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TOPOLOGICAL NECESSARY CONDITIONS OF SMOOTH STABILIZATION IN THE LARGE

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Topological necessary conditions of smooth stabilization in the large[1]

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Abstract: Several topological necessary conditions of smooth stabilization in the large have been obtained. In particular, if a smooth single - input nonlinear system is smoothly stabilizable in the large at some point of a connected component of equilibria set, then the connected component is to be an unknoted, unbounded curve.

Keywords: Nonlinear systems; asymptotic stabilization; feedback control; degree of continuous function; knot.

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1 Introduction

Consider the system Σ_f:

$$\dot{x} = f(x, u),$$

where $x \in R^n$, $u \in R^m$ and R^ℓ is used for ℓ-dimensional Euclidean space. $f(x,u)$ is a complete C^∞ vector field on R^n for every $u \in R^m$ fixed.

The set

$$f^{-1}(0) = \{(x,u) \in R^{n+m} \ ; \ f(x,u) = 0\}$$

is called an equilibria set of the control system.

A system Σ_f is said to be *smoothly stabilizable* at $(x^*, u^*) \in f^{-1}(0)$ in the large, iff there exists a C^∞ function $u = u(x)$, such that $u(x^*) = u^*$ and x^* is an asymptotically stable in the large singular point of the closed loop system

$$\dot{x} = f(x, u(x)),$$

i.e., x^* is stable and

$$\lim_{t \to +\infty} e^{tf} x = x^* \quad \forall\, x \in R^n$$

where e^{tf} is the flow generated by the vector field $f(x, u(x))$.

The smooth stabilizability problem has been considered in many papers (for the list of references see, e.g., [7]). Necessary conditions for local smooth stabilization have been obtained in [1, 2], and for smooth stabilization in the large in [5]. This paper represents a continuation of a line of work started in [5].

2 Some facts about degree of function

This section is devoted to recalling some facts about the degree of continuous functions. For additional details on the degree of continuous functions, see [3, 4, 6]. We start with some notations.

(i) $x = (x_1, \ldots, x_n) \in R^n$; $|x|^2 = \langle x, x \rangle$, where $\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i \quad \forall\, x, y \in R^n$.

(ii) D is a bounded, open subset of R^n. Given $D \subset R^n$, its closure is written $\bar{D}$, its interior $Int D$, its boundary ∂D.

(iii) Given a real positive number r and $y \in \mathbf{R}^n$, $\mathbf{B_r(y)}$ is the closed ball centre y, radius r :

$$B_r(y) = \{x \in \mathbf{R}^n;\ |x - y| \leq r\}.$$

(iv) $C(\bar{D})$ is the linear space of continuous functions from $\bar{D}$ into $\mathbf{R}^n$ with the norm

$$\| f \| = \sup_{x \in \bar{D}} | f(x) | .$$

$C^1(\bar{D})$ is the space of functions having continuous first order partial derivatives in $\bar{D}$; the norm on $C^1(\bar{D})$ is

$$\| f \|_1 = \sup_{x \in \bar{D}} | f(x) | + \sup_{x \in \bar{D}\ 1 \leq j \leq n} | \frac{\partial}{\partial x_j} f(x) |,$$

where

$$\frac{\partial}{\partial x_j} f(x) = (\frac{\partial}{\partial x_j} f_1(x), \ldots, \frac{\partial}{\partial x_j} f_n(x))^T,$$

$\frac{\partial}{\partial x_j} f_i(x)$ is the partial derivative of the i–th entry of the function $f : \bar{D} \to \mathbf{R}^n$, T indicates transpose; $\frac{\partial}{\partial x} f(x)$ is the Jacobian matrix and $det(\frac{\partial}{\partial x} f(x))$ is the Jacobian determinant of f at x.

(v) Let M, N be smooth manifolds of dimension n. Then C^∞ - mapping $\psi : M \to N$ is called diffeomorphism iff ψ is homeomorphism and ψ^{-1} is also C^∞ - mapping.

Now we define the degree of ϕ when function $\phi \in C^1(\bar{D})$ and

$$\forall x \in \phi_{\bar{D}}^{-1}(p) = \{x \in \bar{D};\ \phi(x) = p\} \quad det(\frac{\partial}{\partial x}\phi(x)) \neq 0,$$

i.e., p is not critical value of ϕ on D.

