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INDUSTRIAL MATHEMATICS: GENERAL REMARKS AND SOME CASE STUDIES

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Industrial Mathematics: General Remarks and Some Case Studies

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Industrial mathematics has many faces; but its essential feature is the cooperation of partners - from industry and from universities - with quite different interest (business versus academic carreer), normally working on different time scales. They measure success in a different way (selling rate against citing index), they have different hierarchies of values and are very often distrusting each other. Industry doubts that mathematicians are willing and/or able to produce something real practical and useful (and the mathematicians should not be too much surprised about this attitude, they very often doubt themselves) - mathematicians are afraid to loose their competence (their ideal of scientific truth, to say it more idealistically), to sell their souls.

The only way to convince both sides of the usefulness of such a messalliance is to show examples, which satisfy both: Examples with practical success, where "real" (i.e. publishable) mathematics is involved - even better: where real mathematics has to be invented. As an ideal, at least from a mathematical point of view, one might consider the following remark, which Euler made in a letter from 1736 [1]: "Therefore you may see, most Honourable Sir, that this solution according to its character has almost no connection to mathematics, and I do not understand why such a solution should be expected more from a mathematician than from any other human being. The solution is based only on reason, and principles of mathematics are not necessary in order to find it." The solution he mentions was that of the Königsberg seven bridges problem, the origin of topology.

Our mathematical dreams are certainly not identical with the dreams of those, who pose us the problems - this was even true for Euler, about whom the Prussian king Frederic the Great was complaining in a letter to Euler: "The lifting tackle was built according to mathematical calculations and as yet cannot lift a single drop of water up to 50 feet from its container. Vanity of vanities! Vanity of Mathematics!" The origin of many disharmonies between the partners is a different meaning of the word "solution". Solution for mathematicians means: Existence and uniqueness - a qualitative explanation of a typical behaviour - a numerical algorithm; solution for people in praxis today means very often: Software; they want an instrument for checking an old, designing a new system, they have no time to study carefully "the interior" of such an instrument, it must be reliable and easy to handle. I believe that this idea of what a solution should be is held by a strong majority - inspite of the fact that "the industry" does not exist, as does not "the mathematician". Just to name two extremes: Aeronautical industry and the computational fluid dynamist are partners, who have very little problems with each other. But there are many sometimes quite small companies producing specialities with a highly developed very sophisticated technology, who never thought about mathematics - inspite of the fact that mathematics is hidden everywhere in their production process. And there is on the other hand the pure mathematician, who cannot see any application of his knowledge - inspite of the fact that he would be quite happy if such an application would exist. There are many of these small companies and many pure mathematicians - and both would gain something in a partnership, if they would dare to try.

That's somehow the policy my "Arbeitsgruppe Technomathematik" (Laboratory for Technomathematics) at the University of Kaiserslautern follows. Educated as a pure mathematician and starting the academic carreer with existence theorems for equations of mathematical physics, 15 years ago I became interested in the question what mathematics means outside the university, what our students were doing when leaving the university after graduation. Travelling through German industry of all kind and asking for open problems which might be solvable by mathematical methods (and not selling my special competence), I established a cooperation with until now more than 30 companies, most of them relatively small but also including for example Siemens, AUDI and Marcel Dassault. Cooperation comprises education and research: Companies are posing problems for student seminars and for master thesis - or we make research contracts with them. Contract means that industry pays the salary for the researcher - money is in this case an honest measure for their interest. Taking money certainly creates the danger to become dependent, to do only simple programming or classified work. Being aware of this danger makes it easy to avoid it (for example one should insist in making the mathematical content of the research publishable but should accept to keep all informations about the industrial partner secret).

In order to illustrate the kind of problems, which satisfied both partners in being mathemtically nice and practically successful, I shall now present two examples taken from our work during 1988.

I. **Quality Control for Artificial Fabrics** (see [2])

Artificial fabrics are sometimes produced by air spinning processes. Several hundreds of plastic fibres are drawn by wind, at the end moving turbulently and falling on a rolling ribbon (see figure 1); there they stick together and a texture is produced, which has a huge variety of applications.

