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# MODELLING AND CONTROLLER DESIGN FOR HEAT TREATMENT PROCESSING OF ENAMELLED WIRES

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# 1 The heat treatment system of lacquered wires

Heat treatment is the last procedure of a process of enamelling wires. The main parts of the heat treatment plant consist of a furnace, several containers with lacquer, a ventilation installation, a servo mechanism and a preheating device.

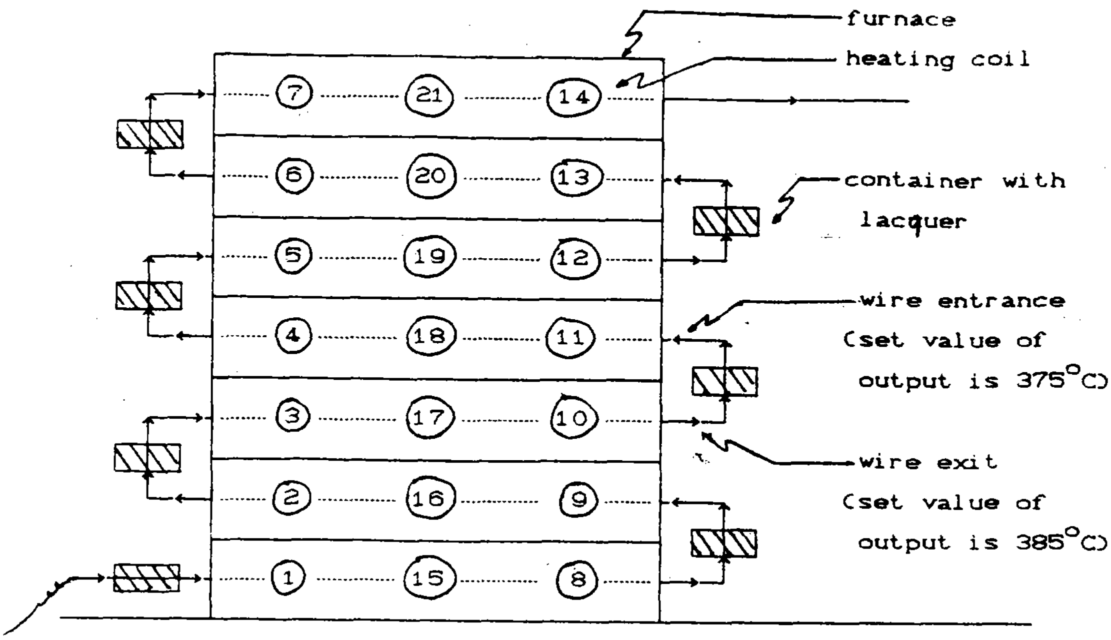



Fig. 1 The heat treatment furnace

A furnace is divided into 7 layers. Each layer is separated from the others by a steel plate (10 mm thick). The sizes of the furnaces are slightly different. A typical example is as following: 6 m long, 1.5 m wide, 1.3 m high. There are three separated heating coils and 3 temperature measuring sensors in each layer. The electric current or/and voltage of the heating coils are considered as inputs of the system and the temperatures at the positions of the sensors as system outputs. There are 21 inputs and 21 outputs. The desired values at the temperature measurement points are $375^0$ C at the layer entrances and $385^0$C at the layer exits. 32-40 untreated copperwires are fed through a pretreating device before the central heating process. Via this preheating the adherence properties of the lacquer are improved. On every furnace layer the preheated copper wires pass through the containers with lacquer into the heating zone with the three seperated heating coils. This way the lacquered wires are moved with a certain speed by a servo mechanism through the furnace from the lowest to the highest layer. This process of enameling and heat treatment is repeated three times. A ventilation installation is placed on the top of the furnace. The main parameters of the ventilation system are fixed during the system is operating. Some parameters, for example the amount of passing air, are properly adjusted by workers on duty based on their experience according to the different temperatures around the furnace. The ventilation system makes the temperature inside the furnace more uniform and at the same time displaces the polluted air inside. Furthermore there is a special flux mixed with the lacquer. When it is heated, the flux will vaporize and release heat. This process helps uniforming the temperature inside of the furnace. The speed of the copper wires is fixed during the heating process however it varies with different wire diameters (0.2 mm to 0.25 mm). In our modelling and controller design the speed is not used as a control variable although it influences the heat treatment process.

# 2 Modelling and identification

In Fig. 1 the numbers 1-14 denote local system loops whose input can be controlled, while the heating coils 15-21 have fixed inputs (voltage supply). If the loops 1-14 are controlled well then also in the interior system measuring points show temperatures which are close to the desired set values and the product quality is satisfying. For the modelling procedure the loops 1-14 are ignored and the process is modelled as a $14 \times 14$ input-output system.

The desired reference temperatures are $375^0$ C at the heating coils 8-14 and $385^0$ C at the heating coils 1-7. Due to the technical restrictions the inputs are of switch on switch off type, i.e. there are only two possible input values, $u(t) \in \{0, 220\}$.
At present the furnace is considered as 14 independent identical local control loops consisting of a heating coil (HC) a temperature sensor (TS) and a fixed PI-controller:

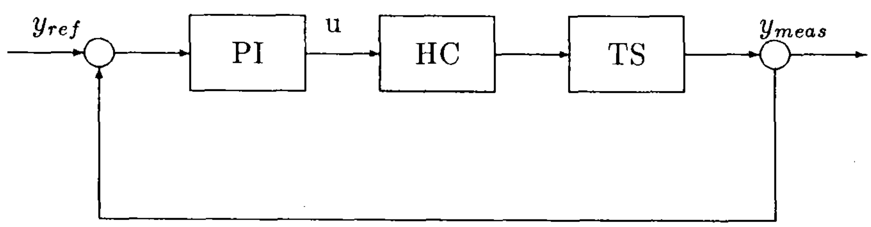



Fig. 2: Single PI-controlled system loop

However because of couplings between the 21 heating coils which are not considered by this decentralized control strategy the process is badly controlled and the error signals $e^i(t) = y_i^{ref}(t) - y_i^{meas}(t)$ often escape from the tolerance bands of size $\Delta T = 10^0$C around the operating temperatures $375^0$C resp. $385^0$C.
Our model of the furnace is based on closed loop identification experiments around the operating point of the system. As a result of these experiments the following two modelling assumptions were applied:

(i) System output $i, i = 2, 3, 4, 5, 6, 9, 10, 11, 12, 13$, depends only on system input $i$ and system outputs $i - 1$ and $i + 1$. System output $i = 1, 7, 8, 14$ depend only on system input $i$ and system outputs $2, 6, 9$ resp. $13$. The influence of the other outputs can be neglected.

(ii) The furnace is symmetric with respect to the vertical axis through the heating coils 15-21, hence instead of a 14 × 14 model it suffices to consider a 7 × 7-input-output-model.

(iii) The process is modeled as a discrete-time, time-invariant linear model where one time-step corresponds to 5 minutes in real time.

According to these assumption we obtain a mathematical model of the following type:

$$A(q^{-1})y(t) = B(q^{-1})u(t) \tag{2.1a}$$

where:

$$A(q^{-1}) = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} & 0 \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} & a_{67} \\ 0 & 0 & 0 & 0 & 0 & a_{76} & a_{77} \end{pmatrix}, \tag{2.1b}$$

$$B(q^{-1}) = diag(b_{11}\ b_{22}\ b_{33}\ b_{44}\ b_{55}\ b_{66}\ b_{77}) \tag{2.1c}$$

$$y(t) = (y_1(t), \cdots, y_7(t))^T,$$
$$u(t) = (u_1(t), \cdots, u_7(t))^T.$$

Here $a_{ij}(q^{-1}), i,j = 1,\ldots,7$ and $b_{ii}(q^{-1}), i = 1,\ldots,7$ are polynomials in the operator $q^{-1}v(t) := v(t-1)$.
Our model is designed as a linearization of the heat treatment process around a operating point. Thus $y_i(t)$ denotes the error signal:

$$y_i(t) := y_i^{meas}(t) - y_i^{ref}(t),\ i = 1,\ldots,7 \tag{2.2}$$

Furthermore $u_i(t)$ denotes a normalized input signal.

$$-1 \le u_i(t) = \frac{(u_i^{meas}(t) - 2.5)}{2.5} \le 1,\ i = 1,\ldots,7 \tag{2.3}$$

where $u_i^{meas}(t)$ is the connecting time of the i-th control device between the time samplings $t-1$ and $t$ (5 minutes).

There are three further a priori requirements on the mathematical model:

(iv) $a_{ij}(q^{-1})$ is a Hurwitz polynomial for $i = 1,\ldots,7$ $\quad A(q^{-1})^{-1}B(q^{-1})$ is a stable transfer matrix (the real system is stable).

(v) $b_{ii}(1) > 0\,,\quad a_{ij}(1) \geq 0$ for $i \neq j$.

The coefficients on $A(q^{-1})$ and $B(q^{-1})$ were determined on the basis of date (cf. Appendix) collected in seven identification experiments, where step inputs successively were applied to the loops $i = 1, \ldots, 7$ while the system was working around an operating point with PI-controlled closed loops $j \neq i$.
In order to reduce the complexity of the model the following structure (system of difference equations of second order) was presupposed:

$$I_7 y(t+2) + A_1 y(t+1) + A_2 y(t) = B_1 u(t+1) + B_2 u(t) \tag{2.4a}$$

resp.

$$A(q^{-1}) = I_7 + A_1 q^{-1} + A_2 q^{-2}, B(q^{-1}) = B_1 q^{-1} + B_2 q^{-2} \tag{2.4b}$$

where

$$A_1, A_2, B_1, B_2 \in \mathbb{R}^{7\times 7}$$
$$y = (y_1, \cdots, y_7)^T \in \mathbb{R}^7, u = (u_1, \cdots u_7) \in \mathbb{R}^7.$$

The following matrices $A_1, A_2, B_1, B_2$ were determined by a least square estimation technique applied to the experimental data (cf. Appendix)

$$A_1 = \begin{pmatrix} -1.5008 & 0.2150 & 0 & 0 & 0 & 0 & 0 \\ -0.1337 & -0.6690 & -0.0996 & 0 & 0 & 0 & 0 \\ 0 & -0.2550 & -0.9810 & -0.2377 & 0 & 0 & 0 \\ 0 & 0 & -0.0443 & -1.1829 & -0.4388 & 0 & 0 \\ 0 & 0 & 0 & -0.1764 & -0.9711 & -0.2157 & 0 \\ 0 & 0 & 0 & 0 & -0.0069 & -1.5224 & -0.1780 \\ 0 & 0 & 0 & 0 & 0 & -0.2249 & -0.9125 \end{pmatrix}, \tag{2.4c}$$

$$A_2 = \begin{pmatrix} 0.5059 & -0.2172 & 0 & 0 & 0 & 0 & 0 \\ 0.0223 & 0.0921 & -0.0401 & 0 & 0 & 0 & 0 \\ 0 & 0.1955 & 0.2410 & 0.0485 & 0 & 0 & 0 \\ 0 & 0 & -0.0565 & 0.5121 & 0.2103 & 0 & 0 \\ 0 & 0 & 0 & 0.1089 & 0.1207 & 0.0609 & 0 \\ 0 & 0 & 0 & 0 & -0.0053 & 0.5484 & 0.1664 \\ 0 & 0 & 0 & 0 & 0 & 0.1918 & 0.0461 \end{pmatrix}, \tag{2.4d}$$

$$b1 = diag(1.6991 \quad 0.7846 \quad 1.1435 \quad 0.5388 \quad -0.0689 \quad 1.2515 \quad 0.4869), \tag{2.4e}$$
$$b2 = diag(1,7732 \quad -0.41 \quad 2.9399 \quad 3.3678 \quad 1.8529 \quad -0.757 \quad 1.9747). \tag{2.4f}$$

This model satisfies the requirements (iv)- (v). In particular we have the following pole configuration inside the unit disc:

Poles of $A(q^{-1}) = \{-0,0123,\quad 0,1996 + 0,1926i,\quad 0,0951,\quad ,0,5455 + 0,3762i,\quad 0,5638 + 0,0930i,\quad 0,9918,\quad 0,9976,\quad 0,6809\}$

# 3 Controller design

Although the obtained model for the heat treatment process is stable there are strong reasons to apply adaptive controllers which are designed on the basis of the model structure (2.4), i.e. tridiagonal (diagonal) structure of $A(q^{-1})(B(q^{-1}))$ and maximal order two of the polynomials in $A(q^{-1})$ and $B(q^{-1})$:

- The obtained model is a linearization of the nonlinear real process around an operating point,
- The parameters of the true systems depend on the wire speed, furnace outside temperature, ventilation speed, etc.,
- There are disturbances originating from the voltage supply, measurement sensor noise and sensor failures.

According to these uncertainties 5 different controllers have been simulated (cf. Harris and Billings (1981), Anderson et al (1986)):

(a) Decentralized self tuning pole assignment regulators for

- the noninteracting model (C1)
- the interconnected model (C2)
- modification of C2 (C4)

(b) simple proportional control (C3)

(c) Centralized self-tuning pole assignment regulator (C5)

In the following we describe in more detail the control algorithms (1)–(5).

## (C1) Decentralized self-tuning pole assignment regulator for the noninteracting model.

Instead of (2.4) we assume a decentralized model of the form:

$$a_{ii}(q^{-1})y_i(t) = b_{ii}(q^{-1})u_i(t), \quad (3.1a)$$
$$a_{ii}(q^{-1}) = 1 + a_i^1 q^{-1} + a_i^2 q^{-2} \quad (3.1b)$$
$$b_{ii}(q^{-1}) = b_i^1 q^{-1} + b_i^1 q^{-2}, \quad i = 1, \ldots, 7 \quad (3.1c)$$

(3.1) is equivalent to:

$$y_i(t) = \phi_i^T(t-1)\Theta_i$$

where

$$\Theta_i = (a_i^1, a_i^2, b_i^1, b_i^2)^T$$

and

$$\Theta_i(t-1) = (-y_i(t-1), -y_i(t-2), u_i(t-1), u_i(t-2))^T$$

**Parameter estimation $\hat{\Theta}_i$:**

The $\Theta_i's$ are recursively estimated by the following algorithm:

$$\hat{\Theta}_i(t) = \hat{\Theta}_i(t-1) + K_i(t)(y_i(t) - \phi_i^T(t-1)\hat{\Theta}_i(t-1)) \qquad (3.2a)$$

$$K_i(t) = \frac{P_i(t-1)\phi_i(t-1)}{1+\phi_i^T(t-1)P_i(t-1)\phi_i(t-1)} \qquad (3.2b)$$

$$P_i(t) = (1 - K_i(t)\phi_i^T(t-1))P_i(t-1) \qquad (3.2c)$$

with initialization: $\hat{\Theta}_i(0) = 0, P_i(0) = 10^6$ for $i = 1, \cdots 7$

**Feedback controller:**

Determine from (3.2):

$$\hat{a}_{ii} = 1 + \hat{a}_i{}^1 q^{-1} + \hat{a}_i{}^2 q^{-2} \qquad (3.3a)$$

$$\hat{b}_{ii} = \hat{b}_i^{\,1} q^{-1} + \hat{b}_i^{\,2} q^{-2} \qquad (3.3b)$$

(3.3c)

Solve

$$\hat{a_{ii}}\hat{g}_i + \hat{b_{ii}}\hat{f}_i = 1, i = 1, \ldots, 7 \qquad (3.4)$$

with $\hat{g}_i$ and $\hat{f}_i$ of the form:

$$\hat{g}_i = 1 + \hat{g}_i{}^1 q^{-1} \qquad (3.5a)$$

$$\hat{f}_i = \hat{f}_i^{\,0} + \hat{f}_i^{\,1} q^{-1} \qquad (3.5b)$$

Determine $u_i(t)$ from the controller equation:

$$\hat{f}_i y_i(t) + \hat{g}_i \overline{u_i}(t) = 0 \qquad (3.6)$$

and

$$u_i(t) = \begin{cases} 1 & if \quad \overline{u_i}(t) > 1 \\ u_i(t) & if \quad -1 < \overline{u_i}(t) < 1 \\ -1 & if \quad \overline{u_i}(t) < -1 \end{cases} \qquad (3.7)$$

**Closed loop equation:**

Let $\hat{A}_1 = diag(\hat{a_{11}}, \cdots, \hat{a_{77}}), \hat{B} = diag(\hat{b_{11}}, \cdots, \hat{b_{77}}), \hat{F} = diag(\hat{f_{11}}, \cdots, \hat{f_{77}})$ and $\hat{G} = diag(\hat{g_{11}}, \cdots, \hat{g_{77}})$ then we obtain from (3.4) and (3.6):

$$\hat{G}\hat{A}_1 + \hat{F}\hat{B} = I_7 \tag{3.8a}$$
$$\hat{F}y + \hat{G}\overline{u} = 0 \tag{3.8b}$$

Now consider the original interconnected model:

$$[A_1(q^{-1}) + A_2(q^{-1})]y(t) = B(q^{-1})\overline{u}(t) \tag{3.9}$$

with

$$A_1(q^{-1}) = diag(a_{11}, \cdots, a_{77}), B = diag(b_{11}, \cdots, b_{77})$$

and

$$A_2 = \begin{pmatrix} 0 & a_{12} & 0 & \cdots & & 0 \\ a_{21} & 0 & a_{23} & \cdots & & 0 \\ 0 & a_{32} & 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & a_{67} \\ 0 & \cdots & \cdots & 0 & a_{76} & 0 \end{pmatrix}$$

Let further $\tilde{A}_1 := A_1 - \hat{A}_1, \tilde{B}_1 := B_1 - \hat{B}_1$. Then we obtain from (3.8) and (3.9) for the closed loop system:

$$(I + \hat{G}\tilde{A}_1 + \tilde{B}\hat{F} + \hat{G}A_2)y = 0 \tag{3.10}$$

## C2 Decentralized self-tuning pole assignment regulator for the interconnected model

We assume that the process model is of the form (2.4) resp. (3.9), i.e. the i'th local loop is given by:

$$a_{ii}(q^{-1})y_i(t) + a_{i,i-1}y_{i-1}(t) + a_{ii+1}y_{i+1}(t) = b_{ii}u_i(t) \ , \qquad i = 1, \ldots, 7 \tag{3.11}$$

or equivalently:

$$\begin{aligned} y_i(t) &= \phi_i^T(t-1)\Theta_i, \quad i = 1, \ldots, 7 && (3.12a) \\ \phi_i(t-1) &= (-y_{i-1}(t-1), -y_{i-1}(t-2), y_i(t-1), \\ &- \ y_i(t-2), -y_{i+1}(t-1), -y_{i+1}(t-2), u_i(t-1), u_i(t-2))^T && (3.12b) \\ \Theta_i &= (\alpha_i^1, \alpha_i^2, a_i^1, a_i^2, \beta_i^1, \beta_i^2, b_i^1, b_i^2) && (3.12c) \end{aligned}$$

where $a_i^1, a_i^2, b_i^1, b_i^2$ are given by (3.1) and $\alpha_i^1, \alpha_i^2, \beta_i^1, \beta_i^2$ denote the coefficients in $a_{ii+1}(q^{-1})$ resp. $a_{ii+1}(q^{-1})$:

$$a_{ii-1} = \alpha_i^1 q^{-1} + \alpha_i^2 q^{-2} \ , \qquad a_{ii+1} = \beta_i^1 q^{-1} + \beta_i^2 q^{-1} \tag{3.13}$$

The estimation of $\Theta_i$ as well as the controller design is analogous to controller $C1$ (cf. (3.2), (3.4), (3.6)). In the closed loop equation (3.10) we now have convergence of the parameters to the true values, i.e.

$$\lim_{t\to\infty} \hat{A}_1(t) = A_1, \lim_{t\to\infty} \hat{B}_1(t) = B_1$$

and asymptotically we obtain the closed loop dynamics:

$$(I_7 + \hat{G}A_2)y = 0 \tag{3.14}$$

## (C3) Simple proportional control

$$u_i(t) = \begin{cases} -\frac{1}{7}y_i(t) & \text{if } |y_i(t)| \le 7 \\ 1 & \text{if } y_i(t) < -7 \\ -1 & \text{if } y_i(t) > 7 \end{cases}$$

If $|y_i(t)| \le 7$ for some $t \ge T$ we have for $t \ge T$ the closed loop system:

$$(A + \frac{1}{7}B)y = 0 \tag{3.15}$$

with pole set $\{0,4179 \pm 0,7183i,\ 0,5566 \pm 0,7129i,\ 0,6110 \pm 0,62445i,\ 0,4761 \pm 0,4140i,\ 0,4937 \pm 0,3321i,\ 0,8024,\ 0,4877,\ 0,3918,\ 0,1137\}$. This controller shifts the poles of the original model a bit further away from the unit circle.

