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Adaptive Orientation of Exponential Finite Elements for a Phase Field Fracture Model

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One technique to describe the failure of mechanical structures is a phase field model for fracture. Phase field models for fracture consider an independent scalar field variable in addition to the mechanical displacement [1]. The phase field ansatz approximates crack surfaces as a continuous transition zone in which the phase field variable varies from a value that indicates intact material to another value that represents cracks. For a good approximation of cracks, these transition zones are required to be narrow, which leads to steep gradients in the fracture field. As a consequence, the required mesh density in a finite element simulation and thus the computational effort increases. In order to circumvent this efficiency problem, exponential shape functions were introduced in the discretization of the phase field variable, see [2]. Compared to the bilinear shape functions these special shape functions allow for a better approximation of the steep transition with less elements. Unfortunately, the exponential shape functions are not symmetric, which requires a certain orientation of elements relative to the crack surfaces. This adaptation is not uniquely determined and needs to be set up in the correct way in order to improve the approximation of smooth cracks. The issue is solved in this work by reorientating the exponential shape functions according to the nodal value of phase field gradient in a particular element. To be precise, this work discusses an adaptive algorithm that implements such a reorientation for 2d and 3d situations.

1 Phase field model for brittle fracture

The regularized phase field model for brittle fracture includes two energy density contributions, the surface energy density ψ^{s} and the elastic energy density ψ^{e}, see (1). The formulation contains two field variables, the displacement field $\mathbf{u}$ in form of the linearized strain tensor $\boldsymbol{\varepsilon} = \frac{1}{2}(\mathbf{u} + (\nabla\mathbf{u})^T)$ and the fracture field s, which describes the integrity of the material (broken: $s = 0$, intact: $s = 1$). The two governing field equations are the well known equilibrium equations

$$\psi(\boldsymbol{\varepsilon}, s) = \underbrace{\frac{1}{2}(s^2+\eta)\boldsymbol{\varepsilon} : [\mathbb{C}\boldsymbol{\varepsilon}]}_{\psi^{\mathrm{e}}(\boldsymbol{\varepsilon},s)} + \underbrace{\mathscr{G}_c\left(\frac{(1-s)^2}{4\epsilon} + \epsilon|\nabla s|^2\right)}_{\psi^{s}(s)}, \quad (1) \qquad \dot{s} = -M\underbrace{\left[s\,\boldsymbol{\varepsilon} : [\mathbb{C}\boldsymbol{\varepsilon}] - \mathscr{G}_c\left(2\epsilon\,\nabla s + \frac{1-s}{2\epsilon}\right)\right]}_{\delta\psi/\delta s}. \quad (2)$$

The characterizing parameters of both equations are the linear elasticity tensor $\mathbb{C}$, the residual stiffness parameter η, the cracking resistance $\mathscr{G}_c$, ϵ and the mobility M. A detailed description can be found in [1]. Of particular interest is ϵ, which determines the transition zone of s and plays a key role in the refinement of the discretization.

2 Adaptivity of shape functions

2.1 Exponential shape functions

In order to approximate cracks without a very fine discretization Kuhn and Müller [2] introduced exponential shape functions for the phase field variable. The 1d exponential shape functions of an element with two nodes in natural coordinates of the interval $\xi \in [-1,\ 1]$ are

$$N_1^{\exp}(\xi,\delta) = 1 - \frac{\exp(-\delta(1+\xi)/4) - 1}{\exp(-\delta/2) - 1} \quad \text{and} \quad N_2^{\exp}(\xi,\delta) = \frac{\exp(-\delta(1+\xi)/4) - 1}{\exp(-\delta/2) - 1}. \quad (3)$$

The parameter δ is the ratio of element size h to ϵ. The shape functions for 2d and 3d are obtained by tensorial multiplication of the 1d shape functions.

