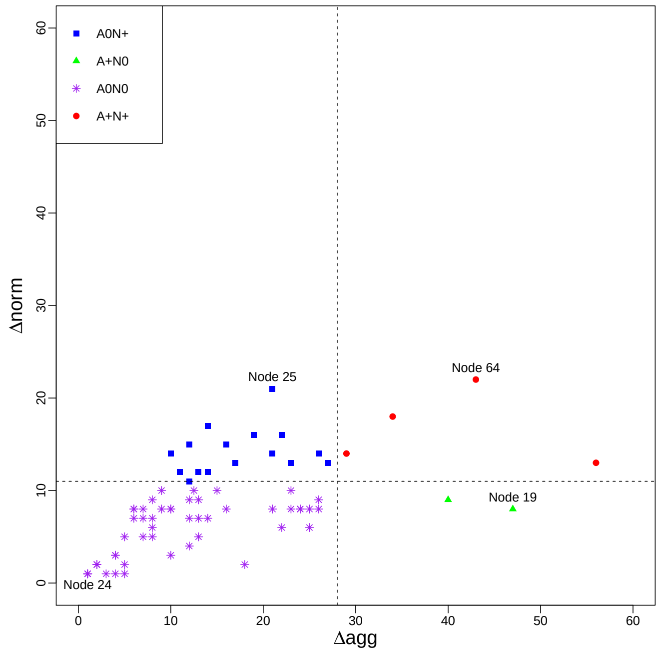


FIGURE 5.8: Law firm network. A scatter plot of the $\Delta norm$ and Δagg values of the nodes is shown here. The four sections of the plot contain the group of nodes sensitive to only one choice ($\boldsymbol{A0N+}$ or $\boldsymbol{A+N0}$), those sensitive to none ($\boldsymbol{A0N0}$), and those sensitive to both ($\boldsymbol{A+N+}$), respectively. The figure is reprinted from [89].

other two methods *NormMethod 2* and *NormMethod 4*, i.e., these produce almost similar rankings. Since the minimum and the maximum *degree* values do not vary much between the layers of this dataset as listed in Table 5.1 (C), and the structure of the layers are comparable, there is not much difference between the four normalization methods.

5.4 Discussion and conclusion

The intuitive exploration of nodes' centrality ranking in this chapter shows that even seemingly simple preprocessing steps, such as choosing a particular normalization prior to aggregations, lead to very different ranking positions of the nodes with respect to their *degree* values in multiplex networks. We conducted a sensitivity analysis using an approach that considers multiple *degree* centrality values of different layers as multiple criteria in an MCDM problem, where values are normalized using four different normalization methods. We showed how choosing different aggregations over the results of all layers yield different rankings for the nodes. The scatter plot of the nodes, based on their sensitivity to different normalization methods versus different aggregations guided by a

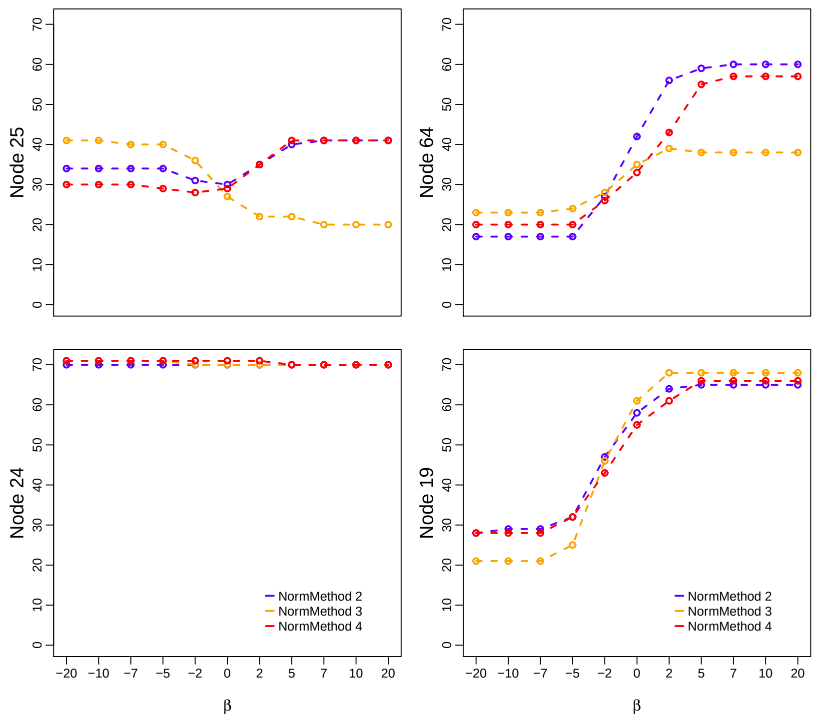


FIGURE 5.9: Law firm network. The ranking details of four nodes, which characterize the four groups of the visualization with respect to Δagg versus $\Delta norm$, are depicted here. The ranking positions were obtained using different aggregations (using the β-parameter) for the aggregation of the three layers *seeking advice*, *co-working*, and *friendship*. The three curves show the three different normalization methods (*NormMethod 1* and *NormMethod 2* yield the same result in this dataset). The figures are reprinted from [89].

parameter, were demonstrated using the two measures Δagg and $\Delta norm$. This way of exploration allows determining the number of nodes in four different groups: *sensitive to different aggregations*; *sensitive to different normalization methods*; *not sensitive to any one*, or *sensitive to both* modeling decisions.

In the experimental results, we inspected those nodes that show more sensitivity or more stability in their rankings. In three multiplex networks — a European airline network, a Twitter network data and a law firm network, we explained different aspects of sensitivity analysis. Regarding the European airline network, we discussed how the competition for rewarding nodes with respect to their highest degree values might be different if multiple ways of normalization exist. The reason is that some network layers (e.g., Lufthansa) are more *centralized* than the other layers and might not get a chance to participate in an aggregation. For example, in the European airline network, the airport of Kos Island is among the airports that are fragile with respect to both modeling decisions, normalization and aggregation. On the other hand, an airport like

Chania is among the nodes that are not sensitive to either one.

In the second dataset, which is a large tweet network, we observed that the nodes are distributed across a wide spectrum, from having no sensitivity to either modeling decision to having no stability in their ranking position. Focusing on the influences of different normalization assumptions, we found that a specific preprocessing model like normalization can be defended only if we study all the other choices that can be considered for its modeling and check whether the ranking results are robust to taking different choices. For instance, the *NormMethod 1* can be used before taking any aggregation when the cumulative distribution of values show that there is no dominant layer. As in the law firm dataset observed, an aggregation over the results of the layers would be meaningful, i.e., using *NormMethod 1* there will be at least one node which obtains the 0 value and at least one node which obtains 1 in each layer, and there might be a node that has always the worst degree in all layers, i.e., $[0,0,0]$ and always the best value,i.e., $[1,1,1]$ for three layers. Similarly, *NormMethod 4* can be applied to solve the problem when there is a dominant layer among the layers as we observed for the Twitter network data. If we consider only the shared nodes, *NormMethod 4* results in a value for the nodes between $[\frac{1}{|V^*|}$,1] in each layer, i.e., it is assumed that the lowest and the highest ranking values are 1 and $|V^*|$ respectively.

In network layers with totally different characteristics and centralization, applying any normalization for a centrality measure should be performed carefully and reproducibly. We discussed this in the Twitter network data in which a high sensitivity was observed for a node.

It can be conjectured that for other types of centrality measures, such as *betweenness* and *closeness*, more sensitivity will be recognized if the network layers have different structures, as these properties globally capture the importance of nodes. The proposed explanatory explorative analysis emphasizes a very important side of any analysis regarding node centrality and centrality ranking in the field of network science.

As mentioned in the beginning of this chapter, a wide range of studies exist on "centrality ranking"; $130,000$ papers were found on Google Scholar in October 2017. However, not all of them provide details about the basic assumptions made in the models used and about whether or not the node rankings are dependent on a particular choice in their proposed models. We believe that for any kind of network analysis, all the different modeling assumptions need to be discussed and documented in order to make them reproducible. Even a small change in the results might put the findings into question and thus require a new interpretation for them.

Note 5.1 *Since the rankings can be sometimes sensitive to the choices of different modeling decisions, and as many networks are based on incomplete data that might increase uncertainty in the outcome, is there any way to present the results of centrality values using different class of centrality with respect to their normalized value?*

In the following chapter, we will focus on a situation where a real dataset is used to construct a multiplex network. We will partition the nodes into a set of centrality classes, each containing a set of nodes with about the same normalized centrality values.