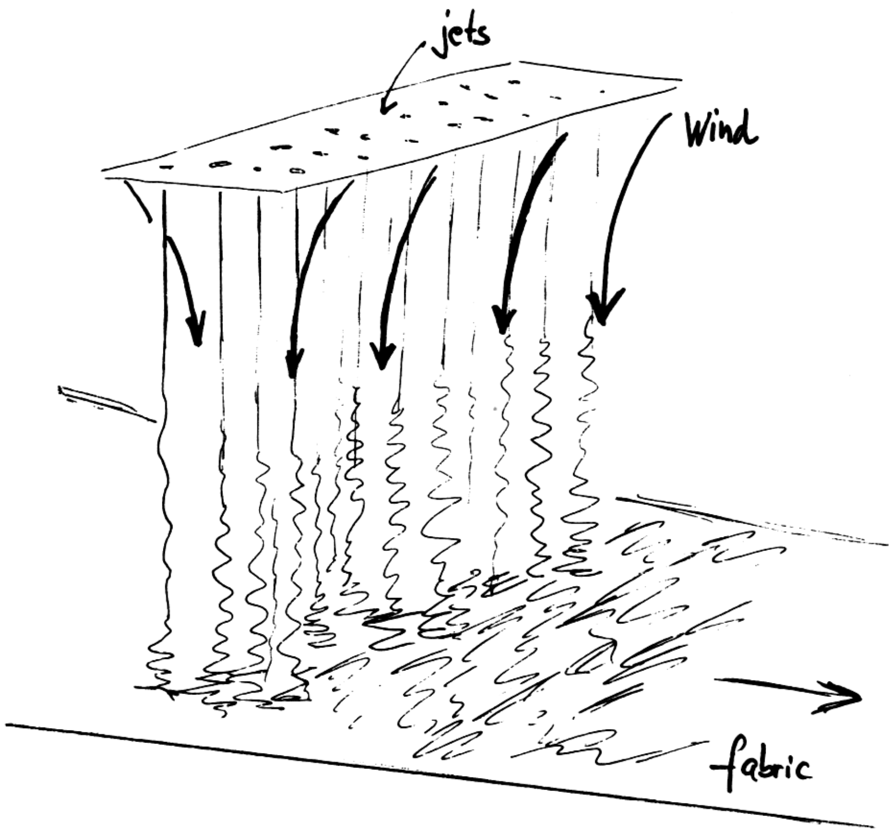


Figure 1

There are many aerodynamical problems involved - some of them extremely difficult. For example: Make a mathematical model for the interaction of fibres and flow, which explains the dependence of the fibre thickness on the air speed and which describes the fluttering of the fibres as elements of the whole ensemble. We are working in this direction but do not have enough results to report on it. But there is a much simpler problem connected: The tissue produced always shows some defects, irregularities which are called clouds (spots of several centimeters diameter which are darker or brighter than the average in through-shining light) or stripes (an anisotrpy in the fibre structure, which appear when several fibres stick together); these defects become more severe, when the plant, i.e. the wind channel becomes polluted during the production. What is needed is a quantitative measure for the irregularity, which can be calculated online and may therefore be used for quality and production control. One may assume that there is a laserbeam used, which gives the absorption through the texture on 4 parallel lines along the tissue. What is delivered are 4 functions showing the absorption along these lines, functions similar to stochastic time functions. But we are not interested in the statistical properties of these functions, but only in the regularity of the material; therefore we need a concept for the uniformity of the tissue.

Consider a piece Ω of the fabric as a pattern, given by the absorption: If M is a subset of Ω, $\mu(M)$ will denote the power of the light passing through M. Since we are only interested in deviations from uniformity, we can normalize $\mu(\Omega)=1$. μ can be considered as a normalized Borel measure on Ω. The ideal pattern, i.e. the uniformity of the fabric is given by $\mathring{\mu}$, where $\mathring{\mu}(M)$ is just the proposition of the area of M compared to the area of Ω

$$\mathring{\mu}(M) = \frac{\text{area}(M)}{\text{area}(\Omega)} \ .$$

$\mathring{\mu}$ has the density $\mathring{f}(x) = \frac{1}{\text{area}(\Omega)}$ for all $x \in \Omega$.