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2.2 Adaptive Algorithm

The main issue of the exponential shape functions is that they are not symmetric. However, with a proper reorientation according to the gradient of the phase field, they provide a significant improvement of the FE approximation. In order to obtain a proper orientation, an adaptive orientation algorithm becomes inevitable. To reduce the computational effort and increase the stability of the algorithm, linear shape function are applied in regions with constant phase field. Furthermore, to ensure continuity across the element boundaries, the condition for the choice of the shape functions are the difference of nodal phase field values and the orientation of the edges, which values are shared by neighbour element. Therefore, every element edge is examined individually for the choice of its shape function, which can become a linear N^{lin} (—), normal exponential N^{exp} (▷) or reversed exponential N^{exp} (◁) shape function, see Fig. 1. As a result, the adaptivity generates blending elements, which have some similarities to [3], but without additional nodes and exponential shape functions instead of shape functions with higher order polynomials.

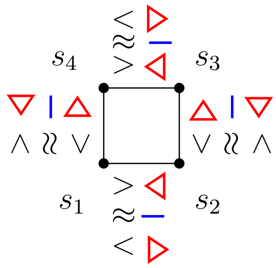

Fig. 1: Adaptive algorithm scheme for a 2d quadrilateral element.

3 Numerical Example

As a test of the adaptive algorithm a setup without mechanical loads and an initial crack ($s = 0$) is analysed, see Fig. 3. The domain is a square and has an initial crack with half of the edge length $2L$ and with linear shape functions. Within an iteration loop a set of shape functions for each edge is established. As a validation of the the new algorithm, the surface energy is observed. The difference between the simulation where the domain discretized with exponential elements compared to the simulation with adaptively approximated elements is small, see Fig. 2, although is only the exponential approximation is only present at the crack surface and only for the direction normal to the crack. It should also be mentioned that the adaptive algorithm modifies the number of quadrature points. So fully linear elements, which are elements with an almost constant phase field, are numerically integrated by 2×2 Gauss points to increase the performance.

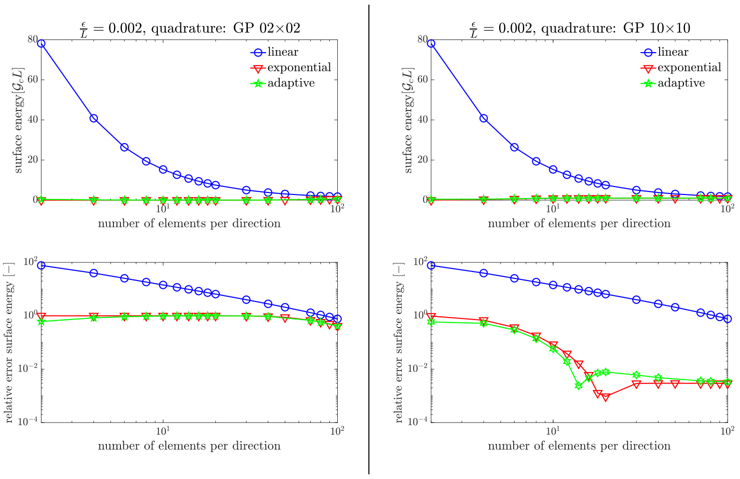

Fig. 2: Comparison of the surface energy computed with linear (blue), exponential (red) and adaptive (green) shape functions.

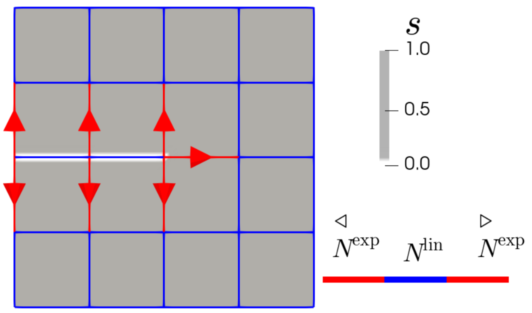

Fig. 3: Fracture field s, edge shape functions $N^{\text{lin/exp}}$ and orientation of the exponential shape functions for four element per direction.

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