Chapter 6

Fuzzy representation of centrality and the assignment of nodes to classes of centrality

Obtaining centrality ranking is a trivial and unavoidable part of analyzing node centrality in simple, weighted, and multiplex networks. However, it has been shown that in multiplex networks, a node's ranking can be sensitive to the choices of different definitions of normalization, which is an essential preprocessing step before doing any aggregation over the centrality results of the nodes in multiple layers. We have observed a high sensitivity of turning a node from being the *most central* to the *least central* in a multiplex network where the layers have different structures and characteristics. The findings emphasize the importance of the choices we make in any type of analysis regarding centrality ranking. To be aware of such sensitivity, any analysis needs to be done carefully, by considering all the assumptions and their possible influence on the results, similar to the explorative analysis in the previous chapter. In some cases of quantifying an individual's centrality, we overestimate the importance of some nodes and ignore some others using a precise ranking. Imagine analyzing the activity of patients in a psychotherapy chat session. Obviously, a discrete ranking showing their activity level, which might be used to demonstrate their improvement, would be too precise when additional factors might have a more leading role in assessing the improvement of the patients. In such cases, we might instead want to check which individuals have about the same activity level. In this chapter we will show that by using a set of centrality classes we are able to do this to a moderate extent.

In addition to the aforementioned problem, since many real networks are constructed based on incomplete data and since centrality measures can be sensitive to edge addition and removal, a very precise ranking is not always the best technique to opt for—we will elaborate this concern in the next section. This motivates us to find a means of representing centrality values using a set of classes, each containing nodes with about the same degree of importance in real networks. A great number of studies shows that fuzzy models are one solution for dealing with uncertainty stemming from imperfection of data and for avoiding information loss in decision making problems [36, 37, 35, 75, 74, 76]. As described in Chapter2.2.2, Herrera in [36] proposed a 2-tuple model to present the results of values by labels. Some studies used this linguistic model to analyze central nodes in fuzzy cognitive maps [66]. In this chapter, we similarly use a set of linguistic terms and a 2-tuple fuzzy model as proposed by Herrera in [36] to label nodes with respect to their centrality index value instead of comparing their importance with a discrete ranking, but in a multiplex network. In addition, we will evaluate the robustness of the model to edge removal and edge addition noises, applying each with several rates to multiple layers of the network [1].

6.1 Imperfection of network data and its possible influence on centrality measures

Consider a situation where person A is asked to name his/her friends. If the answer is imprecise—independent of what the reason is— the number of communication partners of that person, which will be captured by the *degree* of the corresponding node in the deduced network, will not be precise either. Consequently, if we aim at presenting the results of the measured *degree* using a precise ranking, we are neglecting the uncertainty coming from the imprecision of the respective property. This has been shown in an experiment by Brewer and Webster, where the residents of a university residence hall forgot to name 20% of their friends on average. The authors showed the impact of incompleteness on the results of node centrality in a constructed friendship network [16].

Having imprecise logged data with respect to multiple types of interaction—which can be represented in multiple layers— is a similar issue that is of concern in a multiplex network [88]. Multiple investigations have been conducted to show the effects of missing information in collected datasets represented as networks [52, 18]. In a study [52], Kossinets elaborates the effect of missing information in an experiment by simulating uncertainty using three different models and assessing the sensitivity of network measures to the simulated uncertainty. He states three main issues in data sets as follows: the "boundary specification" problem, which considers the effect of missing nodes and missing links in the statistical results; the issue of "non-response effects", which mainly considers missing data due to non-response of examinees (those asked to complete the questionnaire by naming their interactions); and the issue of "fixed-choice designs", meaning that the actors (represented as nodes) are asked to name two (a cut-off value) of their friends, while they might have more than two. In any case, some interactions would be neglected in the deduced network, which can change the result of a measure, as shown in the study [52].

[1] The results in this chapter have been published already in a paper [88].

TABLE 6.1: Noordin network dataset. The structural properties of its three network layers that contain 79 nodes, i.e., $V_i = V^*$, are listed here.

Properties	*Trust Network*	*Operational Network*	*Communication Network*
$\|V_i\|$	79	79	79
$\|E_i\|$	259	437	200
$\max_{v \in V_i}\{deg(v)\}$	28	43	41
$\eta(l_i)$	0.084	0.142	0.065

In contrast to the above-mentioned analyses, the other studies report that centrality measures are almost robust to random network errors. Therefore, it is likely that the confidence interval around a centrality value can be obtained and the changes will be linear [11, 21]. In 2003, Costenbader and Valente analyzed the stability of eleven centrality measures by using a bootstrapping procedure and measuring the correlation of the results between the sampled and the original networks. In their experiments, the most unstable measure among all was a directed version of the *betweenness centrality*. In addition, in 2006 Borgatti showed that the majority of the classical centrality measures are robust to different rates of network errors. He states that the accuracy of the measures declines slightly and predictably with respect to the rates of the applied errors. In general, he concludes that network errors on nodes are negligible in comparison to errors on edges. According to him, edge addition is the "least forgiving" among the network errors. Regarding edge removal, he explains that if the *density* of a network is small, then this error would not greatly affect the accuracy of the centrality measures. In contrast, if the *density* of a network is high, then the error of edge addition has the least effect on the accuracy of most measures [11].

For dealing with the incompleteness of data, several approaches have been proposed in the literature. In 2009 Huisman showed the effects of basic imputation methods on a friendship network in a simulation study [39]. He demonstrated that the imputation of data using simple approaches (as same as neglecting the imprecision of data) has a negative effect on some network properties.

Considering uncertainty in the data—whether due to incompleteness or imprecision— and looking back at the results of the sensitivity analysis of a node's ranking in a multiplex network motivated us to search for a real network data, in which there is always imperfections to some extent, and to aim at representing the result of a centrality index value using a set of labels.

6.2 A real-world network data

Regarding the incompleteness of real network data, we opted for a real dataset that can be represented as a multiplex network and might be representative for an incomplete data. This dataset contains multiple interactions among 79 individuals in the Noordin group. This organization was identified in various report as the group responsible for several attacks between the years 2003 and 2005. The main report that led to the collection of the dataset was published in 2006 [43]. Roberts provided a structured data set based on that report in a study [73]. Moreover, Everton and Cunningham thoroughly explored the information available in the data and studied this so-called "dark network" [27]. Iacovacci et al. also used this data to analyze the activity of members using several multiplex page rank methods [41]. This data as a crucial dataset contains very rich information regarding different types of relations and interactions among the members of the aforementioned organization, which had preplanned attacks. In addition, it encompasses several features for each member, such as information regarding military training, nationality, and level of education. In this dataset, we are interested in observing to which extent these members play an important role in their communications, and in partitioning them based on the respective findings. As the data contain various kinds of information regarding the activity of the members, we can represent it as a multiplex network. We assume three different types of relations among the members as the three layers of a multiplex network [88]. In the represented network, the first layer constitutes the *trust network*, which is an aggregated version of four types of ties, each indicating a relationship between the 79 members, such as friendship, classmate, soulmate, or kinship. As listed in Table 6.1, this layer encompasses 259 edges (excluding multiple edges and self-loops) and is an undirected, unweighted network. The second layer is named *operational network*: The edge between a pair of nodes represents the aggregated version of multiple interactions if the corresponding members have provided the same logistics, participated in the same meetings and/or in common operations, or attended the same training event. This layer is the densest among the three network layers. Note that all constructed networks are simple, meaning two nodes are linked if at least one of the aforementioned relationships exists between the corresponding members. The third layer is named the *communication network*: Between a pair of nodes in this network, there is a link if they had contacts within the organization by means of messages, or communicated via other kinds of tools, such as codes or videos as a means of recruiting other people outside the organization. This network layer has the least density compared with the other network layers.

6.3 Fuzzy representation of centrality

Instead of comparing nodes according to their centrality ranking, we aim at assigning nodes to fuzzy labels exhibiting different degrees of importance in a predefined term set {**Very Peripheral**, **Low**, **Medium**, **High**, **Very Central**}. As input, consider a centrality index, e.g., *degree* or *closeness*, that is computed for all nodes in a layer of a multiplex network. Then normalization is performed for any centrality value by subtracting the minimum from the corresponding value and dividing it by the amount obtained by subtracting the minimum value from the maximum value found among the values of all nodes in the same layer—this will keep all the resulting values of a centrality measure to be in $[0, 1]$.

Then we use a 2-tuple fuzzy representation model as explained in Chapter 2.2.2 to transform a normalized centrality index value onto a set of labels. Then, for any node the corresponding 2-tuple contains a label and the extent to which a node is close to its label.

