Mathematics as a Key to Key Technologies

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In memory of a very precious friend, Henrik Martens, who loved mathematics and its applications, Spanish guitar music, Norwegian mountains and Oberwolfach - even more than I do.

Asking people on the road, in parliaments, in ministries about useful sciences, one cannot expect that mathematics is very often named in a top list. Of course, it is useful to practise the brain of pupils; to be good in mathematics may be regarded as a sign for some intelligence, but for example in my country even many well educated people seem to be proud that mathematics was their horror subject in school ("but see, what I have nevertheless achieved"). There are, however, countries like France, where the prestige of mathematics is really very much higher; I quote a sentence from a letter of the former French president Mitterand to the assembly of mathematicians in 1992: "*Your discussions refer to a very essential domain of research, which commands the scientific and technological progress of a country and they refer to a central discipline of our educational system.*" Mathematics commands the development of science and technology of the country - maybe a reality or at least a dream in France, but what about other countries?

In Germany some of the research directors of big companies see it like Mitterand. In 1994 I asked 5 of them in a letter, what they think about the economic benefits of mathematics; not all replied, but for example Prof. Weule, by then research director of Daimler, wrote: "*The demand for a maximum output of industrial research and development today can only be fulfilled by an increasing use of mathematical methods.*" And he explained why: "*Examples are simulation methods, which allow to reduce the experimental and constructive effort for the development of complex products drastically.*" Simulation here is the key word - computer simulation substitutes more and more real experiments; there is still a huge need of new simulation tools, which of course must always be evaluated by experiments. The main message is that:

Mathematics always is the core of any computer simulation.

Those in industry, who almost daily use a certain software package to simulate a process or the behaviour of a product, sometimes forget this simple fact - a disadvantage for them, since they may forget the limitations of their tool too and a disadvantage for mathematicians, since their key role comes out of view.

We will discuss the role of mathematics in simulation - and therefore in control and optimisation - of technical processes and products more in details. The general task was clearly formulated in a lecture by J. L. Lions in 1994:

"*Mathematics helps to make things*
better, faster, safer, cheaper
by
the simulation of complex phenomena,
the reduction of the flood of data, visualisation."

This means hope and challenge for us! For 20 years I have tried to work in this direction; I had my first projects with industry in 1978 and have, since then, found and worked on over 200 projects from companies from all over the world. In 1980 we developed at Kaiserslautern a new programme called "Technomathematics", which combines proper mathematics with computer science and engineering and is now offered at more than 20 German and some foreign universities. In 1986 I was a co-founder of the European Consortium for Industrial Mathematics, which still bundles the efforts of European institutes and universities in this field. During that time we realised that Industrial Mathematics, with proper modifications, may also be of great interest for scientists in 3^{rd} world countries and started in 1987 a 2 year master programme with today 30 beginners per year. Finally, in 1995 we founded an "Institute for Industrial Mathematics" (ITWM) outside of the university, which has to earn 75% of its total budget through industrial and public projects. For 1999 we expect a total budget of 10 Million DM, of which 7.5 have to be earned; we shall do that with almost 70 fully employed postdoc scientists and about 40 part time scientists and PhD students. We expect to become the first "Fraunhofer-Institut" for Mathematics in 2000.

These experiences are the basis for my analysis of the following questions:

A Why has mathematics become so important for industry and commerce during the last 20 years and which kind of mathematics is needed?

B What are the consequences for research and education in the field of mathematics?

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called Tecmath, which originated 10 years ago from my university group and has already a very successful history.

A Simulation, data reduction, control, optimisation, visualisation: Key words for the role of mathematics in industry. And mathematics as a key to these keywords: all these statements are already made in the introduction. But why just now during the last decades of the 20th century? The only reason is that modern computers allow the evaluation of realistic mathematical models. Therefore, the behaviour of complex, 3-dimensional technical systems can be predicted, designed and optimised in a virtual reality, which is astonishingly near to "real" reality. "Virtual prototyping" is very fashionable in industry - and again: Its core is mathematics. More precise: The main components in virtual prototyping and in industrial mathematics are:

Mathematical Modelling and **Scientific Computing**

By **modelling** we mean the transformation of real objects into mathematics by neglecting details, which are unimportant with respect to the questions we pose. (I know that there are constructivists, who see the world different: Our models are the only reality we have. They may be right - but it is hard to explain this philosophy to an industrialist, who wants to sell his "real" product.) Heinrich Hertz, the famous physicist, explained modelling in his "Introduction to Mechanics" from 1897: "*We make images or symbols of objects in such a way that the logical consequences of the images are again images of the natural consequences of the objects.*" His models should be "correct" (in the sense just defined above) and "logically admissible" (not leading to a contradiction). Models are not unique and it is the art of the scientist to choose the "cheapest" model. And it was clear to Hertz: The raw material of these images is, these models "are made of", mathematics. Of course: The better the raw material, the better the model. One needs good "pure" mathematics to do good modelling: "Engineering mathematics" is not enough. There are hierarchies of models: Very complicated ones, which could be very precise - if you have a very fast computer to evaluate them and if you have information about all the parameters involved. And simpler ones, deduced from the top model by simplifications of geometry or by taking asymptotic limits. This deduction is sometimes called modelling too and is a real mathematical art. Simple models are in general easier to evaluate (often even analytically, the only way of evaluation before the computer age), but provide you only with a qualitative prediction, seldom with reliable data. They are useful to explain certain phenomena, but industrialists often want the optimal design and the optimal parameters of a 3-dimensional technical system. To get these things, you need to evaluate the more complex model and we do that with aid of **scientific computing**. By this we mean to design an algorithm to solve the equation representing the model, to implement it in a proper computer system and to organise the data produced by the computer so that we are able to get the information we want. The Russian mathematician A. A. Samarskii called the whole procedure a "numerical experiment" and claimed it to be "a new scientific method, which determines the style of thinking of a modern scientist as well as the kind of problems he is able to attack."

Model + **A**lgorithm + **P**rogramme = **MAP**,

a map from the real into the virtual, mathematical world.

One word more about the relation between asymptotical and numerical methods (which sometimes seems to create a separation between mathematical cultures): I think that both methods supplement each other and that it is an essential feature of an industrial mathematician to be knowledgeable in both disciplines. Of course we have to choose the simplest model, which is sufficiently exact; and to get it, we need analytical (= asymptotic) methods. "*What is complex is to make things simple*" (Paul Valéry). But very often it is not possible to get sufficiently exact simple models: Then we need scientific computing. It happens quite often that in one region of the (spatial or time) domain of our problem a simpler model may be used, but in another we need to apply a more complex description: This leads to domain decomposition and to the task of matching different algorithms for the different models. The American applied mathematician J. Keller finished his invited lecture at ICIAM 95 in Hamburg by claiming that, while the first half of this century was dominated by asymptotical methods, the second by numerical methods, we are now entering a century dominated by the combination and interaction of these methods (I cite it from my memory, since he has not published the lecture). I will show some applications of these ideas in our context. I agree completely with Keller: Our chances are not "either-or", but "and".

If we want to see how this MAP works, we have to consider examples from the past; even more interesting, however, is the question about the future of this research: What are the most challenging problems to be solved during the first decades of the next century? Of course, I cannot do what David Hilbert did at the International Conference in Paris in 1900: He posed 23 yet unsolved problems mainly in pure mathematics, which influenced the mathematical research during the last century enormously. I even believe that nobody can do something similar today. What is possible are "Reflections on the Future of Mathematics", like Avner Friedman made at the last International Conference in Berlin, 1998 (see Berlin Intelligencer, Springer 1998). He described, in a spirit very near to my own, what mathematics can do in fields like material sciences, biology and multimedia. I shall try to give examples and a paradigmatic problem for the future in the fields of

I. Design of materials

II. Design of products

III. Simulation and Optimisation of production processes

IV. Discovery of Patterns in data for quality control, economic data prediction and medical diagnosis.

I hope to be able to show which kind of questions in these fields mathematics can help to answer. I could also name mathematical research domains, which occur most often in these projects - domains like multiscale analysis, free boundary value problems, computational fluid dynamics, stochastic systems, optimisation or signal- and image processing. And we could make a matrix about what is used where. But this book-keeping does not tell us much - I believe examples to be more illustrative.

1. Design of materials

This is the main goal of material sciences. "*Material sciences is concerned with properties and the use of materials. The objectives are the synthesis and manufacture of new materials, the modification of materials, the understanding and prediction of material properties, and the evolution and control of these properties over a time period. Until recently, material sciences was primarily an empirical study in metallurgy, ceramics and plastics. Today it is a vast growing body of knowledge based on physical sciences, engineering and mathematics.*" (A. Friedman)

One problem we encountered arises in dental technology: We need to compose a material for the filling of tooth cavities out of a certain kind of "cement" and small pieces of glass. The composite should have as little shrinkage and as much stability as possible: How many glass particles of which size should the mixture contain. The process is quite complicated, since the acidity in the cement triggers a kind of sintering in the glass.

How can we predict the behaviour of such a composite, not to talk about finding an optimal design? We can model the material to be linear viscoelastic with aging and isotropic shrinkage characterized by different parameters in glass and cement. We get integrodifferential equations with spatially very inhomogeneous coefficients: Could we really think of FEM approximations, resolving the microstructure of the composite? Of course we can't. This is a point where multiscale analysis comes in: If the microstructure is very fine compared to the total dimensions (the size of the hole), the details will not play a big role and averaged quantities will be sufficient. Mathematically we get them in a limiting process called homogenisation, when the relation of the microscopic to the macroscopic scales tends to zero (see for example: E. Sanchez-Palencia: *Non-homogeneous Media and Vibration Theory*, Springer 1980 or V. V. Jikov, S. Kozlov, O. Oleinik: *Homogenization of Differential Operators and Integral Functionals*, Springer 1994). Homogenisation needs very tricky mathematics, new concepts of convergence; but every new problem poses new tasks - here, to include aging and shrinkage. If this is done, we may talk about the optimal volume fraction of glass etc. Much depends on a good model - dental laboratories often have not enough facilities to measure the material properties.

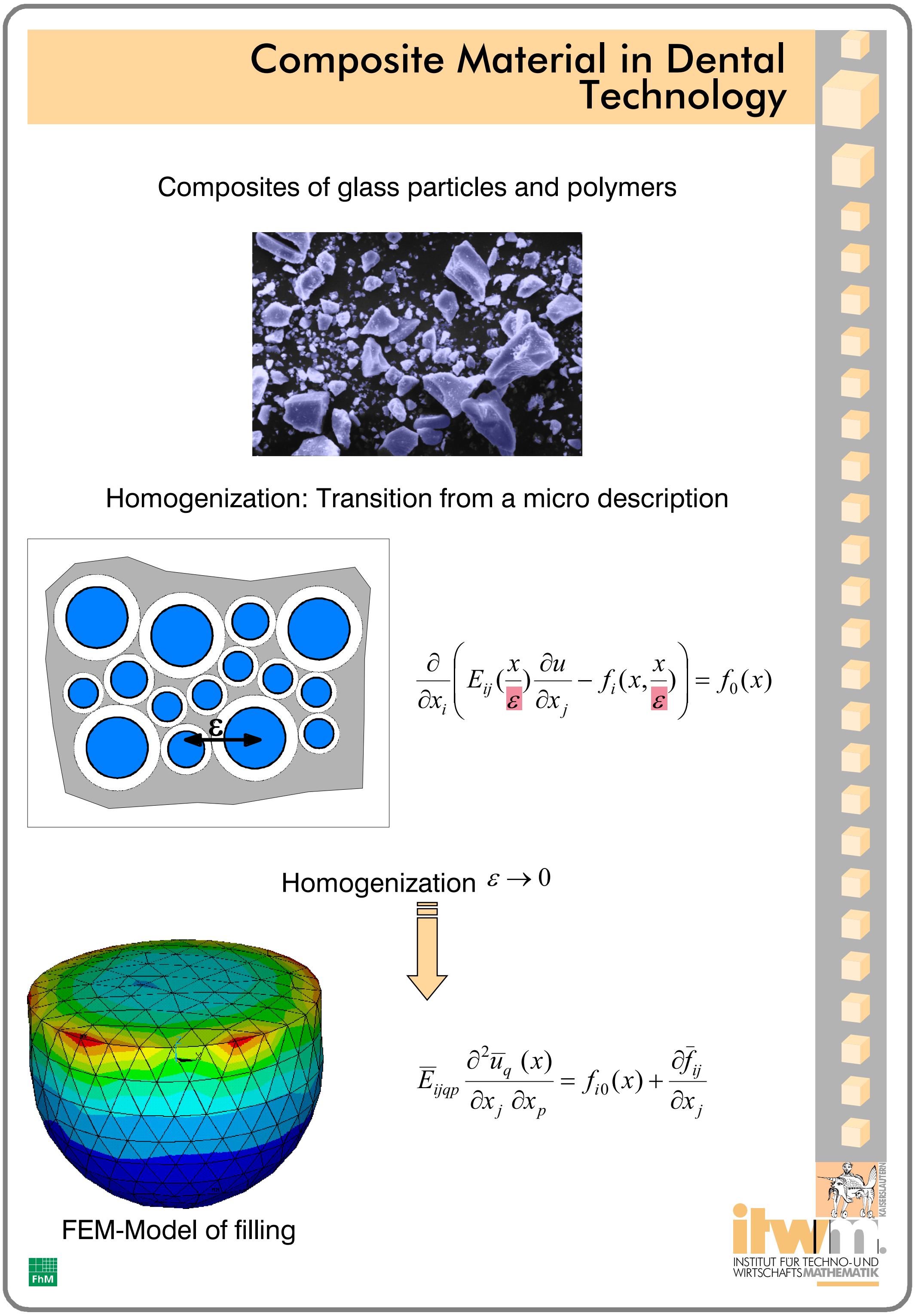


Figure 1: Composite material in dental technology

So much about a problem we are presently working on. As a big challenge for the future research in this area I consider "fatigue life analysis". During a periodic or irregular loading material dissipates energy, which leads to internal fractures and finally to a local break down. The main interest for fatigue life analysis is shown by car industry: How long will critical safety components of a car survive given a "normal" driver on a "normal" road? Damage events are characterised by hysteresis loops in the stress-strain relation; this is fine for one-dimensional strains and stresses and leads to a method called rainflow counting (see e. g. A. Beste, M. Brokate, K. Dressler: *Kann man berechnen, wie lange ein Auto hält*, in "Mathematik in der Praxis", pp. 3-24, Springer 1995).

