V. Hlaváč, R. Šára (Eds.), Computer analysis of images and patterns; Lecture Notes in Computer Science, Vol. 970, Springer, Berlin, pp. 230–237, 1995.

# Multiscale Texture Enhancement

Joachim Weickert[1]
Laboratory of Technomathematics, University of Kaiserslautern,
P.O. Box 3049, D-67653 Kaiserslautern, Germany.
Tel.: +49 631 205 4084. Fax: +49 631 205 3052.
E-mail: weickert@mathematik.uni–kl.de

**Abstract**

The ideas of texture analysis by means of the structure tensor are combined with the scale-space concept of anisotropic diffusion filtering. In contrast to many other nonlinear diffusion techniques, the proposed one uses a diffusion tensor instead of a scalar diffusivity. This allows true anisotropic behaviour. The preferred diffusion direction is determined according to the phase angle of the structure tensor. The diffusivity in this direction is increasing with the local coherence of the signal. This filter is constructed in such a way that it gives a mathematically well-founded scale-space representation of the original image. Experiments demonstrate its usefulness for the processing of interrupted one-dimensional structures such as fingerprint and fabric images.

## 1 Introduction

Textures play an important role in various pictures. In images depicting, e.g. fabrics or fingerprints, the texture consists of locally one-dimensional objects. If the image quality is not sufficient for a direct interpretation the problem appears to close interrupted lines.

The goal of the present paper is to contribute a multiscale method for this purpose. It is based on an anisotropic diffusion process whose activity is driven by a diffusion tensor. This diffusion tensor is adapted to the image in order to enhance coherent structures. To this end, we chose it as a function of a stable and reliable descriptor of local structure, the structure tensor.

The paper is organized as follows.

Section 2 gives a brief review of the idea of anisotropic diffusion filtering, and in Section 3 we recall the essential properties of structure tensor analysis. Equipped with this information, in Section 4 we construct an anisotropic process which enhances locally coherent structures. Section 5 relates the work to other nonlinear filters based on partial differential equations. In Section 6 we apply this multiscale process to fingerprint and fabric images. We conclude with a summary in Section 7.

[1]supported by a grant from "Stiftung Rheinland–Pfalz für Innovation"

## 2 Nonlinear Anisotropic Diffusion Filtering

Let the image domain be an open rectangle $\Omega := (0, a_1) \times (0, a_2)$, $\Gamma := \partial\Omega$ its boundary, and let an image $f(x)$ be represented by a bounded function $f : \Omega \to \mathbb{R}$. Then, one may obtain a processed version $u(x,t)$ of $f(x)$ with a scale parameter $t \geq 0$ as the solution of a diffusion equation with $f$ as initial condition and reflecting boundary conditions:

$$\partial_t u = \operatorname{div}(D\, \nabla u) \quad \text{on} \quad \Omega \times (0, \infty) \tag{1}$$
$$u(x,0) = f(x) \quad \text{on} \quad \Omega \tag{2}$$
$$\langle D\nabla u,\, n\rangle = 0 \quad \text{on} \quad \Gamma \times (0, \infty) \tag{3}$$

Hereby, $n$ denotes the outer normal and $\langle .,. \rangle$ the usual inner product.

If one wants to adapt the diffusion process to the image itself one should choose the symmetric positive definite diffusion tensor $D \in \mathbb{R}^{2\times 2}$ as a function of the local image structure. This may be useful to avoid undesirable effects such as blurring or dislocating of edges. A very simple descriptor of the image structure is the regularized gradient $\nabla u_\sigma$, where

$$K_\sigma(x) \;:=\; \frac{1}{2\pi\sigma^2} \cdot \exp\left(-\frac{|x|^2}{2\sigma^2}\right), \tag{4}$$
$$u_\sigma(x,t) \;:=\; (K_\sigma * \tilde{u}(.,t))\,(x) \qquad (\sigma > 0) \tag{5}$$

and $\tilde{u}$ denotes an extension of $u$ from $\Omega$ to $\mathbb{R}^2$, which may be obtained by mirroring at $\Gamma$. Regularizing by convolving with a Gaussian makes the edge detection insensitive to noise at scales smaller than $\sigma$. Within a flat region, we have $\nabla u_\sigma = 0$. At pronounced edges, $|\nabla u_\sigma|$ is large and $\nabla u_\sigma$ points in the normal direction of the edge.