Deviation from uniformity means distance between μ and $\mathring{\mu}$.

$$\text{Irregularity of } \mu = I(\mu) = \text{distance}(\mu,\mathring{\mu})$$

Which distance? μ and $\mathring{\mu}$ are normalized measures - which concepts for a distance does the theory offer?

To make things simpler (and more realistic), assume that Ω is segmented into N pixels and μ is just given by the values $\mu_1,\dots,\mu_N$ in these pixels $\mu_i \geq 0$ and $\sum_{i=1}^{N} \mu_i = 1$.

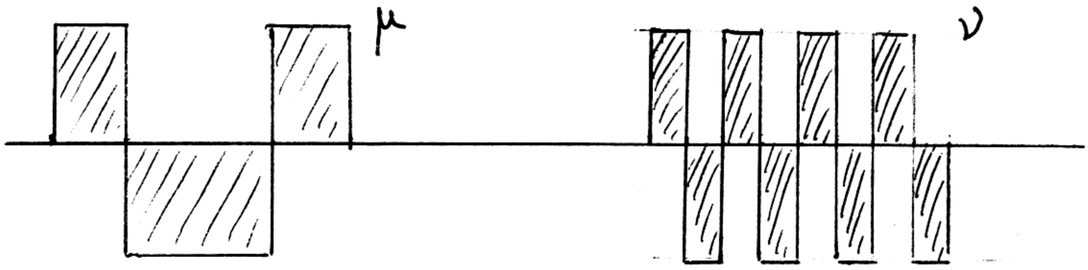
The corresponding $\overset{o}{\mu}$ is $\overset{o}{\mu}_i = \frac{1}{N}$ for all $i=1,\dots,N$.

Now we have the following possibilities:

(a) <u>Relative Entropy</u>, where

$$\text{distance } (\mu,\overset{o}{\mu}) := E(\mu/\overset{o}{\mu}) := \sum_{i=1}^{N} \mu_i \ln \frac{\mu_i}{\overset{o}{\mu}_i} = -\ln N + \sum_{i=1}^{N} \mu_i \ln \mu_i$$

This distance, suggested by statistical physics is in fact a measure for nonuniformity - but not the appropriate one for us: $E(\mu/\overset{o}{\mu})$ is not "influenced by order" and this means here: One "big hole" in the fabric counts as many "small holes".



$E(\mu/\overset{o}{\mu}) = E(\nu/\overset{o}{\mu})$

Figure 2

This does not correspond to the companies idea of quality.

(b) <u>Bounded Lipschitz Distance</u>, where

$$\text{distance } (\mu,\overset{o}{\mu}) = D(\mu,\overset{o}{\mu}) := \max_{\underline{a}\in D} \left| \sum_{i=1}^{N} a_i (\mu_i - \tfrac{1}{N}) \right|$$

with $D := \left\{ \underline{a} \in \mathbb{R}^N / |a_{i+1} - a_i| \leq \frac{1}{N} \right\}$.

One realizes that the definition of this metric corresponds to a quite strange norm in $\mathbb{R}^N$

$$\|\mu\|_D := \max_{\underline{a}\in D} \left| \sum_{i=1}^{N} a_i \mu_i \right|$$

This distance filters somehow the holes (like a car driving over a road with holes on a certain speed), so that large holes are worse than many small ones. Mathematically speak-

ing, D generates the weak topology in the space of probability measures - which means practically that small pattern shifts create small distance deviations.
D is a reasonable measure, but a little bit elaborate to be computed numerically. G. Rote ([3]) realized that the dual problem (in the sense of linear programming) is a curve smoothing problem allowing a quite simple and fast algorithm.
Further and in general as good is the