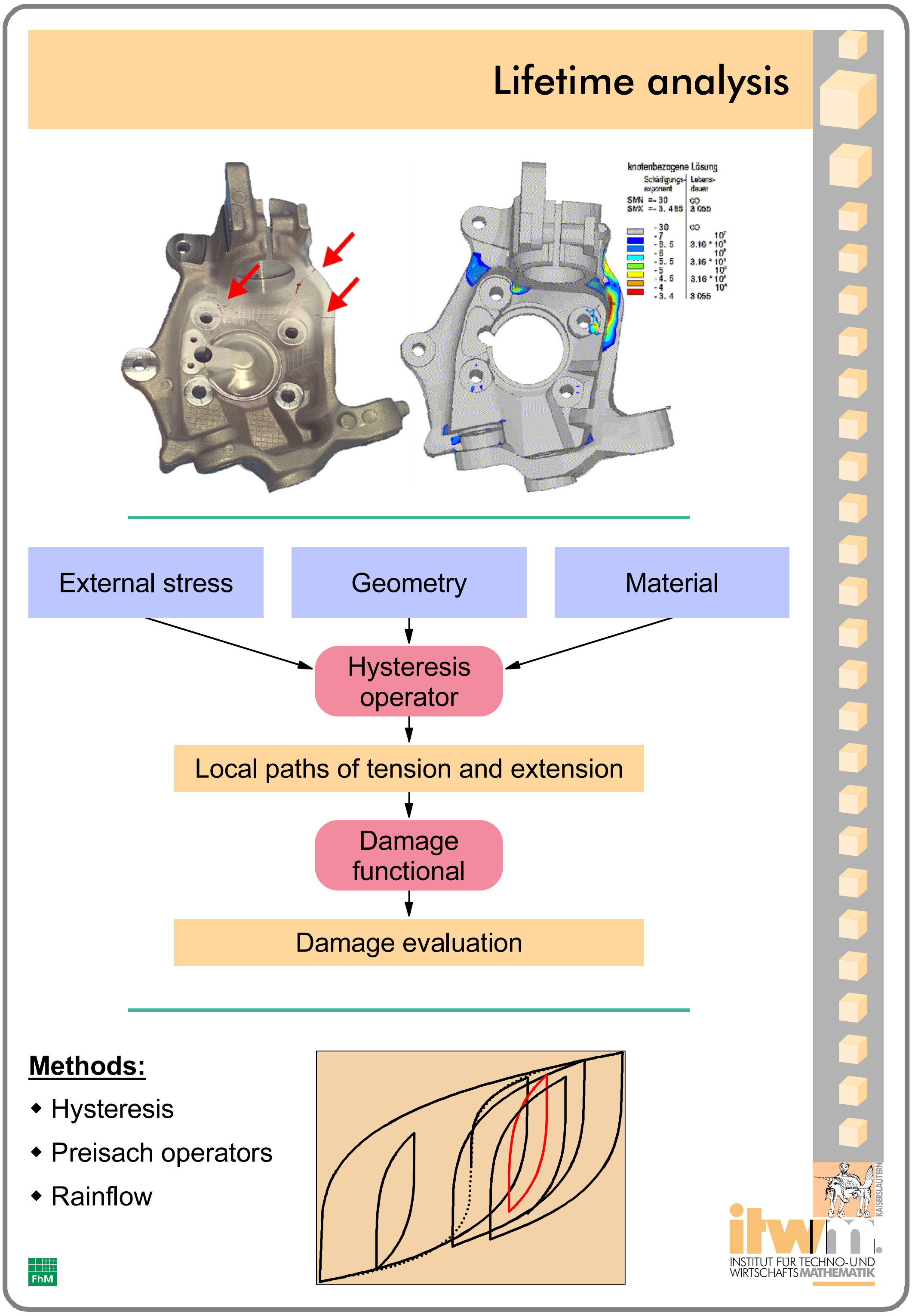


Figure 2: Service life time

For more-dimensional situations one needs new concepts - the research started at TecMath and led to practically very successful tools provided now by an off spring, LMS Durability at Kaiserslautern and used in almost any car company in the world. It is good, but not perfect - and it has to be combined with other research branches like fracture analysis. Cracks show a very complex, a "fractal" geometry and the questions, where they initiate, how they evolve and when they branch into several cracks are still being researched. I don't know about many interactions between fatigue life and fracture analysis. But the big challenge in this area is again to design materials with a maximal duration under given loading. These may again be composite materials and we get a high dimensional parameter manifold, on which we have to find the optimum. Reliable tools to predict the life time of materials and to find an optimal design - a big task not only for mathematicians, but for teams of scientists including mathematicians.

A wide research field inside the material sciences is related to polymers. Polymers in dilute solutions give rise to very high-dimensional parabolic equations (Fokker-Planck equations), where the dimension is related to the "length" of a molecule and to its linear, branched or network structure. As financial mathematics (see IV. has posed new problems how to compute 2000-dimensional integrals, polymers pose problems how to compute solutions of say 100-dimensional parabolic equations. And as in finance mathematics we have to develop numerical methods based on pseudo random numbers (and therefore on number theory) to solve these problems (see: J. F. Traub, A. G. Werschulz: *Complexity and Information*, Cambridge Univ. Press 1998).

If we have a "pure" polymeric substance, we have to study its transition from the fluid to the solid form, which is a kind of crystallisation. This process poses exciting mathematical (mainly stochastical) problems - there is an ECMI "special interest group" based in Milano dealing with this topic (see e. g. V. Capasso et al.: *Stochastic modelling and statistics of polymer crystallization processes*, Surveys on Math. for Industry, Vol. 6, Nr. 2, 1996).

Let me, last not least in this chapter, mention that even classical problems from metallurgy pose new and exciting tasks for mathematicians. Consider casting processes in steel production plants or in "normal foundries". In continuous casting for the production of rolled steel, one has to control the solidification front by a proper cooling; this leads to an inverse, i.e. ill-posed problem, where the "free" boundary, the solidification front is prescribed and the cooling conditions at a fixed boundary have to be determined. "Inverse problems" is, like optimisation and many others, a very important subject, "transversal" to our classification of problems; in ECMI it is mainly represented in Linz (see H. W. Engl, M. Hanke, A. Neubauer: *Regularization of Inverse Problems*, Kluwer 1996).

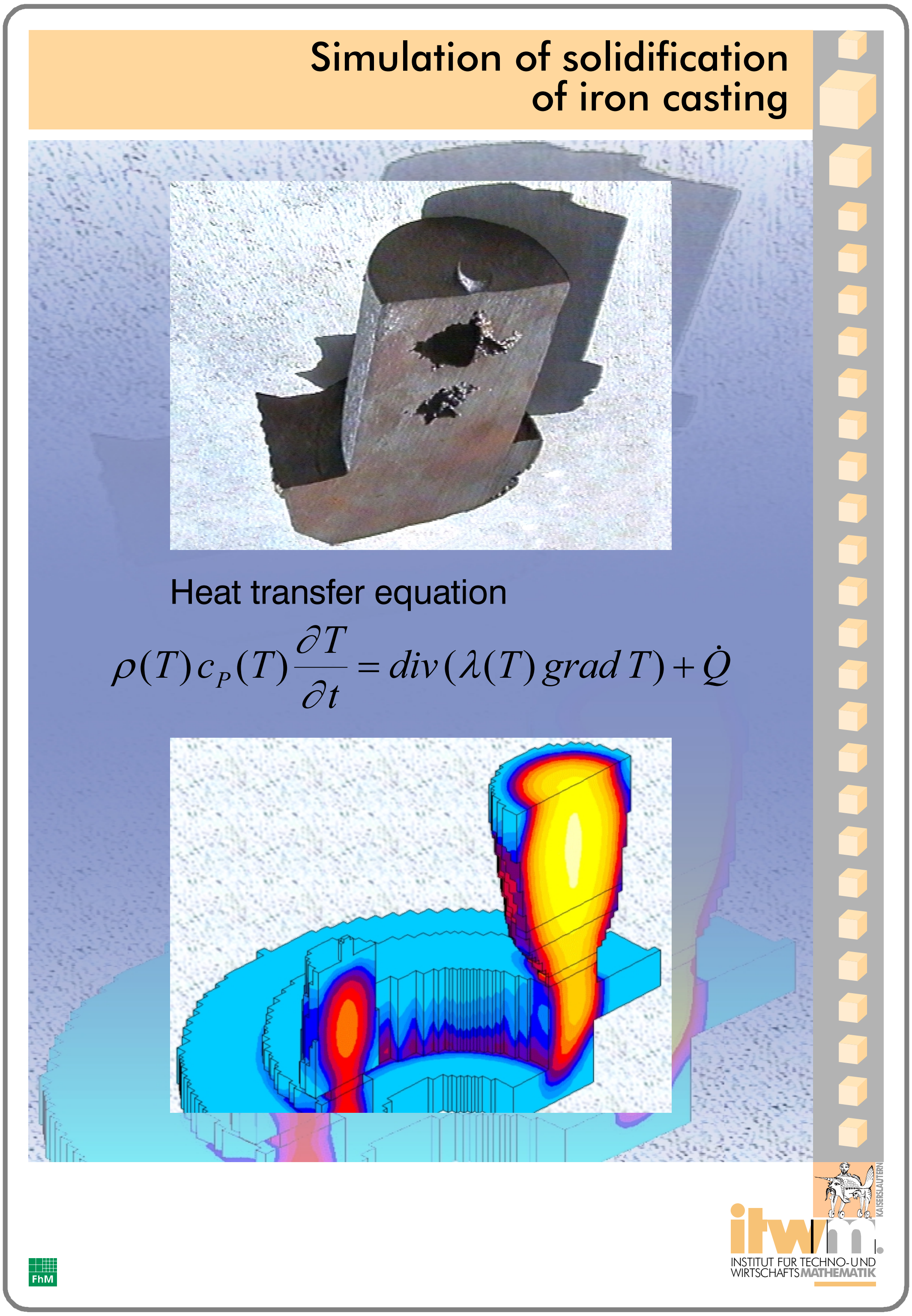


Figure 3: Solidification in iron casting

In my institute we cooperate with the software company MAGMA, which offers a software tool to simulate the casting of iron in complex moulds. The package, while providing excellent simulations and therefore being internationally top, has a draw-back: on normal work stations it may take weeks to get one simulation finished. This defines another task for mathematicians: Find parallel algorithms for work-stations, that do the same job in one night.

You see: Not only the so-called high technology needs mathematics - the basic technologies do it as well. Especially if they try to approach an optimum: Maximal quality at minimal costs (and minimal time). And since even small and mean size enterprises today are forced to do that, the role of mathematics (as I see it) becomes more and more crucial.

II. Design of products

Of course, the border line between the design of materials and the design of products is as "fuzzy" as the border line between products and production processes: The material is part of the product - and the product depends on the production process. But, in the virtual world of simulation, you may first think of an optimal product, where materials and production processes are virtual; then, in a second step, you may try to realize appropriate materials and processes. In this sense, the design of optimal products is the key task. Of course, there are many examples, where mathematics has helped in the design of products and I will only choose some of our experience, which I consider to be quite surprising.

With the first problem - the optimal design of baby diapers - we have struggled for quite a while. Diapers consist of cellulose and absorbing grains; the transport of liquid in diapers is modelled by Darcy's law supplemented by ordinary differential equations describing the absorption process. The permeability, however, depends strongly on the amount of transported or absorbed liquid - the problem is ugly nonlinear. The model has to be checked to be "logically correct" (in the sense of Hertz): We have to show that it defines a solution uniquely. Then we have to develop numerical algorithms able to deal with these nonlinearities; and finally we have to define the proper material (distribution of the granulate, kind of cellulose) in order to get optimal properties of the diaper (minimal surface humidity f.e.). Why is this a never ending story? Told in details it would show many quite typical difficulties for the interaction of industry with mathematics. Often, mathematics is behind the practical needs - sometimes it is ahead. It took us almost ten years until industry believed that our striving for optimality was justified. Even mathematics benefits sometimes from industrial competition.

$$\frac{\partial u}{\partial t} = \nabla \cdot (\lambda(v) D(\tilde{u}) \nabla \tilde{u}) - A(u,v)$$
$$\frac{\partial v}{\partial t} = A(u,v)$$

Figure 4: The diaper problem

Flow in porous media, especially in woven or non-woven fabrics, is very common in industry: Think about the flow of oil in filters, of water in the felts of a paper machine, of sweat in shirts, of resin through a periodic matrix to form a composite, of blood in measuring stripes etc. Darcy's law is not always correct - one may need Darcy with memory or Darcy with inertia or Brinkman's law; the mathematical theory of homogenisation helps a lot, but here again (as in I.) a lot of modelling and computing has to be done (see e. g. U. Hornung, ed: *Homogenization and Porous Media*, Springer 1997). Think only about the problem that a baby in fact deforms the diaper. A simple experiment still to be simulated is: To press out a sponge! Do you see that mathematics literally is everywhere?