We may adapt the orthonormal system of eigenvectors $v_1$, $v_2$ of $D$ to the direction of $\nabla u_\sigma$ by choosing $v_1 \parallel \nabla u_\sigma$ and $v_2 \perp \nabla u_\sigma$. The corresponding eigenvalues $\lambda_1$, $\lambda_2$ prescribe the amount of diffusion in the eigendirections $v_1$ and $v_2$, respectively.

Let us study one example: if one wishes to smooth within each region and aims to reduce diffusion across steep edges one may choose

$$\lambda_1 \;:=\; \begin{cases} 1 & \text{if } \nabla u_\sigma = 0, \\ 1 - \exp\left(\frac{-1}{|\nabla u_\sigma|^2}\right) & \text{else,} \end{cases} \tag{6}$$
$$\lambda_2 \;:=\; 1. \tag{7}$$

Models of this type are useful for denoising of highly degraded images, for visualizing quality-relevant features in computer aided quality control, for preprocessing medical images, and for postprocessing fluctuating numerical data [10, 12]. A theoretical scale-space interpretation of this and related methods is given by Weickert [11].

Nevertheless, since $\nabla u_\sigma$ serves only as an edge detector, the applicability of the previously discussed diffusion filter is restricted to smoothing with edge enhancement. In the sequel, we study an example of how diffusion filters can be modified for enhancing coherent structures.

## 3 The Structure Tensor

In order to identify features like corners or to measure local coherence of structures, we need more sophisticated structure-analysing methods taking into account how the gradient changes within the vicinity of any investigated point.

The *structure tensor* (also called *scatter matrix* or *(windowed second) moment tensor*) is an important representative of this class. It is a useful tool for analysing flow-like textures and spatio–temporal image sequences, see [9], [4, pp. 147–153], [6, pp. 349–382] and the references therein. Let us focus on some aspects which are of importance in our case.

To this end, we reconsider the vector-valued structure descriptor $\nabla u_\sigma$ within a matrix framework. The matrix $J_0$ resulting from the tensor product

$$J_0(\nabla u_\sigma) := \nabla u_\sigma \otimes \nabla u_\sigma := \nabla u_\sigma \nabla u_\sigma^T \tag{8}$$

has an orthonormal basis of eigenvectors $v_1$, $v_2$ with $v_1 \parallel \nabla u_\sigma$ and $v_2 \perp \nabla u_\sigma$. The corresponding eigenvalues $|\nabla u_\sigma|^2$ and 0 give just the contrast (the squared gradient) in the eigendirections. By convolving $J_0(\nabla u_\sigma)$ componentwise with a Gaussian $K_\rho$ we obtain the structure tensor

$$J_\rho(\nabla u_\sigma) := K_\rho * (\nabla u_\sigma \otimes \nabla u_\sigma) \qquad (\rho \geq 0). \tag{9}$$

It is not hard to verify that the symmetric matrix $J_\rho = \begin{pmatrix} j_{11} & j_{12} \\ j_{12} & j_{22} \end{pmatrix}$ is positive semidefinite and possesses orthonormal eigenvectors $w_1$, $w_2$ with

$$w_1 = \begin{pmatrix} \dfrac{2j_{12}}{\sqrt{\left(j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}\right)^2+4j_{12}^2}} \\ \dfrac{j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}}{\sqrt{\left(j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}\right)^2+4j_{12}^2}} \end{pmatrix} \tag{10}$$

if $j_{11} \neq j_{22}$ or $j_{12} \neq 0$. The corresponding eigenvalues are

$$\mu_{1,2} = \frac{1}{2}\left(j_{11}+j_{22} \pm \sqrt{(j_{11}-j_{22})^2 + 4j_{12}^2}\right), \tag{11}$$

where the + sign belongs to $\mu_1$. The eigenvalues integrate the variation of the grey values within a neighbourhood of size $O(\rho)$. They describe the average contrast in the eigendirections. Thus, the *integration scale* $\rho$ should reflect the characteristic size of the texture. The presmoothing for obtaining $\nabla u_\sigma$ makes the structure tensor insensitive to noise and irrelevant details at scales smaller than $O(\sigma)$. The parameter $\sigma$ is called *local scale.*

Constant areas are characterized by $\mu_1 = \mu_2 = 0$, while straight edges give $\mu_1 \gg \mu_2 = 0$, and corners yield $\mu_1 \geq \mu_2 \gg 0$. The difference

$$\mu_1 - \mu_2 = \sqrt{(j_{11}-j_{22})^2 + 4j_{12}^2} \tag{12}$$

becomes large for anisotropic structures. It measures the *coherence* within a window of scale $\rho$. It will play an important role for the construction of our novel diffusion filter.