Definition 2.1. Suppose $\phi \in C^1(\bar{D})$, $p \bar{\in} \phi(\partial D)$ and p is not critical value of ϕ on D. Define the degree of ϕ at p relative to D to be $d(\phi, D, p)$, where

$$d(\phi, D, p) = \sum_{x \in \phi^{-1}(p)} sign[det(\frac{\partial}{\partial x}\phi(x))].$$

If $\phi \in C(\bar{D})$, then the degree of ϕ can be defined as the degree of a sufficiently good C^1 approximation of ϕ (for details, see [4]).

Definition 2.2. Suppose that $\phi \in C(\bar{D})$ and $p \bar{\in} \phi(\partial D)$. Define $d(\phi, D, p)$ to be $d(\psi, D, p)$, where ψ is any function in $C^1(\bar{D})$ satisfying

$$| \phi(x) - \psi(x) | < \rho(p, \phi(\partial D)) \quad \forall x \in \bar{D},$$

where $\rho(x, \phi(\partial D)) = \inf_{y \in \phi(\partial D)} \mid x - y \mid$.

Recall that if X and Y are topological spaces, two continuous functions f and g are said to be homotopic ($f \sim g$) if there is a continuous function (homotopy)

$$H : \ [0,1] \times X \ \rightarrow \ Y$$

such that

$$H(0,x) = f(x), \quad H(1,x) = g(x) \quad (x \in X).$$

We will need the following properties of degree.

Theorem 2.1 *(1) If $H(t,x) \equiv h_t(x)$ is a homotopy and $p \bar{\in} h_t(\partial D)$ for $0 \leq t \leq 1$, then $d(h_t, D, p)$ is independent of $t \in [0,1]$.*

(2) If a smooth feedback $u = u(x)$ stabilizes the system Σ_f at $p \in D \subset \mathrm{R}^n$ in the large, then

$$d(f(u), D, 0) = (-1)^n,$$

where $f(u)$ denotes $f(x, u(x)) : \ \mathrm{R}^n \ \rightarrow \ \mathrm{R}^n$.

(3) Suppose $\phi \in C(\bar{D})$. If $d(\phi, D, p)$ is defined and non-zero, then there is $q \in D$ such that $\phi(q) = p$.

In the definitions 2.1, 2.2 D can be replaced by a smooth, oriented manifold M of dimension n, $dim M = n$ (details of this and related material may be found in [3 , 6]). Given $\phi \in C^1(M)$, $\phi : M \ \rightarrow \ \mathrm{R}^n$, $p \bar{\in} \phi(\partial \mathrm{M})$, $d(\phi, \mathrm{M}, \mathrm{p})$ denotes the degree of ϕ at p relative to M.

An immersion $i : \ M \ \rightarrow \ \mathrm{R}^n_x \times \mathrm{R}^m_u$ which maps M homeomorphically into its image $i(M) \subset \mathrm{R}^n_x \times \mathrm{R}^m_u$ with topology induced by $\mathrm{R}^n_x \times \mathrm{R}^m_u$ is called regular embedding. $i_x : M \ \rightarrow \ \mathrm{R}^n_x$, $i_u ; \mathrm{M} \ \rightarrow \ \mathrm{R}^m_u$ are used to denote $P_x \circ i$ and $P_u \circ i$, respectively, where P_x, P_u are the projections : $P_x(x,u) = x$, $P_u(x,u) = u$.

Lemma 2.1 *Suppose $f : \mathrm{R}^n_x \times \mathrm{R}^m_u \ \rightarrow \ \mathrm{R}^n$ is a smooth function and*

$$rank\{\frac{\partial}{\partial x} f(x,u), \frac{\partial}{\partial u} f(x,u)\} = n \qquad \forall \, (x,u) \ \in \ f^{-1}(0).$$

Suppose further ω is a bounded connected component of $f^{-1}(0)$. If $u = v(x) : \mathrm{R}^n_x \ \rightarrow \ \mathrm{R}^m_u$ is a smooth function such that

$$f^{-1}(0) \cap \{(x,u) \in \mathrm{R}^n_x \times \mathrm{R}^m_u : \mathrm{u} = \mathrm{v(x)}\} = \omega \cap \{(\mathrm{x},\mathrm{u}) \in \mathrm{R}^n_x \times \mathrm{R}^m_u ; \mathrm{u} = \mathrm{v(x)}\},$$

then

$$d((f, P_u - v \circ P_x), B_R(0), 0) = 0$$

where $R > 0$ such that $\omega \subset Int B_R(0)$.

