On the Mass Difference of Neutrinos

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Abstract

We calculate a relative neutrino mass difference of $\frac{\Delta m}{m} = 1.6 \times 10^{-9}$ at the one loop level in a two flavor model. If we combine our result with recently published possible solutions to the solar neutrino problem we can estimate a neutrino mass range of $m = (0.12\text{–}0.19)\,eV$.

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1 Introduction

There is no compelling theoretical reason, why neutrinos should have masses equal to zero. On the other hand the empirical evidence supports the assumption, that these masses are small – if not equal to zero outright.

However the deficit of the solar neutrino flux (the so called solar neutrino puzzle) meassured indepently by four experiments (Chlorine [1], Kaminokande [2], GALLEX [3], SAGE [4]) is most easily interpreted assuming neutrino mixing (a 'la Pontecorvo–Bilenkii [5]), which requires nonzero mass for at least one neutrino flavor. The gauge group structure $SU(2)_L \times U(1)$ of the Standard Model leads to no evidence for the neutrino to be massless or not. Several ways can be choosen to give mass to them. Adding right–handed neutrinos are the means in a lot of models (for details see for example [6]). The next step may be the observation, that the massterm has no reason to be flavor diagonal. So it is possible to find transitions between different neutrino types by only assuming standard couplings of the neutrino to the W^+ and S^+ boson (in the t' Hooft gauge). There is no reason to postulate a new coupling to the gauge bosons at all to find such a transition. We want to stress this particular point in this contribution.

The simplest possibility is to consider a two flavor model. The mass eigenstates ν_1, ν_2 are connected to the flavor eigenstates by the relations

$$\mid \nu_1 \rangle = \cos\vartheta \mid \nu_e \rangle - \sin\vartheta \mid \nu_\mu \rangle \quad (1)$$

$$\mid \nu_2 \rangle = \sin\vartheta \mid \nu_e \rangle + \cos\vartheta \mid \nu_\mu \rangle \quad (2)$$

and masses m_1 and m_2 are generated through (infinite) selfenergy diagramms ("radiative corrections"). In the model transitions between the different neutrino types could occur at the one loop–level too. This transition at the one loop–level gives rise to mass spliting of the neutrinos similar to the case of neutral Kaons. The mass difference Δm and the invariant amplitude of the process are connected through the formula [5] [6]:

$$\triangle m = 2\sqrt{\frac{M_{12}^2}{4\sin^2\vartheta\cos^2\vartheta}} \quad (3)$$

2 Calculation of the mass splitting

The only contribution to the matrix element M_{12} at the one loop-level in Feynman–t‘ Hooft gauge is shown in fig.1 and fig.2. The invariant amplitude for the loop with the W^+–boson and an electron is:

$$
\begin{aligned}
\mathcal{M}_e^W &= \Big(\frac{i\,g}{2\,\sqrt{2}}\Big)^2 \sin\vartheta\cos\vartheta \\
&\int \frac{d^\omega l}{(2\pi)^\omega}\mathbf{D}^W_{\beta\alpha}\overline{u}_2(q)\gamma^\alpha\,(1-\gamma_5)\not{\mathbf{D}}^e\gamma^\beta(1-\gamma_5)u_1(q) \qquad (4)\\
&= \Big(\frac{i\,g}{2\,\sqrt{2}}\Big)^2 \sin\vartheta\cos\vartheta \int \frac{d^\omega l}{(2\pi)^\omega}\frac{-i\,g_{\beta\alpha}}{l^2-M_W^2} \\
&\overline{u}_2(q)\gamma^\alpha\,(1-\gamma_5)\frac{i}{(\not{q}-\not{l})-m_e}\gamma^\beta(1-\gamma_5)u_1(q) \qquad (5)\\
&= -\frac{g^2(2-\omega)}{4}\sin\vartheta\cos\vartheta\int\frac{d^\omega l}{(2\pi)^\omega}\frac{q^\rho-l^\rho}{\big(l^2-M_W^2\big)\big((q-l)^2-m_e^2\big)} \\
&\qquad \overline{u}_2(q)\,\gamma_\rho\,(1-\gamma_5)u_1(q) \qquad (6)
\end{aligned}
$$

where $\mathbf{D}^W$ and $\not{\mathbf{D}}^e$ are the W–boson and the electron propagators, and we use the dimensional regularization (in ω dimensions) to evaluate the Feynman integrals. The formulas used for the dimensional regularization are given in the appendix. Using the formula (18) one finds that:

$$
\begin{aligned}
\mathcal{M}_e^W &= \frac{g^2}{4}\sin\vartheta\cos\vartheta(2-\omega)\frac{i}{(4\pi)^2} \\
&\Big\{\ \int_0^1 d\alpha(1-\alpha)\Big(\frac{2}{\omega-4}-\mathbf{C}+\ln(4\pi)-\ln M_e^2(\alpha)\Big)+\mathcal{O}(4-\omega)\Big\} \qquad (7)\\
&\overline{u}_2(q)\,\not{q}\,(1-\gamma_5)u_1(q)
\end{aligned}
$$

where

$$
M_e^2(\alpha) = (1-\alpha)M_W^2+\alpha m_e^2-(\alpha-\alpha^2)q^2\,;\ q^2=m^2 \quad \text{(neutrino mass)} \qquad (8)
$$

The calculation for the W^+ boson muon–loop is quite similar. One only has to take care of the opposite sign of the contributions. In fact, due to this difference the divergent and constant terms cancel completly. If we introduce a quantity M_μ^2 similar to M_e^2 in the equation (8) we find:

$$\begin{aligned} M^W &= M_\mu^W + M_e^W = -\frac{g^2}{4(4\pi)^2}(2-\omega)\sin\vartheta\cos\vartheta \\ &\cdot \int_0^1 d\alpha(1-\alpha) \quad \ln\Big(\frac{M_e^2(\alpha)}{M_\mu^2(\alpha)}\Big)\ \overline{u}_2(q)\ \not{q}(1-\gamma_5)u_1(q) \end{aligned} \tag{9}$$

The contribution of the S^+ boson to the matrix element are negligible. This is a consequence of the supression of the amplitude by the factor $\mathcal{O}(\frac{m_l^2}{M_w^2})$ at the vertices (m_l^2 denotes the charged lepton mass) in the part of the S^+ amplitude that is proportional to the W^+ contributions; other contribution to the amplitude is futher suppressed by products of nearly chiral projectors with opposite chiralities. Here we mean with nearly chiral operators operators of the type $(1-\varepsilon\gamma_5)(1+\varepsilon\gamma_5)$ where ε is a small number related to the ratio of the neutrino to charged lepton masses.

Averaging over spin states in (9) one finds for the mass difference the expression:

$$\Delta m = 2\sqrt{\frac{|\overline{\mathcal{M}}|^2}{4\sin^2\vartheta\cos^2\vartheta}}$$

$$= \frac{g^2\,(\omega-2)}{4(4\pi)^2}\sqrt{\frac{f(\omega)}{2}\,m^2}\int_0^1 d\alpha(1-\alpha)\quad \ln\left(\frac{M_\mu^2(\alpha)}{M_e^2(\alpha)}\right) \qquad (10)$$

$$\overset{\omega\to 4}{=} \frac{1}{\sqrt{2}(4\pi)^2}\,g^2\,m\int_0^1 d\alpha(1-\alpha)\quad \ln\left(\frac{M_\mu^2(\alpha)}{M_e^2(\alpha)}\right) \qquad (11)$$

For the relative mass difference $\frac{\Delta m}{m}$ we thus obtain a fully determined expression. The mixing angle canceled completly:

$$\frac{\Delta m}{m} = \frac{1}{\sqrt{2}(4\pi)^2}\,g^2\int_0^1 d\alpha(1-\alpha)\quad \ln\left(\frac{M_\mu^2(\alpha)}{M_e^2(\alpha)}\right) \simeq 1.6\times 10^{-9} \qquad (12)$$

The integral itself is evaluated numerically and gives 9×10^{-7}. The same result can be obtained itself by expanding the integrand in a Taylor series and regarding only terms of first order in $\frac{m_\mu^2-m_e^2}{M_W^2}$:

$$\int_0^1 d\alpha(1-\alpha)\quad \ln\left(\frac{M_\mu^2(\alpha)}{M_e^2(\alpha)}\right) \simeq \int_0^1 d\alpha\frac{m_\mu^2-m_e^2}{M_W^2}\alpha \qquad (13)$$

3 Determination of the neutrino mass

The relative mass spliting (12) alone doesn't allow to estimate a value for the neutrino mass m itself. However if we combine our results with the recently published analysis of the solar neutrino experiments by N. Hata [7], and by P. I. Krastev and S. T. Petcov [8] which leads to a value for the difference of squares of the neutrino masses $m_1^2 - m_2^2$ it is possible then to make an estimate for the neutrino mass by using simple arithmetics. One has to be careful because the terms Δm and Δm^2 are sloppyly used in the literature. To point this difference we use the symbol $\Delta(m^2)$ instead of the symbol Δm^2 normally used in literature.

