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SUFFICIENT CONDITIONS FOR ADAPTIVE STABILIZATION AND TRACKING

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Sufficient Conditions for Adaptive Stabilization and Tracking

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Abstract

We consider universal adaptive stabilization and tracking controllers for classes of linear systems. Under the technical assumption of linear scaling invariance necessary and sufficient conditions for adaptive stabilization are derived. For scalar systems sufficient conditions for adaptive tracking of finite dimensional reference signals are explored.

1. INTRODUCTION

In a systematic framework adaptive controllers which require no explicit identification and which stabilize linear systems under very weak assumptions on the system parameters have been developed by e.g. Morse (1983, 1985), Willems and Byrnes (1984), Mårtensson (1985), Owens, Ilchmann and Prätzel-Wolters (1987), Helmke and Prätzel-Wolters (1988). These controllers are called universal, since they achieve their control objective for a whole prescribed class of linear systems and all possible initial conditions. Previous work on universal adaptive controllers was mainly concerned with the adaptive stabilization problem. In Martensson (1986) and Byrnes, Helmke and Morse (1987) first theorems on necessary and sufficient conditions for universal adaptive stabilization were developed.

In this paper we introduce the concept of <u>linear scaling invariance</u> of classes of linear systems and show in section 2 that under this additional assumption known sufficient conditions for adaptive stabilization become necessary too. Furthermore in section 3 we consider universal adaptive tracking controllers for finite dimensional reference signals. By an augmentation argument it is shown how the tracking problem can be interpreted as a stabilization problem for the augmented systems in the scalar case.

2. THE ADAPTIVE STABILIZATION PROBLEM

The purpose of this section is to give necessary and sufficient conditions for the adaptive stabilization problem.

Let $\Sigma \equiv \Sigma(n,m,p)$ denote a <u>class of finite-dimensional linear time-invariant systems</u>

$$\dot{x}(t) = Ax(t) + Bu(t) , \quad y(t) = Cx(t)$$
$$x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^m, \quad y(t) \in \mathbb{R}^p \tag{2.1}$$

where m, p are given and n is fixed but not necessarily known.

A q-dimensional <u>universal adaptive stabilizer</u> (UAS) for Σ is a feedback control system

$$u = f(y,z)$$
$$\dot{z} = g(y,z) \tag{2.2}$$

where $z \in \mathbb{R}^q$ and

$$f: \mathbb{R}^{p+q} \longrightarrow \mathbb{R}^m$$
$$g: \mathbb{R}^{p+q} \longrightarrow \mathbb{R}^q$$

are smooth (C^∞) functions such that for any $(A,B,C) \in \Sigma$ and all initial conditions $x(0) \in \mathbb{R}^n$, $z(0) \in \mathbb{R}^q$ the solution $(x(\cdot),z(\cdot))$ of the closed loop system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(Cx(t),z(t)) \\ \dot{z}(t) &= g(Cx(t),z(t)) \end{aligned} \tag{2.3}$$

exists for all $t \in \mathbb{R}_+$ and satisfies:

$$\lim_{t\to\infty} x(t) = 0 \tag{2.3a}$$

$$\lim_{t\to\infty} z(t) = z_\infty \in \mathbb{R}^q \quad \text{exists.} \tag{2.3b}$$

Necessary and sufficient conditions for the existence of such universal controllers were given by Mårtensson (1986) and Byrnes, Helmke and Morse (1986). They have shown the following results:

Theorem 2.1 (Mårtensson (1986))

Suppose that for any $(A,B,C) \in \Sigma$ there exists a linear dynamic feedback controller of order $\leq q$, which stabilizes (A,B,C). Then there exists a $(q+1)$-dimensional universal stabilizer (f,g) for Σ. □

Thus an upper bound on the orders of stabilizing linear controllers for $(A,B,C) \in \Sigma$ is sufficient a priori assumption for universal adaptive stabilization. This condition is also almost necessary, as shown by the following theorem:

Theorem 2.2 (Byrnes, Helmke, Morse (1986))

Suppose there exists a q-dimensional universal adaptive stabilizer (f,g) for Σ. Then, for each $(A,B,C) \in \Sigma$, there exists a linear dynamic feedback controller of order q such that the resulting closed loop system has its poles in the closed left half plane $\overline{\mathbb{C}^-}=\{z \in \mathbb{C} \mid \text{Re } z \leq 0\}$. □

<u>**Remark**</u>

We emphasize that Theorem 2.2 only says that the poles can be placed within the <u>closed</u> left half plane $\overline{\mathbb{C}^-}$. This is not enough to conclude asymptotic stability and implies stability only if the geometric multiplicities of the eigenvalues on the imaginary axis are 1. Therefore the condition of Theorem 2.1 need not to be necessary and in fact is not known to be. This should be compared with the over-optimistic conclusion of Mårtensson (1986), Thm. 4.1.