(c) **Discrepancy**, where

$$\text{distance}(\overset{o}{\mu},\mu) = \text{Dis}(\overset{o}{\mu},\mu) = \max_{1\leq\alpha\leq\beta\leq N} \left| \sum_{j=\alpha}^{\beta} (\mu_j-\tfrac{1}{N}) \right|$$

This concept originates from numbertheoretic consideration already in papers by Hermann Weyl (1916) (see also [4]) and is related to the Kolmogorov-Smirnov distance in statistics. Again Rote [3] proposed a very simple algorithm:

Put

$$f_j := \mu_j-\tfrac{1}{N},\quad j=1,\dots,N,\qquad F_k := \sum_{i=1}^{k} f_i,\qquad F_o = 0.$$

Then

$$\text{Dis}(\overset{o}{\mu},\mu) = \max_{0\leq i<j\leq N} |F_i-F_j| = \max_i(F_i)-\min_i(F_i).$$

The algorithm works as follows:

```
     MAX := 0,  MIN := 0,  F := 0
for k=1 to N do  F := F+f_k.
    If (F>MAX) : MAX := F
    If (F<MIN) : MIN := F
Then  Dis(μ°,μ) = MAX - MIN.
```

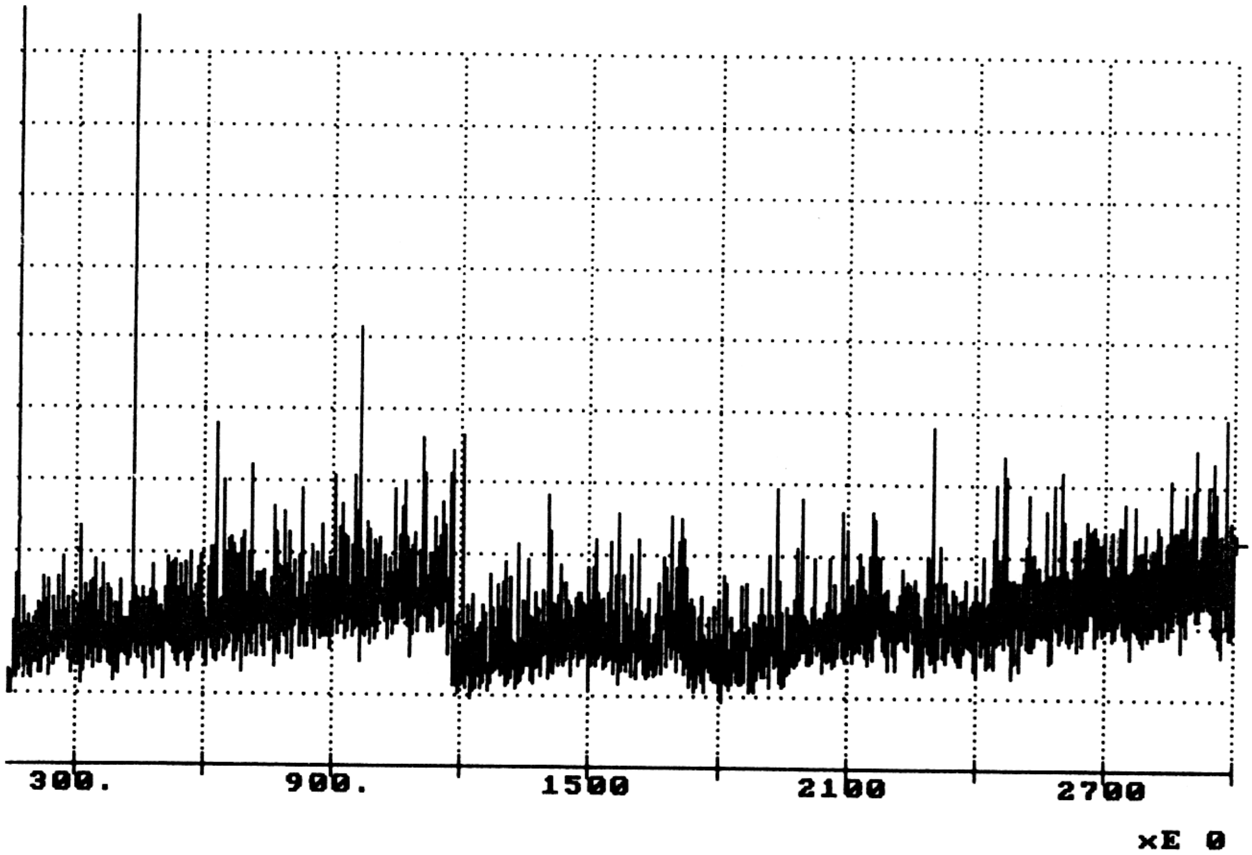


Figure 3

Figure 3 shows discrepancy measurements along only one line of laser scanning; each value of Dis(μ,$\mathring{\mu}$) is calculated for Ω being a ca. 1 m long piece of this line (which means that Ω is one dimensional, which we had implicitly assumed already in our definitions; these definitions can be easily extended to 2 or more dimensions - but not the algorithms!). Each abscissa in the diagram corresponds to such a piece of the line, each ordinate to the corresponding discrepancy. The discrepancy is in average increasing until the channnel is cleaned (x $\approx$ 1200).
Discrepancy proves to be in general a useful measure of irregularity, even with only one scanning track. This is especially true, if clouds are the main source of defects. If stripes are more important, one has to define a measure for anisotropy - here different methods, maybe something like an 2d-wavelet analysis, should be tried. This work is in progress.

II. Computer Aided Design with Worst Case Tolerances (see [5] and [6])

The problem arose in car industry: Construction drawings of gears are normally given with tolerances for the sizes of the construction elements. A construction drawing consists of finitely many points in the plane together with straight lines and circles determined by these points and some additional information like the radii of the circles. These points are constructed recursively in using simple elementary constructions.