# 4 Coherence-Enhancing Anisotropic Diffusion

To adapt the diffusion tensor $D$ to the local structure, we may prescribe that it should possess the same eigenvectors $w_1$, $w_2$ as the structure tensor $J_\rho(\nabla u_\sigma)$. The corresponding eigenvalues of $D$ are chosen as

$$\lambda_1 := \alpha \tag{13}$$

$$\lambda_2 := \begin{cases} \alpha & \text{if } j_{11}=j_{22} \text{ and } j_{12}=0, \\ \alpha + (1-\alpha)\exp\left(\frac{-1}{(j_{11}-j_{22})^2+4j_{12}^2}\right) & \text{else} \end{cases} \tag{14}$$

with some parameter $\alpha \in (0,1)$. This gives

$$D(J_\rho(\nabla u_\sigma)) = \begin{pmatrix} w_1 \mid w_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} w_1^T \\ w_2^T \end{pmatrix}. \tag{15}$$

Thus, our diffusion filter consists in solving the initial boundary value problem (1)–(3) with the preceding diffusion tensor.

We observe that $\lambda_2$ is an increasing function in $(\mu_1-\mu_2)^2$. Since the corresponding eigenvector $w_2$ points in the direction with the highest coherence (the lowest average contrast within an integration scale $\rho$) we have constructed a diffusion process acting preferently along coherent structures.

The exponential function and the positive parameter $\alpha > 0$ were introduced mainly for two theoretical reasons: First, the diffusion tensor is a smooth function in the whole image domain:

$$D \circ J_\rho : \quad [C^\infty(\Omega)]^2 \to [C^\infty(\Omega)]^{2\times 2} \qquad \forall \rho \geq 0.$$

The second reason is that the process never stops: Even if the structure becomes isotropic ($\mu_1 - \mu_2 \to 0$), we still have isotropic diffusion with diffusivity $\alpha > 0$. Thus, the diffusion tensor is uniformly positive definite.

Exploiting these properties, we may get a well-founded scale-space interpretation in a similar way as for the anisotropic filter class proposed in [11]. This multiscale representation simplifies the image with respect to many aspects: maxima decrease, minima increase, all $L^p$-norms ($1 \leq p \leq \infty$) decrease, even central moments are diminished, and the entropy increases. Moreover, the solution depends continuously on the original image. For $t \to \infty$, all images tend to a constant image with the same average grey value. Existence and uniqueness results for this problem can be obtained in a similar way as in [2].

# 5 Relation to Other Partial Differential Equations

Since the work of Perona and Malik [8] numerous nonlinear diffusion filters have been proposed. Nevertheless, most of them use a (spatially varying) scalar diffusivity, not a diffusion tensor. Thus, they act inhomogeneously (nonuniformly) on the image, but – in our terminology – they remain isotropic.

True anisotropic diffusion filtering is studied in the reaction-diffusion model of Cottet and Germain [3]. It uses the eigenvectors $v_1 \parallel \nabla u_\sigma$, $v_2 \perp \nabla u_\sigma$ and its eigenvalues are

$$\lambda_1 \;:=\; 0, \tag{16}$$

$$\lambda_2 \;:=\; \frac{\eta |\nabla u_\sigma|^2}{1 + (|\nabla u_\sigma|/\sigma)^2} \qquad (\eta > 0). \tag{17}$$

This choice is similar in spirit to our method, as it diffuses mainly along strongly anisotropic structures. However, there are two important differences:

First, we observe that this diffusion tensor does not fit into our scale-space framework using uniformly positive definite diffusion tensors. Due to its additional reaction term, the Cottet/Germain model is intended as a restoration method leading to nontrivial steady states. Second, the eigendirections of $D$ are adapted to $\nabla u_\sigma$, not to the eigendirections of the structure tensor. In Section 6, we shall see that the structure tensor can improve the measurement of local orientation.

