

Fatigue Lifetime Estimation based on Rainflow Counted Data using the Local Strain Approach

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Abstract

In the automotive industry both the local strain approach and rainflow counting are well known and approved tools in the numerical estimation of the lifetime of a new developed part especially in the automotive industry. This paper is devoted to the combination of both tools and a new algorithm is given that takes advantage of the inner structure of the most used damage parameters.

1 Lifetime estimation and mathematical tools

Whenever new parts are developed in the automotive industry one has to estimate the lifetime of these parts to verify the security of the whole car. This process has to include both physical tests and application of numerical tools. On one hand the results of calculations just based on the theory and algorithms which are known today are not reliable enough if they are used as the only test. On the other hand physical tests for any part in any stage of development are much too expensive (see [5]).

What is the typical reason for the failure of a part in a car? The parts are usually not destroyed by one large load, but by the accumulation of the

energy dissipated by many (typically several millions) hysteresis loops. It is assumed that the damage of a loop is rate-independent. This means the frequency of the oscillations is not taken into account. The damage induced by the individual loops is accumulated using Miner's rule. (see [9])

If during a test track drive one has measured the strain values at a point one uses the tool of rainflow counting (see [8]), which means classifying and counting of the corresponding hysteresis loops. The damage induced by these loops is used to estimate the lifetime (for a mathematical description see [3] and [1]).

But often one has no information about the local values, since to get real local values you have to built up a specimen put it into a car and drive it around a test track. The way out is to measure outer loadings the part is subjected to during a test track drive. These measurements can be used for any stage of development of the part, especially if one does shape optimization. Of course in this cases the method used to do the estimation of lifetime must not depend on data you need to measure on specimens first. Here the local strain approach is used.

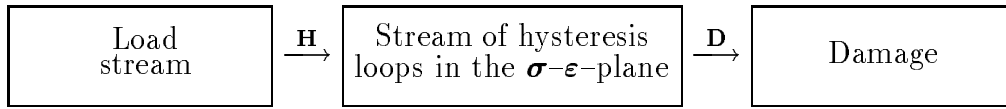
2 Operations in the local strain approach

The local strain approach is used as a mathematical tool to estimate the time to failure of a given part when applying a given unidirectional load. The necessary information are the local maxima and minima of the load. This leads to a string of numbers, we call the stream of turning points. (The mathematical reason for the restriction to this stream is that the damage as a functional on the loading is assumed to be rate-independent. For a mathematical description see [3].)

Using information about the geometry and the material this stream of extremal values is transformed into a stream of hysteresis loops in the stress-strain-plane at a point of interest (see [11]) . (We will call the operator transforming streams of turning points into the stream of hysteresis loops **H**.) These points are chosen as the points where the failure of the part is expected. One can find these points especially in notches. Here the stress and strain induced by the unidirectional load are one-dimensional hence we can speak about the stress-strain-plane and about hysteresis loops.

Having the loop stream a damage is evaluated using material properties, experimental data and a so-called damage parameter. This damage parameter induces a functional \mathbf{D} , that maps the stream of hysteresis loops into its damage value indicating how much the stream of loops has damaged the part. Usually a value of zero means not damaged at all and the value one indicates that the part is broken.

Schematically we get:



Because of the division into two independent steps (the operators \mathbf{H} and \mathbf{D}), it is obvious that the stream of hysteresis loops in the stress-strain-plane gives the whole information to estimate the damage.

The following questions arise:

- (1) Let a damage parameter be given (i.e. an operator \mathbf{D}). Which information about the ordering of the loops in the stream is necessary to estimate the damage?
- (2) Let us assume we do not know the stream of extremal load data but the information about the load is given via a rainflow count of the load. Which information about the stream of hysteresis loops can we gain?

3 Damage estimation using rainflow counted data

Let us at first consider the second question. What means rainflow counting the stream of loading values?

The first definition is due to Endo [8], we want to sketch the hysteresis type of approach. We follow the trajectory in the loading-strain-plane, find all closed hysteresis loops, delete them from the loading signal (i.e. replace the original signal with a signal leading to the same trajectory but the closed loop), classify the loops by the turning points and count the loops having similar

loading values, where the loop opens and turns around. The classification is done by subdividing the set of possible loading values into a finite number of intervals. We finish up in an integer matrix \mathbf{RFM} where the entries denote the number of loops and the part of the signal where no more loops occur which we name the residual \mathbf{RES} . (For a mathematical treatment see [3].)

