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SIMPLE ADAPTIVE CONTROL OF A DISCRETE ALMOST STRICT POSITIVE REAL HEAT TREATMENT SYSTEM

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1. INTRODUCTION

In this paper we consider simple adaptive controllers (SAC) for a heat treatment system of lacquered wires. Heat treatment is the last procedure of a process of enamelling copper wires. Such a production process is for example installed in several furnaces in the factory "China Electro Plant" in Shanghai. The plant model considered here is the result of identification experiments applied to one of the furnaces in this factory (cf. Prätzel-Wolters and Shuzhong Chen (1991)). Our controller is called simple because there are few control parameters to be adapted and no permanent identification of the process model is made. The controller is principally high gain based and can be viewed as a modified discrete version of the continuous time high gain adaptive stabilizers considered in Ilchman et al. (1987). However, the basic system model is non minimum phase. For the treatment of unstable zeros we apply an augmentation procedure proposed in Bar-Kana (1989), which transforms the process into a discrete almost strict positive real system. To avoid gain divergence caused by disturbances in the adaptive high gain controller applied to the augmented system the adaptation law for the real gain matrix K is modified by a stabilizing pole. This idea is also realized in Bar-Kana and Kaufman (1985) and was first proposed by Ioannou et al. (1982).

In section 2 a short description of the heat treatment system and the basic plant-model for the (SAC) is given. Section 3 contains the general description of our adaptive controller and five different closed-loop configurations for the heat treatment system - centralized and decentralized versions of the complete resp. a simplified process model. Finally in section 4 the behaviour of the different controller versions is compared by a variety of simulation studies including output disturbances of the process.

2. LINEARIZED MODELS OF A HEAT TREATMENT SYSTEM

Heat treatment is the last procedure of a process of enamelling wires. The main parts of the heat treatment plant consists of a furnace, several containers with lacquer, a ventilation installation, a servo mechanism and a preheating device.

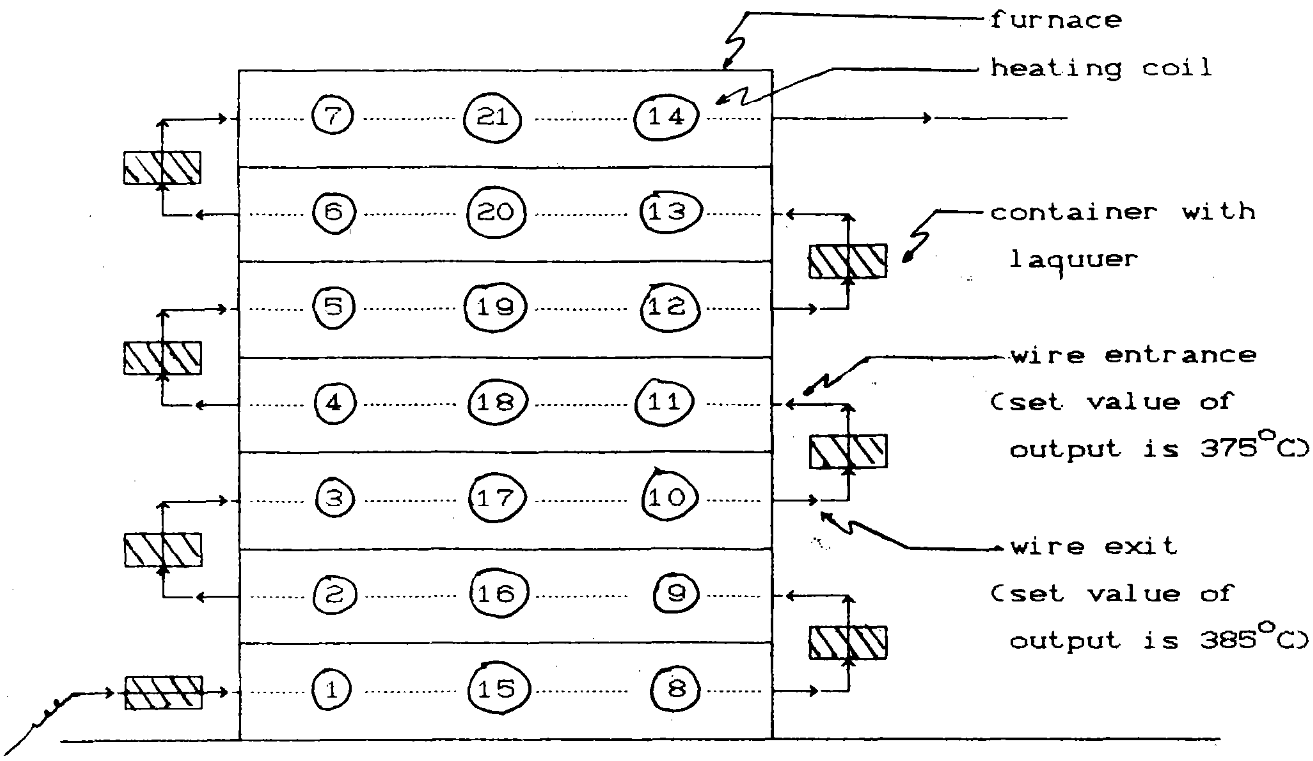

Fig. 1 The heat treatment furnace

The furnace is divided into 7 layers. Each layer is separated from the others by a steel plate (10 mm thick). The sizes of the furnace are slightly different. A typical example is as following: 6 m long, 1.5 m wide, 1.3 m high. There are 3 separated heating coils and 3 temperature measuring sensors in each layer. The electric current resp. voltage of the heating coils are considered as inputs of the system and the temperatures at the positions of the sensors as system outputs. There are 21 inputs and 21 outputs. The desired values at the temperature measurement points are 375°C at the layer entrances and 385°C at the layer exits. 32-40 untreated copper wires are fed through a preheating device before the central heating process. Via this preheating the adherence properties of the lacquer are improved. On every furnace layer the preheated copper wires pass through containers with lacquer into the heating zone with the 3 separated heating coils. In this way the lacquered wires are moved with a certain speed by a servo mechanism through the furnace from the lowest to the highest layer. This process of enameling and heat treatment is repeated 3 times. A ventilation installation is placed on the top of the furnace. The main parameters of the ventilation system are fixed during the system is operating. Some parameters, for example the amount of passing air, are properly adjusted by workers on duty based on their experience according to the different temperatures around the furnace. The ventilation system makes the temperature inside the furnace more uniform and at the same time displaces the polluted air inside. Furthermore there is a special flux mixed with the lacquer. When it is heated, the flux will vaporize and release heat. This process helps uniforming the temperature inside of the furnace.

The speed of the copper wires is fixed during the heating process however it varies with different wire diameters (0.2 mm to 0.25 mm). In our modelling and controller design the speed is not used as a control variable although it influences the heat treatment process. Due to technical restrictions the inputs are of the type switch on or switch off, i.e. there are only two possible input values, $u(t) \in \{0,220\}$.

Our model of the furnace is based on closed loop identification experiments (cf. Prätzel-Wolters and Shuzhong (1991)) around the operating point of the system. As a result of these experiments the following two assumptions were applied:

(i) System output i, i=2,3,4,5,6,9,10,11,12,13 depends only on system input i and system outputs i-1 and i+1. System outputs i=1,7,8, resp. 14 depend only on system input i and system outputs 2, 6,9, resp.13. The influence of the other outputs can be neglected.

(ii) The furnace is <u>symmetric</u> with respect to the vertical axis through the heating coils 15-21, hence instead of a 14×14 model it suffices to consider a 7×7 input-output-model.