The use of structure tensors for diffusion-like filters was proposed by Nitzberg and Shiota [7]. They apply the quadratic form induced by the structure tensor to determine the shape of their anisotropic Gaussian kernel to convolve the image with. For special scaling limits of the parameters, this can be interpreted as an anisotropic diffusion filter. In [11], it is shown how to derive anisotropic diffusion models from isotropic ones using the structure tensor approach. These and the Nitzberg/Shiota model combine isotropic smoothing inside a region with anisotropic diffusion along edges. They may enhance corners, but they are not constructed for enhancing coherent structures.

Other anisotropic partial differential equations for smoothing images rely on morphological methods such as the *mean-curvature motion (geometric heat equation)* [5, 1]

$$\partial_t u \;=\; u_{\xi\xi} \;=\; |\nabla u| \, \mathrm{curv}(u) \tag{18}$$

with $\xi$ being the direction perpendicular to $\nabla u$.

Since the mean-curvature motion propagates isophotes in inner normal direction with curvature-dependent speed, we should not expect such methods to be capable of closing interrupted line-like structures.

# 6 Examples

To approximate the proposed multiscale process, a splitting-based semi-implicit finite difference scheme was implemented. Its computational effort is linear with respect to the pixel number. The following calculations used as grid sizes $\Delta x = \Delta y = 1$, and the time step size is chosen to be $\Delta t = 2$. For $256 \times 256$ images, it takes approximately 5 CPU seconds per time step on an HP 712/80 workstation.

Figure 1(a) shows a fingerprint image of size $256 \times 256$. In order to investigate its local orientation we have calculated the gradient direction of the smoothed image (Fig. 1(b)). Horizontally oriented structures appear black, while vertical structures are depicted in white. We observe high fluctuations in the local orientation. However, when calculating the local orientation by use of the structure tensor, we investigate a much smoother behaviour (Fig. 1(c)). The proposed anisotropic diffusion filter based on this structure analysis is applied in Fig. 1(d). We recognize that this process is really capable of closing interrupted lines without destroying the relevant singularities in the image.

Figure 2 depicts the scale-space behaviour of coherence-enhancing anisotropic diffusion. The original image is taken from a fabric. In order to assess its quality one is interested in visualizing coherent fibre agglomerations (stripes) at different scales (see also [10]). The temporal behaviour of the proposed diffusion filter seems to fit this requirement fairly well. Due to the established scale-space properties, the image becomes gradually simpler with respect to many aspects, before it finally tends to its coarsest representation, a constant image with the same average grey value as the original one.

# 7 Summary and Conclusions

In the present paper, we have treated the problem of enhancing flow-like patterns. For such tasks, a reliable measurement of local orientation is needed. Our experiments demonstrate that the structure tensor satisfies this requirement. Unlike many other applications, we do not restrict it to pure image *analysis*, we use it also as a tool for *steering* a filtering process.

To this end, we evolve the original image by means of a nonlinear anisotropic diffusion equation. Its diffusion tensor reflects the local image structure by using the same set of eigenvectors as the structure tensor. The eigenvalues are chosen in such a way that diffusion acts mainly along the direction with the highest coherence, and becomes stronger when the coherence increases. The proposed filter gives a well-founded scale-space representation, which takes into account the demands to enhance coherent structures.

The considered problem class is a typical example for questions that can be exclusively solved by anisotropic techniques. Isotropic equations, either linear or nonlinear, are not suited for such applications.

