A new look at the RST model

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Abstract

The RST model is augmented by the addition of a scalar field and a boundary term so that it is well-posed and local. Expressing the RST action in terms of the ADM formulation, the constraint structure can be analysed completely. It is shown that from the view point of local field theories, there exists a hidden dynamical field ψ_1 in the RST model. Thanks to the presence of this hidden dynamical field, we can reconstruct the closed algebra of the constraints which guarantee the general invariance of the RST action. The resulting stress tensors $T_{\pm\pm}$ are recovered to be true tensor quantities. Especially, the part of the stress tensors for the hidden dynamical field ψ_1 gives the precise expression for $t_{\pm}$. At the quantum level, the cancellation condition for the total central charge is reexamined. Finally, with the help of the hidden dynamical field ψ_1, the fact that the semi-classical static solution of the RST model has two independent parameters (P,M), whereas for the classical CGHS model there is only one, can be explained.

With the advent of the model proposed by Callan, Giddings, Harvey and Strominger (CGHS) [1], dilaton gravity in two dimensions has been widely recognized as an excellent arena in which a variety of fundamental issues in quantum gravity can be discussed, especially those concerning quantum properties of a black hole. Indeed now a large body of literature on the CGHS model and its variants is available, and the notable model in the study of the black hole evaporation problem is the Russo-Susskind-Thorlacious (RST) model which admits physically sensible evaporating black hole solutions [2]. The RST model has been considered a theoretical laboratory for the study of Hawking radiation [2, 3], black hole entropy [4, 5, 6], critical phenomena [7, 8] and so on.

However, until now, there are some problems which are still unclear in the RST model. For example, the RST action is manifestly invariant under the diffeomorphism transformation, so the constraints should form the closed algebra at the classical level, i.e., ought to be first-class, which guarantees the general covariance or invariance of the theory. Nevertheless, as is well known, the stress tensors $T_{\pm\pm}$ in the earlier semiclassical approach do not transform as tensors but rather as projective connections, which means that under a conformal change of coordinates $T_{\pm\pm}$ pick up an extra term equal to $-\kappa/2$ times the schwarzian derivative of the transition function at the classical level [9]. As a result, the Poisson brackets of the constraints have the classical central extension, that is, the closed algebra of the constraints is destroyed [10], which explicitly contradicts the fact that the RST action is invariant with respect to diffeomorphism transformations.

Usually, arbitrary functions (more precisely, projective connections) $t_{\pm}$ are added to $T_{\pm\pm}^{\rho,\phi}$ by hand, and under a conformal change of coordinates $t_{\pm}$ are assumed to pick up $\kappa/2$ times the schwarzian derivative, so that $T_{\pm\pm} + t_{\pm}$ are true tensor quantities. Since $t_{\pm}$ are introduced by hand, their precise meanings are implicit, so $t_{\pm}$ have various physical explanations. For instance, in [11] $t_{\pm}$ are explained as the stress tensors for the ghost sector, and their central charges are equal to 26, whereas in [3, 9] $t_{\pm}$ are considered as the result of the nonlocality of the Polyakov term, and the corresponding central charges are 12κ. So, until now, it is not clear how these conflicts could be reconciled in a consistent way.

In the present paper, the RST model is first dicussed from the viewpoint of the Dirac quantization method so as to solve the above mentioned problems. Since there is a nonlocal term in the RST model, the RST Lagrangian must first be localized so that Dirac quantization can be performed. For this purpose, the scalar field χ and the boundary term are introduced in order that the reformulated RST model is well-posed and local [6, 15]. Expressing the RST action in terms of the ADM formulation [12, 13, 14], the constraint structure can be easily analysed. It is found that there are four first-class constraints in the RST model, and two of these generate the well-known Virasoro algebra without classical central charge. At the quantum level, the cancellation condition for the total central charge is reexamined. Three types of measures are discussed and the corresponding