## (C4) Controller C4

C4 assumes $\hat{F}$ in (3.8) also in tridiagonal form, however the simulations don't show significant improvements compared with controller C2.

## (C5) Centralized self tuning pole assignment regulator

We assume the complete process model (2.4) in the form:

$$(I + A_1q^{-1} + A_2q^{-2})y = (B_1q^{-1} + B_2q^{-2})u$$

We estimate $A_1, A_2, B_1$ and $B_2$ by (3.12) and solve the equation:

$$\hat{A}(q^{-1})\hat{G}(q^{-1}) + \hat{B}(q^{-1})\hat{F}(q^{-1}) = I_7 \tag{3.16}$$

where $\hat{G}$ and $\hat{F}$ are of the form:

$$\hat{G} = I_7 + \hat{G}_1q^{-1}, \quad \hat{F} = \hat{F}_0 + \hat{F}_1q^{-1} \tag{3.17}$$

(3.16) is equivalent to:

$$\begin{bmatrix} I_7 & \hat{B}_1 & 0 \\ \hat{A}_1 & \hat{B}_2 & \hat{B}_1 \\ \hat{A}_2 & 0 & \hat{B}_2 \end{bmatrix} \begin{bmatrix} \hat{G}_1 \\ \hat{F}_0 \\ \hat{F}_1 \end{bmatrix} = \begin{bmatrix} -\hat{A}_1 \\ -\hat{A}_2 \\ 0 \end{bmatrix} \tag{3.18a}$$

resp.

$$(\hat{A}_1 - \hat{B}_1\hat{B}_2^{-1}\hat{A}_2 - \hat{B}_2\hat{B}_1^{-1})\hat{G}_1 = \hat{B}_2\hat{B}_1^{-1}\hat{A}_1 - \hat{A}_2 \tag{3.18b}$$

$$\hat{F}_0 = -\hat{B}_1^{-1}(\hat{G}_1 + \hat{A}_1) \tag{3.18c}$$

$$\hat{F}_1 = -\hat{B}_2^{-1}\hat{A}_2\hat{G}_1 \tag{3.18d}$$

Determine $\tilde{G} := I + \tilde{G}_1 q^{-1}$ and $\tilde{F} = \tilde{F}_0 + \tilde{F}_1 q^{-1}$ such that

$$\tilde{G}\hat{F} = \tilde{F}\hat{G} \tag{3.19a}$$

$$\Longleftrightarrow \qquad \tilde{F}_0 = \hat{F}_0, \tag{3.19b}$$

$$\tilde{G}_1(\hat{F}_1 - \hat{F}_0\hat{G}_1) = (\hat{F}_1 - \tilde{F}_0\hat{G}_1)\hat{G}_1 \tag{3.19c}$$

$$\tilde{F}_1 = \hat{F}_1 + \tilde{G}_1\hat{F}_0 - \tilde{F}_0\hat{G}_1 \tag{3.19d}$$

Finally $\overline{u}(t)$ is determined by the controller equation:

$$\tilde{G}\overline{u}(t) + \tilde{F}y(t) = 0 \tag{3.20}$$

and $u(t)$ is calculated via (3.7).
From (3.20) and (3.19) we obtain:

$$\overline{u}(t) = -\tilde{G}^{-1}\tilde{F}y(t) = -\hat{F}\hat{G}^{-1}y(t) \tag{3.21}$$

substituting (3.21) into (2.4) we get:

$$(A + B\,\hat{F}\hat{G}^{-1})y(t) = 0 \tag{3.22}$$

which is equivalent to:

$$(\hat{A} + \hat{B}\hat{F}\hat{G}^{-1} + \tilde{A} + \tilde{B}\hat{F}\hat{G}^{-1})y(t) = 0 \tag{3.23}$$

if we insert $\tilde{A} = A - \hat{A}, \tilde{B} = B - \hat{B}$ in (3.22). Now applying (3.16) and assuming that the estimated parameters $\hat{A}, \hat{B}$ converge to the true values $A, B$ the closed loop equation (3.22) asymptotically converges to:

$$y(t) = 0 \tag{3.24}$$

It should be mentioned that in this paper we did not prove the convergence of the adaptive algorithms inherent in the controller design C1, C2, C4 and C5. This is postponed to a forthcoming paper. However the simulations show satisfactory behaviour of the closed loop systems.

# 4 Simulations

Simulated are systems of the form:

$$\begin{aligned} A(q^{-1})y(t) &= B(q^{-1})u(t) + C(q^{-1})e(t) + d \cdot \text{mod}(t,40) \\ e(t) &= \alpha \cdot T \cdot \tilde{e}(t) \end{aligned} \tag{4.1a}$$

where $d, \alpha \in \mathbb{R}_+$ and the disturbance $\text{mod}(t,40)$ is defined as:

$$\text{mod}(t,40) = \begin{cases} 0 & \text{if } \; t \neq 40k, \\ 1 & \text{if } \; t = 40k, \end{cases} \qquad k \in \mathbb{N} \tag{4.1b}$$

The second disturbance $d(t) = C(q^{-1})e(t)$ is a stochastic ARMA-process of the form

$$I_7 d(t+2) + A_1 d(t+1) + A_2 d(t) = I_7 e(t+2) + C_1 e(t+1) + C_2 e(t) \tag{4.2}$$

with $e(t) = \alpha \cdot T\tilde{e}(t)$ where $\tilde{e}(t)$ is white noise ($E\tilde{e}(t) = 0, \quad E\{\tilde{e}(t)\tilde{e}^T(t)\} = I_7$) and $C_1, C_2$ and $T$ are given as:

$$\begin{aligned} C_1 &= \text{diag}\,(-0,6,\ -0,5,\ -0,6,\ -1,\ -1,2,\ -0,31,\ -1,3) \\ C_2 &= \text{diag}\,(0.05,\ 0.06,\ 0.5,\ 0,25\ 0,5\ 0,\ 0.42) \end{aligned}$$

$$T = \begin{pmatrix} 1.165 & 1.7971 & 0.5774 & -0.7989 & 0.4005 & -0.3229 & -0.9235 \\ 0.6268 & 0.2641 & -0.3600 & -0.7652 & -1.3414 & 0.3180 & -0.0705 \\ 0.0751 & 0.8717 & -0.1356 & -0.8617 & 0.3750 & -0.5112 & 0.1479 \\ 0.3516 & -0.7012 & -1.2704 & 0.5135 & 0.7286 & 1.6065 & -0.3367 \\ 1.6961 & 1.2460 & 0.9846 & 0.3967 & -2.3775 & 0.8476 & 0.4152 \\ 0.0591 & -0.6390 & -0.0449 & -0.7562 & -0.2738 & 0.2681 & 1.5578 \end{pmatrix}$$

The initialization for all simulations is:

$$u(0) = 0,6, \quad u(-1) = 0,6, \quad y(0) = 8, \quad y(-1) = 8$$

# Simulation results

The Figures 4.1 show the behaviour of the system (4.1) controlled by C1, C2 resp. C3 without disturbance ($\alpha = 0 = d$).
Although for (C2) the estimated parameters converge to the true values (cf. Fig. 4.2) while this is not true for (C1) the closed loop behaviour of (C1) is better than (C2). In the simulations the model outputs 6 and 7 do not converge to 0 for (C2). The reason is that the term $\hat{G}$ influences the closed loop system. The following table gives the asymptotic

values of $\hat{G}$ for the controllers (C1) and (C2):

| $\hat{G}$ | $C_1$ | $C_2$ |
|---|---|---|
| $\hat{G}_1$ | $1 + 0.7808q^{-1}$ | $1 + 0.6841q^{-1}$ |
| $\hat{G}_2$ | $1 + 1.8528q^{-1}$ | $1 + 8.6274q^{-1}$ |
| $\hat{G}_3$ | $1 + 0.6365q^{-1}$ | $1 + 0.7579q^{-1}$ |
| $\hat{G}_4$ | $1 + 1.0729q^{-1}$ | $1 + 1.0520q^{-1}$ |
| $\hat{G}_5$ | $1 + 1.0975q^{-1}$ | $1 + 1.5027q^{-1}$ |
| $\hat{G}_6$ | $1 + 7.5044q^{-1}$ | $1 + 34.4624q^{-1}$ |
| $\hat{G}_7$ | $1 + 0.7409q^{-1}$ | $1 + 0.7525q^{-1}$ |

As long as the system (2.4) really models our process the simple proportional controller shows acceptable behavior (cf. Fig. 4.1).

The centralized self tuning controller (C5) shows a behaviour (cf. Fig. 4.2) which is similar to (C1). The oscillating limiting dynamics which was observed at the outputs 6 and 7 for controller C2 does not occur for (C5). On the other hand there are no obvious preformance advantages compared with (C1), however the computing time for (C1) is less than half of the computing time needed for (C5).

In Fig. 4.3 the closed loop behaviour of the controllers (C1), (C3) and (C5) are compared when the system outputs are disturbed. For the white noise ARMA-process the results are very similar for all three controllers with slight advantages for the centralized controller (C5). For the deterministic disturbances there are conversely significant advantages for the decentralized controller (C1) compared with (C5).

## References


Harris, C. and S. Billings, 1981: Self tuning and adaptive control Theory and Applicatins, Peregrinus Ltd, London.

Anderson, B.D.O., Bitmead, R.R., Johnson, Jr., C.R., Kokotovic, P.V., Kosut, R.L., Mareels, M.Y., Praly, L. Riedle, B.D., 1986: Stability of Adaptive Systems, Passivity and Averaging Analysis, MIT Press

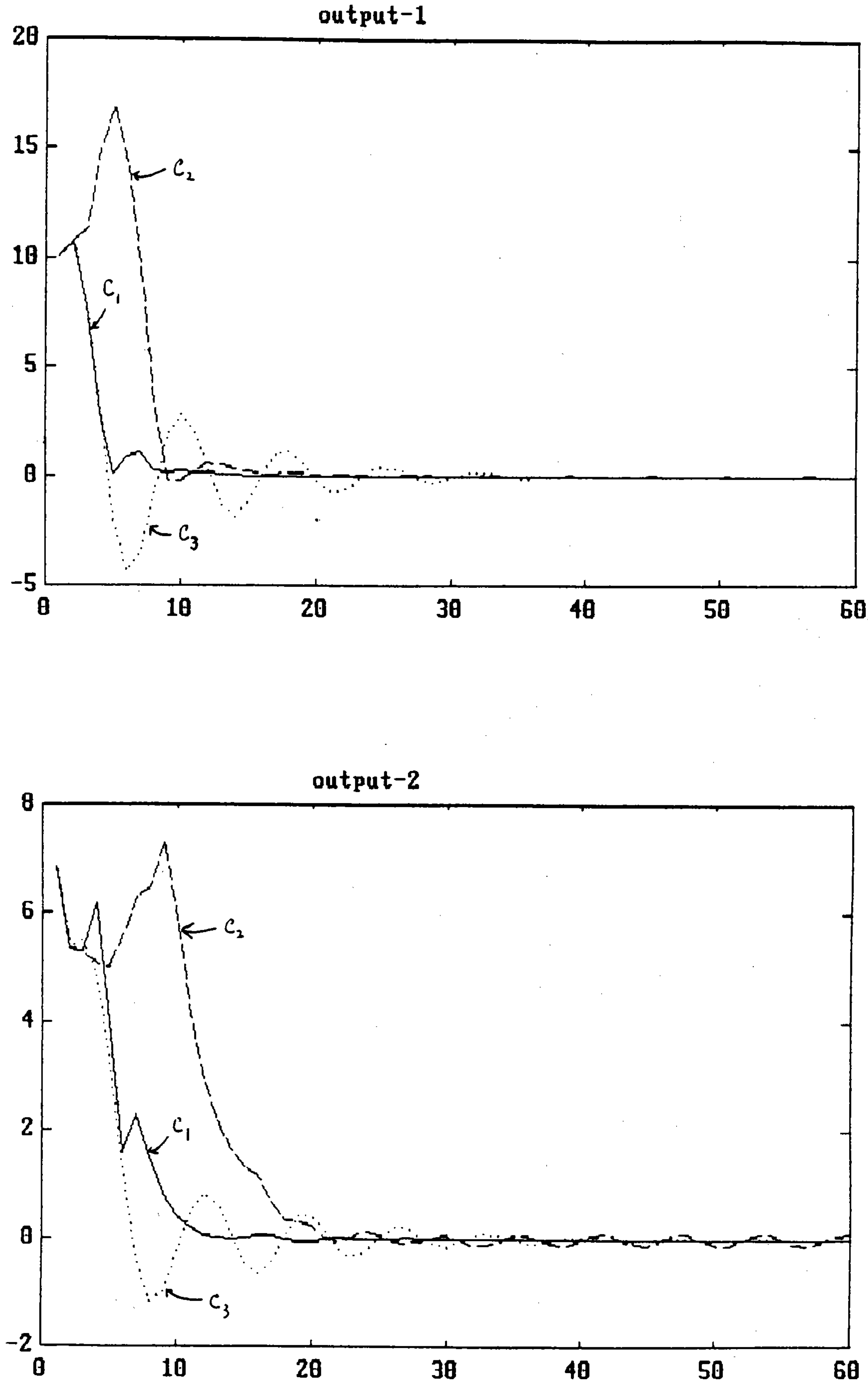
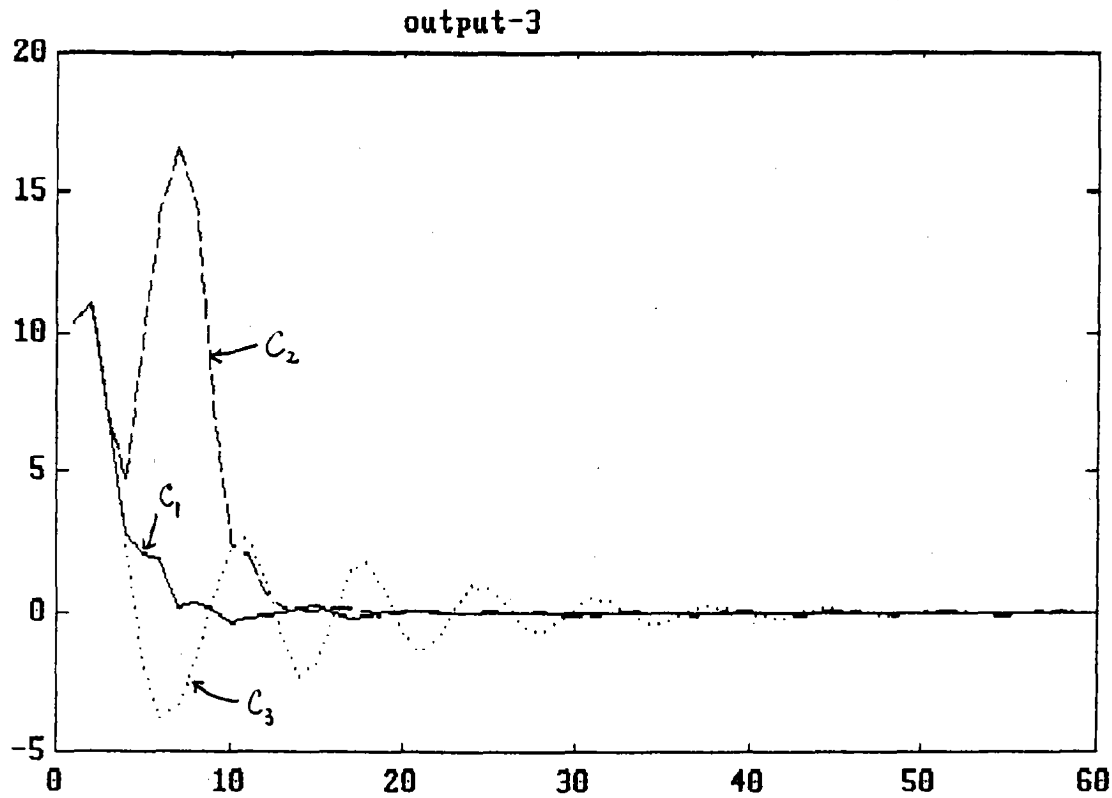

output-1
20
15
10
5
0
-5
0
10
20
30
40
50
60
C2
C1
C3
output-2
8
6
4
2
0
-2
0
10
20
30
40
50
60
C2
C1
C3