**Acknowledgment.** The author thanks Andrea Bechtold for valuable hints.

# References

[1] Alvarez, L., Lions, P.-L., Morel, J.-M.: Image selective smoothing and edge detection by nonlinear diffusion. II. SIAM J. Numer. Anal. **29** (1992) 845–866

[2] Catté, F., Lions, P.-L., Morel, J.-M., Coll, T.: Image selective smoothing and edge detection by nonlinear diffusion. SIAM J. Numer. Anal. **29** (1992) 182–193

[3] Cottet, G.-H., Germain, L.: Image processing through reaction combined with nonlinear diffusion. Math. Comp. **61** (1993) 659–673

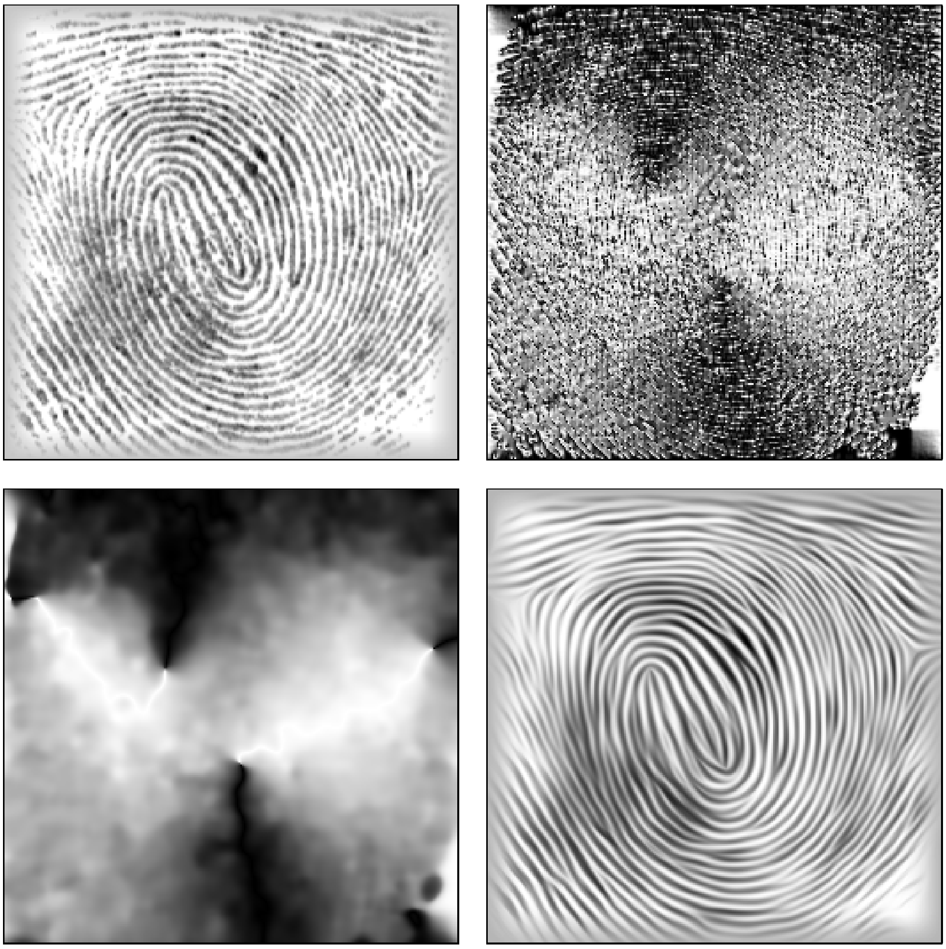

Figure 1: Local orientation in a fingerprint image. (a) **Top Left:** Original fingerprint, $\Omega = (0, 255)^2$. (b) **Top Right:** Orientation of smoothed gradient, $\sigma = 0.5$. (c) **Bottom Left:** Structure tensor orientation, $\sigma = 0.5$, $\rho = 4$. (d) **Bottom Right:** Coherence-enhancing anisotropic diffusion, $\alpha = 0.001$, $\sigma = 0.5$, $\rho = 4$, $t = 20$.