results are obtained. By the Hamiltonian constraint analysis, it is shown that except for the N scalar matter degrees of freedom, the true physical degrees of freedom for gravity, the dilaton and the new field χ, are nonzero. From the viewpoint of the local field theories, there is a hidden dynamical field in the RST model, which was omitted in the usual semiclassical approach. Exploiting the equations of motion, the stress tensors $T_{\pm\pm}$ can be derived from our original constraints $\mathcal{H}_\pm$. In comparison with the known results, it is found that just the stress tensors of the hidden dynamical field ψ_1 give the precise expression for $t_\pm$. Thus we conclude that the previous semiclassical approach is intrinsically inconsistent due to the omission of this hidden dynamical field ψ_1, which results in the above mentioned conflicts. Finally, with the help of the hidden dynamical field ψ_1, the fact that the semi-classical static solution for the RST model has two independent parameters (P,M), whereas for the classical CGHS model it has only one, can be well elucidated.

We now consider the RST model with the action [2]

$$S = \frac{1}{2\pi} \int_\mathcal{M} d^2x \sqrt{-g} \left[e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2 - \frac{\kappa}{4}(R\frac{1}{\nabla^2}R + 2\phi R) \right]$$

(1)

where $g_{\mu\nu}$ is the metric on the 2D manifold $\mathcal{M}$, R is its curvature scalar, ϕ is the dilaton field, and the f^i, i = 1,...,N, are N scalar matter fields. The nonlocal term $R\frac{1}{\nabla^2}R$ comes from the familiar conformal anomaly. The local and covariant term $2\phi R$ is to preserve the simple form of the current $j^\mu = \partial^\mu(\phi - \rho)$, with $\partial_\mu j^\mu = 0$. The coefficient κ has to be positive, since in the case of κ being negative, there is no singularity in gravitational collapse [2]. Obviously, Eq. (1) is invariant with respect to the diffeomorphism transformation.

According to Ref. [15], one can introduce an independent scalar field χ to localize the conformal anomaly term, and add a boundary term to define the variational problem properly. Then Eq. (1) becomes [15]

$$
\begin{aligned}
S &= \frac{1}{2\pi} \int d^2x \sqrt{-g} \{ R\tilde{\chi} + 4[(\nabla\phi)^2 + \lambda^2]e^{-2\phi} - \frac{\kappa}{4}g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi \\
&\quad - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2 \} - \frac{1}{\pi} \int d\Sigma\sqrt{-h}K\tilde{\chi}
\end{aligned}
$$

(2)

where $\tilde{\chi} = e^{-2\phi} - \frac{\kappa}{2}(\phi - \chi)$, h is the induced metric on the boundary of $\mathcal{M}$ (assumed spacelike), and K is the mean extrinsic curvature of $\partial\mathcal{M}$. As in (3+1)-dimensional gravity, the boundary term serves to eliminate second time derivatives of the metric from the action which are contained in R.

Following the ADM formulation, the metric can be parametrized as follows [12, 13, 14]:

$$g_{\mu\nu} = e^{2\rho}\hat{g}_{\mu\nu} \tag{3}$$

$$\hat{g}_{\mu\nu} = \begin{pmatrix} -\sigma^2 + \theta^2 & \theta \\ \theta & 1 \end{pmatrix} \tag{4}$$

where $\sigma(x)$ and $\theta(x)$ are lapse and shift functions respectively, and we factor out the conformal factor $e^{2\rho}$.

In terms of this parametrization, the action (2) can be written as

$$
\begin{aligned}
S = \frac{1}{2}\int d^2x\sqrt{\hat{g}}\Big\{ & \hat{R}\tilde{\chi} + 2\hat{g}^{\alpha\beta}\partial_\alpha\tilde{\chi}\partial_\beta\rho - 2\hat{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta e^{-2\phi} \\
& + 4\lambda^2 e^{2(\rho-\phi)} - \frac{\kappa}{4}\hat{g}^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi - \frac{1}{2}\sum_{i=1}^N \hat{g}^{\alpha\beta}\partial_\alpha f_i\partial_\beta f_i \Big\} \\
& - \int d\Sigma\sqrt{\pm h}\,K\tilde{\chi}
\end{aligned}
\tag{5}
$$

where $\hat{R}$ is the curvature scalar for $\hat{g}_{\mu\nu}$, and for simplicity, the factor π^{-1} in front of action (2) has been omitted.