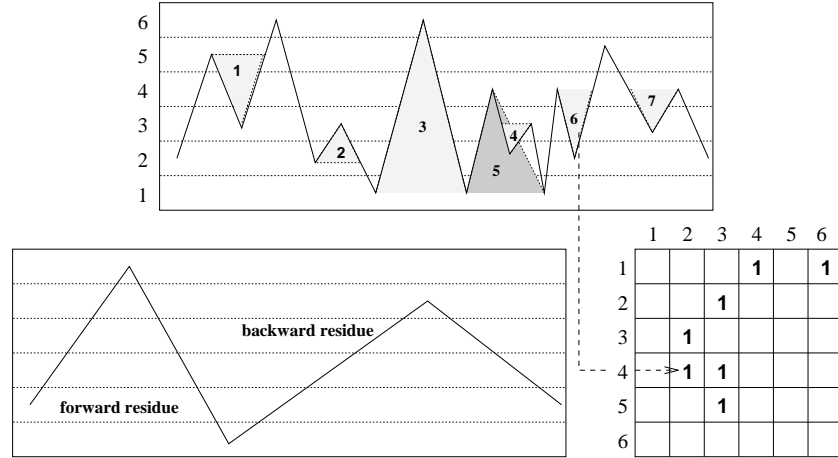


Figure 1: An example for the rainflow count. The loading space is subdivided into six parts. The loops are indicated by the numbered triangles. Each occurrence of a loop starting in class i and turning around in class j increases the entry of $\mathbf{RFM}(i,j)$ by one. E.g. loop 6 is counted in $\mathbf{RFM}(4,2)$. After the deletion of the loops we end up with the residue.

But why are we interested in the damage of a rainflow counted loading, since one needs the loading signal to calculate this count. Why don't we use the original data? Several reasons have to be considered:

- (1) The load can be rainflow counted on-line, i.e. during the drive on the test-track to reduce the arising data, such that much longer drives and much finer measurement can be applied.
- (2) There exist mathematical tools to manipulate the rainflow count, e.g. to extrapolate the data in a way such that the resulting rainflow count has the same distribution of hysteresis loops.

This gives a much better extrapolation of track data than simply to merge several load histories.

As well the rainflow data can be totally artificial to examine special situations.

- (3) We can manipulate the rainflow matrix to simulate special situations.
- (4) We can generate artificial rainflow matrices.

Observation: It is clear that all information on the original order with respect to time of the hysteresis loops (in the plane load vs. local strain) is lost, when the data is rainflow counted. We get a set of loops instead of a stream.

The problem is, that the counting is done on the loading stream. Hence we only can get information on the location of the loops with respect to the loading. We can not read the exact location of the local strains from the rainflow count.

For each class of hysteresis loops we can get the upper and lower loading value $\mathbf{L}_u, \mathbf{L}_l$ and by using formulae from geometry (see [11]) and material properties we can calculate the amplitude of strain $\boldsymbol{\varepsilon}_a$.

By the transformation from hysteresis loops in the loading-strain-plane into such in the stress-strain-plane, we also loose the knowledge on the exact location in the stress-strain-plane. We just have the amplitudes ($\boldsymbol{\sigma}_a$ and $\boldsymbol{\varepsilon}_a$).

Most of the damage parameters do also encounter for the location of the hysteresis loops in the stress-strain-plane. The reason is that the damage induced by a hysteresis loop of a given stress and strain amplitude is larger under tension than under pressure. The crucial value hereby is the mean stress ($\boldsymbol{\sigma}_m$). Therefore one can not speak of the damage of a rainflow count with respect to such an mean stress dependent damage parameter.

But it is possible to come back from a rainflow count to a stream of turning points.

A rainflow count consists of the tuple (RFM, RES), where RFM denotes the rainflow matrix with entries $\text{RFM}(i, j)$ giving the number of hysteresis loops from the level i to the level j and RES is the residual of the count.

