Mode beating of spin wave beams in ferrimagnetic $Lu_{2.04}Bi_{0.96}Fe_5O_{12}$ films

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***Abstract*—We report on measurements of the two-dimensional intensity distribution of linear and non-linear spin wave excitations in a LuBiFeO film. The spin wave intensity was detected with a high-resolution Brillouin light scattering spectroscopy setup. The observed snake-like structure of the spin wave intensity distribution is understood as a mode beating between modes with different lateral spin wave intensity distri-butions. The theoretical treatment of the linear regime is performed analytically, whereas the propagation of non-linear spin waves is simulated by a numerical solution of a non-linear Schrödinger equation with suitable boundary conditions.**

***Index Terms*—Brillouin light scattering spectroscopy, micro-waves, non-linear spin-wave excitation**

I. INTRODUCTION

The excitation of spin-waves in thin films by microwaves is a fruitful field of research in which a wide range of effects of non-linear wave phenomena, from solitons to chaos can be observed [1,2]. However, in conventional microwave experiments the distribution of the spin wave intensity across the film is mostly inaccessible, and the integrated intensity of the wave beam at the position of the output-antenna and the absorbed energy at the input antenna are the experimental quantities from whose the physical properties of the system are deduced. To measure the two dimensional spin-wave distribution an experimental setup which scans along the surface of the sample and measures directly the dynamic part of the magnetisation in the film is needed. This can be done by using a scanning inductive probe [3,4], a space resolved Faraday rotation method [5] or by space resolved Brillouin light scattering spectroscopy (BLS) [6,7]. We use the latter method because of its high spatial resolution of about 50µm, its high dynamic range of currently > 50dB and the possibility to investigate volume modes in ferrimagnetic dielectrics which are not so easy accessible with an inductive probe. Studying the two dimensional spin wave distribution in a lutetium bismuth iron garnet LuBiFeO (BIG) sample we found the expected self focusing effect at high power levels [7]. In addition we also found a snake-like structure of the maximum of the spin wave intensity along the film. Its origin could not be explained so far and it will be subject of this paper.

The evolution of a narrow packet of magnetostatic spin waves along the *z*-direction in a ferrimagnetic film with the antenna directed along the *y*-direction (see Fig. 1) can be described by a two-dimensional non-linear Schrödinger equation [8,9]:

$$i\left(\frac{\partial\varphi}{\partial t}+V_g\frac{\partial\varphi}{\partial z}\right)+\frac{1}{2}D\frac{\partial^2\varphi}{\partial z^2}+S\frac{\partial^2\varphi}{\partial y^2}-N|\varphi|^2\varphi=-i\omega_r\varphi \quad (1)$$

$$V_g=\frac{\partial\omega_k}{\partial k_z},\; D=\frac{\partial^2\omega_k}{\partial k_z^2},\; S=\frac{\partial^2\omega_k}{\partial k_y^2},\; N=\frac{\partial\omega_k}{\partial|\varphi|^2}$$

φ is the dimensionless amplitude of the wave beam, V_g is the group velocity, *D* is the dispersion coefficient, *S* is the diffraction coefficient, *N* is the non-linear parameter and ω_r describes the dissipation. ω_k is the dispersion relation.

For magnetostatic backward volume waves (MSBVW) the dispersion coefficient (*D*) and the diffraction coefficient (*S*) are positive, while the non-linear parameter (*N*) is negative, so that the Lighthill criteria [10] for longitudinal modulational instabilities, $ND<0$, and for transversal modulational instabilities, $NS<0$, are both fulfilled. This leads in the case of the absence of dissipation to a wave collapse at a wave-beam amplitude threshold $|\varphi_{th}|$. With dissipation there is no such collapse, but the wave beam should be focused at some distance from the antenna.

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Manuscript submitted 15. October 1997

In addition, the quantization of the *k*-vector caused by the shape of the sample has to be taken into account. In our case, the sample is quasi-infinite in the *z*-direction, but is bounded in the *x*- and the *y*-direction. Therefore k_z is continuous, whereas k_x, k_y obey the quantization condition

$$k_x = \frac{p\pi}{d}, k_y = \frac{q\pi}{b}, \qquad (2)$$

Here *d* is the thickness of the film in the *x*-direction and *b* is the width of the film in the *y*-direction. The integers *p* and *q* are the mode numbers.

II. EXPERIMENT

In our experiments we studied the evolution of a quasi-plane-wave front initial beam in a BIG-film of the composition $Lu_{2.04}Bi_{0.96}Fe_5O_{12}$. The BIG-Film was epitaxially grown on a single crystalline (111)-oriented gallium gadolinium garnet substrate. The BIG sample was mounted on a standard delay-line-structure with input and output antennas (width r=35 µm, length W=2.5 mm) separated by a distance of 8mm and connected to a network analyzer (Fig. 1). The parameters of the sample are: saturation magnetization $4\pi M_s$=1750 Oe, film thickness d=1.5 µm, ferromagnetic resonance linewidth ΔH=0.9 Oe, sample width b=2 mm and sample length L=10 mm. The applied field had a strength of H=2130 Oe and the microwave-generator worked at a frequency of ν=8,10 GHz, resulting in a carrier wave number of $k_z \cong 370$ cm^{-1}.

