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Phase field modeling of fatigue crack initiation and growth under various loading situations

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In this work we illustrate the ability of a phase field model for fatigue crack growth in terms of extension capability to various amplitude loading and mean stress effects. The additional energy density contribution accounting for the energy associated with fatigue is modified in order to provide a more general model. Results obtained from numerical fatigue crack growth simulations are briefly presented and discussed.

1 Introduction

The application of the phase field method to fracture problems has increasingly gained attention during the last decade. Due to the beneficial properties, as for instance no need of a priori knowledge of the crack path or remeshing algorithms, the phase field method was applied to many different kinds of crack problems. Recently, this framework is also used in the field of fatigue crack growth. With regard to technical structures, consideration of the fatigue phenomenon is of great importance since lots of components are subjected to oscillating loads, which can cause an accumulation of damage in the material even for comparatively low load levels. In [1] we introduced a phase field model for cyclic fatigue crack growth, where the evolution of the phase field order parameter $s(\mathbf{x}, t) \in [0, 1]$, with 1 indicating intact material and zero indicating broken material, is not solely driven by static loading but also by an additional energy density contribution accounting for fatigue related damage. As only constant loading amplitudes were covered, a generalization of this model towards of arbitrary load time histories and mean loads is developed and described in the following.

2 Model Description

The origin of the phase field model for fatigue crack growth is given by the model for brittle fracture proposed by Kuhn and Müller [2], where a regularization of the variational formulation of brittle fracture from [3] is adapted. Within this model the regularization of the free energy of a body consists of two parts, namely the linear elastic stain energy and the crack energy. The integration of the fatigue mechanism was accomplished by introducing an additional energy density component, whose evolution is governed by fatigue damage laws. The regularized energy can be formulated as

$$E_\mathrm{f} = \int\limits_V (s^2 + \eta) W(\boldsymbol{\varepsilon})\,\mathrm{d}V + \int\limits_V s^2 q \left\langle D - D_c \right\rangle^b \,\mathrm{d}V + \int\limits_V \mathcal{G}_c \Gamma(s, \nabla s)\,\mathrm{d}V. \tag{1}$$

In Eq. (1) the first integral represents the contribution of strain energy, where the function s^2 governs the degradation of the stresses in broken regions. The parameter η ensures a residual stiffness for those regions. The last integral can be referred as regularization of Griffith's crack energy with critical energy release rate $\mathcal{G}_c$ and

$$\Gamma = \frac{(1-s)^2}{4\epsilon} + \epsilon |\nabla s|^2, \tag{2}$$

where the length scale ϵ controls the size of the transition zone of s. Suppose only these two contributions are considered within a minimization of the total energy, no cracks will appear in the solution for very small loads. A fatigue crack, however, will initiate if the fatigue associated work is taken into account, which is captured by the second integral. Once the fatigue damage D exceeds the threshold D_c the contribution will increase, governed by the parameters q and b. Note that the Macauley brackets imply $\langle(\cdot)\rangle^n = 0$ for $(\cdot) \leq 0$ and $\langle(\cdot)\rangle^n = (\cdot)^n$ for $(\cdot) > 0$. The fatigue damage must be controlled by an adequate driving force, and hence we propose

$$D = D_0 + \frac{\Delta N}{N_D A_D(L)^k} \left[\bar{\sigma}(1-L)\right]^k, \tag{3}$$

with the previous state of damage D_0 and the Wöhler parameters N_D, A_D and k. In oder to incorporate the accurate assessment of mean stress effects the driving force quantity $\bar{\sigma}$ is degraded by the ratio of the external load $L = \frac{L_\mathrm{mean}}{L_\mathrm{max}}$. Within the

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simulation the cycle increment ΔN is proportional to the time step size and accordingly a certain number of load cycles is simulated within one simulation step. An evolution equation for s can be provided in terms of a generalized Ginzburg-Landau equation [4].