(iii) The process is modeled as a discrete-time, time-invariant linear model where one time-step corresponds to 5 minutes in real time.

According to these assumptions we obtain a mathematical model of the following type:

$$A(q^{-1})y(t) = B(q^{-1})u(t) \qquad (2.2)$$

where:

$$A(q^{-1}) = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} & 0 \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} & a_{67} \\ 0 & 0 & 0 & 0 & 0 & a_{67} & a_{77} \end{pmatrix}, \quad (2.2a)$$

$$B(q^{-1}) = diag\begin{pmatrix} b_{11} & b_{22} & b_{33} & b_{44} & b_{55} & b_{66} & b_{77} \end{pmatrix} \quad (2.2b)$$

and $$y(t) = (y_1(t), \ldots, y_7(t))^T ,$$
$$u(t) = (u_1(t), \ldots, u_7(t))^T .$$

Here $a_{ij}(q^{-1})$, $i,j=1,\ldots,7$ and $b_{ii}(q^{-1})$, $i=1,\ldots,7$ are polynomials in the operator $q^{-1}(v(t)) := v(t-1)$.

(2.2) is designed as a linear model for the heat treatment process around an operating point. Thus $y_i(t)$ denotes the error signal:

$$y_i(t) := y_i^{meas}(t) - y_i^{ref}(t), \quad i=1,\ldots,7 . \quad (2.3)$$

Furthermore $u_i(t)$ denotes a normalized input signal.

$$-1 \leq u_i(t) = \frac{u_i^{meas}(t)-2.5}{2.5} \leq 1, \quad i=1,\ldots,7 \quad (2.4)$$

where $u_i^{meas}(t)$ is the connecting time of the i-th control device between the time samplings t-1 and t (5 minutes).

In order to reduce the complexity of the model the following structure of (2.2) (system of second order difference equations) was <u>presupposed</u>:

$$I_7 y(t+2) + A_1 y(t+1) + A_2 y(t) = B_1 u(t+1) + B_2 u(t) \quad (2.5)$$

respectively

$$A(q^{-1}) = I_7 + A_1 q^{-1} + A_2 q^{-2}, \quad B(q^{-1}) = B_1 q^{-1} + B_2 q^{-2} \quad (2.5a)$$

when $A_1, A_2, B_1, B_2 \in \mathbb{R}^{7\times 7}$

$$y = (y_1, \ldots, y_7)^T \in \mathbb{R}^7, \quad u = (u_1, \ldots, u_7)^T \in \mathbb{R}^7 .$$

The A_1, A_2, B_1, B_2 were determined by identification experiments:

$$A_1 = \begin{pmatrix} -1.5008 & 0.2150 & 0 & 0 & 0 & 0 & 0 \\ -0.1337 & -0.6690 & -0.0996 & 0 & 0 & 0 & 0 \\ 0 & -0.2550 & -0.9810 & -0.2377 & 0 & 0 & 0 \\ 0 & 0 & -0.0443 & -1.1829 & -0.4388 & 0 & 0 \\ 0 & 0 & 0 & -0.1764 & -0.9711 & -0.2157 & 0 \\ 0 & 0 & 0 & 0 & -0.0069 & -1.5224 & -0.1780 \\ 0 & 0 & 0 & 0 & 0 & -0.2249 & -0.9125 \end{pmatrix} \quad (2.5b)$$

$$A_2 = \begin{pmatrix} 0.5059 & -0.2172 & 0 & 0 & 0 & 0 & 0 \\ 0.0223 & 0.0921 & -0.0401 & 0 & 0 & 0 & 0 \\ 0 & 0.1955 & 0.2410 & 0.0485 & 0 & 0 & 0 \\ 0 & 0 & -0.0565 & 0.5121 & 0.2103 & 0 & 0 \\ 0 & 0 & 0 & 0.1089 & 0.1207 & 0.0609 & 0 \\ 0 & 0 & 0 & 0 & -0.0053 & 0.5484 & 0.1664 \\ 0 & 0 & 0 & 0 & 0 & 0.1918 & 0.0461 \end{pmatrix} \quad (2.5c)$$

$$B_1 = \mathrm{diag}(1.6991, 0.7845, 1.1435, 0.5389, -0.0689, 1.2515, 0.4866)$$
$$B_2 = \mathrm{diag}(1.7732, -0.4100, 2.9399, 3.3687, 1.8529, -0.7570, 1.9747) \quad (2.5d)$$

3. A SIMPLE ADAPTIVE CONTROL CONFIGURATION

The identified system (2.5) is stable, however, some of the poles are close to the unit circle (cf. table 3.1) and generate an unsatisfactory transient behaviour - some components of $y_i(t)$ tend to zero very slowly, oscillating with slowly decreasing amplitudes. To improve the performance a feedback controller is desired which should be <u>adaptive</u> for the following reasons:

- The obtained model is a linearization of the nonlinear real process around an operating point.
- The parameters of the true systems depend on the wire speed, furnace outside temperature, ventilation speed, etc.
- There are unknown disturbances originating from the voltage supply, measurement sensor noise and sensor failures.

The demands for the controller are:

- Asymptotically stable closed loop systems,
- Low values for $\sum_{t=0}^{\infty} |y_i(t)|$ resp. $\sum_{t=0}^{\infty} y_i^2(t)$, i=1,...,7, for regulation of y(·) around the operating point,
- Bounded outputs within prescribed tolerance bounds for L_∞ input and output disturbances.

The SAC described in this section satisfies these requirements and works without identification of the plant parameters. Those controllers obviously require certain structural properties of the process to be controlled. For example the adaptive <u>high gain control law</u>:

$$u(t) = k(t)y(t) , \quad k(t+1) = k(t)^2 \tag{3.1}$$

cannot be applied because the model (2.5) has unstable zeros and hence is not minimum phase.

However, it is easily verified that there exist constant invertible feedback matrices $K \in \mathbb{R}^{7\times 7}$ such that the closed loop transfer function

$$G_c(z) = [I_7+G(z)K]^{-1}G(z) \tag{3.2}$$

is asymptotically stable.
It is shown in Bar-Kana (1986) that the augmented system

$$G_a(z) = G(z) + K^{-1} \tag{3.3}$$

then is <u>discrete almost strict positive real</u> (DASPR). A proper rational transfer function G(z) is DASPR if there exists a constant matrix F such that $H(s) = [I+G(z)F]^{-1}G(z)$ is <u>discrete strict positive real</u> (DSPR), i.e. there exists a ρ, $0<\rho<1$, such that $H(\rho z)$ is <u>discrete positive real</u> (DPR). A transfer function H(z) is called (DPR), if it satisfies the conditions:

(DPRI) The entries $h_{ij}(s)$ of H(s) are analytic in $\{z \in \mathbb{C} \mid |z| \geq 1\}$.

(DPRII) $G(z)+\overline{G(z)}^T$ is positive semidefinite hermitian.

In our context it is important that DASPR implies high gain stability. Therefore after a suitable augmentation we can apply an adaptive high gain feedback concept. The following simple adaptive controller (SAC) is a special version of a discrete adaptive model reference controller developed by Bar-Kana and Kaufman (1983) and Bar-Kana (1989).