To analyze the spin wave intensity we used a high contrast, computer controlled Sandercock type (3+3)-pass tandem Fabry-Perot interferometer and an Ar$^+$-ion laser (wavelength λ=514,5 nm).The measurements were performed in forward scattering geometry with a spatial resolution of 50 µm. In this geometry it was possible to measure *k*-vectors in the range 0-10^4 cm^{-1}.

Each of the reported measurements contains 14×14 points with a spacing of 0.1 mm parallel to the antenna and 0.5 mm perpendicular to the antenna. The data acquisition time was about 45 s per point so that each complete scan took 2.5 hours. To avoid drifts special care was taken to keep the sample at a constant temperature and the microwave input signal was kept below 100 mW by a modulation of the input signal at a frequency of about 4 kHz with variable duty cycles to avoid heating of the sample by the microwaves.

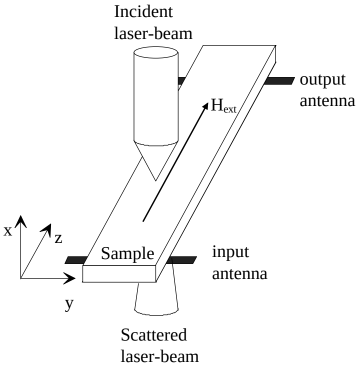

Fig. 1. Experimental setup. The antennas are connected to a network analyser. The sample is scanned in the *y*- and *z*-direction.

The results of the measurements are shown in Fig. 2 as two-dimensional-plots of the light scattering intensity which is proportional to the spin wave intensity $|\varphi|^2$. Bright/dark areas indicate high/low intensities. The raw data shows an exponential decay with increasing distance from the input antenna due to the attenuation in the film, which was corrected in the representation in Fig. 2 to display in a clearer way the lateral distribution of the wave beam (for details see [7]). The four panels show the value of $|\varphi|^2 \bullet \exp(2\omega_r/V_g z)$ for a microwave input power of 30 mW, 100 mW, 800 mW and 1000 mW, respectively. The numbers above the panels denote the absolute values of the light scattering intensity (in counts/ms), measured near the input antenna. A comparison of the absolute values of the BLS-intensities shows that they are not proportional to the input power, even near the input antenna. A reason for that is not clear at present. At the lowest power level a snake-like structure of high spin wave intensity can clearly be identified. At higher input power the snake-like structure starts to break up into several islands of high intensity which still lie on the path of the wave beam at low power. In addition, the self-focusing-effect can be seen at the input power level of 800 mW at a distance from the antenna of about 1.5 mm.

III. DISCUSSION

As discussed above several modes characterized by the mode numbers *p* and *q* (cf. (2)) can propagate in thin films. From the modes quantized along the *x*-direction (perpendicular to the film) only the lowest order mode with p=0 is excited. In contrary several modes quantized along the *y*-direction (called lateral width modes hereafter) can be excited. The intrinsic assymmetry of the input antenna (one side is grounded), allows to excite lateral width modes both of even

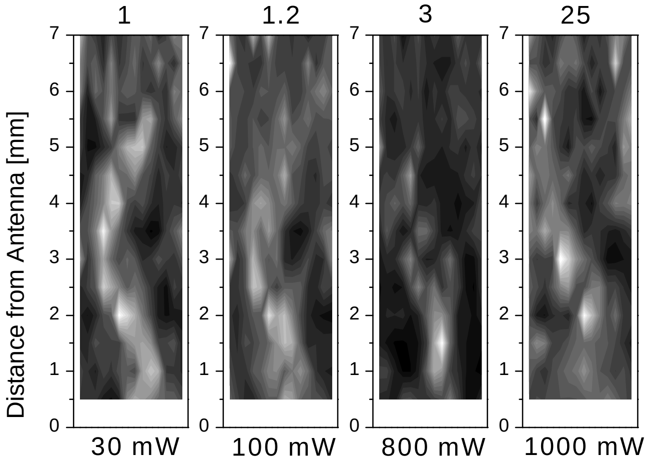

Fig. 2. Measured MSBVW intensity distribution in a ferrimagnetic BIG-film at the microwave input powers as indicated at the bottom of the figure. The origin of the *z*-axis denotes the position of the input antenna. The frame indicates the width of the sample of 2 mm. Bright/dark intensities indicate high/low spin-wave intensities. The numbers above the panels indicate the value of the maximum spin wave count rate for each measurement, which is a measure of the spin wave in-tensity. The snake-like structure of the wave beam along the film can

be seen at all input power levels. At a power of 800 mW the self-focusing of the wave beam appears at 1.5 mm distance from the input antenna.

and odd mode numbers *q*. From the panels shown in Fig. 2 it is evident that the lateral width modes are pinned since the spin wave intensity goes to zero very close to the film boundaries. This pinning effect is essential in this case due to the smallness of the wavevector component in the *y*-direction, which is $q\pi/b$.