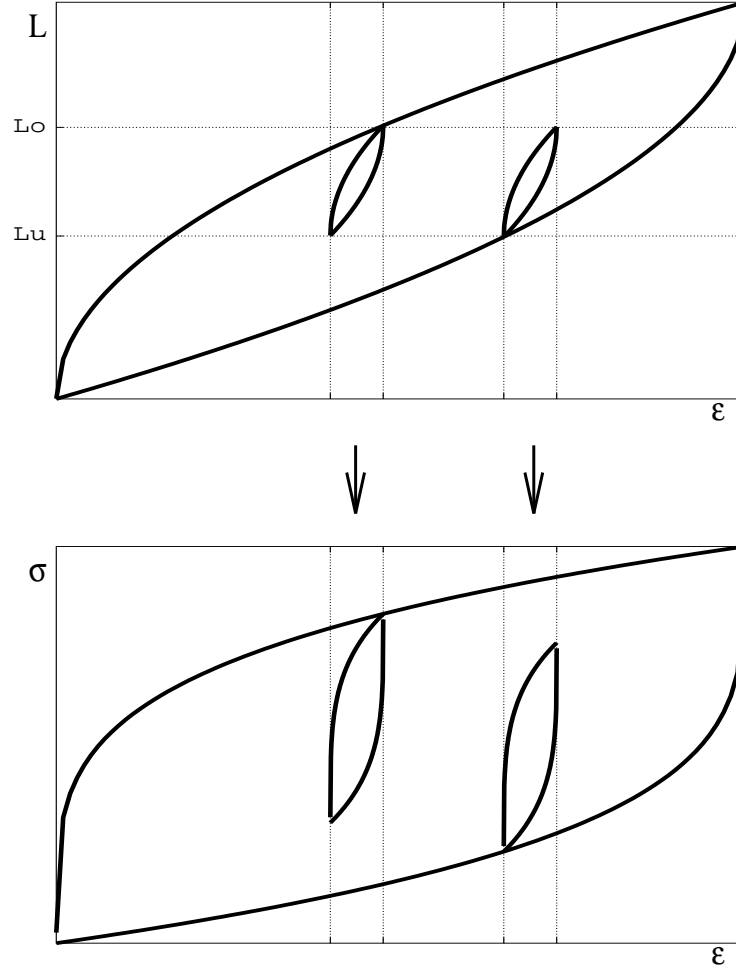


Figure 2: Let us assume that L_{min} and L_{max} are the extremal loading values. If a hysteresis loop is classified in the loading space to occur between L_u and L_o then its strain amplitude is fixed, but its location in the strain space can vary between the two indicated extremal locations. Since hysteresis loops can be nested it can be situated anywhere between, too. By the transformation into the stress-strain-plane the different strain locations lead to different mean stresses.

We consider the set of streams of loading turning points which lead to the given rainflow count (RFM, RES). If \mathbf{RC} denotes the rainflow count operator, we write formally

$$\mathcal{L} := \mathbf{RC}^{-1}(\text{RFM}, \text{RES})$$

For each stream $L \in \mathcal{L}$ we can calculate the damage $\mathbf{D}(\mathbf{H}(L))$. Hence we can canonically define the damage of \mathcal{L} and for (RFM, RES) by the expectation value of the damage of the stream of turning points in \mathcal{L} , where we assume that every stream has the same probability. The assumption makes sense, whenever we have not got any further information on the loading.

$$\mathbf{D}(\text{RFM}, \text{RES}) := \frac{1}{|\mathcal{L}|} \sum_{L \in \mathcal{L}} \mathbf{D}(\mathbf{H}(L))$$

If we use the tool of rainflow reconstruction on (RFM, RES) we get a “stochastically optimal” stream L (see [6]). This means that we get each $L \in \mathcal{L}$ with the same probability, to avoid a systematics. Of course one won’t score all streams in \mathcal{L} , hence one will use a sample mean-value as an estimate of the expectation value. One chooses $L_1, \dots, L_N \in \mathcal{L}$ and considers

$$\overline{\mathbf{D}(\text{RFM}, \text{RES})} = \frac{1}{N} \sum_{i=1}^N \mathbf{D}(\mathbf{H}(L_i))$$

