THE SIMULATION OF THE CHARGE CYCLE
IN A CYLINDER OF A COMBUSTION ENGINE

H.-G. Stark, H. Trinkaus, Ch. Jansson

I. INTRODUCTION

The performance of a combustion engine is essentially determined by the charge cycle, i.e. by the inflow of fresh air through the inlet pipe into the cylinder after a combustion cycle. The amount of air, exchanged during this process, depends on many factors, e.g. the number of revolutions per minute, the temperature, the engine and valve geometry. In order to have a tool in designing the engine one is interested in calculating this amount.

The proper calculation would involve the solution of three-dimensional hydrodynamical equations governing the gas flow including chemical reactions in a complicated geometry, consisting of the cylinder, valves, inlet and outlet pipe. Since this is clearly too ambitious, we consider a simplified model, consisting of the following elements:

1. The working substance is ideal gas, whose thermodynamical properties remain the same before and after combustion (i.e. we do not discriminate between different types of gases).
2. In the cylinder mass balance and the first law of thermo dynamics yield a system consisting of two ordinary differen tial equations.
3. In the pipe system mass, momentum and energy balance lead to partial differential equations describing one dimensional isentropic gas flow.
4. Both equation systems are coupled together in a suitable manner.
5. Combustion is taken into account by empirical functions, describing heat and mass influx.

The method developed here, relies on a similar procedure in [1]. The most subtle question concerns the coupling condition mentioned above. In the following we formulate this condition and study it with a model consisting of a cylinder, inlet and outlet valves and an inlet pipe. We simulate pure charge cycles with this model, i.e. combustion is not yet taken into account. The generalization of this system to include combustion and exhaust pipes should be straightforward and remains to be done.

In the next sections we derive the model equations and describe the coupling condition. The resulting discretized system is solved by using an explicit Runge-Kutta scheme. Stability problems encountered with this method can perhaps be avoided by using implicit schemes, which will be investigated in the future.

2. THE MODEL

The model has the following structure (the inlet pipe is straight and of constant cross section):
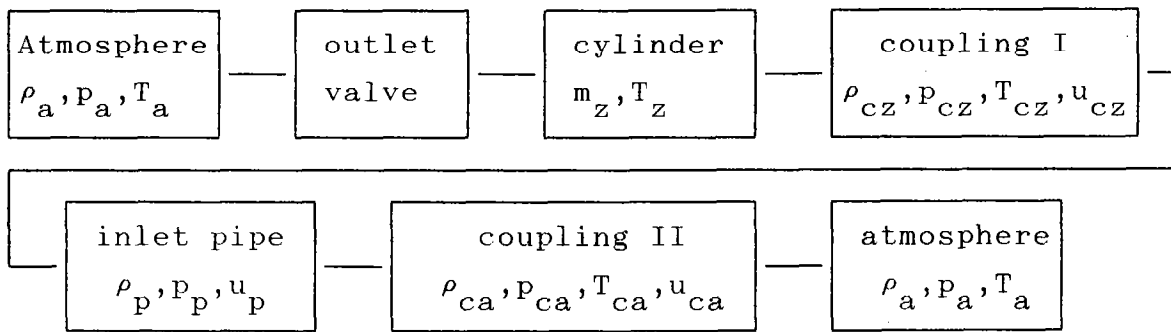

Fig. 1: Model structure

m_z and T_z denote mass and temperature in the cylinder; ρ, p and u density, pressure and velocity in the corresponding blocks (we remark that if one would include an exhaust pipe, the "out" block left to the cylinder would have the same structure as the corresponding "in" block to the right). The quantities $m_z(t)$, $T_z(t)$, $\rho_p(t,x)$, $p_p(t,x)$, $u_p(t,x)$ obey equations (7)-(11) below (the pipe quantities are assumed to be constant over the cross section, so they spatially depend only on the coordinate x along the pipe axis. x=0 corresponds to the boundary with the inlet valve, x=L to the boundary with the atmosphere). The dependent functions p_z (pressure in the cylinder) and T (temperature in the corresponding blocks) are given by the ideal gas law

$$p_z = R \cdot m_z \cdot T_z / V_z$$

or

$$T = p/(R \cdot \rho).$$

R denotes the ideal gas constant (normed on the unit mass). The cylinder volume $V_z(t)$ is known, when the angular velocity $\phi(t)$ of the crank shaft is given and serves together with the cylinder, valve and pipe geometry and the atmosphere values ρ_a, p_a and T_a as input to the model. In particular $A_{in}(t)$ and $A_{out}(t)$ denote the cross section areas of inlet and outlet valve respectively, open to gas exchange. The functions also are given, when $\phi(t)$ it given.

The coupling quantities are calculated from the corresponding pipe boundary values and cylinder or atmospherical values respectively as described in the next chapter.

Then the mass flow per unit of time through the inlet valve is given by (negative velocities correspond to inflow)

$$\dot{m}_{in} = -\rho_{cz} \cdot u_{cz} \cdot A_{in} . \tag{1}$$

With κ denoting the adiabatic coefficient of the gas we define $\psi(p_1, p_2)$ by

$$\psi(p_1, p_2) = \begin{cases} \left\{ \dfrac{\kappa}{\kappa-1} \left(\left(\dfrac{p_1}{p_2}\right)^{2/\kappa} - \left(\dfrac{p_1}{p_2}\right)^{\frac{\kappa+1}{\kappa}} \right) \right\}^{1/2} & \text{for } \dfrac{p_1}{p_2} \geqslant \left(\dfrac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \\[4mm] \left(\dfrac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}} \left(\dfrac{\kappa}{\kappa+1}\right)^{1/2} & \text{for } \dfrac{p_1}{p_2} \leq \left(\dfrac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \end{cases} \tag{2}$$

Then the mass flow through the outlet valve is obtained as (see e.g. [2]).

$$\dot{m}_{out} = \begin{cases} -A_{out} \sqrt{2\rho_z p_z} \; \psi(p_a, p_z) & \text{for } p_z \geqslant p_a \\[3mm] A_{out} \sqrt{2\rho_a p_a} \; \psi(p_z, p_a) & \text{else} \end{cases} \tag{3}$$

(the first line corresponds to outflow, the second to inflow through the outlet valve). Of course this expression has to be replaced by an expression of type (1), if one wants to consider an exhaust pipe also.

The mass flow through the inlet valve gives rise to the following enthalpy flow:

$$\dot{h}_{in} = \begin{cases} \dot{m}_{in} \cdot c_p \cdot T_z & \text{if } \dot{m}_{in} < 0 \\ \\ \dot{m}_{in}(c_p \cdot T_{cz} + \dfrac{u_{cz}^2}{2}) & \text{else.} \end{cases} \tag{4}$$

A similar consideration for the enthalpy flow through the outlet valve leads to

$$\dot{h}_{out} = \begin{cases} \dot{m}_{out} \cdot c_p \cdot T_z & \text{if } \dot{m}_{out} < 0 \\ \\ \dot{m}_{out}\left[c_p T_a + \left(\dfrac{\dot{m}_{out}}{A_{out}}\dfrac{V_z}{m_z}\right)^2 \Big/ 2 \right] & \text{else} \end{cases} \tag{5}$$

c_p denotes the specific heat capacity of the gas at constant pressure.

The heat losses through the cylinder wall (with the temperature T_w) empirically [3] are found to be of the form

$$\dot{q} = f(t) \, T_z^{-0.53} \, p_z^{0.8}(T_z - T_w) \tag{6}$$

(for the detailed form of f and its dependence on angular velocity refer to [3]).

Then the mass balance for the cylinder reads

$$\frac{dm_z}{dt} = \dot{m}_{in} + \dot{m}_{out} \tag{7}$$

The first law of thermodynamics (i.e. the balance of internal energy) for an ideal gas, applied to the cylinder, leads to

$$\frac{dT_z}{dt} = \frac{1}{m_z \, c_v} \left(-p_z \cdot \frac{dV_z}{dt} + \dot{h}_{in} + \dot{h}_{out} + \dot{q} - c_v T_z(\dot{m}_{in} + \dot{m}_{out})\right) \tag{8}$$

Note that through the coupling quantities in (1) and (4) the r.h.s. of equations (7) and (8) depend not only on m_z and T_z, but also on the boundary values $\rho_p(t,0)$, $p_p(t,0)$ and $u_p(t,0)$ of the pipe (see below)! c_v in (8) denotes the specific heat capacity at constant volume.

To obtain the equations for the gas flow in the pipe, one can set up the mass balance on a resting control volume and

momentum and energy balances, both performed on a moving control volume. We assume no heat transfer through the pipe wall (adiabatic flow). If one accounts for frictional effects by introducing a frictional force in the momentum balance, proportional to the square of the velocity, and if one includes frictional work in the energy balance, one obtains after some manipulations the following equations, describing adiabatic gas flow in the pipe:

$$\frac{\partial \rho_p}{\partial t} = -u_p \frac{\partial \rho_p}{\partial x} - \rho_p \frac{\partial u_p}{\partial x} \tag{9}$$

$$\frac{\partial u_p}{\partial t} = -\frac{1}{\rho_p}(\frac{\partial p_p}{\partial x} + \alpha u_p^2) - u_p \frac{\partial u_p}{\partial x} \tag{10}$$

$$\frac{\partial p_p}{\partial t} = -\kappa p_p \frac{\partial u_p}{\partial x} - u_p \frac{\partial p_p}{\partial x} \tag{11}$$

α is some empirical friction constant, remember that κ denotes the adiabatic coefficient.

