Received: 15 June 2021 | Accepted: 5 September 2021

DOI: 10.1002/pamm.202100077

Adaptive Exponential Finite Elements for a Phase Field Fracture Model

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Fracture phenomena can be described by a phase field model in which an independent scalar field variable in addition to the mechanical displacement is considered [3]. This field approximates crack surfaces as a continuous transition zone from a value that indicates intact material to another value that represents the crack. For an accurate approximation of cracks, narrow transition zones resulting in steep gradients of the fracture field are required. This necessitates a high mesh density in finite element simulations, which leads to an increased computational effort. In order to circumvent this problem without forfeiting accuracy, exponential shape functions were introduced in the discretization of the phase field variable, see [4]. These special shape functions allow for a better approximation of steep gradients of the phase field with less elements as compared to standard Lagrange elements. Unfortunately, the orientation of the exponential shape functions is not uniquely determined and needs to be set up in the correct way in order to improve the approximation of smooth cracks. This work solves the issue by adaptively reorientating the exponential shape functions according to the nodal values of the phase field gradient in each element. Furthermore, a local approach is pursued that uses exponential shape function only in the vicinity of the crack, whereas standard bilinear shape function are used away from the crack.

1 Approximation of brittle cracks by a phase field

The phase field model for brittle fracture proposed by Kuhn and Müller [3] was first introduced in Bourdin [1]. In this model, the energy density functional contains two contributions, the surface energy density ψ^{s} and the elastic energy density ψ^{e}. Additionally, a volumetric-deviatoric split in positive and negative elastic energy density $\psi^{\mathrm{e}}_{+/-}$ for the strain according to Amor et al. [2] is introduced in (1). The formulation includes two field variables, the displacement field $\mathbf{u}$ in form of the linearized strain tensor $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ and its deviator $\mathbf{e}$ and the fracture field s. The two governing field equations are the well known equilibrium equation (div $\boldsymbol{\sigma} = \vec{0}$)

$$\psi(\boldsymbol{\varepsilon}, s) = \underbrace{(s^2+\eta)\left(\underbrace{\frac{K}{2}\langle\mathrm{tr}(\varepsilon)\rangle_+^2 + \mu(\mathbf{e}:\mathbf{e})}_{\psi^{\mathrm{e}}_+}\right) + \underbrace{\frac{K}{2}\langle\mathrm{tr}(\varepsilon)\rangle_-^2}_{\psi^{\mathrm{e}}_-}}_{=\psi^{\mathrm{e}}} + \underbrace{\mathscr{G}_c\left(\frac{(1-s)^2}{4\epsilon} + \epsilon|\nabla s|^2\right)}_{\psi^{\mathrm{s}}(s)}, \tag{1}$$

and the Ginzburg Landau type evolution equation of the phase field

$$\dot{s} = -M\underbrace{\left[2s\,\psi^{\mathrm{e}}_+ - \mathscr{G}_c\left(2\epsilon\,\nabla s + \frac{1-s}{2\epsilon}\right)\right]}_{\delta\psi/\delta s}. \tag{2}$$

Eq. (2) describes the evolution of the crack field. The characterizing parameters of both equations are the bulk modulus K, shear modulus μ, the residual stiffness parameter η, the cracking resistance $\mathscr{G}_c$, the internal lenght scale ϵ and the mobility parameter M. A detailed description can be found in [3]. Of particular interest is ϵ, which determines the width of the transition zone of s and plays a key role in the refinement of the discretization.

2 Adaptive shape functions

In general, phase field models describe different material constituents with an additional field variable, which has different values in different "phases" of the material. In this work, the crack field describes the integrity of the material (cracked: s = 0, sound: s = 1). At the phase interfaces a steep transition from one characteristic value of the phase field to another occurs. For an improved approximation of this zone Kuhn and Müller [4] introduced exponential shape functions for the phase field variable. They enable an accurate approximation the phase field transition in the zones with only one element regardless of the

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regularization parameter ϵ. This is achieved by an additional parameter δ, which is constructed as the ratio of element size h to ϵ. It enables an elementsize independent transition zone. These shape functions can be extended for 2d and 3d by tensorial multiplication of 1d shape functions [4].
The advantage of the exponential shape functions, comes at the expense of adaptability. The orientation of the shape functions needs to differ on opposing crack faces [4]. This issue can be solved by reorienting elements at opposite crack surfaces according to sign of the spatial gradient of s. Though the reorientation can be changed manually for existing cracks, crack nucleation and unpredictable crack growth requires an adaptive strategy to change the shape functions dynamically. In order to lower the computational costs and improve the stability of the algorithm, linear shape function are applied in regions with an almost constant phase field [5]. The choice of the shape function is evaluated by a marking strategy and distance function. In the first step of the algorithm, all elements are initiated as linear Lagrange elements. Then the relative difference of the surface energy of an exponential and linear approximation for each element edge is evaluated. If the value reaches a certain limit the algorithm changes to an exponential shape function. In addition, a distance function also modifies the shape functions of all elements in the vicinity of a modified element, see [5] for details.

3 Numerical Example

The adaptive strategy is analyzed for a simple shear test, see Fig. 1. The model is a quadratic 2D domain and has an initial crack of the half length of the body which is vertically bounded. The crack progresses just in the domain with a tensile volumetric stress, see Fig. 2, as can be expected from Eq. (1). The adaptive algorithm chooses exponential shape functions (red) around the crack and the rest of the domain is approximated by linear shape function (blue), see Fig. 3. As an example, Fig. 4 shows the region A of the crack field and shape functions. The red arrows indicate the orientation of the exponential shape functions for each edge. As can be observed in Fig. 4, the algorithm produced a mirrored orientation at the crack surfaces leading to a symmetric crack like it is intended.

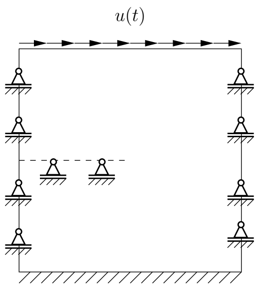

Fig. 1: Simple shear test with boundary at the crack.

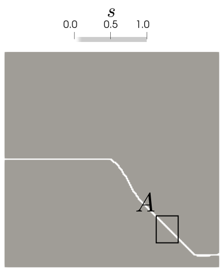

Fig. 2: Crack phase field s after rupture.

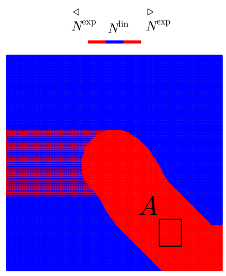

Fig. 3: Shape function for the crack field.

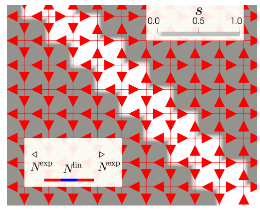

Fig. 4: Superposition of Fig. 2 and Fig. 3 for region A.

Acknowledgements The authors gratefully acknowledge support by the Deutsche Forschungsgemeinschaft in the Priority Program 1748 *Reliable Simulation Techniques in Solid Mechanics. Development of Non-standard Discretization Methods, Mechanical and Mathematical Analysis* under the project *Advanced Finite Element Modelling of 3D Crack Propagation by a Phase Field Approach* (project number 255846293). Open access funding enabled and organized by Projekt DEAL.

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