Proof. Consider the function

$$\varphi(x,u) = \begin{pmatrix} f(x,u) \\ u - v(x) \end{pmatrix}$$

If $\omega \subset IntB_R(0)$, then $0\overline{\in}\varphi(\partial B_R(0))$ and $d((f, P_u - v \circ P_x), B_R(0), 0)$ is defined.
The set

$$V = \varphi([B_R(0) \cap \{f^{-1}(0) \setminus \omega\}] \cup \partial B_R(0))$$

is compact and $0\overline{\in}V$. Thus making use of Sard's theorem (see, e.g., [3]), for any $\varepsilon > 0$ we can choose so a point $p \in R^n_x \times R^m_u$, that p is not critical value of ϕ on $B_R(0)$, $| p |< \varepsilon$, $P_x(p)$ is not critical value of $f(x,u)$ and p is in the connected component of the set $(R^n_x \times R^m_u) \setminus V$ containing zero.
For any $\delta > 0$ one can find a positive number ε such that

$$f^{-1}(B_\varepsilon(0)) \cap B_R(0) \subseteq \bigcup_{x \in f^{-1}(0) \cap B_R(0)} B_\delta(x).$$

Let us choose $\varepsilon > 0$, $\delta > 0$ such that

$$\{(x,u) \in B_R(0); \ | u - v(x) |< \varepsilon\} \cap [\bigcup_{x \in (f^{-1}(0)\setminus\omega) \cap B_R(0)} B_\delta(x)] = \emptyset,$$

$$\bigcup_{x \in \omega} B_\delta(x) \subset IntB_R(0),$$

$$\{\bigcup_{x \in \omega} B_\delta(x)\} \cap [\bigcup_{x \in (f^{-1}(0)\setminus\omega) \cap B_R(0)} B_\delta(x)] = \emptyset.$$

. Then the property (1) (Theorem 2.1) implies

$$d(\phi, B_R(0), 0) = d(\phi, B_R(0), p)$$

and according to the definition 2.1.

$$d(\phi, B_R(0), p) = \sum_{(x,u) \in \phi^{-1}(p)} sign[det \begin{pmatrix} \frac{\partial}{\partial x} f(x,u) & \frac{\partial}{\partial u} f(x,u) \\ -\frac{\partial}{\partial x} v(x) & I_m \end{pmatrix}],$$

where I_m is the identity $m \times m$ -matrix.
Let

$$\omega_p = f^{-1}(P_x(p)) \cap [\bigcup_{x \in \omega} B_\delta(x)]$$

and $i : \omega_p \rightarrow R^n_x \times R^m_u$ be regular embedding z will be used to denote local coordinates on ω_p. Due to

$$rank(\frac{\partial}{\partial x} f(x,u), \frac{\partial}{\partial u} f(x,u)) = n \quad \forall (x,u) \in f^{-1}(P_x(p))$$

one can choose local coordinates on ω_p so that

$$det\begin{pmatrix} (\frac{\partial}{\partial x}f)\circ i(z) & (\frac{\partial}{\partial u}f)\circ i(z) \\ (\frac{\partial}{\partial z}i_x(z))^T & (\frac{\partial}{\partial z}i_u(z))^T \end{pmatrix} > 0 \quad \forall z \in \omega_p.$$

Thus

$$sign[det\begin{pmatrix} \frac{\partial}{\partial x}f(x,u) & \frac{\partial}{\partial u}f(x,u) \\ -\frac{\partial}{\partial x}v(x) & I_m \end{pmatrix}] = sign[det[\begin{pmatrix} \frac{\partial}{\partial x}f(x,u) & \frac{\partial}{\partial u}f(x,u) \\ -\frac{\partial}{\partial x}v(x) & I_m \end{pmatrix} \times$$

$$\begin{pmatrix} (\frac{\partial}{\partial x}f(x,u))^T & \frac{\partial}{\partial z}i_x(z) \\ (\frac{\partial}{\partial u}f(x,u))^T & \frac{\partial}{\partial z}i_u(z) \end{pmatrix}] = sign[det(\frac{\partial}{\partial x}f(x,u)(\frac{\partial}{\partial x}f(x,u))^T + \frac{\partial}{\partial u}f(x,u)(\frac{\partial}{\partial u}f(x,u))^T)] \times$$

$$sign(det[\frac{\partial}{\partial z}i_u(z) - \frac{\partial}{\partial x}v(x)\cdot\frac{\partial}{\partial z}i_x(z)]) \quad \forall z \in \omega_p \text{ and } x = i_x(z), \quad u = i_u(z).$$