3. THE COUPLING BLOCKS

We motivate and describe the calculation of the coupling values only for block I in fig. 1 with the cylinder values $m_z(t)$, $V_z(t)$ and the pipe boundary values $\rho_b(t) \equiv \rho_p(t,0)$ (p_b, u_b analogously defined) as inputs. To simplify writing, the explicit t-dependence is omitted from now on. The same analysis applies for block II with cylinder entities replaced by the atmospheric values and boundary values taken at x=L.

Consider equations (9)-(11) in the stationary and frictionless case:

$$u\rho_x + \rho u_x = 0 \tag{12}$$

$$uu_x + \frac{p_x}{\rho} = 0 \tag{13}$$

$$\kappa p u_x + u p_x = 0 \tag{14}$$

Then one can verify directly that $p\rho^{-\kappa}$ and $\frac{\kappa}{\kappa-1} \frac{p}{\rho} + \frac{u^2}{2}$ are constant. The latter of these quantities we shall write in a

more convenient way as $c_p T + \frac{u^2}{2}$ (using that for ideal gas $c_p - c_v = R$, $c_p/c_v = \kappa$ and therefore $p/\rho = (c_p - c_v)T$). In computing the coupling values we shall use these conservation laws, thus assuming stationary, frictionless, adiabatic flow from the cylinder or the pipe respectively to the coupling block, as follows.

First we note that by the ideal gas law conservation of $p\rho^{-\kappa}$ implies conservation of $p^{\frac{1-\kappa}{\kappa}} T$.

For $p_z \geq p_b$ we assume gas flow from the cylinder to the pipe and for the pressure p_{cz} in coupling I (see fig. 1) we set [1]

$$p_{cz} = p_b \tag{15}$$

Conservation of $p^{\frac{1-\kappa}{\kappa}} T$ implies

$$T_{cz} = T_z \left(\frac{p_b}{p_z}\right)^{\frac{\kappa-1}{\kappa}} . \tag{16}$$

Since the gas starts with zero velocity in the cylinder, conservation of $c_p T + \frac{u^2}{2}$ together with the restriction to the subsonic region leads to

$$u_{cz} = \min(\sqrt{2c_p(T_z - T_{cz})}, \ \sqrt{\kappa R T_{cz}}) \tag{17}$$

($\sqrt{\kappa R T_{cz}}$ is the sonic velocity in the coupling block). The ideal gas law then gives

$$\rho_{cz} = \frac{p_{cz}}{R T_{cz}} \tag{18}$$

for the coupling density.

If $p_z \leq p_b$ we put [1]

$$p_{cz} = p_z \tag{19}$$

$$u_{cz} = u_b . \tag{20}$$

Conservation of $p^{\frac{1-\kappa}{\kappa}} T$ leads to

$$T_{cz} = T_b \left(\frac{p_z}{p_b}\right)^{\frac{\kappa-1}{\kappa}} \tag{21}$$

and ρ_{cz} is given as in (18).

As mentioned above, formulae (15)-(21), with the boundary values taken at x=L and the cylinder quantities replaced by the atmospherical values apply also to the coupling II block in fig. 1.

At coupling I we in addition have to account for closed valves (i.e. $A_{in} = 0$), which we do by

$$u_{cz} = 0 \tag{22}$$

From conservation of $c_p T + \frac{u^2}{2}$ then follows

$$T_{cz} = T_b + \frac{u_b^2}{2c_p} \tag{23}$$

The conservation of $p^{\frac{1-\kappa}{\kappa}} T$ gives

$$P_{cz} = P_b \left(\frac{T_b}{T_{cz}}\right)^{\frac{\kappa}{1-\kappa}} \tag{24}$$

and ρ_{cz} again is calculated from the ideal gas law.

As we shall see in the next section, in a spatially discretized version of (9)-(11) the coupling values determine the increments of the pipe values in the boundary cells, thus "driving" the pipe by cylinder and atmosphere respectively. At coupling I they serve a twofold purpose, since moreover they determine the "input" to the cylinder equations (7)-(8).

4. DISCRETIZATION

We divide the pipe in N cells of length Δx such that $N\Delta x=L$. The procedure (see [1]) is explained for the density, the other pipe quantities are treated analogously. We assume the density to be constant in the i-th cell and thus obtain the vector $\rho_{pi}(t)$ (i=0,...,N+1) with $\rho_{po}(t) := \rho_{cz}(t)$ (see fig. 1) and $\rho_{p\,N+1}(t) := \rho_{ca}(t)$. i=1 denotes the cell at the boundary

with coupling I, i=N at the boundary with coupling II (see
fig. 1) and the coupling quantities are obtained from those at
i=1 and the cylinder values and from those at i=N and the
atmospheric values respectively as described in the last sec-
tion. Following [1] we approximate the spatial derivatives by

$$\Delta_x \rho_{pi}(t) = \frac{\rho_{p\ i+1}(t) - \rho_{p\ i-1}(t)}{2\Delta x} \qquad (i=2,\ldots,N-1)$$

$$\Delta_x \rho_{p1}(t) = \frac{(\rho_{p1}(t) + \rho_{p2}(t))/2 - \rho_{po}(t)}{\Delta x} \qquad (25)$$

$$\Delta_x \rho_{pN}(t) = \frac{\rho_{p\ N+1}(t) - (\rho_{pN}(t) + \rho_{p\ N-1}(t))/2}{\Delta x}$$

With these definitions we obtain a system of differential
equations in time for $m_z(t)$, $T_z(t)$, $\rho_{pi}(t)$, $p_{pi}(t)$, $u_{pi}(t)$
$(i=1,\ldots,N)$, consisting of equations (7), (8) and the
spatially discretized version of (9)-(11), i.e.

$$\frac{d\rho_{pi}}{dt} = -u_{pi}\,\Delta_x\rho_{pi} - \rho_{pi}\,\Delta_x u_{pi} \qquad (26)$$

$$\frac{du_{pi}}{dt} = -1/\rho_{pi}(\Delta_x p_{pi} + \alpha u^2_{pi}) - u_{pi}\Delta_x u_{pi} \qquad (27)$$

$$\frac{dp_{pi}}{dt} = -\kappa p_{pi}\Delta_x u_{pi} - u_{pi}\Delta_x p_{pi} \qquad (28)$$

To solve this system, a 4th order Runge-Kutta-scheme is
employed (see [1]). We collect the cylinder quantities $m_z(t)$,
$T_z(t)$ to a vector CYL(j,t) (j denotes the order of the Runge-
Kutta step, see below) and the pipe quantities $\rho_{pi}(t)$, $p_{pi}(t)$,
$u_{pi}(t)$ to PIPE(i,j,t) $(i=1,\ldots,N)$. PIPE(0,j,t) is calculated
from CYL(j,t) and PIPE(1,j,t) as described in the last sec-
tion, analogously PIPE(N+1,j,t) is obtained from the
atmospheric values ρ_a, p_a, T_a and PIPE(N,j,t).
With Δt denoting the time step size, the time discretized
version of (7) and (8) determining the increments ΔCYL(j,t)
may be written as

$$\Delta CYL(j,t) = f_{CYL}(CYL(j,t),PIPE(0,j,t),t)\Delta t \qquad (29)$$

and analogously we collect (26)-(28) to

$$\Delta PIPE(j,t) = f_{PIPE}(PIPE(0,j,t),PIPE(1,j,t),\ldots,$$

$$PIPE(N,j,t),PIPE(N+1,j,t)). \qquad (30)$$

Of course all geometrical and thermodynamical properties as $V_z(t)$, $A_{in}(t)$, $A_{out}(t)$, n (number of revolutions per time unit), R, c_p, c_v,... enter the functions f_{cyl} and f_{pipe} respectively (thus leading to the explicit time dependence in (29)).

Then the Runge-Kutta-scheme leads to the following diagram for M time steps:
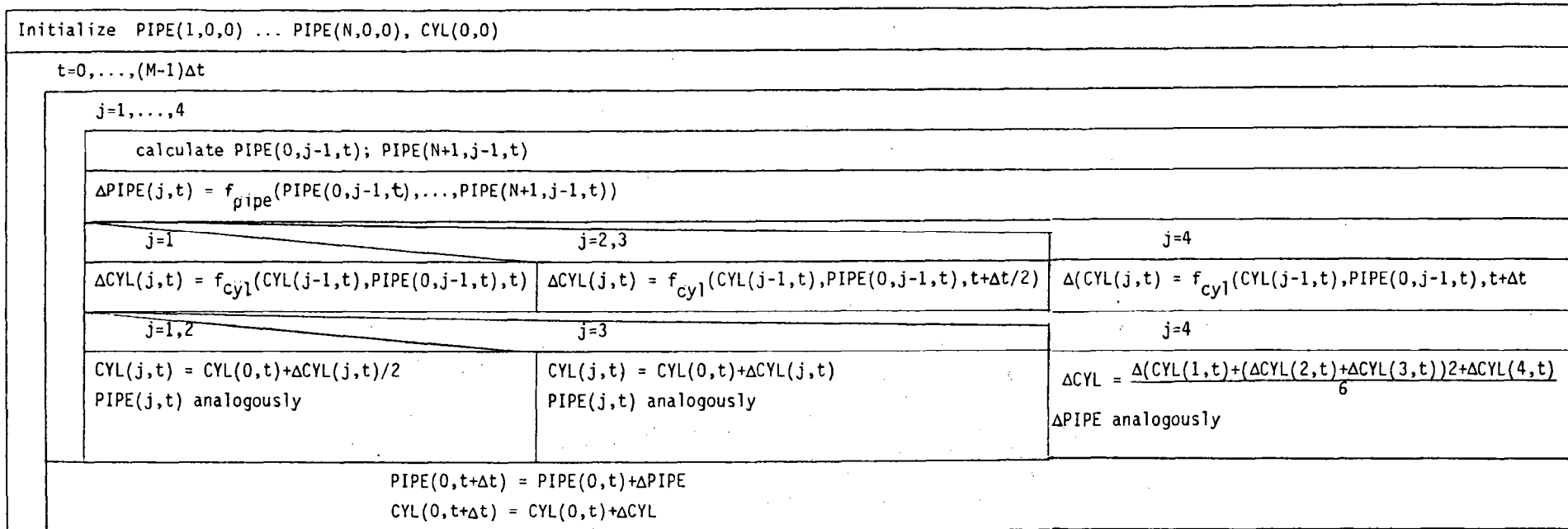
Initialize PIPE(1,0,0) ... PIPE(N,0,0), CYL(0,0)

t=0,...,(M-1)Δt

    j=1,...,4

        calculate PIPE(0,j-1,t); PIPE(N+1,j-1,t)