I want to show a second example, from Tecmath. It originated about 12 years ago, when, after my talk at AUDI about the "possibilities of mathematics in car industry", a responsible engineer formulated the need of a computer model of a human being, including a detailed skin model and including postural comfort. If you scan a surface of a human body you get a cloud of millions of points; if you want to simulate a change of the posture in a computer, you cannot do it operating on this cloud. At least not directly: You need a data reduction - and this can only be done by finding a "lower dimensional basis" i.e. typical modes of posture; these modes make a data reduction to less than 100 parameters possible.

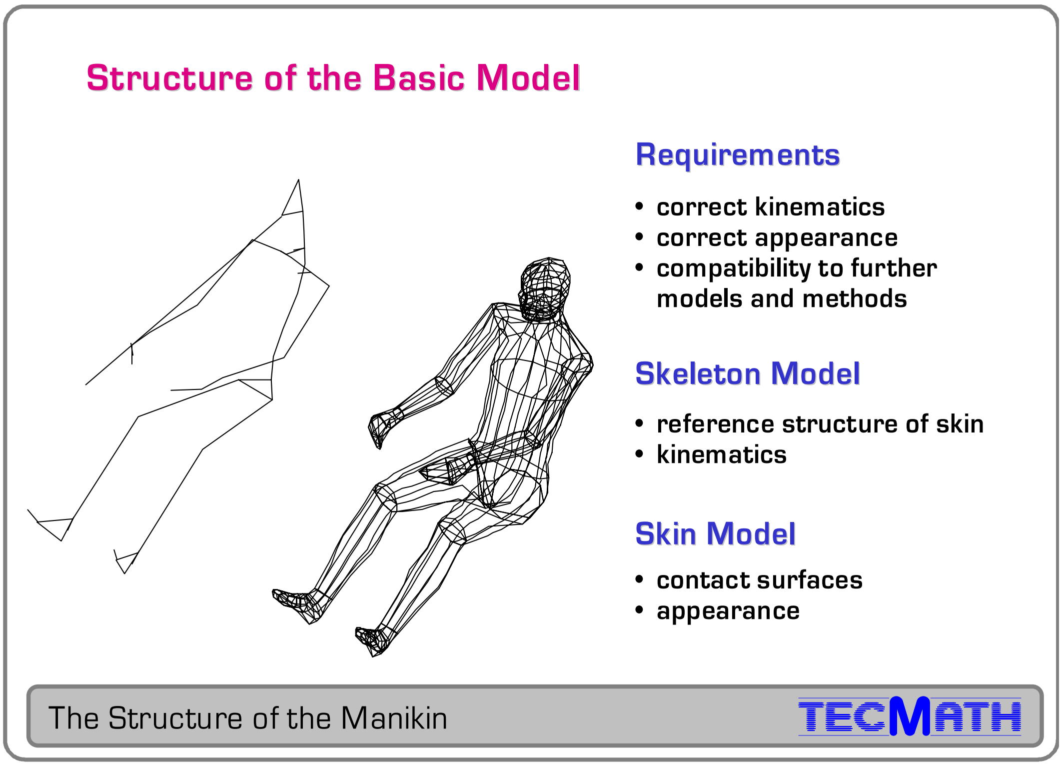


Figure 5: The basic modes: RAMSIS

RAMSIS was developed using careful anthropometric measurements. If this reduction is done, one may define comfort functions (depending on these parameters) and find postures maximising the comfort.

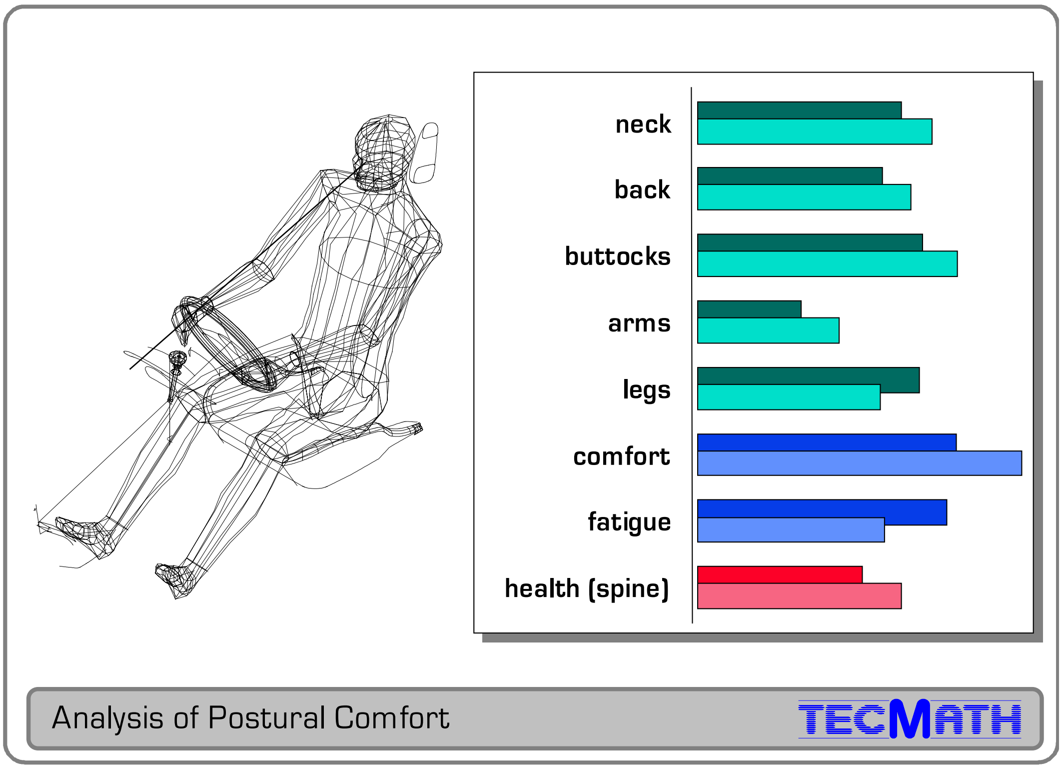


Figure 6: Postural comfort

Of course, it is a multicriterial optimisation and you need tricky algorithms (and representation of surfaces) to do it properly. Then, you may use it to validate the ergonomy of car seat: What does the driver see, what can he reach - even, how is the secular change of the data of a typical driver? How are belts operating - and maybe even airbags (if our institute will be able to simulate the expansion of an airbag - see III.).

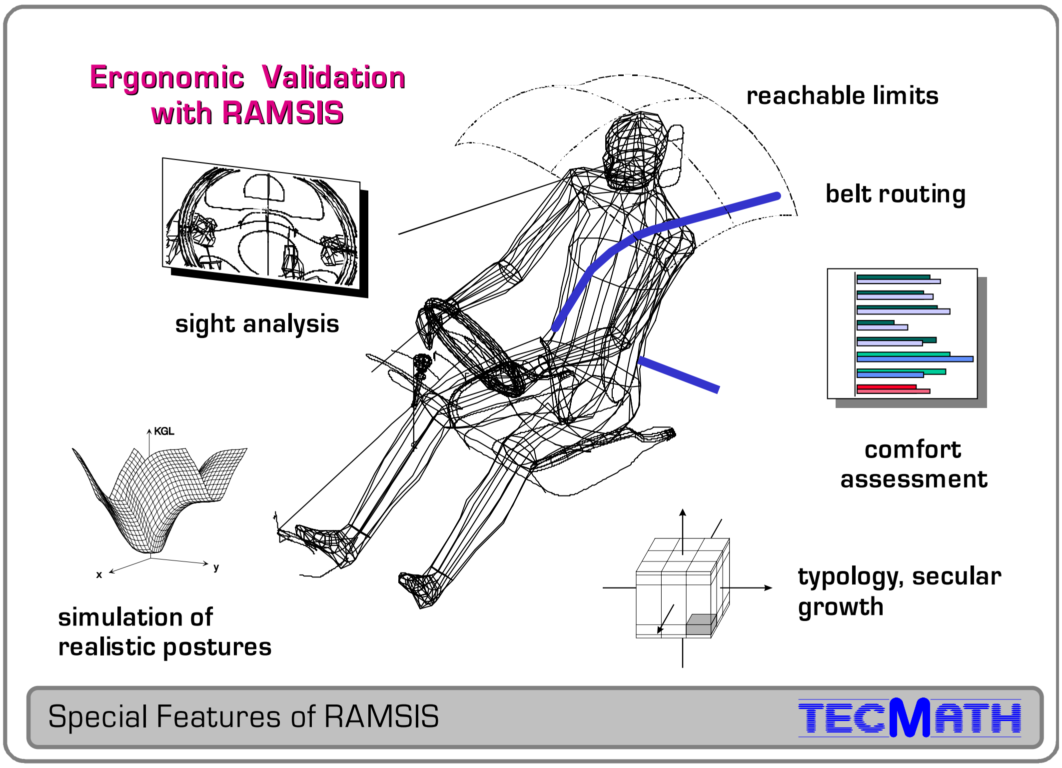


Figure 7: Features of RAMSIS

The tool is now operating in almost every car company in the world. But not only that: To have a good model for the human body is useful for many other things: To study characteristic postures say in hobby and working, at fishing, biking etc.

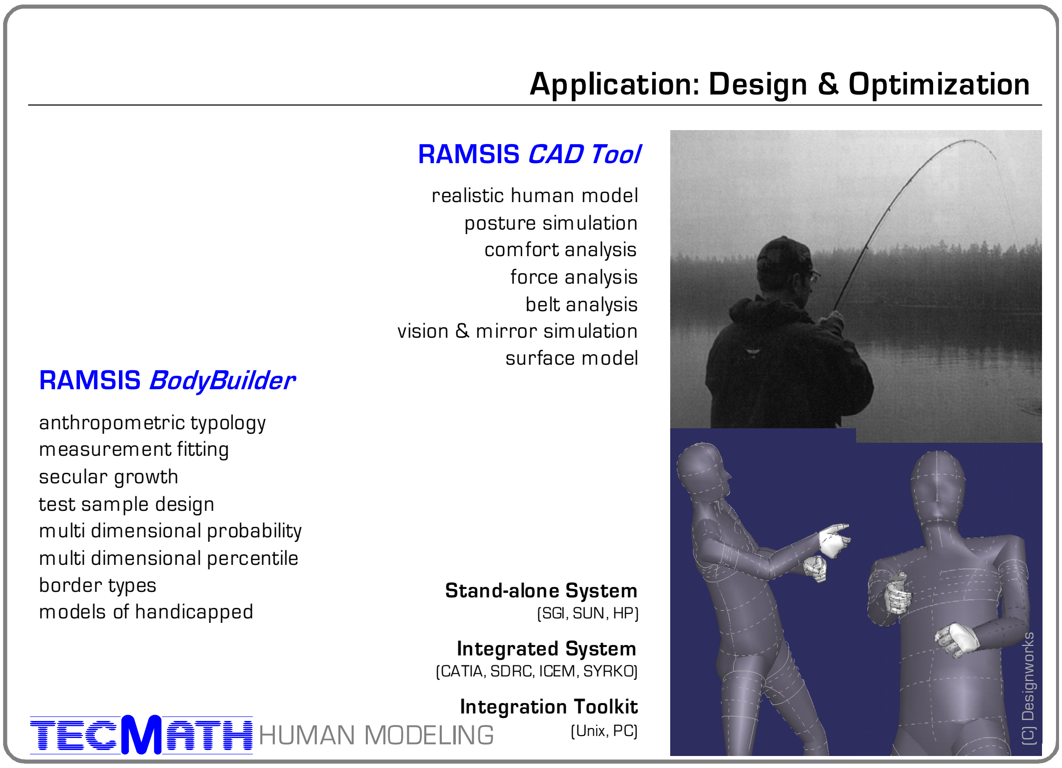


Figure 8: Applications

A new and very profitable application is in tailoring: The body is scanned (the point cloud is produced), the scan is projected to the manifold of RAMSIS modes, clothes are virtually put on the body, they are designed and inspected in different postures,