"Chains" of points $P_o, P_1, \ldots, P_N$ can be recursively constructed, if the (cartesian or polar) coordinates of P_j with respect to a coordinate system with a fixed basis origin at P_{j-1} are given; $P_o = O$ is fixed "general origin". Each point can now be used as center of a circle with given radii, two points can be used to determine a straight line. Circles and straight lines may be now used again to define further points through intersection. There fore, if we consider only construction by circle and straight edge, a point is recursively defined by previously given points through the following data:

(a) the coordinates with respect to a coordinate system with another point as origin,

(b) as intersection of straight lines and/or circles defined by pairs of points and by point and radius respectively.

The problem of tolerances is now easily defined: The data (coordinates, radii) are subject to given tolerances. Therefore the positions of the points are not uniquely defined. One may think of probability distribution instead of points - but the recursion leads quickly to hopelessly complicated formulae for these distributions, so that only simulation can be considered, if the number of points is more than say 10. Moreover, in the case of the gears, one is mainly interested in worst case considerations: Which positions of the point are in accordance with data and tolerances?

Instead of points P we consider tolerance sets Tol(P), which are also depending on the tolerance sets for the previously constructed points, the data and their tolerances and certainly the construction method (i.e. (a) or (b)).

As example we consider only constructions without circle. In (a), the point P_1 is given with respect to a fixed origin O with $Tol(O) = \{O\}$ by his coordinates. We write here $Tol_o(P_1)$ -

certainly a rectangle for cartesian coordinates but a set like

for polar coordinates.

If a point Q is given with respect to a coordinate system, whose origin P is again in a tolerance set, then we get

$$Tol(Q) = Tol(P) \oplus Tol_o(Q)$$

where $A \oplus B$ denotes the Minkovski sum $\{a+b/a \in A, b \in B\}$.
For chains we get

$$Tol(P_N) = Tol_o(P_1) \oplus \ldots \oplus Tol_o(P_{N-1}).$$

Figure 4 shows such a tolerance set with Tol(P) as the interior of a circle and $Tol_o(Q)$ as the tolerance domain for polar coordinates.