If we introduce momenta π_ρ, π_ϕ, π_χ respectively for the fields ρ, ϕ, χ, the Hamiltonian would become so complicated that we cannot quantize the theory. Thus we need a field redefinition to diagonalize the kinetic term of action (5), which is first given by

$$
\begin{aligned}
\psi_0 &= \frac{1}{\sqrt{\kappa}}e^{-2\phi} - \frac{\sqrt{\kappa}}{2}\phi + \sqrt{\kappa}\rho \\
\psi_1 &= -\frac{\sqrt{\kappa}}{2}\chi + \sqrt{\kappa}\rho \\
\psi_2 &= \frac{1}{\sqrt{\kappa}}e^{-2\phi} + \frac{\sqrt{\kappa}}{2}\phi
\end{aligned}
\tag{6}
$$

Here we should point out that the physical value of ψ_2 is restricted, i.e., it is a non-negative quantity. If this restriction is ignored, the semi-classical solution of the model is unstable [16]. The black holes radiate forever at a fixed rate and the Bondi mass tends to negative infinity. This feature will not be changed by the addition of the auxilliary field to the model [17]. Then we have

$$
\begin{aligned}
S = \int d^2x\Big\{ & \frac{\sqrt{\kappa}}{\sigma}(\dot{\psi}_0 - \dot{\psi}_1)\theta' + \frac{\sqrt{\kappa}}{\sigma}(\psi_0' - \psi_1')(\sigma\sigma' - \theta\theta') \\
& + \frac{1}{2}\sigma\hat{g}^{\alpha\beta}\partial_\alpha\psi_\mu\partial_\beta\psi_\nu\eta^{\mu\nu} + 2\lambda^2\sigma e^{\frac{2}{\sqrt{\kappa}}(\psi_0-\psi_2)} - \frac{1}{4}\sigma\sum_{i=1}^N \hat{g}^{\alpha\beta}\partial_\alpha f_i\partial_\beta f_i \Big\}
\end{aligned}
\tag{7}
$$

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where $\mu, \nu = 0, 1, 2$ with $\eta^{\mu\nu} = (1, -1, -1)$, $\hat{g}^{00} = -\sigma^2$, $\hat{g}^{01} = \hat{g}^{10} = \theta\sigma^{-2}$, $\hat{g}^{11} = (\sigma^2 - \theta^2)\sigma^{-2}$. In the above, dots and primes denote differentiation with respect to time and space respectively. The canonical momenta associated with the fields $\{\sigma, \theta, \psi_\mu, f_i\}$ are

$$P_\sigma = 0 \tag{8}$$

$$P_\theta = 0 \tag{9}$$

$$P_0 = -\frac{\dot{\psi}_0}{\sigma} + \frac{\theta\psi_0'}{\sigma} + \frac{\sqrt{\kappa}\theta'}{\sigma} \tag{10}$$

$$P_1 = \frac{\dot{\psi}_1}{\sigma} - \frac{\theta\psi_1'}{\sigma} - \frac{\sqrt{\kappa}\theta'}{\sigma} \tag{11}$$

$$P_2 = \frac{\dot{\psi}_2}{\sigma} - \frac{\theta\psi_2'}{\sigma} \tag{12}$$

$$\pi_i = \frac{\dot{f}_i}{2\sigma} - \frac{\theta f_i'}{2\sigma} \tag{13}$$

with

$$\{\sigma(x), P_\sigma(y)\} = \{\theta(x), P_\theta(y)\} = \delta(x - y)$$
$$\{\psi_\mu(x), P_\nu(y)\} = \delta_{\mu\nu}\delta(x - y)$$
$$\{f_i(x), \pi_j(y)\} = \delta_{ij}\delta(x - y) \tag{14}$$