In order to obtain a single necessary and sufficient condition for universal adaptive stabilization we make a technical assumption.

<u>**Definition 2.3**</u>

Σ is called <u>locally scaling invariant</u> (LSI), if for every $(A,B,C) \in \Sigma$ there exists $\varepsilon > 0$ such that

$$(A+aI,B,C) \in \Sigma \quad \text{for all } 0 \leq a < \varepsilon \ . \qquad (2.4)$$

<u>**Examples**</u> for (LSI)-sets Σ are:

(1) Any open subset $\Sigma \subset \mathbb{R}^{n(n+m+p)}$ is locally scaling invariant.

(2) The set of minimum phase systems (A,B,C) with

$$\det\begin{bmatrix} sI-A & B \\ C & 0 \end{bmatrix} \quad \text{a Hurwitz polynomial}$$

is locally scaling invariant.

(3) The set of observable and controllable minimum phase systems with a fixed bound on the relative degree is locally scaling invariant.

(4) The class of systems (A,B,C) which can be (asymptotically) stabilized by linear compensators of order $\leq q$ is locally scaling invariant.

Theorem 2.4

Let $\Sigma=\Sigma(n,m,p)$ be locally scaling invariant and let (f,g) be a q-dimensional universal stabilizer for Σ. Then there exists for each $(A,B,C) \in \Sigma$ a linear dynamic feedback controller of order q which stabilizes (A,B,C).

Proof

Let $(x(\cdot),z(\cdot))$ be the solution of the nonlinear closed loop system

$$\dot{x} = Ax + Bf(Cx,z)$$

$$\dot{z} = g(Cx,z)$$

with

$$\lim_{t\to\infty} x(t) = 0$$

$$\lim_{t\to\infty} z(t) = z_\infty \in \mathbb{R}^q .$$

Thus $(0,z_\infty)$ is an equilibrium point of (2.3) and the linearization of (2.3) around $(0,z_\infty)$ is given by

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = J(A,B,C) \cdot \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

where

$$J(A,B,C) = \left[\begin{array}{c|c} A+B\frac{\partial f}{\partial y}(0,z_\infty)C & B\frac{\partial f}{\partial z}(0,z_\infty) \\ \hline \frac{\partial g}{\partial y}(0,z_\infty)C & \frac{\partial g}{\partial z}(0,z_\infty) \end{array}\right] \tag{2.5}$$

is the transition matrix for the closed loop system of (A,B,C) with the linear compensator given by

$$\begin{aligned} \dot{z} &= \frac{\partial g}{\partial z}(0,z_\infty)z + \frac{\partial g}{\partial y}(0,z_\infty)y \\ u &= \frac{\partial f}{\partial z}(0,z_\infty)z + \frac{\partial f}{\partial y}(0,z_\infty)y \end{aligned} \tag{2.6}$$

Let $\varepsilon > 0$ such that $(A+\varepsilon I,B,C) \in \Sigma$. By Byrnes, Helmke and Morse (1986)

$$\text{Spec } J(A+\varepsilon I,B,C) \subset \overline{\mathbb{C}^-}$$

for an appropriate choice of the initial condition $(x(0),z(0))$. Thus the linear controller

$$\begin{bmatrix} \dot{z} \\ u \end{bmatrix} = K_\varepsilon \begin{bmatrix} z \\ y \end{bmatrix}$$

with

$$K_\varepsilon = \left[\begin{array}{c|c} \frac{\partial g}{\partial z}(0,z_\infty)-\varepsilon & \frac{\partial g}{\partial y}(0,z_\infty) \\ \hline \frac{\partial f}{\partial z}(0,z_\infty) & \frac{\partial f}{\partial y}(0,z_\infty) \end{array}\right]$$

places the poles of (A,B,C) within $\overline{\mathbb{C}^-_\varepsilon} = \left\{z \in \mathbb{C} \;\middle|\; \text{Re } z \leq -\varepsilon\right\} \subset \mathbb{C}^-$ and we are done. □

As an immediate consequence of this theorem we obtain the following corollaries:

Corollary 2.5

Suppose there exists $a>0$ such that $(A+aI,B,C) \in \Sigma$ for all $(A,B,C) \in \Sigma$. Let (f,g) be a q-dimensional adaptive stabilizer for Σ. Then there exists for each $(A,B,C) \in \Sigma$ a linear controller of order q which places the poles within

$$\overline{\mathbb{C}_a^-} = \left\{ z \in \mathbb{C} \;\middle|\; \mathrm{Re}\, z \le -a \right\}.$$ □

Corollary 2.5a

a) Let Σ be a class of scalar minimum phase systems. There exists a UAS for Σ if and only if there exists an upper bound $r*$ for the relative degrees of all $(A,b,c) \in \Sigma$.

b) Let $\Sigma_+(n,1,1)$ be the set of all scalar minimum phase systems. There exists a q-dimensional UAS for Σ_+ if and only if $n \le q-1$.