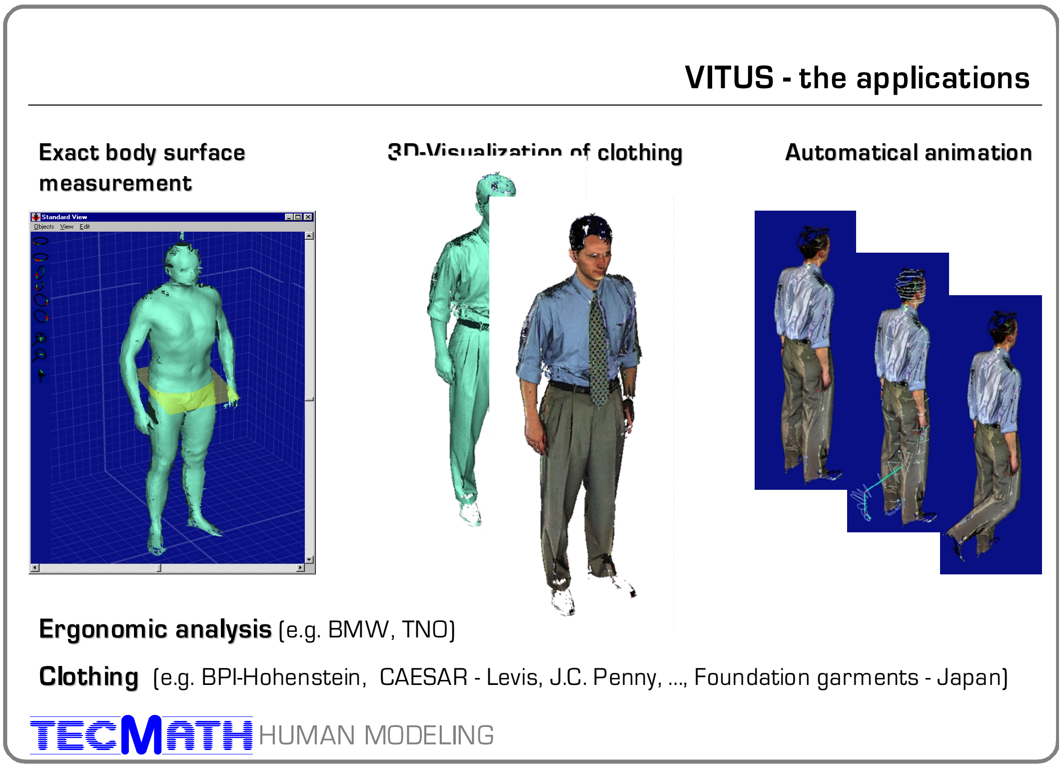


Figure 9: Made-to-measure clothing

the patterns for tailoring are designed by the computer and the clothes are produced: made-to-measure clothes, 20% more expensive than industrially produced ones.

And of course possibilities for extensions are infinite, for example to shoe making.

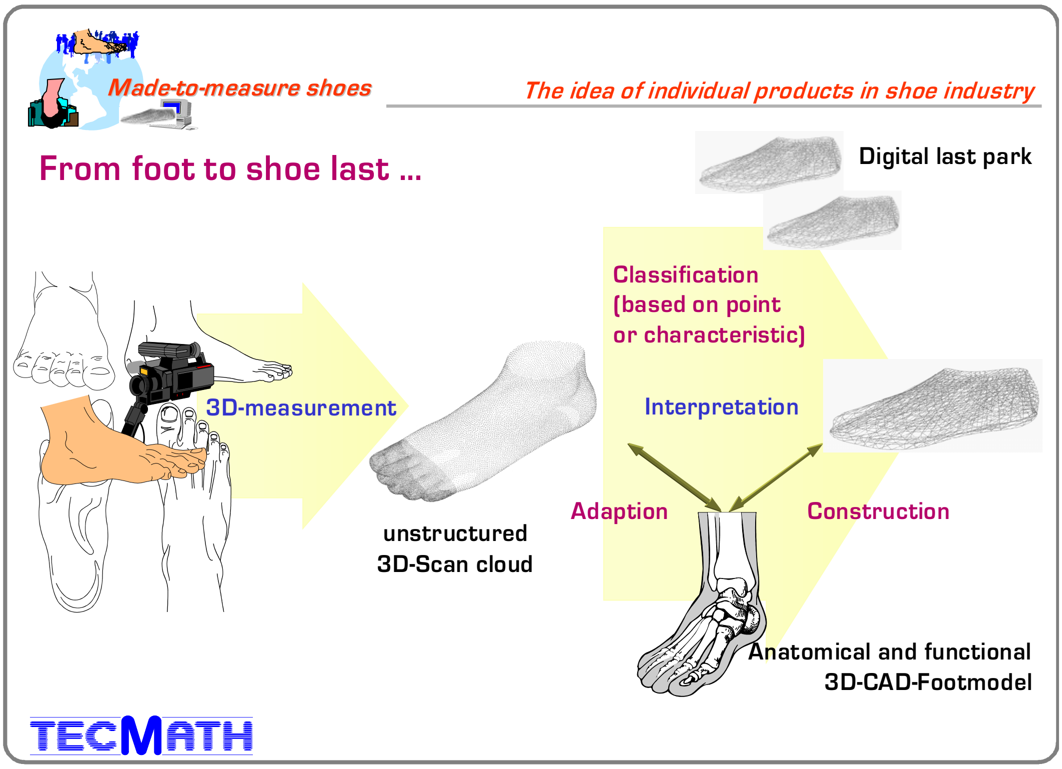


Figure 10: Made-to-measure shoes

I believe that this computer model of a human body is just a very typical example for what will be an important task during the next century: Individual products, designed according to the needs of individual persons (think of children or old people) and producible at costs not much higher than mass ready-made products. Of course the geometric shape of the human body is important, but by far it is not everything: Operating the products and the functioning of the product may be adapted to the individual capacities of the user.

A field, where mathematics has become very popular during the last 10 years, finance mathematics, seems to have similar goals: To design financial products like options, block of shares according to the wishes of the customer. Mathematics here is different: Time series analysis, theory of martingales, free boundary value problems for Black Sholes equations. Mathematical chairs for and books on this subject shoot up at present, new study programmes are created and high salaries in banks offered to our graduates. It is wonderful that quite advanced mathematics found a new, prosperous field of application; but I believe that quite soon we will reach a more modest saturation level. Altogether, the design of individual products (who have to be produced nearby, not at far east) may essentially contribute to the bright future of mathematics (see e. g. P. Wilmott, J. Dewynne, S. Howison: *Option Pricing*, Oxford 1993).

III. Simulation and optimisation of production processes

When the product is designed, it has to be produced. I said already that design and production are strongly correlated. And, of course, the machines for production are again products of another company, products, which should be designed optimally with respect to their task, to produce these products. To improve the quality of paper is a task for the paper manufacturer as well as for the paper machine manufacturer.

I want to take again examples from our own experience: from paper and from glass industry.

If we want to model complex production processes, we have to realise that again we have a hierarchy of models. Look for example at a paper machine:

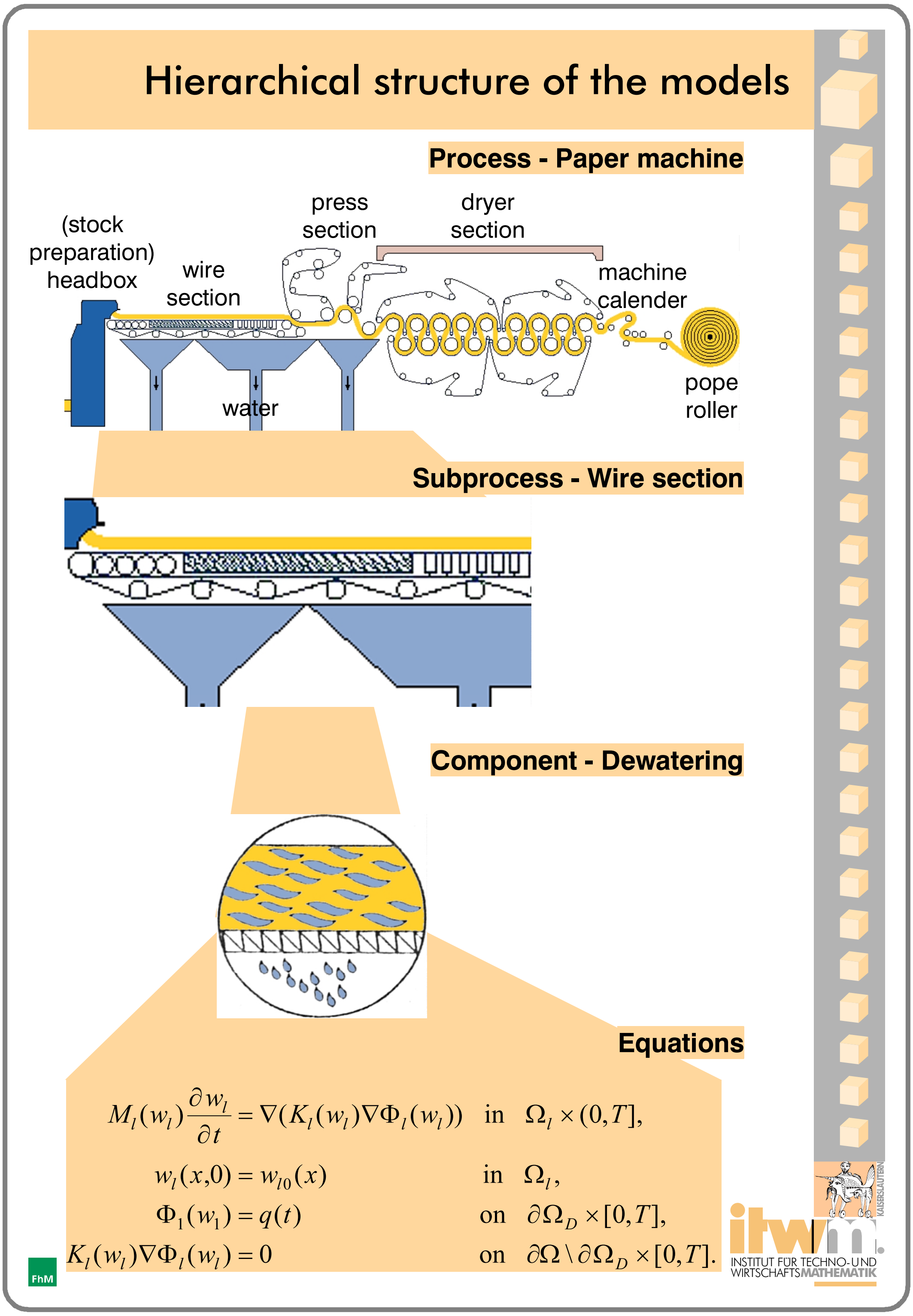


Figure 11: Hierarchical structure of models for a paper machine

You may have the whole process in your focus - we call that again the macroscopic view. Then you may consider the sub-processes just as input-output systems with capacities and buffers in between, forming a network with stochastic perturbations etc. Stochastic manufacturing systems may be considered as a network of queues and simulated as such; under the so-called heavy traffic assumption (the buffers are never empty), a diffusion approximation for the network may be derived, which has as many variables as the system buffers. Again we end up with 50-dimensional diffusion equations - an exciting task and similar to the polymer problem of section 1.

(A1) Simulation

Simplification under heavy traffic assumption
(buffers empty with probability zero):

Diffusion approximation for

$f(t, x_1, \ldots, x_5) =$ probability density for buffer X_i

to have a surplus x_i, $i = 1,\ldots,5$ at time t

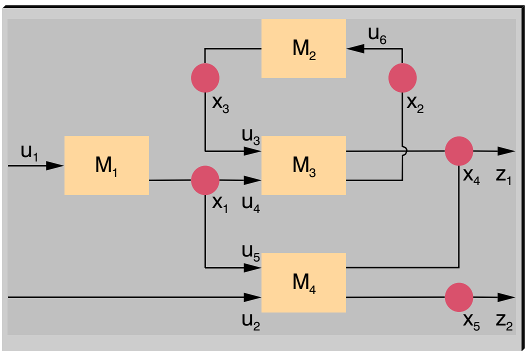
$$\frac{\partial f}{\partial t} = -\sum_{i=1}^{5} b_i \frac{\partial f}{\partial x_i} + \sum_{i,j=1}^{5} a_{ij} \frac{\partial^2 f}{\partial x_i \, \partial x_j}$$

(b_i depending on the capacities,

a_{ij} reflects the graph of the network)

High-dimensional diffusion:

May be used for scheduling of the network –
if the dimension is not too high.



ITWM research:

Numerical methods for processes with up to 100 buffers.

FhM

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Figure 12: Simulation of manufacturing systems by diffusion

We may also think of an optimal control of the whole system, where we have only one control variable per input-output system (machine unit).