$\Delta PIPE(j,t) = f_{pipe}(PIPE(0,j-1,t),...,PIPE(N+1,j-1,t))$

j=1	j=2,3	j=4
$\Delta CYL(j,t) = f_{cyl}(CYL(j-1,t),PIPE(0,j-1,t),t)$	$\Delta CYL(j,t) = f_{cyl}(CYL(j-1,t),PIPE(0,j-1,t),t+\Delta t/2)$	$\Delta(CYL(j,t) = f_{cyl}(CYL(j-1,t),PIPE(0,j-1,t),t+\Delta t$

j=1,2	j=3	j=4
$CYL(j,t) = CYL(0,t)+\Delta CYL(j,t)/2$ PIPE(j,t) analogously	$CYL(j,t) = CYL(0,t)+\Delta CYL(j,t)$ PIPE(j,t) analogously	$\Delta CYL = \dfrac{\Delta(CYL(1,t)+(\Delta CYL(2,t)+\Delta CYL(3,t))2+\Delta CYL(4,t)}{6}$ ΔPIPE analogously

$$PIPE(0,t+\Delta t) = PIPE(0,t)+\Delta PIPE$$
$$CYL(0,t+\Delta t) = CYL(0,t)+\Delta CYL$$

Fig. 2: Runge-Kutta-scheme

In an obvious manner we have sometimes omitted the cell index i (running from 1 to n) in the above figure. Stability considerations [1] lead to the following restriction on Δt and Δx:

$$a \Delta t / \Delta x \overset{!}{\leq} 1 \tag{30}$$

with a denoting the sonic velocity.

5. EXAMPLE AND OUTLOOK

We show the results of some charge cycle simulations. The engine geometry was based on reasonable assumptions, in particular the pipe length was 1m and the areas $A_{in}(t)$, $A_{out}(t)$ were based on real life valve data. We assumed a constant angular velocity of the crank shaft with (n denotes the number of revolutions per second) $n=17$ s^{-1} ($\hat{=}$ 1020 min^{-1}) and replaced the time t by the crank shaft angle Φ in the following plots. The inlet valve starts opening at -30 and 690, respectively, (in the plots the corresponding points are denoted with I.O.) and is completely closed at 240 (denoted with I.C.). Analogously the output valve is completely closed at 60 (O.C.) and starts opening at 510 (O.O).
For figure (3)-(10) we took N=80 for the cell number and $\Delta \Phi = 0.003$. The plots show the cylinder volume, the amount of mass in the cylinder, density, temperature and pressure in the cylinder and in the pipe and the gas velocity in the pipe. The pipe values were taken at five equidistant points