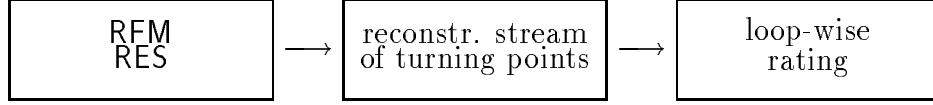
Remark: Because of the special structure of the reconstruction algorithm [6], which distributes all hysteresis loops stochastically and there usually are lots of hysteresis loops of the same kind, there is an intrinsic averaging even if we only reconstruct one stream of turning points. Therefore often one does only one reconstruction, but we can not exclude that the damage of a reconstructed stream is a exception value with respect to the set \mathcal{L} .

This method does not depend on \mathbf{H} or on \mathbf{D} , since we always construct a full stream of loading turning points, which is used as input to the usual local strain approach.

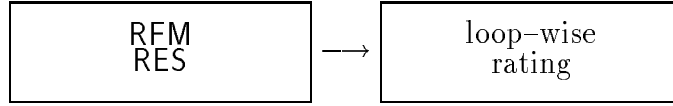
If we examine the most used damage parameters, we find additional properties not used by the latter procedure [9]:

- (i) The damage is linearly accumulated.
- (ii) The rating of the damage is done hysteresis loop by hysteresis loop.

Especially the latter point shows, that the full reconstruction of the stream is too much:



Since the information we use to construct the stream of turning points is completely contained in (RFM, RES), it has to be possible to do a damage evaluation without reconstructing a complete stream of turning points:



Using the latter assumptions (i.e. linear accumulation and loop-wise rating) we get the damage of the whole stream of turning point as follows:

Let the stream of turning points consist of the hysteresis loops $\{h_\tau | \tau = 1, \dots, T\}$, the total damage is evaluated as

$$\mathbf{D}_{tot} = \sum_{\tau=1}^T \mathbf{d}(h_\tau)$$

where $\mathbf{d}(h_\tau)$ denotes the damage induced by the hysteresis loop h_τ .

If we have a rainflow count instead, we can not any more speak of the total damage but of the expectation value of it. Again we use the individual damages of each loop. For each class of loops h_{ij} , we have $\text{RFM}(i, j)$ representatives which we name $h_{ij}^{(\nu)}$, $\nu = 1, \dots, \text{RFM}(i, j)$. We get the expectation value of the damage as:

$$\mathbf{D}(\text{RFM}, \text{RES}) = \mathcal{E} \left(\sum_{i,j=1}^{n_{Kl}} \sum_{\nu=1}^{\text{RFM}(i,j)} \mathbf{d}(h_{ij}^{(\nu)}) \right)$$

where n_{Kl} denotes the number of classes used for the rainflow count.

We assume the damage of the single hysteresis loops to be stochastically independent. Hence the expectation value coincides with the linear superposition of the single expectation values:

$$\begin{aligned}
 \mathbf{D}(\text{RFM}, \text{RES}) &= \sum_{i,j=1}^{n_{Kl}} \sum_{\nu=1}^{\text{RFM}(i,j)} \mathcal{E} \mathbf{d}(h_{ij}^{(\nu)}) \\
 &= \sum_{i,j=1}^{n_{Kl}} \text{RFM}(i,j) \mathcal{E} \mathbf{d}(h_{ij}) \\
 &=: \sum_{i,j=1}^{n_{Kl}} \text{RFM}(i,j) \mathcal{E} \mathbf{d}_{ij}
 \end{aligned}$$

Hence we need for the calculation of the damage of a rainflow count (RFM, RES) the expectation values of the damage of the different hysteresis loop types that occur in the matrix.

In the following we will see depending on the damage parameter whether and how we can calculate $\mathcal{E} \mathbf{d}_{ij}$.