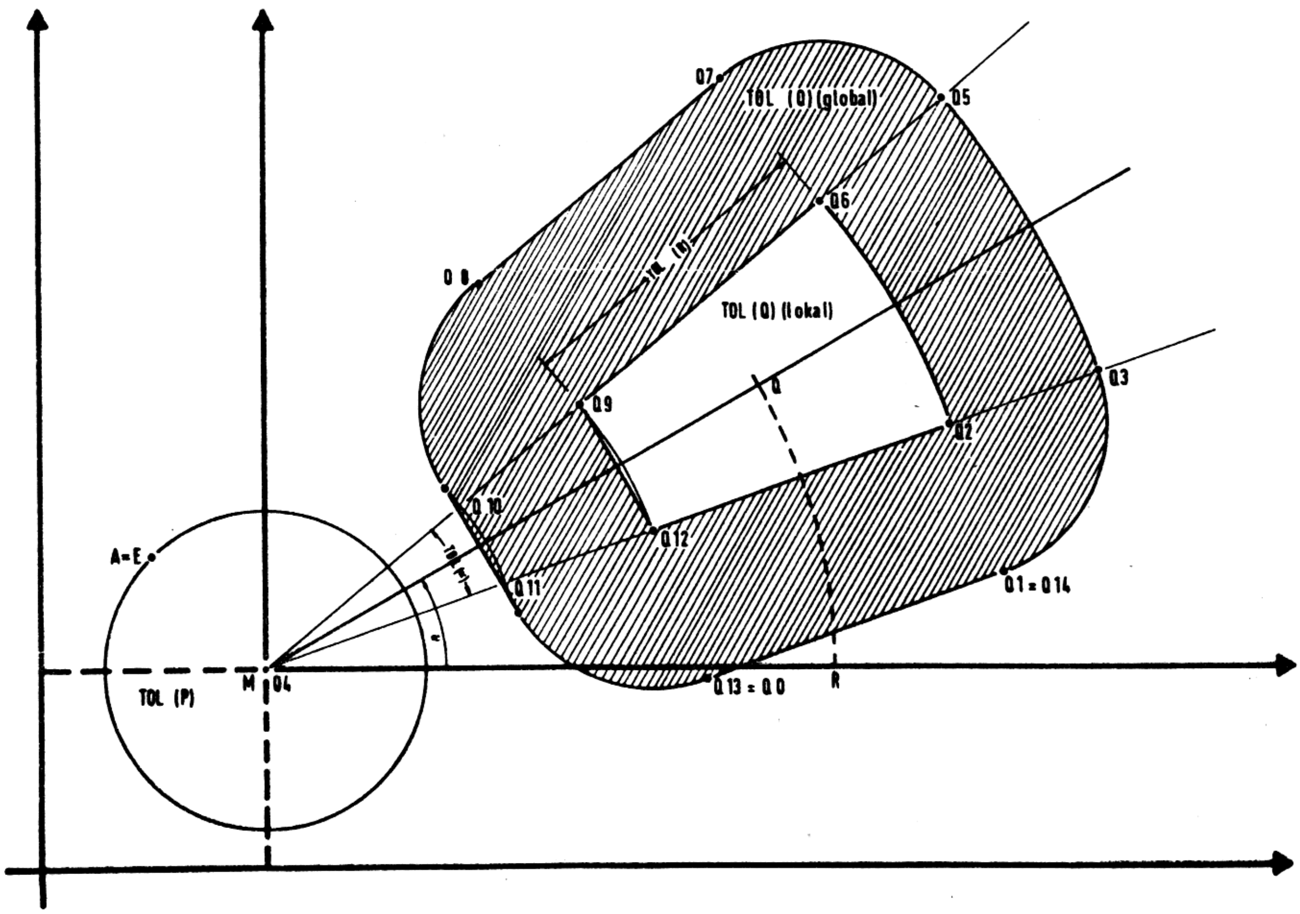


Figure 4

One may realize that the computational effort will soon increase enormously. The essential idea now is to enlarge the tolerance domains a little bit by taking the convex hull as new tolerance set. This changes a little bit $Tol_o(P)$ for P given by polar coordinates. The enlargement is not very significant and since the Minkovski sum of convex sets remains convex, everything else discussed above has not to be changed.
Let's now consider straight lines g, defined by the tolerance set for two points P_1, P_2. One get again sets, which may be denoted by

$$Tol(g) := \{x/x \text{ lies on a straight line, which intersects } Tol(P_1) \text{ and } Tol(P_2)\}.$$