Clearly (8) and (9) are primary constraints and $\sigma(x)$ and $\theta(x)$ play the role of Lagrange multipliers. The canonical Hamiltonian, up to surface terms, is

$$\mathcal{H}_c = \int dx(\sigma\mathcal{H}_\sigma + \theta\mathcal{H}_\theta) \tag{15}$$

where

$$\mathcal{H}_\sigma = -\frac{1}{2}(P_0^2 + \psi_0'^2) + \frac{1}{2}(P_1^2 + \psi_1'^2) + \frac{1}{2}(P_2^2 + \psi_2'^2)$$
$$+ \sqrt{\kappa}(\psi_0'' - \psi_1'') - 2\lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} + \sum_{i=1}^{N}(\pi_i^2 + \frac{1}{4}f_i'^2) = 0 \tag{16}$$

$$\mathcal{H}_\theta = P_0\psi_0' + P_1\psi_1' + P_2\psi_2' - \sqrt{\kappa}(P_0' + P_1') + \sum_{i=1}^{N}\pi_i f_i' = 0 \tag{17}$$

are secondary constraints. $\mathcal{H}_\theta$ is the generator of spatial diffeomorphisms, but $\mathcal{H}_\sigma$ does not exactly correspond to the generator of temporal diffeomorphisms [18]. Since the constraint $\mathcal{H}_\sigma$ is nonlinear in the momenta, it does not generate a transformation which corresponds to a symmetry of the corresponding

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Lagrangian system. Rather it is responsible for the dynamics of the system. On the other hand, the transformation generated by $\mathcal{H}_\sigma$ is indeed a symmetry of the Hamiltonian system (which cannot be identified with a Lagrangian symmetry in gravity theory) [18]. However, all Lagrangian symmetries can be recovered in the Hamiltonian formalism only if we consider the transformation generated by $\mathcal{H}_\sigma$ in a very special combination with a particular "trivial" transformation [18].

We now calculate the Poisson brackets of the constraints $\mathcal{H}_\sigma$, $\mathcal{H}_\theta$, and after a series of steps, we have

$$\{\mathcal{H}_\sigma(x), \mathcal{H}_\sigma(y)\} = [\mathcal{H}_\theta(x) + \mathcal{H}_\theta(y)]\partial_1^x \delta(x^1 - y^1) \tag{18}$$

$$\{\mathcal{H}_\theta(x), \mathcal{H}_\theta(y)\} = [\mathcal{H}_\theta(x) + \mathcal{H}_\theta(y)]\partial_1^x \delta(x^1 - y^1) \tag{19}$$

$$\{\mathcal{H}_\sigma(x), \mathcal{H}_\theta(y)\} = [\mathcal{H}_\sigma(x) + \mathcal{H}_\sigma(y)]\partial_1^x \delta(x^1 - y^1) \tag{20}$$

Eqs. (18-20) show that $\mathcal{H}_\sigma$, $\mathcal{H}_\theta$ form a closed algebra under Poisson brackets, that is, they are first-class constraints at the classical level. Here we emphasize that thanks to the existence of the scalar field χ, the closed algebra is recoverd, which guarantees the general invariance of the RST action.

It is obvious that the total number of degrees of freedom is $5+N (i.e., \sigma, \theta, \psi_\mu, f_i)$, while there are four first-class constraints, so the true number of physical degrees of freedom is 1+N, i.e., $(5 + N) - (2 + 2) = 1 + N$. From the view point of local field theories, we find that except for the N scalar matter fields, there is another dynamical field ψ_1, and we call it a hidden dynamical field, which was omitted in the previous semiclassical approach. In the present case, due to the presence of this hidden dynamical variable, the constraints $\mathcal{H}_\sigma$, $\mathcal{H}_\theta$ are recovered to be first-class.

According to Dirac's algorithm, the conditions of a physical state Ψ can be expressed as

$$\begin{aligned}
\mathcal{H}_\sigma \Psi &= \Big\{ -\frac{1}{2}(P_0^2 + \psi_0'^2) + \frac{1}{2}(P_1^2 + \psi_1'^2) + \frac{1}{2}(P_2^2 + \psi_2'^2) + \sqrt{\kappa}(\psi_0'' - \psi_1'') \\
&\quad -2\lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} + \sum_{i=1}^{N}(\pi_i^2 + \frac{1}{4}f_i'^2) \Big\} \Psi = 0
\end{aligned} \tag{21}$$

$$\mathcal{H}_\theta \Psi = \Big\{ P_0\psi_0' + P_1\psi_1' + P_2\psi_2' - \sqrt{\kappa}(P_0' + P_1') + \sum_{i=1}^{N}\pi_i f_i' \Big\} \Psi = 0 \tag{22}$$

Eqs. (21,22) are just modified versions of the Wheeler-DeWitt equation [19,20]. Here we note that the constraints $P_\sigma = 0$ and $P_\theta = 0$ require the wave functional Ψ to be independent of the Lagrange multipliers $\sigma(x)$ and $\theta(x)$. So the physical state will have the form

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$$\Psi = \Psi(\psi_\mu, f_i) \tag{23}$$

in the functional Schrödinger representation.