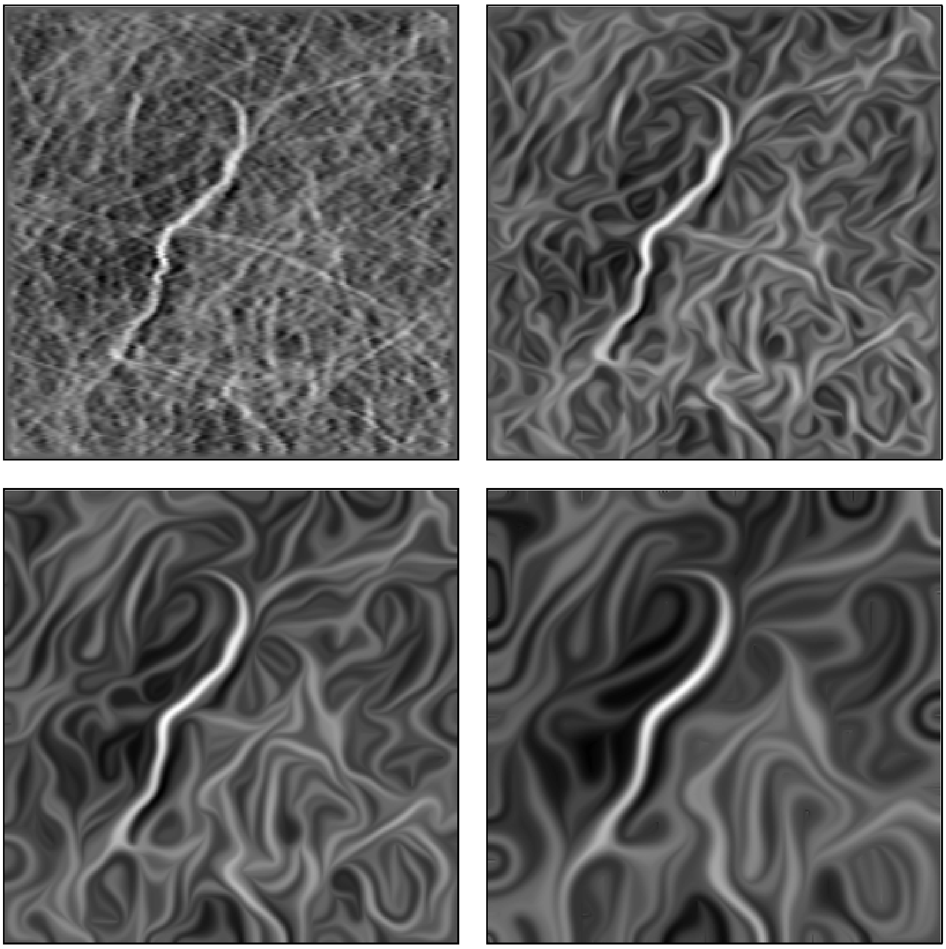

Figure 2: Scale-space behaviour of coherence-enhancing diffusion ($\alpha = 0.001$, $\sigma = 0.5$, $\rho = 2$). (a) **Top Left:** Original fabric image, $\Omega = (0, 256)^2$. (b) **Top Right:** $t = 20$. (c) **Bottom Left:** $t = 120$. (d) **Bottom Right:** $t = 640$.

[4] Jähne, B.: Spatio-temporal image processing. Lecture Notes in Comp. Science, Vol. 751, Springer, Berlin, 1993

[5] Kimia, B.B., Tannenbaum, A., Zucker, S.W.: Toward a computational theory of shape: An overview. O. Faugeras (Ed.), Proc. ECCV 90, Lecture Notes in Comp. Science, Vol. 427, Springer, Berlin, 402–407, 1990

[6] Lindeberg, T.: Scale-space theory in computer vision. Kluwer, Boston, 1994

[7] Nitzberg, M., Shiota, T.: Nonlinear image filtering with edge and corner enhancement. IEEE Trans. Pattern Anal. Mach. Intell. **14** (1992) 826–833

[8] Perona, P., Malik, J.: Scale space and edge detection using anisotropic diffusion. IEEE Trans. Pattern Anal. Mach. Intell. **12** (1990) 629–639

[9] Rao, A.R., Schunck, B.G.: Computing oriented texture fields. CVGIP: Graphical Models and Image Processing, **53** (1991) 157–185

[10] Weickert, J.: Anisotropic diffusion filters for image processing based quality control. In Fasano, A., Primicerio, M. (Eds.): Proc. Seventh European Conf. on Mathematics in Industry. Teubner, Stuttgart, 355–362, 1994

[11] Weickert, J.: Scale-space properties of nonlinear diffusion filtering with a diffusion tensor. Report No. 110. Laboratory of Technomathematics, University of Kaiserslautern, P.O. Box 3049, 67653 Kaiserslautern, Germany, 1994 (submitted)

[12] Weickert, J.: Theoretical foundations of anisotropic diffusion in image processing. Computing Supplement (in press)