□

Corollary 2.6

Let Σ be a locally scaling invariant class of systems. The necessary and sufficient a priori knowledge for universal adaptive stabilization is knowledge of an integer ℓ such that for any $(A,B,C) \in \Sigma$ there exists a fixed linear controller of order ℓ which stabilizes (A,B,C). □

See Mårtensson (1986) for the sufficiency part.

Our next result shows that the class Σ of systems for which a universal adaptive stabilizer exists is necessarily output feedback invariant.

Corollary 2.7

Let $\Sigma \subset \mathbb{R}^{n(n+m+p)}$ be the largest LSI class of systems for which a q-dimensional universal adaptive stabilizer exists. Then for each $(A,B,C) \in \Sigma$:

(i) $(A+BFC, BU^{-1}, VC) \in \Sigma \quad \forall\, F \in \mathbb{R}^{m\times p},\ U \in GL(m),\ V \in GL(p)$

(ii) $(SAS^{-1}, SB, CS^{-1}) \in \Sigma \quad \forall\, S \in GL(n)$.

Proof

Let $(A,B,C) \in \Sigma$. By Corollary 2.5, (A,B,C) can be stabilized by a linear controller of order q and thus also $(A+BFC, BU^{-1}, VC)$, (SAS^{-1}, SB, CS^{-1}). Let $\Sigma^* \supset \Sigma$ be the saturation of Σ with respect to (i), (ii). Since Σ is LSI, so is Σ^*. By Mårtensson's Theorem (1986) there exists a UAS for Σ^*. Since Σ was maximal, $\Sigma^* \subset \Sigma$ and thus $\Sigma^*=\Sigma$ is output feedback invariant.

□

3. SUFFICIENT CONDITIONS FOR ADAPTIVE TRACKING (SCALAR SYSTEMS)

In this section we derive sufficient conditions for universal adaptive tracking of finite dimensional reference signals.

Let $\Sigma(1,1)$ be a given class of scalar linear systems

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= cx(t) \end{aligned} \tag{3.1}$$

and let

$$R_{\psi}, \quad \psi(s) = s^{\ell}+\psi_{\ell-1}s^{\ell-1} + \dots + \psi_{o} \in \mathbb{R}[s]$$

denote the solution space of the differential equation

$$\psi(\tfrac{d}{dt})r(\cdot) \equiv 0 \ . \tag{3.2}$$

For the tracking problem it is required to find an adaptive controller which forces the tracking error $e(t) = y(t)-r(t)$ to go to zero as $t\to\infty$:

$$\lim_{t\to\infty} e(t) = 0 \ . \tag{3.3}$$

The following lemma is well known:

<u>**Lemma 3.1**</u>

Let $r: \mathbb{R}_+ \to \mathbb{R}$ be a solution of $\psi(\frac{d}{dt})r(\cdot) \equiv 0$ with $\psi(s) \in \mathbb{R}[s]$ monic of degree ℓ. Then there exists an observable system $(A,c) \in \mathbb{R}^{\ell\times\ell} \times \mathbb{R}^{\ell}$ such that

$$r(t) = c\bar{x}(t) \tag{3.4a}$$

$$\dot{\bar{x}}(t) = A\bar{x}(t) \tag{3.4b}$$

for an appropriate initial state $\bar{x}(0)$. Moreover, if (A,c) is any scalar observable pair such that ψ divides the characteristic polynomial of A, then (3.4) holds for an appropriate choice of the initial state $\bar{x}(0)$. □

Let now $(A_r,c_r) \in \mathbb{R}^{\ell\times\ell} \times \mathbb{R}^{\ell}$ be observable and satisfy (3.4) with initial state $\bar{x}(0)$. Choose any $b_r \in \mathbb{R}^{\ell\times\ell}$ such that (A_r,b_r,c_r) is controllable and observable and define

$$g_r(s) := 1-c_r[sI-A_r]^{-1}b_r \ . \tag{3.5}$$

Given any $(A,b,c) \in \Sigma(n,1,1)$ with transfer function

$$g(s) = c[sI-A]^{-1}b \qquad 3.6)$$

the state space equations for the augmented system $g(s)g_r(s)$ are:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{b}v \tag{3.7a}$$

$$y = \hat{c}\ \hat{x} \tag{3.7b}$$

where

$$\hat{A} = \begin{bmatrix} A & b_r c_r \\ 0 & A_r \end{bmatrix}, \qquad \hat{b} = \begin{bmatrix} b \\ b_r \end{bmatrix}, \tag{3.8a}$$

$$\hat{c} = [c \;\; 0], \qquad \hat{x} = \begin{bmatrix} x \\ x_r \end{bmatrix} . \tag{3.8b}$$