Owing to the algebra (18-20) being isomorphic to two commuting copies of the 1D diffeomorphism algebra, we can construct the constraints in terms of the light cone ones:

$$
\begin{aligned}
\mathcal{H}_\pm &= \frac{1}{2}(\mathcal{H}_\sigma \pm \mathcal{H}_\theta) = -\frac{1}{4}(P_0 \mp \psi_0')^2 + \frac{1}{4}(P_1 \pm \psi_1')^2 + \frac{1}{4}(P_2 \pm \psi_2')^2 \\
&\quad + \frac{\sqrt{\kappa}}{2}(\mp P_0' + \psi_0'') - \frac{\sqrt{\kappa}}{2}(\pm P_1' + \psi_1'') - \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}(\pi_i \pm \frac{1}{2}f_i')^2 = 0
\end{aligned}
\tag{24}
$$

¿From Eqs. (18-20,24), we immediately recognize $\mathcal{H}_\pm$ obeying the Virasoro algebra [21].

In the conformal gauge (which means $\sigma = 1, \theta = 0$), $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\rho}$, the action (7) can be written as

$$
\begin{aligned}
S &= \int d^2 x \Big[-\partial_+\psi_0\partial_-\psi_0 + \partial_+\psi_1\partial_-\psi_1 + \partial_+\psi_2\partial_-\psi_2 + \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} \partial_+f_i\partial_-f_i \Big]
\end{aligned}
\tag{25}
$$

and the constraints (24) become

$$
\begin{aligned}
\mathcal{H}_\pm &= -\partial_\pm\psi_0\partial_\pm\psi_0 + \sqrt{\kappa}\partial_\pm^2\psi_0 + \partial_\pm\psi_2\partial_\pm\psi_2 \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} \partial_\pm f_i\partial_\pm f_i + \partial_\pm\psi_1\partial_\pm\psi_1 - \sqrt{\kappa}\partial_\pm^2\psi_1 \\
&\quad - \sqrt{\kappa}\partial_+\partial_-\psi_0 + \sqrt{\kappa}\partial_+\partial_-\psi_1 - \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} = 0
\end{aligned}
\tag{26}
$$

The equations of motion derived from action (25) are

$$\partial_+\partial_-\psi_0 = -\frac{\lambda^2}{\sqrt{\kappa}}e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} \tag{27}$$

$$\partial_+\partial_-\psi_1 = 0 \tag{28}$$

$$\partial_+\partial_-\psi_2 = -\frac{\lambda^2}{\sqrt{\kappa}}e^{\frac{2}{\sqrt{\kappa}}(\psi_0 - \psi_2)} \tag{29}$$

$$\partial_+\partial_-f_i = 0 \tag{30}$$

With Eqs. (27) and (28), the constraints $\mathcal{H}_\pm$ can be reduced to

$$\begin{aligned}
\mathcal{H}_\pm &= -\partial_\pm \psi_0 \partial_\pm \psi_0 + \sqrt{\kappa} \partial_\pm^2 \psi_0 + \partial_\pm \psi_2 \partial_\pm \psi_2 \\
&\quad + \frac{1}{2} \sum_{i=1}^N \partial_\pm f_i \partial_\pm f_i + \partial_\pm \psi_1 \partial_\pm \psi_1 - \sqrt{\kappa} \partial_\pm^2 \psi_1 = 0
\end{aligned}$$
(31)

In comparison with previous results [2, 3, 9, 11], $\mathcal{H}_\pm$ are nothing but the stress tensors $T_{\pm\pm}$ with added contributions

$$t_\pm = \partial_\pm \psi_1 \partial_\pm \psi_1 - \sqrt{\kappa} \partial_\pm^2 \psi_1$$
(32)

Eqs. (31,32) show that $T_{\pm\pm}$ are true tensor, and under a conformal change of coordinates $t_\pm$ indeed pick up $-\kappa/2$ times the schwarzian derivative. In our derivation, $t_\pm$ appear in a natural way, as a matter of fact, $t_\pm$ are just the stress tensors for the hidden dynamical field ψ_1. If the hidden dynamical field ψ_1 is omitted, the above mentioned conflicts will arise, that is, the original stress tensors $T_{\pm\pm}$ will turn out to be nontensor, and the algebra (18-20) will not be closed.