It follows from

$$rank[\frac{\partial}{\partial x}f(x,u), \frac{\partial}{\partial u}f(x,u)] = n \quad \forall\ (x,u) \in \omega_p,$$

that

$$det(\frac{\partial}{\partial x}f(x,u)(\frac{\partial}{\partial x}f(x,u))^T + \frac{\partial}{\partial u}f(x,u)(\frac{\partial}{\partial u}f(x,u))^T) \neq 0 \quad \forall (x,u) \in \omega_p.$$

Hence we obtain

$$\mid d(\phi, B_R(0), p) \mid = \mid d(i_u - v\circ i_x, \omega_p, P_u(p)) \mid .$$

Since ω_p is a compact manifold without boundary and $i_u - v \circ i_x : \ \omega_p \rightarrow R^m_u$ is a continuous function on ω_p, it implies (see , e.g., [3,6]) $d(i_u - v\circ i_x, \omega_p, P_u(p)) = 0$. The proof is completed.

3 Main results

3.1 Multi - input systems

We start with the following necessary condition of smooth stabilization in the large.

Theorem 3.1 *Suppose* $f : R^n_x \times R^m_u \rightarrow R^n$ *is a smooth function and*

$$\lim_{|x|^2+|u|^2 \rightarrow \infty} . \ \inf . < f(x,u), f(x,u) > \ > 0. \qquad (1)$$

Then the system Σ_f *is not smoothly stabilizable in the large at any point* $(x^*, u^*) \in f^{-1}(0)$.

Proof. It follows from (1) that there is a positive real number R, such that

$$f^{-1}(0) \subset Int(B_R(0)).$$

Hence

$$f(x,u) \neq 0 \quad \forall u \in R^m_u, \ \mid x \mid = R$$

and properties (1), (3) (Theorem 2.1) imply

$$d(f(x,u(x)),P_x(B_R(0)),0) = d(f(x,\bar{u})\ P_x(B_R(0)),0) = 0,$$

where $|\bar{u}| = R$ and $u = u(x)$ is any C^∞ - function, while

$$d(f(x,u(x)),P_x(B_R(0)),0)$$

is to be equal to $(-1)^n$ whenever $u = u(x)$ is a smooth feedback stabilizing the system in the large. Thus the system can not be smoothly stabilized in the large at any point $(x^*,u^*) \in f^{-1}(0)$. The contention of the theorem is proved.

If $f^{-1}(0)$ is a regularly embedded submanifold of $\mathrm{R}^n_x \times \mathrm{R}^m_u$, then Theorem 3.1 can be replaced by the following stronger result.

Theorem 3.2 *Suppose* $f : \mathrm{R}^n_x \times \mathrm{R}^m_u \rightarrow \mathrm{R}^n$ *is a smooth function and*

$$rank\{\frac{\partial}{\partial x}f(x,u), \frac{\partial}{\partial u}f(x,u)\} = n \quad \forall\, (x,u) \in f^{-1}(0).$$

If the system Σ_f *is smoothly stabilizable in the large at a point* $(x^*,u^*) \in \omega$, *where* ω *is a connected component of* $f^{-1}(0)$, *then* ω *is to be unbounded.*

Proof. If $u = v(x)$ is a smooth feedback stabilizing in the large the system Σ_f at a point $(x^*,u^*) \in \omega$ and ω is bounded, then there is $B_R(0)$ such that

$$\omega \subset Int B_R(0)$$

and

$$d(f(v),P_x(B_R(0)),0) = (-1)^n.$$

Note that

$$f^{-1}(0) \cap \{(x,u) \in \mathrm{R}^n_x \times \mathrm{R}^m_u \,;\, \mathrm{u} = \mathrm{v(x)}\} = \omega \cap \{(\mathrm{x},\mathrm{u}) \in \mathrm{R}^n_x \times \mathrm{R}^m_u \,;\, \mathrm{u} = \mathrm{v(x)}\}$$

and

$$| d(f(v),P_x(B_R(0)),0) | = | d((f, P_u - v \circ P_x), B_R(0), 0) | .$$

Thus making use of Lemma 2.1 we obtain the contradiction which proves the theorem.