We restricted ourself to solutions with large ϑ and so only the results of the long wavelength oscillations are of interest. It is $\Delta(m^2) = m_1^2 - m_2^2 = (m_1 + m_2)(m_1 - m_2) \simeq 2m\Delta m$. If we take the value $\Delta(m_{exp}^2)$ in the allowed range of (0.45–1.2)$10^{-10} eV$ calculated by N. Hata [7] we arrive at a neutrino mass prediction:

$$m \quad = \quad \sqrt{\frac{\Delta(m_{exp}^2)}{2\;\Delta m/m}} \quad = \quad (0.12\text{–}0.19) \quad eV \tag{14}$$

In the paper published by P. I. Krastev and S. T. Petcov [8] they found a possible range for vakuum oscillations quite similar to the range that was found by N. Hata.

4 Conclusion

By connecting our calculations with experimental data it is possible to predict a mass range for the neutrino. We are aware of the problem that the data of the solar neutrino experiments shows no strong evidence that long wavelength oscillations are the preferable solutions to the solar neutrino problem. Future experiments and improved statistics can help to clarify this problem.

Our calculation was done with a minimal extension of the standard model in a two flavor system by only assuming the existence of right handed neutrinos. So there is no need to fix more parameters than the neutrino mass. Every other modell that takes care of new exotic interactions would need further data (not only exacter data) to fix the parameter. Our modell shows that if all other interactions are neglegible at the requiered energy range we can find a prediction for the neutrino mass in the available data. The mass range in (14) is too small to solve the astrophysical problem of the missing dark mass.

A Appendix

For calculations in dimension ω we use the convention that γ_5 anticommute with all other γ matrices.

$$\{\gamma_5, \gamma_\mu\} = 0 \tag{15}$$

The Dirac states are normalized to

$$\sum_{s=\pm 1/2} u_s(p)\overline{u}_s(p) = \frac{\not{p} + m}{2m} \tag{16}$$

and traces are calculated using equations like

$$\mathbf{Tr}(\gamma_\mu \gamma_\nu) \quad = \quad f(\omega) g_{\mu\nu} \qquad \text{where} \qquad f(\omega) \stackrel{\omega \to 4}{\longrightarrow} 4 \tag{17}$$

The following formulas are used to evaluate the Integrals in (6). At first we use a Feynman parametrization.

$$I_e^\rho \quad = \quad \int \frac{d^\omega l}{(2\pi)^\omega} \qquad \frac{q^\rho - l^\rho}{\Big(l^2 - M_W^2\Big)\Big((q-l)^2 - m_e^2\Big)} \tag{18}$$

$$= \quad \int \frac{d^\omega l}{(2\pi)^\omega} \int_0^1 d\alpha \frac{q^\rho - l^\rho}{\Big[\alpha\Big((q-l)^2 - m_e^2\Big) + (1-\alpha)\Big(l^2 - M_W^2\Big)\Big]^2} \tag{19}$$

By using standard techniques as described by Leibbrandt [9] one finds for the Integral:

$$I_e^\rho \quad = \quad \frac{i}{(4\pi)^2}\, q^\rho \int_0^1 d\alpha (1-\alpha)\Big(\frac{2}{4-\omega} - \mathbf{C} + \ln(4\pi) - \ln M_e^2(\alpha)\Big) \tag{20}$$

$$+\mathcal{O}(4-\omega)$$

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Figure captions

W^+

ν_1 e^-/μ^- ν_2

Figure 1: Loop for a W^+ boson and electron respectively muon. The coupling at the vertices can be read of formula (1) and (2).

S^+

ν_1 e^-/μ^- ν_2

Figure 2: Loop for a S^+ boson and electron respectively muon. The coupling at the vertices can be read of formula (1) and (2).

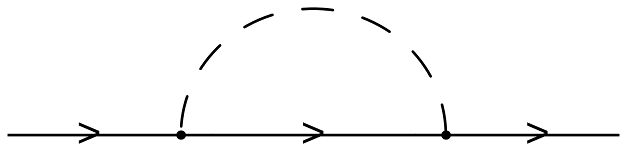

Figure 1

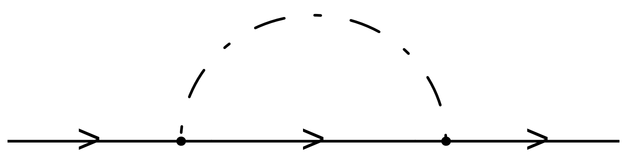

Figure 2