Fig. 4.1


output-3
20
15
10
5
0
-5
0
10
20
30
40
50
60
C2
C1
C3

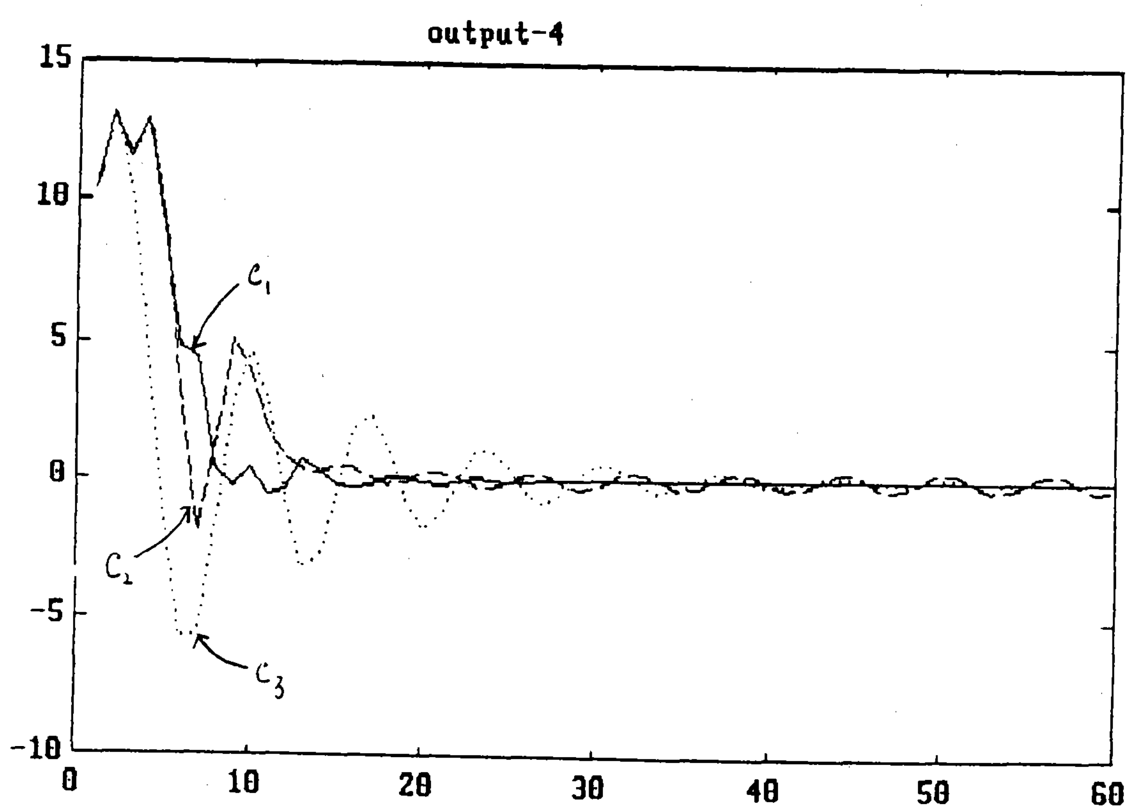
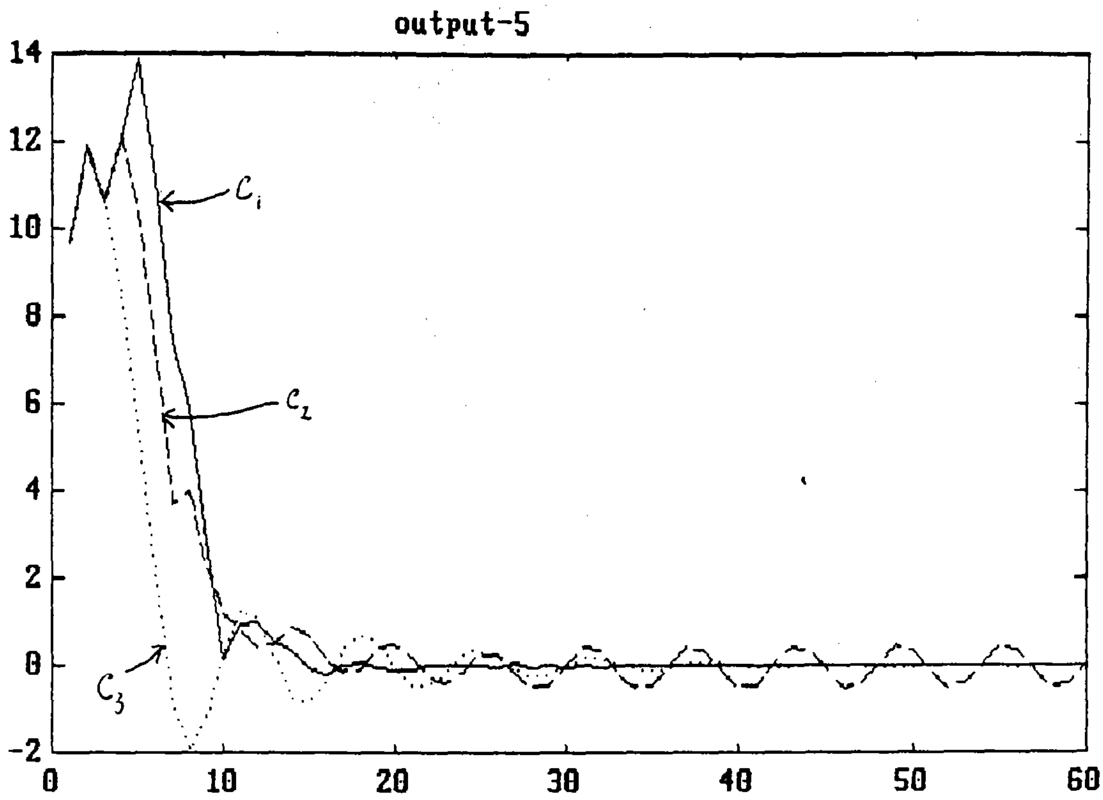