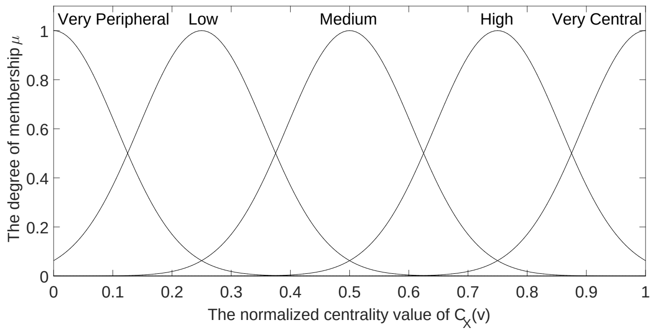


FIGURE 6.1: Five linguistic terms and their semantics are described using five overlapping Gaussian membership functions. The figure is reprinted from [88].

Example: for a centrality index value of $C_X(v) = 0.15$, the fuzzy set is $\{(s_0, 0.369), (s_1, 0.642), (s_2, 0.004), (s_3, 0), (s_4, 0)\}$

6.3.1 Assigning nodes to a set of centrality classes

To obtain the centrality class of node v in layer l_i, in the first step, we fuzzify its normalized centrality index value (measured by *degree* or *closeness*) through Gaussian curve membership functions, each representing a label in the corresponding linguistic term set. The classes are distributed symmetrically and equally between the least normalized centrality index value and the highest normalized centrality index value. As described in Chapter 2.2.2, any value x in the interval $[0, 1]$ can be fuzzified using several membership functions as proposed by Herrera [37].

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $s_i \in S$ a linguistic term described with a symmetric Gaussian function[2]. Assume a set of five ordered classes (labels) as shown in Figure 6.1. The membership value μ_{s_i} of a normalized centrality value of node v, denoted by $C_x(v)$, is then determined by the intersection of the index value in the corresponding class of s_i. According to the definition of membership functions in Chapter 2.2.2, we obtain the membership values for each node in several labels. This outcome is a fuzzy set that contains the membership values for a node's centrality index and all classes, as illustrated in Figure 6.1 for an exemplified value of 0.15.

Then the membership values for node v over the fuzzy set using a symbolic aggregation operator are aggregated as defined in Equation 2.6. For any node v, the outcome of the corresponding aggregation θ_v is a value in $\in [0, g]$, for which an equivalent information in the predefined term set S, can be represented using the function $\Delta(\theta_v)$ (s. Eq.2.7). This function results in a 2-tuple (s_j, α) that provides information about the centrality class of node v and the extent $\alpha \in [-0.5, 0.5)$ to which the node is close to its assigned class s_j. After we have obtained all the labels for all nodes in multiple layers, we visualize the

[2]The set of Gaussian membership functions were built in a fuzzy toolbox in Matlab by setting the standard division to 0.106, and the centers to 0, 0.25, 0.5, 0.75, and 1, respectively.

assignments of the nodes to the classes—ranging from **Very Peripheral** to **Very Central**— separately for each layer.

From a multiplex perspective, in order to answer the question of which centrality class node v belongs to with respect to its best and worst normalized centrality index values within multiple layers, we can employ the same model accordingly: Using the three operators min, max, and average, we get the overall importance of the nodes with respect to their least value over all layers, their best value over all the layers, and their average value over all layers[3]. Then a 2-tuple model is similarly applied in order to assign a node to the ordered set of centrality classes with respect to its overall importance value.

6.3.2 Applying edge addition and removal

After assigning the nodes to the classes, we aim at analyzing the robustness of the model by simulating uncertainty in the network data using network noises through random edge removal and edge addition with rates of 10%, 20%, and $30\%|E_i|$. To apply the additional edges, we randomly select a pair of nodes for each and check whether they are connected or not. If they are not linked but have a common neighbor, then the edge is added to the layer—this condition is considered based on the assumption that if some individuals are asked to name their friends (or acquaintances) and they forgot to recall some of them. For each rate of noise, we get an average over the result of a centrality measure after 50 iterations of noise application. To apply the edge removal noise, we randomly select a sequence of edges and delete them from a network layer; then we compute the centrality of the nodes in the largest connected component in the corresponding layer including noises [88].

Since we use five linguistic terms to present the classes of centrality, the minimum and maximum possible degrees of changes that a node can have within its class in the original network and in the network with the noise equal 1 and 4, respectively. For this we can use a distance measure to calculate the extent of the changes between classes in the following way: Assume $d_{class}(l_i, l'_i) := \frac{\sum_{v=1}^{|V_i|} |(s,\alpha)_v - (s',\alpha')_v|}{g \cdot |V_i|}$ as a dissimilarity measure [88], where l_i denotes the original network layer and l'_i is the network layer with the noise and where normalization is performed by the number of nodes $|v_i|$ and the maximum degree of changes that a node can have in its classes—this yields values between $[0, 1]$. Having a maximum of four possible change for all nodes will be however hard to achieve with respect to our assumptions in the noise application. This might only happen in a network that is very sparse, such that after the application of a noise, e.g., adding edges with a very high rate, every node becomes connected to all others in the resulting network. In contrast, we are interested in low rate noises in order to model a small rate of uncertainty in the available data—we assume that for any dataset regarding groups or friends, it will be almost unexpected to have a very high degree of uncertainty caused, e.g., by incompleteness: As shown in a friendship network in an experiment and explained at the beginning of this chapter, people forget to name their friends

[3]This can be done using an OWA aggregation operator; however, to avoid repetition, we leave its explanation out of this chapter and will just present the results of the min, max, and $average$ cases.

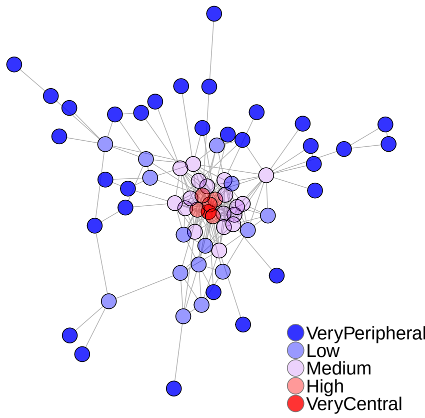
with a low but unavoidable rate. Instead, we compare the results of the percentage of nodes that change their classes between the original network and the network including the noise with a low degree.

6.4 Results

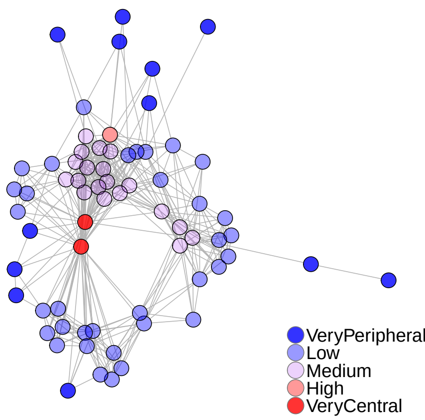
As we mentioned in the introduction, the data we chose for the current analysis is a real dataset, which is represented as a multiplex network including three layers. The results of the *degree centrality* with respect to the fuzzy centrality classes in the three layers *trust network*, *operational network*, and *communication network* are depicted using a color code in Figure 6.2. The isolated nodes—the members without any communication—-in the aforementioned network layers, are removed in the visualizations. We focus on the analysis of those members that were recognized as the key individuals of the organization in the study by Iacovacci et al. [41]. Only the abbreviation of the members' names are used in the visualizations. The person whose name is abbreviated as A.S.R. is a member in the center of the *trust network* who belongs to the highest class of centrality (**Very Central**) with respect to the fuzzy *degree centrality* as depicted in Figure 6.2 (A). It can be conjectured that he has a very important role in the group and consequently in the other activities; however, we will explain in the following that this is not the case here and that this person is just a strategist known to some of the members as someone giving advice.

Five members, including the strategist in the visualization Figure 6.2 (A), are assigned to the class of **High** centrality. These members are: the leader of the group and the four remaining members M.R., T.R., A.B.B., and F.A.Gh., who trusted the leader or were trusted. Note that using a precise ranking, they can be considered differently, as their pure *degree* values range from 28 to 18. Obviously, as recognized and expected, the leader is always in the best class with respect to operational and communicational activities, which are represented in two layers, the *operational network* and the *communication network*, in Figures 6.2 (B)-(C). In addition, A.H., whose role was the bomb expert in the organization, is assigned to the class of *Very Central* nodes and is the closest member to the leader in terms of operational activity. When we look at the importance of the members with respect to the same activity, I.D., whose role was coordinator for attacks and logistics, is the only member assigned to the class of **High** centrality. While most nodes are distributed over the five classes of centrality in the *trust network* and *operational network* layers, the nodes in the *communication network* layer are condensed in the classes of **Very Peripheral** and **Low** centrality.