4 Damage parameters with hysteresis memory — Example of the \mathbf{P}_{SWT} -parameter

In physical tests one can observe that a part which is under tension can not sustain as many hysteresis loops of a given stress-amplitude as the same part under tension. Hence damage parameters were introduced that deal with the influence of the mean stress. One of the most used is the so-called \mathbf{P}_{SWT} -parameter (see [7]), which is given by the formula

$$\mathbf{P}_{SWT} := \sqrt{(\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_m)^+ \boldsymbol{\varepsilon}_a \mathbf{E}}$$

where

- $\boldsymbol{\sigma}_a$: amplitude of stress
- $\boldsymbol{\varepsilon}_a$: amplitude of strain
- $\boldsymbol{\sigma}_m$: mean stress
- \mathbf{E} : modulus of elasticity
- $()^+ : :$ positive function branch

Observations:

- (1) We only need to know the set of hysteresis loops in the stress-strain-plane: we do not need to know the ordering of these loops.
- (2) We need to know the location of the hysteresis loops in the stress space (we need the mean stress σ_m).

The location of the hysteresis loops does not only depend on the given values of the loading but the history of the loading has an influence, too. If we use the reconstruction algorithm, we construct globally for all hysteresis loops one and only one ordering (history). The idea of the method of *hysteresis loop reconstruction* is to build up an individual history for each hysteresis loop.

To estimate $\mathcal{E}\mathbf{d}_{ij}$, we do the following

- (i) For each class of hysteresis loops from class i to j construct some histories.
- (ii) Calculate the location of the hysteresis loop in the stress space, depending on the constructed history.
- (iii) Rate the hysteresis loop using the \mathbf{P}_{SWT} -parameter.
- (iv) Build the mean of all damages corresponding to the histories constructed in (i).

The result is an estimate of $\mathcal{E}\mathbf{d}_{ij}$.

At first glance this procedure seems not to be able to compete with the reconstruction algorithm, since we construct a lot of histories instead of one respectively a few. The key point is that the histories here are local histories — they only need to reflect the hysteresis history at the point where the given hysteresis loop is situated.

Therefore we examine, what is the minimal information about the history to be able to calculate $\mathcal{E}\mathbf{d}_{ij}$.

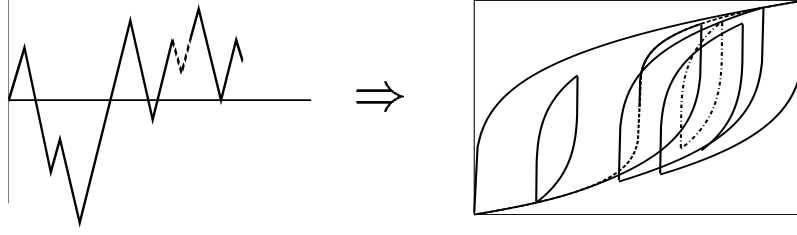


Figure 3: Example: We are interested in the dotted loop in the right picture. To this loop corresponds the dotted part of the time signal in the left half of the picture

Notation: We use the notation $[a, b]$ for the convex hull of a and b , i.e. the interval $[a, b]$ if $a \leq b$ and the interval $[b, a]$ else.

Observations:

- (1) Obviously the location of the hysteresis loop can not depend on the future: Let $(L_\tau)_{\tau=1}^\mu$ denote the loading stream and L_t the point where our given hysteresis loop ends its opening branch. Then the location of the hysteresis loop can only depend on $(L_\tau)_{\tau=1}^t$.

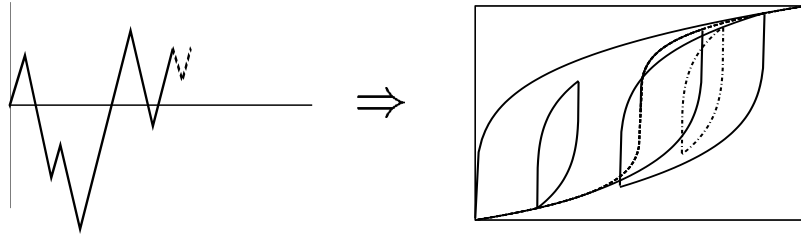


Figure 4: We truncate the future of the loop.