Consider a linear discrete time-invariant process S(A,B,C,D):

$$\begin{aligned} x(t+1) &= Ax(t)+B(u(t)+d_i(t)), \quad x(0) = x_o \\ y(t) &= Cx(t)+Du(t)+d_o(t) \end{aligned} \tag{3.4}$$

$x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p \ni y(t)$, $t \in \mathbb{N}_o$

$d_i(t), d_o(t) \in \mathbb{R}^p$ disturbances

and let:

$$G(z) = C[zI-A]^{-1}B+D \in \mathbb{R}(z)^{p\times p} \tag{3.5}$$

the transfer function of the undisturbed system.
For the controller let

$$u(t) = K(t+1)y(t) \ . \tag{3.6}$$

The adaptation law for K(t) consists of an integral and a proportional part:

$$K(t) = K_I(t)+K_p(t) \tag{3.6a}$$

$$K_p(t+1) = y(t)y^T(t)\theta \tag{3.6b}$$

$$K_I(t+1) = K_I(t)+y(t)y^T(t)\Gamma-\rho K_I(t+1) \tag{3.6c}$$

$$K_I(1) = K_{I0}$$

θ symmetric positive semidefinite (3.6d)

Γ symmetric positive definite (3.6e)

$\rho \in \mathbb{R}$, $\rho \geq 0$. (3.6f)

(3.6a) is equivalent to

$$K(t+1) = y(t)y^T(t)\theta + K_{I0}(1+\rho)^{-t} + \sum_{\tau=1}^{t} (1+\rho)^{-(t-\tau+1)} y(\tau)y^T(\tau)\Gamma \tag{3.7}$$

3.1 Theorem

(I) Under the assumptions:

(i) $G(z)$ is DASPR

(ii) $rk_{\mathbb{R}(z)}(G(z)+G(z)^T) = p$

(iii) $d_i, d_o \in L_\infty(\mathbf{N}_o, \mathbb{R}^p)$

The closed loop system (3.4), (3.6) satisfies for all initial values K_{I0}, $x(0) = x_o$:

$$u(\cdot), y(\cdot) \in L_\infty(\mathbf{N}_o, \mathbb{R}^p), \; K(\cdot) \in L_\infty(\mathbf{N}_o, \mathbb{R}^{p\times p}) \tag{3.8}$$

(II) If $d_i \equiv d_o \equiv 0$ and $\rho = 0$, then additionally:

$$\lim_{t\to\infty} y(t) = 0 \tag{3.9}$$

for all initial values K_{I0}, $x(0) = x_o$.

Proof:

(cf. Bar-Kana (1983), Bar-Kana and Kaufman (1983), Bar-Kana (1989), Prätzel-Wolters and Reinke (1991).)

3.2 Remark:

If input and output disturbances $d_i(\cdot)$ resp. $d_o(\cdot)$ enter the system perfect tracking to 0 is not possible. To consider (3.6c) instead of the simpler adaptation law $K_I(t+1) = K_I(t)+y(t)y(t)^T\Gamma$ is a strategy for avoiding divergence of integral gains. Without this term the integral gains of (3.6c) would steadily diverge to infinity, although the closed loop system remains stable. However, diverging gains are unacceptable for a "realistic" controller.

Theorem 3.1 (I) guarantees globally bounded outputs in the presence of bounded disturbances, however, perfect stabilization ($y(t) \xrightarrow{t\to\infty} 0$) is not possible in noisy environment, although the finite tracking errors $y(\infty)$ may be very small.

To apply now the (SAC) to our heat treatment system the I/O-model (2.5) is transformed into state-space form. Furthermore the input restrictions have to be considered. Our implementation works with $\bar{u}(t)$ instead of $u(t)$, where:

$$\bar{u}(t) = \begin{cases} 1 & \text{if } u(t) \geq 1 \\ u(t) & \text{if } |u(t)| < 1 \\ -1 & \text{if } u(t) \leq -1 \end{cases} \qquad (3.10)$$

We distinguish between the complete model (CM) and a simplified decentralized model (SM), where only the diagonals of A_1 and A_2 are considered:

$$\begin{aligned} A_1 &= \mathrm{diag}(-1.5008, -0.6690, -0.9810, -1.1829, -0.9711, -1.5224, -0.9125) \\ A_2 &= \mathrm{diag}(0.5059, 0.0921, 0.2410, 0.5121, 0.1207, 0.5484, 0.0461) \\ B_1 &= \mathrm{diag}(1.6991, 0.7845, 1.1435, 0.5389, -0.0689, 1.2515, 0.4866) \\ B_2 &= \mathrm{diag}(1.7732, -0.4100, 2.9399, 3.3687, 1.8529, -0.7570, 1.9747) \end{aligned} \qquad \text{(SM)}$$

(SM) is a direct sum of seven independent subsystems.
For the complete model (CM) B_1 and B_2 coincide with B_1 and B_2 in (SM) and A_1 and A_2 are of the form:

$$A_1 = \begin{pmatrix} -1.5008 & 0.2150 & 0 & 0 & 0 & 0 & 0 \\ -0.1337 & -0.6690 & -0.0996 & 0 & 0 & 0 & 0 \\ 0 & -0.2550 & -0.9810 & -0.2377 & 0 & 0 & 0 \\ 0 & 0 & -0.0443 & -1.1829 & -0.4388 & 0 & 0 \\ 0 & 0 & 0 & -0.1764 & -0.9711 & -0.2157 & 0 \\ 0 & 0 & 0 & 0 & -0.0069 & -1.5224 & -0.1780 \\ 0 & 0 & 0 & 0 & 0 & -0.2249 & -0.9125 \end{pmatrix}$$

$$\text{(CM)}$$

$$A_2 = \begin{pmatrix} 0.5059 & -0.2172 & 0 & 0 & 0 & 0 & 0 \\ 0.0223 & 0.0921 & -0.0401 & 0 & 0 & 0 & 0 \\ 0 & 0.1955 & 0.2410 & 0.0485 & 0 & 0 & 0 \\ 0 & 0 & -0.0565 & 0.5121 & 0.2103 & 0 & 0 \\ 0 & 0 & 0 & 0.1089 & 0.1207 & 0.0609 & 0 \\ 0 & 0 & 0 & 0 & -0.0053 & 0.5484 & 0.1664 \\ 0 & 0 & 0 & 0 & 0 & 0.1918 & 0.0461 \end{pmatrix}.$$

The following table contains the poles and zeros of (SM) and (CM):

subsystem	poles (SM)	zeros (SM) and (CM)
1	0.9896 0.5112	-1.0436
2	0.4752 0.1938	0.5226
3	0.4905 ± 0.0202i	-2.5710
4	0.5915 ± 0.4028i	-6.2511
5	0.8248 0.1463	26.8926
6	0.9373 0.5851	0.6049
7	0.8588 0.0537	-4.0582

poles (CM)
-0.0123 0.0951
0.1996 ± 0.1926i
0.5455 ± 0.3762i
0.5638 ± 0.0930i
0.9918 0.9976
0.6809 ± 0.0278i
0.8301 0.8577

Table 3.1 Poles and Zeros of (SM) and (CM)

The subsystems of (SM) are all of relative degree 1. Both systems (SM) and (CM) are asymptotically stable, however with unstable zeros, hence neither (SM) nor (CM) is DASPR. For both systems an augmentation matrix D has to be determined such that the augmented system is DASPR.

To select D two conflicting strategies have to be taken into account:

- To keep the stationary error small $\|D\|$ should be as small as possible,
- Stable zeros close to the unit circle have to be avoided, because the simple adaptive control algorithm places the closed loop poles for high gain $k(t)\to\infty$ onto the zeros. Hence $\|D\|$ should be taken not too small.