output-4
15
10
5
0
-5
-10
0
10
20
30
40
50
60
C1
C2
C3

Fig. 4.1


output-5
14
12
10
8
6
4
2
0
-2
0
10
20
30
40
50
60
$C_1$
$C_2$
$C_3$

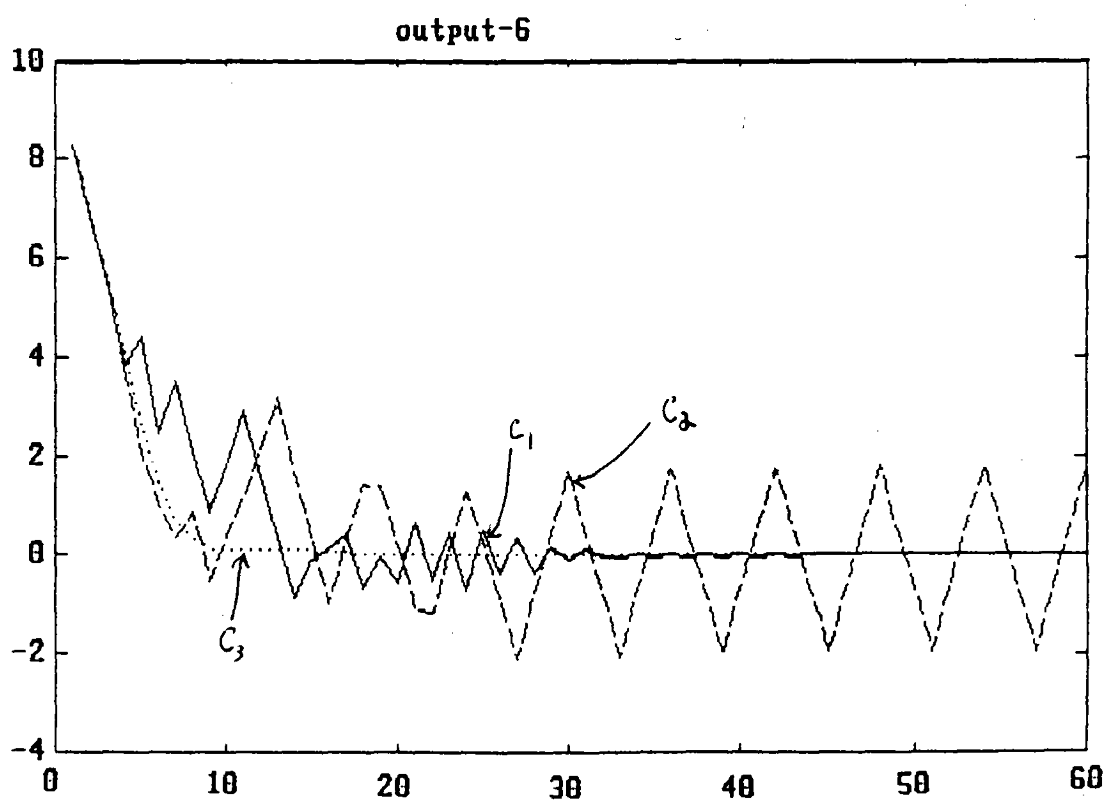

output-6
10
8
6
4
2
0
-2
-4
0
10
20
30
40
50
60
$C_1$
$C_2$
$C_3$

Fig. 4.1

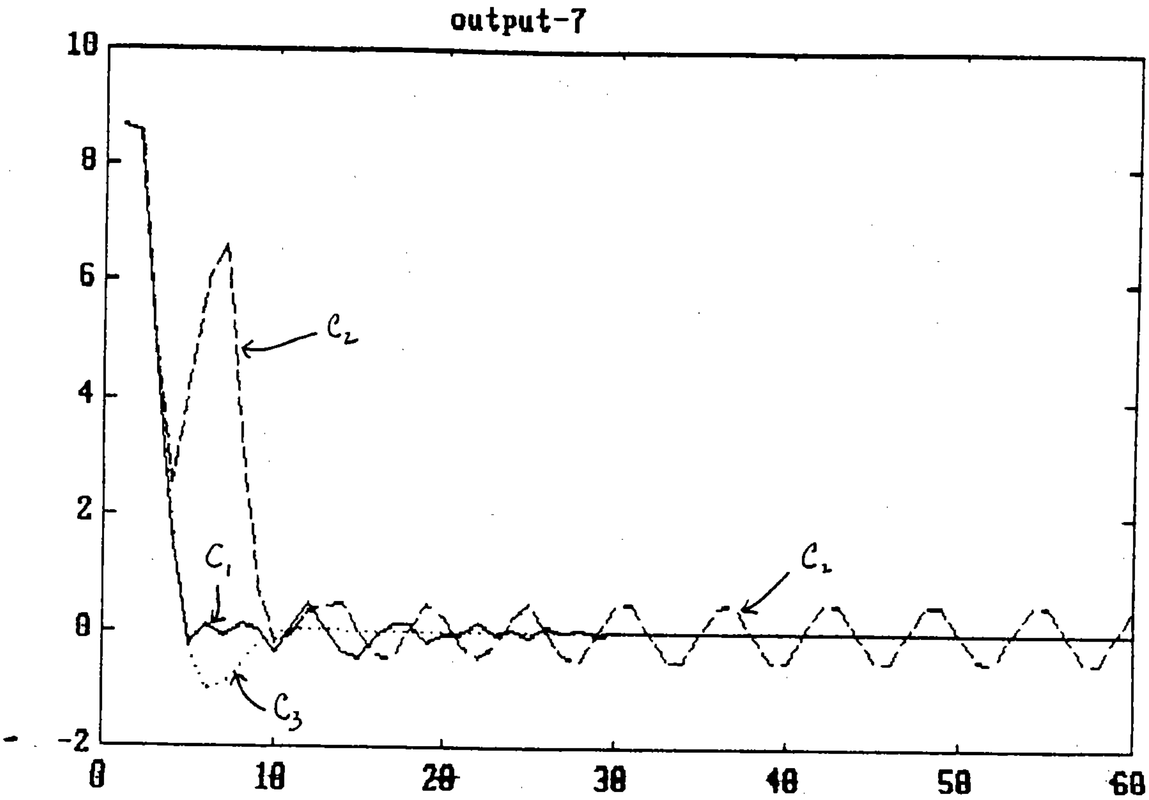



Fig. 4.1

Output 1

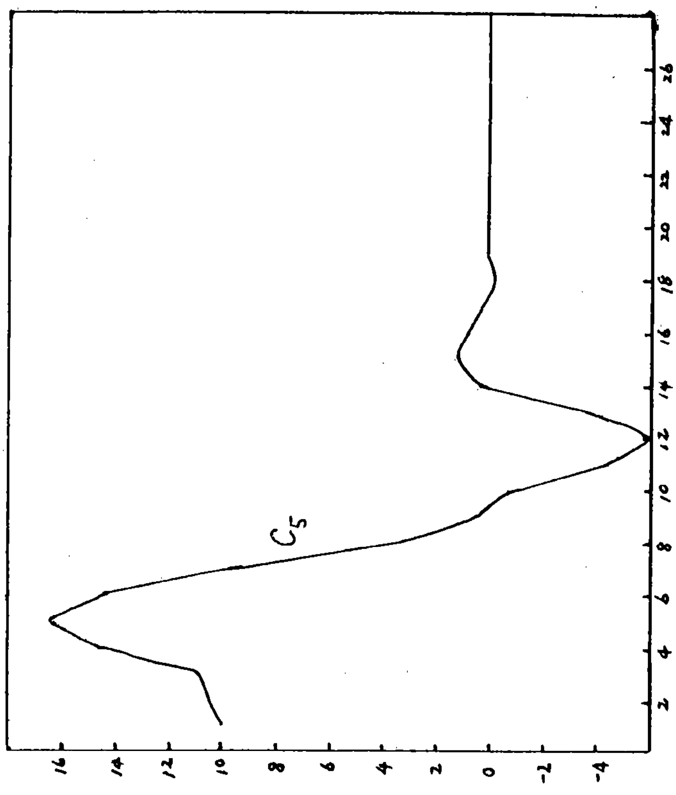



Fig. 4.2

Output 2

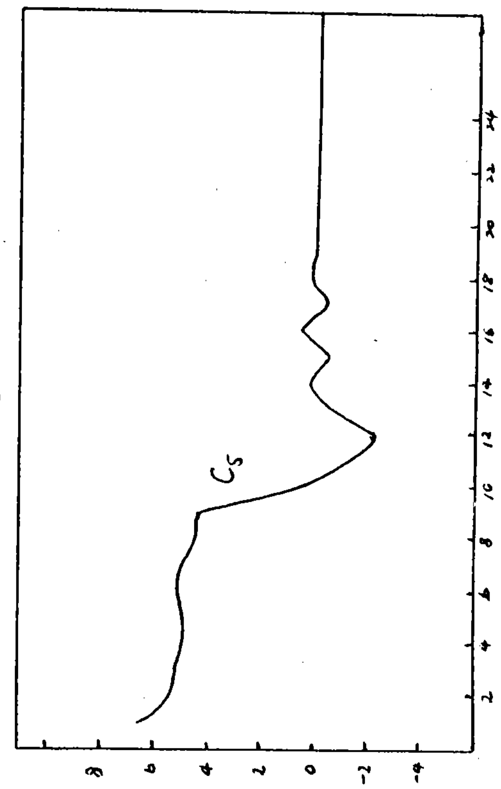



Fig. 4.2

Output 3

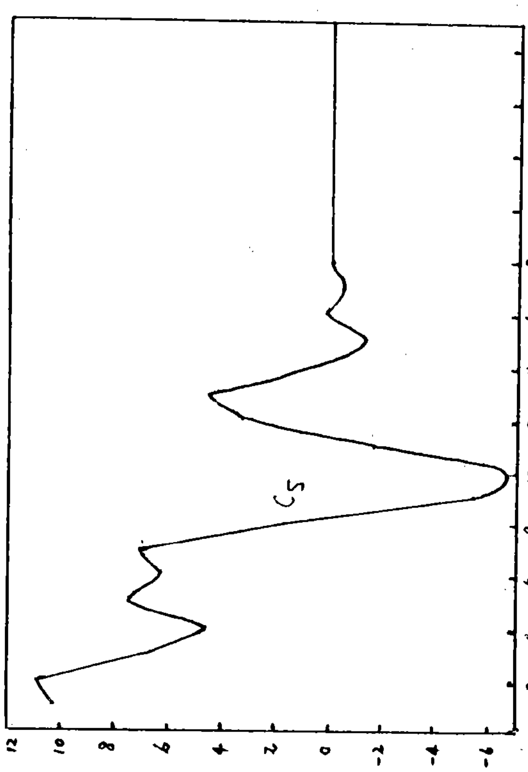



Fig. 4.2

Output 4

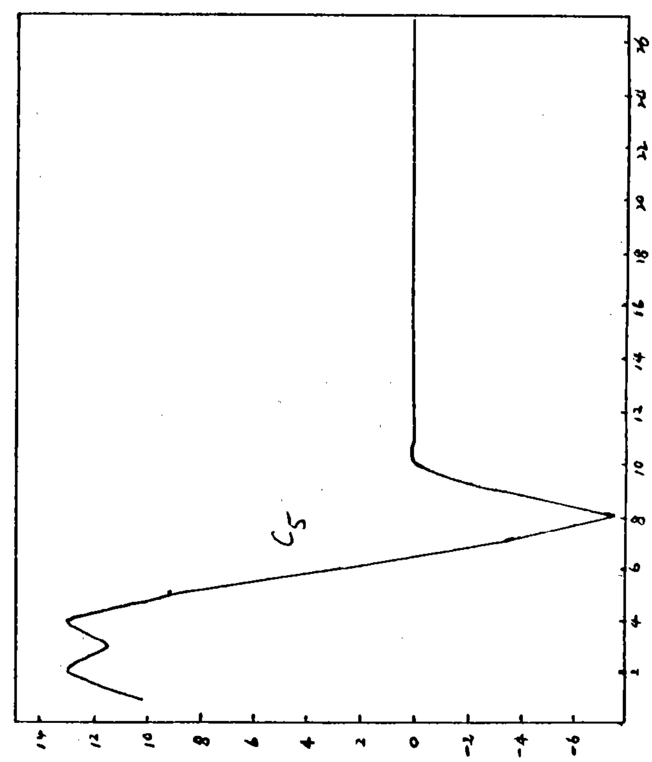



Fig. 4.2

Output 5

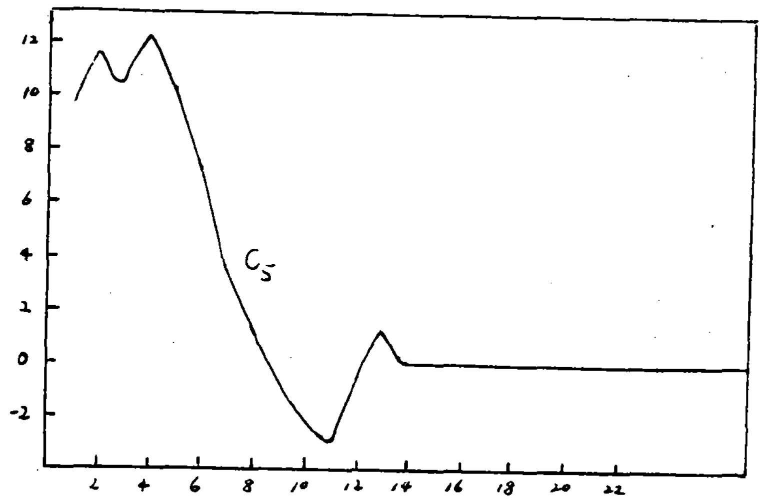



Fig. 4.2

Output 7

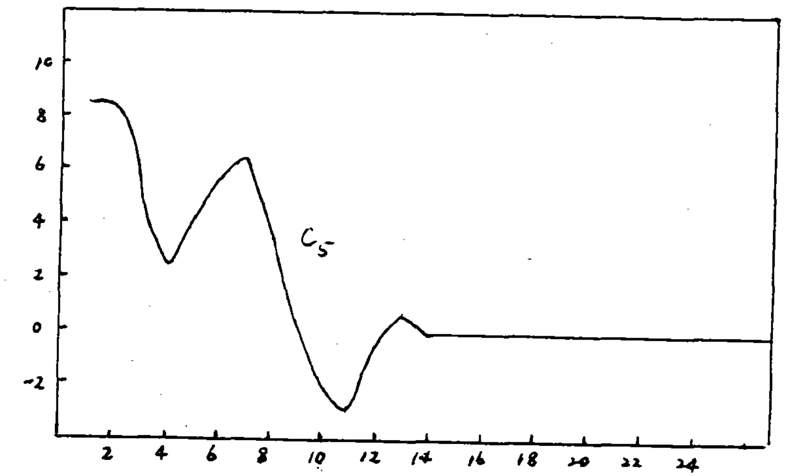



Fig. 4.2

Output 6

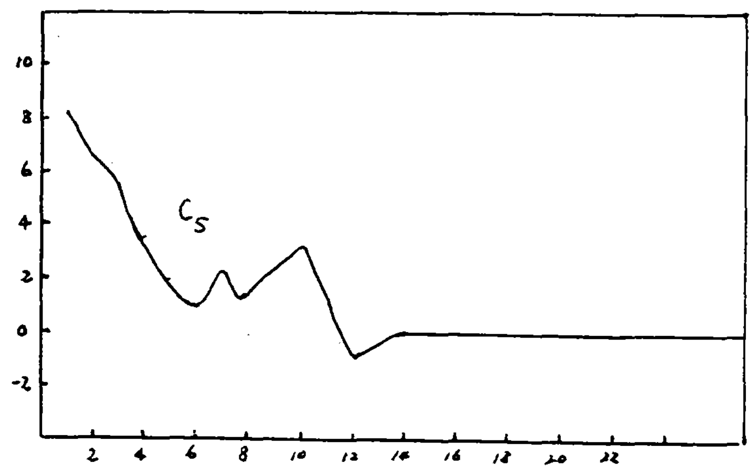



Fig. 4.2