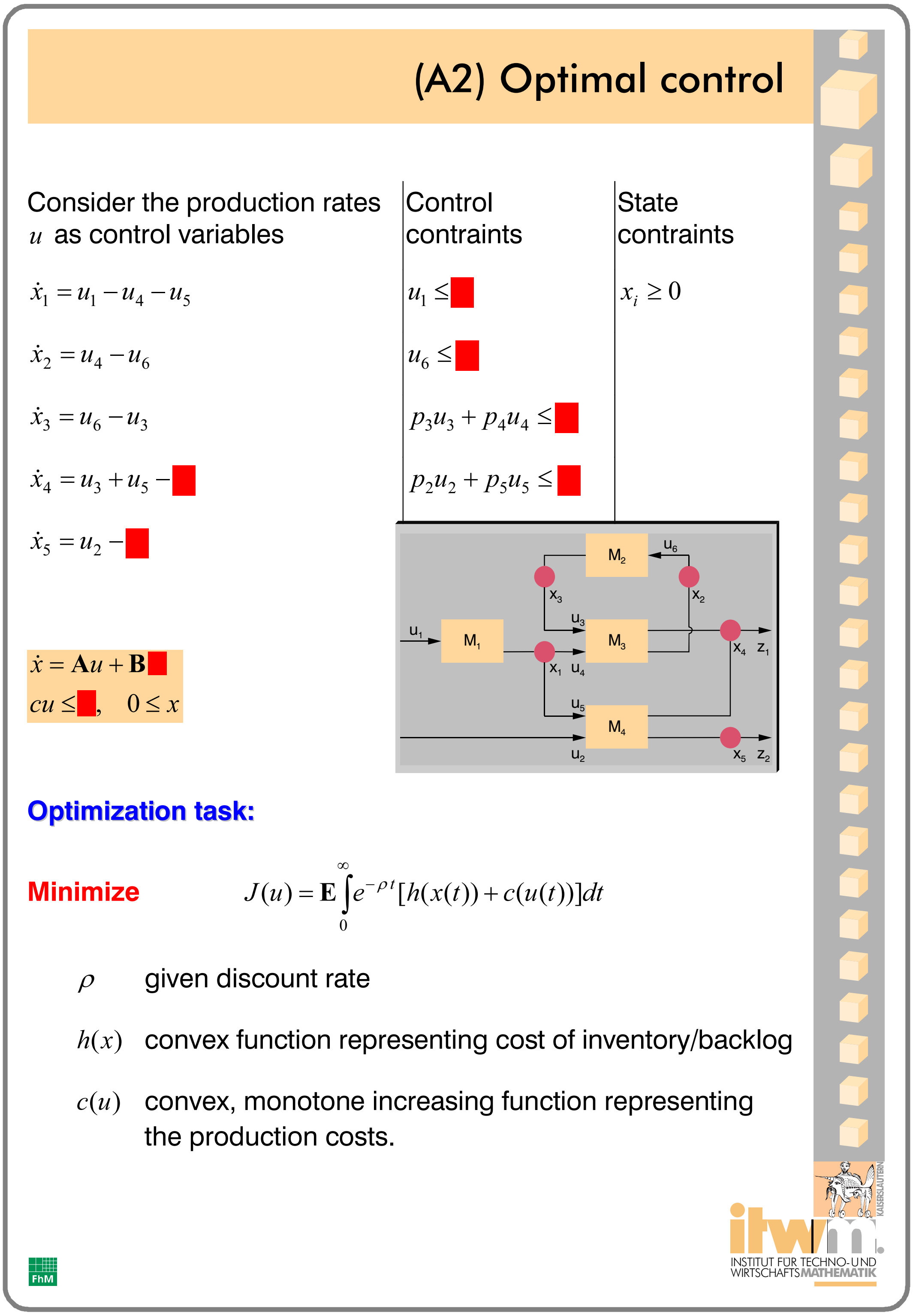


Figure 13: Optimal control of a manufacturing system

This leads to Hamilton-Jacobi-Bellman equations and a lot of tricky analysis and numerics. Even questions concerning the optimal time to buy a new machine could be integrated and management decision could be supported (see e. g. S. P. Sethi & Q. Zhang: *Hierarchical Decision Making in Stochastic Manufacturing Systems*, Birkhäuser 1994).

On an intermediate level we look at the subsystems more closely; instead of modelling them as a black box, we try to model the flow of energy and material in order to derive a dynamical system with controls like input of material, temperature etc. A lot is done on this level, which we call the mesoscopic view. New research turns into the direction of adaptive or learning systems, where unknown or changing parameters of the system are updated automatically. System theory, neuronal networks and fuzzy logic to integrate expert rules are modern key words in this field.

Finally we may look at the finest level, the physical processes occurring in a paper machine; we call it the microscopic view.

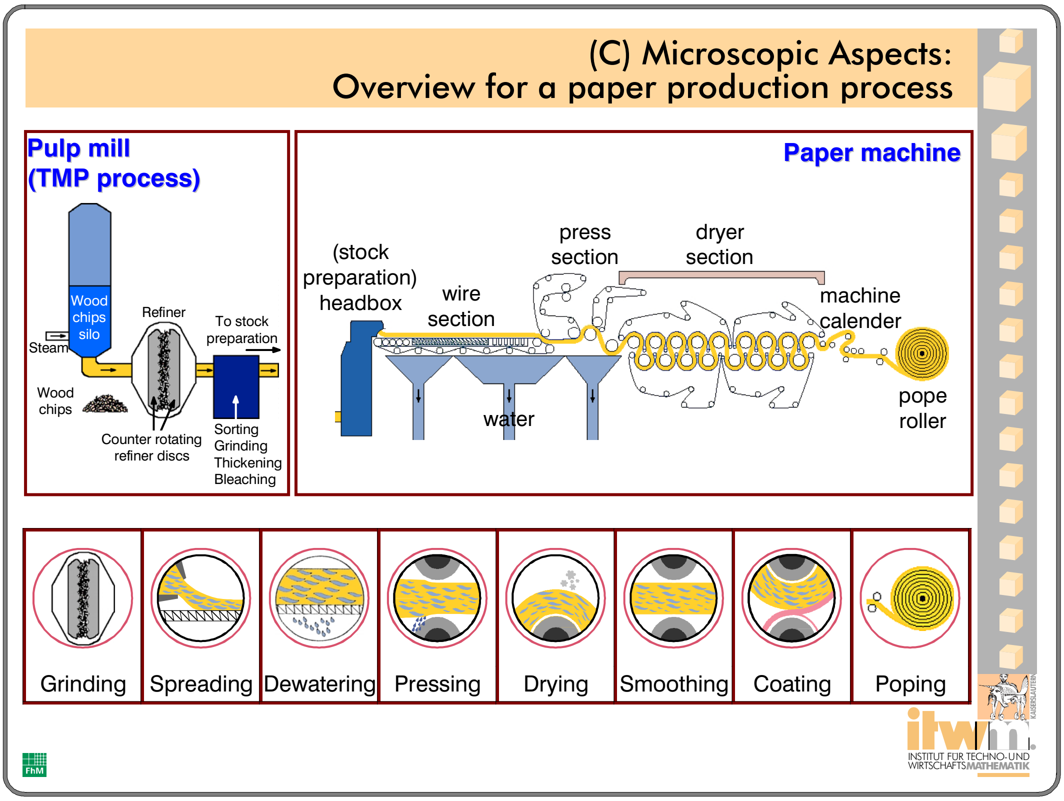


Figure 14: Microscopic aspects for a paper machine

None of these processes - grinding, spreading, dewatering, pressing, drying, smoothing, coating and poping - is simple to model. In spite of the fact that quite a lot of work is done, for example at the ECMI centre in Finland at Jyväskylä, there is even still more to do: Multiphase flows with free boundaries, sedimentation, flow in deformed porous media, again free boundary value problems, thin layer approximation, ... If you really want to understand the task, you have to look at a paper-mill, a huge monster, which you want to simulate.

What is really needed is a proper combination of all these hierarchical levels in order to get a **tool for diagnosis**: How can you influence the quality of the final product, how can you avoid failures like tearing. Risk-, or better diagnosis tools for the total system: That is what industry is looking for, not only in paper-mills or printing machines or polymer fibre plants - even in systems like planes or fast trains etc.

Much of the work industry or research institutes like ours are doing in this field is classified. It would be a bad sign for the commercial value of our work, if it would always be pre-competitive. Sometimes we are not even allowed to name the company we are cooperating with, sometimes only the tricky details are kept secret. Therefore I select an example, a cooperation of our institute with a company, which runs already over more than 3 years and has been so successful that we may tell about the cooperation - maybe not the latest, but at least the fundamental ideas. The companies name is Schott and it is one of the best known glass producers in Germany; and the problem is old - so old that the German physicist Fraunhofer, who gave the name to the research association we are related to, already struggled with it 200 years ago: During the cooling of glass, heat is to an essential part transported by radiation; if one wants to control the cooling process (in order to avoid cracks), one has to predict the temperature evolution and therefore the radiation. Glass is a semitransparent medium: Each point in the glass absorbs and emits rays at the same time. Ray tracing, relatively easy for surface-to-surface radiation, where the points in between do not influence anything, becomes very expensive, in realistic 3-dimensional configurations it is not feasible. Mathematically we have to solve a 6-dimensional integrodifferential equation, coupled with the heat equation.

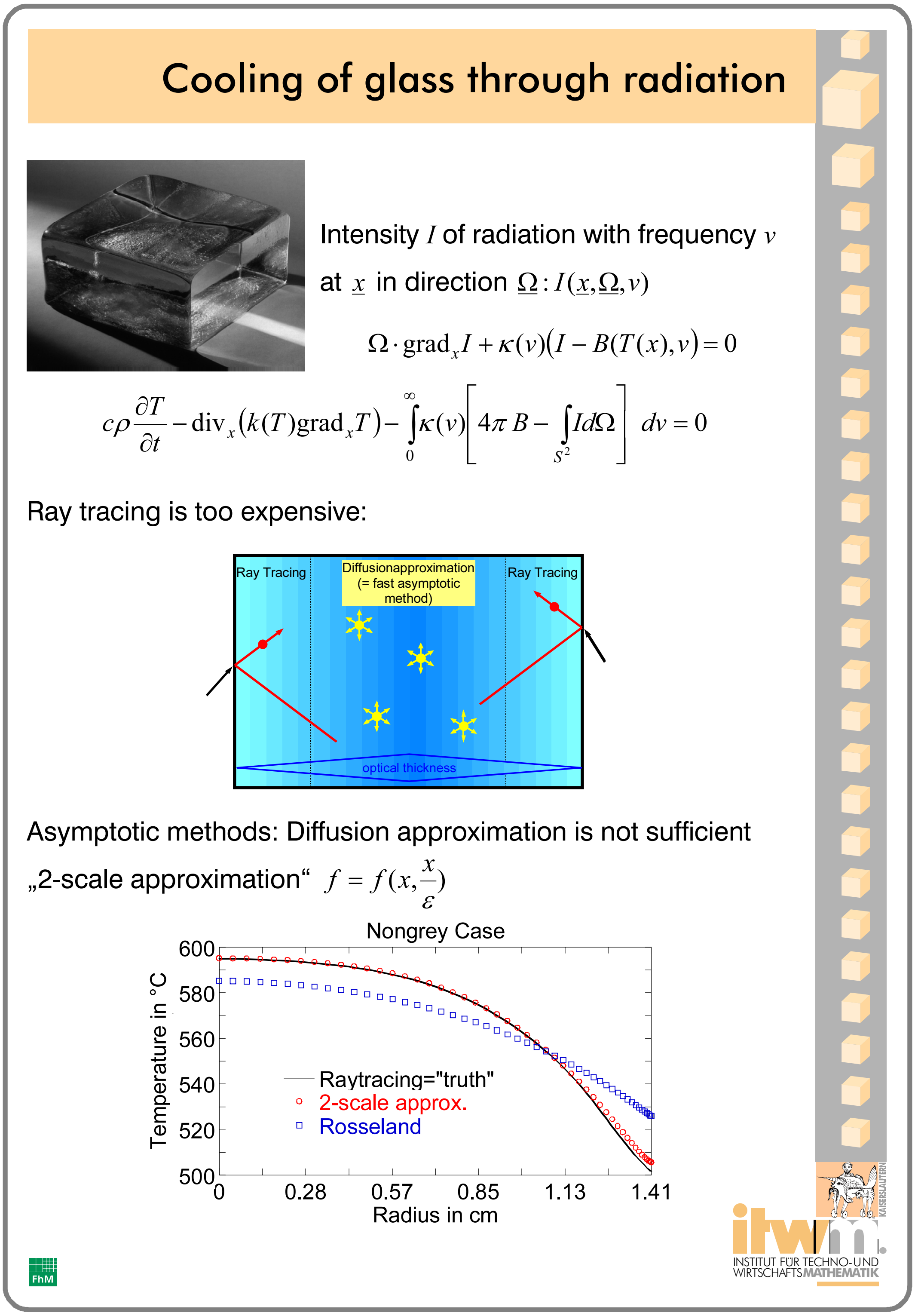


Figure 15: Cooling of glass through radiation

There are many asymptotic methods, using the optical thickness as small parameter; the most often used Rosseland approximation is quick, but not sufficiently accurate. Combination of ray tracing, where it is necessary (i.e. at the boundary) and diffusion approximation, whenever it is possible, is in the spirit of combining analytical with numerical methods; a proper 2-scale Ansatz is a more elegant similar approach, which permits us now to do accurate 3-dimensional calculations in a reasonable time; this gives our partner a lead and us (besides the money) scientific success. Moreover, radiation in semitransparent medium happens not only in ordinary glass production; even in medicine for example the human body is semitransparent for certain rays - a fact, which is used in radiation therapy. Many more applications are at hand (see e. g. F. Zingsheim: *Numerical Solution Methods for Radiative Transfer in Semitransparent Media*, PhD-Thesis, Kaiserslautern 1999).

A completely different kind of mathematical problem is posed by the production of plastic fibres. Here, air streams flow through an ensemble of very thin fibres, through a filament. How does the air flow interact with the fibres? These fibres are almost one-dimensional objects in a 3-dimensional flow: An Euler model would not even notice them. For wet spinning, where the fibres are drawn by means of rotating rollers, nice work has been done at another ECMI centre, at OCIAM in Oxford.