There does not exist a systematic algorithm for an optimal choice of $\|D\|$ (together with Γ, θ and ρ) (optimal for examples in the sense of minimizing $\sum_t |y_i(t)|$ or $\sum_t y_i^2(t)$). For our system, which is already asymptotically stable, the design objective $y(t)\to 0$ can be achieved with $u(t)\to 0$, i.e. it is not necessary to keep $\|D\|$ as small as possible.

For the implementation of our controllers the selection of D, Γ, θ and ρ is based on a "trial-and-error-strategy". We distinguish between four controllers:

(C_1) centralized control of (SM)
(C_2) decentralized control of (SM)
(C_3) centralized control of (CM)
(C_4,C_5) decentralized control of (CM)

<u>(C_1) and (C_2)</u>

The following table 3.2 contains the maximal gains k_i for which the seven subsystems remain stable and the associated lower bounds $d_{ii}=k_i^{-1}$ for the entries of the augmentation matrix.

subsystem i	maximum gain k_i	$d_{ii} = k_i^{-1}$
1	0.2786	3.5888
2	1.4743	0.6783
3	0.2582	3.8734
4	0.1448	6.9045
5	0.4746	2.1072
6	1.5289	0.6541
7	0.4831	2.0701

Table 3.2 Maximal admissible gains for Σ_i, i=1,...,7

As already mentioned small values of d_{ii} generate oscillating behaviour of y(t) and $y_a(t)$ with high, slowly decreasing amplitudes. The following choice of the d_{ii}, i=1,...,7, turned out to be satisfactory:

subsystem i	d_{ii}	zeros	maximum modulus of zeros
1	13	0.6850 ± 0.4159i	0.8014
2	0.92	0.5098 -0.6835	0.6935
3	14	0.4497 ± 0.4988i	0.6716
4	150	0.5897 ± 0.4323i	0.7311
5	5.1	0.4923 ± 0.4916i	0.6957
6	0.78	0.6100 -0.6920	0.6920
7	4.5	0.4220 ± 0.5685i	0.6964

Table 3.3 Zeros of augmented system

For (C_1) the model (SM) is considered as one system of state-space dimension 14. The parameters for the centralized controller are:

(C_1) $$\Gamma = \theta = 100 I_7 \ , \ \rho=0$$

For (C_2) every subsystem Σ_i, $i=1,...,7$, of (SM) is separately controlled with the following parameters:

subsystem i	d_{ii}	Γ	Θ
1	13	10^5	10^5
2	0.92	1	1
3	14	10	10
4	150	100	100
5	5.1	100	100
6	0.78	10^{-1}	10^{-1}
7	4.5	10^{-2}	10^{-2}

Table 3.4 SAC-parameters

<u>(C_3)-(C_5)</u>

(C_3) denotes the implementation of the (centralized) simple adaptive control algorithm for the complete model (CM) with the parameters:

(C_3) $$D = \mathrm{diag}(12,0.82,8,24,9.3,0.78,4.9)$$
$$\Gamma = \theta = 100 I_7 \ , \ \rho=0$$

(C_4) and (C_5) are two different decentralized implementations of (2.5). It is assumed that the inputs and outputs of the seven subsystems Σ_{ii}, $i=1,...,7$, are available for the controller. Only these diagonal subsystems are controlled, while the coupling off diagonal subsystems remain uncontrolled. (C_4) denotes the controller where Σ_{ii}, $i=1,...,7$, are regulated by one central controller (like (C_1)) and for (C_5) every subsystem Σ_{ii} is controlled separately (like (C_2)). The parameters are:

(C_4) $$D = \mathrm{diag}(12,0.82,8,24,9.3,9.78,4.9)$$
$$\Gamma = \theta = 10^{-2} I_7 \ , \ \rho=0$$

(C_5) parameters of (C_2) (cf. Table 3.4)

4. SIMULATION RESULTS

Table 4.1 contains $\sum_{i=1}^{60} |y_i(t)|$ and $\sum_{i=1}^{60} y_i^2(t)$ for (C_1)-(C_5). Fig. 4.1 shows graphically the output behaviour y(t) for (C_1)-(C_5).

i	C_1: $\sum_{i=1}^{60} \lvert y_i(t)\rvert$	C_1: $\sum_{i=1}^{60} y_i^2(t)$	C_2: $\sum_{i=1}^{60} \lvert y_i(t)\rvert$	C_2: $\sum_{i=1}^{60} y_i^2(t)$	C_3: $\sum_{i=1}^{60} \lvert y_i(t)\rvert$	C_3: $\sum_{i=1}^{60} y_i^2(t)$
1	50.9201	353.1498	50.9183	352.8524	45.2739	332.5530
2	16.0645	82.7291	16.1386	82.7330	28.6038	140.0530
3	35.1142	282.4773	34.9264	283.7231	40.4575	297.4632
4	50.5792	448.0869	50.5805	448.1640	69.2261	612.6908
5	41.7933	340.8888	39.3582	304.3078	58.0921	484.4143
6	28.4028	167.4685	28.3359	167.4654	27.3261	164.8746
7	28.1340	179.7831	25.7507	180.2368	30.2750	183.1465

i	C_4: $\sum_{i=1}^{60} \lvert y_i(t)\rvert$	C_4: $\sum_{i=1}^{60} y_i^2(t)$	C_5: $\sum_{i=1}^{60} \lvert y_i(t)\rvert$	C_5: $\sum_{i=1}^{60} y_i^2(t)$
1	153.5876	603.4181	159.4276	621.8611
2	64.1383	176.5999	61.8414	191.8380
3	78.2839	388.8098	112.1965	566.1891
4	99.4210	659.5906	139.4614	959.3393
5	99.3031	551.2757	133.5706	630.2551
6	63.1809	194.8087	81.7881	224.3501
7	45.3374	197.1827	45.5985	191.3886

Table 4.1 Performance numbers for (C_1)-(C_5)

For the simplified model (SM) the behaviour of (C_1) and (C_2) is nearly identically (with small advantages for (C_2) - for y_5 and y_7). For the complete model (CM) the centralized controller (C_3) shows similar results to (C_1) and (C_2). The regulation behaviour of (C_1) and (C_2) is slightly better than that of (C_3). All three controllers are very satisfactory, contrary to the decentralized controller (C_4) and (C_5) for (CM). For (C_4) and (C_5) the initial errors don't vanish fast enough for $t \to \infty$ (for $t \geq 30$ we have $u(t) \approx 0$ and the closed loop behaviour is dominated by the poles close to the unit circle). This non acceptable behaviour is a consequence of the fact that the off diagonal terms in (2.5) are not considered in u(t).