Tol(g) is not convex and is not convexified. The construction (b) defines a point P as intersection of two straight lines g and h, and we may try to define

$$Tol(P) := \text{convex hull of } Tol(g) \cap Tol(h) \;.$$

Figure 5 shows what may happen: $Tol(g) \cap Tol(h)$ is a convex or nonconvex polyeder, maybe unbounded, even disconnected. In the first two cases Tol(P) is really defined as above; if $Tol(g) \cap Tol(h)$ becomes unbounded, the construction is not acceptable, a CAD programme will inform the user about the rejection.

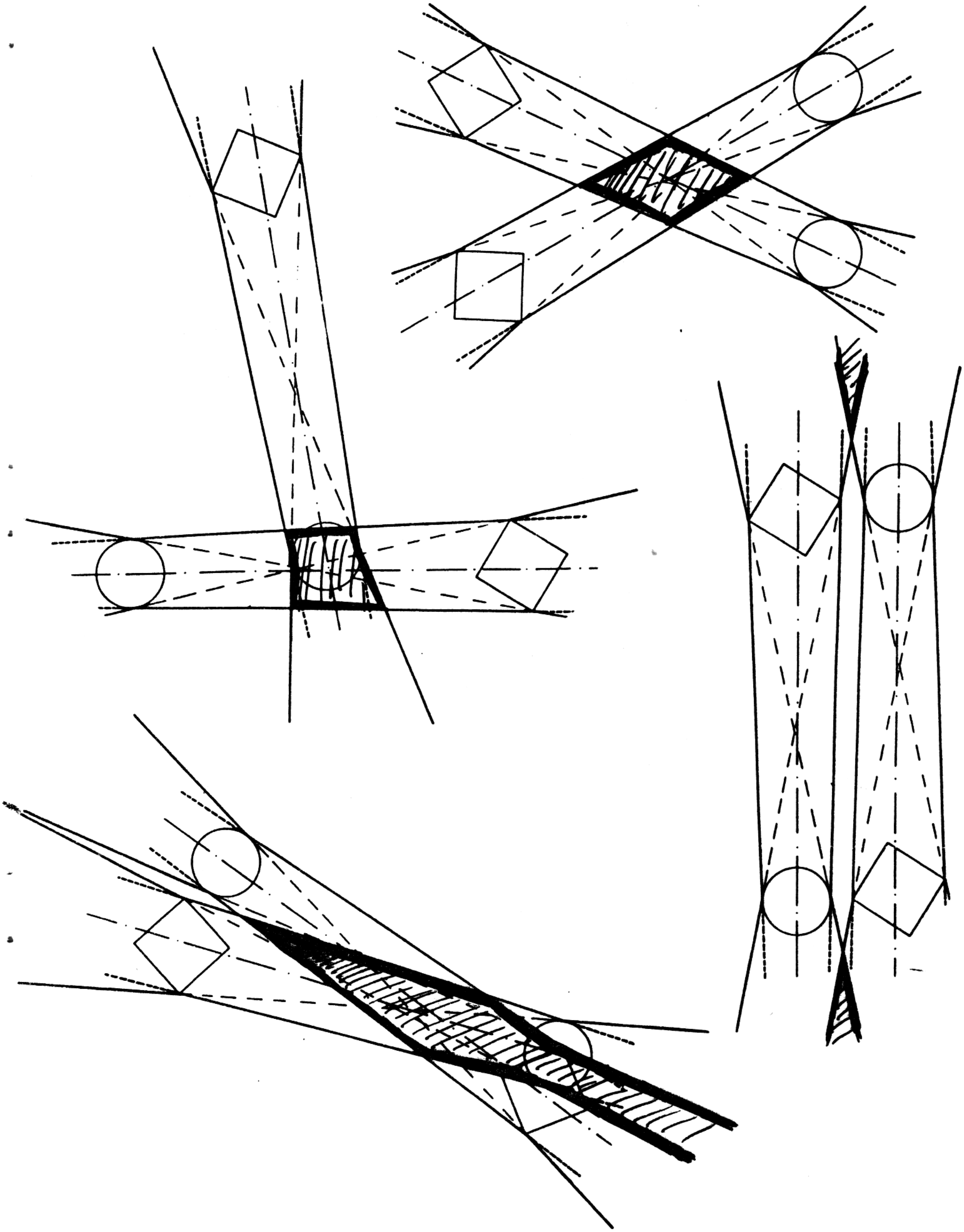


Figure 5

In any case, the computational effort lies in determining the convex hulls and the Minkovski sums. But since the set is polyedric (or, if we use circles, "quasipolyedric", with boundaries consisting of pieces of straight lines or circular arcs),it is enough to find the extremal points. This is in general a linear programming task; the extremal points of the Minkovski sum are given as sums of the extremal points of the terms, if the interior of the normal cones at those points have nonempty intersection. There is some convex geometry involved - not deep, but nice - and one finds that there are very quick simple linear programming algorithms sufficient to do the computations. The state of art now is, that we have an algorithm and a simple CAD programme to determine the tolerance domains for 2d construction drawings, if circle-circle are excluded. To complete the task means some programming effort (even new rejection criteria will become necessary).

I think that the practical relevance of this idea is evident. Where is the mathematical fun? Now, there have been many attempts to solve the problem of toleranced CAD, all more or less failing, if the drawing has a high complexity, since the effort increases very strongly with the numbers of points (for our method, it's essentially linear). Moreover, one can do a little bit axiomatic geometry - with convex sets instead of points etc. The situation is similar to what is called interval arithmetic - a theory much further developed but maybe even less rich and exciting in structure.