For quite a while we have worked on air-spinning processes, and tried to find numerical schemes computing the flow and the motions of the fibres at the same time. The main problem of course is the complex geometry of the flow region: The filament forms a moving, complex, interior boundary.

Similar problems appear if you want to predict the behaviour of sails or flags in wind or other similar industrial processes. To compute flows in complex regions with quickly moving (free) boundaries we would prefer to use grid free methods. There are two main ideas to develop those methods.

1. Particle methods like "Smoothed Particle Hydrodynamics". Here "particles", mathematically understood as δ-measures, carry information like velocity, energy etc.; they move with their velocities, but change these velocities, energies etc. according to ordinary differential equations (with the number of variables corresponding to the number of particles) in such a way that the particle number density, the particle momentum and energy densities approximate air density, momentum and energy, which solve the Euler or Navier-Stokes equations.

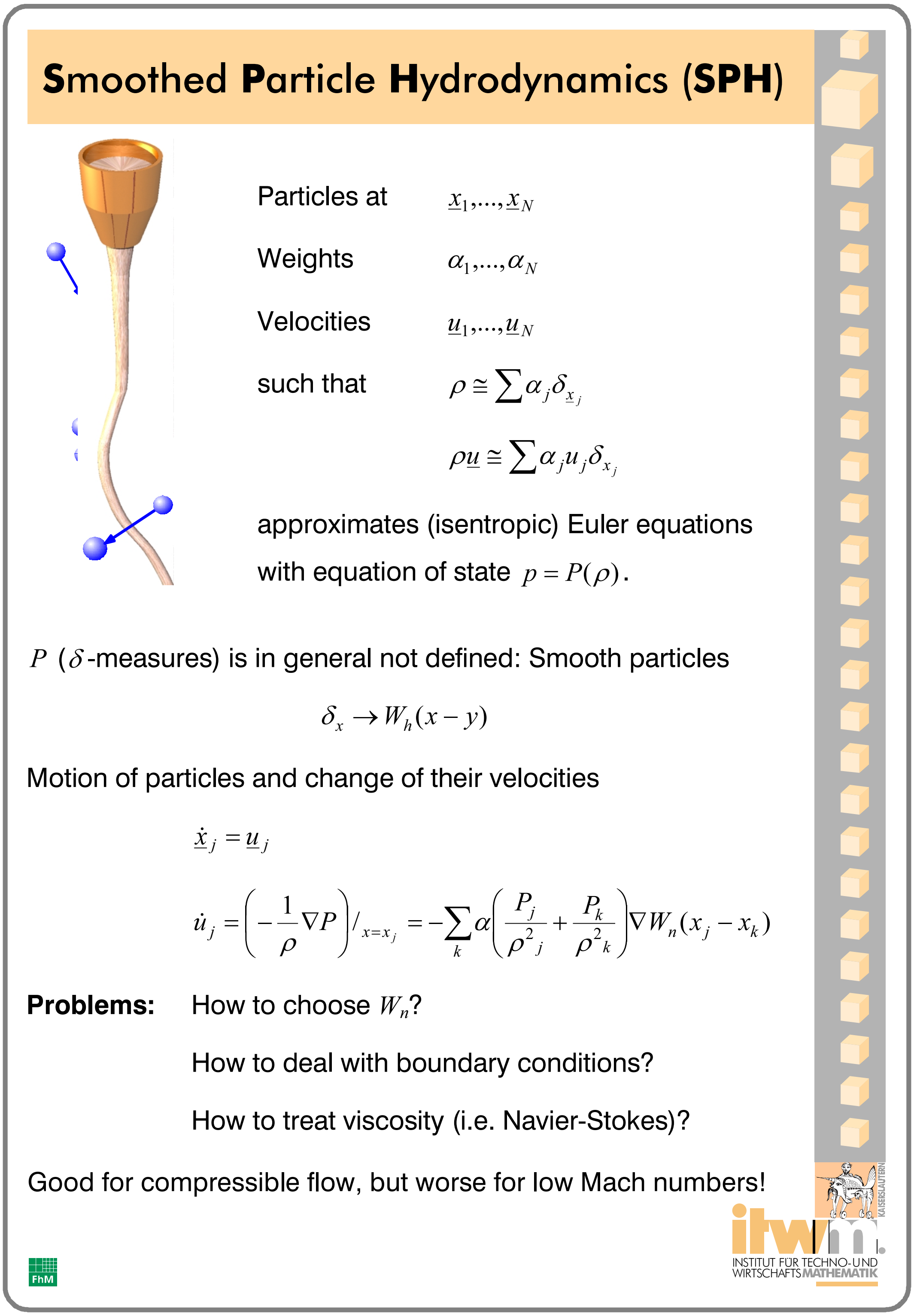


Figure 16: Smoothed particle hydrodynamics

The method, very popular in astrophysics, is not very accurate, but working even for difficult problems - as long as the flow does not get incompressible, i.e. for sufficiently large Mach numbers. Incompressible Navier-Stokes equations should not be attacked with SPH. There is an alternative method called

2. Lattice Boltzmann Method (LB). It is by far not grid free. Opposite: As the name says, it works on a lattice in space and we consider particles flowing through the lattice but only with velocities built-in in the lattice.

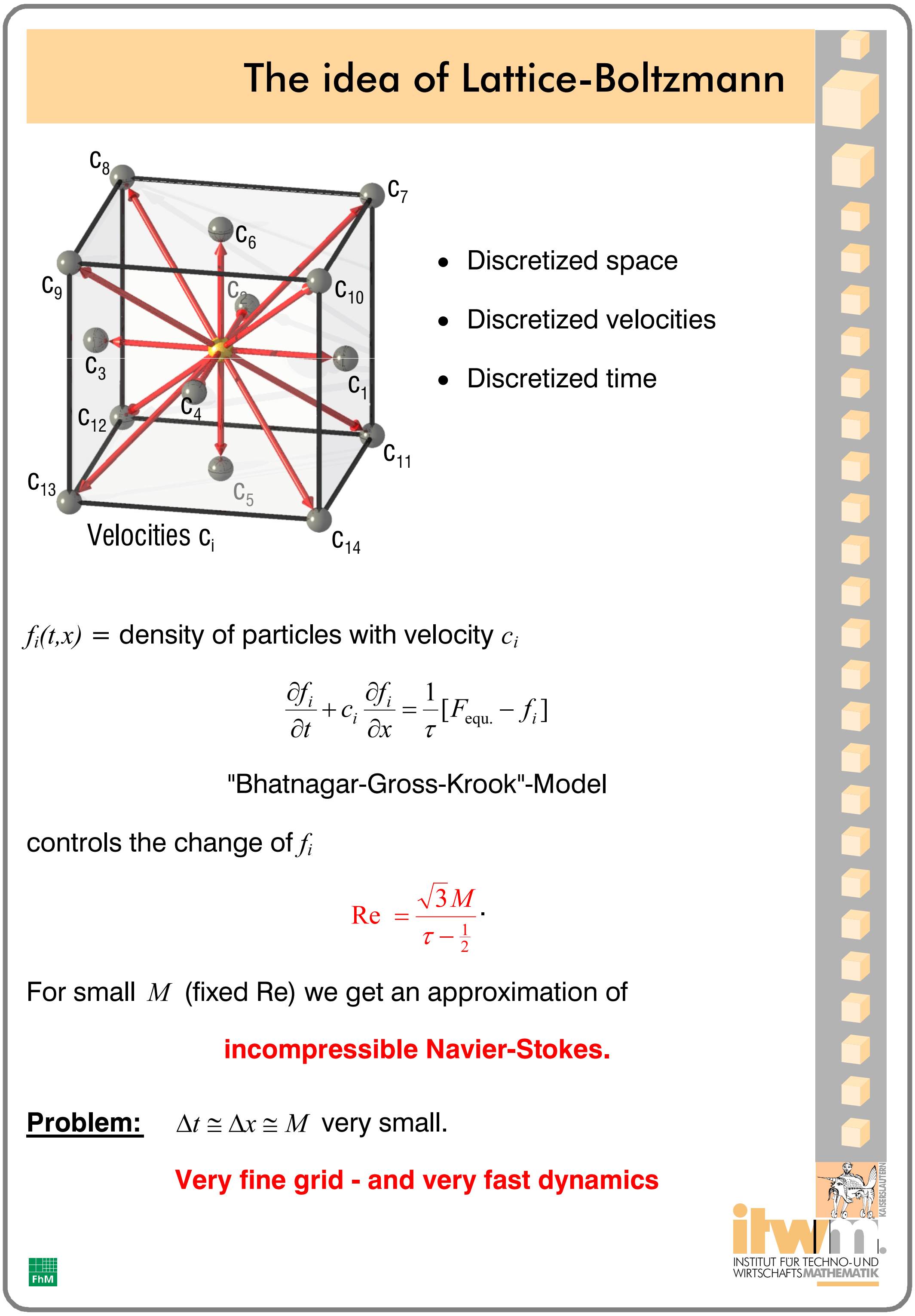


Figure 17: The idea of Lattice Boltzmann

And we use a kinetic Boltzmann equation like description, which considers the evolution of densities $f(t,x,v)$ in position - and velocity space. Normal density, momentum and energy are moments of f with respect to v. Everything is now discrete: time, position and velocity; the kinetic equation contains an equilibrium F_{equ} and a relaxation parameter - both can be chosen such that the moments approximate incompressible Navier-Stokes equations (Shiyi Chen, G. D. Doolen: *Lattice Boltzmann Methods for Fluid Flows*, Annual Rev. Fluid Mechanics 30, pp. 329-364, 1998).

But again: Δt and even Δx have to be chosen small, when the Mach number of the flow is small - and since we want to describe incompressible gas now, the Mach number is small. So, why is it nevertheless a good idea? The evolution of f is extremely fast; therefore we can use very fine lattices, millions of cells - and a very fine lattice is of course as good as no grid at all. It is able to resolve complicated structures. But still, there are many problems to solve and ITWM has a group of 5 scientists working on the feasibility of these methods for industrial flow problems. The combination of codes for compressible and incompressible flows is necessary, since this is not a real alternative: Industrial flows may well be compressible and incompressible at the same time, but in different regions. There is hope to get progress in this direction using the idea of "kinetic schemes" (M. Junk: *Kinetic Schemes*, PhD-Thesis, Kaiserslautern 1997).

Of course, there are many more production processes in completely different fields: Think for example of oil reservoir modelling, where again flow in complex porous media has to be modelled and simulated. ECMI groups in the north of Europe (Oslo) and in the south (Sardegna) provide European expertise. Or think of the production of a good Italian espresso: The ECMI group in Florence has done marvellous work - you may even taste it.

IV. Discovery of patterns for quality control, prediction of finance data and diagnosis of medical data

This is an area where a very efficient tool is already available: our brain. Evolution has taught it to discover patterns in audible or visual signals and to classify them. But to teach computers to have the same capacity is still out of scope.

Think only of walking through a pedestrial area in your home town, obviously chek-king hundreds of faces in seconds and discovering that a certain face belongs to a person you knew very well but haven't seen since 10 years. The brain realises the similarity between the face we just see and another, younger face, which we had seen long time ago -one of the maybe 1000 faces it has stored since then. How long will mathematics and/or computer science have to make intense research until a computer can do the same?

An easier example from our projects: Look at a veneer and try to classify it according to its quality; that is what men and women do in a furniture manufactory every day at a high speed. But how can we tell a computer what is quality and to determine it?

The main problem is the amount of data: The image given by a veneer consists for example of 256x256 pixels with 256 grey values at each pixel. How should we classify points in 256^3-dimensional spaces? The first problem is always data reduction - as it was with RAMSIS, the model of a human body: We need to find "modes", an underlying structure, which allows us to describe a single image by say 20 (instead of 256^3) parameters. In the veneer case the best way to do it was to ask the biologists: A tree trunk is a perturbed cone and the annual rings as well; a veneer consists therefore of perturbations of conic sections. The perturbations are caused by the axis of the tree being a curve (and not a straight line) and by knots. Using these informations we may reduce the number of parameters in fact to about 25. The next problem is an inverse problem again: Given a real veneer and our 25-dimensional model class - find the 25 parameter values giving the best approximation of our real object. This problem can be solved, but we are still far too slow in comparison to what industry is asking for. The last problem would then be to classify these points on our 25-dimensional manifold according to the quality; this task can be learnt from experienced persons by training a neuronal network according to their judgements. This would be quick and at least as reliable as human beings.