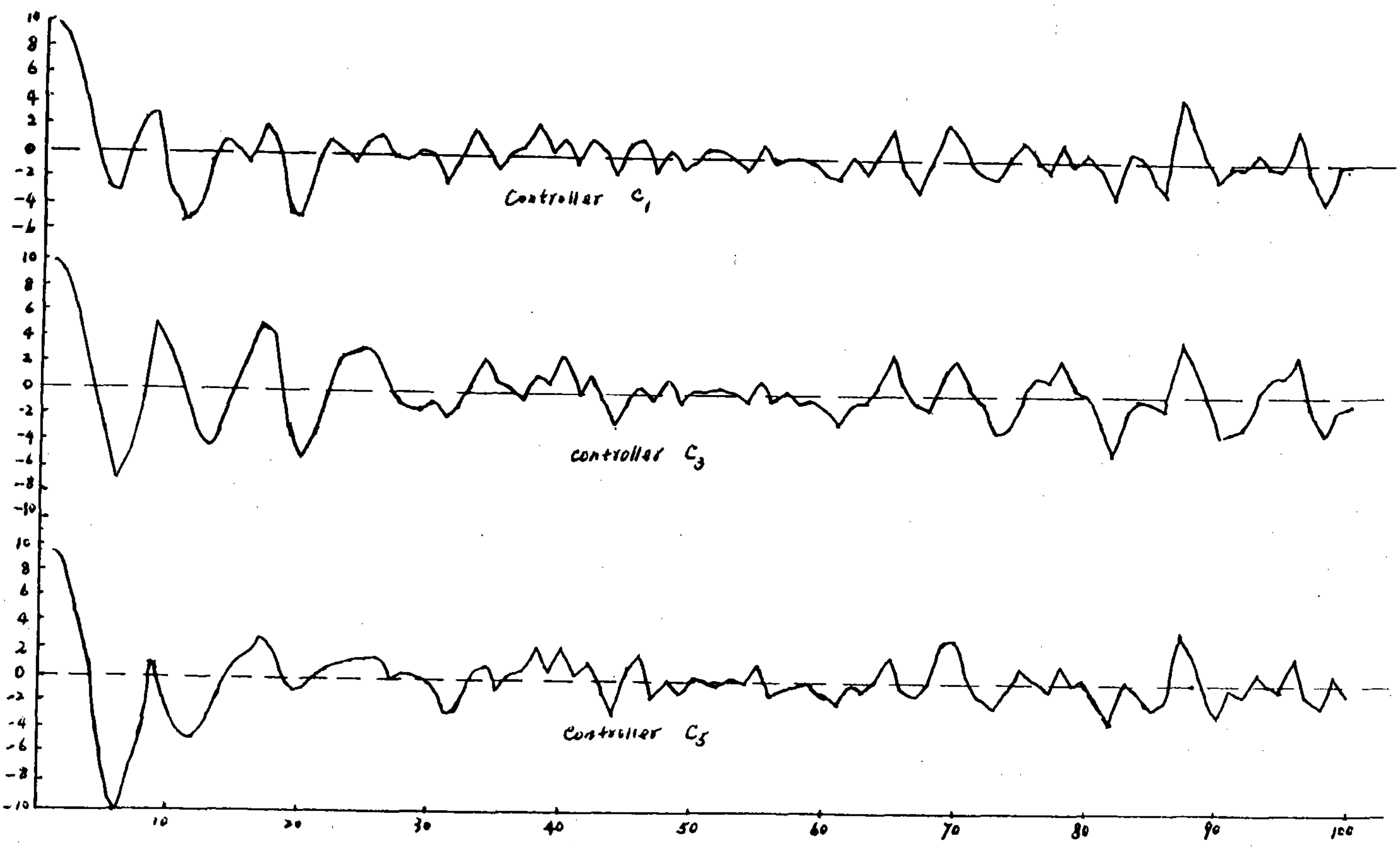



Fig. 4.3 output-1 , $\alpha=0.5$ , $d=0$

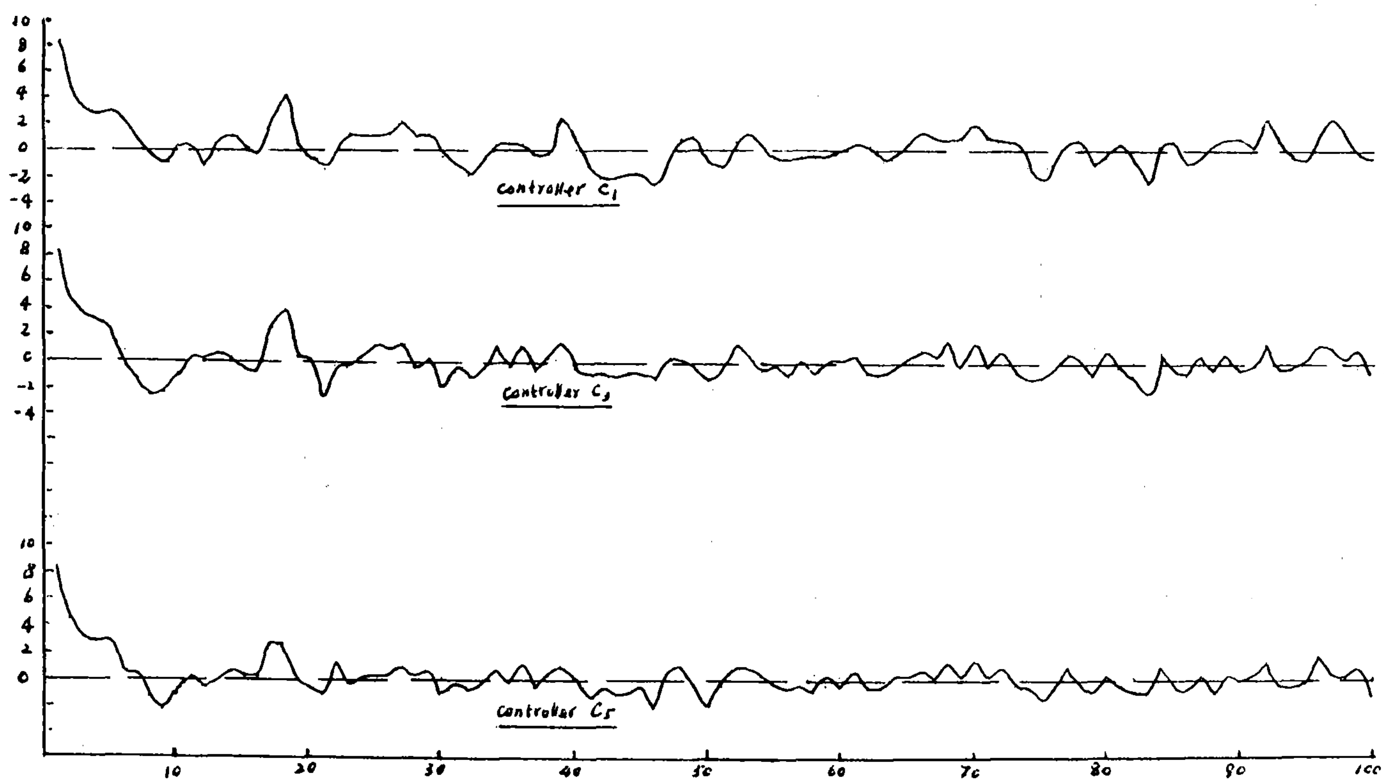



Fig. 4.3 output-2, $\alpha=0.5$ , $d=0$

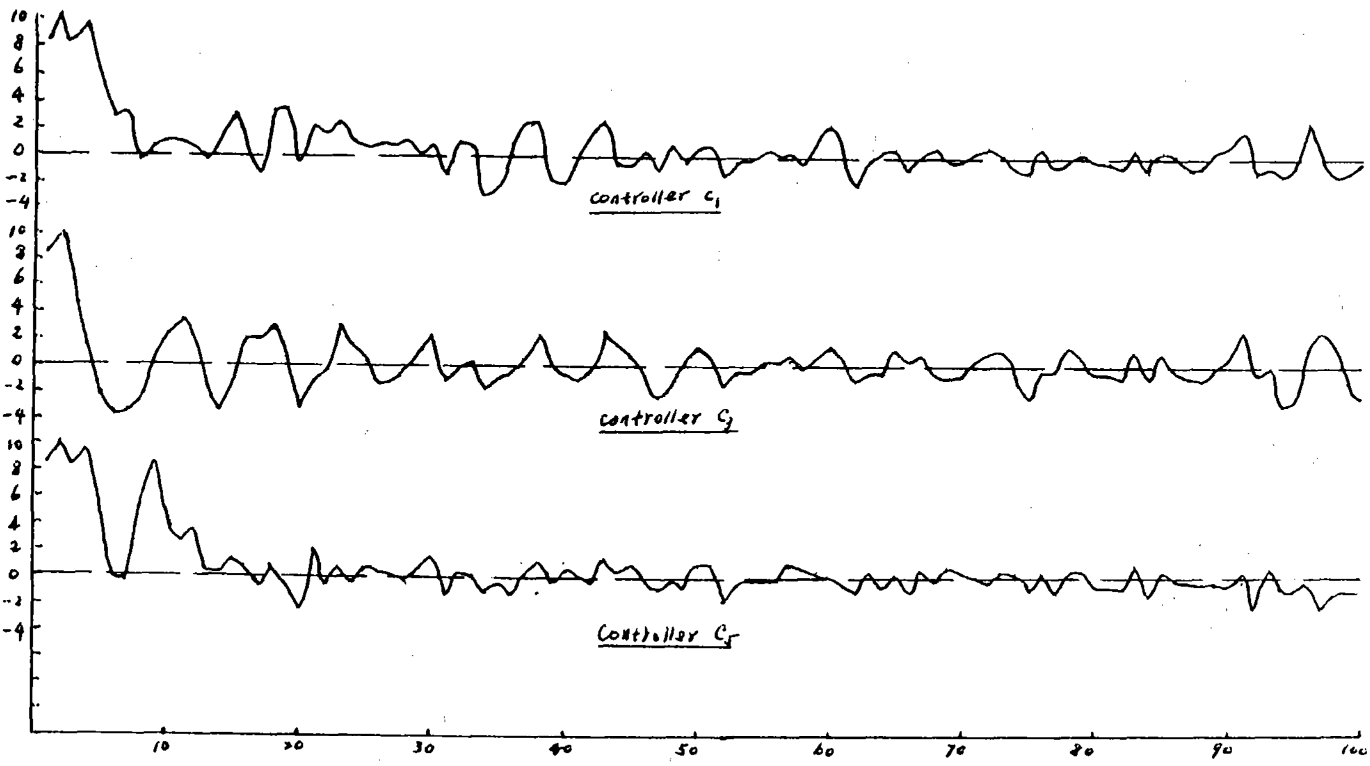



Fig. 4.3 output-3, $\alpha = 0.5$, $d = 0$

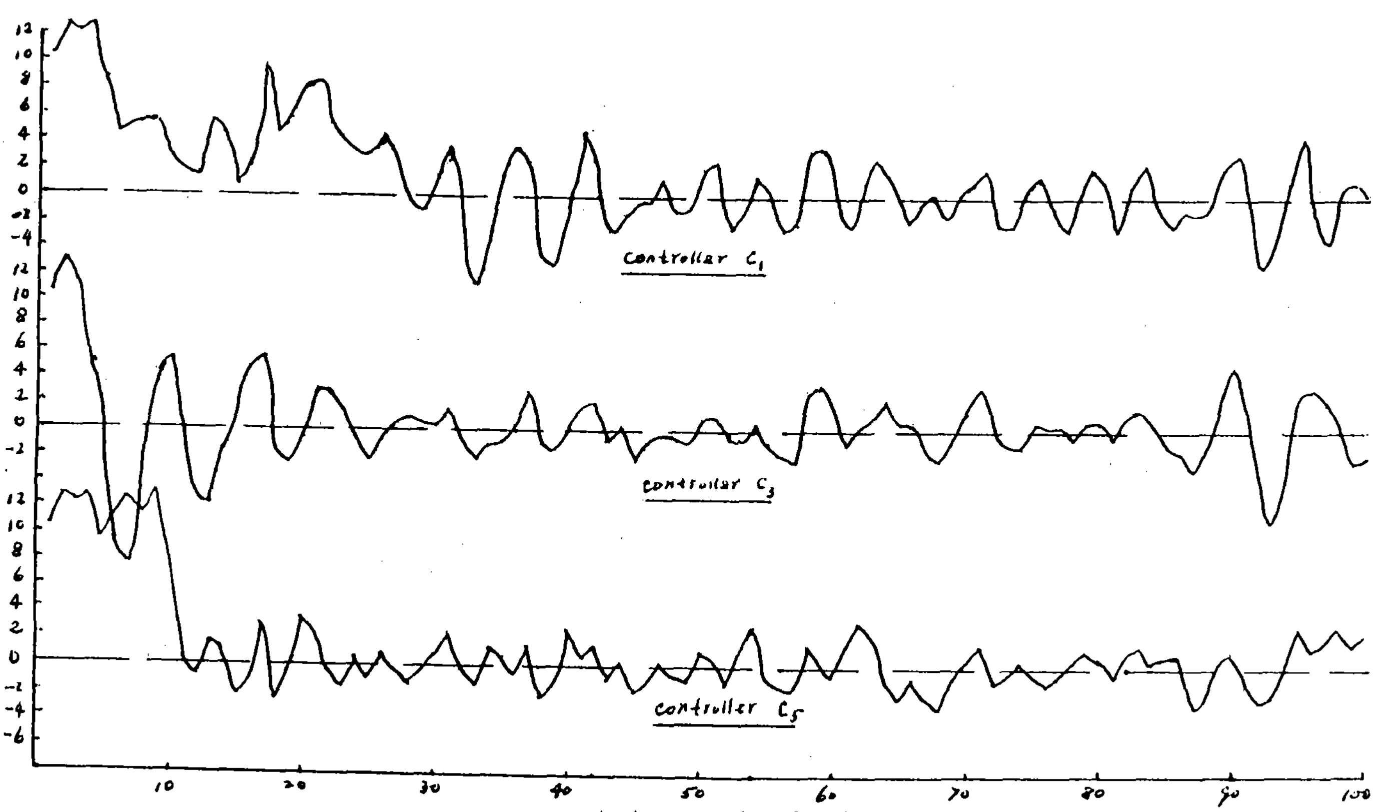



Fig. 4.3 output-4, $\alpha = 0.5$, $d = 0$

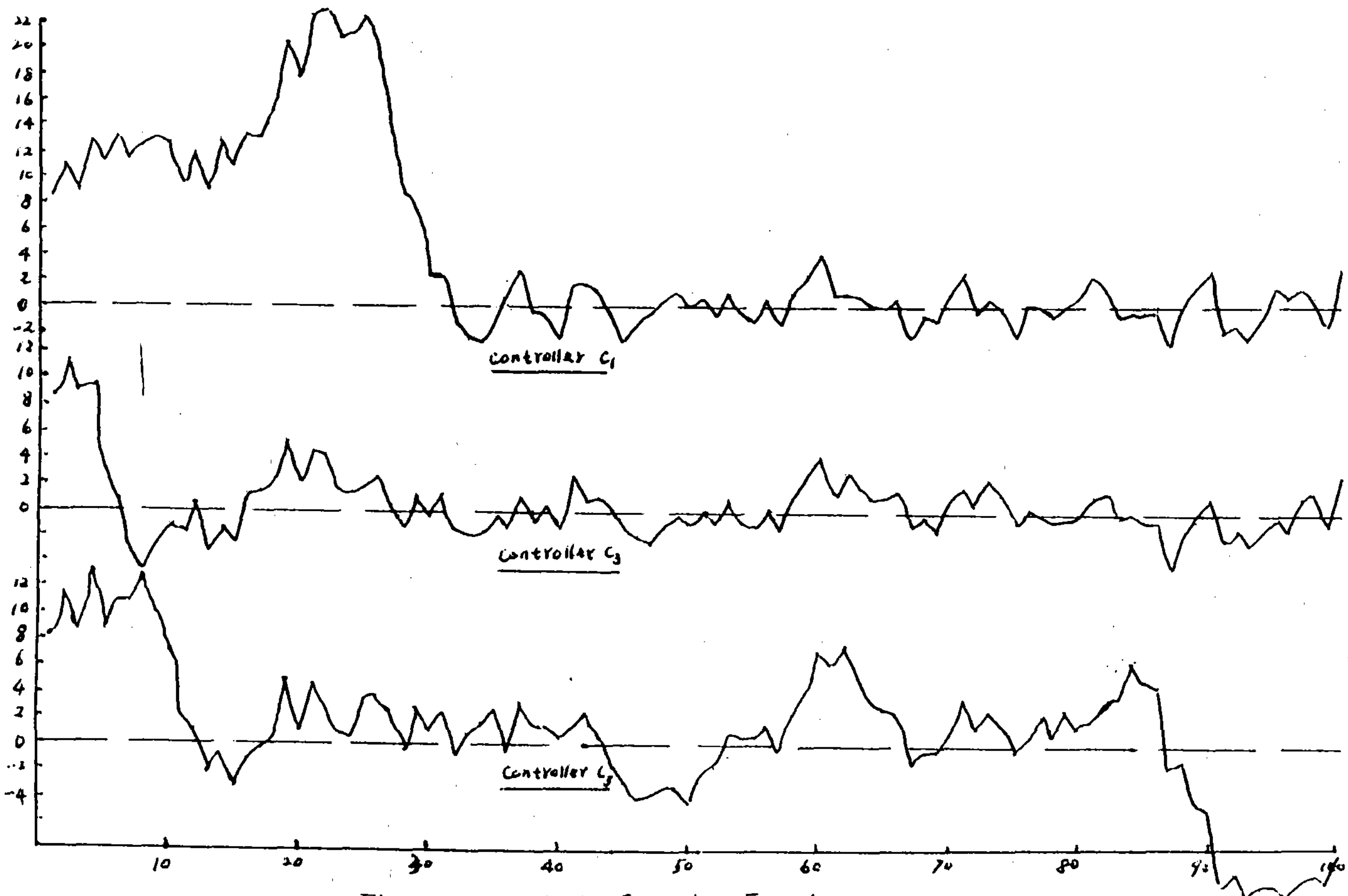



Fig. 4.3 output - 5 , ν = 0.5 , d = 0

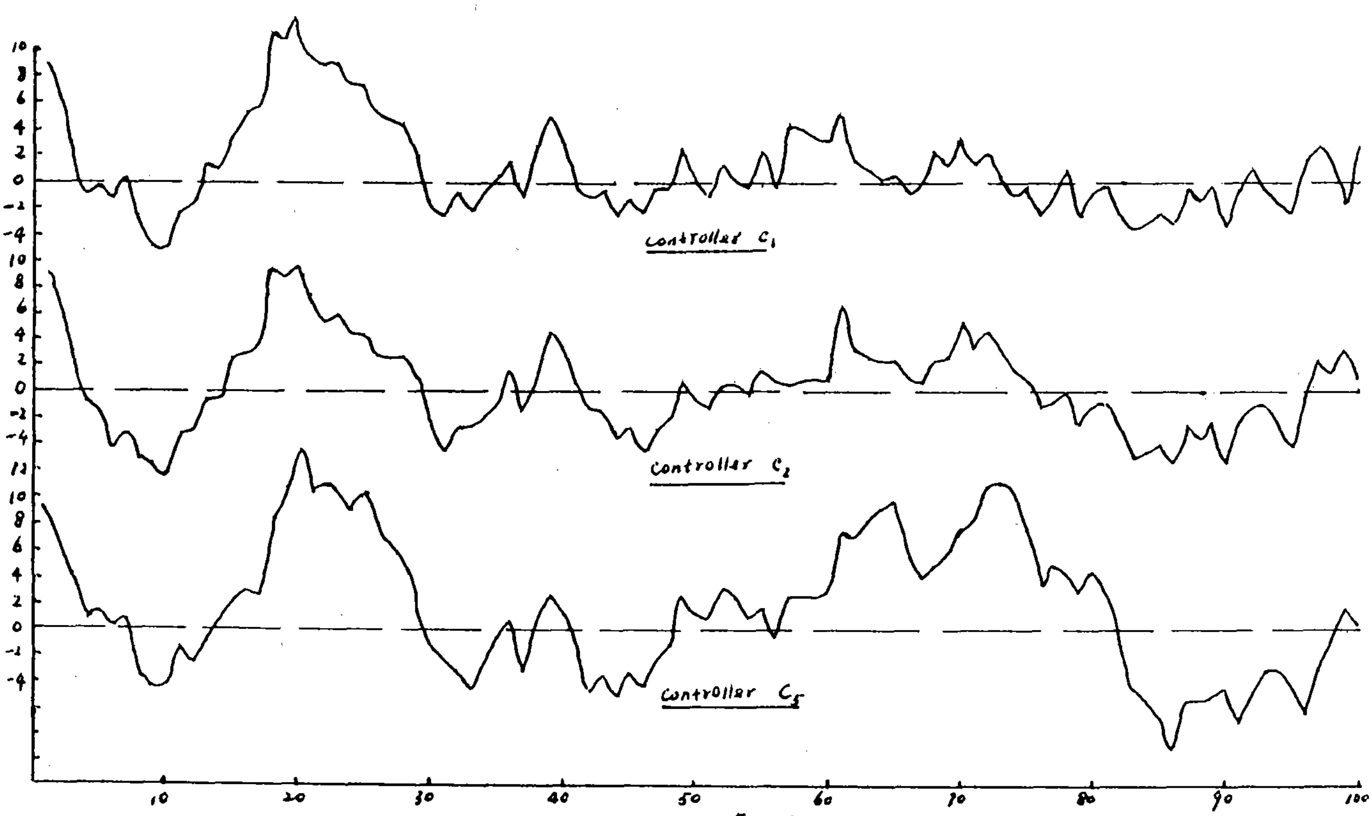



Fig. 4.3 output - 6 , ν = 0.5 , d = 0

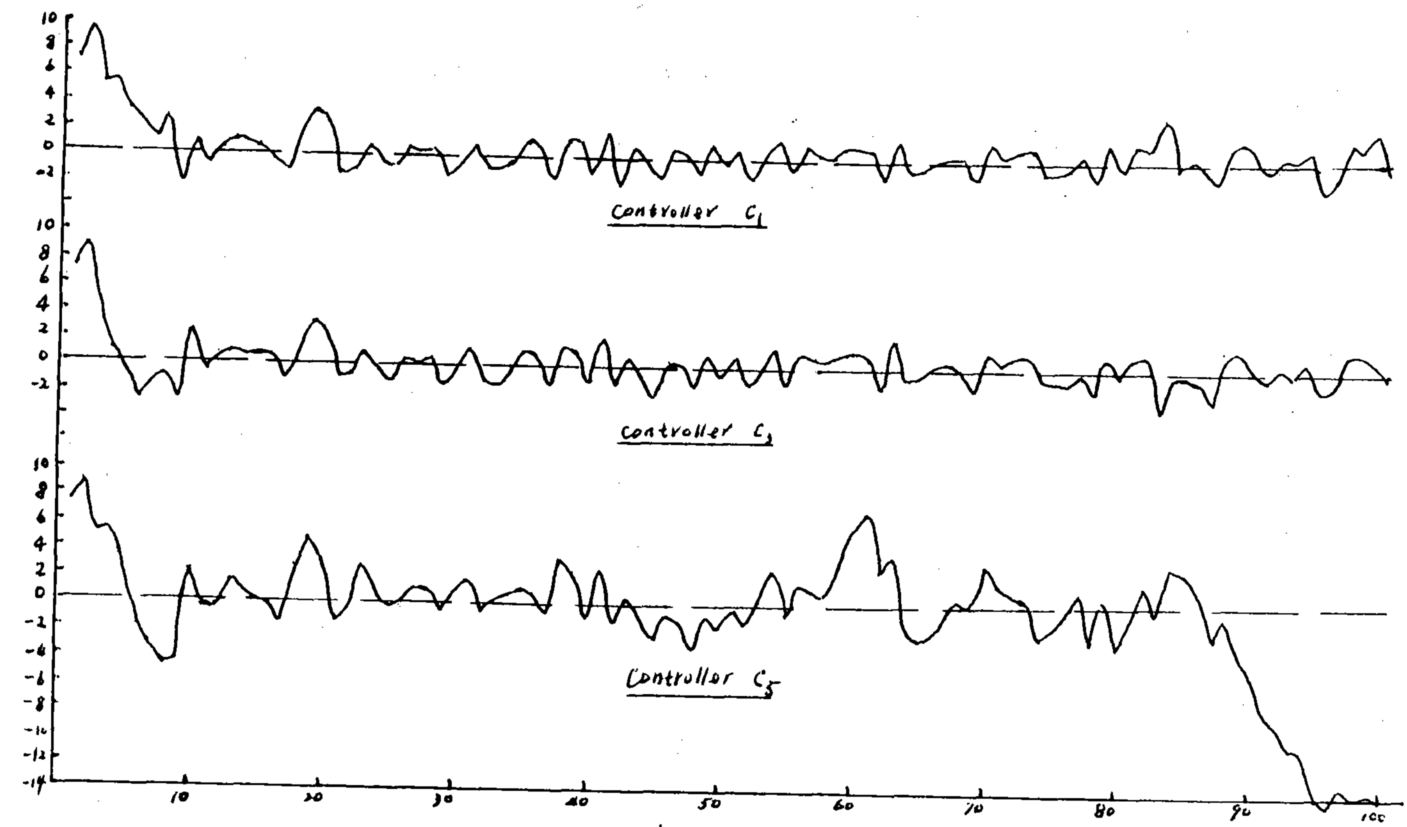



Fig. 4.3 output - 7, α=0.5, d=0

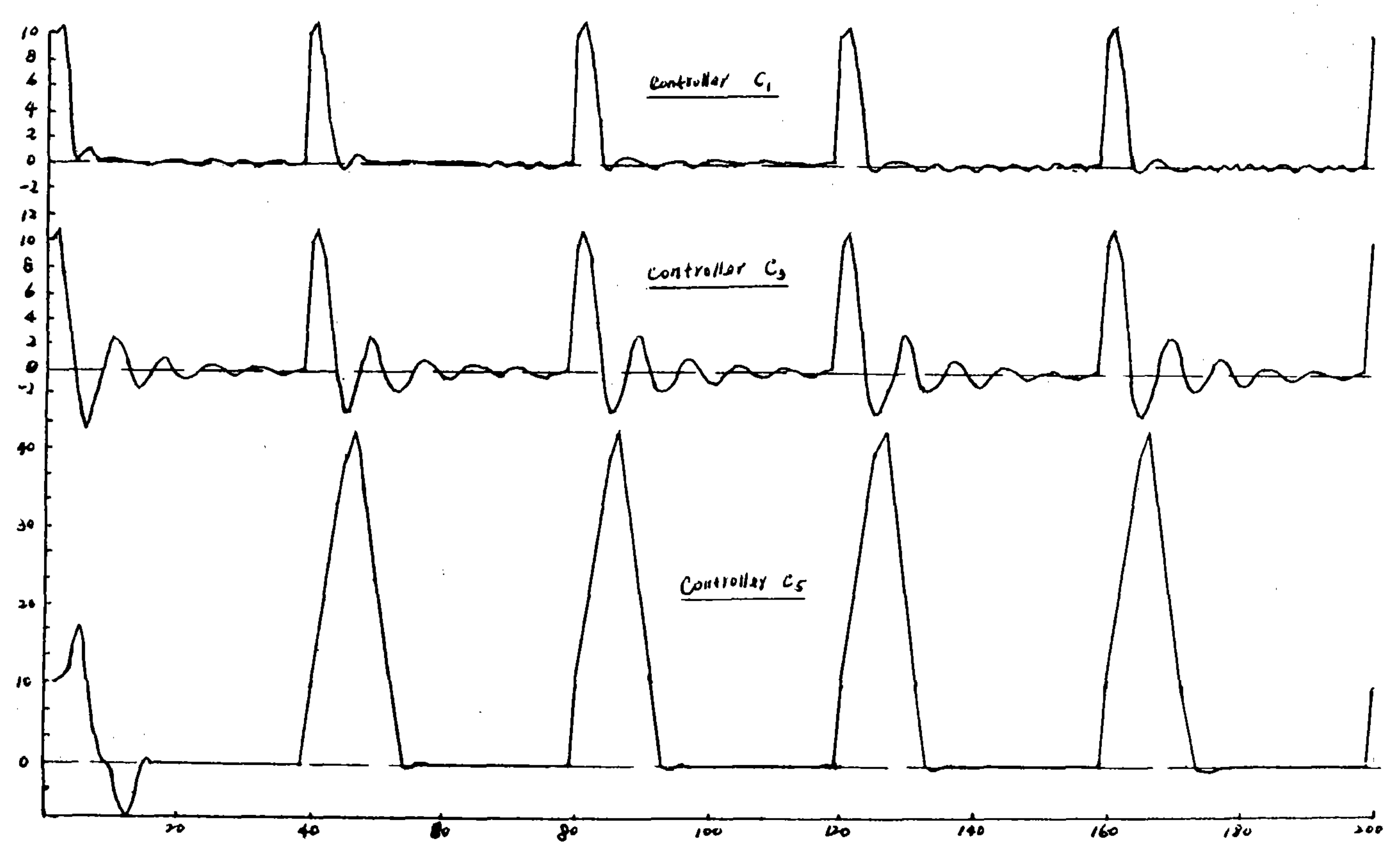



Fig. 4.3 output - 1, d=10, α=0

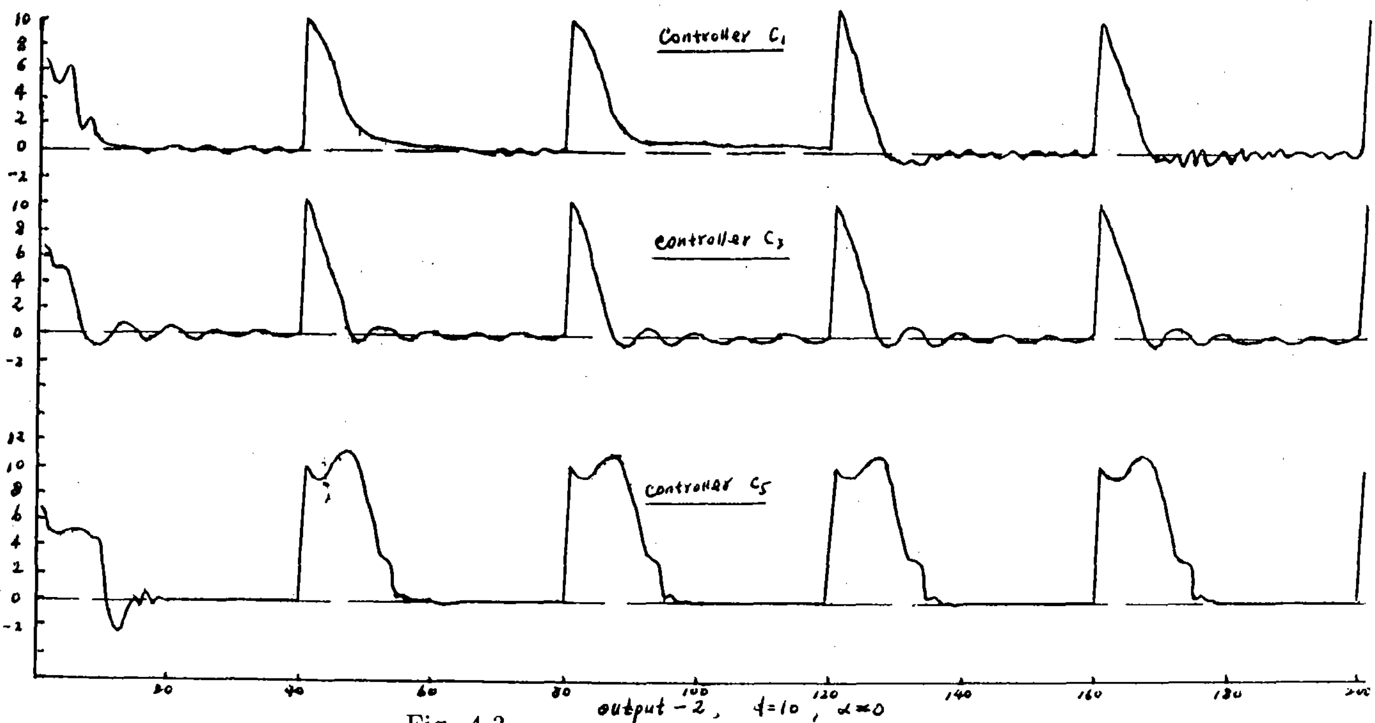

Controller $C_1$
Controller $C_3$
Controller $C_5$
output - 2, d=10, α=0

Fig. 4.3