Disturbances

Two kinds of output disturbances are considered:

$$\text{(i)}\quad d_{o1}(t) := \begin{cases} (1)_{7\times 1} & : t = 40k \\ (0)_{7\times 1} & : t \neq 40k \end{cases}, \quad k \in \mathbf{N}_o$$

(ii) $d_{o2}(t)$ is a stochastic ARMA-process of the form:

$$I_7 d_{o2}(t+2) + A_1 d_{o2}(t+1) + A_2 d_{o2}(t)$$
$$= I_7 w(t+2) + C_1 w(t+1) + C_2 w(t)$$

$$w(t) = 0.5T\,\tilde{w}(t)$$

where $\tilde{w}(t)$ is white noise with $Ew(t) = 0$ and $E\{\tilde{w}(t)\tilde{w}^T(t)\} = I_7$. A_1 and A_2 are as in (CM),

$$C_1 = \mathrm{diag}(-0.6, -0.5, -0.6, -1, -1.2, -0.31, -1.3)$$
$$C_2 = \mathrm{diag}(0.05, 0.06, 0.5, 0.25, 0.5, 0, 0.42)$$
$$T = \begin{pmatrix} 1.1650 & 1.7971 & 0.5774 & -0.7989 & 0.4005 & -0.3229 & -0.9235 \\ 0.6268 & 0.2641 & -0.3600 & -0.7652 & -1.3414 & 0.3180 & -0.0705 \\ 0.0751 & 0.8717 & -0.1356 & -0.8617 & 0.3750 & -0.5112 & 0.1479 \\ 0.3516 & -1.4462 & -1.3493 & -0.0562 & 1.1252 & -0.0020 & -0.5571 \\ -0.6965 & -0.7012 & -1.2704 & 0.5135 & 0.7286 & 1.6065 & -0.3367 \\ 1.6961 & 1.2460 & 0.9846 & 0.3967 & -2.3775 & 0.8476 & 0.4152 \\ 0.0591 & -0.6390 & -0.0449 & -0.7562 & -0.2738 & 0.2681 & 1.5578 \end{pmatrix}$$

The parameters for (C_1), (C_2) and (C_3) are identical to those in the disturbance free simulations (but $\rho=0.01$ is selected instead of $\rho=0$).

Fig. 4.2 shows the seven components of d_{o2}, Fig. 4.3 and Fig. 4.4 show the behaviour of $y_i(t)$, $i=1,\ldots,7$, if $d_{o1}(t)$ resp. $d_{o2}(t)$ is applied.

Table 4.2 contains $\sum_{i=1}^{160} |y_i(t)|$ and $\sum_{i=1}^{160} y_i^2(t)$ for (C_1), (C_2) and (C_3) and $d_{o1}(t)$ resp. $d_{o2}(t)$.

i	c_1 disturbance d_{o1} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_1 disturbance d_{o1} $\sum_{i=1}^{160} y_i^2(t)$	c_2 disturbance d_{o1} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_2 disturbance d_{o1} $\sum_{i=1}^{160} y_i^2(t)$	c_3 disturbance d_{o1} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_3 disturbance d_{o1} $\sum_{i=1}^{160} y_i^2(t)$
1	142.7370	862.6601	142.8902	862.5816	134.5089	842.3290
2	63.6579	486.2835	63.7001	486.2696	82.7288	550.4965
3	100.3076	726.1270	99.4714	725.7056	117.9883	785.0111
4	93.9706	848.7431	93.9498	848.8152	161.8776	1115.801
5	99.5881	763.7535	88.6642	712.0772	131.2883	934.9157
6	73.2656	572.2366	72.8516	572.2151	74.3432	570.4993
7	88.5619	608.4826	84.3373	609.6167	92.0852	616.5935
i	c_1 disturbance d_{o2} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_1 disturbance d_{o2} $\sum_{i=1}^{160} y_i^2(t)$	c_2 disturbance d_{o2} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_2 disturbance d_{o2} $\sum_{i=1}^{160} y_i^2(t)$	c_3 disturbance d_{o2} $\sum_{i=1}^{160}\lvert y_i(t)\rvert$	c_3 disturbance d_{o2} $\sum_{i=1}^{160} y_i^2(t)$
1	282.6125	996.7547	281.6264	992.6683	269.0638	927.7774
2	307.4604	1047.957	307.1939	1047.389	153.4852	343.4632
3	442.0752	2091.824	443.7912	2106.561	170.6492	557.7613
4	1057.056	11467.45	1057.089	11468.25	285.1048	1060.096
5	529.7932	3260.612	676.6102	5418.604	262.1168	867.1789
6	306.9017	952.2822	307.2699	955.6666	332.2325	1075.433
7	184.3424	428.0020	188.1831	434.4412	171.6152	400.9852

Table 4.2 Performance numbers for the disturbed system

For d_{o1} (C_1) and (C_2) show nearly identical control behaviour. The complete model (CM) is stronger affected by d_{o1} than the simplified model (SM). For d_{o2} we have opposite results. (C_3) is a lot better than (C_1) resp. (C_2). For y_2, y_4 and y_5 the control error is approximately identical to the disturbance if (C_1) and (C_2) are considered, for y_3 it is relatively high. The disturbance $d_{o2}(t)$ is not really rejected for y_2-y_5. For y_2 and y_5 the reason is the input restriction (3.10), for y_3 and y_4 the relatively high values $d_{33}=14$ and $d_{44}=150$ are responsible.

