Bloch electron in presence of dc and ac fields

M. Glück, A. R. Kolovsky[1] and H. J. Korsch

Fachbereich Physik, Universität Kaiserslautern, D-67653 Kaiserslautern, Germany

and [1]L. V. Kirensky Institute of Physics, 660036 Krasnoyarsk, Russia

fax: +631 2053907; e-mail: kolovsky@physik.uni-kl.de

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Abstract

The paper studies metastable states of a Bloch electron in the presence of external ac and dc fields. Provided resonance condition between period of the driving frequency and the Bloch period, the complex quasienergies are numerically calculated for two qualitatively different regimes (quasiregular and chaotic) of the system dynamics. For the chaotic regime an effect of quantum stabilization, which suppresses the classical decay mechanism, is found. This effect is demonstrated to be a kind of quantum interference phenomenon sensitive to the resonance condition.

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In this letter we study quantum states of a Bloch particle in the presence of external dc and ac fields:

$$H = p^2/2 + \cos x + Fx + F_\omega x \sin(\omega t) \ . \tag{1}$$

The system (1) has been discussed in context of solid state physics for at least 50 years but is still of some recent interest [1]. The main bulk of results so far known about it concerns the case of static field $F_\omega = 0$ (see reviews [2,3], for example) or pure oscillatory field $F = 0$ [4–6]. In the general case, $F \neq 0$, $F_\omega \neq 0$, our knowledge about the system is rather poor and mainly based on studies of the tight-binding model [7–10]. However, the use of the tight-binding model, which amounts to using a one-band approximation, can leave outside a number of phenomena. For example, neglecting interband transitions in the case of a pure dc field transforms the Wannier-Stark resonances (metastable states [11]) into stationary, exponentially localized eigenfunctions. In the pure oscillatory case, using a one-band approximation reduces the phase space of the generally chaotic system (1) to a cylinder and, thus, eliminates the phenomenon of chaotic diffusion of the momentum. In this present paper we study the system states *beyond* the one-band approximation. From the preliminary remarks above these states are expected to be a kind of “chaotic” metastable states [12].

We begin with the classical analysis. It is convenient to include the ac term in the Hamiltonian (1) in the periodic potential, which is done by the usual gauge transformation $p \to p + (F_\omega/\omega)\cos(\omega t)$, $x \to x + (F_\omega/\omega^2)\sin(\omega t)$. Then the Hamiltonian (1) takes the form

$$H = \frac{p^2}{2} + \cos[x - \epsilon \sin(\omega t)] + Fx \ , \quad \epsilon = \frac{F_\omega}{\omega^2} \ . \tag{2}$$

It is also useful to expand the “new” time-dependent potential in the Fourier series

$$\cos[x - \epsilon\sin(\omega t)] = J_0(\epsilon)\cos x + \sum_{m=1}^{\infty} J_m(\epsilon)[\cos(x - m\omega t) + (-1)^m \cos(x + m\omega t)] \ . \tag{3}$$

It follows from Eqs. (2–3), that for $F = 0$ the system (1) is a system of many interacting nonlinear resonances and, therefore, its dynamics can be either quasiregular or chaotic depending on a particular choice of the parameters ω and ϵ [13]. Here we restrict ourselves to the choice (a) $\omega = 10/6$, $\epsilon = 0.1$, where the system dynamics is almost regular, and (b) $\omega = 10/6$, $\epsilon = 1.5$, where a developed chaos exists [14]. Figure 1 shows phase portraits of the system for these two cases. For small ϵ only three terms in Eq. (3), $J_0(\epsilon)\cos x$ and $J_{\pm 1}\cos(x \pm \omega t)$, are important [15], – three nonlinear resonances originated by these terms are well seen in Fig. 1(a). For large ϵ many of such resonances overlap and chaotic diffusion is possible. We note, however, that the classical motion remains always bounded in momentum space for $F = 0$.

Adding a weak dc field cardinally changes the system dynamics. The static field destroys the invariant curves separating the chaotic component from the outer region of regular motion. Thus, the chaotic component can not support a bounded (in momentum space) motion for an infinite time. However, it can support a bounded motion temporally. The dotted line in Fig. 2 shows the probability $P_{cl}(t)$ of a classical particle to stay within the chaotic region for the parameters of Fig. 1(b) and $F \approx 0.13$. (The classical probability was calculated by simulating the dynamics of $N = 40000$ particles with initial conditions around $x = 0$, $p = 0$. Then the function $P_{cl}(t)$ denotes the relative number of the particles with momentum $|p| < 6$.) It is seen that the probability decays exponentially with a classical decay time $\tau_{cl} \approx 10T_\omega$. This transient chaotic (actually diffusive) dynamics of the system is in contrast with the accelerated motion observed in the outer region of negative momentum (we fix $F > 0$, so that a particle is accelerated towards minus infinity). It should also be noted that the homogeneous static field not necessarily destroys the invariant curves separating the chaotic component from embedded regions of regular motion. Thus, large stability islands [like those in Fig. 1(a)] typically survive after applying a moderate dc field.