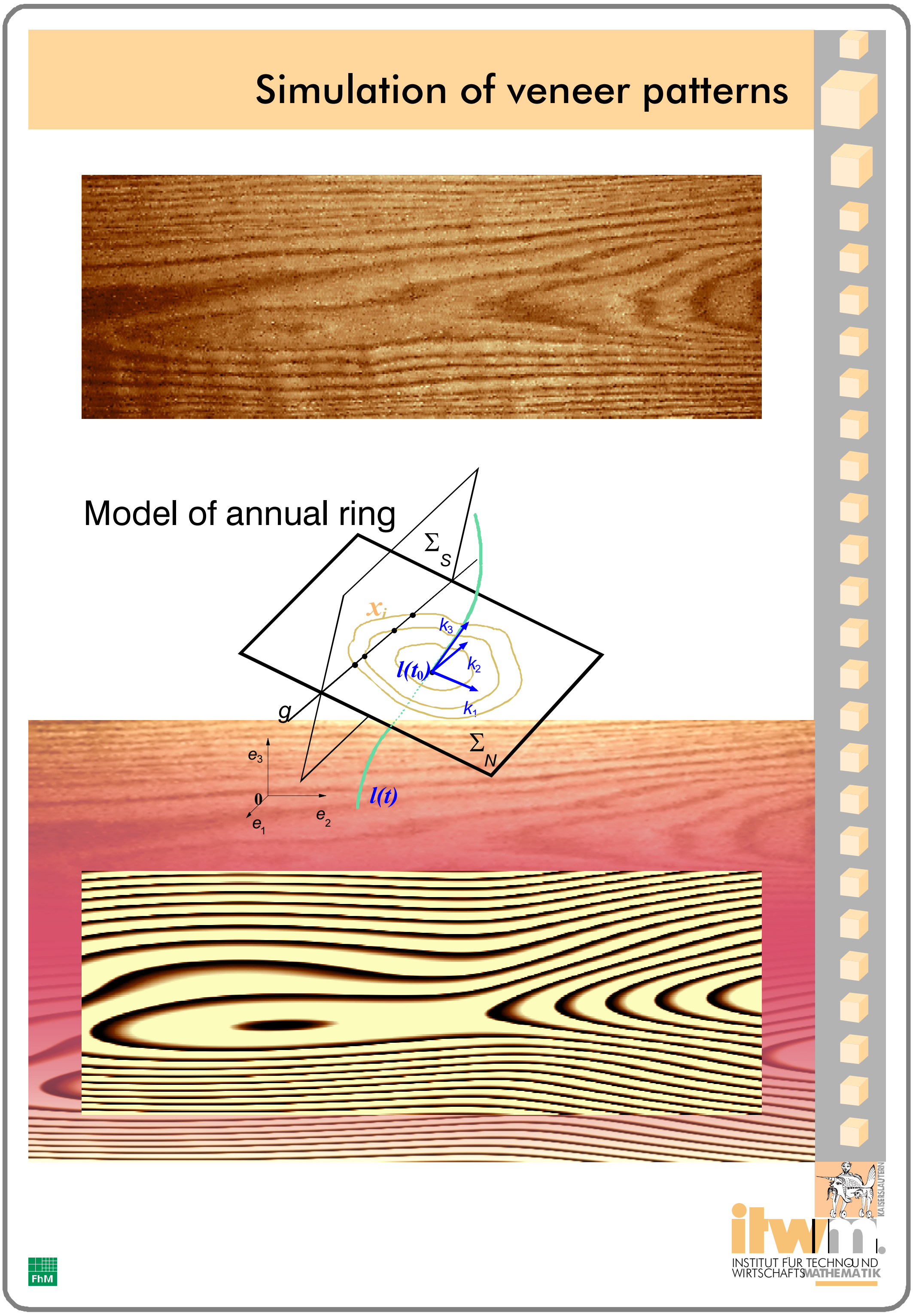
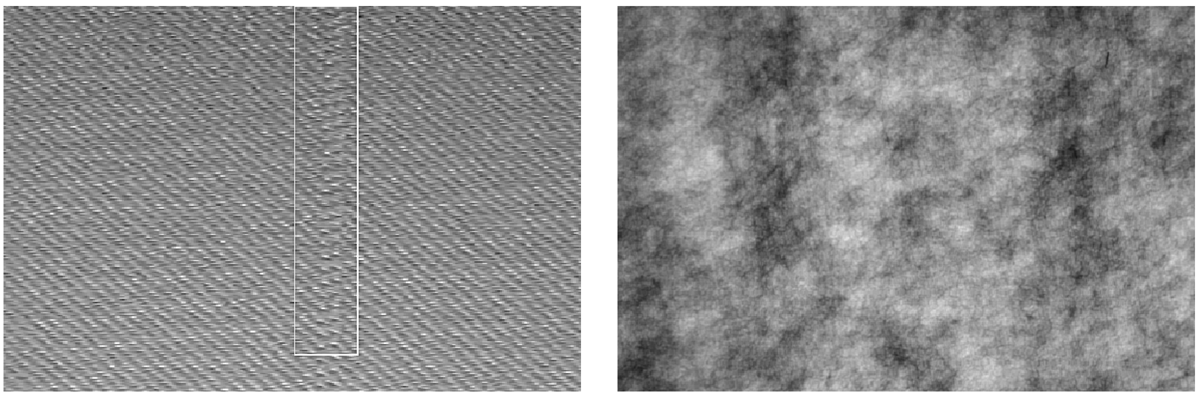


Figure 18: Simulation of veneer patterns

The steps above are typical: A model class, a set of modes, has to be constructed, which allows the reduction of data to a feasible amount; the identification of the best approximation in the model class to a given real object has to be performed; the classification rules for elements in the model class must be learnt.

We will find this job pattern in any pattern recognition job again; there seem to be ideas in neurophysiology, how our brain does it - but besides this very modest transfer of ideas, which is intimated in notions as "neuronal networks", not much is yet done. To increase this transfer is, in my view, one of the big tasks for future research.

Other problems of quality control are similar and conceptually even simpler: Find defects in a woven fabric (for example knitwear), judge the quality of a non woven fabric, given as a properly chosen distance to uniformity. ("Properly chosen" means to find a metric measuring the distance between two images in such a way that the judgement of experts is reflected in this distance; I believe that a large part of modelling consists in finding proper norms or distances.) In these cases the model class is given by wavelet filters or a similar local multiscale analysis, the identification is done through a fast wavelet decomposition or another pyramidal algorithm (see e. g. J. Weickert: *Anisotropic Diffusion in Image Processing*, Teubner 1998).

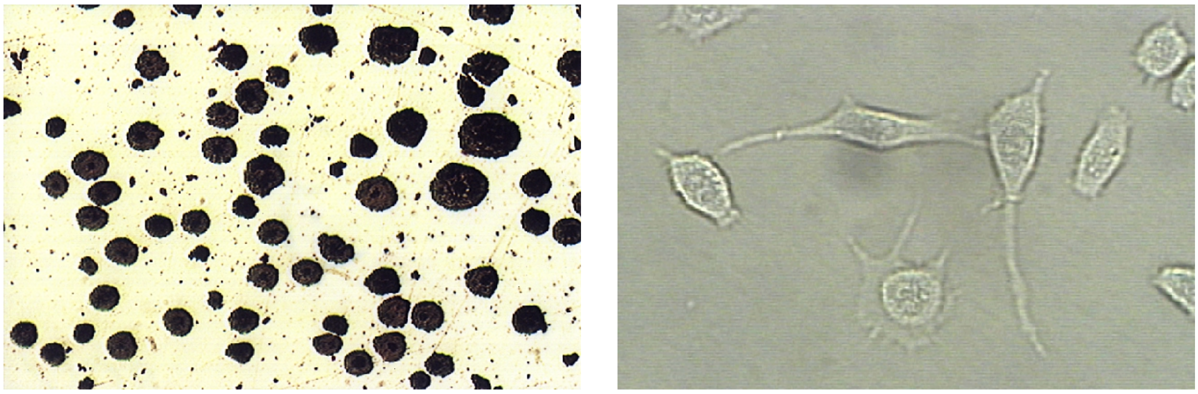


Textile type: Flat fabric

Non woven fabric

Figure 19a: Inspection of fabrics

Sometimes, the quality of a "surface" or an image is determined by certain properties of details. For example, size and density of inclusions in a cut through cast iron may determine the quality of the iron; the model class here is provided by stochastic geometry ("grain processes"), the identification of the parameters is a typical parameter estimation task in statistics.



Modular cast iron | Size and shape of protozoon's

Figure 19b: Inspection of surfaces

More complicated is a problem from biology: The state of protozoons is judged from the length of their flagellum and the size of the "body". Since we did not find a proper model class we tried to study the "intensity mountain", given by the image of a single bacterium. A lot of nonlinear smoothing has to be done to arrive at a mountain which we can deal with.

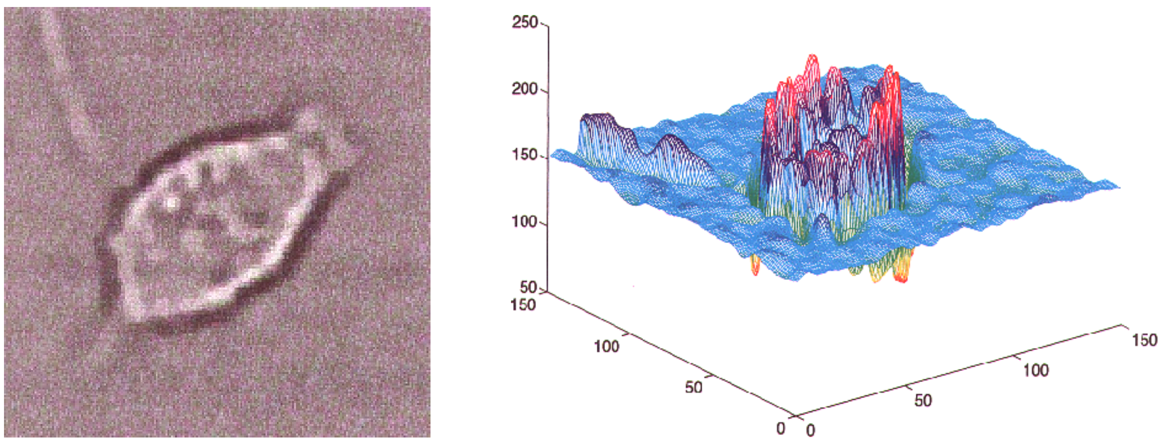


Figure 19c: Size and shape of bacteria

Now we have to decide: Is this "side mountain" on the left a flagellum? How do we determine the "ground plan" and by that the size of the mountain area? Even tricky methods as "Active Contour" or "Snake" methods do not work without errors. People in biological laboratories do the job with a lot of stress, but without many errors.

Since we are already near to medicine, we may switch to this field, which I believe provides enormous challenges for the future. The situation in medicine is new with respect to the amount of available data: The use of data bases allows to store the long-term observation data of patients.

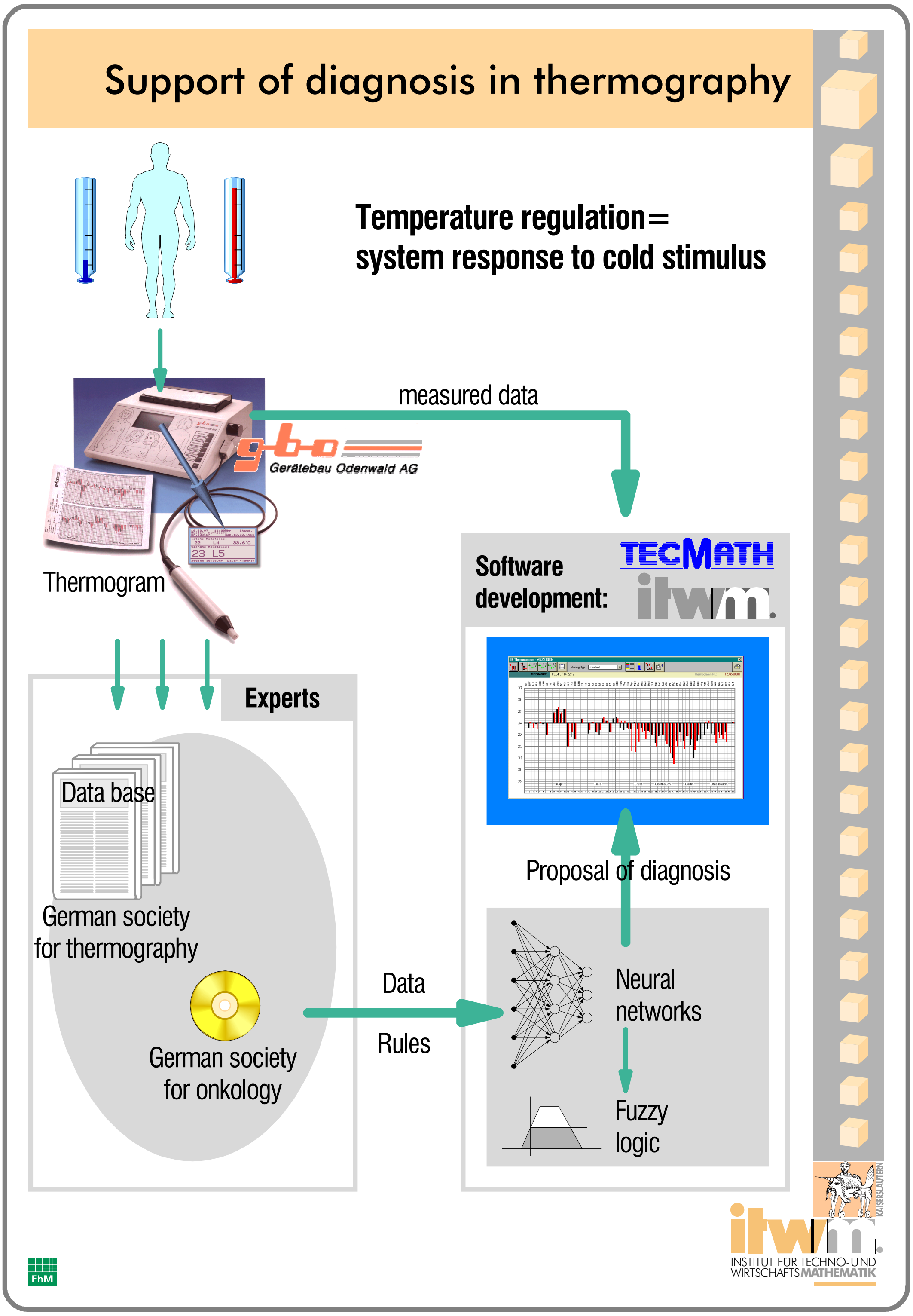


Figure 20: Support of diagnosis in thermography

Consider a human body as an input-output system, for which we have collected many input and output data as medication, food etc. and temperature, heart beat, blood pressure etc. respectively. From a careful long term observation we may have gathered hundreds of thousands of data. Now we choose a model class for this input-to-output mapping: For example linear control systems or neuronal networks. To identify an individual patient in this model class means the classical task of identification of systems or training of networks. Finally we have to classify the "identified" patients according to their state of health; here we may use experiences from experts, which can be often formulated in a fuzzy setting.