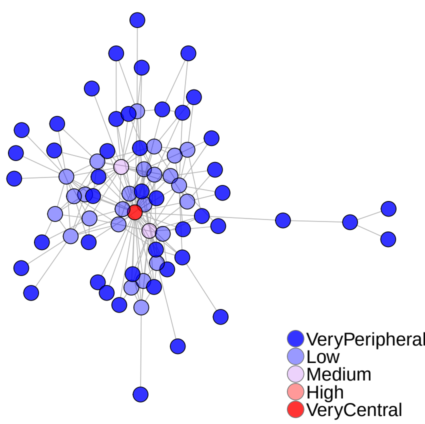
To get a general view of the aggregated class of node centrality, the nodes in Figure 6.3 are assigned to all five previously defined classes of centrality. However, to provide more detail, we see the classes in the $x - axis$ and the $|\alpha|$-value, which denotes the distance of staying before or after a class, in the $y - axis$ [88]. For the sake of clarity and to avoid overlapping the names, the α-values obtained for each node are also used in the $x - axis$. Thus, a negative α-value of a node causes the node to stay before the middle line (|) of its label and a positive α-value positions the node ahead of the assigned label. Since a similar centrality class might be obtained for multiple nodes, in the visualization, a point can indicate the result of multiple nodes.



(A) *Trust network*

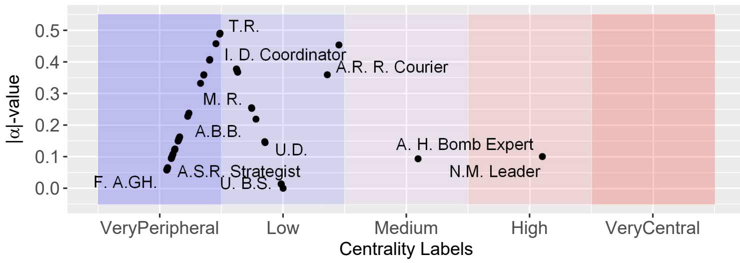


(B) *Operational network*

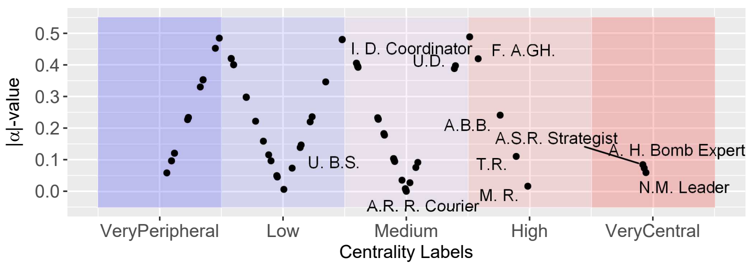


(C) *Communication network*

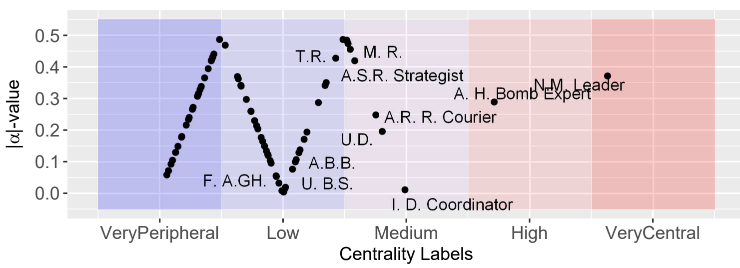
FIGURE 6.2: The assignments of nodes to the five classes of fuzzy *degree centrality* demonstrated separately for the three layers of the network. Figures are reprinted from [88].



(A) min



(B) max



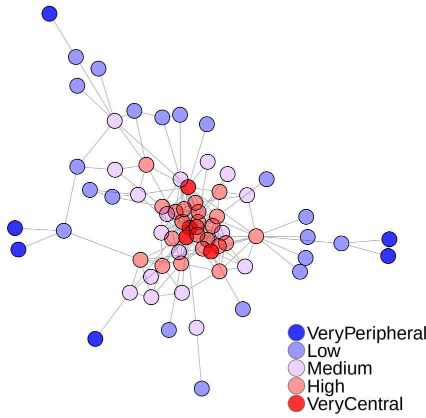
(C) $average$

FIGURE 6.3: The assignment of 79 individuals to the classes of fuzzy *degree centrality* is depicted here based on the results of the basic aggregations over the three layers. Figures are reprinted from [88].

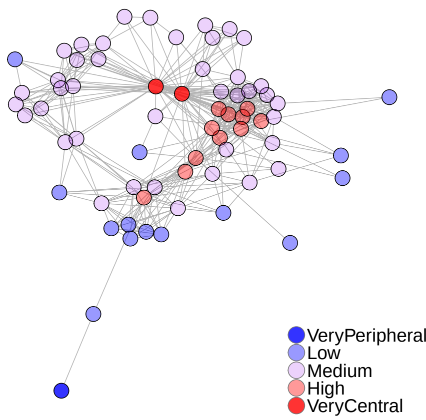
Assume three different aggregations, namely taking the minimum, maximum, and an average over the results of *degree centrality* obtained for the nodes in the three layers. The importance of the nodes regarding their class of aggregated *degree centrality* is depicted in Figure 6.3(A) to (C). In the first aggregation, the nodes are assessed with their minimum value of centrality among the three layers. It is revealed that four members of the organization in this network are identified: The courier (A.R.R.), the coordinator of attacks and logistics, the bomb expert, and the leader of the group, are the key members, even if we assess their importance as their least important role. Similarly, Iacovacci et al. have shown in their paper that the aforementioned members are always at the top of the activities [41]. Back to the case of A.S.R., who had the role of strategist and was discussed above, he is no longer identified as one of the top key members with respect to the least value of importance; he is assigned to the class of **Very Peripheral** nodes. He is in the second aggregation—which assess the nodes with their best role (max value) over all layers— among the **Very Central** nodes as shown in Figure 6.3 (B); his most satisfying class of centrality is obtained in only one layer (the *trust network*), not in *all the layers*. Similarly, the four members M.R., T.R., A.B.B., and F.A.Gh., who had a good relationship with the leader in terms of trustee/trustor, are not among the group of nodes with **Medium** centrality to **Very Central** in the first aggregation, where they need to have a satisfying value of fuzzy *degree* with respect to their worst role (min) over all the layers. On average, only M.R. is assigned to the class of **Medium** centrality. Looking at the second basic aggregation, it turns out that these cases are identified as the key members in the groups **Medium** to **High**. One interpretation of this result can be that the trustee/trustor relationship in the *trust network* plays a leading role in the overall activity of the members.

The results of the fuzzy *closeness centrality* evaluation for all members in the three layers are demonstrated in Figure 6.4(A) to (C). In addition, the classes and the detailed $|\alpha| - value$ are depicted in Figures 6.5 (A), (B), and (C), respectively, for the three layers. A member of the organization (with the abbreviated name U.D.) who was jailed after four months of being in the organization, the strategist, the leader, and the courier are assigned to the class of **Very Central** nodes regarding their communications in the *trust network*. The member mentioned first did not communicate directly with the leader as logged in the interactions of this layer. However, he was connected to a couple of key members, such as the strategist, the courier, a member whose role was facilitator for materials (U.B.S.), and the coordinator of attacks and logistics. As can be seen in Figure 6.4(B) and Figure 6.5(B), the bomb expert and the leader are assigned to the class of **Very Central** nodes, which means they were easily reachable for the other members with respect to operational activities. The one who had the highest level of activity and was in the center of communications with all the members is the leader, as he is assigned to the best class with respect to the fuzzy *closeness centrality* value in the *communication network* layer.

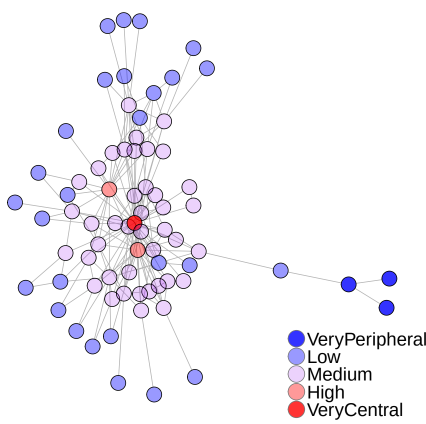
Considering different aggregations over all three layers, as shown in Figure 6.6(A), reveals that the leader is assigned to the best class of *closeness centrality* in *all* three types of relations. After him, three key members—the bomb expert, the courier, and the attack coordinator, respectively— are located in the classes of **High** and **Medium** centrality. Considering only their most satisfying activity, it turns out that the strategist and the member who was jailed after a short time (U.D.) are in the class of **Very Central** nodes as shown in Figure 6.6(B)— U.D. has many connections to the other key members in the *trust network* and *operational network* layers. As shown in Figure 6.6(A), however, with respect to their worst activity, the strategist is identified as a **Very Peripheral** node since he had no operational activity and a very low number of communications with other members via internal and external mediums. *On average*, he is among the nodes with **Medium** centrality; in fact, this shows that getting an average over all layers will some time overestimate the actual importance of a node.