- (2) In $(L_\tau)_{\tau=1}^t$ we can apply the following deletion rules, such that the stream after applying one of the rules leads to the same location of the hysteresis loop:
 - (D1) We can delete L_i if $L_i \in [L_{i-1}, L_{i+1}]$ (monotonous deletion)

- (D2) We can delete L_i and L_{i+1} if $[L_i, L_{i+1}] \subset [L_{i-1}, L_{i+2}]$ (Madelung deletion)

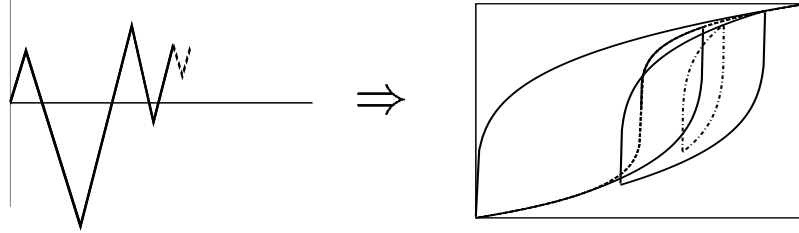


Figure 5: Madelung deletions correspond to the deletion of closed hysteresis loops.

- (D3) We can delete L_1 if $|L_1| < |L_2|$

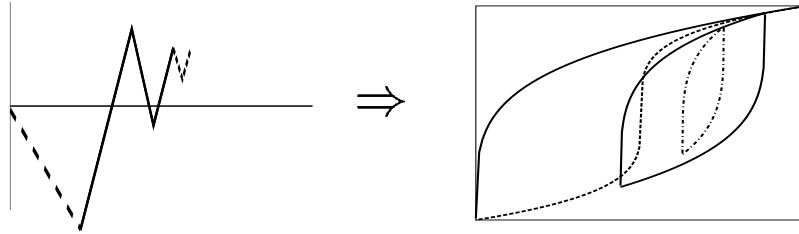


Figure 6: This last rule corresponds to the behavior of metals and alloys, that maxima in the absolute values delete all the memory up to this point (see [10]).

- (3) To any given stream of turning-points leading to a location of our hysteresis loop there exists one of minimal length leading to the same location. This minimal stream is of the form (L_1, \dots, L_n) , where

- (a) $|L_1| > |L_2|$
- (b) $L_i \notin [L_{i+1}, L_{i+2}]$ for $i < n - 2$

(it follows that $|L_i| > |L_{i+1}|$ for $i < n - 1$)

- (4) If the hysteresis loop we are looking for has its opening branch from class i to class j , then $L_{n-1} = i, L_n = j$, since we have truncated the future and hence the opening branch of our loop is the last part of the signal.

If we follow the trajectory in the loading–strain-plane induced by this minimal stream, we get just the nesting of the hysteresis loops around the hysteresis loop, we are interested in.

5 Construction of a minimal stream

We construct a minimal stream by building up the nesting of the loops from inside to outside. We assume that we have an asymmetric rainflow count, i.e. we can distinguish between hanging hysteresis loops, which are classified in $\text{RFM}(i, j)$ where $i > j$ and standing ones ($i < j$)

We have to verify that the probability distribution of our minimal streams corresponds to the inverse image of the rainflow count. Let us first consider the case to construct the whole stream. We know (see [6]) that the number of possibilities to insert a loop (i, j) into a stream where all larger loops have been randomly inserted is

$i > j$

$$\begin{aligned} P(i, j) &= \#\{\text{possible hysteresis loops which } h_{ij} \text{ can hang in}\} \\ &= \sum_{k=1}^{i-1} \sum_{\ell=j}^N \text{RFM}(k, \ell) + \sum_{k=i+1}^N \sum_{\ell=1}^j \text{RFM}(k, \ell) \\ &+ \#\{\nu \in \{1, \dots, M-1\} | \text{RES}_\nu \leq j, \text{RES}_{\nu+1} \geq i\} \end{aligned}$$

$i < j$

$$\begin{aligned} P(i, j) &= \#\{\text{possible hysteresis loops which } h_{ij} \text{ can stand in}\} \\ &= \sum_{k=1}^{i-1} \sum_{\ell=j}^N \text{RFM}(k, \ell) + \sum_{k=j+1}^N \sum_{\ell=1}^i \text{RFM}(k, \ell) \\ &+ \#\{\nu \in \{1, \dots, M-1\} | \text{RES}_\nu \geq j, \text{RES}_{\nu+1} \leq i\} \end{aligned}$$

Remark: We see that this number does not depend on how the larger loops have been inserted.