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output 1

output 2

output 3

output 4

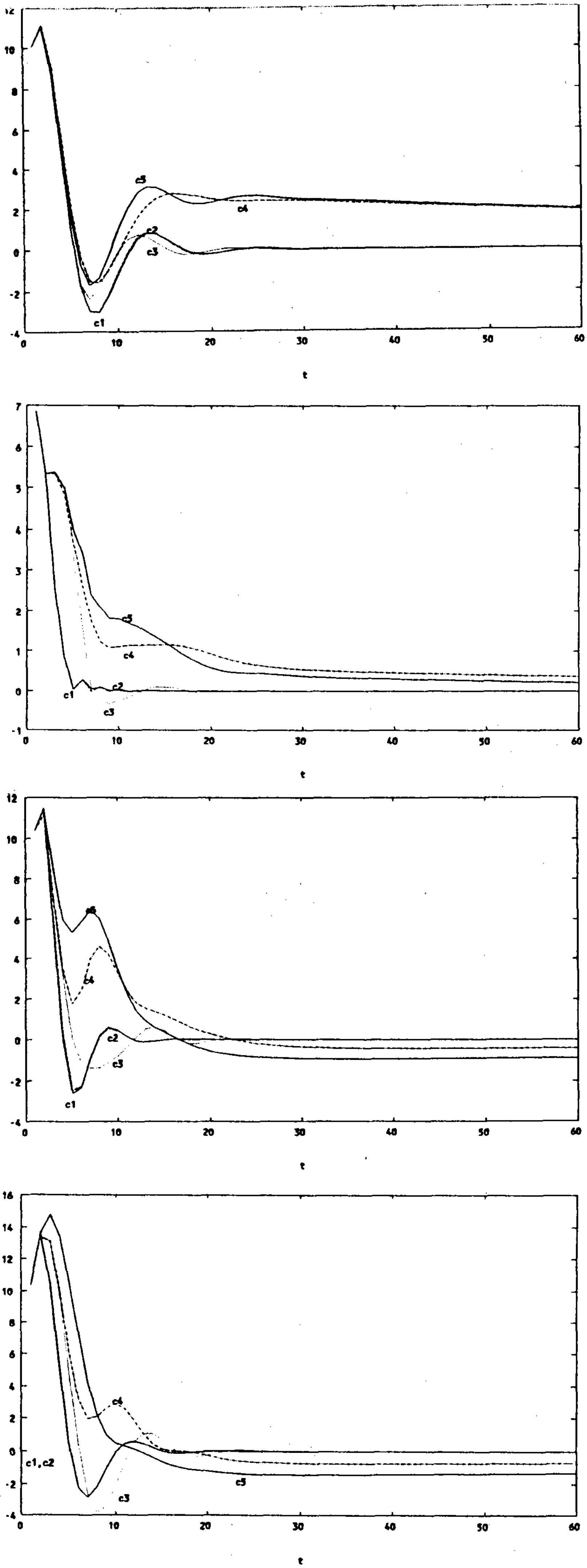
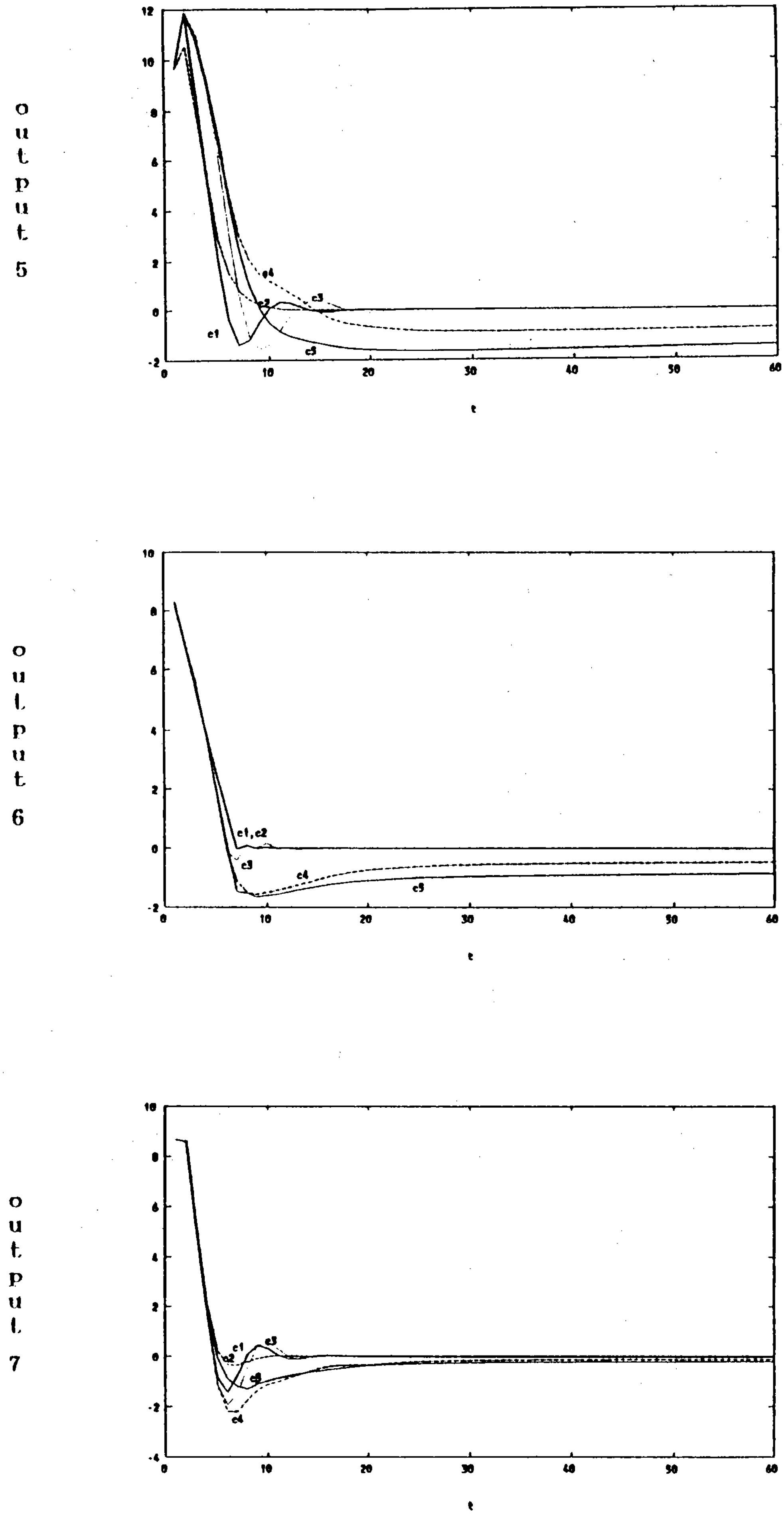


FIGURE 4.1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t
c1
c2
c3
c4
c5
output 6
t
c1,c2
c3
c4
c5
output 7
t
c1
c2
c3
c4
c5

FIGURE 4.1

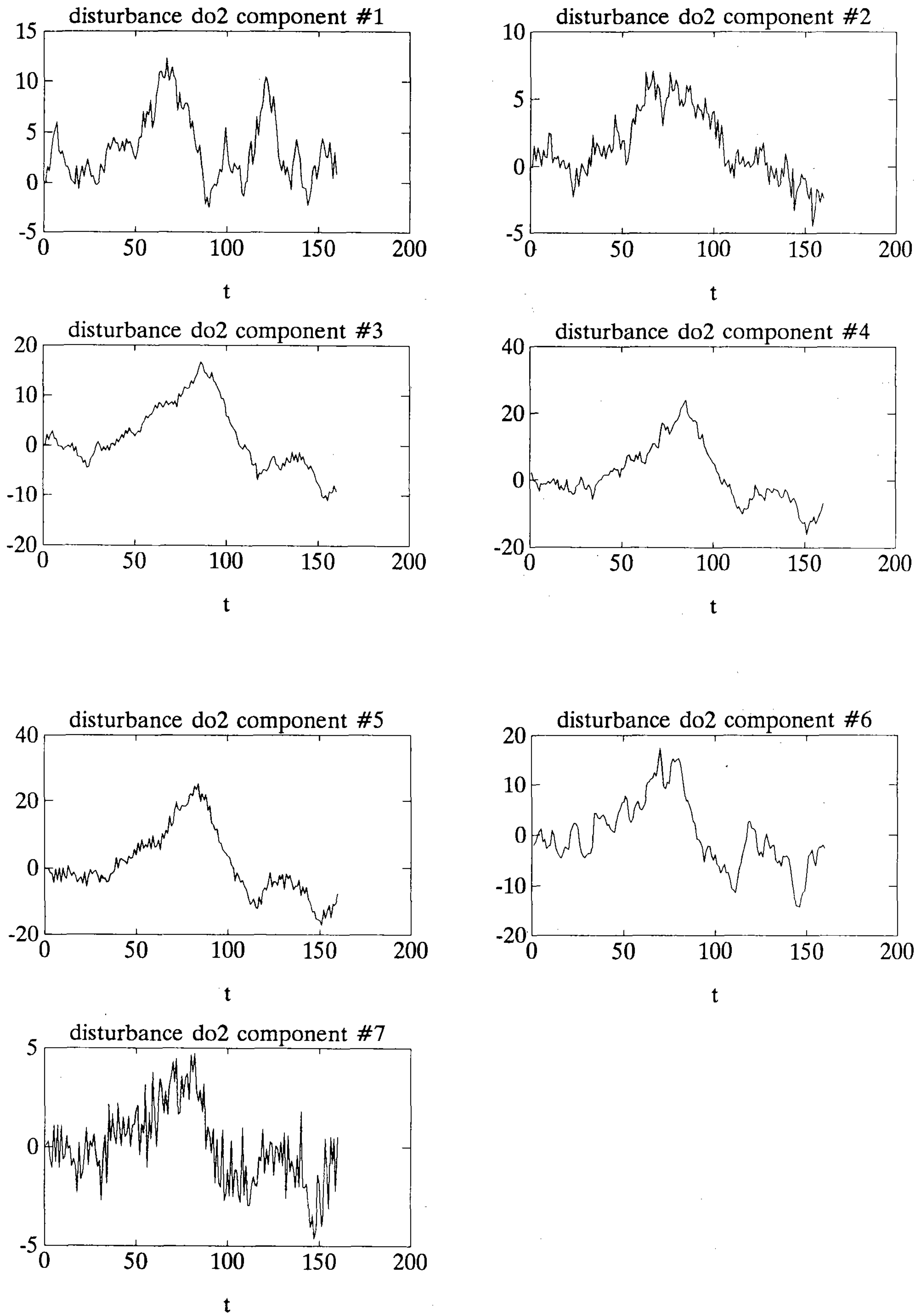

disturbance do2 component #1
disturbance do2 component #2
disturbance do2 component #3
disturbance do2 component #4
disturbance do2 component #5
disturbance do2 component #6
disturbance do2 component #7
t
0
50
100
150
200