The quantum analysis of the system (1–2) is based on the notion of quasienergy states $\psi_{l,k}(x)$, which by definition are the eigenstates of the system evolution operator $\widehat{U}$ over the period T_ω of the driving frequency ω:

$$\widehat{U}\psi_{l,k}(x) = \exp[-iE_l(k)T_\omega/\hbar]\psi_{l,k}(x)\ , \tag{4}$$

$$\widehat{U} = \widehat{\exp}\left(-\frac{i}{\hbar}\int_0^{T_\omega}\widehat{H}(t)\mathrm{d}t\right)\ , \quad T_\omega = \frac{2\pi}{\omega}\ . \tag{5}$$

In the pure oscillatory case the operator (5) obviously commutes with the translational operator over the lattice period, $\hat{a} = \exp(-2\pi\partial/\partial x)$, and, therefore, the quasimomentum k is a good quantum number.

Figure 3 shows the quasienergy spectrum $\lambda = E_l(k)T_\omega/\hbar$ of the system for the parameters of Fig. 1 and (scaled) Planck constant $\hbar = 0.5$. For the almost regular case $\epsilon = 0.1$ in Fig. 3(a), the spectrum is dominated by the three classical nonlinear resonances seen in Fig. 1(a). The horizontal bands are associated with the central resonance, while the bands with the slope 2π (forming a rhombus) originate from the primary resonances with $m = \pm 1$ [see Eq. (3)]. The presence of the chaotic component separating the primary resonances reflects the system local nonintegrability and manifests itself in clear avoided crossings between "primary resonance bands". The bands looking like straight lines correspond to classically ballistic (almost free) motion. For the chaotic case, $\epsilon = 1.5$, the quasienergy spectrum is typical for a globally nonintegrable quantum system and contains many avoided crossings. A statistical analysis of the spectrum is an appropriate approach in this case [16,17]. We also note that by closer inspection of the spectrum depicted in Fig. 3(b) one can also find a sign of the resonances with $m = \pm 1$ and $m = \pm 2$, although these primary resonances are hard to detect in Fig. 1(b).

We proceed with the case $F \neq 0$. For $F \neq 0$ evolution operator (5) generally does not commute with the operator of lattice translation. An exception is the case, where the period of the driving frequency T_ω is commensurable with the so-called Bloch period $T_B = \hbar/F$, i.e., $rT_\omega = qT_B$, r, q are integer [18]. In this paper we mainly consider the case $r = 1$, $q = 1$, where the explicit form of the evolution operator over the common period $T = T_\omega = T_B$ is [12,19]

$$\widehat{U} = e^{-ix}\widehat{\exp}\left(-\frac{i}{\hbar}\int_0^T \widetilde{H}(t)\mathrm{d}t\right) , \tag{6}$$

with

$$\widetilde{H}(t) = (\hat{p} - Ft)^2/2 + \cos[x - \epsilon\sin(\omega t)] . \tag{7}$$

Because we are interested in the metastable states, the operator (6) should be diagonalized with the resonance boundary condition, which corresponds to a vanishing probability to find a particle with a positive momentum:

$$\lim_{p\to\infty} |\psi_{l,k}(p)|^2 \to 0 \ . \tag{8}$$

In a numerical calculation the condition (8) can be automatically satisfied by truncating the operator $\widehat{U}$ in momentum space [19,20].

In the quasiregular case (a) spectrum of the system metastable states is shown in Fig. 4 [21]. The parameters are the same as in Fig. 3(a) and $F \approx 0.13$ is chosen to satisfy the condition $T_\omega = T_B$. The circular diagram on the left depicts the complex quasienergy spectrum in polar coordinates and the plot on the right presents the k-dependence of the real part of the quasienergy. It is seen that the complex quasienergies are arranged in bands which resembles the nonlinear resonance band structure in Fig. 3(a). Thus, we can conclude that the most stable states are those associated with primary nonlinear resonances. This conclusion is in agreement with the classical picture where, as mentioned above, the islands of regular motion do survive after applying a static field. We would also like to stress that the crossing of the bands seen in Fig. 4 (and later on in Fig. 5) is an artificial fact due to the 2-dimensional presentation of 3-dimensional space spanned by the quasienergies (see Fig. 5).