(A) *Trust network*

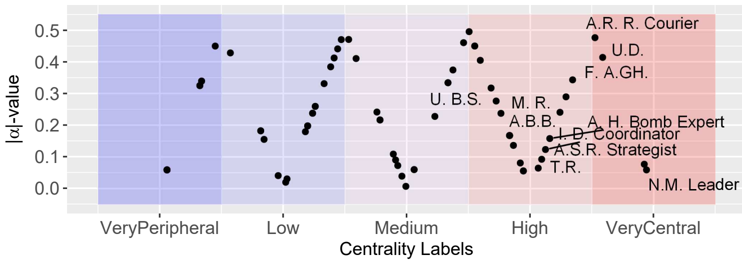


(B) *Operational network*

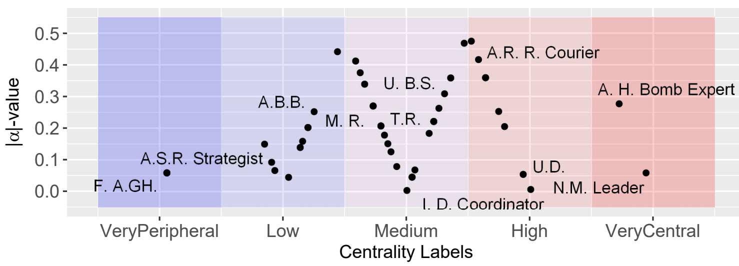


(C) *Communication network*

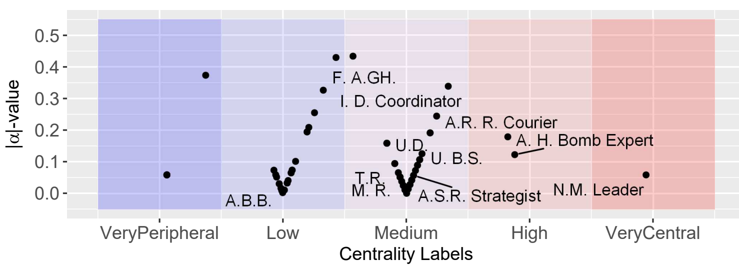
FIGURE 6.4: The assignments of the nodes to the five classes of closeness centrality are separately demonstrated for the three layers of the network. Figures are reprinted from [88].



(A) *Trust network*



(B) *Operational network*



(C) *Communication network*

FIGURE 6.5: The assignment of the nodes to the classes of *closeness* centrality in the layers are shown separately here. Figures are reprinted from [88].

To show how robust the nodes assigned to the classes are against edge addition and edge removal, we apply these to the network layers using several different rates. The result is shown in Table 6.2 for the fuzzy *closeness centrality* for several rates of noises.The results show that a high percentage of nodes stay in the same class of centrality in the *communication network* layer when various rates of edge removal and edge addition are applied. That is in accordance with the low density of this layer (s.Table 6.1) and as explained by Borgatti in his study[11], if a network has low density, edge removal has the least effect on the results. In contrast, both edge removal and edge addition have the greatest impact on the *operational network* layer, which is the densest network among the three layers—still, the changes are very low, less than 10% for a noise rate of 10% for edge addition. Obviously, if we use a ranking over the results of the

TABLE 6.2: The ratios of nodes that stayed in the same class of centrality after different noise applications out of all nodes in each layer.

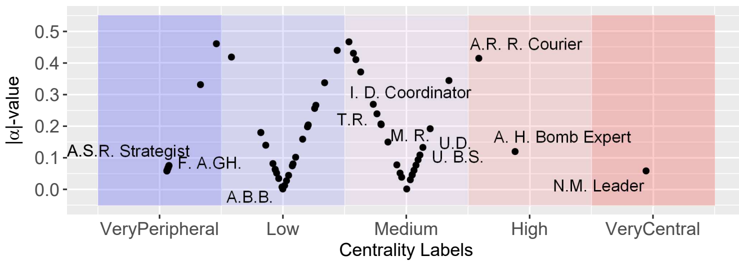
	Edge removal			Edge addition		
	$10\%\lvert E_i\rvert$	$20\%\lvert E_i\rvert$	$30\%\lvert E_i\rvert$	$10\%\lvert E_i\rvert$	$20\%\lvert E_i\rvert$	$30\%\lvert E_i\rvert$
trust network	0.772	0.747	0.722	0.924	0.911	0.722
operational network	0.747	0.646	0.620	0.924	0.823	0.785
communication network	0.911	0.823	0.658	0.987	0.962	0.937

normalized *closeness centrality* values, we would get changes in the rankings of a higher number of nodes after the application of additional edges with the same noise rate. This is because even a slight increase or decrease in the result of a normalized value of a node will be considered as a change in its ranking.

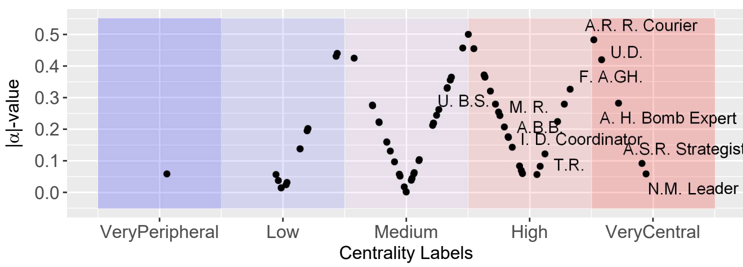
6.5 Discussions

The usage of a 2-tuple fuzzy representation model to assign nodes to the different centrality classes has been studied in this chapter. We aim at showing the importance of nodes in a set of classes instead of presenting the result of a centrality measure using a discrete ranking and categorizing nodes as most or least central. We used a real network data set, which may be prone to uncertainty caused by incompleteness. We investigated the differences in the assignments after the application of several types of noises with different rates to the network layers. The results show that as long as the noise rates are small and the resulting changes in a node's importance (captured either by the *degree* or the *closeness* centrality indices) are rather low, the nodes stay in the same class of centrality.

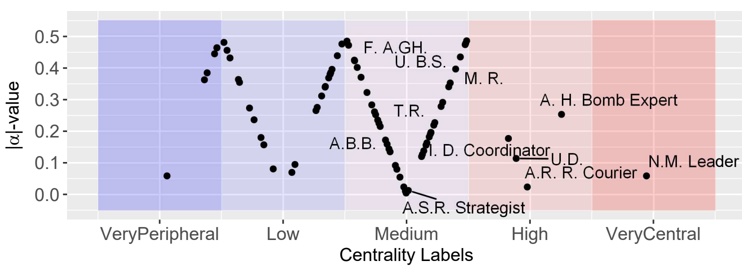
This representation needs some improvements as described as follows: The classes in the model are distributed symmetrically and equally between the least normalized centrality index value and the highest index value and the model does not consider the whole information about how the nodes' centrality values are distributed; whether or not a network layer is strongly centralized, and whether a small number of nodes have a high normalized value while the majority of the nodes have very low index values, i.e., *communication network* is a more centralized layer than *trust network* layer. This problem can be improved by deducing classes and generating membership functions based on the distribution of the actual centrality values.



(A) *min*



(B) *max*



(C) *average*

FIGURE 6.6: The assignments of the nodes to the classes of closeness centrality after the aggregation of the results over the layers. Figures are reprinted from [88].

Chapter 7

Summary and conclusion

Network analysis and the corresponding methods have greatly enhanced our understanding of systems that are complicated in nature and structure. The reason for this is that a network representation reveals the interactions or relations between the entities of the corresponding system. Thus, it is of interest for researchers in many fields to analyze such systems at the structural and/or behavioral level using network analytic methods. In many studies, network analysis has to deal with a multitude of information regarding nodes and/or edges. One of the fundamental analyses is to quantify node centrality in a network using those indices that characterize the important ones among all nodes with respect to their role in a network. Each centrality index can be used to capture a specific property of nodes in a network, e.g., *degree* quantifies a node's potential for having a direct influence on other nodes. We explained a situation in network analysis where the usage of only one property might not be sufficient for identifying important nodes. However, several challenges exist regarding the inclusion of multiple properties in the evaluation of node centrality—The idea argued by Opsahl, Agneessens, and Skvoretz was the first paper that considered two properties of nodes and used a tuning parameter for scaling between having a high value in *degree* and low value in *strength* is favored, and vice versa.