FIGURE 4.2

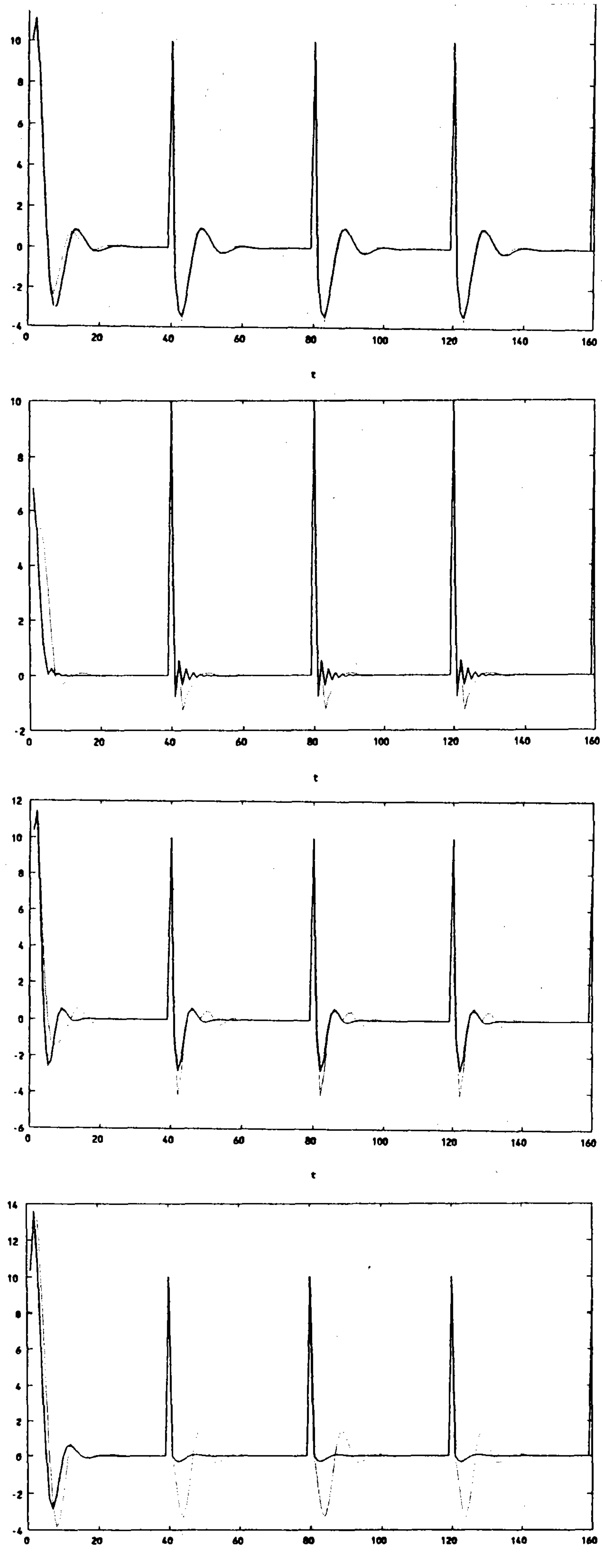
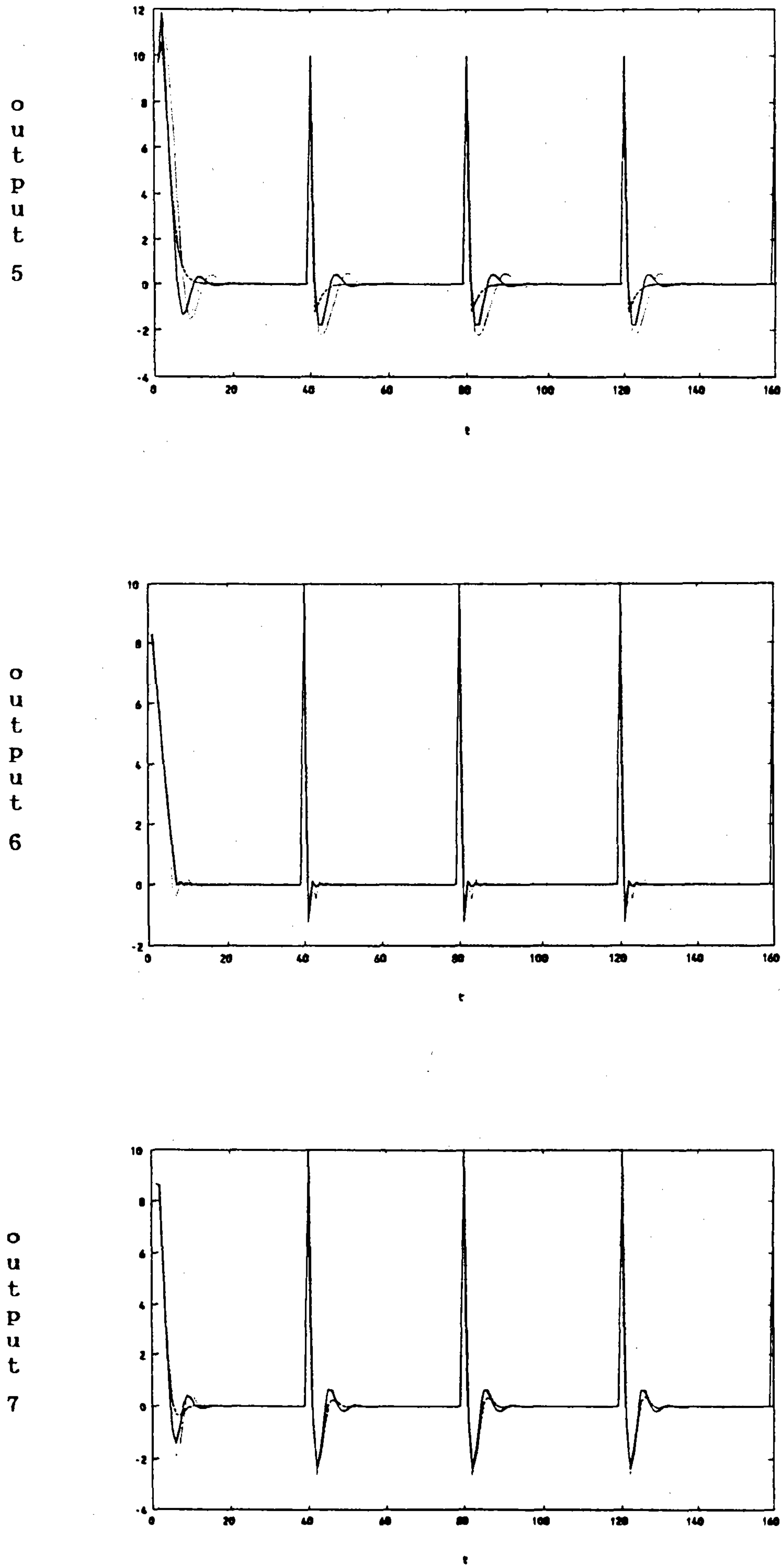