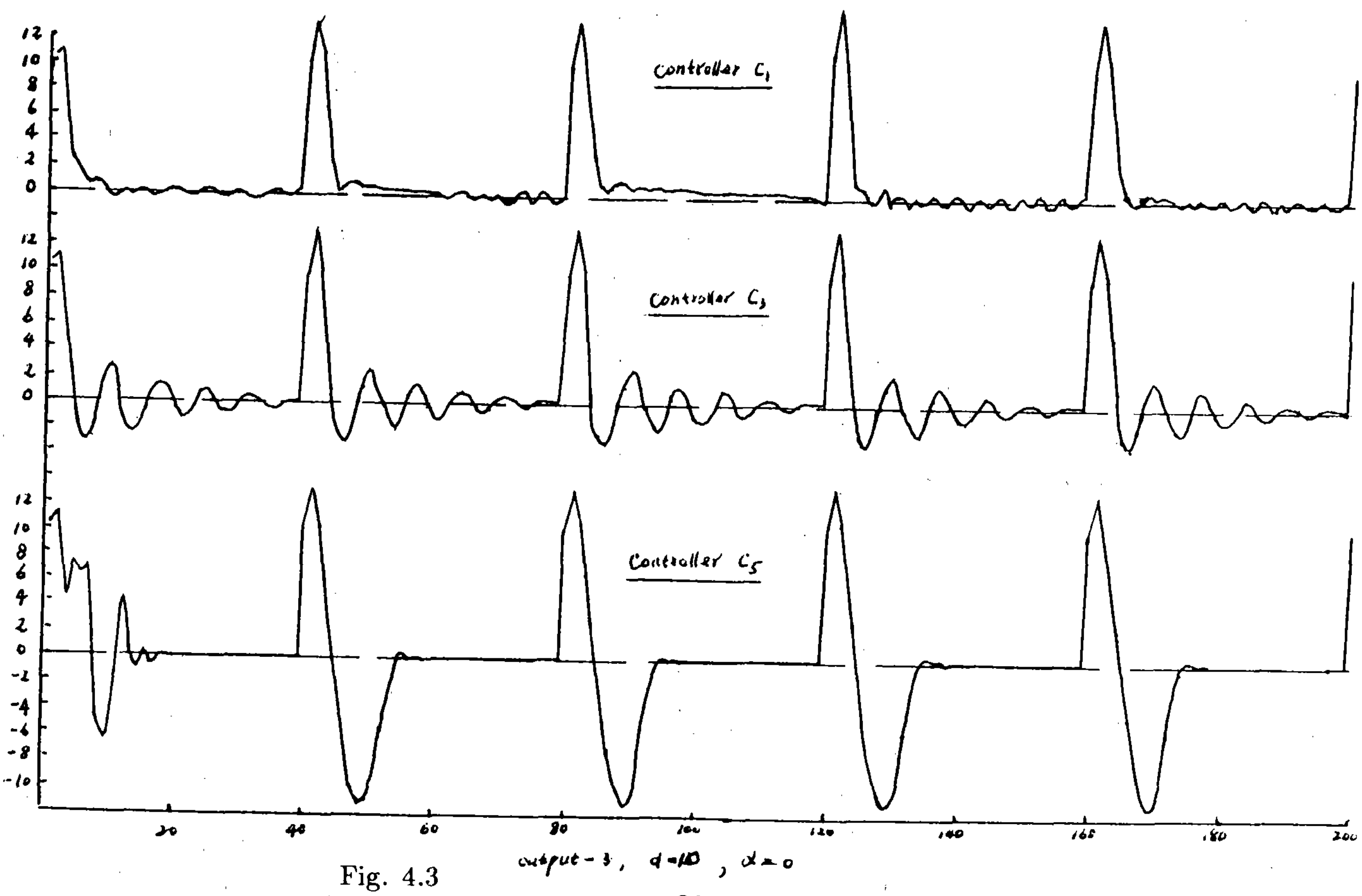

Controller $C_1$
Controller $C_3$
Controller $C_5$
output - 3, d=10, α=0

Fig. 4.3

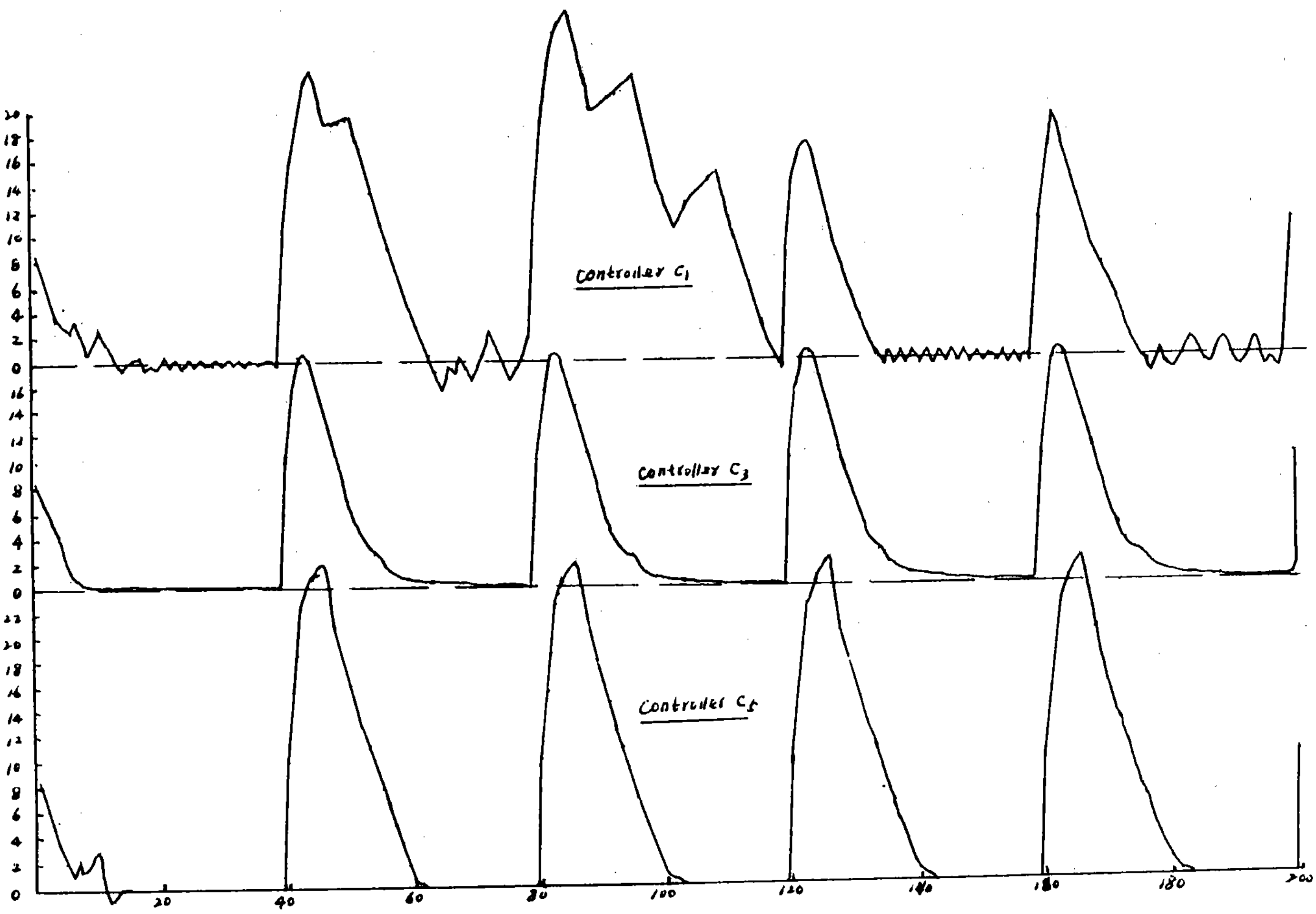



Fig. 4.3 output-6, d=10, $\alpha=0$

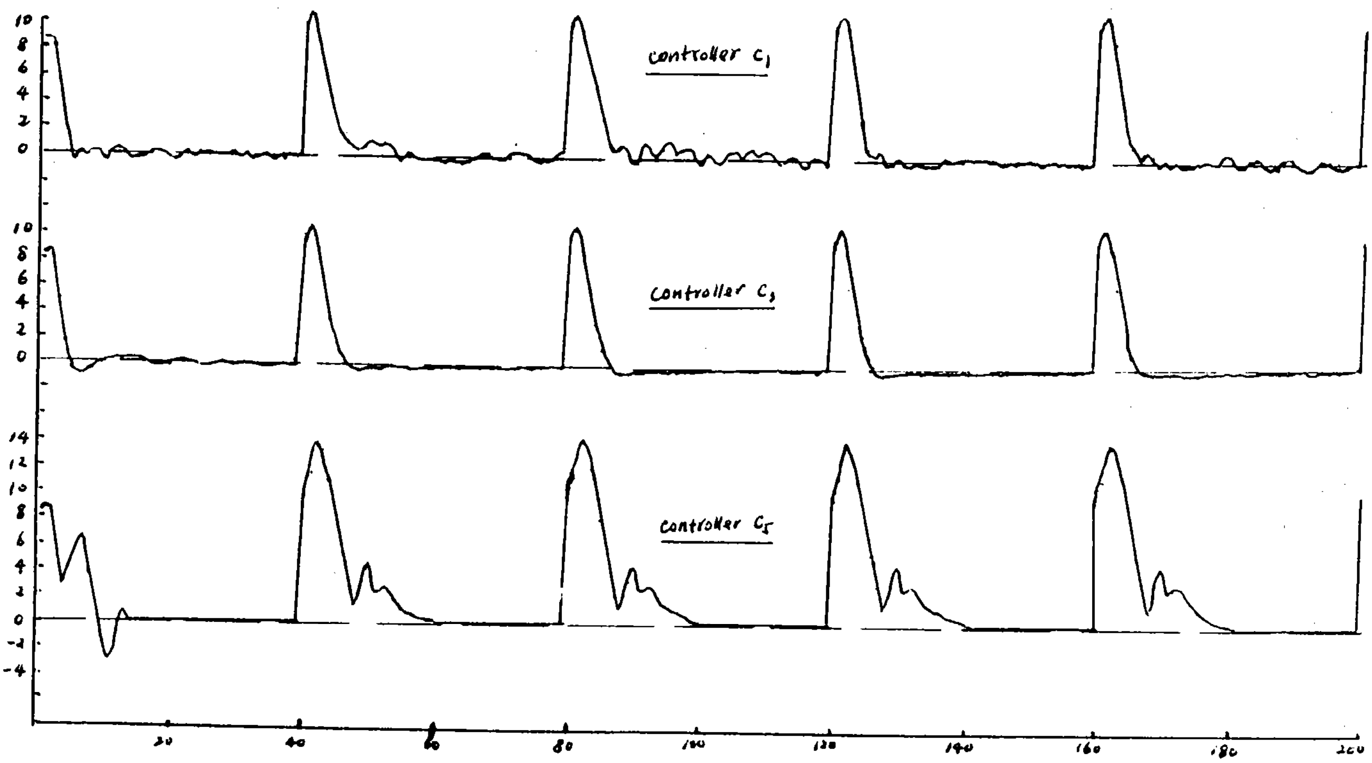



Fig. 4.3 output-7, d=10, $\alpha=0$

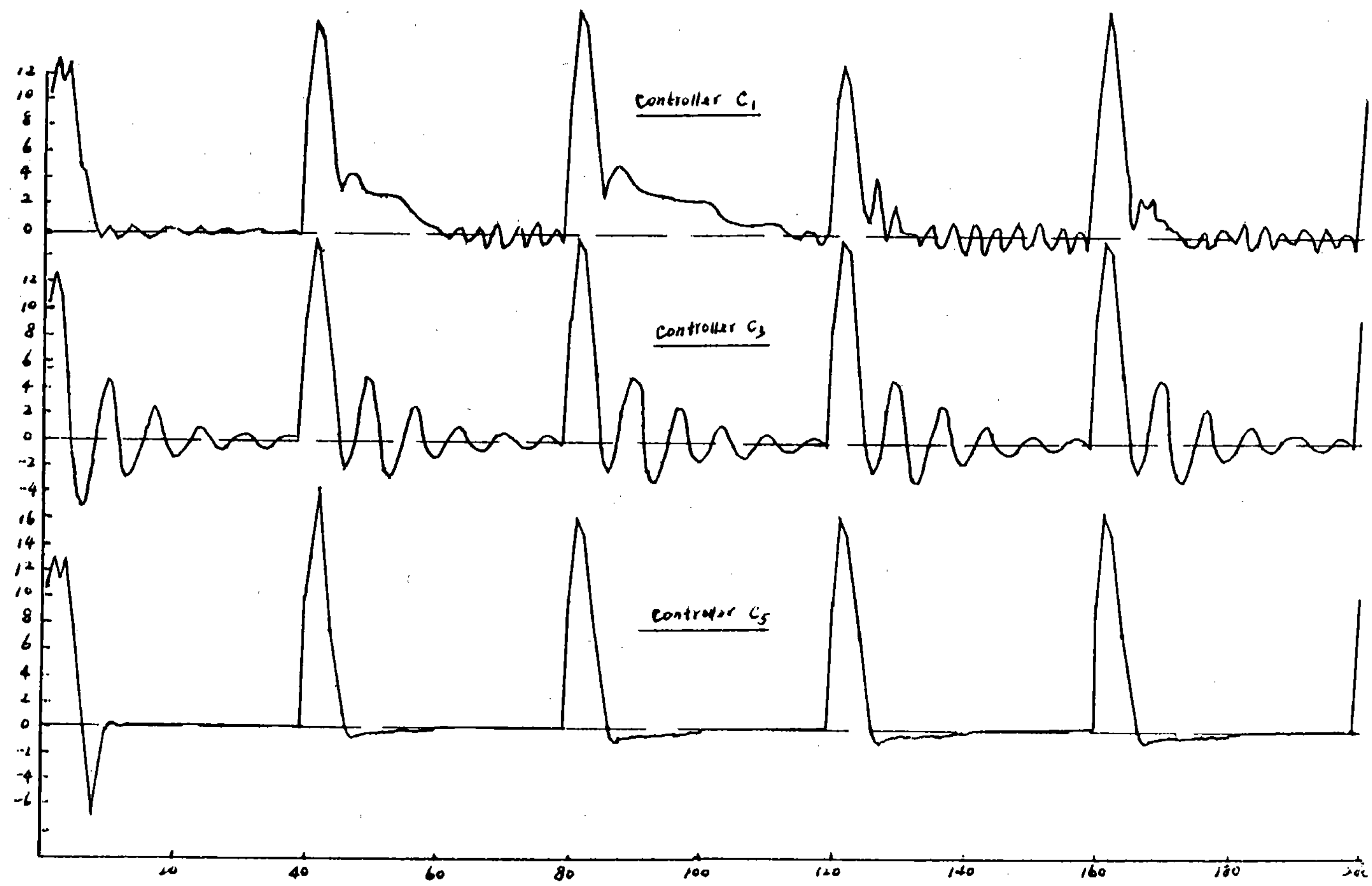



Fig. 4.3 output - 4 , d=10 , α=0

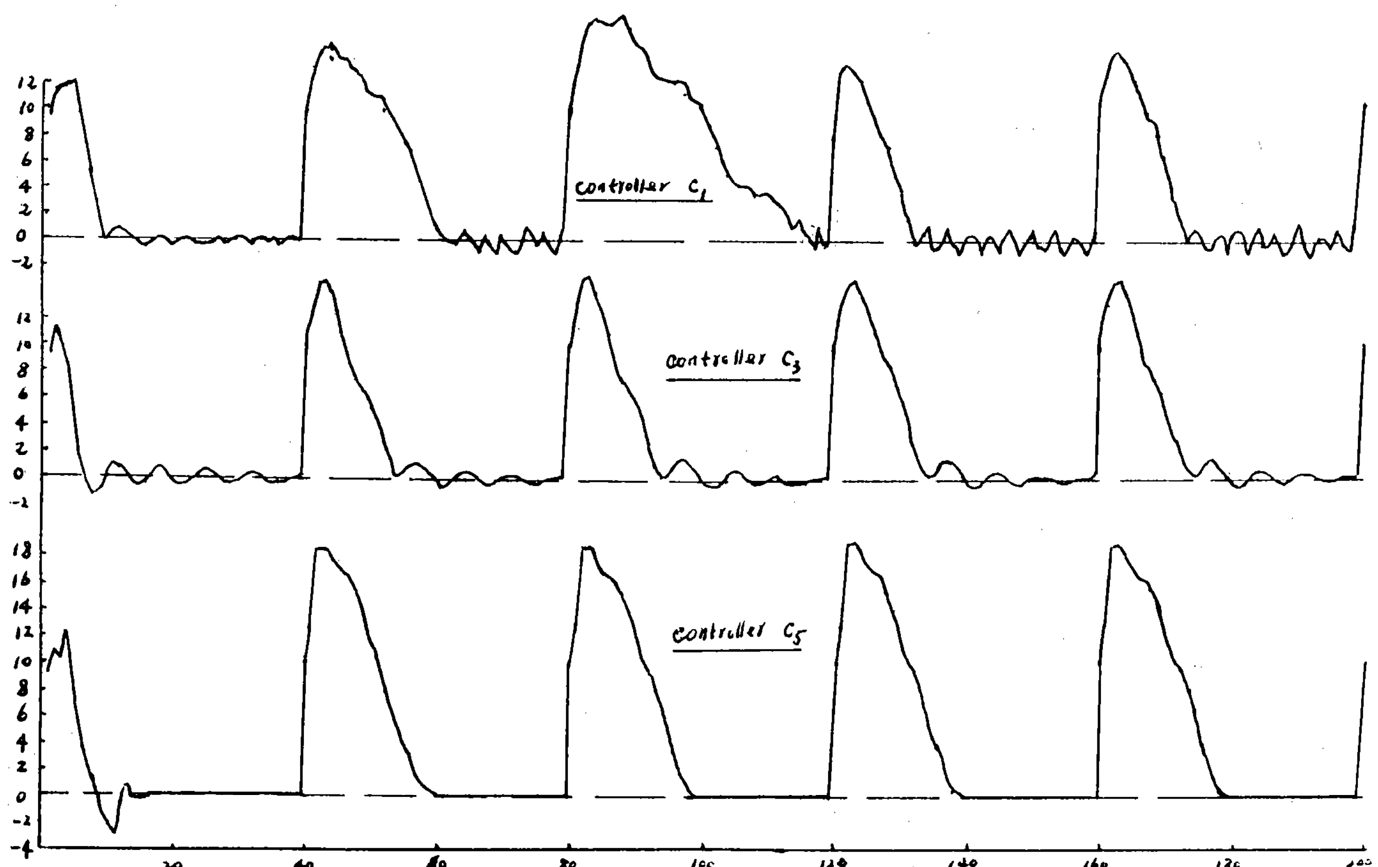
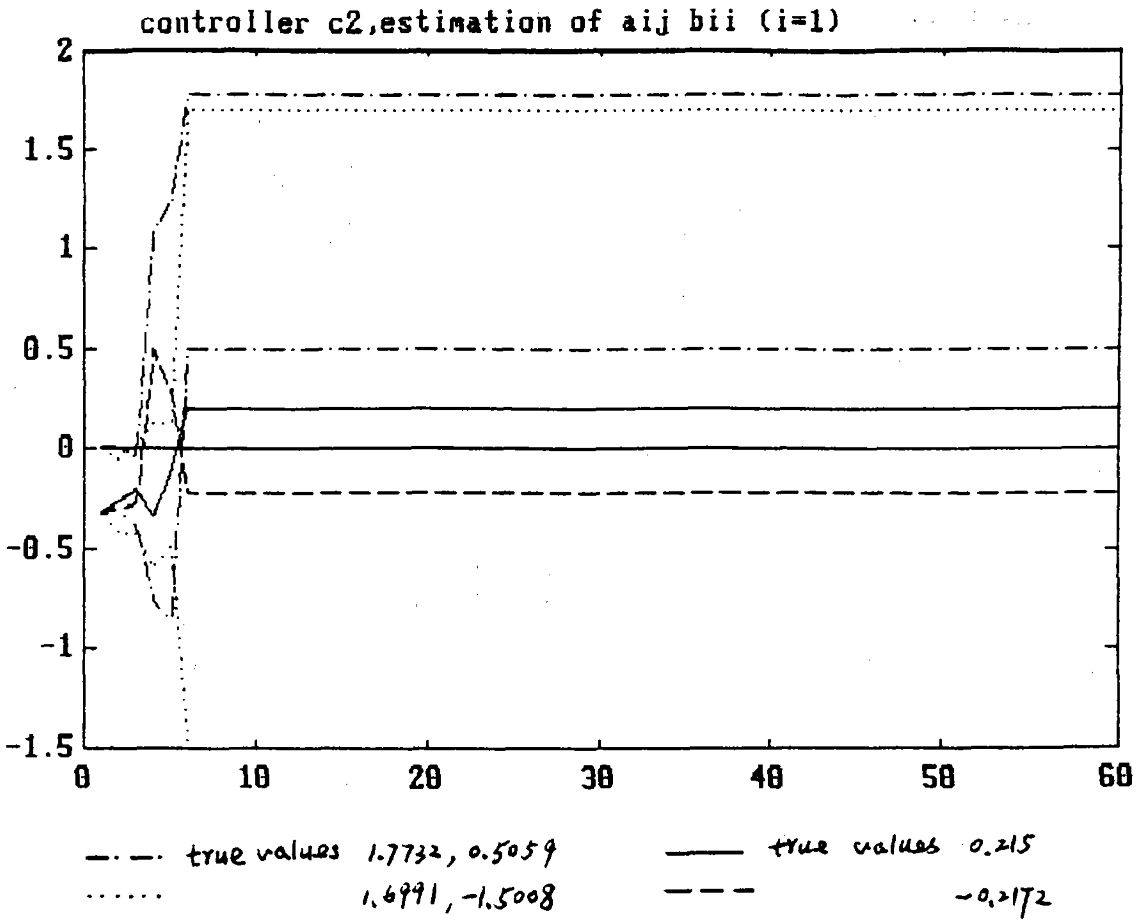



Fig. 4.3 output - 5 , d=10, α=0


controller c2,estimation of aij bii (i=1)
true values 1.7732, 0.5059
1.6991, -1.5008
true values 0.215
-0.2172

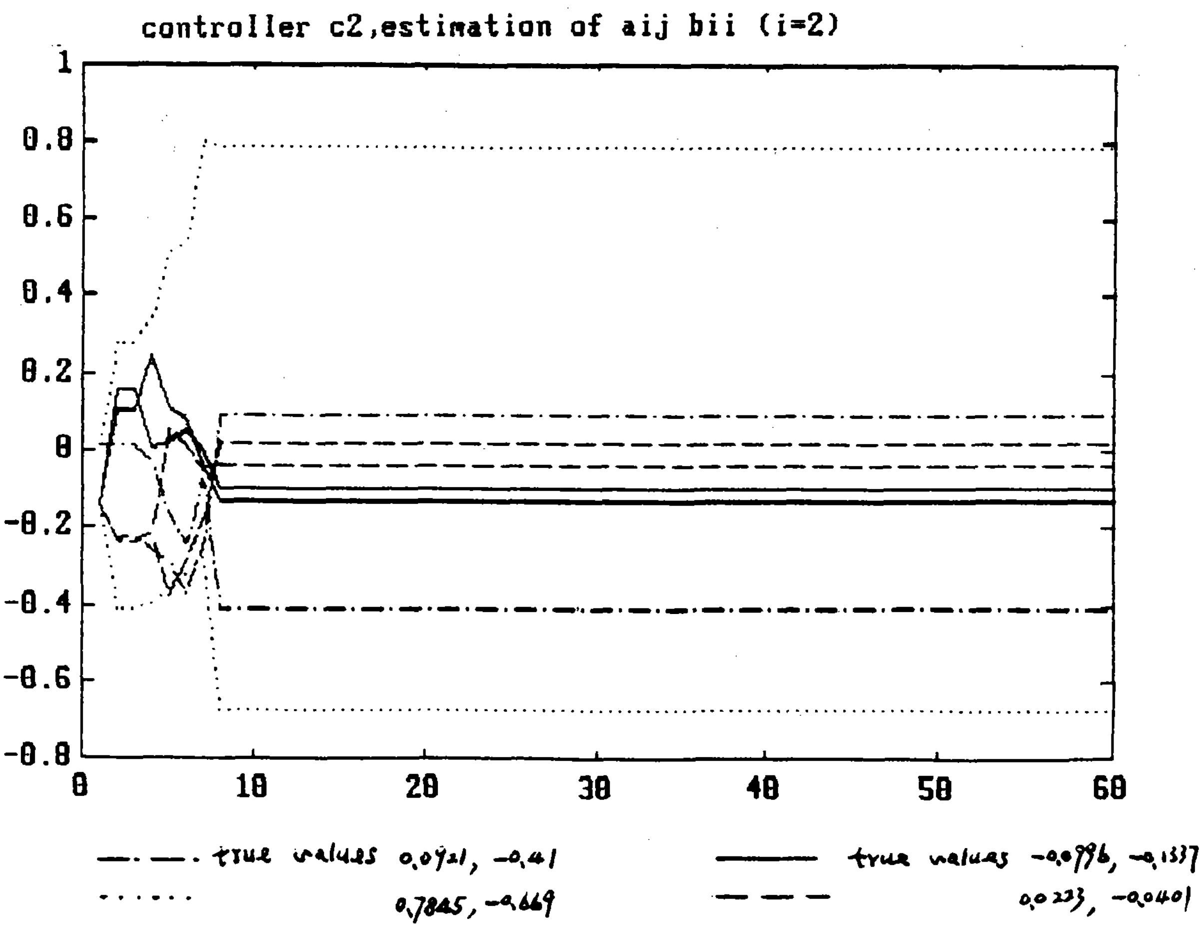

controller c2,estimation of aij bii (i=2)
true values 0.0921, -0.41
0.7845, -0.669
true values -0.0996, -0.1337
0.0233, -0.0401