¿From the above discussion, we find that due to the presence of the hidden dynamical field ψ_1, the constraints form the closed algebra without classical central extension. At the quantum level, we now apply the Bilal– Callan method [9] to analyse the quantum central charge. From the expression for the stress tensors $T_{\pm\pm}$

$$\begin{aligned}
T_{\pm\pm} &= -\partial_\pm \psi_0 \partial_\pm \psi_0 + \sqrt{\kappa} \partial_\pm^2 \psi_0 + \partial_\pm \psi_2 \partial_\pm \psi_2 \\
&\quad + \frac{1}{2} \sum_{i=1} \partial_\pm f_i \partial_\pm f_i + \partial_\pm \psi_1 \partial_\pm \psi_1 - \sqrt{\kappa} \partial_\pm^2 \psi_1 = 0
\end{aligned}$$
(33)

one can easily obtain the cancellation condition for the total quantum central charge:

$$\begin{aligned}
C &= C_{\psi_0} + C_{\psi_1} + C_{\psi_2} + C_M + C_{ghost} \\
&= (1 - 12\kappa) + (1 + 12\kappa) + 1 + N - 26 = 0
\end{aligned}$$
(34)

with

$$N = 23$$
(35)

At first sight, this result seems somewhat surprising, Eq. (34) cannot determine the value of κ, but gives the restriction on N. This is because the stress tensors in the present case have no classical central charge, which is similar to the classical

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CGHS model where the stress tensors are true tensor, so the condition without conformal anomaly in the CGHS model is N = 24 [16]. Our result can also be understood from the measure definition. Suppose we start with action (7) and take ψ_μ, f_i as our fundamental fields. The condition (34) then means the functional measures are defined by the following norms:

$$\parallel \delta\psi_\mu \parallel_{\hat{g}}^2 \; = \; \int d^2x\sqrt{-\hat{g}}\delta^{\mu\nu}\delta\psi_\mu\delta\psi_\nu \tag{36}$$

$$\parallel \delta f_i \parallel_{\hat{g}}^2 \; = \; \int d^2x\sqrt{-\hat{g}}\delta^{ij}\delta f_i\delta f_j \tag{37}$$

If we take the functional measures for the fields $g_{\mu\nu}, \phi, f_i$ to be those defined by the norms

$$\parallel \delta g \parallel_g^2 = \int d^2x\sqrt{-g}g^{\alpha\gamma}g^{\beta\delta}(\delta g_{\alpha\beta}\delta g_{\gamma\delta} + \delta g_{\alpha\gamma}\delta g_{\beta\delta}) \tag{38}$$

$$\parallel \delta\phi \parallel_g^2 = \int d^2x\sqrt{-g}(\delta\phi)^2 \tag{39}$$

$$\parallel \delta f_i \parallel_g^2 = \int d^2x\sqrt{-g}\delta^{ij}\delta f_i\delta f_j \tag{40}$$

and consider the classical CGHS action as our starting point, one might argue à la David, Distler and Kawai (DDK) [22] about the measure in the path integral; then the condition for a vanishing central charge is

$$\kappa = \frac{N - 24}{12} \tag{41}$$

which is just the approach adopted in Refs. [11, 23]. However, if we replace (39) by [24]

$$\parallel \delta\eta \parallel_g^2 = \int d^2x\sqrt{-g}(\delta\eta)^2 \tag{42}$$

with

$$\eta = e^{-\phi} \tag{43}$$

i.e., consider $e^{-\phi}$ as original field, then the corresponding condition becomes [10, 24]

$$\kappa = \frac{N - \frac{51}{2}}{12} \tag{44}$$

Generally speaking, different types of measures used will result in different conditions for the total quantum central charge to vanish [24].