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              (1)      (2)      (3)     (4)      (5)
               .        .        .       .        .
cylinder |======================================| atmosphere
          |_____|
                       pipe
```

The curves ——— belong to point 1 and analogously —*—*— to 2, —o—o— to 3, —+—+— to 4, ——— to 5.
The same applies for figures (11)-(18) taken with N=10 and $\Delta \Phi = 0.05$. The qualitative features of the pictures seem to be satisfying, in particular note the occurrence of oscillations in the pipe after closing the valves and the propagation of excitations, coming from the movement of the piston or reflections at the pipe ends, through the pipe. The occurrence

of small oscillations, superimposed on the "physical" oscillations in the pipe indicates the presence of stability problems, which are of course worse in the second set (note the pipe density) due to the chosen step widths in length and time. Since these stability problems occur in the region of closed valves, where the pipe is "decoupled" from the cylinder, one maybe can separate the procedure and use a more effective scheme for solving the Euler equations there. Another possibility would be, to replace the Runge-Kutta-scheme by an implicit scheme.

We do not claim that the procedure described here is new. Basically the same ideas are present in [1]. But we were unable to decide on the detailed way of coupling cylinder and pipe together in [1], based on the information given there.

References

[1]   P.A. Lakshminarayanan, P.A. Janakiraman, M.K. Gajendra Babu, B.S. Murthy: Prediction of Gas Exchange Processes in a Single Cylinder Internal Combustion Engine; SAE paper 790359, 1979

[2]   Becker: Technische Thermodynamik, Teubner, Stuttgart 1985

      Dubbel: Taschenbuch für den Maschinenbau, 14. ed. Springer, Berlin, Heidelberg, New York, 1981

[3]   G. Woschni: MTZ 31 (1970) 12, pp. 491-499

      G. Woschni, J. Fieger: MTZ 42 (1981) 6, pp. 229-234
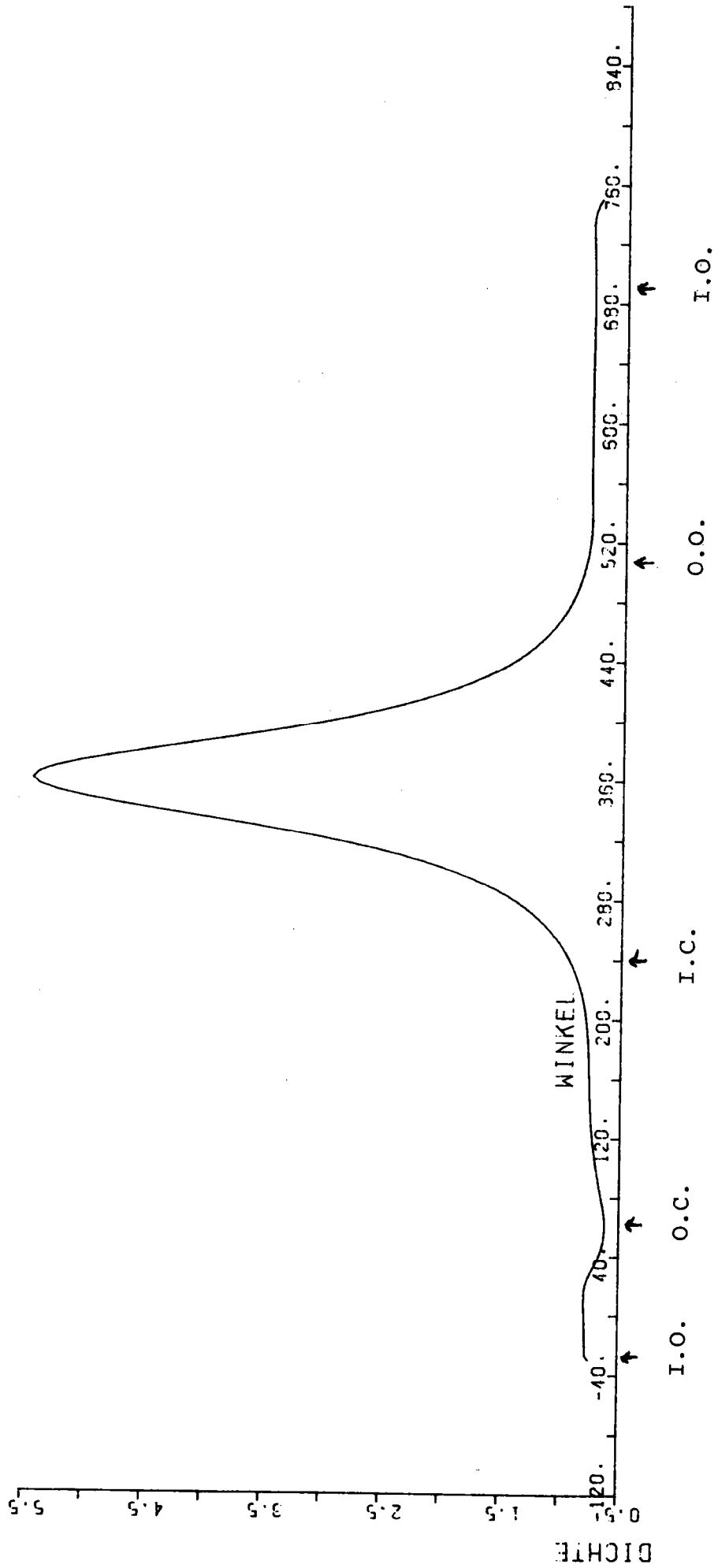
fig. 3

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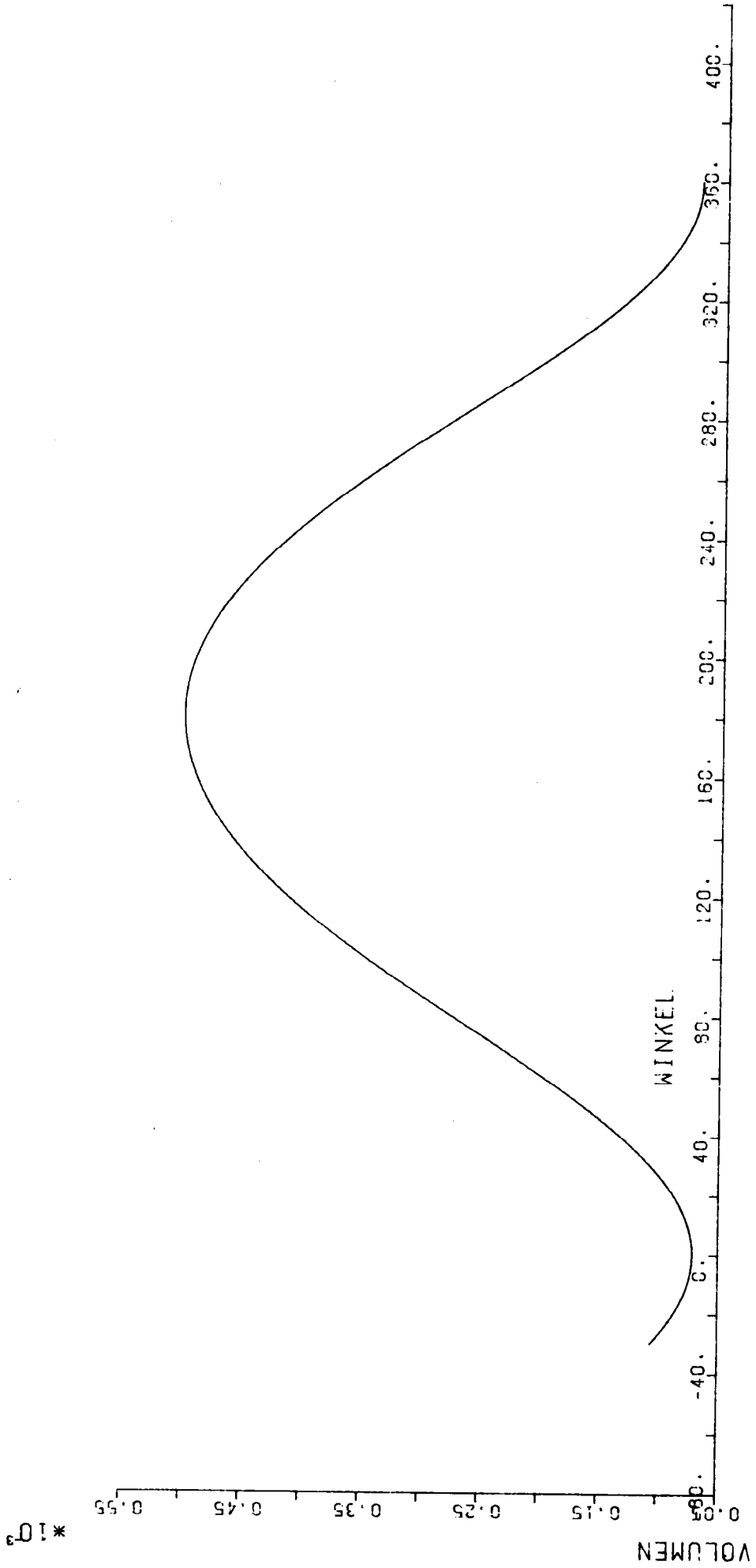
fig. 4

ZYLINDERWERTE-VERSION 12

*10³
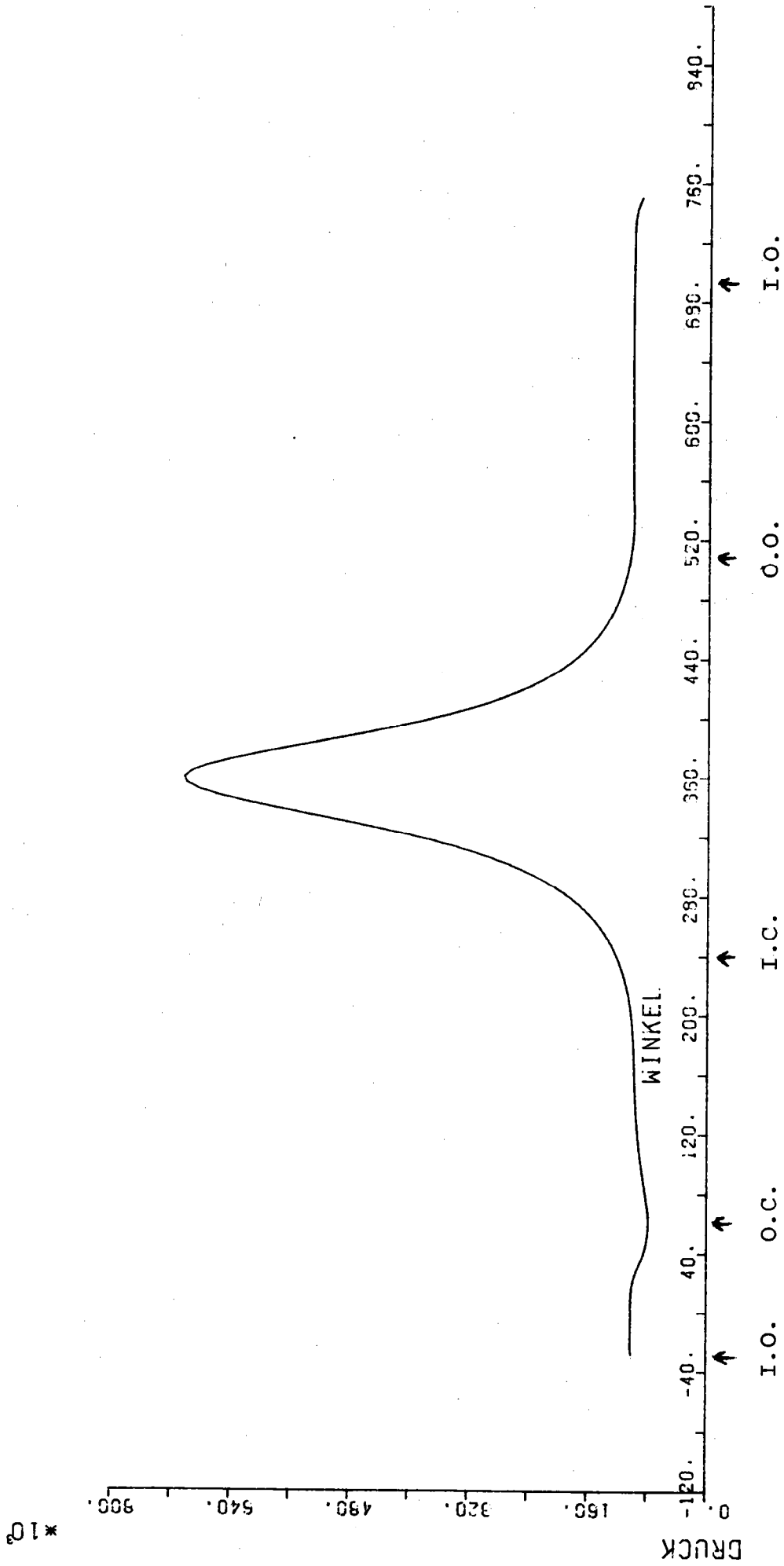
MASSE

0.44 0.35 0.29 0.2 0.12 0.04

-40. 0. 40. 80. 120. 160. 200. 240. 280. 320. 360. 400.