Therefore we can first choose randomly a position for our smaller loop. If this position is again a loop, we can choose the position of this larger loop

now independently. Hence we can do the same process like with the small loop. The only fixed positions are those in the residue. If we find a position in the residue we fix all the nested loops.

We can formulate the algorithm in pseudo-code:

Method: Construction of a nesting

given: RFM : *rainflow matrix*
 RES : *residue*
 (i,j) : *a hysteresis loop classified in RFM(i,j)*

 Goal: *randomly chosen nesting of hysteresis loops*
 where (i,j) is the innermost hysteresis loop.
 for any RFM(i,j) $\neq 0$ *(if equal zero we need no expectation*
 value of the damage of these hysteresis loops).
 nesting := (i,j)
 repeat
 Calculate the number $\mathbf{P}(i,j)$ of all possible hysteresis
 loops where (i,j) can be nested in.
 Choose randomly one of these positions.
 if *the position is a hysteresis loop (k,ℓ) in the rainflow*
 matrix **then**
 add (k,ℓ) to nesting
 $(i,j) := (k,\ell)$
 until *position is part of the residue.*

Since we used the probabilities $P(i,j)$ and using the fact that these probabilities do not depend on the outer loops we get the result:

Theorem 5.1 *The above algorithm constructs each possible nesting with the same probability.*

How to transform the nesting into a minimal stream?

We now start with the residue and add the largest loop. Since we do not need to know anything of the future of the innermost loop and the closure of the larger loops will occur in the future, we only need the resulting string of turning points until the turning point of the inserted largest loop.

Now we can apply deletion rule (*D3*) and delete all points before the maximal amplitude of the resulting string.

Clearly we have to add the nested loops from outside to inside. We have to distinguish, whether the loop is inserted in the first half of the outer loop or the closing part:

Lemma 5.2 *Hanging loops start in the opening branch of sitting loops and in the closing branch of hanging loops. The opposite holds for sitting loops.*

Proof:

The start of a loop corresponds to a turning point in the string, i.e. the direction has to change direction. Since the first half of a hanging loop is decreasing it must start on the increasing part of the outer loop. The same argument holds for sitting loops. \square

Again since we can neglect the future we need only the starting and turning point of the loop. If the loop is situated in the opening branch of the outer loop we can delete the turning point of the outer loop.

We can summarize

Method: Construction of the minimal stream

Insert outermost loop into residue

Delete everything after turning point of the loop

*Apply deletion rule (*D3*) until first point is point of maximal amplitude*

while *there are still loops to insert*

Insert loop using the latter lemma

If necessary delete turning point of outer loop

end while

Now we have just to construct the trajectory to this minimal stream and we get a sample value of \mathbf{d}_{ij} .

Remarks:

- (a) The complexity of the algorithm to estimate $\mathbf{D}(\text{RFM}, \text{RES})$ does only depend on the number of classes of the RFM and the sparsity of the matrix. It does not depend on the overall number of loops counted in the rainflow process. This implies that this method is especially useful for data of long drives or for extrapolated data.

With slight changes we can use it with matrices that represent just the distribution of the loops rather than real rainflow counts.

- (b) Since the individual $\mathcal{E}\mathbf{d}_{ij}$ are independent, we can calculate them in parallel and use modern computer architecture.
- (c) We see that we can calculate all the $\mathbf{P}(i, j)$ in advance.
- (d) If we use the global construction method, the expense that is necessary for the individual loop classes is fixed by the number of their occurrence. But we know that for the damage the large loops are much more important than the small loops which is just opposite to the usual distribution.

In our method we can handle the sample size for each class of loops individually and can give the larger loops their importance.

Hence we get an effective method to estimate $\mathcal{E}\mathbf{d}_{ij}$. Hereby we can estimate $\mathbf{D}(\text{RFM}, \text{RES})$. We can apply the algorithm to any damage parameter that uses besides the amplitudes of stress and strain only the position of the hysteresis loop with respect to the stress to evaluate the damage of this hysteresis loop.

6 Result

We see, that we can use the damage estimation based on rainflow counted data, if damage parameters with hysteresis memory are involved. The calculation of the total damage can be done independently and in parallel for each class of hysteresis loops. By the independence we can give higher importance to the larger loops and less importance (less computational effort) to the smaller ones as those do not contribute too much to the damage.

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