We aimed at dealing with challenges regarding the usage of multiple indices for evaluating node centrality in different types of network representations such as simple, weighted, and multiplex networks: If multiple *centrality indices* are applied to a simple or weighted network, then multiple values regarding a node's centrality will be obtained, which can be used to assess whether the different values (each of which shows an importance degree) have conflicting views about a node's centrality or not. Similarly, if one centrality index is applied to all nodes in multiple layers of a network, then again multiple values will be obtained regarding a node's centrality. In such cases, the usage of a single ranking with respect to one aspect of importance will not provide any insights about how the ranking changes if the other aspects have different views regarding a node's centrality. We thus categorized the main questions in this thesis as follows: Which indices determine important nodes in a network? How can multiple indices providing information about the importance of a node contribute jointly to the identification of important nodes? Which nodes show stability in staying among the top-ranking nodes and which ones show more sensitivity to different types of aggregation performed over the values? Can basic preprocessing methods such as normalization change the results of any aggregation in this evaluation? If so, why is this happening?

To address these questions, we propose an approach that considers the evaluation of node centrality and the identification of most central nodes using multiple indices as an MCDM problem, and where the best nodes are those that always stay among the top-ranking ones in a stable manner. The approach uses a parameter in an aggregation operator that scales between two extreme cases of selecting nodes: (1) where at least one of the measured indices has a high value, and (2) where the least value found among multiple measured indices is high. This consideration allows exploring many interesting cases that could not be found easily using other methods in various type of datasets, such as communication networks, collaboration networks, transportation networks, a social network, and a preplanned dark network.

In communication datasets, we showed that the proposed approach enables us to see whether a node's centrality ranking is robust to the choice of different types of aggregations over the centrality values. We conclude that depending on the type of dataset, we first need to identify which indices meaningfully describe the importance of a node, e.g., the activity level of a participant in a human group. We observed in the ranking of a chat-log dataset that the moderator of the chat was always among the top-ranking nodes with a stable manner in its ranking. In Freeman's EIES communication network, we showed some cases that if only *degree centrality* alone was used, might be totally ignored even if they were very important with respect to the other aspects of centrality. In the datasets represented as a simple network, e.g., an air transportation network, we conclude that the choice of aggregations for analyzing node centrality is very important, e.g., selecting nodes whose at least one centrality index is maximal. In such a case that sensitivity is because some nodes are very central with respect to only one aspect, e.g., the potential of a node in passing a process to all other nodes captured by *closeness*, but they do not have high importance with respect to the other centrality indices.

We emphasized that the intuition behind this idea is not meant to be showing which aggregation is better to use or which parameter value is the optimal one. Rather, we aimed at revealing whether, if a set of different aggregations is created over the values of multiple indices for analyzing node centrality, the rankings of which nodes are robust enough to stay always stable among the top-ranking nodes and which ones shows very sensitive behavior.

While performing this analysis, it became clear that rankings of some nodes can be very sensitive and very dependent on the choices we make regarding their aggregation in a multiplex network. This is an important point as there is great interest in analyzing node centrality in multiplex networks. We explained that if the results of a network layer have a leading role in the results of the aggregation, other normalization methods need to be used before any aggregation. We proposed four different normalization methods for *degree centrality* before applying different types of aggregations. The experiments show that findings vary between the network datasets. Applying the approach to the Twitter network, it was revealed that a node can heavily change its ranking from being among most central to being among least central nodes. However, in the medium-size network of a law firm, the sensitivity to the choices was negligible. We elaborated in which cases we get less sensitivity for a node's centrality ranking. For instance, the structure of the layers in the law firm data indicate that they are almost comparable, which means we observe less sensitivity to the choice of normalization methods. Then, considering any choice on the aggregations, the behaviour of the nodes' rankings will be almost similar. We emphasized that documenting all the preprocessing steps—even if they may seem to be inconsequential—will allow better understanding and interpretation of any network analysis since it mostly has to deal with a multitude of information in complex systems.

Moreover, we consider that since most real networks are deduced from imperfect (incomplete and/or imprecise) data, the usage of a precise ranking to represent the results of *centrality indices* might not be the best option to opt for. We explained that instead of representing the values of *centrality indices* using ranking, nodes can be labeled using a set of classes of centrality. We showed that in a terrorist network dataset, containing multiple types of relations between the members, that satisfies the idea of imperfection, such consideration facilitates representing the results of node centrality.

The evaluation proposed in this thesis will also be useful for other fields that employ network analytic methods, such as psychology and sociology. If they aim at evaluating the behavior of individuals and the data is represented as a network, then the experts in the field can discuss the choices and explore behavioral patterns related to the different roles of the individuals.

7.1 Future work

The idea proposed in this thesis has other interesting aspects that are the topic of future work, as explained in the following:

- In the sensitivity analysis of the ranking, we considered the maximum sensitivity that a node's ranking has with respect to different choices of normalization methods and aggregations. As mentioned in Chapter 5, this can be extended to the minimum and average sensitivity degree of a node's ranking as well. The proposed idea can be extended in terms of analyzing the sensitivity of other centrality measures, such as *betweenness* and *closeness* in multiplex networks, which in recent years have been at the center of attention in many studies. In addition, the partitioning of nodes into four clusters with respect to their sensitivity degree can be improved by fuzzy clustering methods.

- In our future work, we aim at developing a package based on the idea proposed in this thesis in order to analyze the sensitivity of *centrality indices* to the choices of different modeling decisions. This will be very helpful for anyone who wants to check all conflicting rankings for nodes in a network data and identify those nodes that are very sensitive to particular choices.

- A very important analysis regarding *centrality indices* is to investigate which decision is most suitable for analyzing node centrality with respect to the type of network processes, such as disease transmission and information diffusion. This is an interesting topic that will be studied in the future work.

Appendix A

Supplementary results for Chapter 3 and Chapter 5

A.1 Results obtained using OWA based on a quantifier and MEOWA

As described in Chapter 3 in box 3.2, we can also use a quantifier to produce weights for the OWA operator to get an overall importance *score* for the nodes in the three network datasets (see the definition in 2.4). In the quantifier, a set of values $\{0, 0.5, 1, \cdots, 5, \infty\}$ is used for the β-parameter, where 0 results in a max operator, 1 gives an average and ∞ results in a min operator. Note that in the computation, ∞ is used to show a much larger value than the value before that. If we choose the three classic aggregations, which are created when $\beta = 0$, $\beta = 1$, and $\beta = \infty$, the results are the same as those we obtain for the $MEOWA$ operator in Chapter3. Following the steps of the evaluation process results in the ranking of nodes with respect to a set of β-value used in Eq 2.4. The visualization of Fig. A.1 for the chat-log data set, starts with the rankings of the nodes in the extreme case of $\beta \rightarrow 0$ (simply put, a max operator over the values) until the extreme case of $\beta \rightarrow \infty$ is reached (a min operator). From $\beta = 1$ until ∞ is reached for the β-parameter, the aggregations rather reward nodes with high minimal values. As shown in Figure A.1 for the chat-log data, similar to the results discussed in Chapter 3, the Therapist has the most *robust* importance with the highest stability; and $P28$ is the person who stays very few times among the top nodes and has a less stable ranking than $P14$, who is recognized as a medium active person. The patients $P44$ and $P38$ are always among the bottom five nodes. As can be seen in Fig. A.2 Lin Freeman, Mullin, and Wellman are always among the top six nodes.

Also, similar to the results we obtained in the air transportation network, the EDDL (Düsseldorf) and EDDT (Berlin) airports are among the top-ranking airports over all different types of aggregations as can be seen in Fig. A.3.