Example 3.1. Consider the system

$$\dot{x}_1 = x_1^2 + x_2^2 - 1,$$

$$\dot{x}_2 = u.$$

It is easy to see that all condition of Theorem 3.1 are met. Therefore the system is not smoothly stabilizable in the large at any point of the equilibria set defined by $x_1^2 + x_2^2 = 1$.

3.2 Single-input systems

Consider the single - input system Σ_f

$$\dot{x} = f(x,u),$$

where $u \in R$ and f is defined above. If

$$rank(\frac{\partial}{\partial x}f(x,u), \frac{\partial}{\partial u}f(x,u)) = n \quad \forall\ (x,u) \in f^{-1}(0),$$

then the equilibria set of Σ_f consists of regular curves:

$$f^{-1}(0) = \bigcup_i \omega_i,$$

where $\omega_i = \{(x_{\omega_i}(\tau), u_{\omega_i}(\tau)) : \tau \in R\}$ and $\mid \frac{d}{d\tau}x_{\omega_i}(\tau) \mid^2 + \mid \frac{d}{d\tau}u_{\omega_i}(\tau) \mid^2 \neq 0$ for all $\tau \in R$ and i.

Definition 3.1. A parametrization

$$\{(x_{\omega_i}(\tau), u_{\omega_i}(\tau)); \tau \in R\}$$

of the curve $\omega_i \subset f^{-1}(0)$ will be called normal iff for any smooth feedback $u = v(x)$ stabilizing in the large the system Σ_f at any point $(x^*, u^*) \in \omega_i$ the following inequalities hold:

$$u_{\omega_i}(\tau) - v(x_{\omega_i}(\tau)) > 0 \quad \text{for} \quad \tau > \tau^*$$

and

$$u_{\omega_i}(\tau) - v(x_{\omega_i}(\tau)) < 0 \quad \text{for} \quad \tau < \tau^*,$$

where $\tau^* \in R$ such that $x_{\omega_i}(\tau^*) = x^*, \quad u_{\omega_i}(\tau^*) = u^*$.

Proposition. *Let $\omega \subset f^{-1}(0)$, the system Σ_f be smoothly stabilizable in the large at some point in ω and*

$$rank(\frac{\partial}{\partial x}f(x,u), \frac{\partial}{\partial u}f(x,u)) = n \quad \forall\ (x,u) \in f^{-1}(0).$$

Then there is a normal parametrization on ω.

Proof. Assume there is a smooth feedback $u = v(x)$ stabilizing in the large the system Σ_f at some point $(x^*, u^*) \in \omega$. Then following the proof of Lemma 2.1 we conclude that

$$d(i_u - v \circ i_x, \omega, 0) = sign(det[\frac{\partial}{\partial x}f(x,u)(\frac{\partial}{\partial x}f(x,u))^T + \frac{\partial}{\partial u}f(x,u)(\frac{\partial}{\partial u}f(x,u))^T]) \cdot (-1)^n.$$

and the righthand side does not depend on $v(x)$. Therefore we can choose the parametrization so that

$$d(i_u - v \circ i_x, \omega, 0) = 1.$$

That means $(u_\omega(\tau) - v(x_\omega(\tau))) \cdot (\tau - \tau^*) > 0$ whenever $\tau \neq \tau^*$. The proof is finished.

Using the normal parametrization we can formulate the following necessary condition of smooth stabilization in the large.