WINKEL

I.O.   O.C.   I.C.
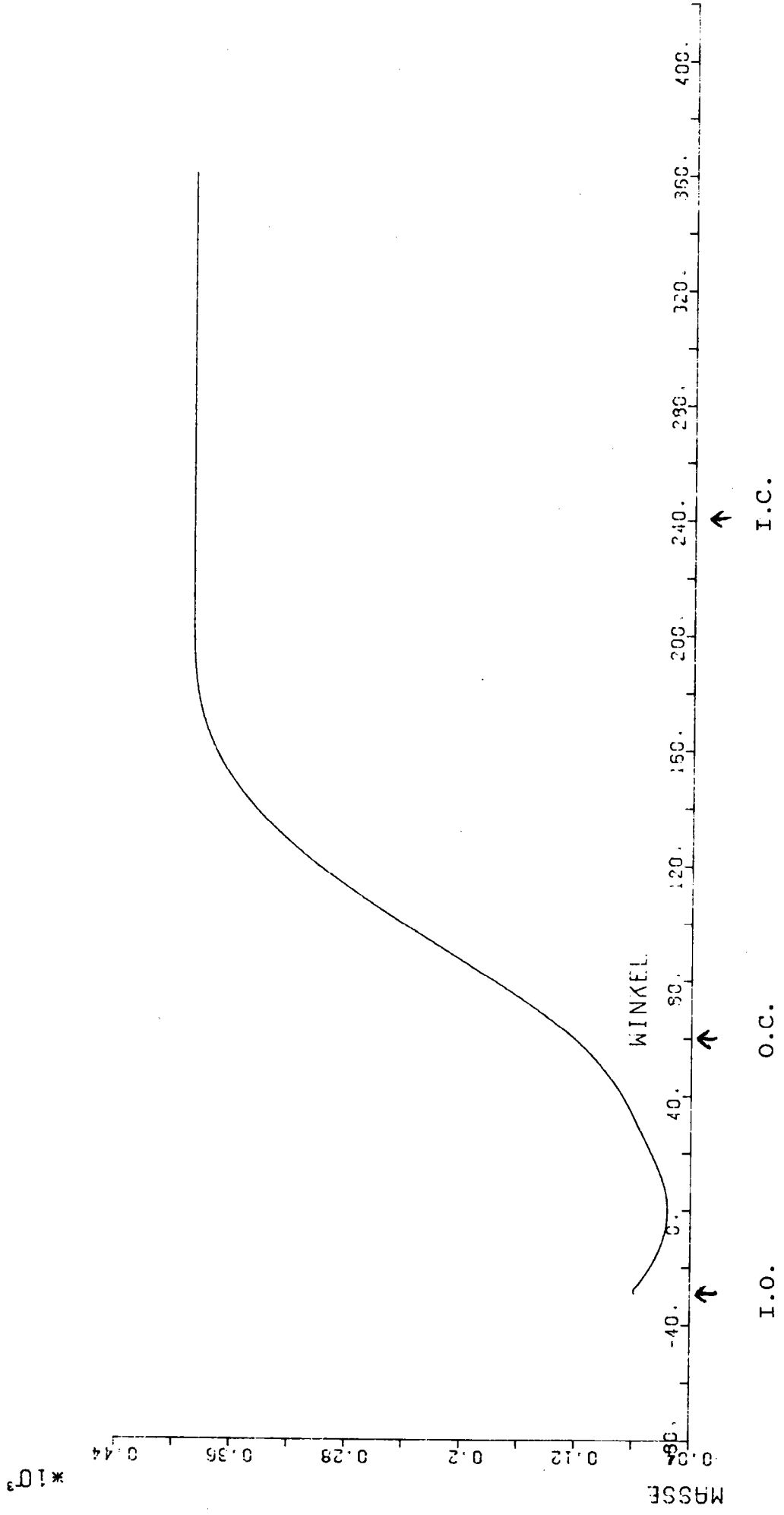
fig. 5
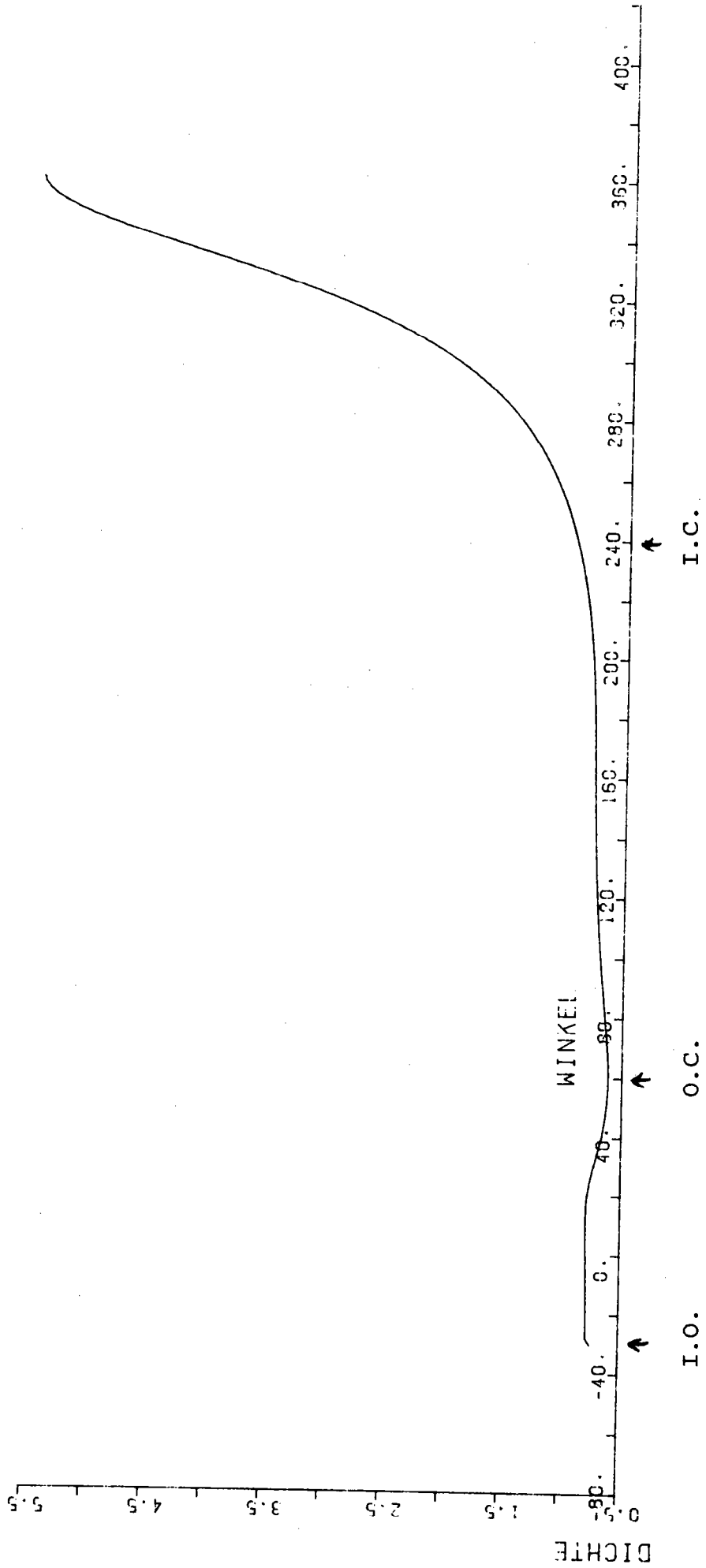
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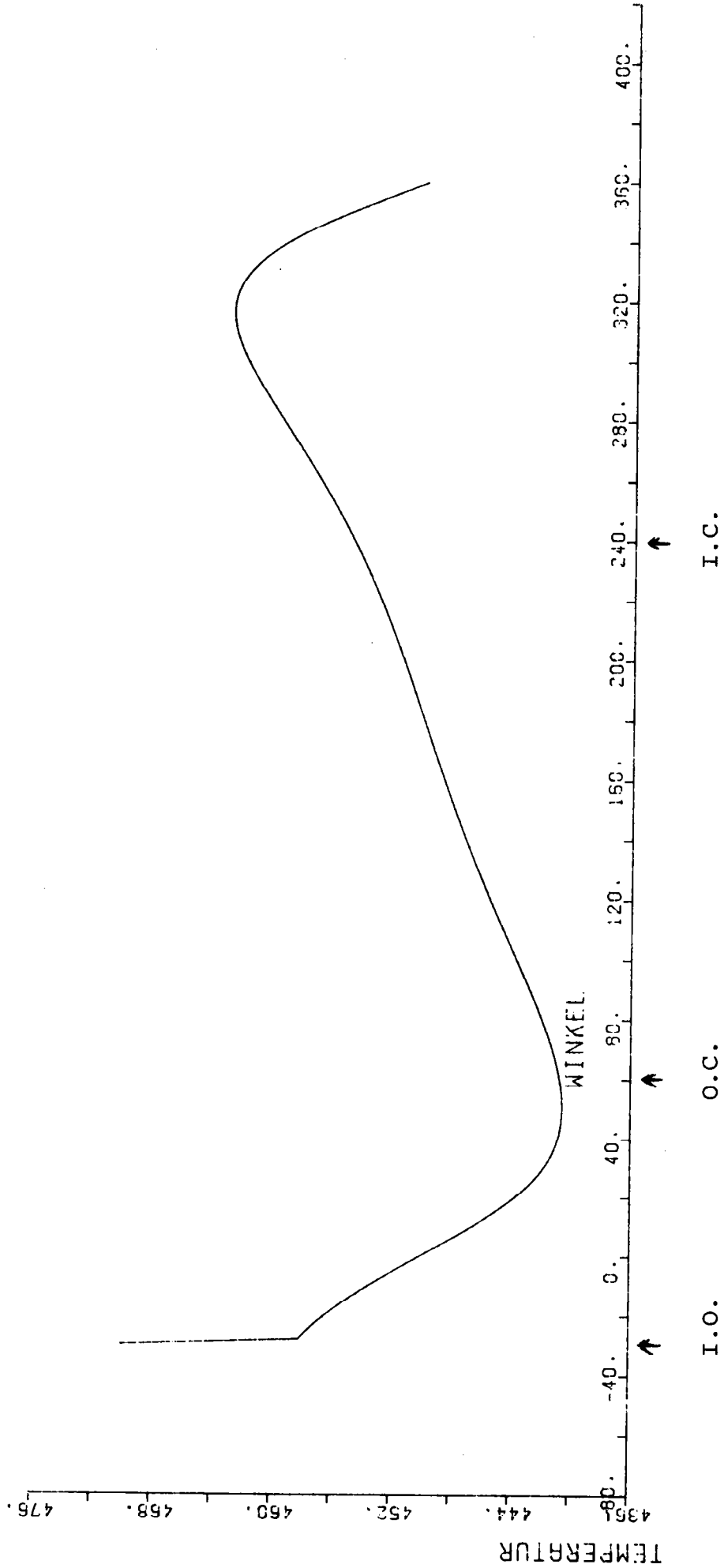
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fig. 7
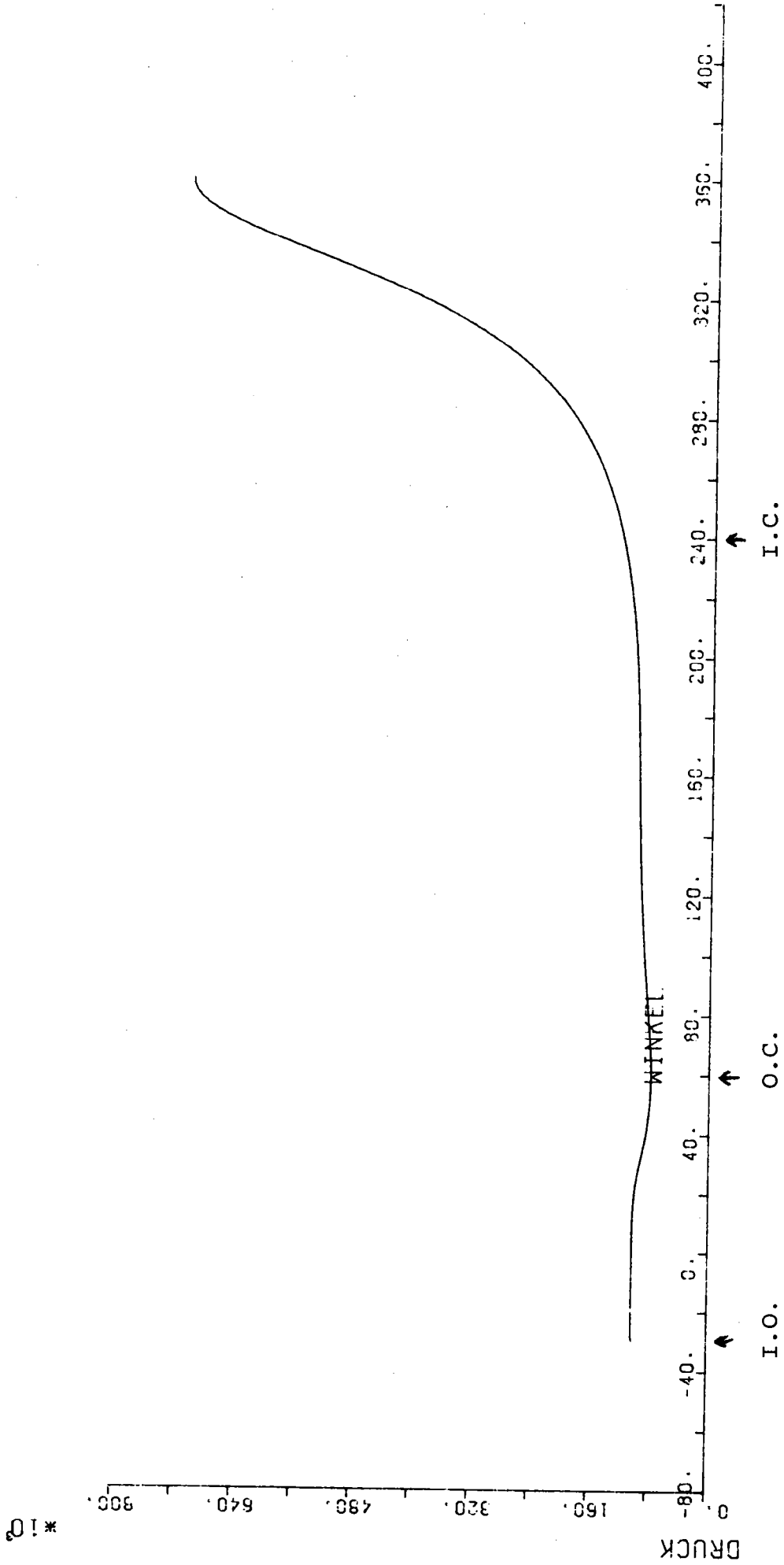
fig. 8

ROHRWERTE-VERSION12

I.O.

I.C.

WINKEL

DRUCK

*10²

-40.   0.   40.   80.   120.   150.   200.   240.   280.   320.   360.   400.

800.   900.   1000.   1100.   1200.   1300.
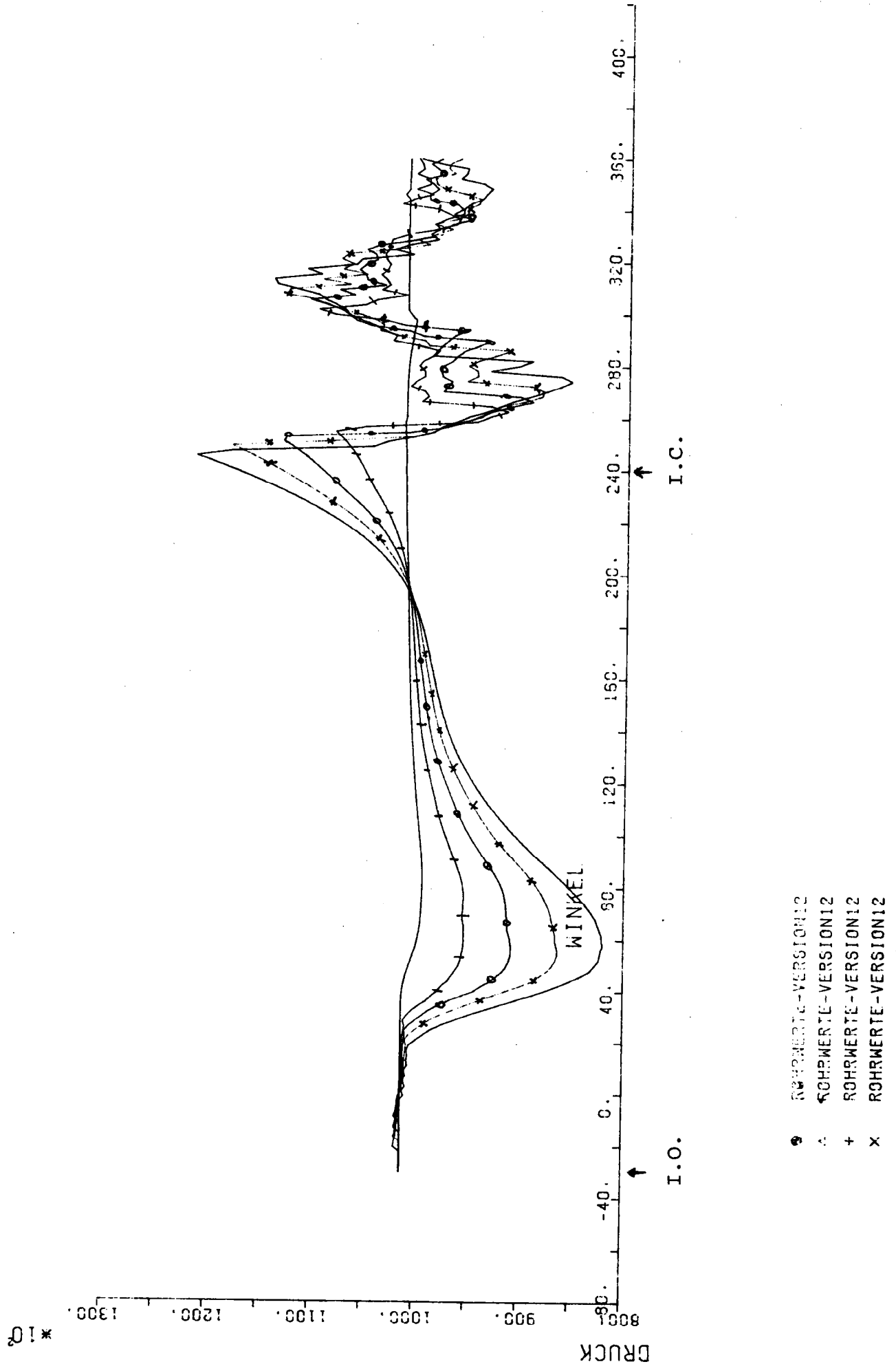
●   ROHRWERTE-VERSION12
△   ROHRWERTE-VERSION12
+   ROHRWERTE-VERSION12
×   ROHRWERTE-VERSION12

fig. 9
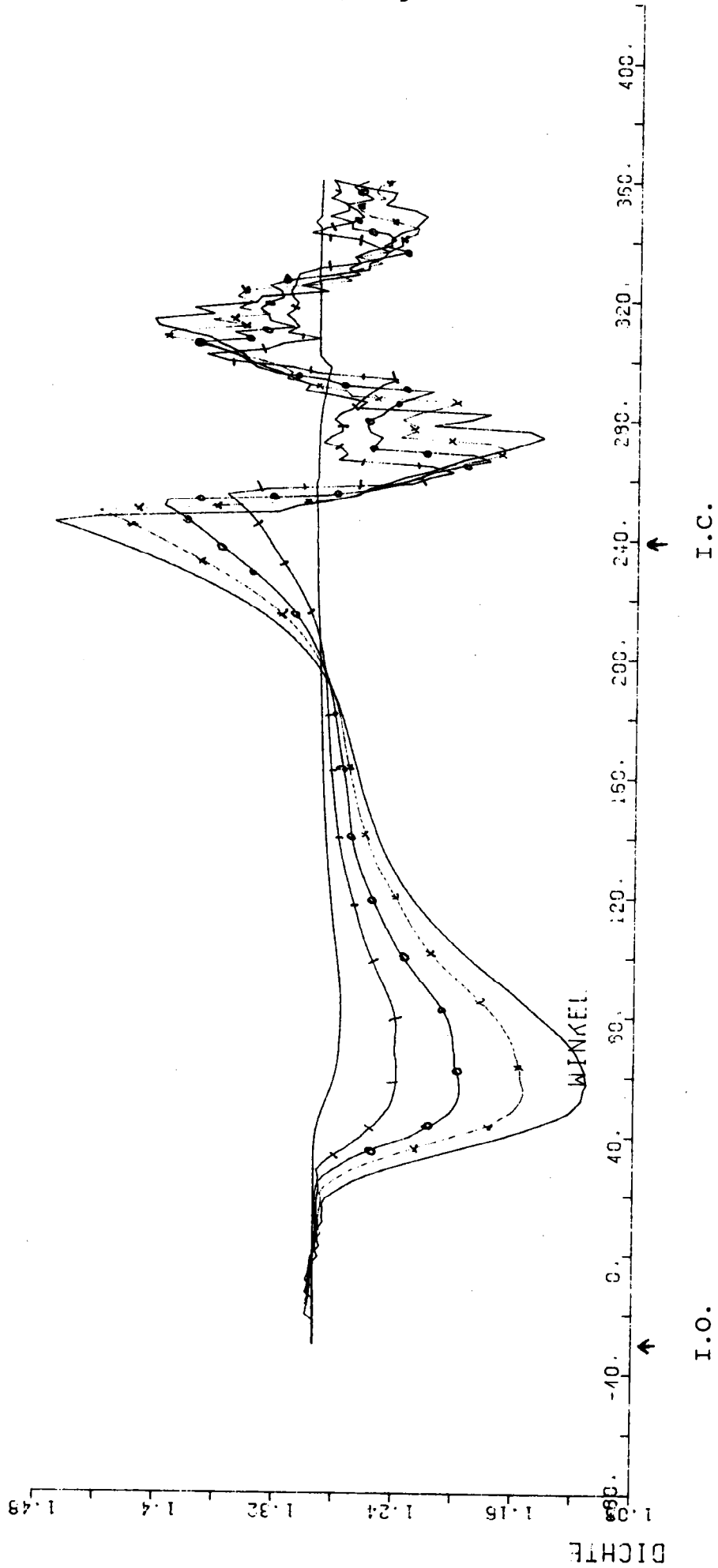
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fig. 10