In Chapter3, we also mentioned that the normalized *closeness* values of 79% of the nodes in the air transportation network are greater than the *degree* and *betweenness* values. We explained in Chapter 5, that in such cases, one of the criterion will have a more important role in the result of the aggregation. To deal with this problem, we propose the usage of a ranking over the actual measured values of a centrality index. The aforementioned rate can be decreased to 29%

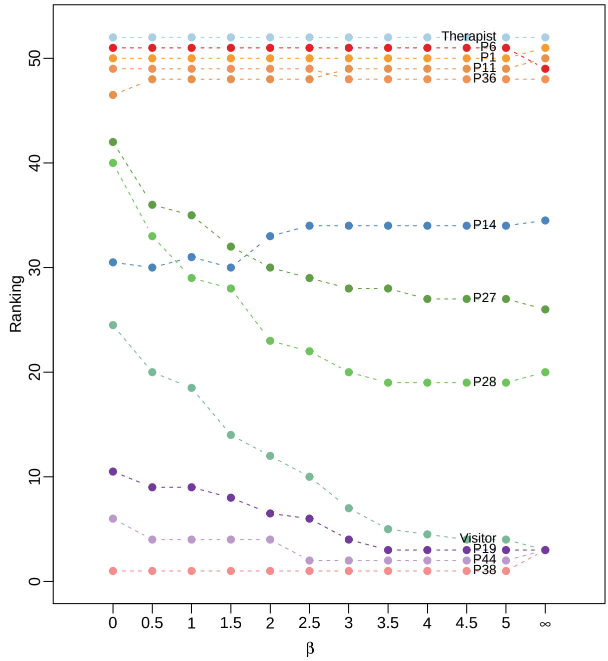


FIGURE A.1: Chat-log network. The detailed rankings of the nodes are depicted over a set of different aggregations guided by the β parameter in the quantifier guided OWA. The details of the four criteria used in the evaluation are listed in Table 3.1.

if we rank the actual values of the centrality measures (e.g., $\{75, 74, 73, \cdots, 1\}$) and normalize them with the number of nodes in the network layer $|V_i|$. Then values between $\frac{1}{|V_i|}$ and 1 are achieved for each criterion. The results of the proposed approach are shown in Figure A.4 for the airports discussed in Section 3.3. As can be seen here, there is no change in the finding regarding those nodes that stay stable among top-ranking and bottom-ranking nodes even if the *NormMethod 4* is used. However, for those nodes that are sensitive to the choice of different types of aggregation, such as LICJ and LICC, using this kind of normalization appears to cause a different behavior considering a set of different aggregations. However, the interpretation depends on what we mean by ranking the actual values. If we want to select some nodes that are always on top, these nodes will definitely not be among them, regardless of what kinds of normalization is used. Using the *NormMethod 4* we rather focus on the position of the nodes with respect to a centrality index and not on the actual index value itself. In such cases we will thus get a different view of the ranking behavior. As we see there is a more stable upward trend in the ranking behavior of these nodes, in contrast to what we saw in Chapter3, where they had a downward trend from left to right side of the plot. Thus, the interpretation of the finding depends on what we want to select, i.e., whether we want to select nodes that are among the top 15 positions (from 75 to 60), or whether we want to select nodes using a cut-off *score* (e.g., those who have a value greater than 0.8). In both cases, these nodes will never appear among selected nodes.

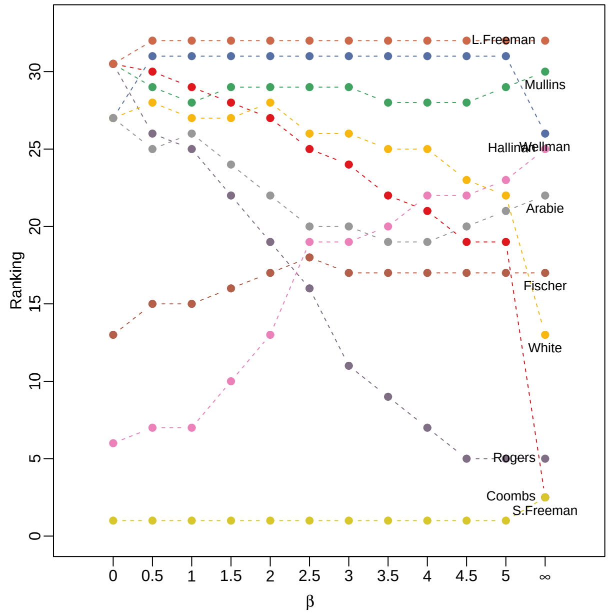


FIGURE A.2: Freeman's EIES network. The detailed rankings of some nodes are depicted over the different aggregations guided by the β parameter in the quantifier guided OWA. The values of three criteria are listed in Table 3.2.

A.1.1 Comparisons of the results of Quantifier guided OWA with the reproduced results of the method by Opsahl et al.

Here we reproduce the results of the method proposed by Opsahl, Agneessen, and Skvoretz in [70] for Freeman's EIES data. As explained in Chapter 2 in Equation 2.1, their method uses an α parameter to deal with the trade-off between nodes that have a high *strength* value but a low *degree* value and nodes that have a low *strength* value but a high *degree* value. In Table A.1, we see that when $\alpha = 0$, the nodes are ranked based on degree. When $\alpha = 0.5$, the nodes with high *degree* are favored; when $\alpha = 1$, the nodes are ranked by their *strength* values, and any value above 1, e.g., $\alpha = 1.5$, rewards nodes with low *degree*. Using two properties, Coombs, Foster, and Ev. Rogers are often among the bottom-ranking nodes. Using our approach, the ranking for Rogers was unstable, as discussed in Chapter 3. He had the highest number for citation value—meaning when the aggregation rewards nodes with at least one high value, he was among the top-ranking nodes—-, but he was not among top-rankings regarding all types of aggregations and his ranking was sensitive. Similarly using our approach, but with three properties, Lin Freeman, Mullins and Wellman are frequently among top-ranking nodes.

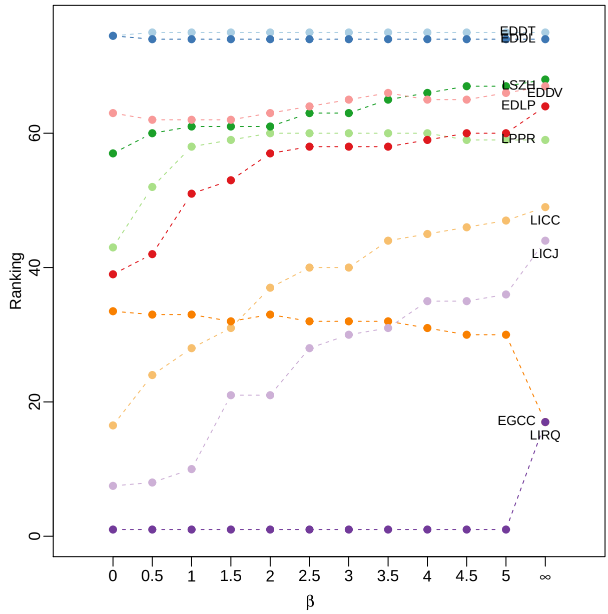


FIGURE A.3: Air transportation network. The detailed ranking of some nodes is depicted over the different aggregation strategies guided by the β parameter in the quantifier guided OWA. The values of the three criteria are listed in Table 3.3.

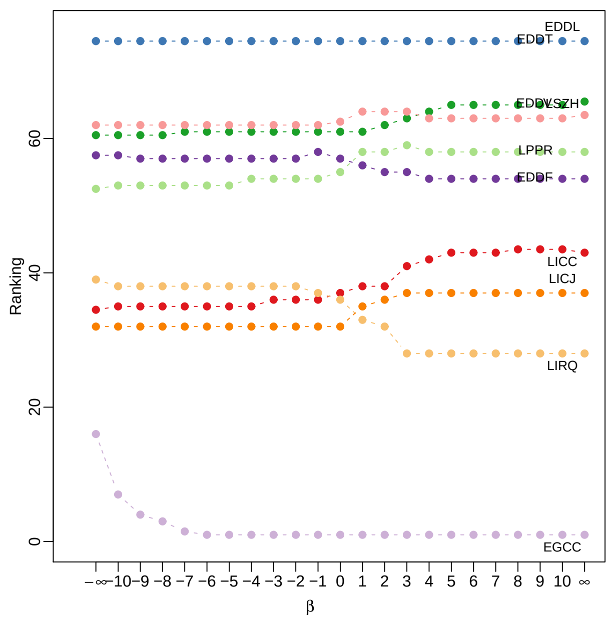


FIGURE A.4: Air transportation network. The detailed *scores* of the airports are depicted over the different aggregations guided by the β parameter in the MEOWA operator.

TABLE A.1: Freeman's EIES network. The reproduced results of the method proposed by Opsahl et al. using the different values of α parameter in Equation 2.1.