Since $\psi(\frac{d}{dt})r(\cdot) \equiv 0$ also $\chi(\frac{d}{dt})r(\cdot) \equiv 0$ where χ is the characteristic polynomial of $\hat{A}$. Assuming observability of $(\hat{A},\hat{c})$ Lemma 3.1 implies that $r(\cdot)$ can be generated by $(\hat{A},\hat{c})$ through an appropriate initial state $\bar{x}(0)$:

$$\dot{\bar{x}}(t) = \hat{A}\bar{x}(t) \tag{3.9a}$$

$$r(t) = \hat{c}\bar{x}(t) \tag{3.9b}$$

Let

$$\xi := \hat{x}-\bar{x}$$

$$e := y-r \; .$$

Thus

$$\begin{aligned} \dot{\xi} &= \hat{A}\xi+\hat{b}v \\ e &= \hat{c}\xi \end{aligned} \tag{3.10}$$

Now let $\Sigma_r(\ell,1,1)$ denote a set of order ℓ reference models $(A_r,b_r,c_r,1)$ constructed for a class of reference signals R. Let Σ be a given class of controllable and observable systems (A,b,c) and let $(\Sigma_r(\ell,1,1),\Sigma)$ denote the set of augmented systems $(\hat{A},\hat{b},\hat{c})$ of the form (3.10).

Theorem 3.2

Suppose that (f,g) is a q-dimensional universal stabilizer for the class of observable augmented systems $(\Sigma_r(\ell,1,1),\Sigma)$ of the form (3.10). Let $r(\cdot)$ be a reference signal generated as the output of some $(A_r,b_r,c_r,1) \in \Sigma_r(\ell,1,1)$:

$$r = c_r\bar{x}_r \ , \ \dot{\bar{x}}_r = A_r\bar{x}_r$$

Then the closed loop system

$$\dot{x} = Ax + b(c_r x_r + f(y-r,z))$$
$$\dot{z} = g(y-r,z) \tag{3.11}$$

defines a $(q+\ell)$-dimensional universal tracking controller for Σ:

$$\lim_{t\to\infty} (x(t)-\bar{x}(t)) = 0$$

$$\lim_{t\to\infty} (cx(t)-r(t)) = 0$$

$$\lim_{t\to\infty} z(t) = z_\infty \quad \text{exists.}$$

<u>Proof</u>

By assumption (f,g) stabilizes all systems (3.10), i.e.

$$\dot{\xi} = \hat{A}\xi + \hat{b}f(\hat{c}\xi,z)$$
$$\dot{z} = g(\hat{c}\xi,z) \tag{3.12}$$

satisfies:

$$\lim_{t\to\infty} \xi(t) = 0 \tag{3.13a}$$

$$\lim_{t\to\infty} z(t) = z_\infty \quad \text{exists.} \tag{3.13b}$$

Rewriting (3.12) yields

$$\frac{d}{dt}(\hat{x}-\bar{x}) = \hat{A}(\hat{x}-\bar{x}) + \hat{b}f(y-r,z)$$
$$\dot{z} = g(y-r,z)$$

and thus

$$\dot{x} = Ax+bc_r x_r+bf(y-r,z)$$
$$\dot{z} = g(y-r,z) \ .$$

But then (3.13) implies:

$$\lim_{t\to\infty} (x(t)-\bar{x}(t)) = 0$$

and
$$\lim_{t\to\infty} (\hat{c}\ \xi(t)) = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} (y(t)-r(t)) = 0 \ .$$

□

The following corollary is an immediate consequence of Thm. 3.2.

<u>Corollary 3.3</u>

Let Σ be a class of scalar minimum phase systems and let $r(\cdot)$ be a reference signal generated as the output of some minimum phase system $(A_r,b_r,c_r,1) \in \Sigma_r(\ell,1,1)$. Then there exists a $(r*+\ell-1)$-dimensional tracking controller for Σ if there exists an upper bound $r*$ for the relative degrees of all $(A,b,c) \in \Sigma$.

□

In Helmke, Prätzel-Wolters and Schmid (1989) explicit constructions of such tracking controllers are given for the case of relative degree one, minimum phase systems.

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