Conclusion

The two examples given above are chosen in order to illustrate the flavour of what we call industrial mathematics. The mathematics there is borrowed not from the classical fields of applications like pde, statistics etc.; it can be done by every mathematician, whatever his strict field of interest is - but (the engineers may apologize) it can be done only by mathematicians. Therefore it's worthwhile for the industrialist to discuss with mathematicians - they may be a source of innovation. And its worthwhile for the mathematician to consider the problems posed by industry - they give fun, prestige, even some money for his group. It's quite rare for scientists that fun and profit goes together.

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- die Optimierung von Kurbelgetrieben, Nocken und Felgen;
- die analytische und numerische Untersuchung spezieller strömungsdynamischer und akustischer Probleme;
- die Simulation stochastischer Prozesse in der Zuverlässigkeitsanalyse.

PRAXISORIENTIERTE GESTALTUNG DER MATHEMATISCHEN AUSBILDUNG IM HINBLICK AUF EINE BESSERE VORBEREITUNG DER ABSOLVENTEN AUF DIE BERUFSWIRKLICHKEIT.

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Auch in die Mathematikausbildung der Ingenieure sollen Modellbildung und moderne, insbesondere numerische und stochastische Methoden verstärkt integriert werden.

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- Anpassung an den neuesten wissenschaftlichen Erkenntnisstand,
- Einordnung des praktisch-beruflichen Wissens in einen theoretisch-wissenschaftlichen Rahmen,
- Auffrischung von Hochschulwissen.

Die Arbeitsgruppe Technomathematik setzt sich aus Professoren und Mitarbeitern der Universität Kaiserslautern und einer Gruppe von Mathematikern an der Technischen Hochschule Darmstadt unter der Leitung von Prof. Dr. Törnig zusammen.