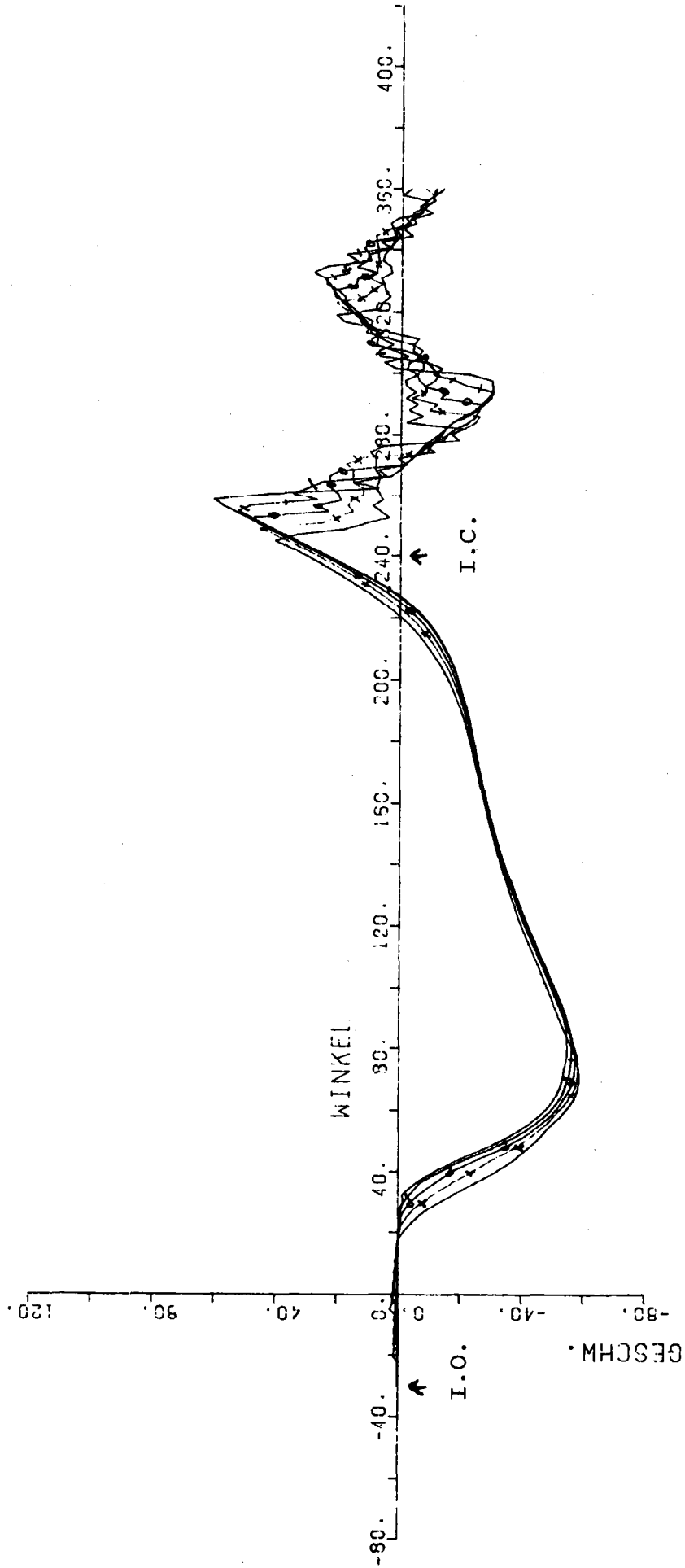
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fig. 11

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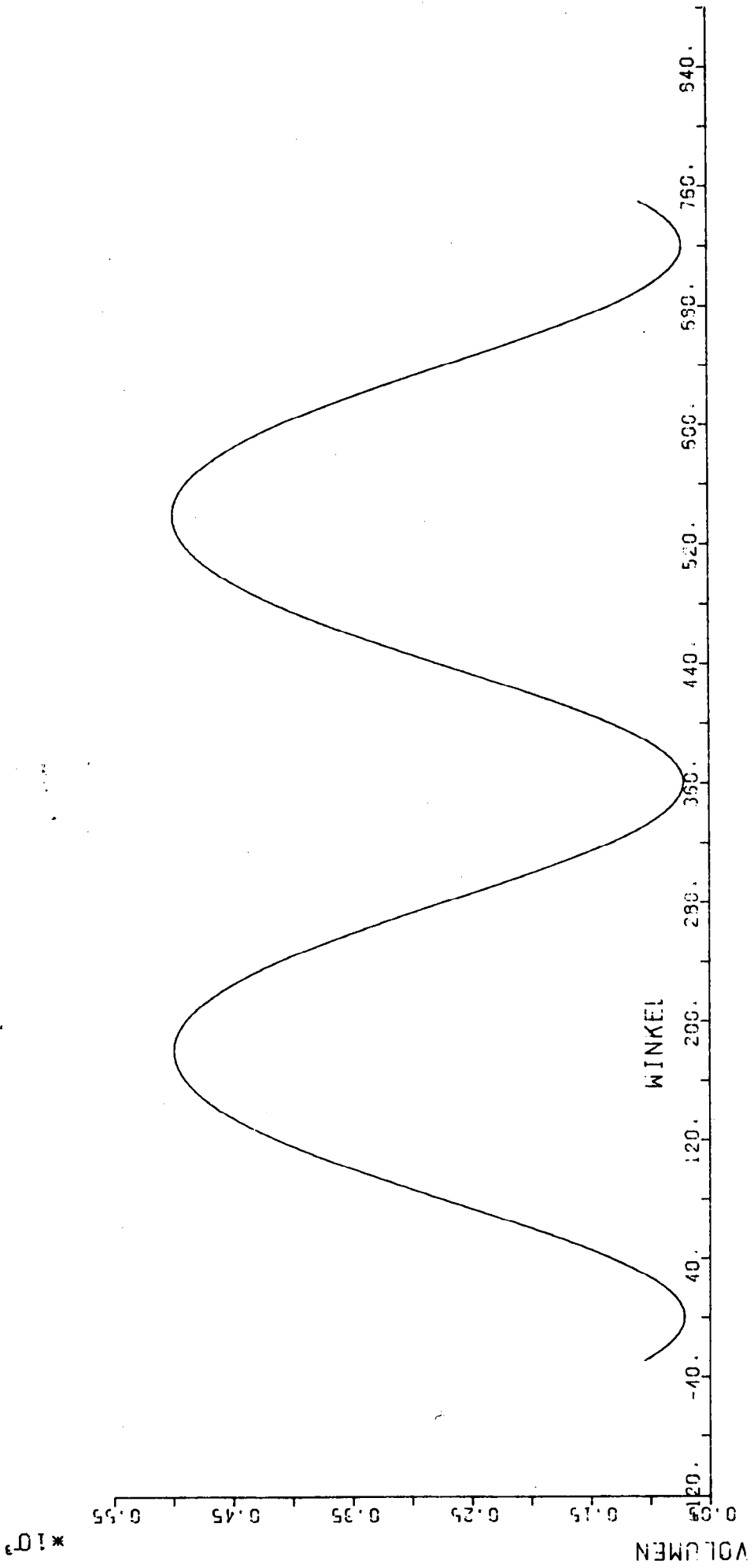
fig. 12

ZYLINDERWERTE-VERSION 12

MASSE *10³

WINKEL
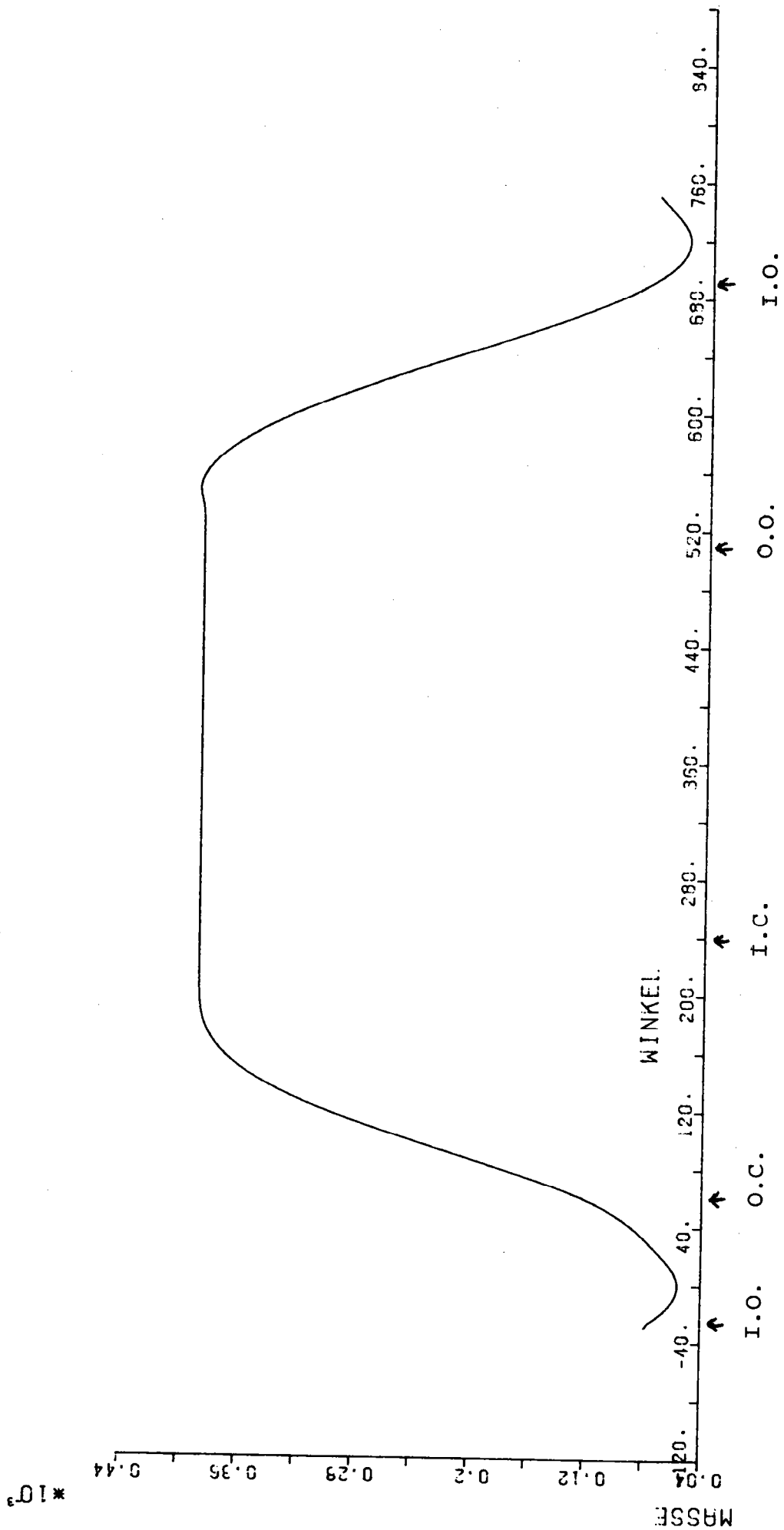
fig. 13

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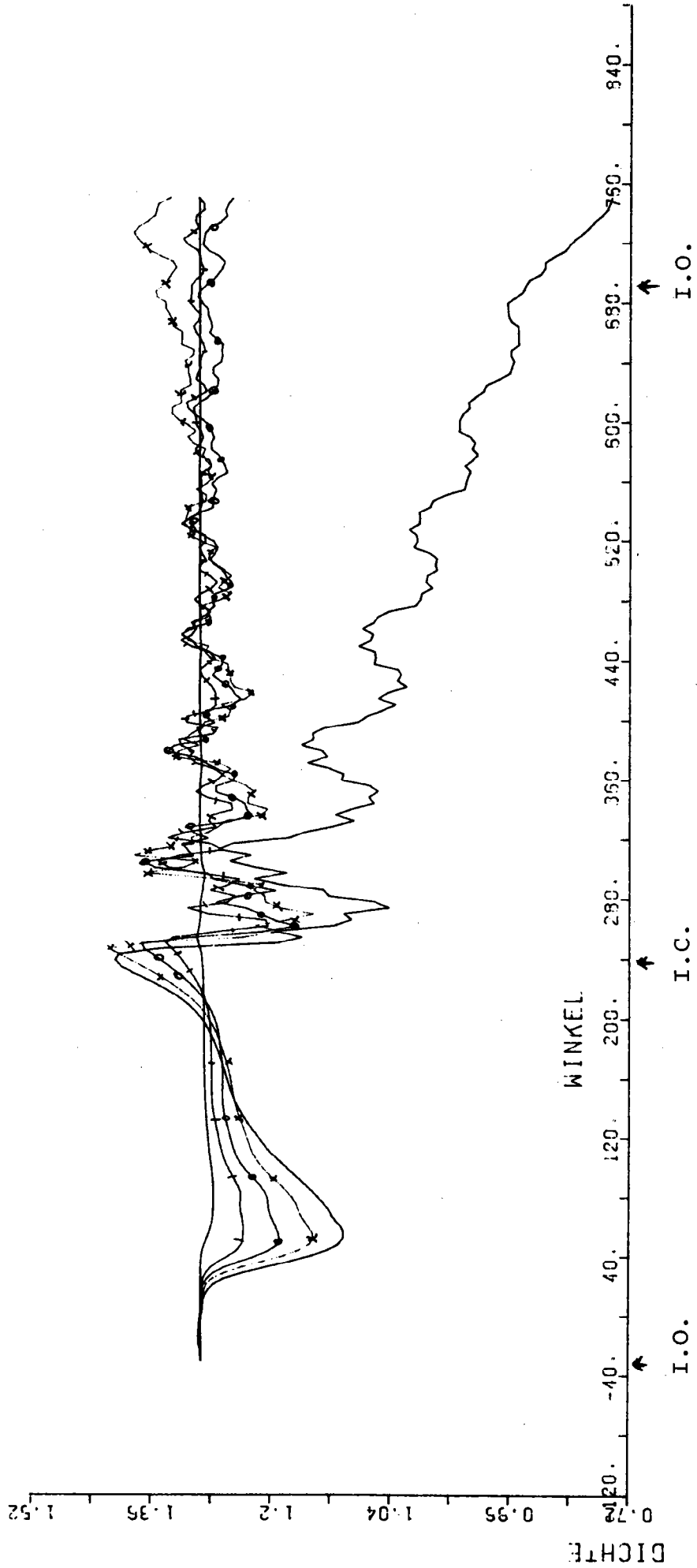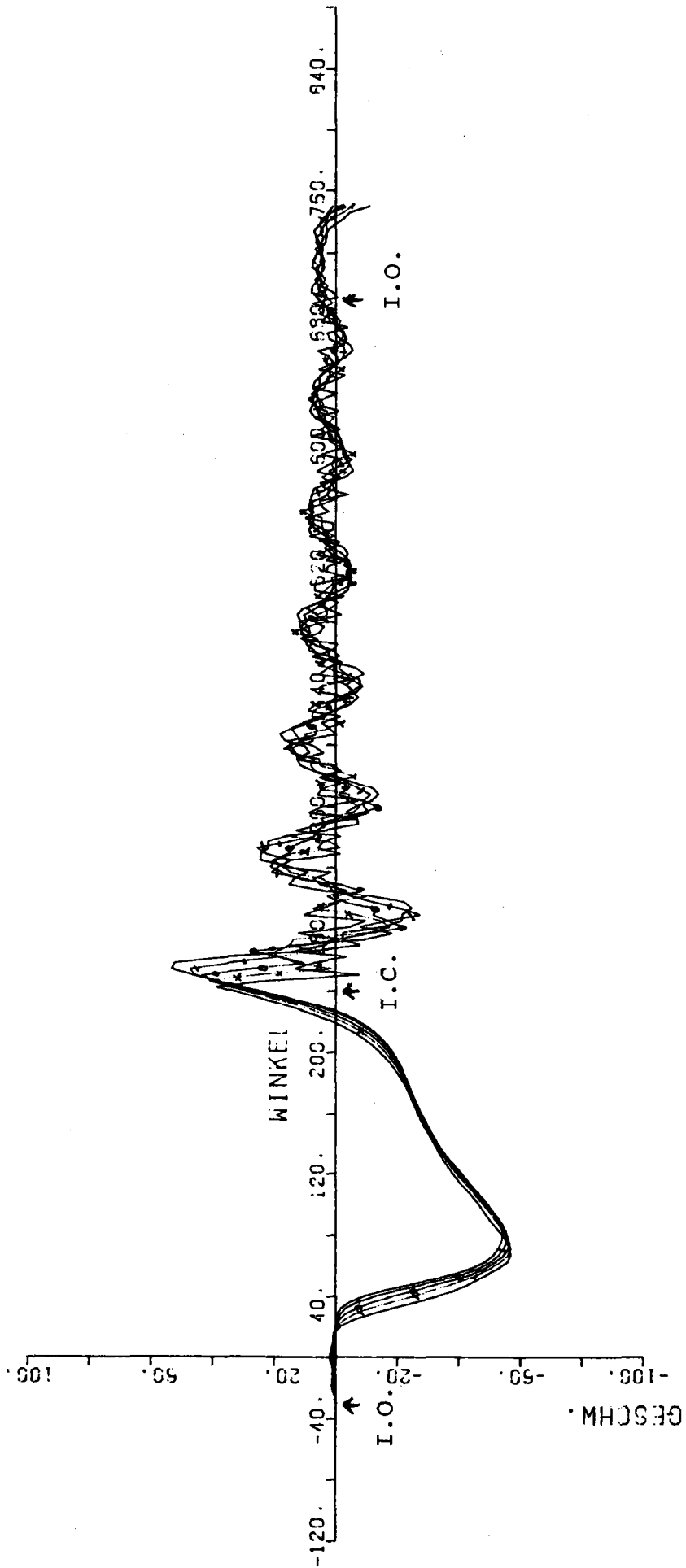
fig. 14

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fig. 15

fig. 17

fig. 18

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WINKEL

GESCHW.

I.O.

I.C.

I.O.

100. 50. 20. -20. -50. -100.

-40.

-120.

40. 120. 200. 840. 750.

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Θ
Δ
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