The general $f_i = 0$ solution for the classical CGHS model is [1]

$$e^{-2\phi} = e^{-2\rho} = \frac{M}{\lambda} - \lambda^2 x^+ x^- \tag{45}$$

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where only one global parameter M exists. The semi-classical static solution of the RST model is [2]

$$\frac{\sqrt{\kappa}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{\kappa}} = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa}} + P\sqrt{\kappa}\ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda\sqrt{\kappa}} \qquad (46)$$

where P and M parametrize different solutions, i.e., there are two global and independent parameters P,M. However, as we know, the equations of motion for both models are differential equations of the same order. One may wonder why both models do not have the same number of global parameters. The reason is that the classical CGHS model has no local degrees of freedom [15] when f_i are zero, whereas the RST model has a hidden dynamical field ψ_1 which is responsible for the parameter P.

¿From Eqs. (27-30,31), we have [2, 11]

$$\psi_0 = \psi_2 = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa}} + \left[\int dx^+ \int dx^+ \left(\partial_+^2 \psi_1 - \frac{1}{\sqrt{\kappa}}(\partial_+ \psi_1)^2\right)\right.$$
$$\left. + \int dx^- \int dx^- \left(\partial_-^2 \psi_1 - \frac{1}{\sqrt{\kappa}}(\partial_- \psi_1)^2\right)\right] + \frac{m}{\lambda\sqrt{\kappa}} \qquad (47)$$

Eq. (28) shows that ψ_1 satisfies a free massless scalar field equation with solution $\psi_1 = \psi_1^+(x^+) + \psi_1^-(x^-)$, so we have the freedom to choose

$$\psi_-^+(x^+) = c\ln(\lambda x^+), \psi_1^-(x^-) = c\ln(-\lambda x^-) \qquad (48)$$

where c is an arbitrary constant. Then Eq. (47) reduces to

$$\psi_0 = \psi_2 = -\frac{\lambda^2 x^+ x^-}{\sqrt{\kappa}} + P\sqrt{\kappa}\ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda\sqrt{\kappa}} \qquad (49)$$

with

$$P = \left(c + \frac{c^2}{\sqrt{\kappa}}\right)/\sqrt{\kappa} \qquad (50)$$

$$M = m - P\sqrt{\kappa}\ln\lambda^2 \qquad (51)$$

Eqs. (49,50) show that the hidden dynamical field ψ_1 induces the parameter P. This result is consistent with the fact that in the semi-classical CGHS model including the conformal anomaly, the static solution (which can be studied numerically) have two parameters, one of which corresponds to the energy density in the asymptotic region.

In summary, we have reconstructed the closed algebra for the constraints with the help a of hidden dynamical field ψ_1, and the resulting stress tensors $T_{\pm\pm}$ are true tensor. If the hidden dynamical field ψ_1 is omitted as in the usual

semiclassical approach, the theory will be inconsistent. For example, under a conformal change of coordinates the stress tensors $T_{\pm\pm}$ will pick up an extra term equal to $-\kappa/2$ times the schwarzian derivative of the transition function at the classical level. Thus the Poisson brackets of the constraints will have the classical extension, i.e., the closed algebra of the constraints (18-20) will be destroyed, in contradiction with the fact that the RST action is manifestly invariant under the diffeormorphism transformation. Thanks to the existence of the hidden dynamical field, the stress tensors $t_{\pm}$ can be endowed with precise meaning, and the contradictions mentioned in the introduction can be resolved in a perfect manner. Now with the diagonalized action (7) and a clear constraint structure at hand, we hope to understand some quantum physics in the strong coupling regime with the path integral approach [25], and meanwhile we can also shed some new light on the physical meaning of the hidden dynamical field [26]. Another aspect of interest is to solve the Wheeler-DeWitt equation (21,22) in the functional Schrödinger representation to obtain the physical wave functional Ψ, from which we can obtain the entropy of the RST model [27] in order to understand the origin of the black hole entropy more deeply. These problems are presently under investigation and we hope to be able to report our progress elsewhere.

Acknowledgement

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