An example from our institute is the task to support the diagnosis from a thermogram. In thermography one measures the surface temperatures of a patient at ca. 100 points; the measurements are done twice, the second ones after cooling down the body a little bit. In this way one gets the system response to a cold stimulus; we see the temperature regulation of the body. A thermogram consists therefore of 2 times 100 data. And we repeat this thermogram and do it with many patients. This data base, together with expert rules, provide an input for a learning system, which at the end is able to make a diagnosis incorporating the knowledge stored in the data and in the expert rules.

The method is now used in oncology and seems to be very promising.

Another medical project we are involved in concerns the evaluation of electrocardiogram data. There again is a lot of expert knowledge, but are there more than fuzzy rules?

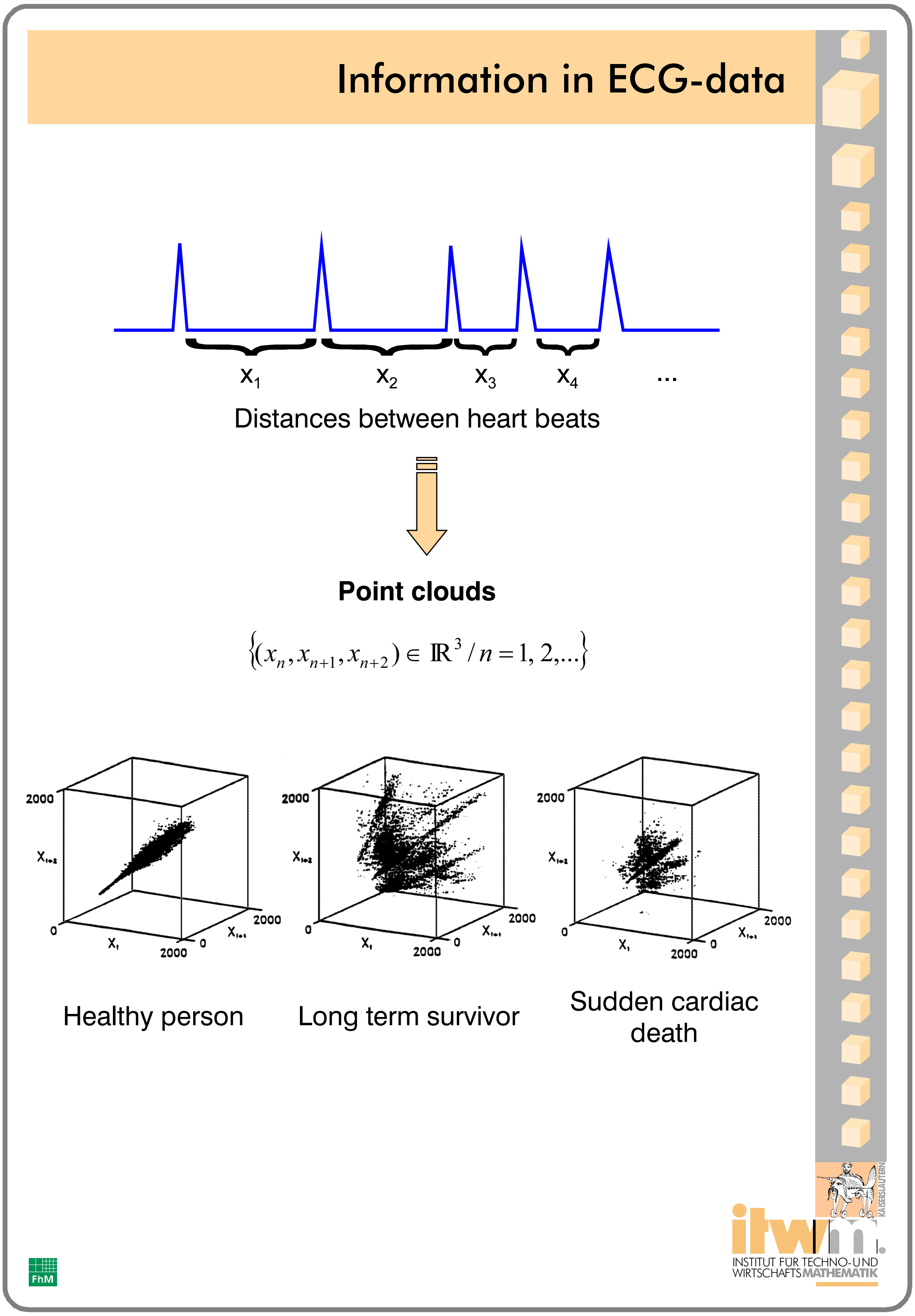


Figure 21: Information in ECG data

It seems general belief that the series of distances of heart beats contains most of the information; but how do we extract this information from the series $x_1, x_2 \ldots$ of beat length? It would be optimal to find a characteristic number assigned to the series, which tells us about the health or illness of the patient. A regular beat would mean $x_1 = x_2 = \ldots$; but is regularity a good sign?

More information is visible, if we consider clouds of points in $\mathbb{R}^2$ or $\mathbb{R}^3$, which are made of 2 or 3 consecutive beat length: $\{(x_n, x_{n+1}, x_{n+1}) \in \mathbb{R}^3 / n = 1, 2, ...\}$. The cloud degenerates to a point on the diagonal $x = y = z$ for complete regularity and it will be a "slim club" along this diagonal, if the beat changes slowly. If the club is too slim, experts consider it as a bad sign.

But what are the proper characteristics? Again, we have much long term data, for which we know the fate of the patient. Therefore, if we would have a suggestion, we can check, how good it is. Astrophysicists have proposed a number - they used their experience in dealing with point clouds; and it works rather well, but optimal? One may again think of many model classes - those from stochastic geometry (my favourite) or from fractal geometry (there are many attempts in this direction - they speak of "chaos" in heart beat; but it seems that they are not better than the astrophysicists) or something completely new. A very exciting research field: We have to find order, structure in seeming by unstructured patterns: Who else if not mathematicians, whose profession it is to find ordering structures, should try to solve those problems.

Another field of finding patterns in data belongs to finance and commerce, which I already mentioned in . The data are provided by the time evolution of the prices of financial assets; the models are taken from stochastics. The evaluation of the models, the answer to questions about arbitrage opportunities or about option pricing leads to new concepts and theories in probability theory, functional analysis, partial differential equations etc.

Even completely new questions, maybe from other fields, where actions of many human beings are decisive, may come up: Nobody would have predicted the boom of finance mathematics 25 years ago! Maybe we have "sociological mathematics" after 25 years from now? The Austrian novelist Robert Musil in his "Man without Properties" (chapter 103) talks about thermodynamics of moral: In societies, the stochastic behaviour of the individuals, in spite of being very important for the individuals themselves, does not contribute very much to the evolution of the ideas in the society. Individuals as Brownian particles, the society as a kind of gas of particles: Are more mathematical models in sociology possible and even useful? Finance mathematics seems only to be a start in this direction.

B The task ahead

A few months before he died in 1993, the ECMI president, Henrik Martens from Trondheim in Norway, wrote an article with just this title "The task ahead" (see Surveys on Mathematics for Industry, Vol. 4, Nr. 1, 1994), where he focussed on a problem he considered to be of primary interest: How do we prepare our students for a profession as an "industrial mathematician"; I tried to describe the corresponding "discipline" in this article. The consequence of this new discipline is a profession with special "*attitudes, habits and skills, which are not usually emphasised in the mathematical community, and often quite foreign to the research mathematicians. We may well see the emergence of industrial mathematicians as members of a new technological profession, solidly rooted in the mathematical sciences, but with its own professional profile and goals*" (H. Martens).

Figure 22: The emergence of an industrial mathematician

These attitudes, habits and skills of a mathematician **in** industry (not necessarily an "industrial mathematician", who makes mathematics **for** industry) were described on a solid base of success stories in US Industry by the SIAM Report on Mathematics in Industry, 1995. I just cite a few sentences from the SIAM NEWS article about this report; it runs under the headline "Change is important" and evaluates the responses of mathematicians in industry: "More than 30% of PhD respondents, however, assessed their preparation (at the university) as less than good in some of the very areas identified as crucial for success in a non-academic setting: computing, the flexibility to work effectively on a range of varied problems, breadth of scientific background, teamwork and communication. An overwhelming majority of mathematicians surveyed (90%) indicated that change in the graduate education of mathematical scientists is important." Even if we are willing to change: How should we educate our mathematics students (or even our engineering students during their mathematical education)? The key words are again "modelling" and "scientific computing". When the students, after the first two years, are "solidly rooted in the mathematical sciences", modelling should be taught by doing. We call that "modelling seminars" or "project seminars": Students work in (if possible: interdisciplinary) groups of 5 students during one semester on a real world problem, which originates from our multiple contacts with industry. The problems are given in their original non-mathematical settings, they have to be modelled, to be evaluated with a computer and presented in a proper oral and written form. These seminars are the key features of our education and lead often to master theses with subjects from industry but supervision at the university. Communication skills, normally not a strong point of mathematicians, have to be better developed: Mathematicians have to communicate with people from industry, who pose the problem, with other scientists, even with "common people". The obligation for someone engaged in teaching mathematics to engineers was put quite simply again by Martens "*To seek out whatever mathematics is relevant to technology, and make it available to the engineer*". Why not to the mathematician too?

So much about the primary task ahead: education.

But there is a task for science (or university) politics too: Research in this interdisciplinary field, research, which "falls among all stools", must be academically rewarding. To develop and implement a tricky algorithm should be acknowledged as a scientific achievement completely equivalent to proving a new theorem. Do not misunderstand: Not every mathematician should work that way. But "*it seems to me that you have to do very good theoretical mathematics in order to justify a total lack of interests in the problems of society*" (Martens). We need pure mathematicians, we need them urgently - but only the very good ones. And we need more "bridge builders", people with knowledge of both sides, of mathematics and of industrial requests; we need them in industry, but also as professors at universities. Academia has to reconsider its criteria for the promotion of young mathematicians; if not, I do not think that modern society will stand in awe of such a situation for very long.

Finally there is a task for the responsible managers in industry and commerce: Not many of them have yet realised the power of mathematics. Whenever I visit a new company I experience an attitude of scepticism with respect to mathematicians. "How can these theoreticians help us in solving our practical problems?" The image of mathematics is often formed by school mathematics, and it appears therefore to be an ivory tower science. It is an advantage that we can show the benefits other companies have from mathematics; how could we earn several millions of Euro every year from industry, mainly from small and mean size enterprises, if these companies would not benefit from our work! Others do not trust in simulations at all - an experiment has to be real and not virtual. And sometimes these sceptics are right: "Mathematical modellers look to an uncertain future" headlines the last issue of SIAM NEWS, December 1998. What it means is that complex models, like models for global climate, multiphase flow or urban traffic, carry some uncertainty, which may be quite prominent. If problems are inherently stochastic and/or highly nonlinear, we cannot make the models as correct as we like just by adding more terms. The request to find methods to quantify and predict this uncertainty may even reshape our scientific methods.

Even if the uncertainty is small, each model has its assumptions; a model is only correct (in the sense of Hertz) if these assumptions are justified. Scientists have to be honest in stressing these restrictions, especially if their results become relevant in political discussions. I think that most mathematicians are!

But, finally, many many simulations are astonishingly correct. In most cases I have seen, the errors of simulation were much smaller and not larger than I had expected: Industrialists rarely ask for results with a precision better than 1%; they are often content with a 5% error, sometimes even with a qualitative understanding of a process.

And therefore, the managers should give a chance to simulation: If they really want to optimise a process or product (of course: in order to be better than the competitor), they have to develop a simulation tool timely. If the competitor has already succeeded in improving his product, it is too late to start with simulation.

Therefore we need

- ➟ open-minded mathematicians with a spirit of adventure,
- ➟ industrial managers with curiosity for and confidence in science and mathematics and the will to invest in future,
- ➟ students well trained in modelling and scientific computing with ability to formulate and solve problems in a wide variety of areas, skills in communicating and the ability to work in interdisciplinary teams.

If we are successful in achieving these goals, mathematics will not only be the key to key technologies, but one of the most important technologies itself.

Figure 23