3 Numerical Example

An evaluation of fatigue material properties is usually done by the analysis of crack growth rates. To validate our model in terms of crack growth rates, a so called CT-specimen according to the ASTM E399 standard (see Fig. 1 a)) is simulated. The proposed phase field model was implemented in a 2d user element routine within the finite element software FEAP. Experimental researchers commonly use the stress ratio $R = \frac{\sigma_{\min}}{\sigma_{\max}}$ for classification of different load sequences. It was shown (see e.g. [5]) that higher stress ratios generally lead to higher crack growth rates. Three sequences with different stress ratios are illustrated at the top of Fig. 1b). In simulations with the CT-specimen these stress ratios were used as load time sequence. The simulations were run until a crack was initiated and has grown to the length also shown in Fig. 1a). The results from all simulations are presented by means of a plot of crack growth rate versus the stress intensity factor range within double logarithmic scales, see Fig. 1b) . It was shown by Paris [6], that in such a diagram data points from crack growth experiments lie on a straight line for the range of stable crack propagation. As the plot in Fig. 1b) shows, this behavior is reproduced by our phase field model. The second very convincing property of the model is the correct prediction of the trend of the crack growth rate curves with respect to the applied stress ratio. As the results show, the crack growth rates are shifted to higher levels with increasing stress ratio.

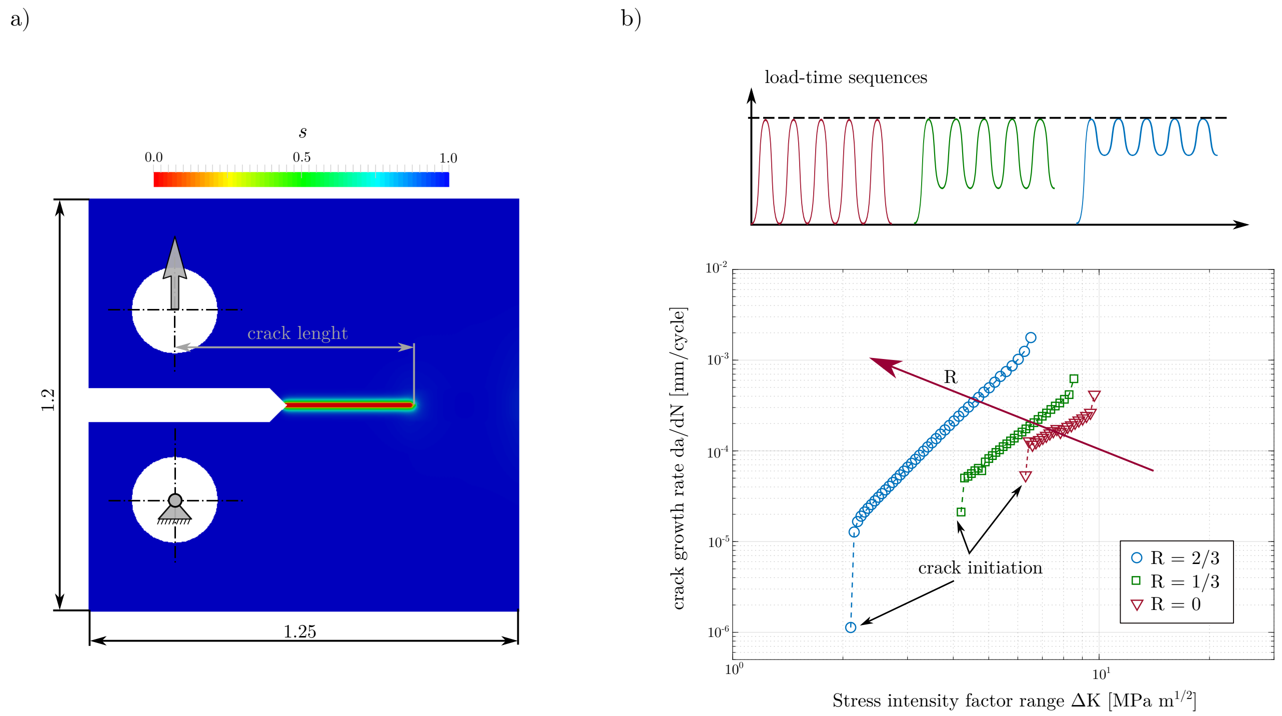

Fig. 1: Results from phase field simulations: a) contour plot of phase field variable s over the simulated CT-specimen after a certain number of load cycles was applied, b) plot of crack growth rate with respect to the stress intensity factor range for simulations with load sequences of different R-ratios

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