Fig. 4.4

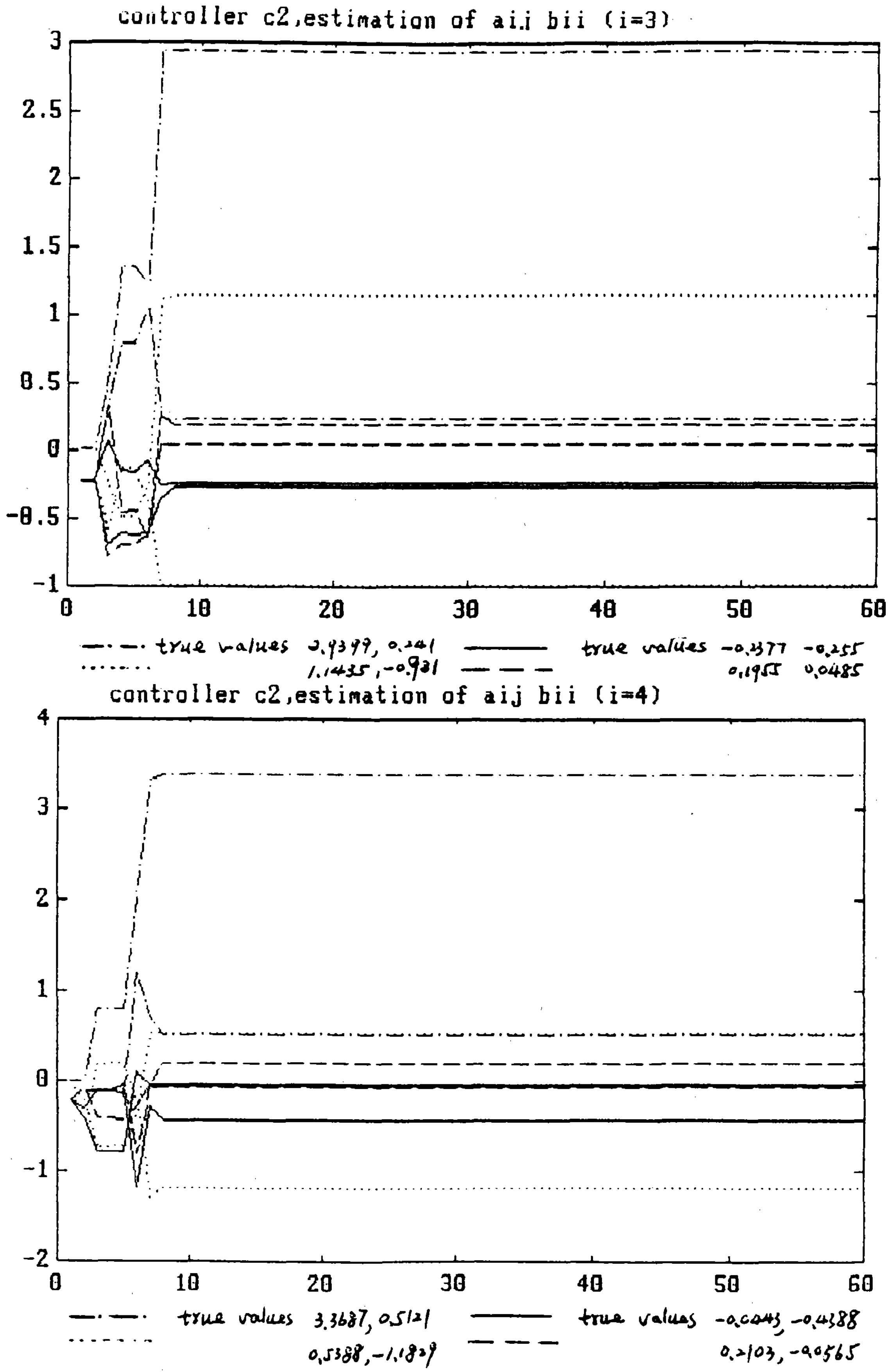
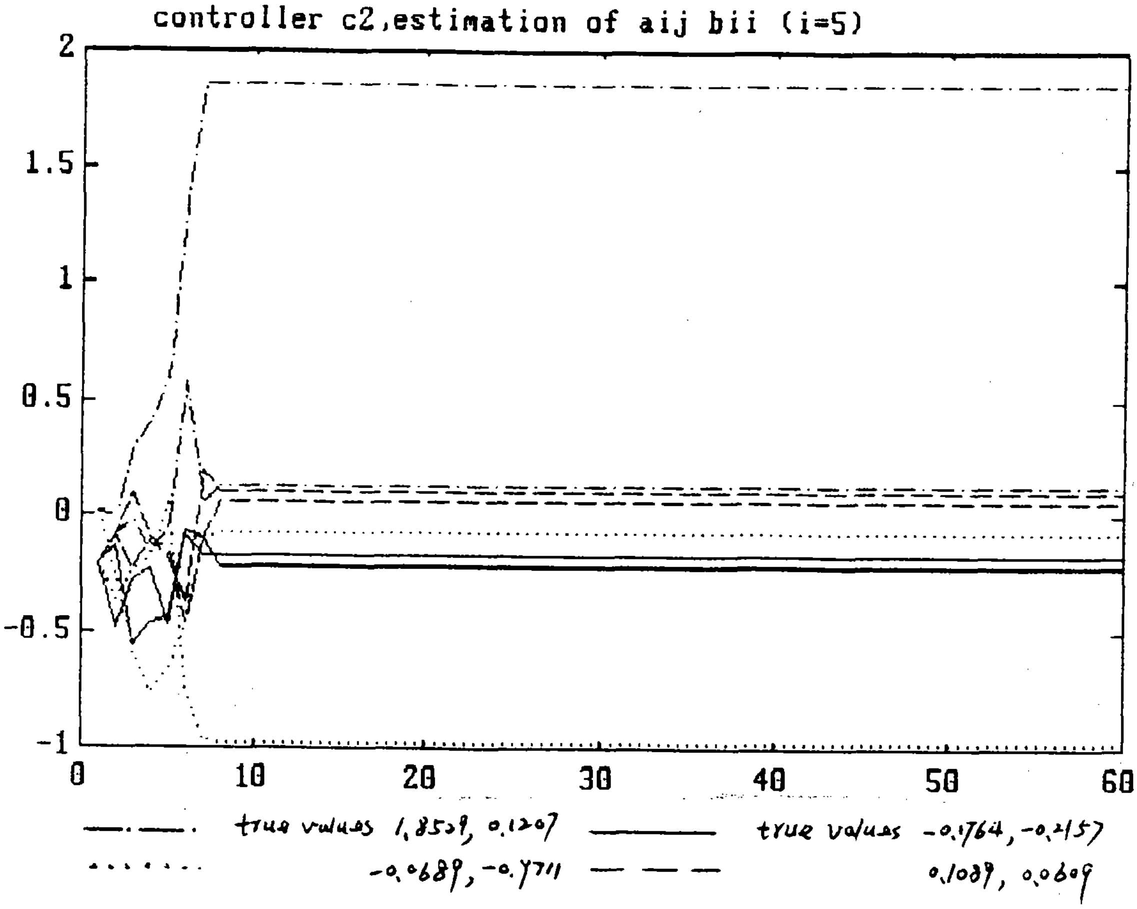

controller c2,estimation of ai.j bii (i=3)
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
true values 2.9399, 0.241
1.1435, -0.931
true values -0.2377 -0.255
0.1955 0.0485
controller c2,estimation of aij bii (i=4)
4
3
2
1
0
-1
-2
0
10
20
30
40
50
60
true values 3.3687, 0.5121
0.5388, -1.1829
true values -0.0443, -0.4388
0.2103, -0.0565

Fig. 4.4


controller c2,estimation of aij bii (i=5)
2
1.5
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
true values 1,8529, 0,1207
true values -0,1764, -0,2157
-0,0689, -0,9711
0,1089, 0,0609

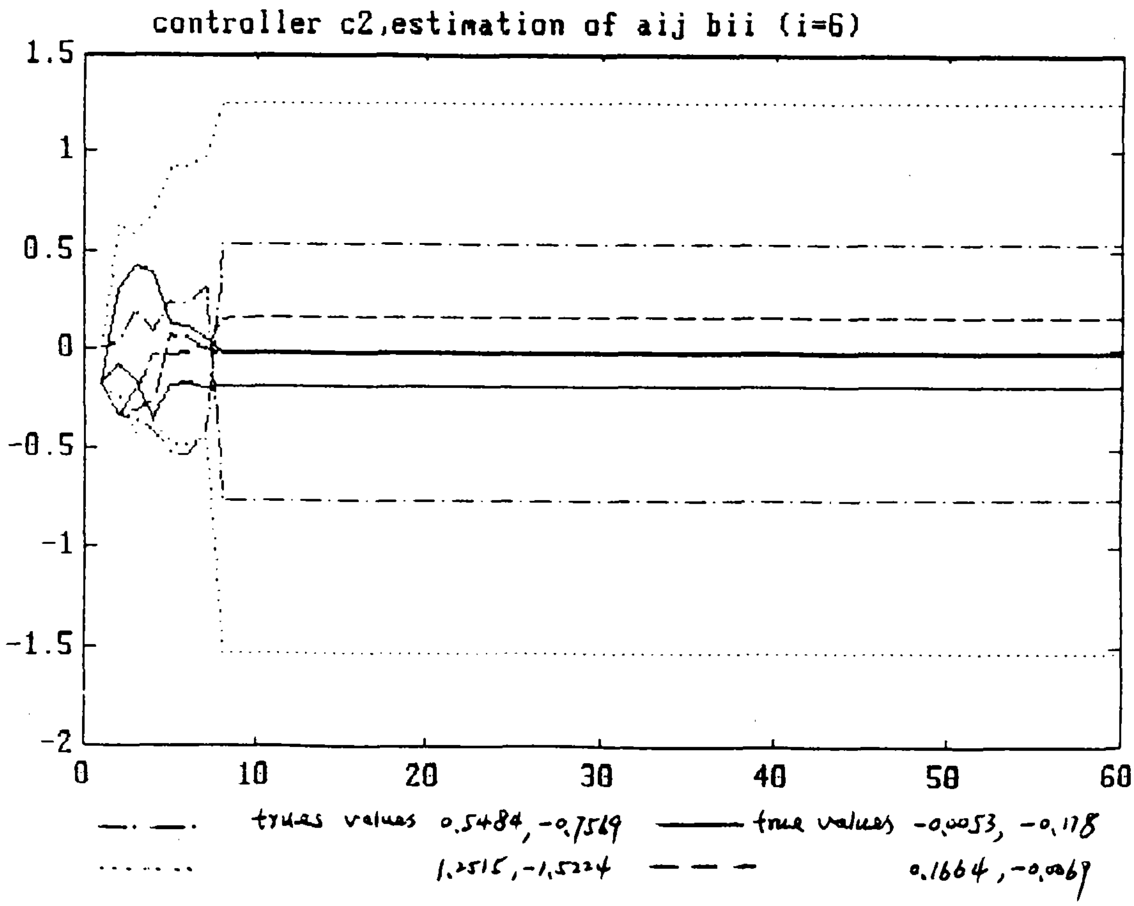

controller c2,estimation of aij bii (i=6)
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
trues values 0,5484, -0,7519
true values -0,0053, -0,118
1,2515, -1,5224
0,1664, -0,0069

Fig. 4.4

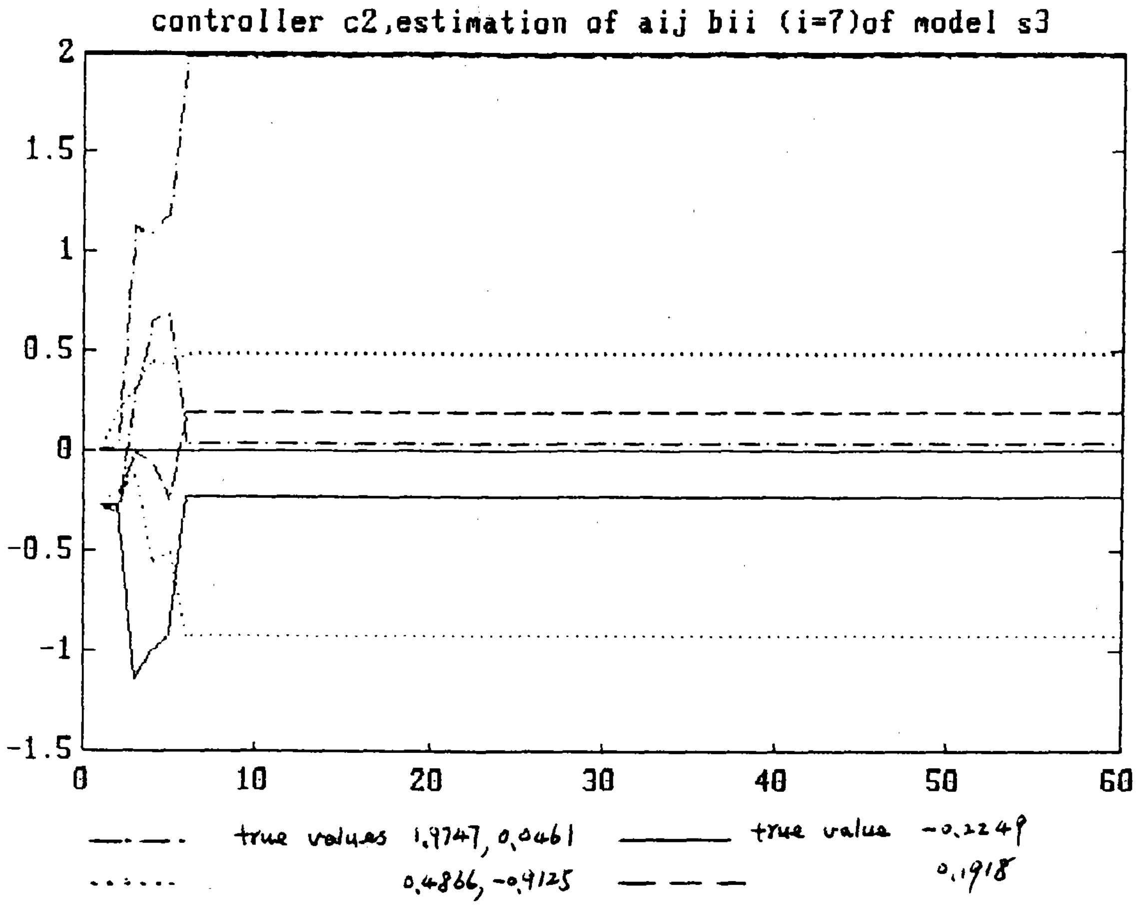

controller c2,estimation of aij bii (i=7)of model s3
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
10
20
30
40
50
60
true values 1,9747, 0,0461
true value -0,2249
0,4866, -0,9125
0,1918

Fig. 4.4

## Appendix (Data collected in the identification experiment)

Data were collected in 7 identification experiments. The corresponding data lists are denoted by cep 2,..., cep 8. In every experiment the output (temperatures) of the 21 heating units were recorded. Sampling time was 5 minutes. In every test only one unit was open loop controlled while the others were PI-closed loop controlled. The open loop controllers are of switch on - switch off type.

| name of data file | unit in open-loop state | number of samplings k |
|---|---|---|
| Cep2.dat | 5 | 21 |
| Cep3.dat | 8 | 19 |
| Cep4.dat | 4 | 16 |
| Cep5.dat | 2 | 17 |
| Cep6.dat | 13 | 16 |
| Cep7.dat | 10 | 16 |
| Cep8.dat | 7 | 16 |

In every data file the first two rows contain the input values applied to the open loop units at $t = 0, 5, 10, \ldots, k \cdot 5$. Here input 1(-1)means that the heating coil is switched on (off) and input 0 means that the voltage supply was connected for 2.5 minutes in every sampling intervall. The remaining 42 rows contain the temperatures for the 21 heating coils.

type a:cep2.dat —— *Experiment 1.*

```
   1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0
   1.0    1.0   -1.0   -1.0   -1.0   -1.0   -1.0   -1.0
 375.0  376.0  376.5  375.0  376.0  377.0  377.0  375.5  376.0  375.0  375.5  376.0  377.5
 377.0  376.0  377.0  378.0  379.0  380.0  377.5  376.5
 385.0  385.0  386.0  385.0  384.0  386.0  385.0  386.0  387.0  386.5  388.0  387.0  385.5
 386.0  388.0  389.0  390.0  388.0  385.0  386.0  385.5
 375.0  375.5  376.0  376.5  378.0  378.0  379.0  380.0  379.0  377.0  378.0  380.5  381.0
 380.5  381.0  379.0  377.0  378.0  379.0  377.5  377.0
 385.0  385.0  386.0  387.0  387.5  388.0  390.0  390.0  391.0  391.0  391.0  391.0  391.5
 392.0  392.0  391.0  390.0  388.0  387.0  385.5  384.0
 375.0  374.0  375.0  378.0  381.0  385.0  386.0  388.0  390.0  392.0  394.0  395.0  396.0
 397.0  396.0  394.0  394.0  391.0  387.0  382.0  379.0
 385.0  387.0  388.0  390.0  391.0  392.0  392.0  393.0  393.0  393.0  394.0  395.0  393.0
 391.0  388.0  387.0  389.0  389.0  387.0  387.0  388.0
 375.0  375.0  376.0  376.0  377.0  376.0  377.5  378.0  378.0  380.0  379.0  380.0  380.0
 379.0  379.0  379.5  380.0  380.0  379.0  378.0  379.0
 385.0  384.0  383.0  382.0  382.5  382.5  382.0  382.5  383.0  384.0  383.0  382.5  384.0
 387.0  388.0  390.5  391.0  389.5  389.0  387.5  388.0
 375.0  375.0  374.0  374.0  373.0  374.0  375.0  375.0  376.5  377.0  377.0  377.0  377.0
 379.0  381.0  383.0  383.0  381.0  381.0  381.0  381.0
 385.0  386.0  386.5  388.0  388.5  388.5  389.0  389.5  394.0  394.0  393.5  394.0  395.0
 395.5  397.0  393.5  393.5  394.5  391.5  392.0  393.5
 375.0  373.0  372.0  372.5  372.0  372.0  373.0  373.0  373.5  374.0  372.5  372.0  372.5
 374.0  375.5  375.5  375.5  375.0  374.5  374.5  374.5
 385.0  383.0  382.0  382.0  384.0  384.0  383.0  384.0  384.0  383.0  382.0  383.0  385.0
 385.0  383.0  383.0  385.0  383.0  382.0  384.0  385.0
 375.0  378.0  379.0  380.0  380.0  381.0  381.0  382.0  381.0  383.0  382.0  381.0  380.0
 385.0  385.0  386.0  385.0  384.0  384.0  383.0  383.0
 385.0  387.0  383.0  382.0  385.0  387.0  386.0  384.0  386.0  387.0  385.0  388.0  388.0
 386.0  385.0  386.0  388.0  386.0  384.0  386.0  388.0
 380.0  381.0  380.0  382.0  382.0  381.0  380.0  382.0  383.0  382.0  381.0  380.0  380.0
 380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 380.0  382.0  380.0  382.0  378.0  379.0  380.0  380.0  380.0  382.0  380.0  382.0  384.0
 386.0  390.0  390.0  386.0  384.0  384.0  386.0  390.0
 380.0  380.0  382.0  382.0  380.0  380.0  379.0  378.0  380.0  380.0  380.0  380.0  381.0
 382.0  381.0  380.0  382.0  382.0  384.0  384.0  384.0
 380.0  379.0  379.0  379.0  378.0  379.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 382.0  382.0  382.0  382.0  382.0  381.0  381.0  382.0
 380.0  380.0  381.0  381.0  382.0  383.0  382.0  384.0  385.0  385.0  386.0  387.0  387.0
 388.0  389.0  389.0  388.0  386.0  385.0  384.0  384.0
 380.0  380.0  380.0  379.0  379.0  378.0  377.0  377.0  376.0  377.0  376.0  374.0  376.0
 374.0  374.0  374.0  376.0  376.0  376.0  374.0  372.0
 380.0  380.0  381.0  381.0  383.0  381.0  380.0  380.0  379.0  378.0  374.0  376.0  378.0
 376.0  378.0  378.0  378.0  379.0  379.0  379.0  379.0
```

```
C:\>type a:cep3.dat —— Experiment 2
  -1.0  -1.0  -1.0  -1.0  -1.0  -1.0  -1.0   1.0   1.0   1.0   1.0   1.0   1.0
   1.0   1.0   1.0   1.0   1.0   1.0
 375.0 374.0 372.0 370.0 369.0 370.0 371.0 373.0 371.0 370.0 370.0 370.0 369.0
 371.0 369.0 370.0 369.0 371.0 371.0
 385.0 383.0 381.0 381.0 383.0 385.0 385.0 383.0 383.0 384.0 384.0 385.0 385.0
 385.0 385.0 385.0 385.0 384.0 383.0
 375.0 372.0 371.0 372.0 373.0 374.0 374.0 374.0 374.0 374.0 372.0 374.0 374.0
 374.0 374.0 374.0 374.0 373.0 375.0
 385.0 382.0 384.0 387.0 387.0 384.0 384.0 387.0 386.0 384.0 385.0 385.0 384.0
 385.0 385.0 385.0 383.0 384.5 386.0
 375.0 376.0 378.0 382.0 384.0 385.0 385.0 384.0 385.0 385.0 384.0 384.0 384.0
 385.0 386.0 384.0 384.0 383.0 383.0
 385.0 387.0 391.0 392.0 394.0 398.0 400.0 399.0 397.0 397.0 398.0 398.0 398.0
 399.0 399.0 399.0 400.0 398.0 398.0
 375.0 377.0 380.0 381.0 382.0 384.0 386.0 387.0 386.0 386.0 386.0 386.0 386.0
 387.0 388.0 388.0 388.0 388.0 388.0
 385.0 383.0 372.0 362.0 354.0 347.0 338.0 332.0 328.0 331.0 338.0 348.0 358.0
 364.0 369.5 374.0 377.0 380.0 383.0
 375.0 375.0 374.0 371.0 371.0 370.0 368.0 366.0 364.0 360.0 361.0 363.0 365.0
 366.0 369.0 370.0 372.0 373.0 374.0
 385.0 382.0 382.0 384.0 386.0 385.5 387.0 386.0 384.0 382.0 384.0 387.0 386.0
 384.0 383.0 385.0 387.0 390.0 388.0
 375.0 374.0 374.0 373.0 372.0 372.0 372.0 372.0 372.0 372.0 372.0 374.0 374.0
 374.0 374.0 374.0 375.0 376.0 377.0
 385.0 387.0 389.0 386.0 385.0 388.0 388.0 387.0 386.0 388.0 388.0 387.0 386.0
 388.0 388.0 387.0 386.0 389.0 389.0
 375.0 375.0 377.0 378.0 379.0 379.0 379.0 379.0 379.0 379.0 379.0 380.0 379.0
 378.0 378.0 379.0 379.0 379.0 379.0
 385.0 387.0 392.0 395.0 395.0 391.0 392.0 395.0 394.0 392.0 393.0 395.0 394.0
 391.0 392.0 395.0 394.0 392.0 393.0
 380.0 378.0 368.0 364.0 362.0 364.0 364.0 366.0 368.0 370.0 375.0 378.0 380.0
 380.0 378.0 379.0 380.0 380.0 380.0
 380.0 380.0 379.0 378.0 379.0 380.0 380.0 380.0 380.0 380.0 380.0 382.0 384.0
 383.0 384.0 382.0 382.0 381.0 380.0
 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 381.0
 381.0 380.0 381.0 380.0 381.0 381.0
 380.0 380.0 380.0 380.0 380.0 382.0 380.0 380.0 380.0 380.0 382.0 380.0 380.0
 380.0 380.0 382.0 382.0 384.0 384.0
 380.0 378.0 379.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0
 381.0 380.0 380.0 380.0 380.0 380.0
 380.0 379.0 379.0 380.0 380.0 382.0 382.0 382.0 382.0 382.0 382.0 382.0 382.0
 382.0 381.0 382.0 382.0 384.0 384.0
 380.0 380.0 380.0 382.0 384.0 386.0 390.0 390.0 390.0 390.0 390.0 388.0 388.0
 390.0 390.0 390.0 390.0 390.0 389.0
```