FIGURE 4.3

—— C_1

---- C_2

.... C_3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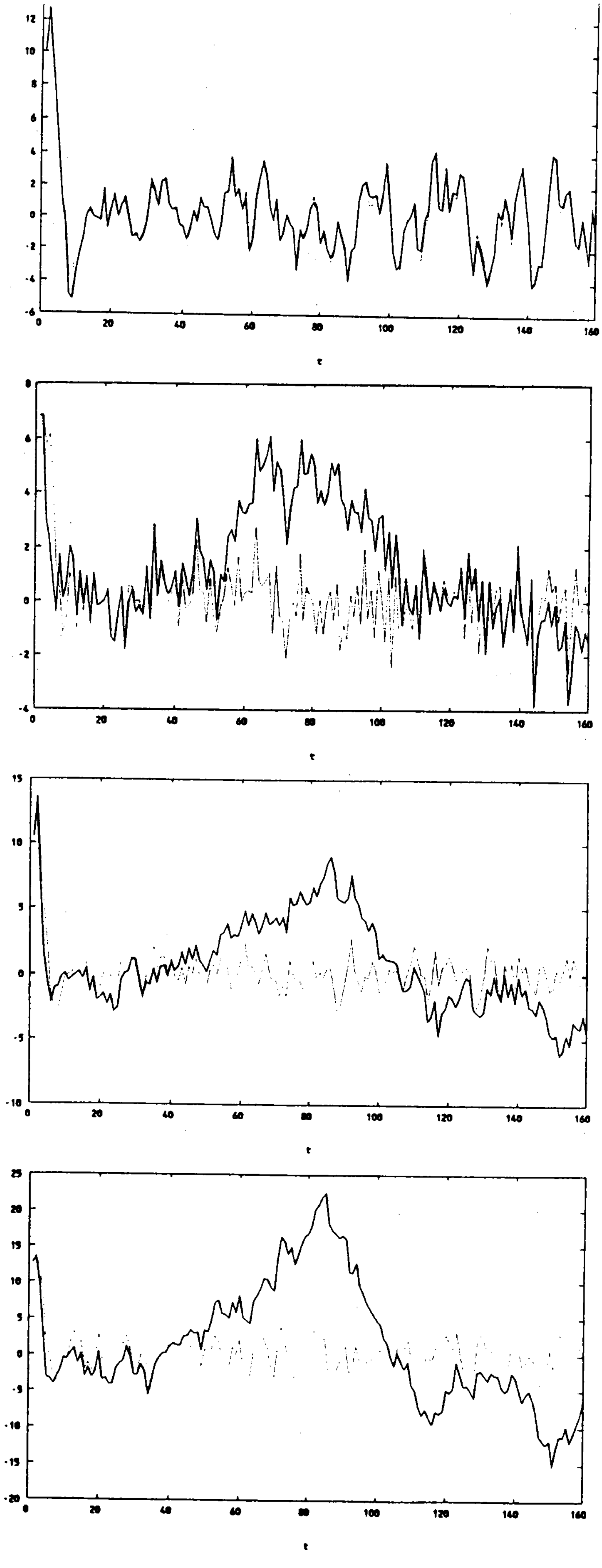
12
10
8
6
4
2
0
-2
-4
0
20
40
60
80
100
120
140
160
t
output 6
10
8
6
4
2
0
-2
0
20
40
60
80
100
120
140
160
t
output 7
10
8
6
4
2
0
-2
-4
0
20
40
60
80
100
120
140
160
t

FIGURE 4.3



output 1

output 2

output 3

output 4

FIGURE 4.4

—— C_1
---- C_2
.... C_3

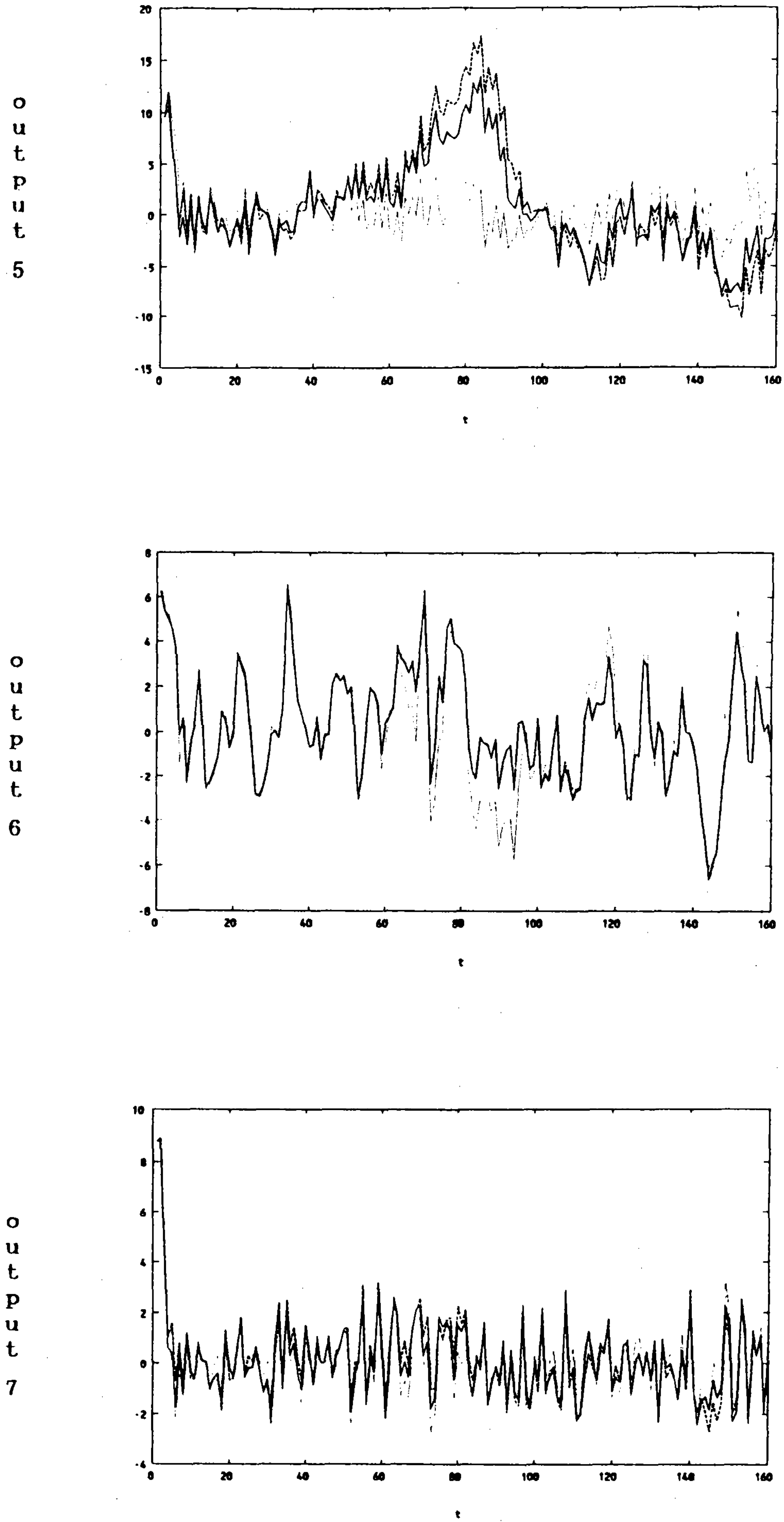


FIGURE 4.4