Figure 6 shows the spectrum of the metastable states in the chaotic case (b). It is seen that the k-dependence of both real and imaginary parts of the quasienergies $E_l(k)$ is quite irregular and a statistical approach seems to be an appropriate one to analyze the spectrum. Such a statistical analysis will be the subject of a forthcoming study. Here we only present the distribution of the imaginary parts of the quasienergies, more precisely, the distribution $f(\tau)$ of decay times of the quantum metastable states

$$\tau_{qu} = \hbar/2\mathrm{Im}[E_l(k)] \tag{9}$$

in Fig. 7. A remarkable feature of the distribution shown is the existence of states with extremely large decay time τ_{qu} in comparison to the classical decay time $\tau_{cl} \approx 10T$. (The numerical data suggests asymptotic $f(\tau) \sim \tau^{-2}$ for $\tau \gg T$.) Thus the quantum system is essentially more stable against a static field than the classical one. This statement is confirmed by direct numerical simulation of wave packet dynamics. The solid line in Fig. 2 shows the probability of a quantum wave packet to "stay within the chaotic region"

$$P_{qu}(t) = \int_{|p|<6} |\psi(p,t)|^2 \mathrm{d}p \ .$$

Here the initial wave function was chosen as a minimal uncertainty Gaussian packet centered at $x = 0$, $p = 0$. A clear deviation from the classical behavior (dotted line) is noticed. For the moment we have no explanation for this quantum stabilization phenomenon. However, it can be stated for sure that this quantum stabilization is a consequence of the resonance condition between the period of the driving frequency and the Bloch period. The dashed line in Fig. 1 shows the function $P_{qu}(t)$ for the *same* values of ω, ϵ and F, however for $\hbar = 0.5109531$ instead of $\hbar = 0.5$. It is seen that in this case the quantum behavior seems to follow the classical one. We also would like to stress that the quantum stabilization discussed is not a particular feature of the system (1). The same phenomenon was noticed earlier for a different chaotic system with the Hamiltonian $H = p^2/2 + \cos(\omega t)\cos x + Fx$ [12].

In the conclusion we compare our approach with that based on the tight-binding model in some more detail. For the problem considered the tight-binding Hamiltonian has the form

$$\hat{H} = -\frac{\Delta}{4}\sum_l (|l+1\rangle\langle l| + |l\rangle\langle l+1|) + [F + F_\omega \sin(\omega t)] \sum_l 2\pi l |l\rangle\langle l| \ , \qquad (10)$$

where l labels the lattice cites and Δ is the width of a single Bloch band. The advantage of the model (10) is that (provided the resonance condition is satisfied) it can be solved analytically [7–10]. However, if we wish to relate the problem (10) to the general system (1), a condition justifying that interband transitions may be neglected should be met. It is easy to see that this is *not* the case for the system parameters presently used. In fact, as it is already seen in Fig. 3, the system is in a regime of strong band coupling induced by the periodic field (every avoided crossing can be identified with a particular multiphoton transition). Thus, the regimes of the system dynamics discussed in this paper can *not* be studied by using a tight-binding model in principle.

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FIGURES

FIG. 1. Phase portrait of the system (2) for $F = 0$. Coordinate $x(t_n)$ and momentum $p(t_n)$ of a classical particle for times t_n being multiple of the driving frequency period is plotted for 50 different initial conditions. The driving frequency $\omega = 10/6$, scaled driving amplitude $\epsilon = 0.1$ (a), and $\epsilon = 1.5$ (b).

FIG. 2. Comparison between classical and quantum decay process for $F = \omega/4\pi \approx 0.13$ in the chaotic case (b). The dotted line shows the classical decay, solid line is the quantum decay for the resonance condition $T_\omega = T_B$ ($\hbar = 1/2$), and dashed line is slightly off resonance case ($\hbar = 0.5109531$).

FIG. 3. Quasienergy spectrum $\lambda = E_l(k)T_\omega/\hbar$ of the system (2) for $F = 0$. The parameters are the same as in Fig. 1 and $\hbar = 0.5$. Only the states with mean kinetic less than $25\hbar^2$ (a) and less than $35\hbar^2$ (b) are shown.

FIG. 4. Complex quasienergies $E_l(k)$ in the quasiregular case (a). The circular plot on the left shows the phase $\lambda = \mathrm{Re}[E_l(k)]T/\hbar$ against the decay coefficient $\Gamma = \exp(-\mathrm{Im}[E_l(k)]T/\hbar)$. The plot on the right presents k-dependence of the phase λ, only the states with $\Gamma > \exp(-2)$ are shown.

FIG. 5. The same as in Fig. 4 but using 3D presentation for the complex quasienergies.

FIG. 6. Complex quasienergies in the chaotic case (b). The phases in the right hand plot are shown only for the states with $\Gamma > \exp(-1)$.

FIG. 7. Integrated distribution function $I(\tau_{qu}) = \int_T^\tau f(\tau')\mathrm{d}\tau'$ of quantum decay times $\tau_{qu} = \hbar/2\mathrm{Im}[E_l(k)]$ in the chaotic case (b). Note that τ-axis is in logarithmic scale. Insert shows the integrated distribution for $\gamma = T/\tau$.