*Experiment 3.*

```
C:\>type a:cep4.dat
   1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0   -1.0   -1.0   -1.0   -1.0   -1.0
  -1.0   -1.0   -1.0
 375.0  375.0  375.0  373.0  372.0  372.0  373.0  373.0  372.0  372.0  373.0  373.0  373.0
 373.0  373.5  373.0
 385.0  386.0  386.0  385.0  385.0  385.0  386.0  387.0  386.0  385.0  385.0  386.0  387.0
 386.0  385.0  385.5
 375.0  376.0  377.0  378.0  380.0  381.0  382.0  380.0  380.0  379.0  377.0  374.0  373.0
 373.5  372.0  371.0
 385.0  385.0  388.0  395.0  399.5  404.0  405.5  406.0  405.0  403.0  393.0  378.0  368.0
 367.0  366.5  365.0
 375.0  376.0  378.0  380.0  382.0  382.0  382.0  380.0  380.0  380.0  378.0  376.0  373.0
 372.0  372.0  371.0
 385.0  387.0  387.0  388.0  388.0  389.0  388.0  388.0  388.0  388.0  387.0  387.0  387.0
 387.0  386.0  387.0
 375.0  374.0  374.0  375.0  376.0  377.0  378.0  379.0  379.0  380.0  380.0  380.0  380.0
 379.0  377.0  378.0
 385.0  382.0  378.0  376.0  374.0  376.0  377.0  379.0  378.0  377.0  377.0  378.0  378.0
 379.0  379.0  380.0
 375.0  373.0  374.0  373.0  372.0  371.0  371.0  372.0  373.0  373.0  373.0  373.0  372.0
 373.0  372.0  373.0
 385.0  386.0  390.0  388.0  386.0  384.0  385.0  386.0  387.0  388.0  391.0  384.0  383.0
 383.0  384.0  384.0
 375.0  375.0  375.0  375.0  374.0  373.0  372.0  373.0  372.0  372.0  372.5  373.0  373.0
 373.0  372.0  373.0
 385.0  386.0  388.0  387.0  386.0  387.0  388.0  387.0  386.0  387.0  388.0  387.0  386.0
 386.0  387.0  387.0
 375.0  374.0  374.0  374.0  374.0  374.0  374.0  374.0  374.0  374.0  375.0  374.0  374.0
 374.0  375.0  374.0
 385.0  383.0  382.0  384.0  385.0  383.0  381.0  384.0  385.0  383.0  382.0  385.0  384.0
 384.0  383.0  384.0
 380.0  380.0  380.0  380.0  378.0  378.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 380.0  381.0  380.0
 380.0  380.0  380.0  380.0  380.0  380.0  380.0  384.0  386.0  388.0  388.0  386.0  380.0
 380.0  381.0  382.0
 381.0  380.0  380.0  380.0  380.0  382.0  384.0  384.0  384.0  381.0  380.0  380.0  380.0
 380.0  380.0  381.0
 380.0  380.0  380.0  381.0  382.0  383.0  385.0  387.0  389.0  386.0  385.0  382.0  380.0
 380.0  382.0  381.0
 380.0  383.0  385.0  386.0  386.0  385.0  382.0  380.0  378.0  376.0  374.0  374.0  377.0
 379.0  380.0  379.0
 380.0  380.0  380.0  380.0  380.0  378.0  378.0  379.0  380.0  380.0  380.0  379.0  378.0
 379.0  380.0  379.0
 380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 380.0  381.0  380.0
```

```
C:\>type a:cep5.dat —— Experiment 4
   1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0
  -1.0   -1.0   -1.0   -1.0
 375.0  376.0  376.0  375.0  374.5  375.0  375.0  375.5  375.0  375.0  375.0  375.0  375.0
 375.0  375.0  375.0  373.0
 386.0  388.0  389.0  386.0  386.0  386.0  386.0  386.5  387.0  387.0  387.5  387.0  386.5
 386.0  386.0  386.0  360.0
 375.0  376.0  376.0  375.0  374.0  373.5  374.5  375.0  376.0  375.5  375.0  374.0  374.0
 374.5  375.0  374.0  373.0
 385.0  383.5  383.0  383.0  383.0  383.0  383.0  384.0  384.5  384.0  383.0  382.0  383.0
 383.0  383.5  383.0  383.0
 375.0  376.0  376.0  375.0  374.0  374.0  374.0  375.0  376.0  376.5  376.0  375.5  375.0
 375.0  376.0  376.0  376.0
 385.0  385.0  384.0  383.0  383.0  383.0  384.0  384.0  385.0  384.5  384.0  384.0  384.0
 384.0  385.0  385.0  384.0
 375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0  375.0
 375.0  375.0  375.0  375.0
 384.0  384.0  384.0  384.0  385.5  384.0  384.0  384.0  383.5  383.0  383.5  385.5  383.0
 383.0  383.0  385.0  385.0
 375.0  374.0  373.0  372.0  373.0  375.0  375.5  375.0  374.5  373.0  373.0  373.0  374.0
 375.0  375.0  375.0  375.0
 384.0  382.0  383.0  385.0  386.0  385.0  384.5  384.0  385.0  384.5  383.0  385.5  386.5
 385.5  384.0  384.0  386.0
 375.0  374.0  373.0  373.0  373.0  376.0  377.0  377.0  377.0  376.0  375.5  376.5  377.5
 379.0  378.0  376.0  377.0
 386.0  387.0  387.0  386.0  387.0  387.0  388.0  386.0  386.0  388.0  387.0  386.0  387.0
 387.0  386.0  388.0  388.0
 373.0  373.0  373.0  372.0  373.0  373.0  373.0  373.0  373.0  373.0  373.0  373.0  373.0
 374.0  374.0  373.0  374.0
 384.0  383.0  382.0  384.0  384.0  384.0  383.0  384.0  385.0  384.0  383.0  384.0  385.0
 384.0  383.0  383.0  385.0
 380.0  380.0  380.0  380.0  380.0  380.0  381.0  381.0  381.0  380.0  380.0  380.0  380.0
 380.0  380.0  380.0  380.0
 379.0  378.0  377.0  379.0  380.0  380.0  380.0  379.0  378.0  376.0  376.0  380.0  380.0
 380.0  380.0  378.0  370.0
 379.0  381.0  381.0  381.0  379.0  379.0  381.0  381.0  381.0  381.0  381.0  381.0  381.0
 381.0  381.0  381.0  379.0
 380.0  380.0  382.0  382.0  386.0  387.0  389.0  389.0  388.0  389.0  388.0  388.0  389.0
 389.0  389.0  386.0  380.0
 379.0  379.0  378.0  378.0  374.0  376.0  378.0  380.0  380.0  380.0  379.0  378.0  377.0
 379.0  380.0  380.0  378.0
 379.0  380.0  385.0  385.0  385.0  387.0  393.0  392.0  391.0  393.0  394.0  393.0  393.0
 394.0  395.0  395.0  393.0
 380.0  380.0  380.0  380.0  388.0  390.0  392.0  400.0  400.0  402.0  402.0  408.0  408.0
 409.0  410.0  410.0  408.0
```

```
C:\>type a:cep6.dat —— Experiment 5
   1.0    1.0    1.0    1.0    1.0    1.0    1.0   -1.0   -1.0   -1.0   -1.0   -1.0   -1.0
  -1.0   -1.0   -1.0
 375.0 376.0 375.0 375.5 376.5 378.0 379.5 381.5 384.0 385.5 388.0 385.0 382.0
 383.0 382.0 381.0
 385.0 383.0 384.0 385.0 386.0 385.5 384.0 385.0 385.0 384.0 383.0 384.0 383.0
 383.0 384.0 383.0
 375.0 375.0 375.0 376.0 376.0 376.0 376.0 376.0 377.0 377.0 376.5 377.0 376.0
 376.0 377.0 376.0
 385.0 384.0 385.5 386.5 386.0 385.0 385.0 386.0 386.0 385.0 385.0 385.5 385.5
 385.0 385.0 385.0
 375.0 375.0 375.0 376.0 376.0 376.0 376.0 376.0 376.0 376.0 376.0 376.0 376.0
 376.0 375.0 376.0
 385.0 387.0 388.0 388.0 388.0 387.0 388.0 388.0 388.0 388.0 388.0 388.0 388.0
 388.0 388.0 387.0
 375.0 375.0 375.0 376.0 375.0 375.0 376.0 376.0 376.0 376.0 376.0 376.0 376.0
 376.0 376.0 376.0
 384.0 383.5 383.5 384.5 384.5 383.0 383.0 384.0 384.0 383.0 383.5 385.0 384.0
 383.0 384.0 383.0
 375.0 377.0 377.0 375.0 375.0 377.0 376.5 375.0 374.5 375.0 375.0 375.5 376.0
 376.0 376.0 375.0
 385.0 385.0 384.0 384.0 386.5 386.5 385.0 385.5 384.5 386.0 385.5 384.0 385.0
 387.0 386.0 385.0
 375.0 377.0 377.5 377.0 376.5 377.5 376.5 376.5 376.5 376.0 376.0 376.0 377.5
 379.0 380.0 380.0
 385.0 386.0 385.0 387.0 388.0 390.0 390.0 390.5 391.0 390.0 392.0 390.0 388.0
 387.0 387.0 386.0
 375.0 377.0 378.0 383.0 387.0 390.0 393.0 394.0 393.0 389.0 385.0 381.0 379.0
 375.0 376.0 375.0
 385.0 384.0 385.0 385.0 386.0 385.0 387.0 388.0 389.0 387.0 386.0 385.0 385.0
 383.0 383.0 384.0
 380.0 380.0 380.0 380.0 380.0 380.0 380.0 381.0 382.0 382.0 382.0 380.0 380.0
 380.0 380.0 380.0
 380.0 380.0 380.0 376.0 376.0 382.0 386.0 388.0 388.0 390.0 390.0 390.0 390.0
 389.0 388.0 387.0
 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 381.0 380.0 380.0 380.0
 380.0 380.0 380.0
 380.0 380.0 380.0 380.0 382.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 381.0
 383.0 381.0 380.0
 380.0 379.0 379.0 380.0 380.0 380.0 379.0 378.0 378.0 378.0 376.0 378.0 378.0
 378.0 378.0 379.0
 380.0 380.0 382.0 384.0 385.0 387.0 389.0 390.0 390.0 388.0 387.0 386.0 384.0
 382.0 380.0 379.0
 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0
 380.0 380.0 381.0
```

```
C:\>type a:cep7.dat —— Experiment 6
   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0  -1.0  -1.0  -1.0
  -1.0  -1.0  -1.0
 375.0 375.0 374.0 373.5 374.0 375.0 374.5 374.0 374.0 374.0 375.0 373.5 373.5
 374.0 374.0 373.0
 385.0 384.0 382.5 383.0 384.5 385.5 384.0 383.0 384.0 387.0 387.0 385.5 385.0
 384.0 385.0 385.0
 375.0 375.0 374.5 375.0 374.0 374.0 374.0 374.0 375.0 375.5 375.0 374.0 374.0
 374.0 374.0 374.0
 385.0 385.5 385.5 386.0 385.0 385.0 386.0 386.0 387.0 388.0 385.0 386.0 387.0
 386.0 387.0 386.0
 375.0 376.0 375.0 376.0 375.5 375.0 376.0 376.0 377.0 377.0 376.0 375.0 375.0
 376.0 375.0 376.0
 385.0 384.0 384.5 385.0 384.5 384.0 384.5 384.5 385.0 385.0 384.5 384.5 384.5
 384.5 384.0 384.0
 375.0 375.0 374.5 375.0 374.5 374.0 374.0 374.0 374.0 374.0 374.5 374.5 374.0
 374.0 375.0 374.0
 385.0 385.0 384.0 383.0 384.0 386.0 385.0 384.0 384.0 384.0 385.0 384.5 385.0
 385.5 385.0 385.0
 375.0 376.0 377.0 378.0 379.0 378.0 379.0 380.0 380.0 381.0 381.0 380.0 379.0
 379.0 378.0 378.0
 385.0 387.0 390.0 392.0 397.0 398.0 400.0 402.0 404.0 405.0 407.0 405.0 396.0
 382.0 378.0 376.0
 375.0 374.0 374.5 376.0 376.0 377.0 378.0 377.5 380.0 378.0 379.5 378.0 376.0
 376.0 377.0 377.5
 385.0 388.0 388.0 388.0 388.0 389.0 388.0 387.0 386.0 389.0 389.0 387.5 387.0
 388.5 388.0 387.0
 375.0 376.0 376.5 377.0 377.0 377.5 377.0 377.0 377.0 377.0 377.5 377.5 377.0
 377.0 377.0 377.0
 385.0 385.5 385.0 384.0 384.0 383.5 383.5 383.5 383.0 385.0 384.5 383.5 383.5
 384.5 384.0 385.0
 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 380.0 379.0
 379.0 380.0 381.0
 380.0 380.0 379.0 378.0 378.0 378.0 376.0 374.0 372.0 370.0 370.0 370.0 371.0
 372.0 373.0 374.0
 380.0 382.0 382.0 384.0 385.0 387.0 388.0 389.0 389.0 390.0 390.0 391.0 389.0
 387.0 386.0 384.0
 382.0 381.0 380.0 380.0 380.0 380.0 380.0 382.0 384.0 384.0 384.0 384.0 382.0
 380.0 381.0 380.0
 380.0 380.0 380.0 380.0 380.0 380.0 382.0 382.0 382.0 382.0 381.0 381.0 382.0
 382.0 381.0 381.0
 380.0 380.0 380.0 380.0 380.0 380.0 382.0 382.0 382.0 382.0 382.0 381.0 382.0
 382.0 381.0 381.0
 380.0 380.0 378.0 378.0 378.0 378.0 380.0 380.0 380.0 380.0 380.0 379.0 380.0
 380.0 380.0 380.0
```

```
C:\>type a:cep8.dat —— Experiment 7
   1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0    1.0   -1.0   -1.0   -1.0   -1.0
  -1.0   -1.0   -1.0
 375.0  375.5  376.0  375.0  378.0  377.0  377.0  377.0  377.0  377.0  376.0  375.0  377.0
 376.0  376.0  376.0
 385.0  385.5  385.0  384.0  388.0  386.0  388.0  385.0  386.0  384.5  384.0  385.0  386.0
 387.0  386.0  385.0
 375.0  374.5  374.0  375.0  375.0  375.0  374.0  374.0  375.0  375.5  375.0  375.0  375.0
 375.5  375.5  375.0
 385.0  385.0  386.0  387.0  386.5  385.0  385.0  385.5  386.5  386.5  386.0  386.0  386.0
 386.5  386.5  386.0
 375.0  375.0  375.0  376.0  377.0  376.0  375.0  375.0  376.0  376.0  378.0  378.0  378.0
 378.0  379.0  378.0
 385.0  385.5  386.0  386.0  388.0  389.0  392.0  390.5  389.0  390.0  389.0  388.0  387.0
 387.0  386.0  386.0
 375.0  376.0  377.0  382.0  382.0  386.0  388.0  388.0  388.0  389.0  389.0  386.0  381.0
 377.0  373.0  370.5
 385.0  380.5  380.0  380.0  379.0  378.0  379.0  380.0  380.0  379.0  379.0  378.0  377.0
 379.0  380.0  380.0
 375.0  379.0  377.0  376.0  374.0  373.0  374.0  376.0  376.0  375.0  374.0  373.0  373.0
 373.0  372.0  373.0
 385.0  389.0  387.0  383.0  381.0  383.0  386.0  387.0  384.0  384.0  382.5  384.0  384.0
 383.0  383.0  383.0
 375.0  374.0  374.0  373.0  373.0  373.0  374.0  374.0  374.0  374.0  374.5  373.0  372.0
 372.0  372.5  372.0
 385.0  385.0  382.0  383.0  385.0  384.0  383.0  383.0  384.0  383.0  383.0  383.0  383.0
 382.0  383.0  382.0
 375.0  374.5  374.0  374.0  374.0  374.0  375.0  374.0  374.0  374.0  374.0  374.0  374.0
 374.0  374.0  374.0
 385.0  384.0  383.0  383.0  384.0  385.0  384.0  383.0  383.0  384.0  384.0  384.0  382.0
 384.0  384.0  384.0
 380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 380.0  380.0  380.0
 380.0  382.0  380.0  376.0  372.0  374.0  380.0  380.0  380.0  378.0  374.0  374.0  380.0
 382.0  382.0  382.0
 380.0  381.0  381.0  381.0  380.0  381.0  381.0  381.0  381.0  381.0  381.0  381.0  381.0
 381.0  381.0  381.0
 380.0  380.0  378.0  378.0  378.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0  380.0
 380.0  380.0  380.0
 380.0  380.0  381.0  382.0  382.0  381.0  380.0  380.0  381.0  382.0  382.0  381.0  382.0
 381.0  381.0  380.0
 380.0  380.0  381.0  382.0  380.0  382.0  380.0  380.0  381.0  382.0  382.0  381.0  382.0
 381.0  381.0  381.0
 380.0  380.0  381.0  382.0  384.0  386.0  387.0  389.0  392.0  392.0  390.0  384.0  382.0
 381.0  381.0  381.0
```