Theorem 3.3 *Let Σ_f be a smooth system such that*

$$rank(\frac{\partial}{\partial x} f(x,u), \frac{\partial}{\partial u} f(x,u)) = n \quad \forall\ (x,u) \in f^{-1}(0)$$

and $\omega \subset f^{-1}(0)$ connected component with the normal parametrization $\{(x_\omega(\tau), u_\omega(\tau)) \in \mathrm{R}^n_x \times \mathrm{R}_u;\ \ \tau \in \mathrm{R}\}$. Then the system Σ_f is not smoothly stabilizable in the large at a point $(x^*, u^*) \in \omega$ whenever either ω is bounded or there is a connected component $\tilde{\omega} \subset f^{-1}(0)$ such that one can find points $(\tilde{x}_2, \tilde{u}_2)$, $(\tilde{x}_1, \tilde{u}_1) \in \tilde{\omega}$ such that

$$x_\omega(\tau_1) = \tilde{x}_1 \qquad \tau_1 \leq \tau^*,$$

$$x_\omega(\tau_2) = \tilde{x}_2 \qquad \tau_2 \geq \tau^*.$$

and

$$u_\omega(\tau_1) > \tilde{u}_1,$$

$$u_\omega(\tau_2) < \tilde{u}_2, \tag{2}$$

where $\tau^* \in \mathrm{R}$ and $x_\omega(\tau^*) = x^*$, $u_\omega(\tau^*) = u^*$.

Proof. If ω is bounded, then the theorem follows from Theorem 3.2. According the definition of normal parametrization

$$(u_\omega(\tau) - v(x_\omega(\tau))) \cdot (\tau - \tau^*) > 0 \quad \forall\, \tau \neq \tau^*$$

where $u = v(x)$ is a smooth feedback stabilizing Σ_f at (x^*, u^*) in the large. If the inequalities (2) hold, then

$$v(x_\omega(\tau_1)) > \tilde{u}_1$$

$$v(x_\omega(\tau_2)) < \tilde{u}_2.$$

Since $\tilde{\omega}$ is a connected component of $f^{-1}(0)$ we obtain the existence of $(\tilde{x}^*, \tilde{u}^*) \in \tilde{\omega}$ such that

$$\tilde{u}^* = v(\tilde{x}^*).$$

That means the closed loop system

$$\dot{x} = f(x, v(x))$$

has two different equilibria points: (x^*, u^*), $(\tilde{x}^*, \tilde{u}^*)$. Therefore the feedback $u = v(x)$ can not stabilize the system Σ_f in the large at the point (x^*, u^*). The proof is completed.

In Theorem 3.3 it is possible also that $\tilde{\omega} = \omega$. In this case we have the following proposition.

Theorem 3.4 *Suppose Σ_f be a smooth system such that*

$$rank(\frac{\partial}{\partial x}f(x,u), \frac{\partial}{\partial u}f(x,u)) = n \quad \forall\ (x,u) \in f^{-1}(0)$$

and $\omega \subset f^{-1}(0)$ a connected component with the normal parametrization

$$\omega = \{(x_\omega(\tau), u_\omega(\tau));\quad \tau \in \mathrm{R}\}.$$

Suppose further there are $\tau_1,\ \tau_2\ \in \mathrm{R}$ for which it is true that either $\tilde{\tau}_1 < \tilde{\tau}_2 \leq \tau^*$ or $\tilde{\tau}_2 > \tilde{\tau}_1 \geq \tau^*$ and

$$x_\omega(\tau_1) = x_\omega(\tilde{\tau}_1) \quad \tau_1 \leq \tau^*,$$
$$x_\omega(\tau_2) = x_\omega(\tilde{\tau}_2) \quad \tau_2 \geq \tau^*,$$
$$u_\omega(\tau_1) > u_\omega(\tilde{\tau}_1),$$
$$u_\omega(\tau_2) < u_\omega(\tilde{\tau}_2),$$

where $\tau^* \in \mathrm{R}$ and $x_\omega(\tau^*) = x^*$, $u_\omega(\tau^*) = u^*$. Then the system Σ_f is not smoothly stabilizable in the large at $(x^*, u^*) \in \omega$.

Geometrically Theorem 3.4 together with Theorem 3.2 mean that if a system Σ_f is smoothly stabilizable in the large at every point of $\omega \subset f^{-1}(0)$, then ω is to be an unknoted, unbounded curve in $\mathrm{R}^n_x \times \mathrm{R}_u$.

Example 3.2. Consider the system

$$\dot{x}_1 = -(x_1 - 2)(x_1 - u^2 - 1) - x_2(x_1 - u^2 - 1)^2 u,$$
$$\dot{x}_2 = -x_2 + x_1(x_1 - u^2 - 1)u.$$

Using Theorem 3.3 we obtain that the system is not smoothly stabilizable in the large at the point $x_1 = 2,\ x_2 = 0,\ u = 0$.

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