We have simulated the spin wave intensity distribution assuming that we excite lateral width modes with $q=1$ and $q=2$. We analytically consider the stationary ($d\varphi/dt=0$) propagation of the spin wave with an initial distribution of the spin-wave-intensity of the form

$$\varphi\,(y,z=0)=A\left\{\sin\frac{\pi y}{b}+\frac{1}{3}\sin\frac{2\pi y}{b}\right\} \qquad (3)$$

for small values of $|\varphi|$, corresponding to the linear regime. The non-linear regime was simulated by the numerically calculated solution of (1). The theoretical analysis yields the following results:

First, even in the linear regime one observes the snake-like structure. It is caused by an interference of the two components in (3). Since these components have the same frequency, but different k_y, they also have a different k_z, resulting in a mode beating and causing the snake-like structure. Second, the self-focusing which appears at a value of the input power of 800 mW at a distance of 1.5 mm from the input-antenna can be clearly seen at a slightly different distance of 1mm from the antenna. This fact has already been shown in the calculations reported in [7]. The self-focusing point is closer to the input antenna because of the higher input power in the simulation. The maximum of the spin-wave intensity follows a snake-like structure which shows a splitting into two branches beyond the self-focusing-point which is also corroborated by the measurements. Furthermore the splitting into several islands at high power levels observed for the input powers of 800 mW (obtained count rate is 3 counts/ms) and of 1000 mW (25 counts/ms) is also evident for the calculations.

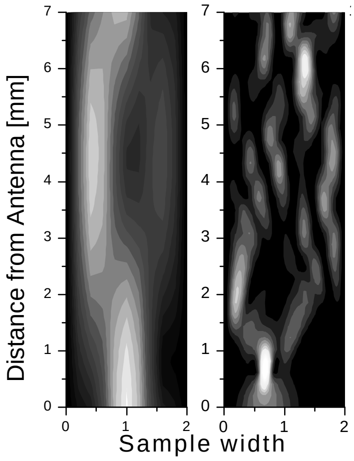

Fig. 3. The left panel shows the results of the analytical theory for small spin wave intensities corresponding to the linear regime. The right panel shows the result of our numerical calculations for $|\varphi_0/\varphi_{th}|=5$. The origin of the *z*-axis is the position of the input antenna.

IV CONCLUSIONS

We show, that the snake-like structure of the maximum of the spin wave intensity which appears in the propagation of a spin-wave beam in a dielectric, ferrimagnetic film can be understood by the superposition of two spin-wave modes of different lateral mode order propagating with different k_y and respectively with different k_z. The intensity distribution in the film at high power levels was numerically simulated by a solution of the non-linear Schrödinger equation which includes two propagating modes and adequate boundary conditions.

ACKNOWLEDGMENTS

We would like to thank C.E. Patton, P. Kabos and B. Kalinikos for fruitful discussions. Support by the Deutsche Forschungsgemeinschaft is greatfully acknowledged. P.A.K. would like to thank the Alexander von Humboldt Foundation for a fellowship. A.N.S. is grateful to the Oakland University Foundation and the Research Corporation for partial financial support.

REFERENCES

[1] ‘Non-linear Phenomena and Chaos in magnetic materials’, P.E. Wigen (ed.), World Scientific, Singapore, 1994.

[2] ‘Linear and Non-linear spin-waves in magnetic films and superlattices’, M.G. Cottam (ed.), World Scientific, Singapore, 1994.

[3] N.P. Vlannes, J. Appl. Phys. **61**, 416 (1987).

[4] A.V. Vashkovskii, A.V. Stal’makhov and D.G. Shakhnazaryan, Izverstiya Vysshikh Uchebnykh Zavedenii, Fisika **11**, 67 (1988).

[5] J.P. Jamet and H. Le Gall, Phys. Stat. Sol. A **31**, 547 (1975).

[6] J.W. Boyle, S.A. Nikitov, A.D. Boardman, J.G. Booth and K. Booth, Phys. Rev. B **53**, 12173 (1996).

[7] M. Bauer, C. Mathieu, S.O. Demokritov, B. Hillebrands, P.A. Kolodin, S. Sure, H. Dötsch, V. Grimalsky, Yu. Rapoport and A.N. Slavin, Phys. Rev. B **56**, R8483 (1997).

[8] V.P. Lukomskii, Sov. Phys.-JETP **57**, 297 (1983).

[9] A.K. Zvezdin and A.F. Popkov, Sov. Phys.-JETP **57**, 350 (1983).

[10] M.J. Lighthill, J. Inst. Math. Appl. **1**,269 (1965).