Rank	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 1.5$
32	Lin Freeman	Lin Freeman	Lin Freeman	Lin Freeman
31	Nick Mullins	Barry Wellman	Barry Wellman	Barry Wellman
30	Sue Freeman	Russ Bernard	Russ Bernard	Russ Bernard
29	Doug White	Sue Freeman	Doug White	Lee Sailer
28	Phipps Arabie	Doug White	Lee Sailer	Doug White
27	Barry Wellman	Nick Mullins	Sue Freeman	Pat Doreian
26	Russ Bernard	Pat Doreian	Pat Doreian	Sue Freeman
25	Ron Burt	Lee Sailer	Nick Mullins	Nick Mullins
24	Pat Doreian	Ron Burt	Ron Burt	Al Wolfe
23	Richard Alba	Richard Alba	Richard Alba	Maureen Hallinan
22	Jack Hunter	Steve Seidman	Steve Seidman	Ron Burt
21	Lee Sailer	Phipps Arabie	Al Wolfe	Richard Alba
20	Steve Seidman	Jack Hunter	Carol Barner-Barry	Steve Seidman
19	Carol Barner-Barry	Carol Barner-Barry	Jack Hunter	Carol Barner-Barry
18	Al Wolfe	Al Wolfe	Maureen Hallinan	Jack Hunter
17	Paul Holland	Paul Holland	Paul Holland	Davor Jedlicka
16	John Boyd	John Boyd	John Boyd	Paul Holland
15	Davor Jedlicka	Davor Jedlicka	Davor Jedlicka	John Boyd
14	Charles Kadushin	Maureen Hallinan	Phipps Arabie	Don Ploch
13	Nan Lin	Don Ploch	Don Ploch	Claude Fischer
12	Don Ploch	Mark Granovetter	Claude Fischer	Phipps Arabie
11	Claude Fischer	Charles Kadushin	Mark Granovetter	Joel Levine
10	Mark Granovetter	Nan Lin	Joel Levine	Mark Granovetter
9	Maureen Hallinan	Claude Fischer	Nick Poushinsky	Nick Poushinsky
8	Nick Poushinsky	Nick Poushinsky	Charles Kadushin	Charles Kadushin
7	Sam Leinhardt	Joel Levine	Nan Lin	Nan Lin
6	Joel Levine	John Sonquist	John Sonquist	Gary Coombs
5	John Sonquist	Sam Leinhardt	Sam Leinhardt	John Sonquist
4	Ev Rogers	Brian Foster	Gary Coombs	Sam Leinhardt
3	Brian Foster	Ev Rogers	Brian Foster	Brian Foster
2	Gary Coombs	Gary Coombs	Ev Rogers	Ev Rogers
1	Ed Laumann	Ed Laumann	Ed Laumann	Ed Laumann

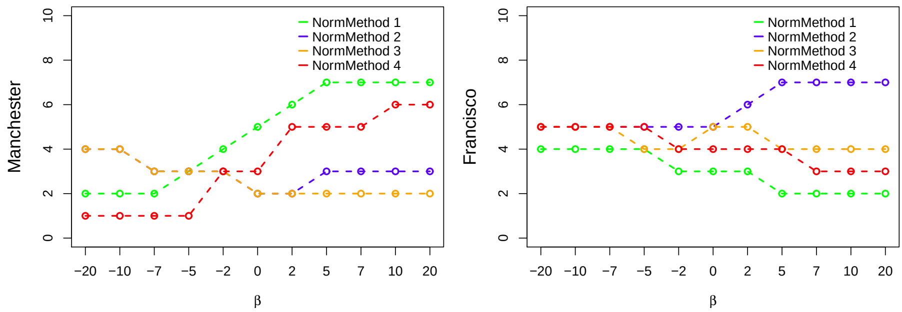


FIGURE A.5: Rankings of two airports chosen from nine nodes shared between the four layers representing AirBerlin, Easyjet, Lufthansa, and Ryanair. The resulting normalized values using the four methods are elaborated below.

A.2 Results obtained using the normalization methods in the European airlines network (including four layers)

In Chapter5 we discussed the European airline transportation network including four layer: AirBerlin, Easyjet, Ryanair, and Lufthansa. While only nine nodes are common among the four layers of airlines, a comparison of the behavior of all four layers is interesting because the Lufthansa airline network is much more centralized than the other networks. We compare the aggregation of the results of the four layers—including Lufthansa, which has a different normalized degree cumulative distribution—with the aggregation of the results of three layers after excluding Lufthansa from the scenario. Figure A.5 depicts the resulting rankings of two airports demonstrated by one curve for each normalization method and visualized against a set of β $\{-20,-10,-7,-5,-2,0,2,5,10,20\}$ values, each of which guides an aggregation.

The actual *degree*s in the four layers of the airlines for Manchester airport are: $(1,12,5,5)$, respectively, while the maximal *degree*s of all shared nodes are: $(26,17,5,28)$ and the maximal *degree*s of all nodes in the respective layers are computed as: $(37,67,78,85)$ (as listed in Table 5.1 (A)).

$$\mathcal{C}_1(v) : 0, 0.667, \boxed{1}, 0$$
$$\mathcal{C}_2(v) : 0, \tfrac{11}{66}, \tfrac{4}{77}, \tfrac{4}{84} \rightarrow 0, \boxed{0.167}, 0.052, 0.048$$
$$\mathcal{C}_3(v) : \mathcal{C}_2(v) \cdot (\tfrac{37}{85}, \tfrac{67}{85}, \tfrac{78}{85}, \tfrac{85}{85}) \rightarrow 0, \boxed{0.131}, 0.048, 0.048$$
$$\mathcal{C}_4(v) : 0.093, 0.818, \boxed{0.887}, 0.461$$

If we follow the curve of *NormMethod 1* along the different β-values, Manchester airport increases its importance from rank 2 (among the 9 common nodes) to rank 7. For the sake of clarity, recall that *NormMethod 1* normalizes the degree value with the maximal degrees of all common nodes in the same layer. Thus, in the Lufthansa layer, the corresponding node gets a normalized degree of 1 (denoted by a box) as it has the highest degree among the nine nodes within the Lufthansa layer. For $\beta = 20$ and $n = 4$, the weight vector in the MEOWA operator multiplies the highest normalized degree by (0.9933) and the second highest by (0.0067). Two other nodes exist with the same maximal normalized *degree* of 1 and a higher second-highest normalized degree than Manchester; thus, Manchester gets the rank 7 for $\beta = 20$ because its minimal normalized degree is very low and results in rank 2 when $\beta = -20$. Similarly, we compute the values of the different normalized degrees using the four methods for Francisco Sá Carneiro (Porto) airport.

Its pure degree values are: $(12, 5, 1, 15)$. As can be seen, using *NormMethod 4*, it gets a normalized index value of 0.833 in the AirBerlin layer, as it is connected to twelve airports, which is less than what we obtain for Manchester airport in the Lufthansa layer, namely 0.887 (marked with a box). When $\beta = 20$, the aggregation favors nodes with at least one high value; in this case Manchester gets a better rank than Francisco as the curve of *NormMethod 4* shows in Figure A.5.

$$\begin{aligned}
\mathcal{C}_1(v) &: \boxed{0.44}, 0.2, 0, 0.435\\
\mathcal{C}_2(v) &: \boxed{0.306}, 0.061, 0, 0.167\\
\mathcal{C}_3(v) &: 0.133, 0.048, 0, \boxed{0.167}\\
\mathcal{C}_4(v) &: \boxed{0.833}, 0.611, 0.184, 0.789
\end{aligned}$$

With respect to sensitivity, we observe that Manchester is quite sensitive to the chosen aggregation; if we fix a normalization method, then we can compute the sensitivity of its ranking to different kinds of aggregations. Once we have found all the sensitivity extents for Manchester, we see that $\Delta agg(Manchester) = 5$, while for Francisco, sensitivity is only $\Delta agg(Francisco) = 2$. Considering sensitivity to the choice of normalization, it turns out that Francisco at $\beta = 20$ shows a maximal difference in the ranking positions of $7 - 2 = 5$. Thus, Manchester and Francisco both result in a $\Delta norm$-value of 5.

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Publications

1. Tavassoli, S. and Zweig, K.A., Constructing social networks from semi-structured chat-log data. In *Advances in Social Networks Analysis and Mining (ASONAM)*, 2014 IEEE/ACM International Conference, pp.146-149, 17-20 Aug. 2014. doi: https://doi.org/10.1109/ASONAM.2014.6921575

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3. Tavassoli, S. and Zweig, K.A., Analyzing multiple rankings of influential nodes in multiplex networks. In *the proceedings of the 5th International Workshop on Complex Networks and their Applications*, pp. 135-146. Springer International Publishing, 2016. doi: https://doi.org/10.1007/978-3-319-50901-3_11

4. Tavassoli, S. and Zweig, K.A., Most central or least central? How much modeling decisions influence a node's centrality ranking in multiplex networks. In *the proceedings of the 3rd European Network Intelligence Conference*, pp. 25-32, 2016. doi: http://dx.doi.org/10.1109/ENIC.2016.012

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6. Tavassoli, S. and Zweig, K.A., Exploratory identification of influential nodes based on multiple aspects of centrality using fuzzy operators. To submit to a journal (in Preparation).